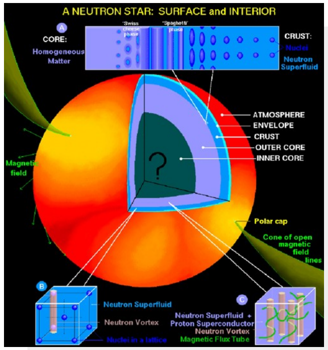
Order-by-order convergence of chiral nuclear forces in neutron matter
(arXiv:2302.07285)

Alex Gezerlis

UNIVERSITY of GUELPH

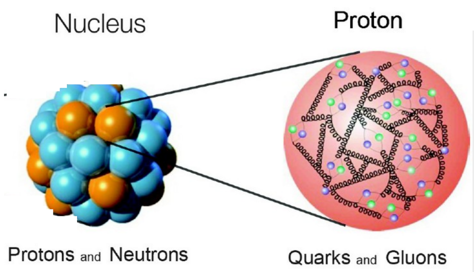
NuSym23, XIth international symposium on nuclear symmetry energy
GSI, Darmstadt, Germany
September 20, 2023

Outline

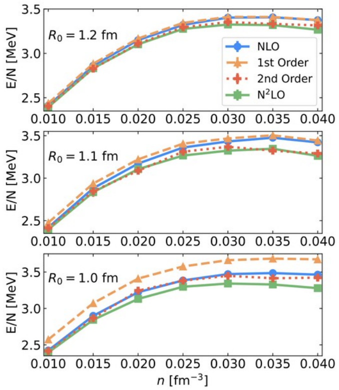


Credit: Dany Page

Motivation

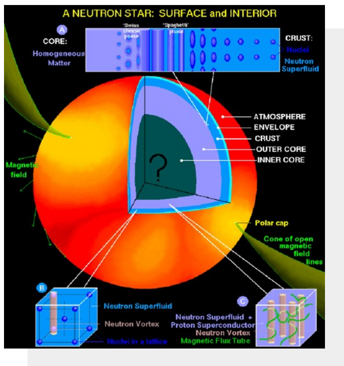


Nuclear methods



Recent results

Outline



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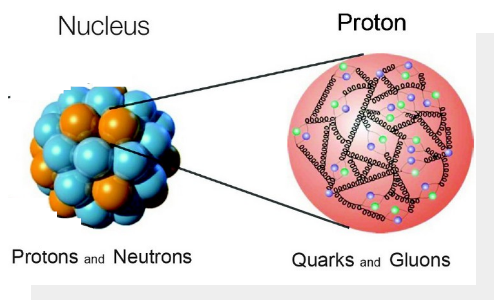

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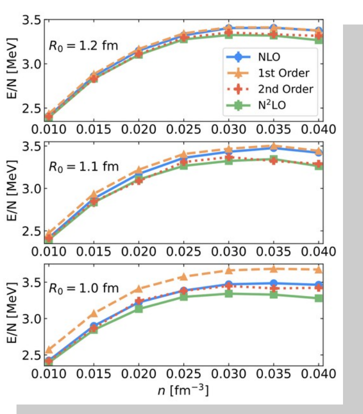
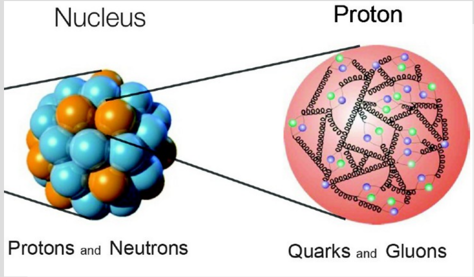


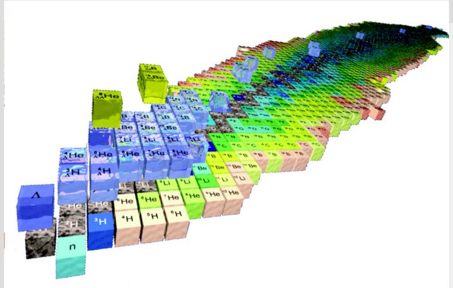
Recent results

Physical systems studied

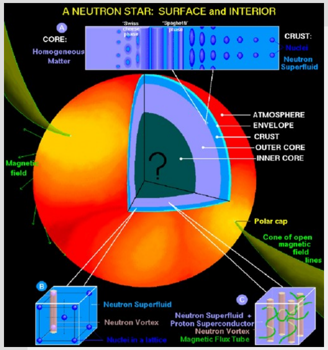
Nuclear forces



Nuclear structure

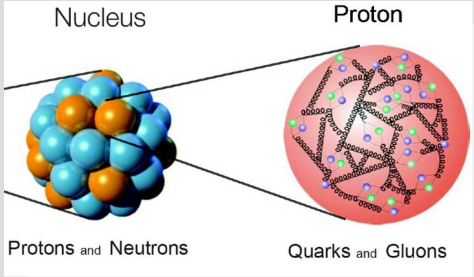


Nuclear astrophysics



Physical systems studied

Nuclear forces



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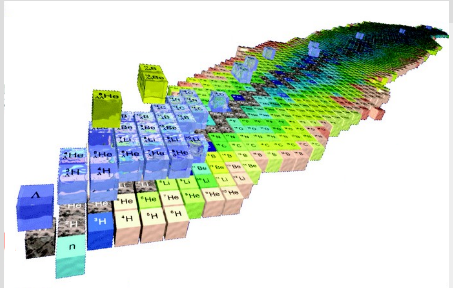

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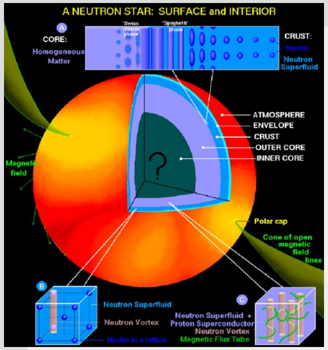

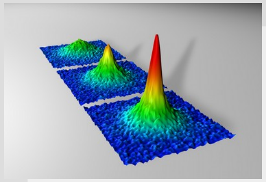

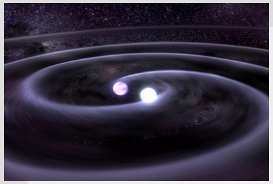

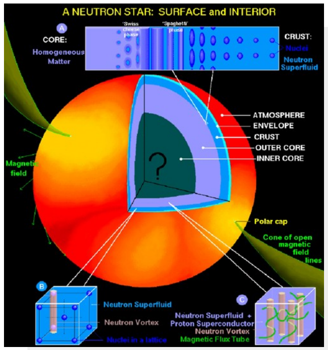

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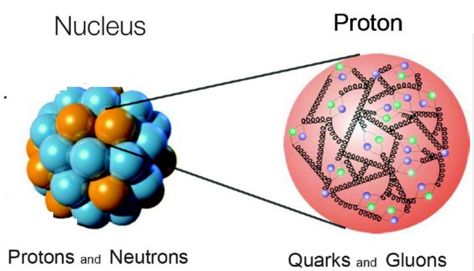
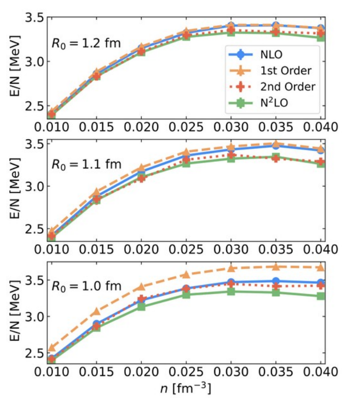


Nuclear methods



Recent results

Nuclear physics is difficult

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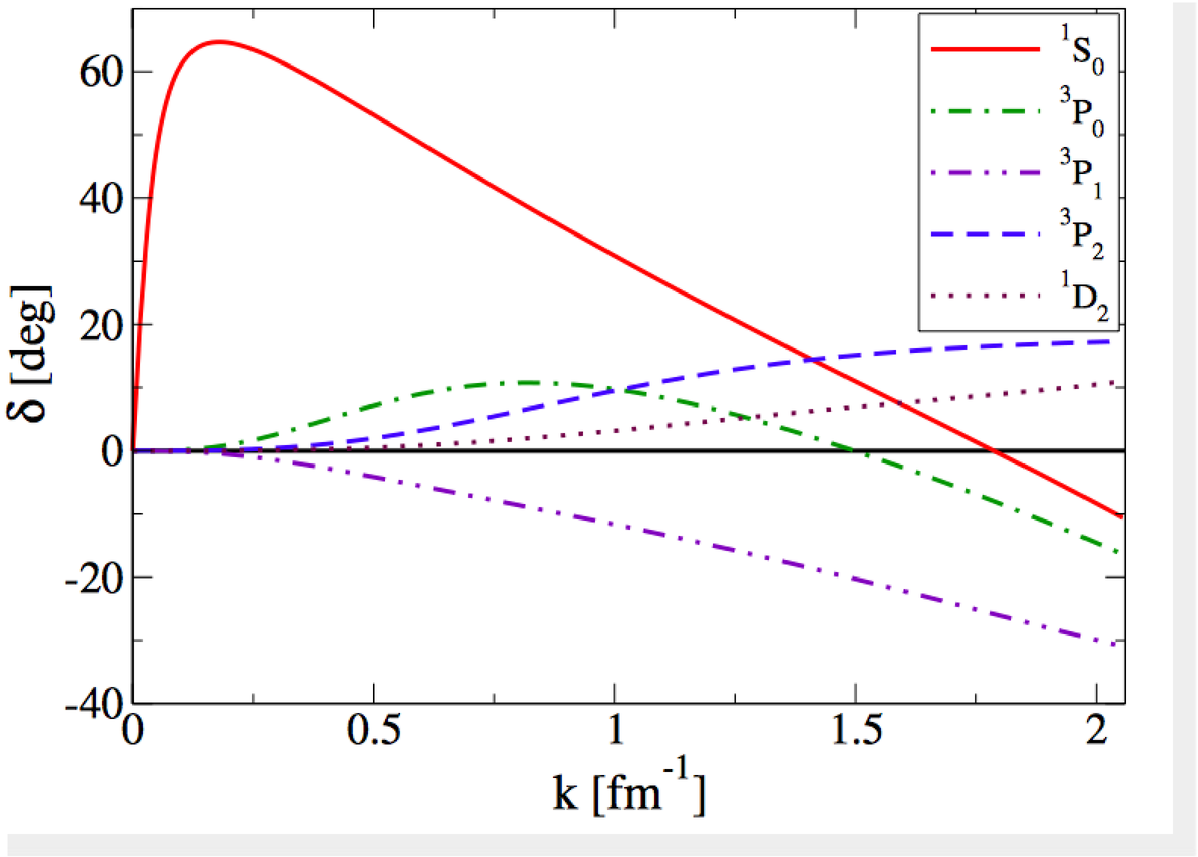
Scattering phase shifts: different “channels” have different behavior.



Any potential that reproduces them must be spin (and isospin) dependent

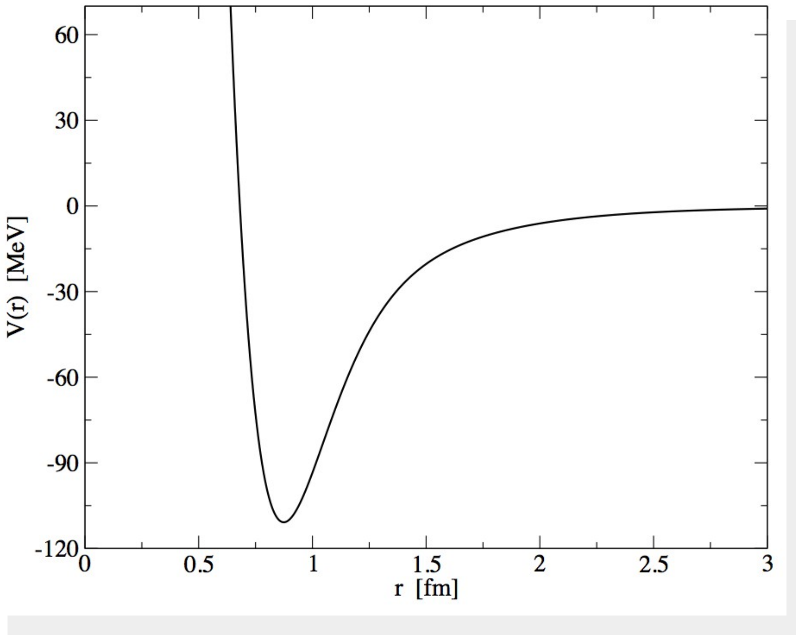
Nuclear interactions

Meson exchanges: phenomenology treats NN scattering without connecting with the underlying level

$$V_2 = \sum_{j<k} v_{jk} = \sum_{j<k} \sum_{p=1}^{8} v_p(r_{jk}) O^{(p)}(j,k)$$

$$O^{p=1,8}(j,k) = (1, \sigma_j \cdot \sigma_k, S_{jk}, \mathbf{L}_{jk} \cdot \mathbf{S}_{jk}) \otimes (1, \tau_j \cdot \tau_k)$$

Such potentials are hard, making them non-perturbative at the many-body level



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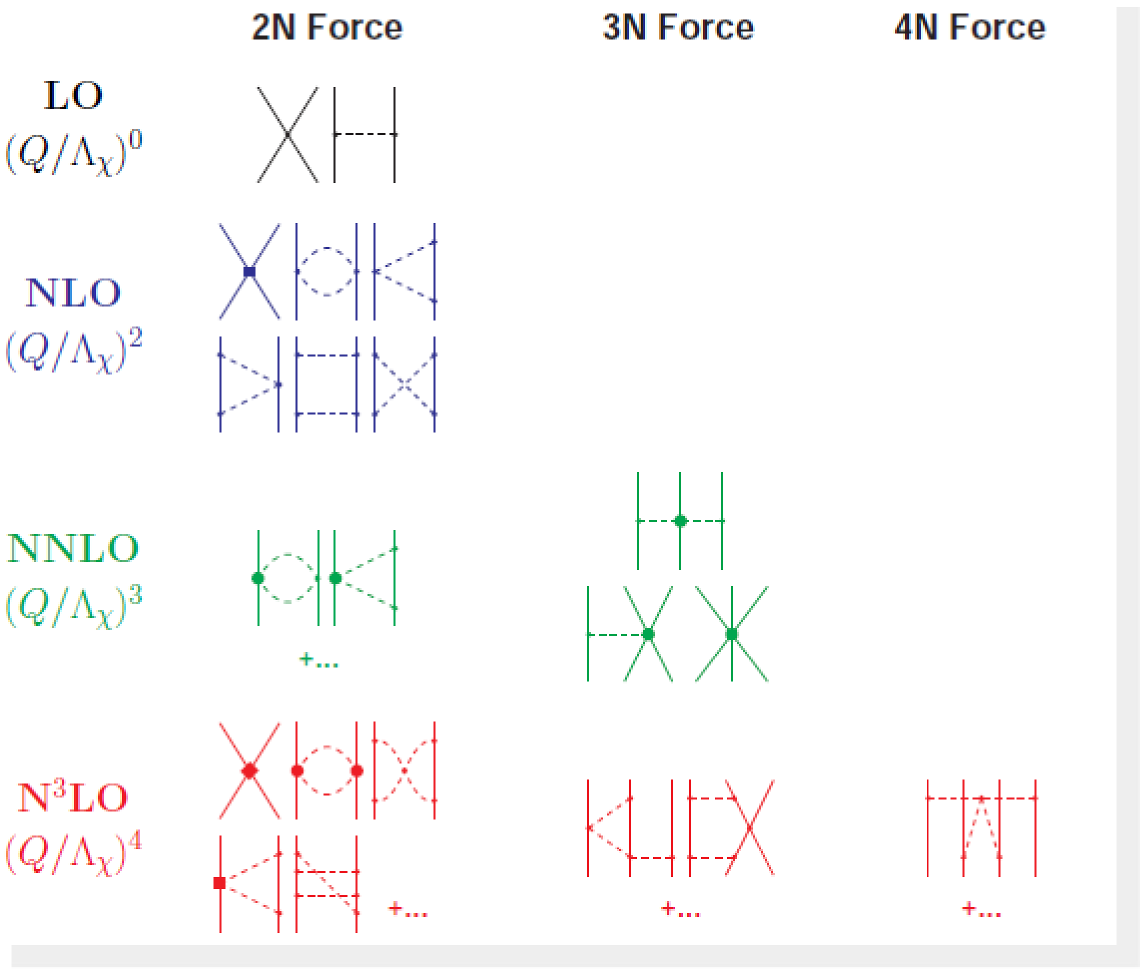
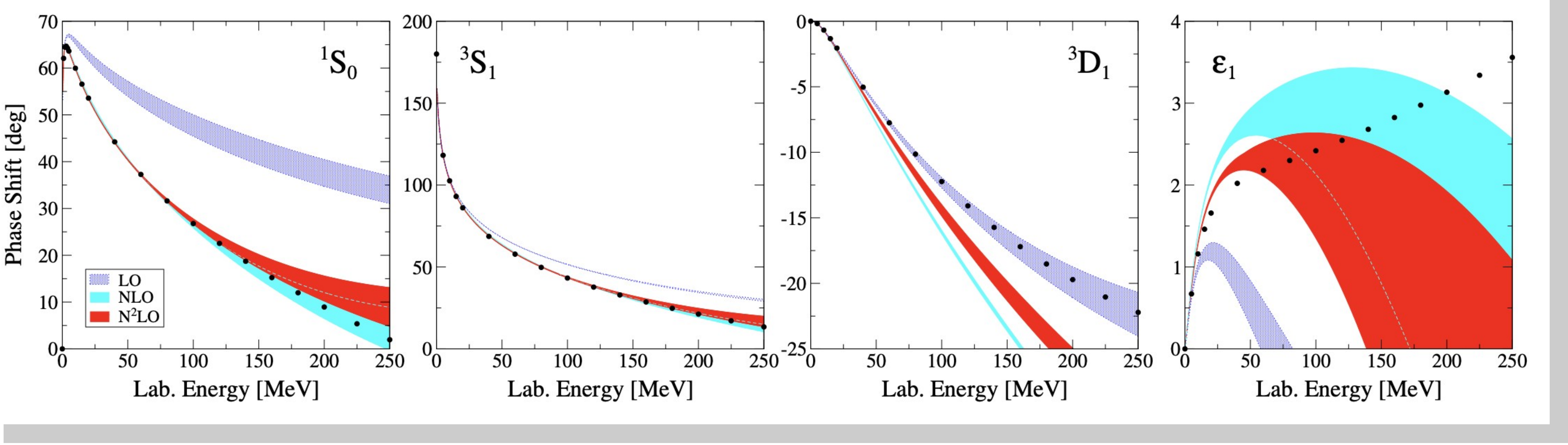


- Attempts to connect with underlying theory (QCD)
- Low-momentum expansion
- Naturally emerging many-body forces
- Low-energy constants from experiment or lattice QCD
- Now available in non-local, local, or semi-local varieties
- Power counting's relation to renormalization actively investigated

S. Weinberg, U. van Kolck, E. Epelbaum, N. Kaiser ...

Local chiral EFT



A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, and A. Schwenk, Phys. Rev. C **90**, 054323 (2014)

Local chiral EFT

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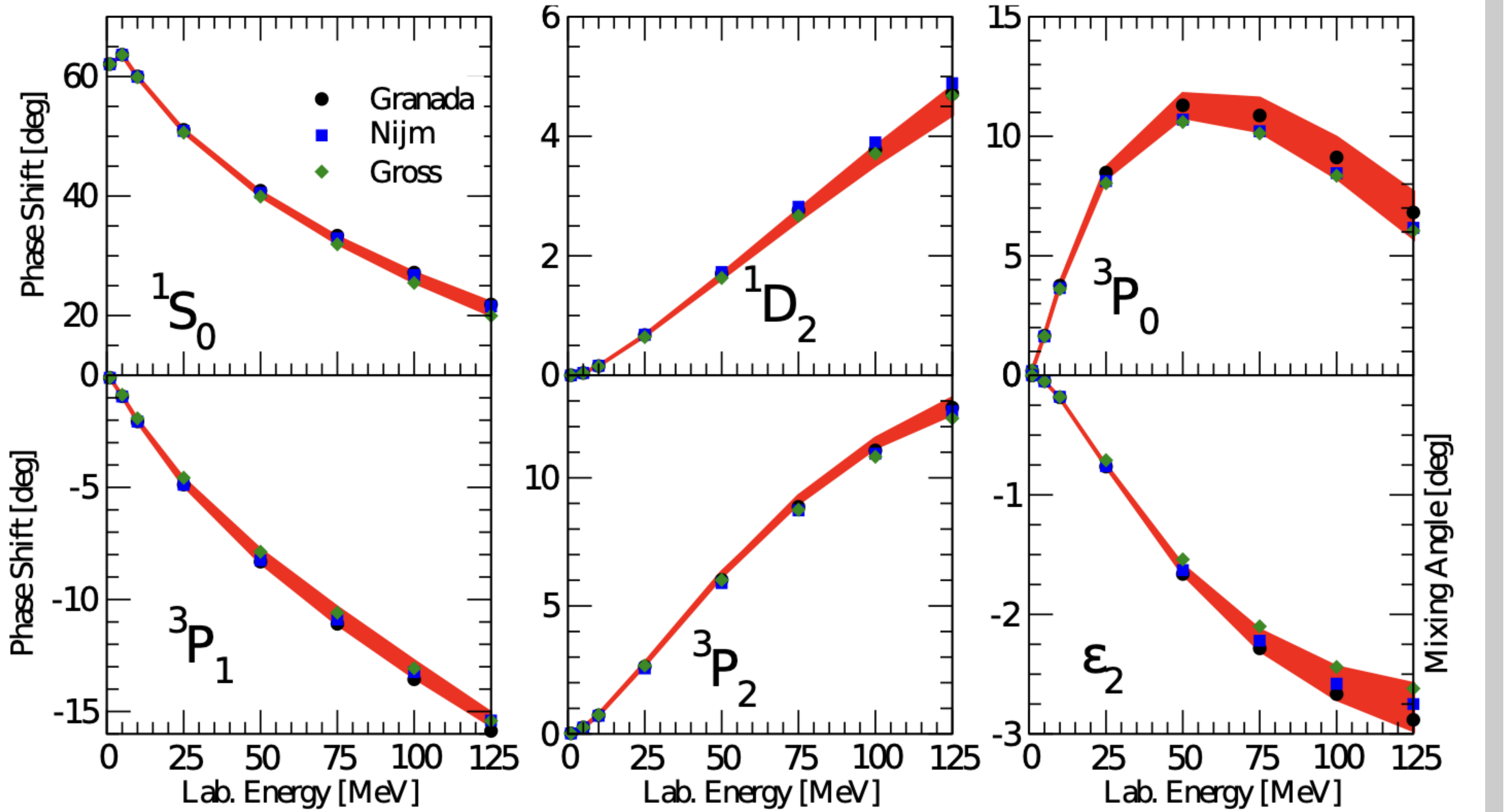

M. Piarulli, L. Girlanda, R. Schiavilla, A. Kievsky, A. Lovato, L. E. Marcucci, S. C. Pieper, M. Viviani, and R. B. Wiringa, Phys. Rev. C **94**, 054007 (2016)

But even with the interaction in place,
how do you solve the many-body problem?

Nuclear many-body problem

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$$H\Psi = E\Psi$$

where $$H = \sum_i K_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

Wave function depends on coordinates, spin projections, and isospin projections, so we are faced with a large number of complex coupled second-order differential equations

Nuclear many-body methods

- Phenomenological
- Ab initio

Two complementary approaches

Phenomenological

- *Shell model*
  mainstay of nuclear physics, still very important
- *Hartree-Fock/Hartree-Fock-Bogoliubov (HF/HFB)*
  mean-field theory, a priori inapplicable, unreasonably effective
- *Energy-density functionals (EDF)*
  like mean-field but with wider applicability

Two complementary approaches

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Ab initio

- *Exact diagonalization techniques (e.g., HH or NCSM)*
  fully ab initio, in contradistinction to traditional SM

- *Quantum Monte Carlo (QMC), continuum or lattice*
  stochastically propagate in imaginary time

- *Perturbative Theories (PT)*
  first few orders only

- *Resummation schemes (e.g. SCGF)*
  selected class of diagrams up to infinite order

- *Coupled cluster (CC) and In-Medium Similarity Renormalization Group (IMSRG)*
  decoupling transformation of Hamiltonian/generate np-nh excitations of a reference state

Outline

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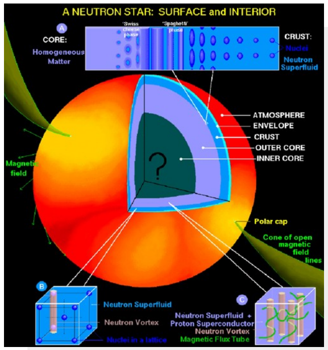

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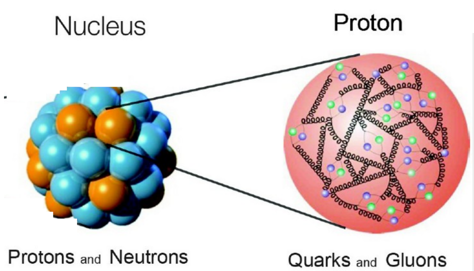

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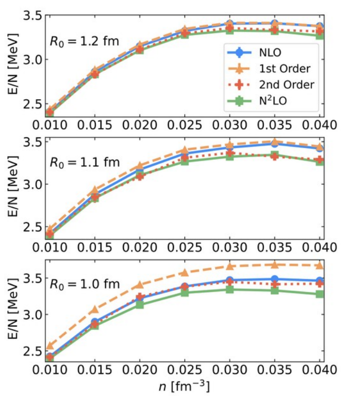
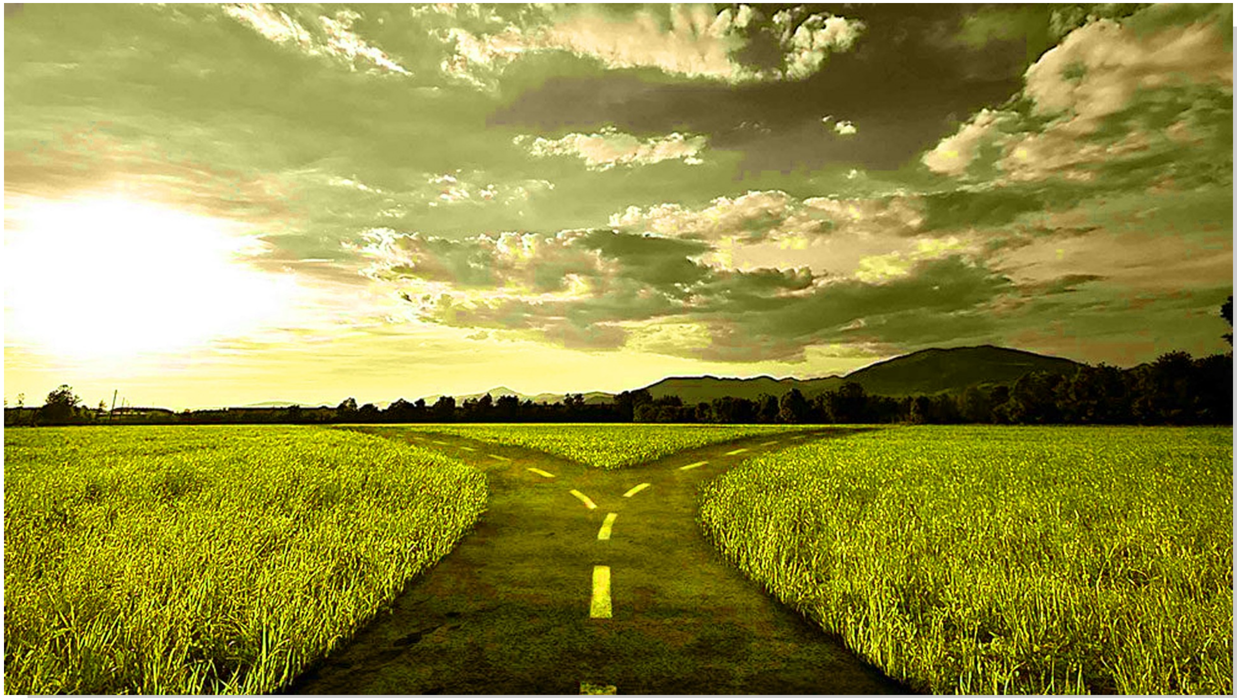


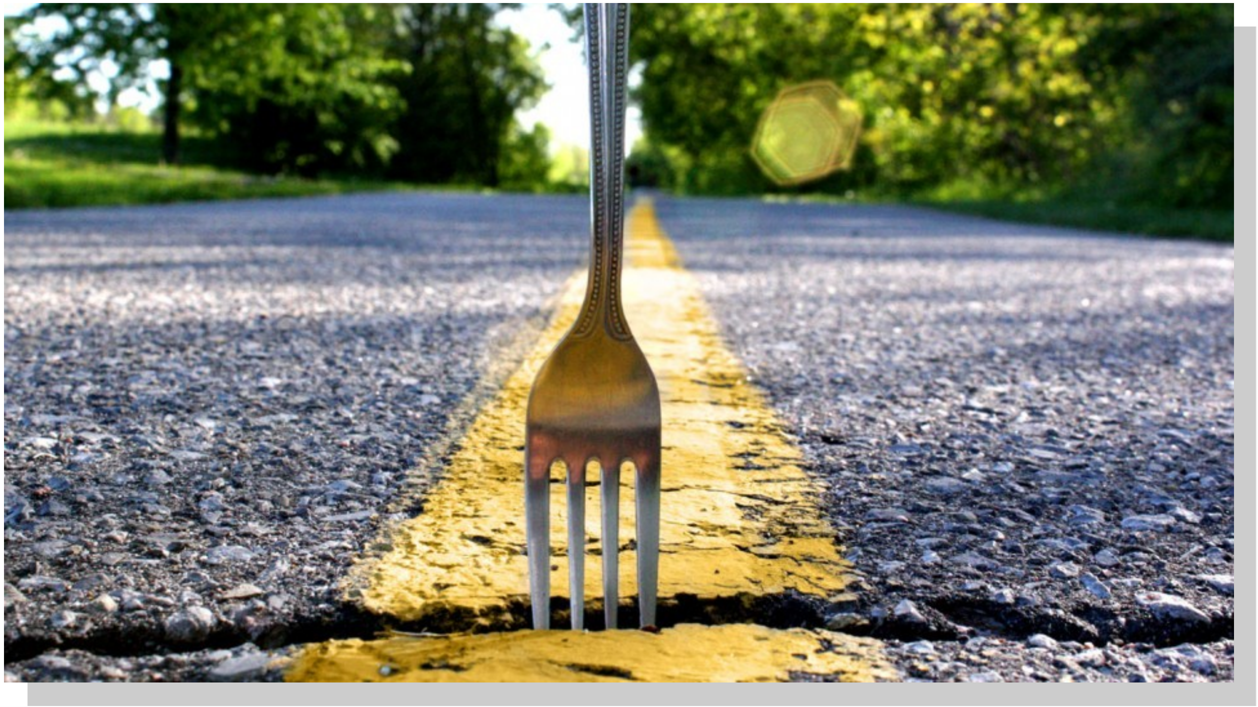
Recent results

Perturbative or non-perturbative?



“When you come to a fork in the road, take it”

Perturbative or non-perturbative?



"When you come to a fork in the road, take it"

Setting the stage

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Split your Hamiltonian:

$$\hat{H} = \hat{H}_0 + V'$$

Nearly everyone can do a first-order perturbation:

$$E_0^{(1)} = \frac{\left\langle \psi_0^{(0)} \middle| V' \middle| \psi_0^{(0)} \right\rangle}{\left\langle \psi_0^{(0)} \middle| \psi_0^{(0)} \right\rangle}$$

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Things not so easy when it comes to the second order:

$$E_0^{(2)} = -\sum_{k \neq 0} \frac{\left| \left\langle \psi_0^{(0)} \middle| V' \middle| \psi_k^{(0)} \right\rangle \right|^2}{E_k^{(0)} - E_0^{(0)}}$$

Deuteron

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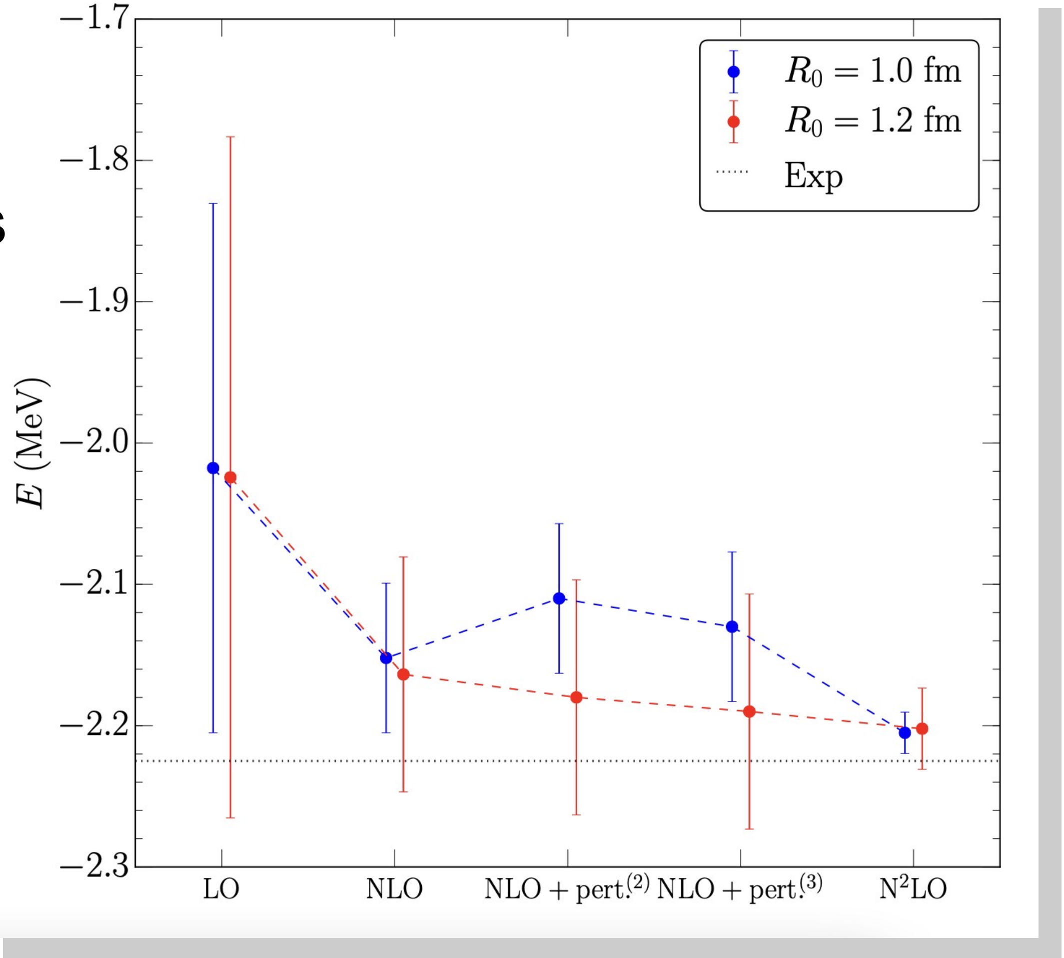
Up to 3rd order

Two zeroth-order Hamiltonians (and two cutoffs).

Speed of convergence appears to depend on softness of interaction.



J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, Phys. Rev. C **96**, 054007 (2017)

Deuteron

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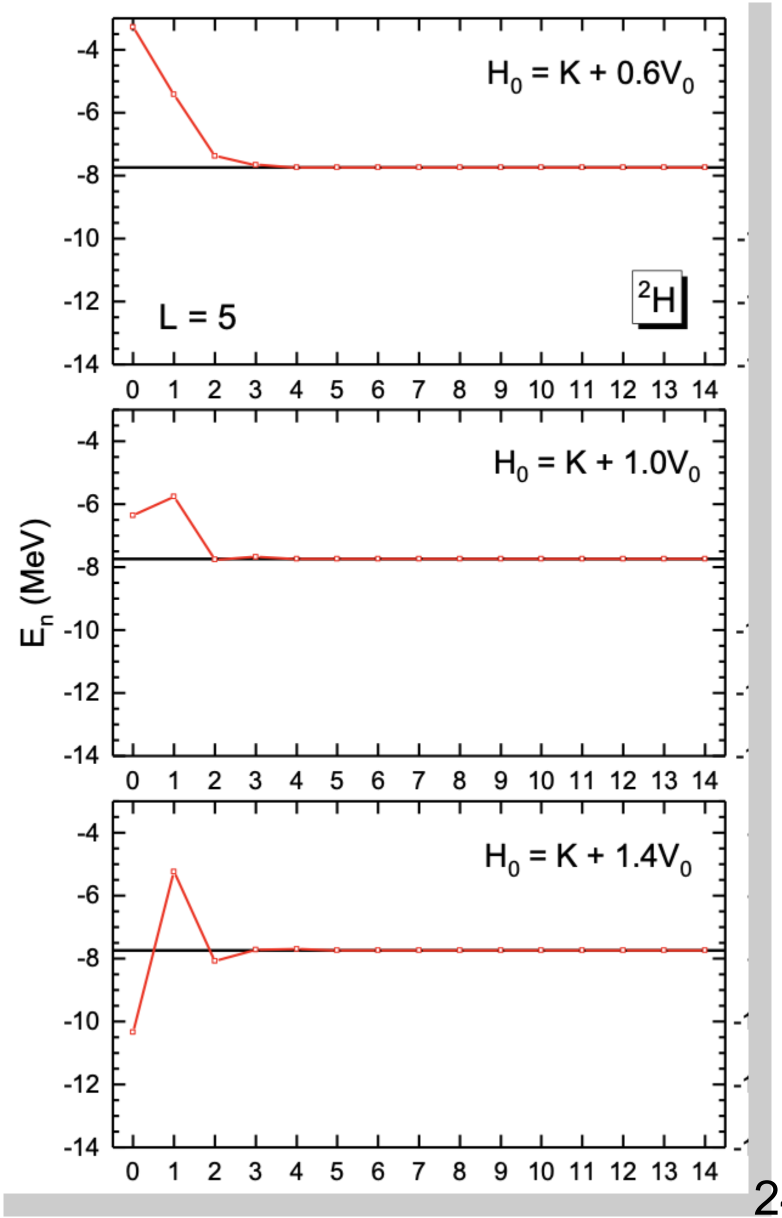
Up to 14th order

Many zeroth-order Hamiltonians (but single momentum cutoff).

Beyond second-order effects small.



B.-N. Lu, N. Li, S. Elhatisari, Y.-Z. Ma, D. Lee, U.-G. Meissner, Phys. Rev. Lett. **128**, 242501 (2022)

Beyond the deuteron

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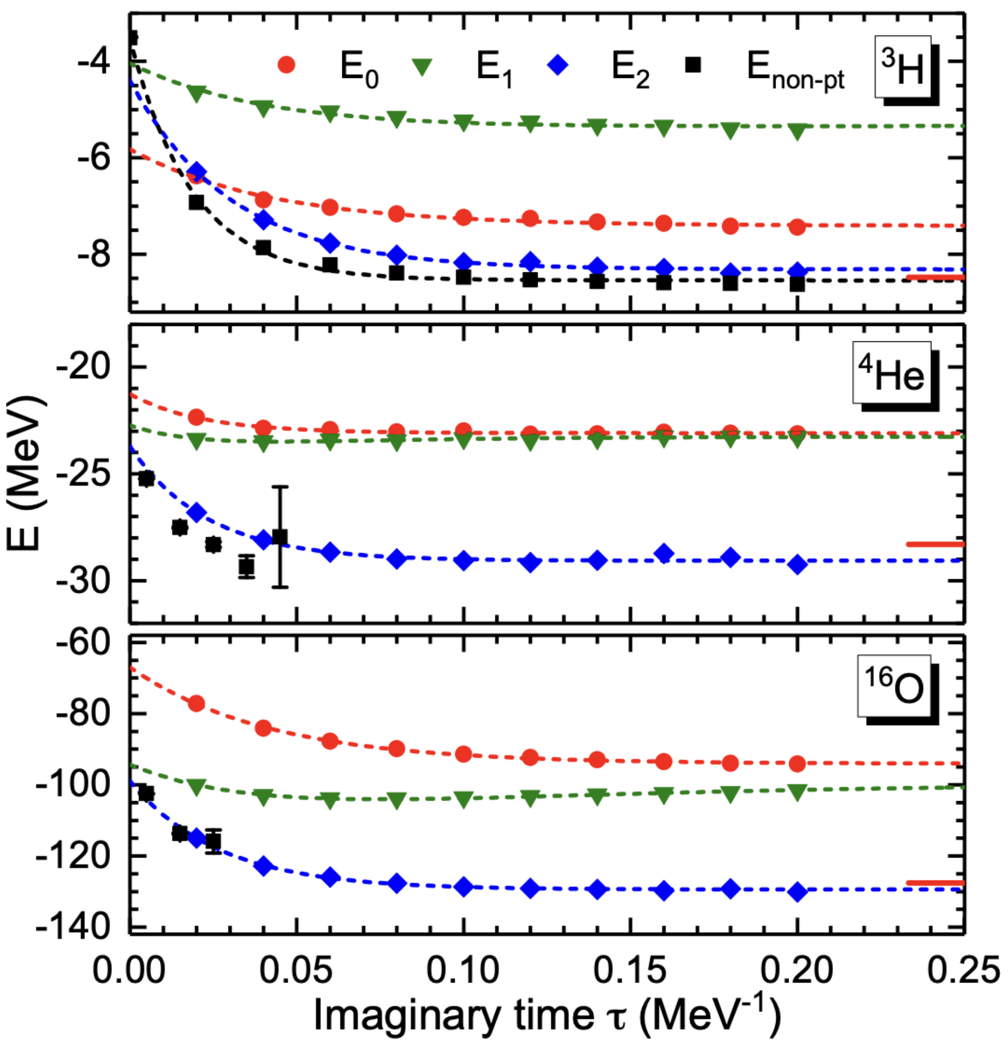
Lattice quantum Monte Carlo: up to 2nd order



- Lattice QMC is QMC (expressed in terms of Euclidean/imaginary time)
- Applied to several nuclei
- Hamiltonian expanded around the Wigner SU(4) limit
- Due to nature of approach, interaction is cast into transfer-matrix form

B.-N. Lu, N. Li, S. Elhatisari, Y.-Z. Ma, D. Lee, U.-G. Meissner, Phys. Rev. Lett. **128**, 242501 (2022)

Beyond the deuteron

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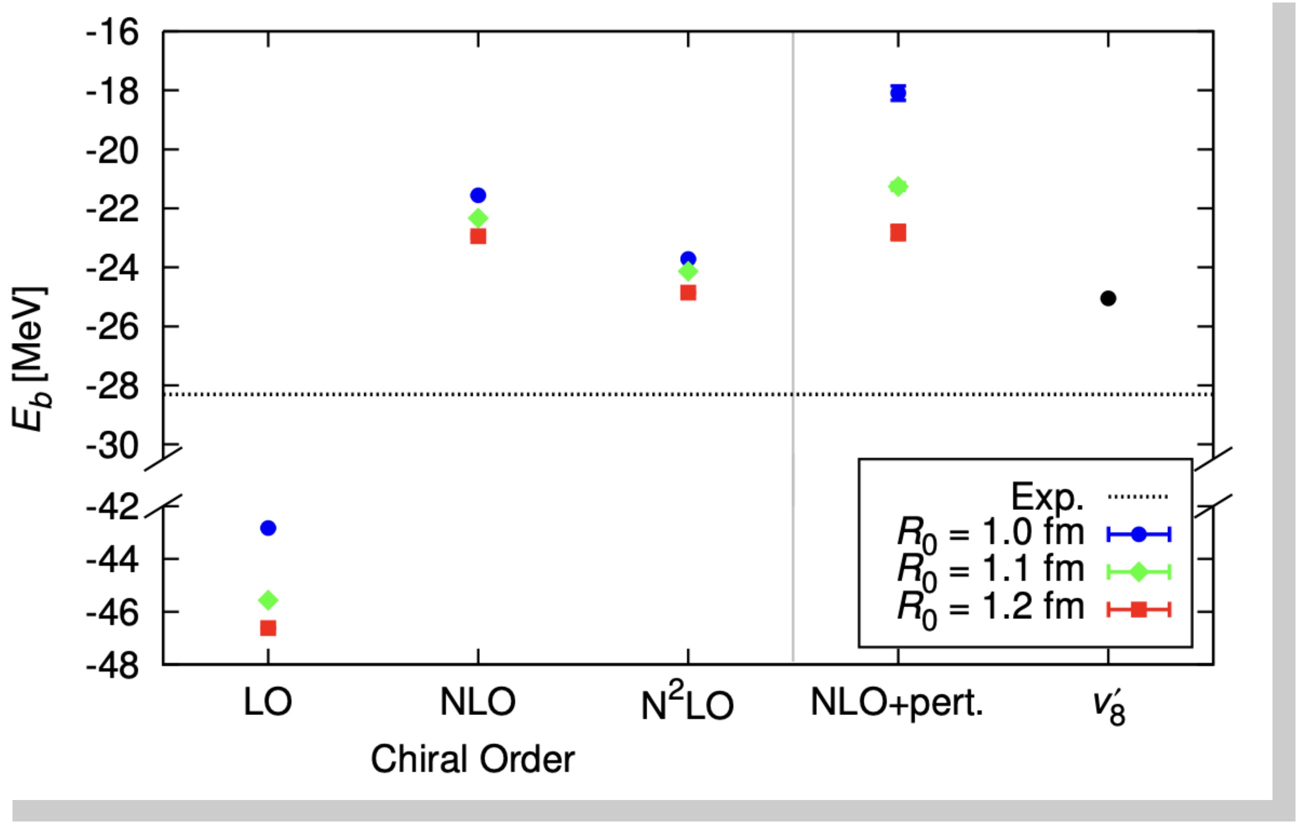
Continuum quantum Monte Carlo: up to 1st order

4He (no TNI)

Three zeroth-order Hamiltonians (and three cutoffs).

Again, speed of convergence appears to depend on softness of interaction.



J. E. Lynn, I. Tews, J. Carlson, E. Epelbaum, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. **113**, 192501 (2014)

Setting the stage

Desiderata: ***approach should be able to***

- straightforwardly go beyond a two-body problem
- use a non-perturbative many-body technique to treat an interaction perturbatively
- straightforwardly handle different momentum cutoffs

R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

Our approach

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Inspired by generic quantum Monte Carlo

$$\lim_{\tau\to\infty} \psi(\tau) = \lim_{\tau\to\infty} \exp[-(\hat{H}_0 - E_T)\tau]\psi_T \propto \psi_0^{(0)}$$

Consider the quantity $$I(\mathcal{T}) = \int_0^{\mathcal{T}} d\tau \left\langle \psi_0^{(0)} \middle| V' e^{-[\hat{H}_0 - E_0^{(0)}]\tau} V' \middle| \psi_0^{(0)} \right\rangle$$

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Recast as

$$I(\mathcal{T}) = (E_0^{(1)})^2\mathcal{T} - \sum_{k\neq 0}^{\infty} \frac{\left|\left\langle \psi_k^{(0)} \right| V' \left| \psi_0^{(0)} \right\rangle\right|^2}{E_k^{(0)} - E_0^{(0)}} \left[e^{-[E_k^{(0)} - E_0^{(0)}]\mathcal{T}} - 1\right]$$

With limiting value $I(\mathcal{T}\to\infty) = (E_0^{(1)})^2\mathcal{T} - E_0^{(2)}$

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With limiting value $I(\mathcal{T} \to \infty) = (E_0^{(1)})^2\mathcal{T} - E_0^{(2)}$

So we can extract the 2nd-order correction from the imaginary time propagation without doing a sum!

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Recast as

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With limiting value

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Our approach

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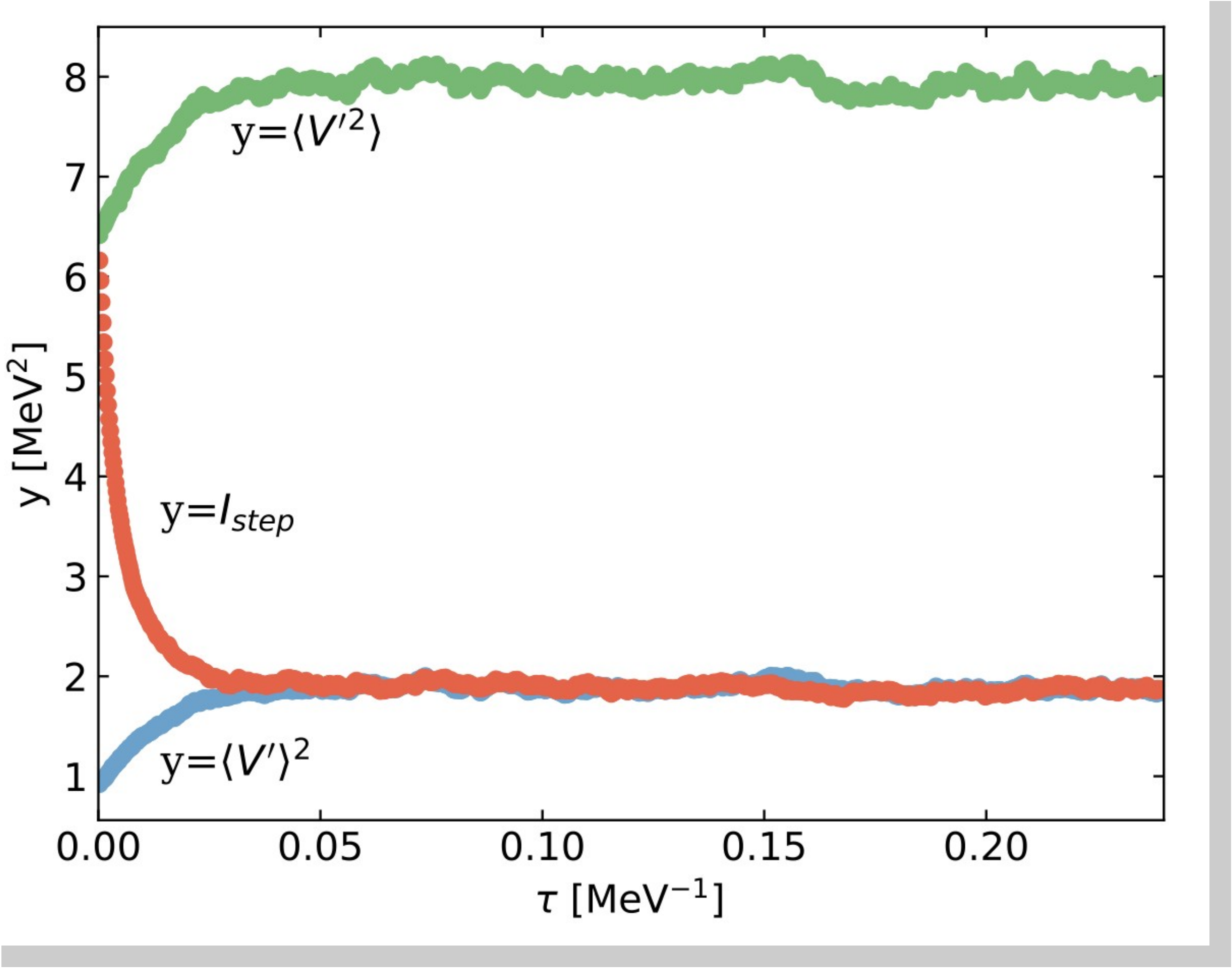
Plot the step-by step evolution of $I(\mathcal{T}) = \int_0^{\mathcal{T}} d\tau \left\langle \psi_0^{(0)} \middle| V' e^{-[\hat{H}_0 - E_0^{(0)}]\tau} V' \middle| \psi_0^{(0)} \right\rangle$



Start at trial values, end up at ground-state values

R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

Applications

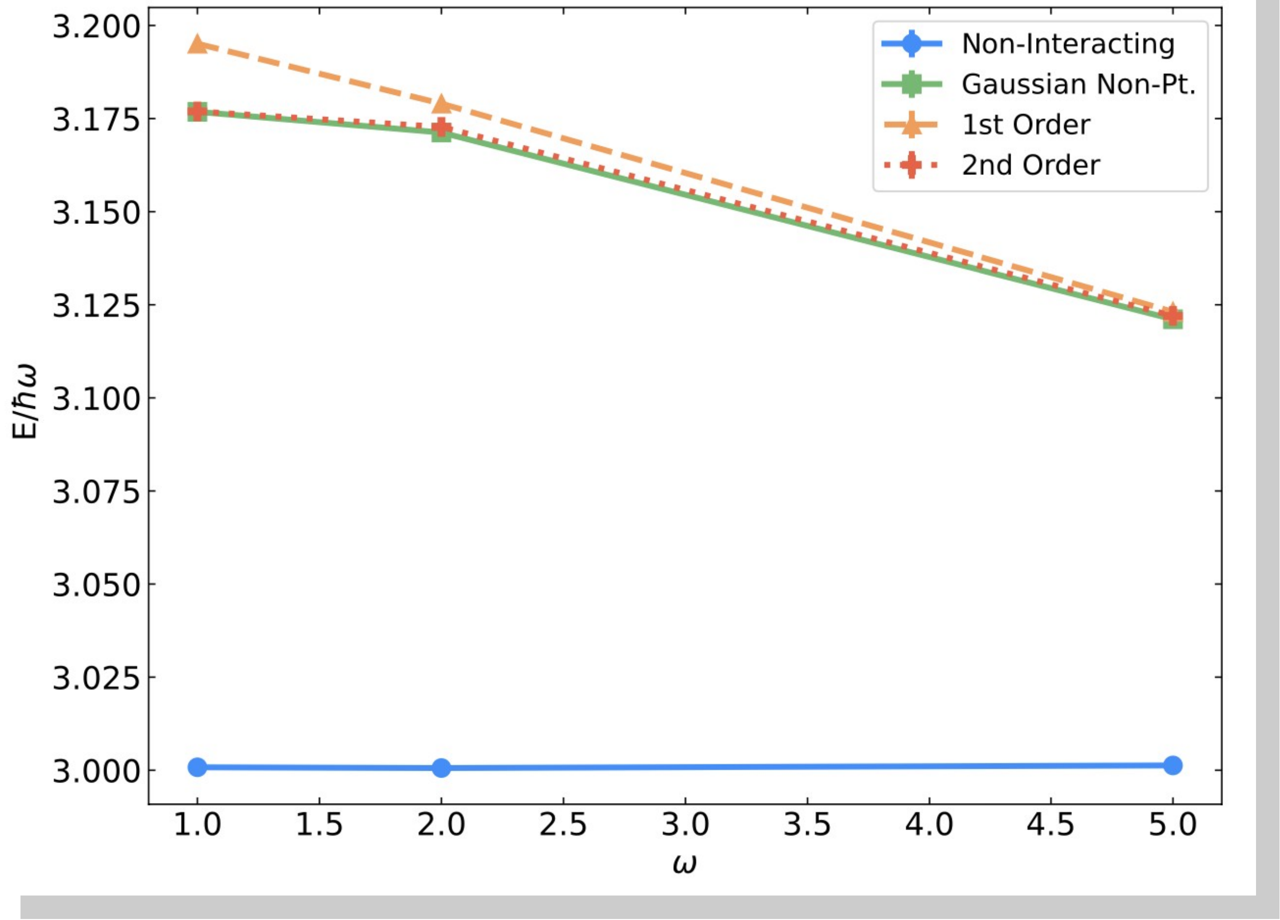
1) Two particles in a trap (Gaussian perturbation)

2) Few neutrons in a trap (charge-independence breaking perturbation)

3) Many neutrons in a box (order-by-order perturbation)

R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

Two particles in a trap (Gaussian perturbation)



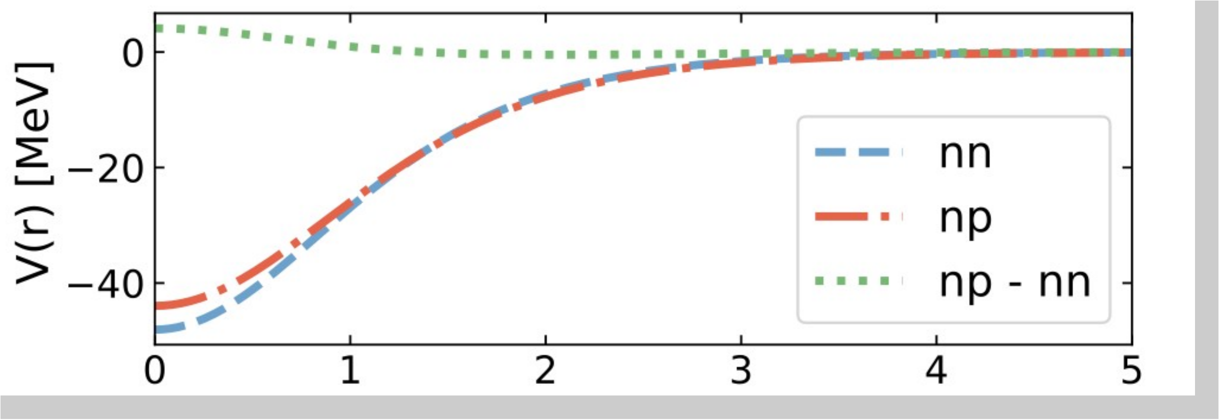
$$V' = ae^{-q^2(\boldsymbol{r}_2-\boldsymbol{r}_1)^2}$$

2nd order gets us to non-perturbative value

R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

Application 2

Few neutrons in a trap (charge-independence breaking perturbation)



Like in Application 1, this is a small perturbation

R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

Application 2

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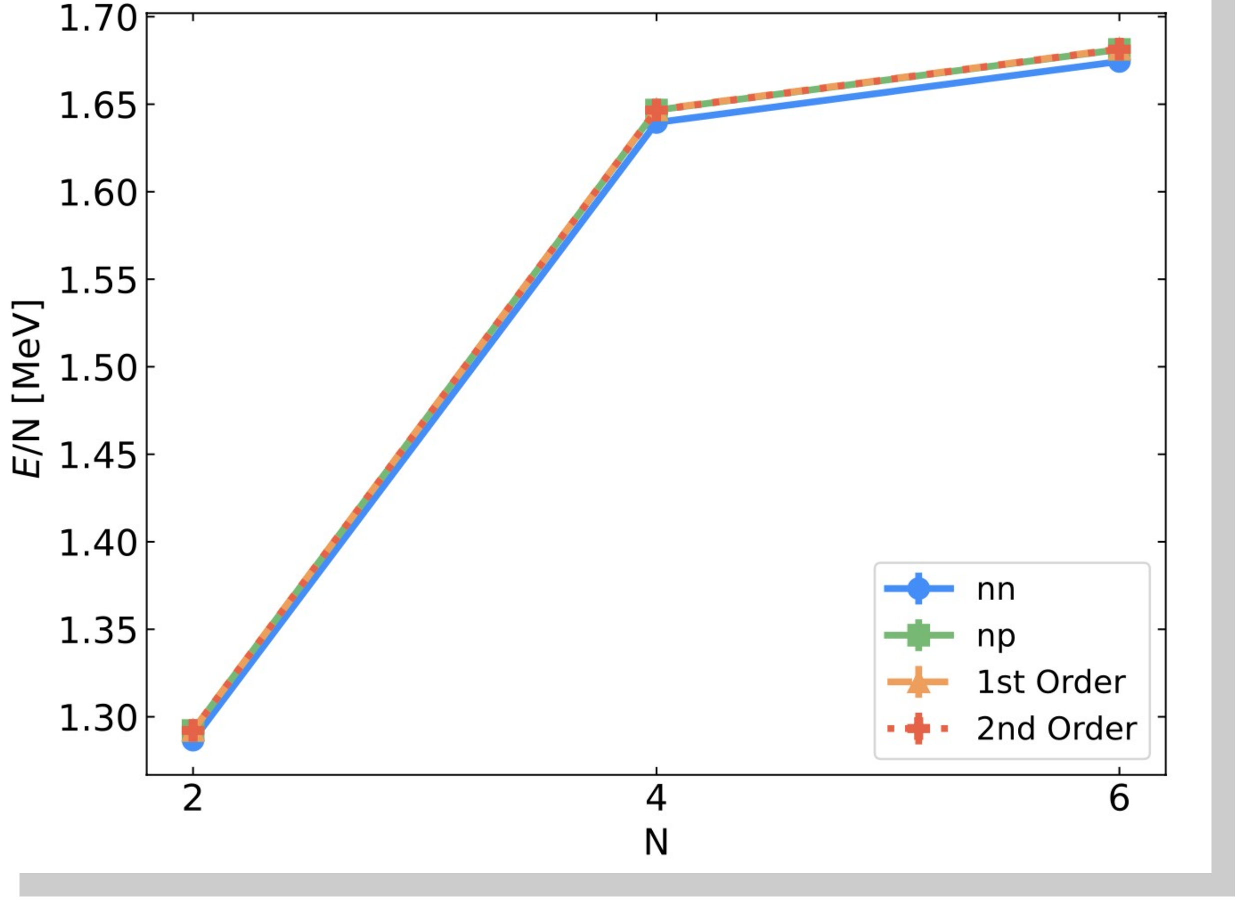

1st order is already good, so it's hard to see what's going on

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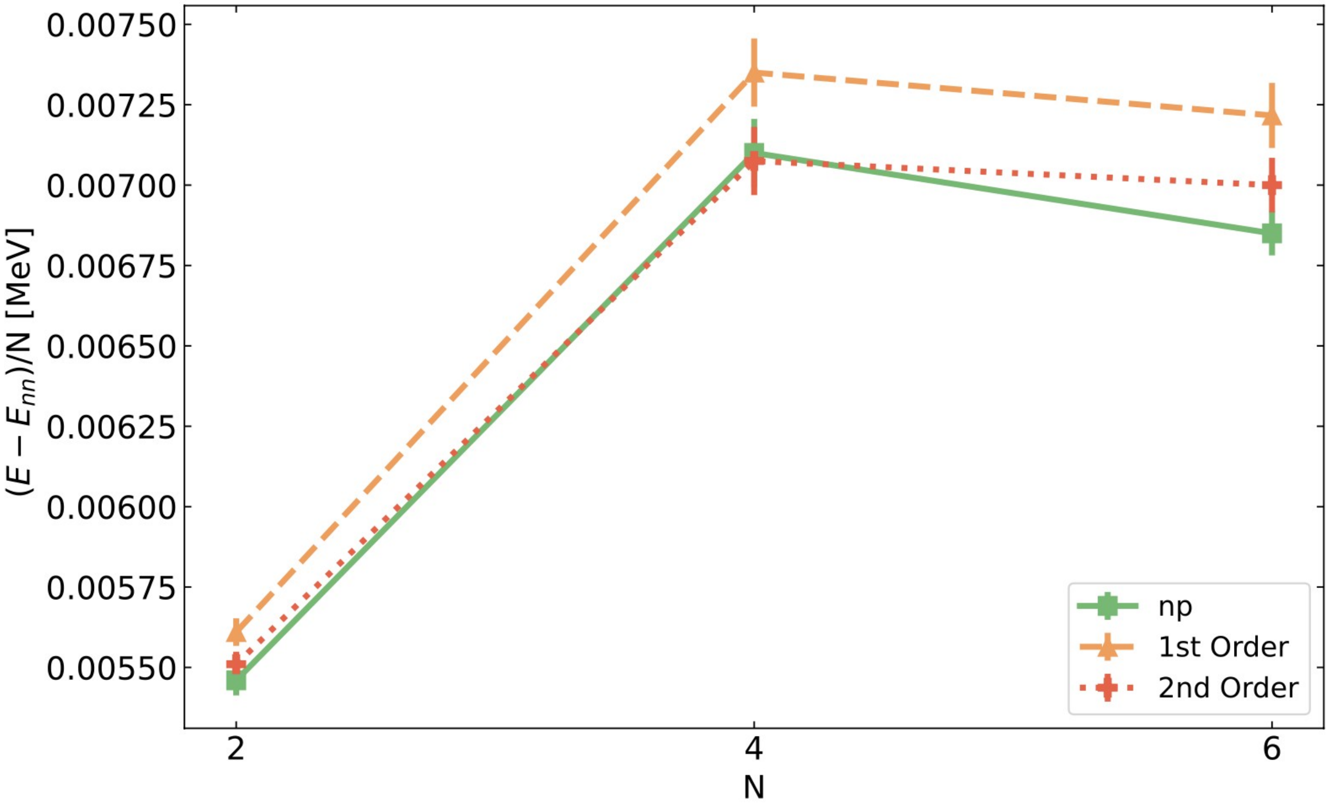

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Application 3

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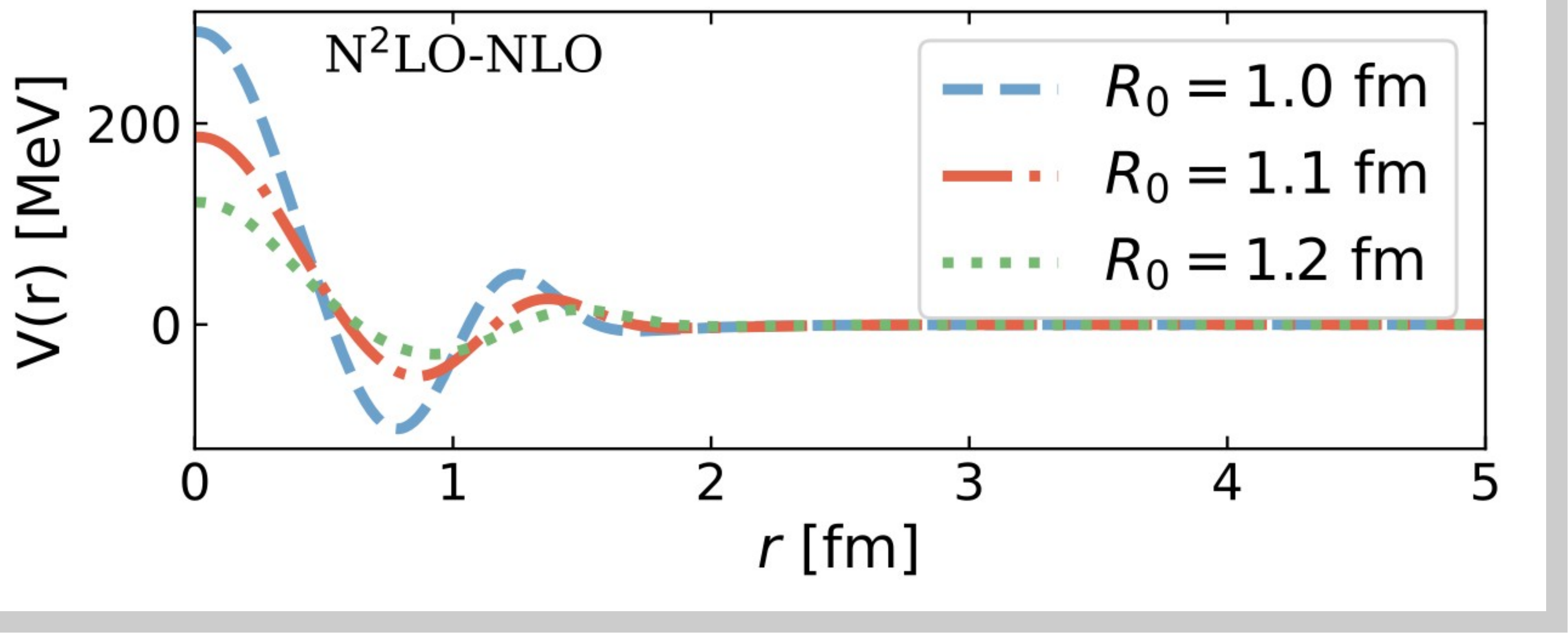

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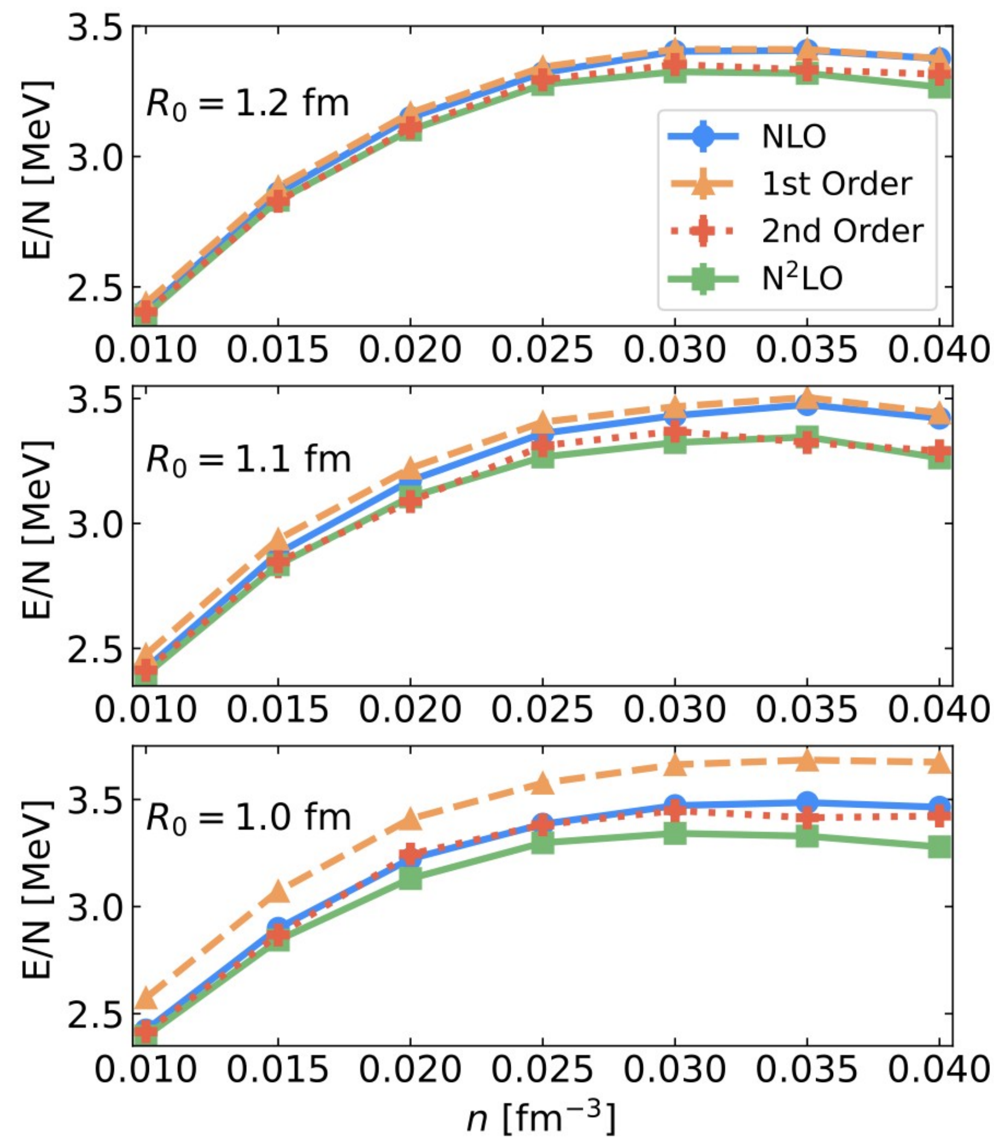
Many neutrons in a box (order-by-order perturbation)

- 66 neutrons
- combination of DMC and PT
- three different cutoffs



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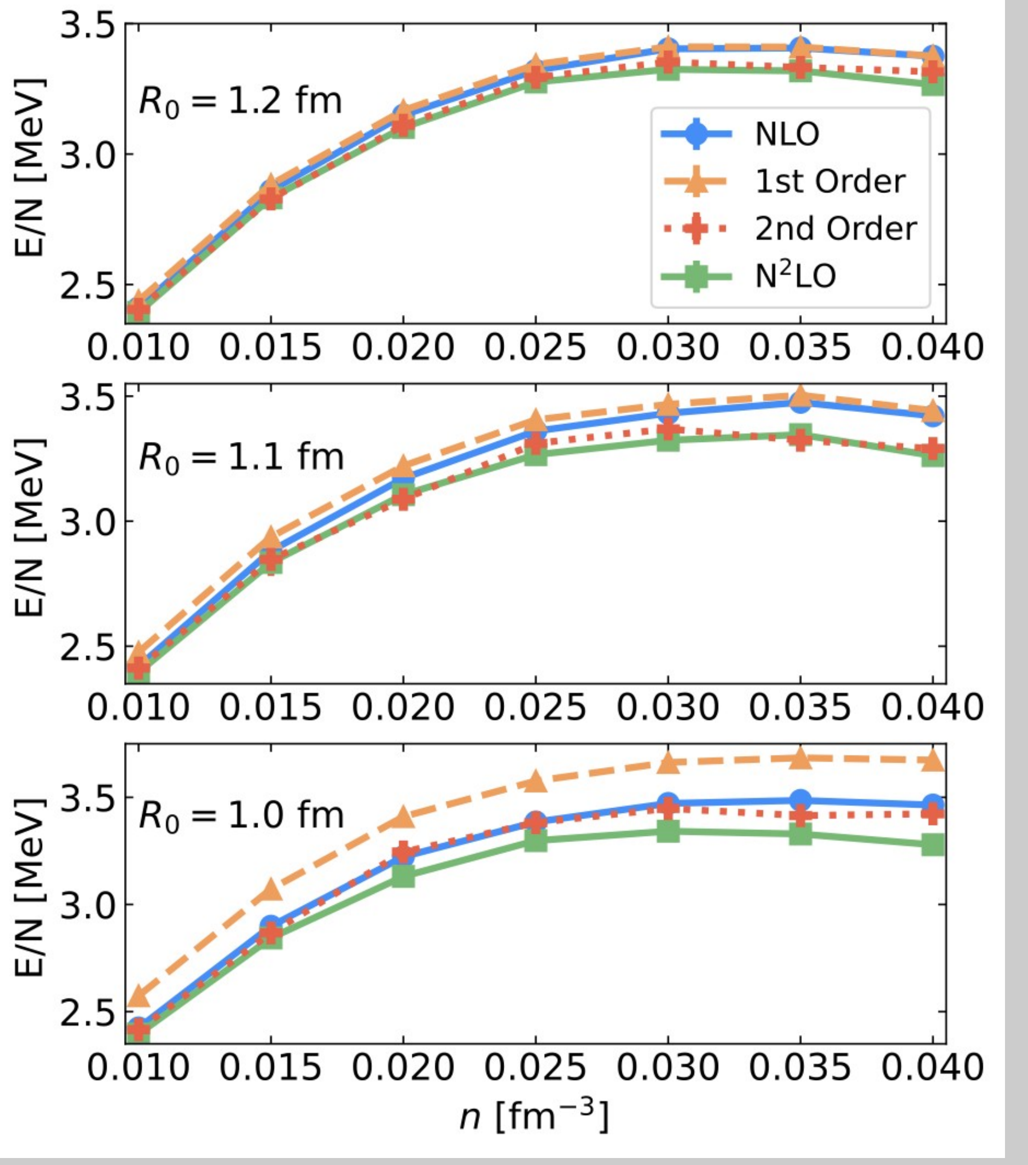
(recall our desiderata)



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- straightforwardly handle different momentum cutoffs

R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

Application 3

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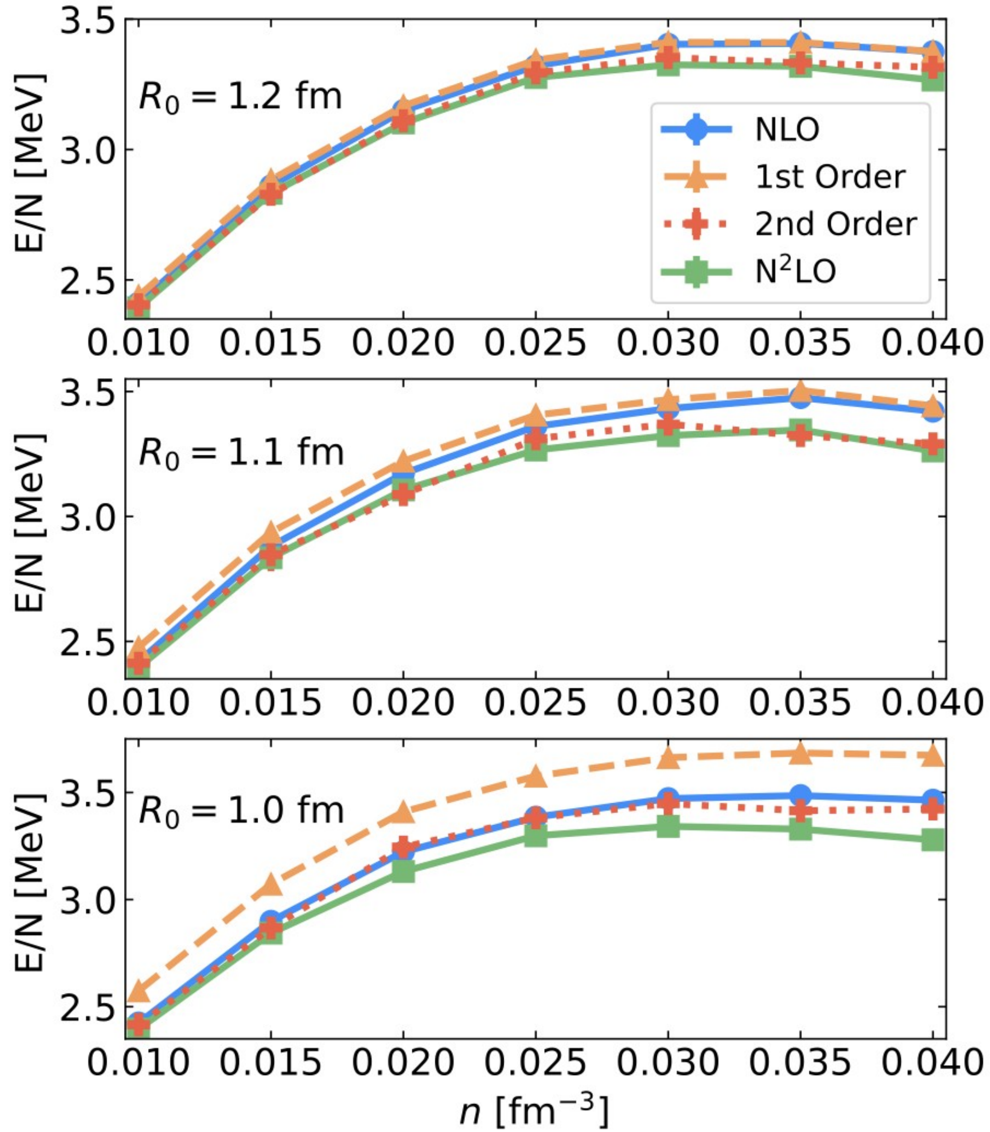
Many neutrons in a box (order-by-order perturbation)

Once again,
speed of convergence
depends on softness
of interaction.



R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis, arXiv:2302.07285

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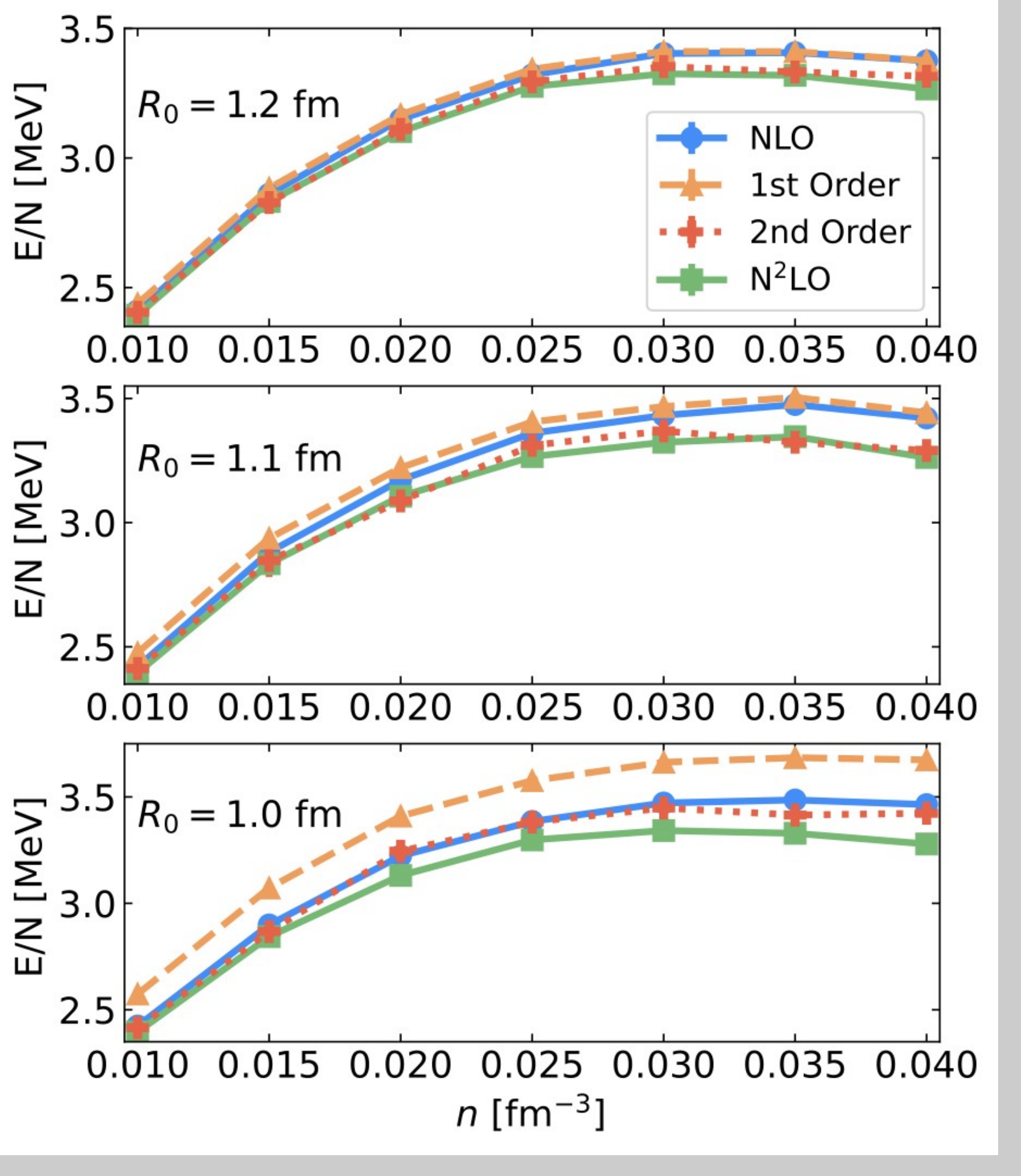
(*The end is in the beginning and yet you go on*.)



R. Curry, J. E. Lynn, K. E. Schmidt, A. Gezerlis,
arXiv:2302.07285

Current outlook

• Certainly worthwhile to go beyond *low-density* neutron matter

• This approach can naturally be applied to the perturbative use of non-local contact operators in chiral EFT

$$\begin{aligned}V^{(4)}_{\text{cont,nonlocal}} &= \tilde{D}_{11}\,\boldsymbol{L}^2 + \tilde{D}_{12}\,\boldsymbol{L}^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\ &+ \tilde{D}_{13}\,\boldsymbol{k}^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q} \\ &+ \tilde{D}_{15}\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{L})(\boldsymbol{\sigma}_2\cdot\boldsymbol{L})\,,\end{aligned}$$

(R. Somasundaram *et al*, arXiv: 2306.13579)

• Worth exploring if a generalization of the above can also be applied to third-order corrections

Conclusions

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- Exciting time in terms of interplay between nuclear interactions and many-body approaches

- Non-perturbative and perturbative approaches are being fruitfully combined

- Detailed probe of well-behavedness of interactions with different cutoffs at the many-body level

Acknowledgments

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