

Kinetic approach of light nuclei in intermediate-energy heavy-ion collisions

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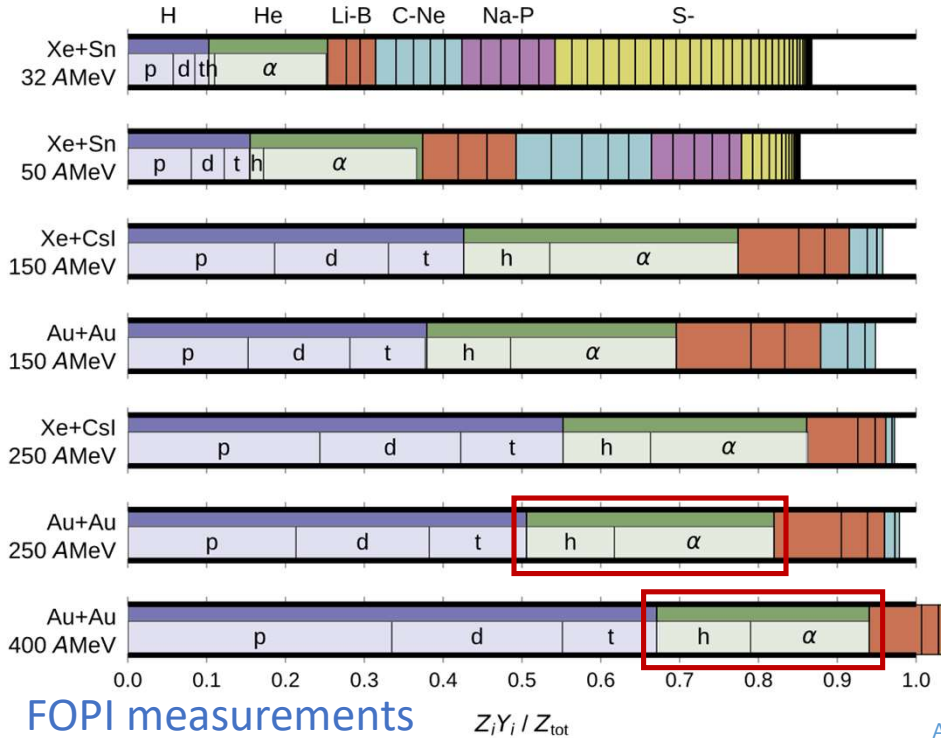
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- 1 Light nuclei in heavy-ion collisions
- 2 Kinetic approach on light nuclei & FOPI measurements
- 3 α -particle fraction in warm and dense nuclear matter

Light nuclei in heavy-ion collisions



FOPi measurements

$Z_i Y_i / Z_{tot}$

Light nuclei account for a large portion of the measured final state charged particles

- Their production mechanism
- Their effects on nucleon/pion observables
- They may provide more efficient probes of nuclear equation of state
-

One interesting feature is that more α -particles are produced than helium-3 (h).

FOPi Collaboration, Nuclear Physics A 848 (2010) 366–427

A. Ono, Progress in Particle and Nuclear Physics 105 (2019) 139–179

Light nuclei in heavy-ion collisions

Dynamical treatment

Mean-field

Spinodal decomposition

- e.g., Statistical mean-field (SMF) or Boltzmann–Langevin equation

M. Colonna, et. al, Nuclear Physics A 642 (1998) 449-460

Low incident energies

Large fragments

Secondary decay
e.g. SMM or GEMINI

Many-body correlations

Many-body scatterings like $NNN \leftrightarrow Nd$

- Anti-symmetrized molecular dynamics (AMD) (Below pion threshold)

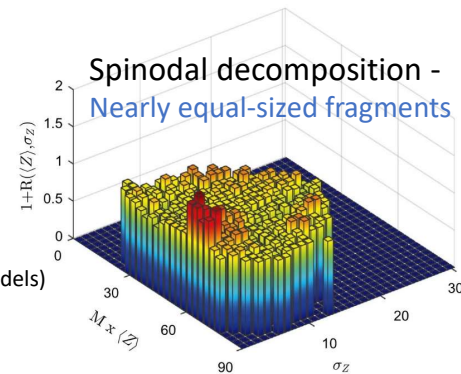
A. Ono, Journal of Physics: Conference Series 420 (2013) 012103

- Kinetic approach/Boltzmann–Uehling–Uhlenbeck equation

Light nuclei with $A = 2$ (d) and $A = 3$ (t, h) have been included.

P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, (1991) 712-748

Light nuclei



Cluster recognition

- Coalescing nucleons according to Δr and/or Δp
e.g., A. S. Botvina, et. al, Physical Review C 103, 064602 (2021) (Combining with decay models)
- Minimum spanning tree/plus most negative E_B
e.g., FRIGA, A. Le Fèvre, et. al, Physical Review C 100, 034904 (2019)
-

B. Borderie, et al., INDRA Collaboration, PRL86, 3252 (2001)
B. Borderie, et al., INDRA Collaboration, PLB782 (2018) 291–296

Light nuclei in kinetic approach

Kinetic equations are derived based on the [closed time-path Green's function formalism](#)

For example, in the deuteron case, the two-body Green's function G_2 satisfies an equation

$$G_2 = \mathcal{G}_2 + \frac{1}{4}\mathcal{G}_2 v G_2$$

The [light nuclei are realized as poles](#) of the many-body Green's function.

In the vicinity of the pole, we have

$$i\langle x|G_2^<(P, \Omega, R, T)|x'\rangle \sim \langle x|\phi(P, R, T)|\rangle\langle\phi(P, R, T)x'\rangle f_2(P, R, T)2\pi\delta[\Omega - E(P, R, T)]$$

$$i\langle x|G_2^>(P, \Omega, R, T)|x'\rangle \sim \langle x|\phi(P, R, T)|\rangle\langle\phi(P, R, T)x'\rangle[1 + f_2(P, R, T)]2\pi\delta[\Omega - E(P, R, T)]$$

[P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, 712-748 \(1991\)](#)

Finally leads to equations of the occupation number f_τ of light nuclei

$$(\partial_t + \vec{\nabla}_p \epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon_\tau \cdot \vec{\nabla}_p) f_\tau = \mathcal{K}_\tau^<[f_n, f_p, f_d, \dots](1 \pm f_\tau) - \mathcal{K}_\tau^>[f_n, f_p, f_d, \dots] f_\tau, \quad \tau = n, p, d, t, h, \alpha$$

Light nuclei in kinetic approach

For example, the loss term of the α particle

$$\begin{aligned}
 K_{\alpha}^{\geq} f_{\alpha} &= \frac{\mathcal{S}_{5'} f_{\alpha}}{2E_{\alpha}} \int \prod_{i=1'}^{5'} \frac{d\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{d\vec{p}_N}{(2\pi\hbar)^3 2E_N} |\overline{\mathcal{M}_{N\alpha \rightarrow NNNNN}}|^2 g_N f_N \prod_{i=1'}^{5'} (1 \pm f_i) (2\pi)^4 \delta^4 \left(\sum_{i=1'}^{5'} p_i - p_N - p_{\alpha} \right) \\
 &+ \frac{\mathcal{S}_{3'} f_{\alpha}}{2E_{\alpha}} \int \prod_{i=1'}^{3'} \frac{d\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{d\vec{p}_N}{(2\pi\hbar)^3 2E_N} |\overline{\mathcal{M}_{N\alpha \rightarrow NNNt}}|^2 g_N f_N \prod_{i=1'}^{3'} (1 \pm f_i) (2\pi)^4 \delta^4 \left(\sum_{i=1'}^{3'} p_i - p_N - p_{\alpha} \right) + t \rightarrow h \\
 &+ \frac{\mathcal{S}_{2'} f_{\alpha}}{2E_{\alpha}} \int \prod_{i=1'}^{2'} \frac{d\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{d\vec{p}_N}{(2\pi\hbar)^3 2E_N} |\overline{\mathcal{M}_{N\alpha \rightarrow dt}}|^2 g_N f_N \prod_{i=1'}^{2'} (1 \pm f_i) (2\pi)^4 \delta^4 \left(\sum_{i=1'}^{2'} p_i - p_N - p_{\alpha} \right) + t \rightarrow h \\
 &+ \text{elastic part.}
 \end{aligned}$$

Light nuclei can be produced and dissociated through many-body scatterings (currently we have included the red ones)

- $A = 2$ $\pi NN \leftrightarrow \pi d$, $NNN \leftrightarrow Nd$
- $A = 3$ $\pi NNN \leftrightarrow \pi t(h)$, $\pi Nd \leftrightarrow \pi t(h)$, $NNNN \leftrightarrow Nt(h)$, $NNd \leftrightarrow Nt(h)$
- $A = 4$ $\pi NNNN \leftrightarrow \pi \alpha$, $\pi NNd \leftrightarrow \pi \alpha$, $\pi Nt(h) \leftrightarrow \pi \alpha$, $NNNN \leftrightarrow N\alpha$, $NNNd \leftrightarrow N\alpha$, $NNt(h) \leftrightarrow N\alpha$, $dt(h) \leftrightarrow N\alpha$

- Many body **transition amplitudes** e.g., $|\overline{M_{Npn \leftrightarrow Nd}}|^2$
- The medium effect of light nuclei – **Mott effect**

R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, Z. Zhang, arXiv:2305.02988

Impulse approximation

$$|\mathcal{M}_{Nd \rightarrow NNN}|^2 \approx F(\sqrt{s}) \sum_{\text{spectator nucleons}} |\langle \vec{k} | \phi_d \rangle|^2 |\mathcal{M}_{NN \rightarrow NN}|^2$$

Detailed balance

$$|\mathcal{M}_{NNN \rightarrow Nd}|^2$$

Internal wave function of light nuclei

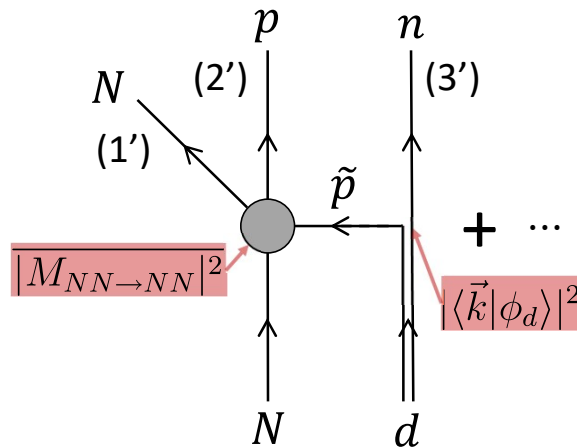
\vec{k} are the standard Jacobian relative momenta

Under IA, $\sigma_{Nd}^{total} \sim \sigma_{Np}^{total} + \sigma_{Nn}^{total}$

$F(\sqrt{s})$ is a factor to 1) account for the inadequacy of IA, 2) exclude the elastic part of N-d from the total amplitude.

They should be determined by comparing with experimental N-d N-t N- α in-elastic cross sections.

In transport approach, under IA the scattering $NNN \leftrightarrow Nd$ can be divided into **two subprocess**

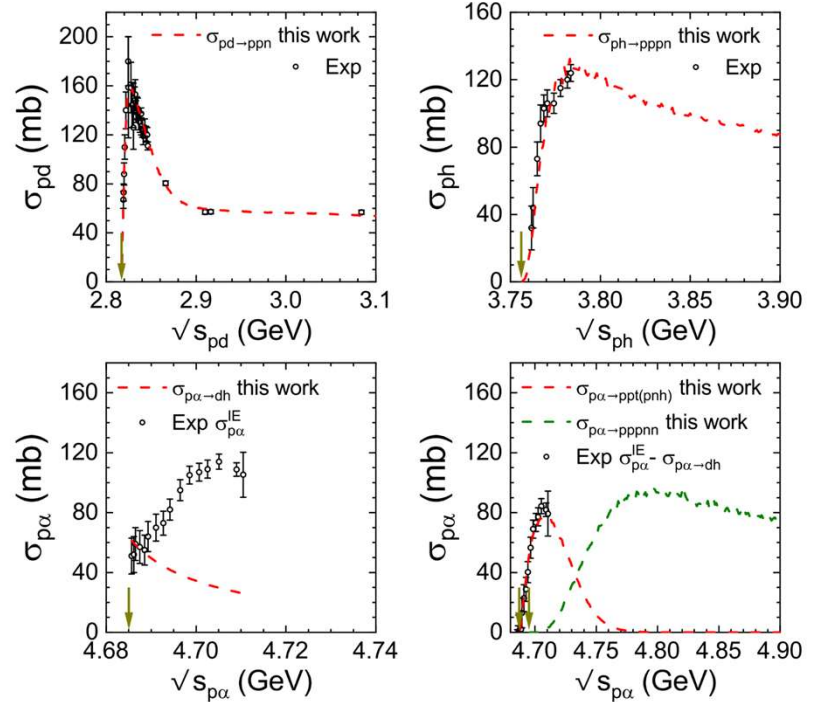


IA could ensure the **detailed balance condition** of many-body scatterings like $NNN \leftrightarrow Nd$

Cross sections

- A natural feature of F is it **approaches to 1 as \sqrt{s} increases**. (For large incident energy, IA becomes very good, and the reaction is dominated by in-elastic channels)
- Different parameterizations of $F(\sqrt{s})$** for different many-body scattering channels are adopted to properly reproduce the experimental N-d, N-h and N- α inelastic cross sections.
- $N\alpha \leftrightarrow NNt(h)$ and $N\alpha \leftrightarrow dt(h)$** should be included to reproduce the experimental N- α inelastic cross sections at small \sqrt{s} . Assumption has to be made of the **branching ratios** of $N\alpha$ scattering

$$|\overline{\mathcal{M}}_{N\alpha \rightarrow NNNNN}|^2 \approx F(\sqrt{s}) \sum_{\text{spectator nucleons}} |\langle \vec{k} \vec{k}_\lambda \vec{k}_\mu | \phi_\alpha \rangle|^2 |\overline{\mathcal{M}}_{NN \rightarrow NN}|^2$$



R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, Z. Zhang, arXiv:2305.02988

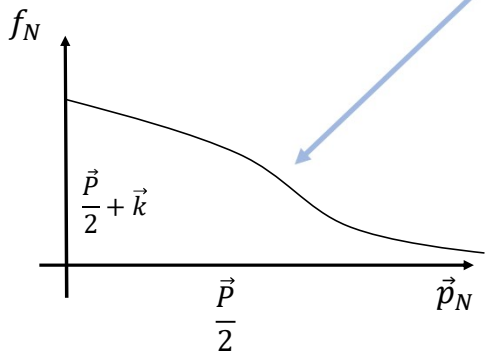
Mott effect

A light nucleus can not bind if its surrounding nucleon phase space is too dense

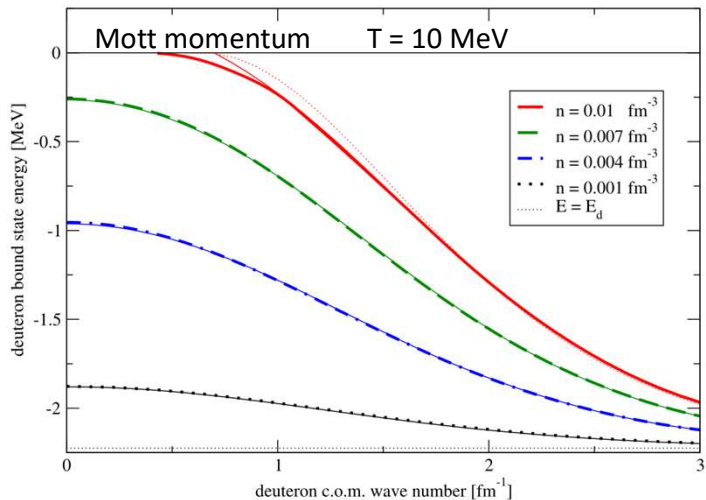
In-medium Schrodinger equation

$$\left[E_n\left(\frac{1}{2}\vec{P}+\vec{q}\right)+E_p\left(\frac{1}{2}\vec{P}-\vec{q}\right) \right] \Psi_{\vec{P}}(\vec{q}) + \left[1 - f_n\left(\frac{1}{2}\vec{P}+\vec{q}\right) - f_p\left(\frac{1}{2}\vec{P}-\vec{q}\right) \right] \int \frac{d\vec{q}'}{(2\pi\hbar)^3} \langle \vec{q}' | v | \vec{q}' \rangle \Psi_{\vec{P}}(\vec{q}') = E(\vec{P}) \Psi_{\vec{P}}(\vec{q})$$

The binding energy of a light nucleus (with momentum P) in nuclear medium, the Mott point is recognized as where the binding energy becomes negative.



Nucleon - Fermi distribution



G. Röpke, Nuclear Physics A 867 (2011) 66–80

Mott effect

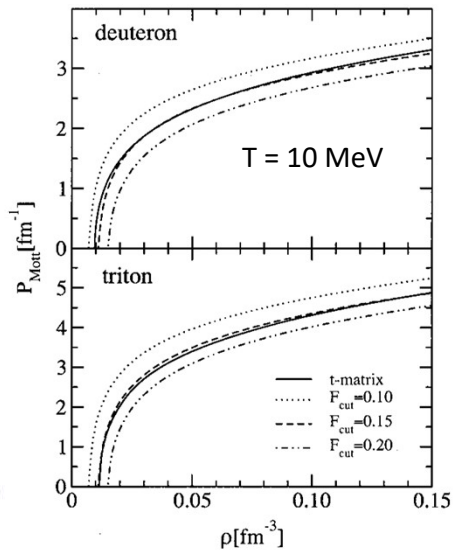
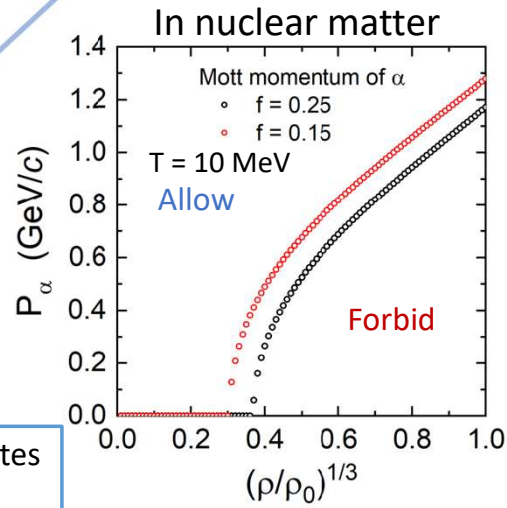
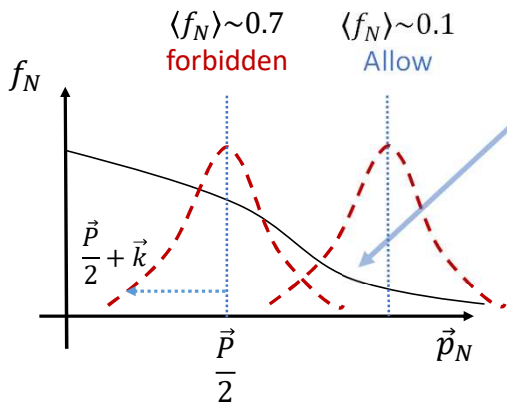
In kinetic approach

The Mott effect can be **effectively** introduced into the kinetic approach through a **phase-space cutoff parameter**.

Momentum distribution of the constituent nucleon, related to the **light-nuclei internal wave function**

$$\langle f_N \rangle_i(\vec{P}) \equiv \int f_N^{\text{tot}} \left(\frac{\vec{P}}{A_i} + \vec{p} \right) |\phi_i(\vec{p})|^2 d\vec{p} \leq f_{A=A_i}^{\text{cut}}$$

A larger $f_{A=A_i}^{\text{cut}}$ corresponds to a weaker Mott effect



$f_{A=2}^{\text{cut}}$, $f_{A=3}^{\text{cut}}$, and $f_{A=4}^{\text{cut}}$ can be treated as surrogates of the strength of light-nuclei Mott effect

C. Kührts, et al., Physical Review C 63, 034605 (2001)

Light-nuclei yields

Central Au+Au collisions at 0.4A GeV

Multiplicities and charge balance for Au + Au at $E/A = 0.40$ GeV and $b_0 < 0.15$.

$Z = 1$	106.0 ± 4.0	p	52.9 ± 2.7	$Z = 2$	21.3 ± 1.9	${}^3\text{He}$	9.4 ± 0.9
		d	34.2 ± 2.1			${}^4\text{He}$	11.9 ± 1.1
		t	19.4 ± 1.8				
π^+	0.95 ± 0.08	π^-	2.80 ± 0.14				
		Li	3.5 ± 0.4	Be	0.84 ± 0.09		
		B	0.27 ± 0.03	C	0.096 ± 0.01		

Charge balance LCP: 148.6 or 94.1%

Charge balance LCP + pions: 146.8 or 92.9%

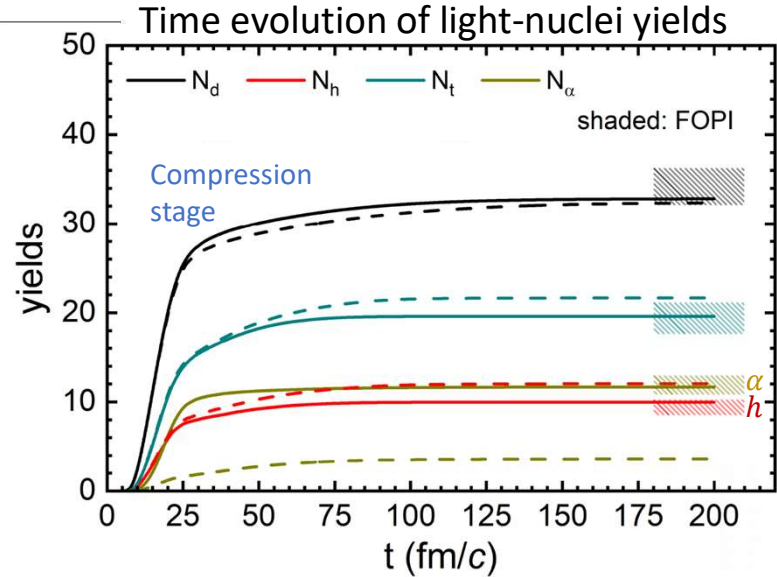
Charge in HC ($Z3-Z6$): 15.77 ± 1.6 or $(10.0 \pm 1.0)\%$

Total balance: 102.9%

To reproduce the measured light-nuclei yields (Note N_α is larger than N_h), one needs to use a larger $f_{A=4}^{cut}$ compared with $f_{A=3}^{cut}$, which means the Mott effect of α -particle should be weaker than that of triton/helium3.

A smaller $f_{A=4}^{cut}$ leads to a significant decrease of N_α .

FOPI data at 0.4A GeV



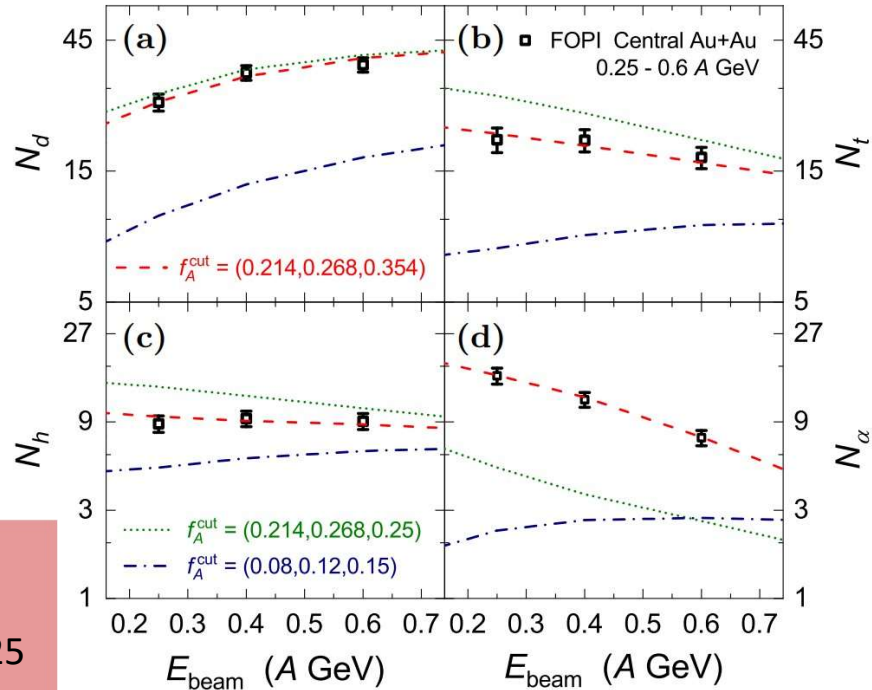
R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, Z. Zhang, arXiv:2305.02988

Light-nuclei yields

Bayesian analysis of f_A^{cut} from light-nuclei yields in Au+Au central collisions at energies of $0.25A$ to $0.6A$ GeV.

- Influence of spinodal decomposition (larger fragments) in the low-energy region.
- Pion catalysis reactions, i.e., $\pi NN \leftrightarrow \pi d$, may contribute to the light nuclei yields at $E_{beam} > 0.6A$ GeV

The cutoff values $f_{A=2}^{cut} = 0.214$, $f_{A=3}^{cut} = 0.268$, and $f_{A=4}^{cut} = 0.354$ reasonably reproduce the FOPI data at energies of 0.25 to $0.6A$ GeV.



R. Wang, Z. Zhang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, in preparation

α -particle fraction

- According to the present kinetic approach, in intermediate-energy heavy-ion collisions, light nuclei are mainly formatted and freeze-out chemically at high densities ($NNNNN \leftrightarrow N\alpha$), especially for α -particles.

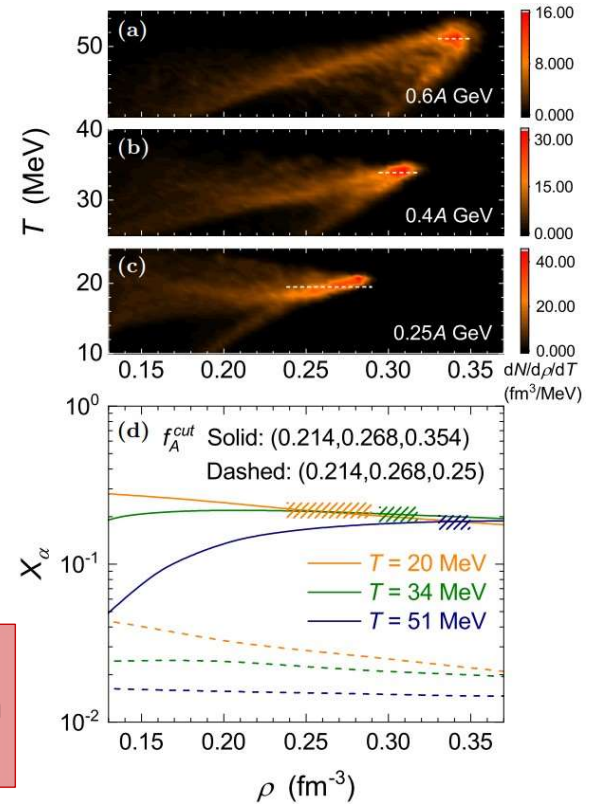
$$\text{Collision rate} \sim \rho^5$$



Contradict!

- It is generally thought that the dense nuclear matter can be regarded almost as a **uniform nucleon liquid**.

α -particle fraction ~ 0.2 for in nuclear matter at around $\rho = 0.266 \text{ fm}^{-3}$ with $T = 19.5 \text{ MeV}$, $\rho = 0.306 \text{ fm}^{-3}$ with $T = 33.9 \text{ MeV}$, and $\rho = 0.340 \text{ fm}^{-3}$ with $T = 51.1 \text{ MeV}$.

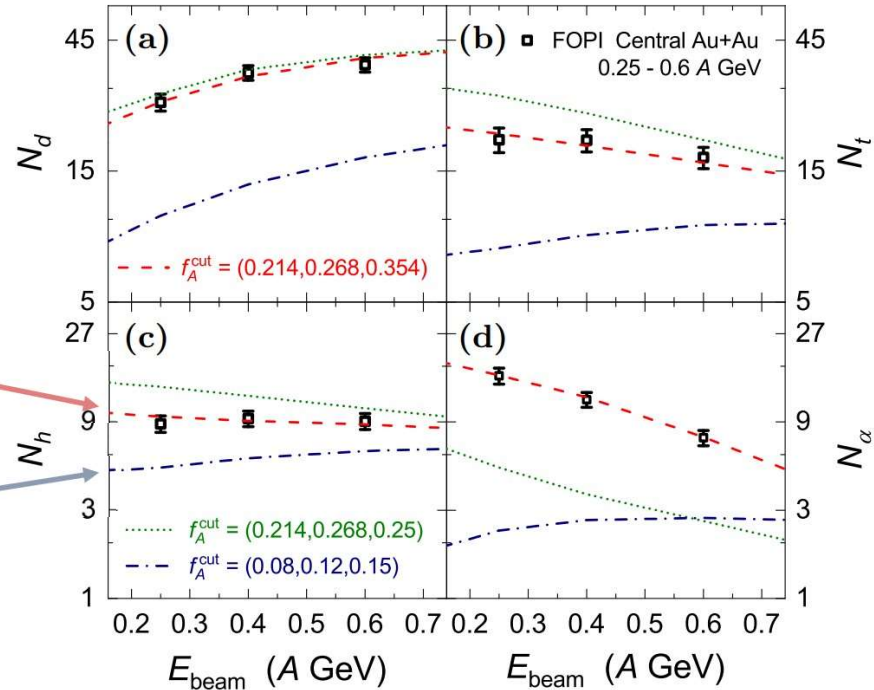
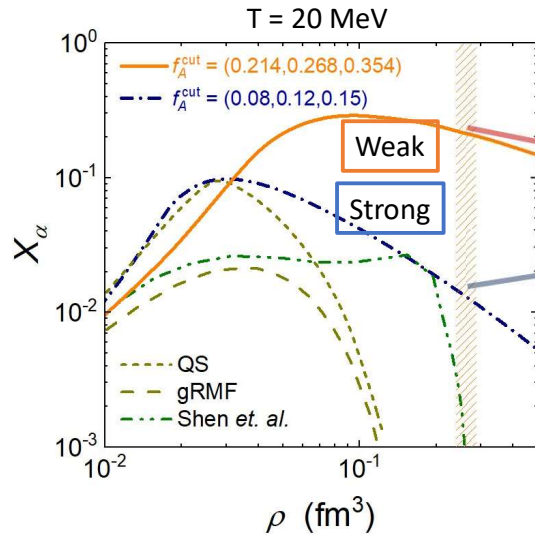


R. Wang, Z. Zhang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, in preparation

α -particle fraction

Comparison with other approaches

- QS & gRMF - Mott momentum from in-medium Schrodinger equation in low density regions
- Shen et. al - excluded volume prescription of α



R. Wang, Z. Zhang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, in preparation

Summary

- The FOPI data on light-nuclei yields can be reasonably reproduced within the present kinetic approach which incorporates dynamically all the light-nuclei (up to $A=4$) degrees of freedom.
- Our results indicate that the enhancement of α -particle yield is a consequence of its weaker Mott effect.
- Based on our approach, the FOPI data of light-nuclei yields indicate an unexpectedly high α -particle fraction in warm and dense nuclear matter.

Key question: Figure out the interplay between light nuclei and large fragments to achieve a more comprehensive description of the cluster production in heavy-ion collisions.

Thank you