Three-nucleon forces at low energy

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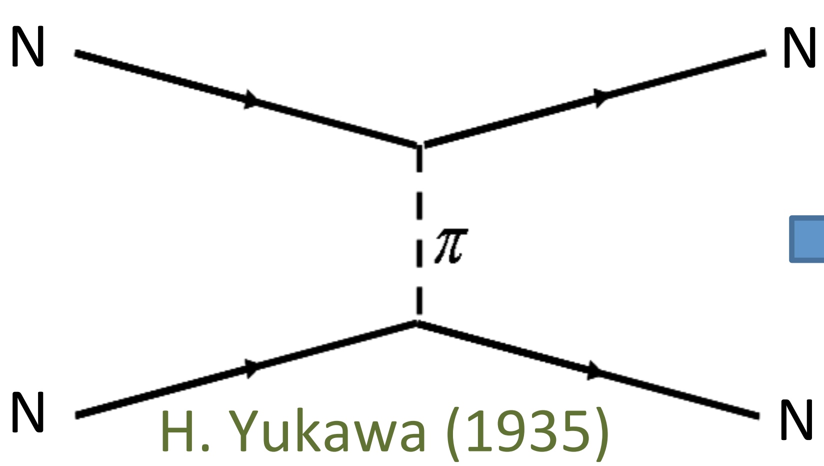
RUB

Outline

- ChPT and low energy QCD
- Nuclear forces in chiral EFT
- Three-nucleon forces up to N^3LO
- Long-range part of three-nucleon forces up to N^4LO
- Summary & Outlook

Nucleon-Nucleon forces

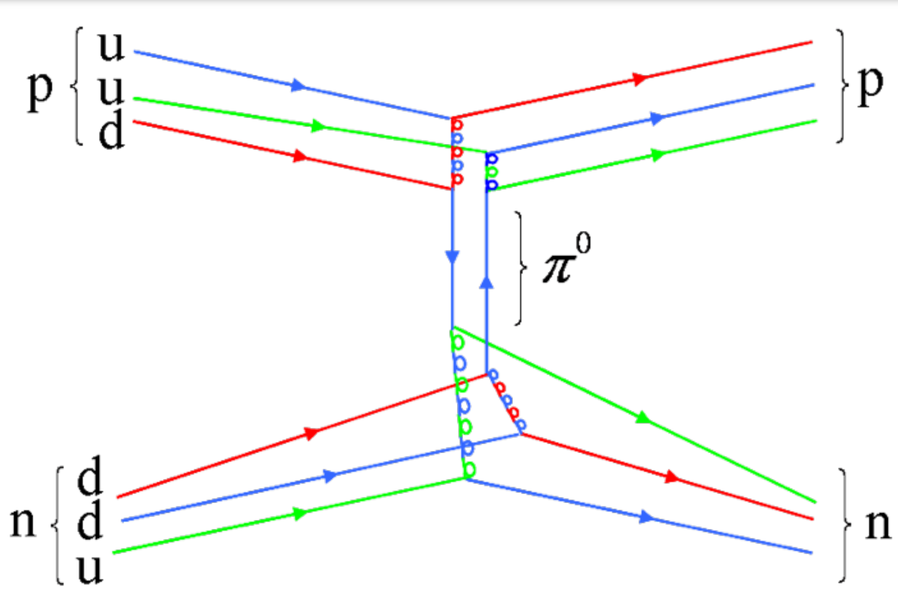
Phenomenological description by Meson-exchange



- Boson-Exchange Models as basis for NN-force
- Highly sophisticated phen. NN potentials
- Excellent description of many experimental data
- Connection to QCD is unclear

QCD Interpretation of NN forces

- NN force as residual strong interaction between hadrons



Chiral EFT Interpretation of NN forces

- Model independent treatment
- At low energies NN force dominated by Goldstone Boson dynamics + short range int.
- Systematic perturbative description of few nucleon potentials
- Underlying QCD symmetries implemented by construction

ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

⇓

Existence of light weakly interacting Goldstone bosons

⇓

Chiral Perturbation theory (ChPT)
Expansion in small momenta and masses of Goldstone bosons

⇓

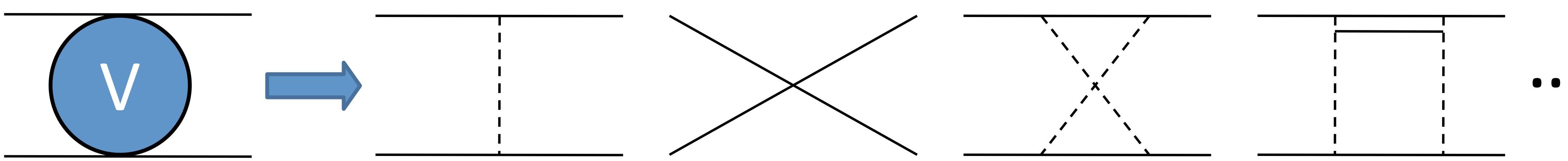
Systematic description of QCD by ChPT in low energy sector
(low momenta $q \ll \Lambda_\chi \simeq 1\,\mathrm{GeV}$)

Weinberg's scheme for NN

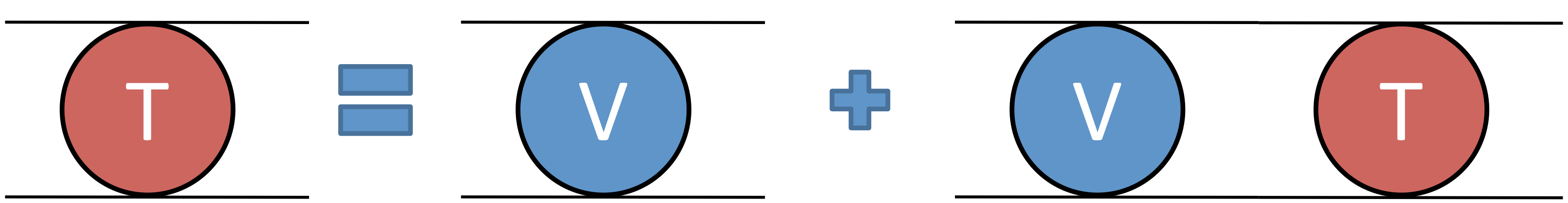
Weinberg, Nucl. Phys. B 363: 3 (1991)

- No perturbative description for bound states

- Construct effective potential perturbatively



- Solve Lippmann-Schwinger equation nonperturbatively



Effective potential

- Decomposition of the Fock space $\mathcal{H}$

Projector operators: $\eta + \lambda = 1$

$$\mathcal{H}_M \overset{\eta}{\leftarrow} \mathcal{H} = \mathcal{H}_M \oplus \mathcal{H}_R \overset{\lambda}{\rightarrow} \mathcal{H}_R$$

Model-space including only pure nucleon states

Remainder-space including states with at least one pion

$$H|\Psi\rangle = (H_0 + H_I)|\Psi\rangle = E|\Psi\rangle \Longleftrightarrow \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix}$$

- Block-diagonalization by applying unitary transformation

$$\tilde{H} = U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda H \lambda \end{pmatrix}$$

$$V_{\rm eff} = \eta(\tilde{H} - H_0)\eta$$

$V_{\rm eff}$ is E - indep.⟹ important for few-nucleon simulations

Possible parametrization by Okubo '54

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

With decoupling eq. $\lambda(H - [A, H] - AHA)\eta = 0$

Can be solved perturbatively within ChPT

Epelbaum et al. '98

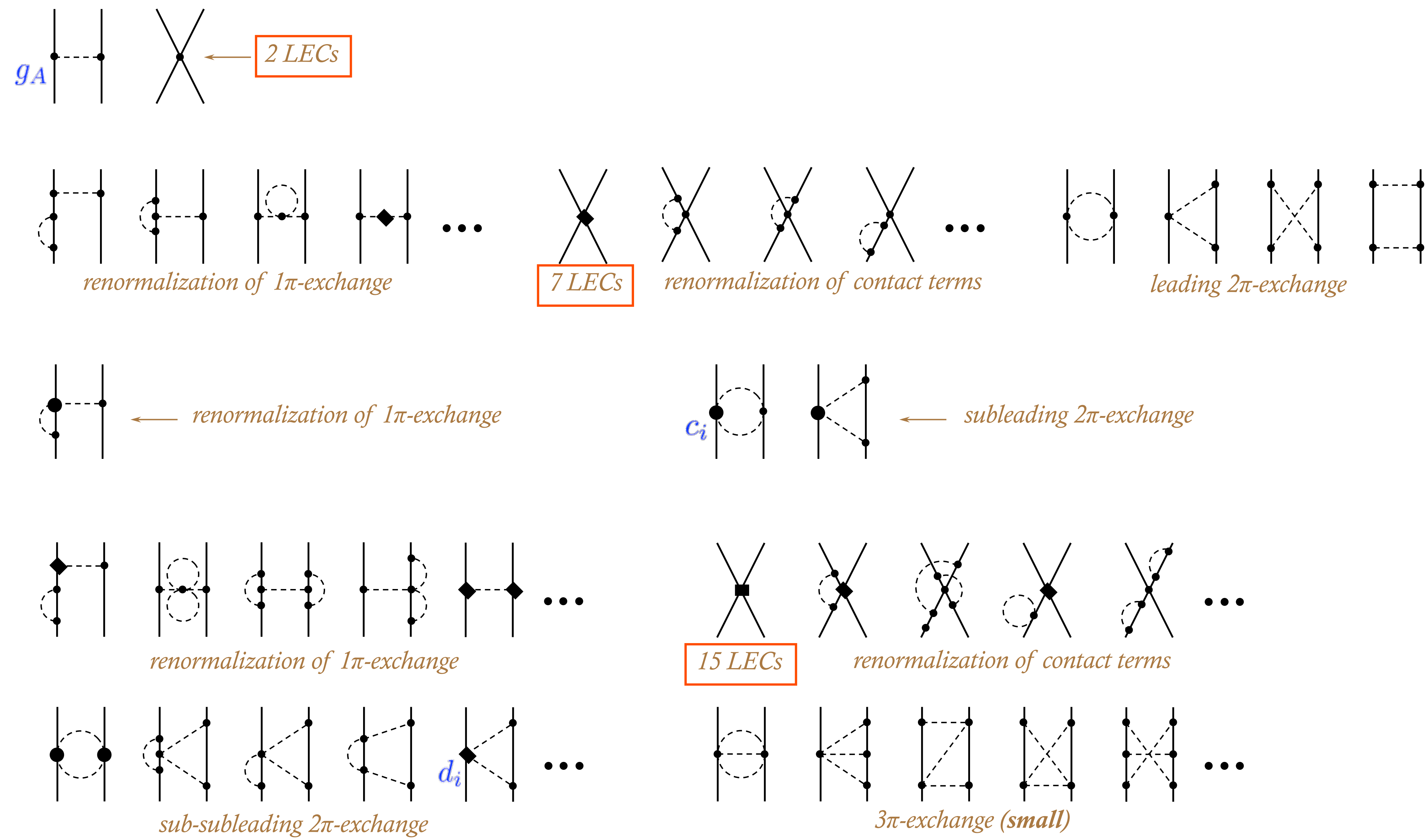
Nucleon-nucleon force up to N³LO

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; Epelbaum et al. ’98,‘03; Kaiser ’99-’01; Higa et al. ’03; …

Chiral expansion for the 2N force:

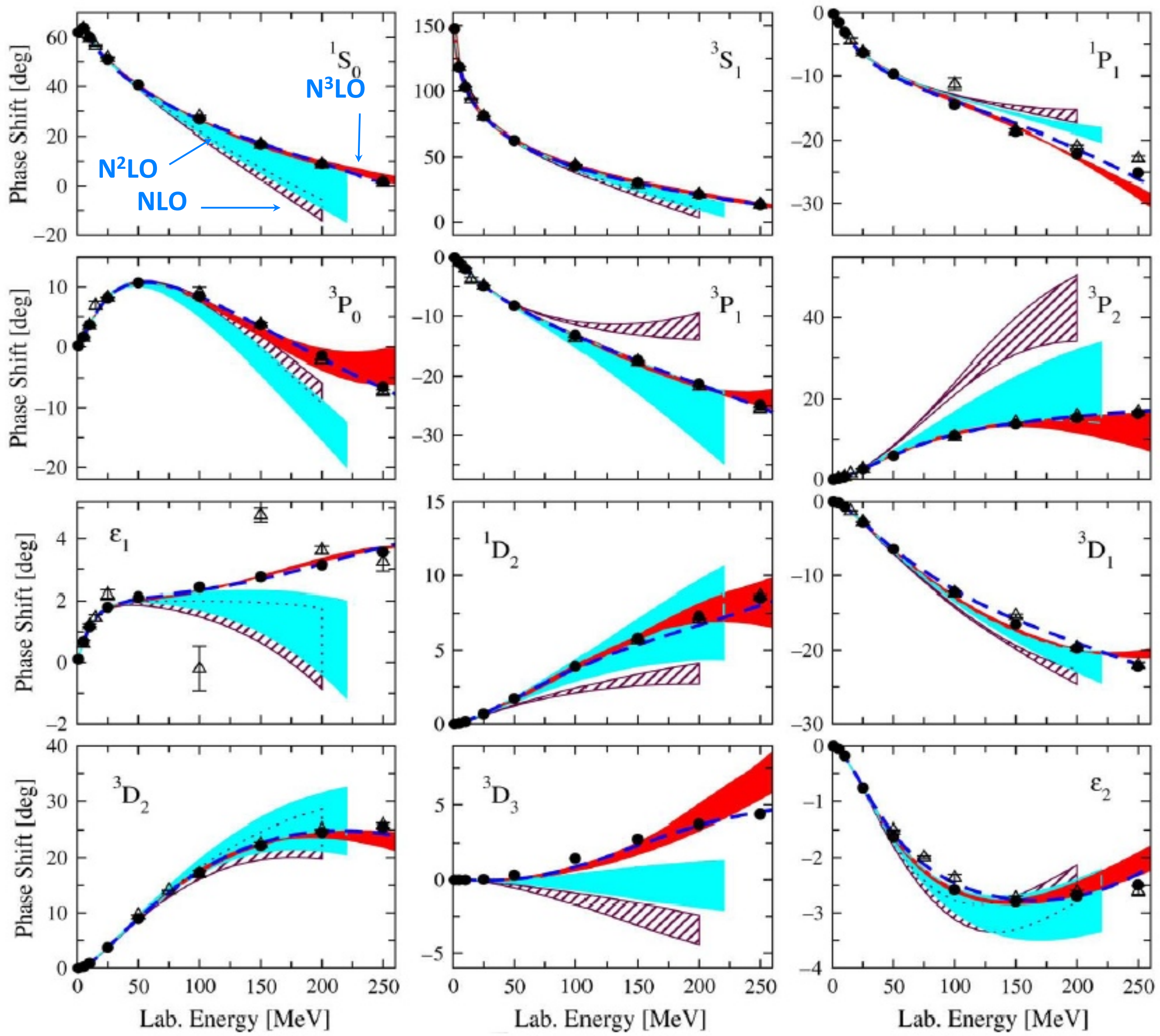
$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$$

- LO:
- NLO:
- N²LO:
- N³LO:

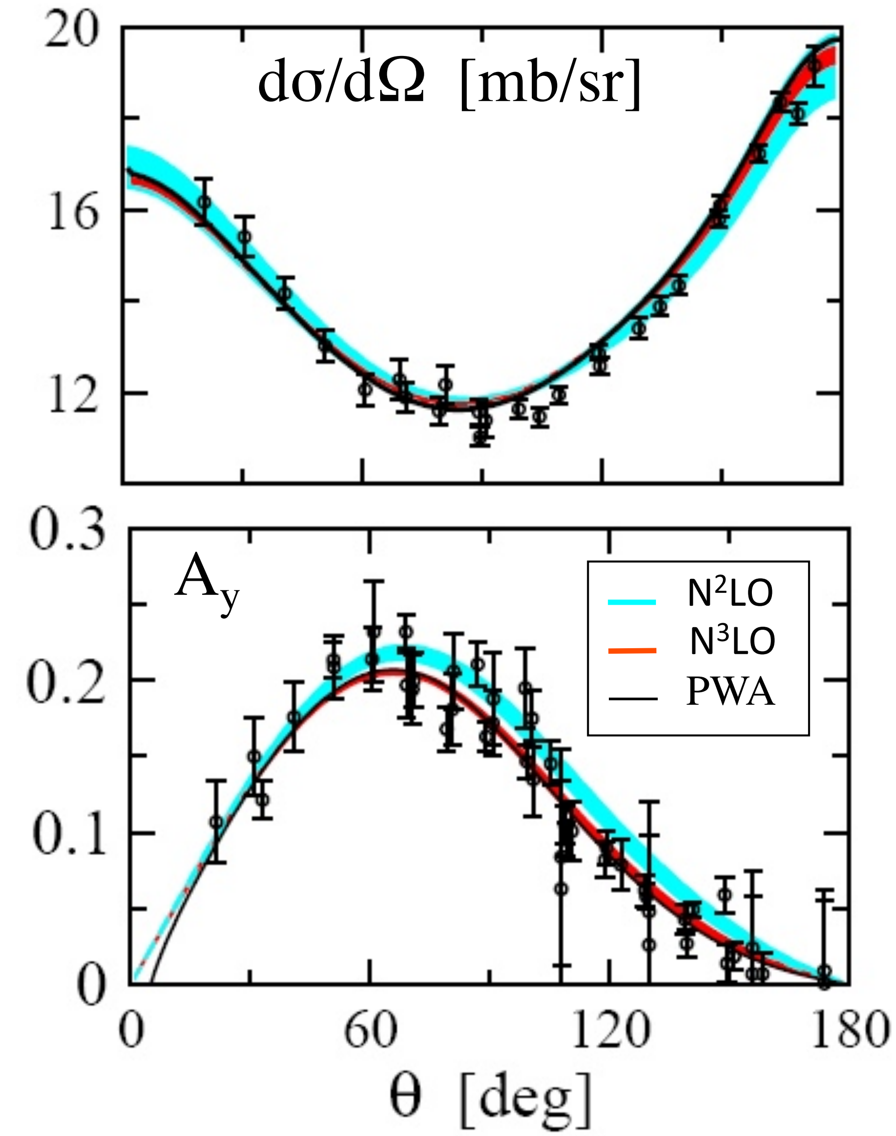


+ 1/m and isospin-breaking corrections…

Neutron-proton phase shifts up to N^{3}LO



np scattering at 50 MeV



Deuteron binding energy & asymptotic normalizations A_s and η_d

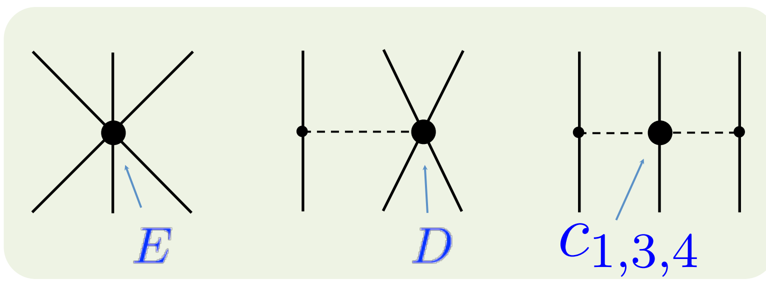
	NLO	N^{2}LO	N^{3}LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [fm$^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

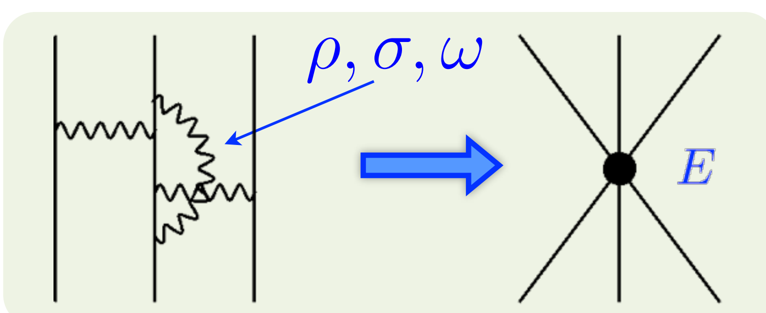
  (U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)



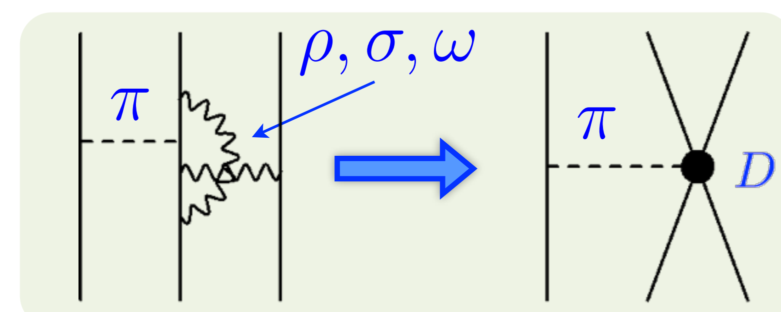
$c_{1,3,4}$ from the fit to πN-scattering data

D, E from $^3\mathrm{H}, ^4\mathrm{He}, ^{10}\mathrm{B}$ binding energy + coherent nd - scattering length

- LECs D and E incorporate short-range contr.



Resonance saturation interpretation of LECs



- Delta contributions encoded in LECs

  (Bernard, Kaiser & Meißner '97)

Delta-resonance saturation

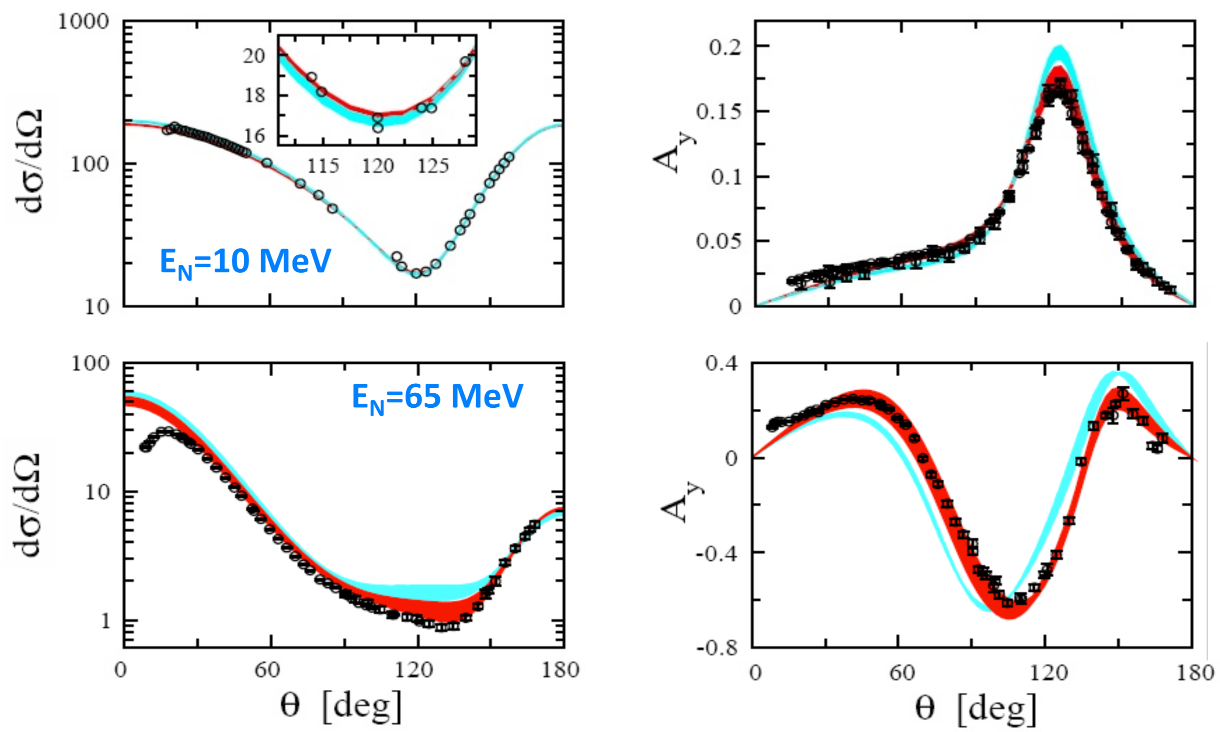
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

Nd elastic scattering

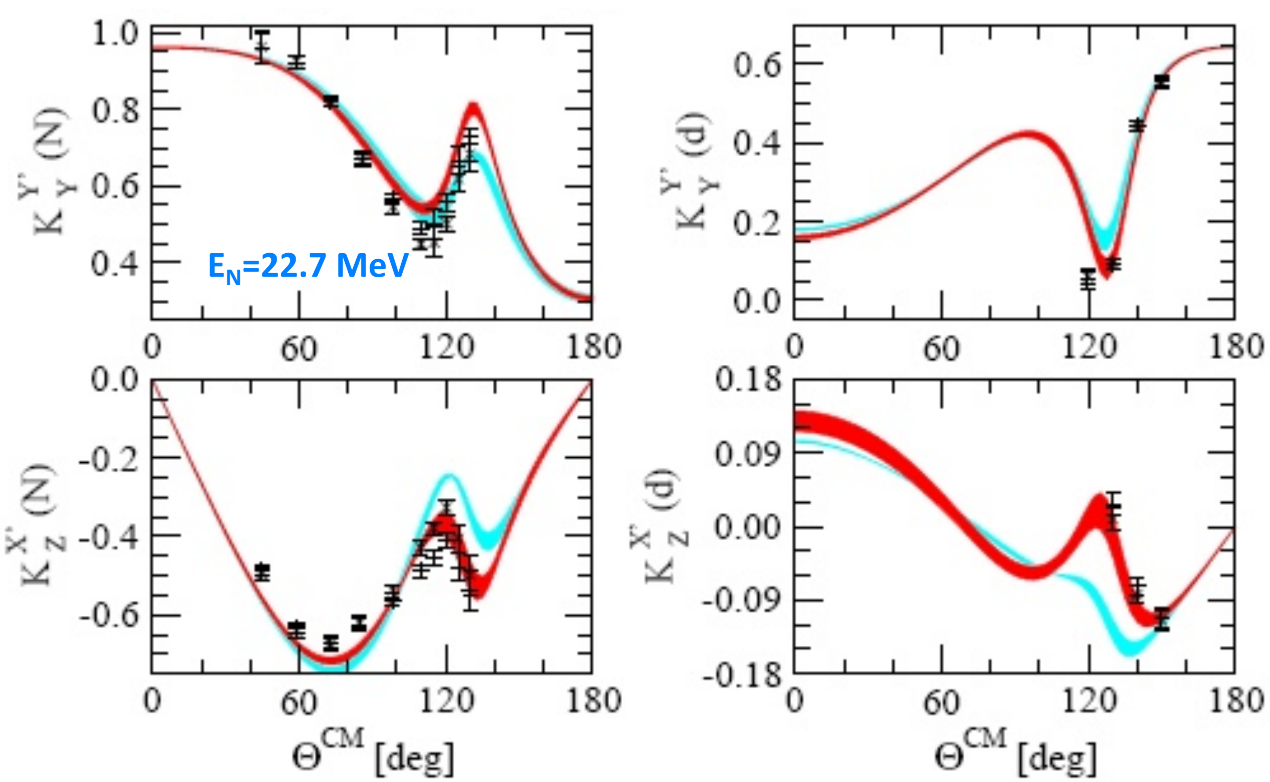
Cross section & vector analyzing power

E.pelbaum, PPNP 57 (2006) 654



Polarization transfer coefficients

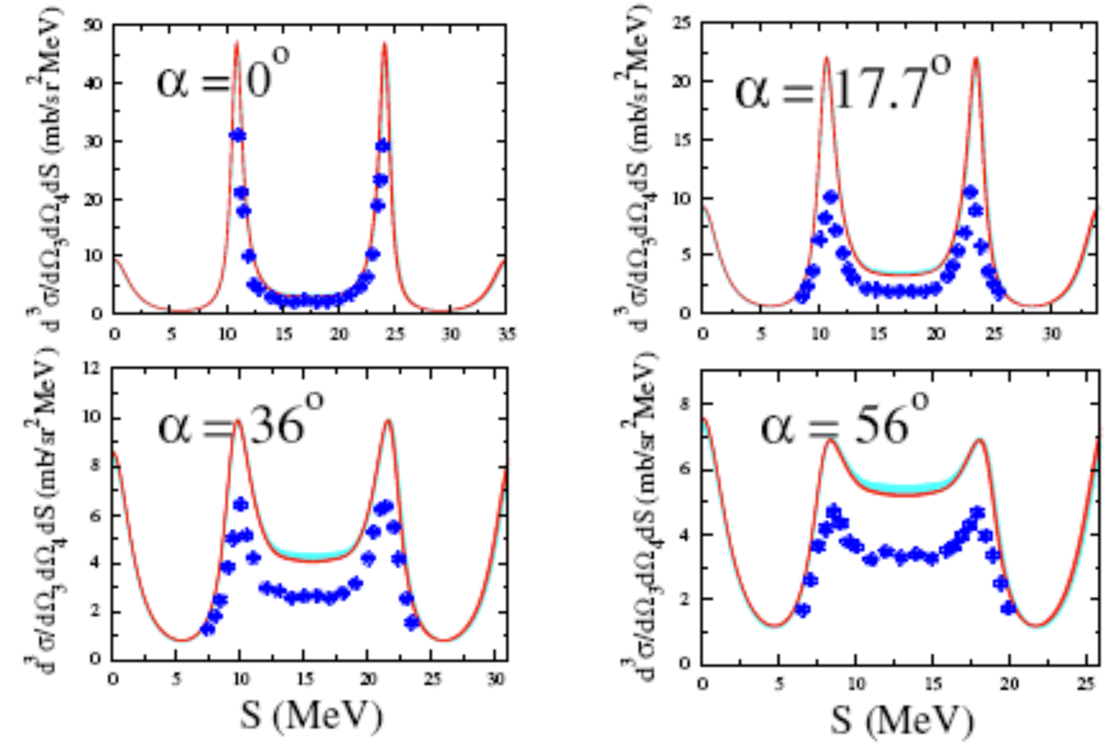
Witała et al., PRC 73 (2006) 044004



Deuteron break-up

SCRE configuration at E_d=19 MeV

Ley et al., PRC 73 (2006) 064001



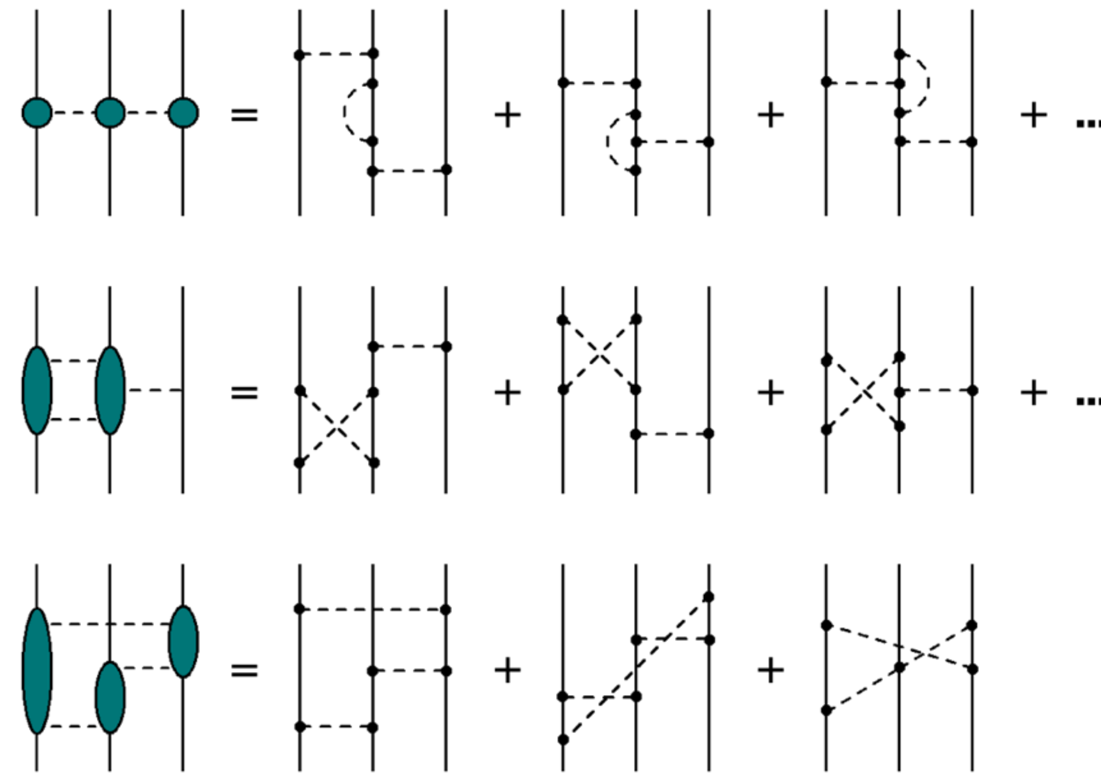
- Promising NNLO results for Nd elastic scattering
- Generally good description of break-up observables except for SCRE/SST break-up configuration at low energy
- Hope for improvement at N^3LO

Three-nucleon forces

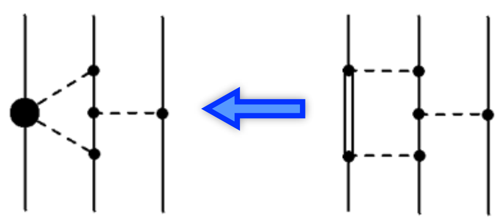
- Three-nucleon forces at N^{3}LO

Long range contributions *Bernard, Epelbaum, H.K., Meißner ´08; Ishikawa, Robilotta ´07*

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important



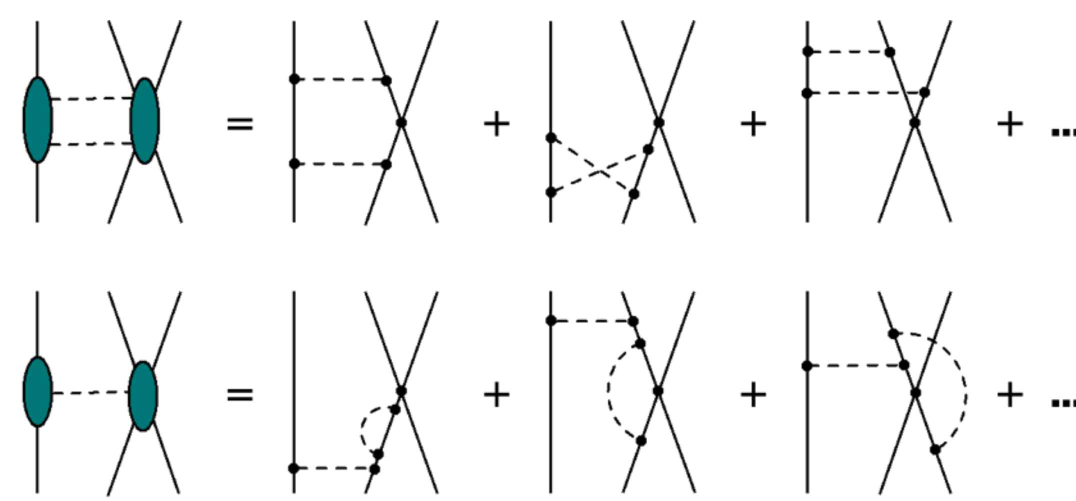
Large c_i $\sim$



Shorter range contributions *Bernard, Epelbaum, H.K., Meißner ´11*

- LECs needed for shorter range contr.

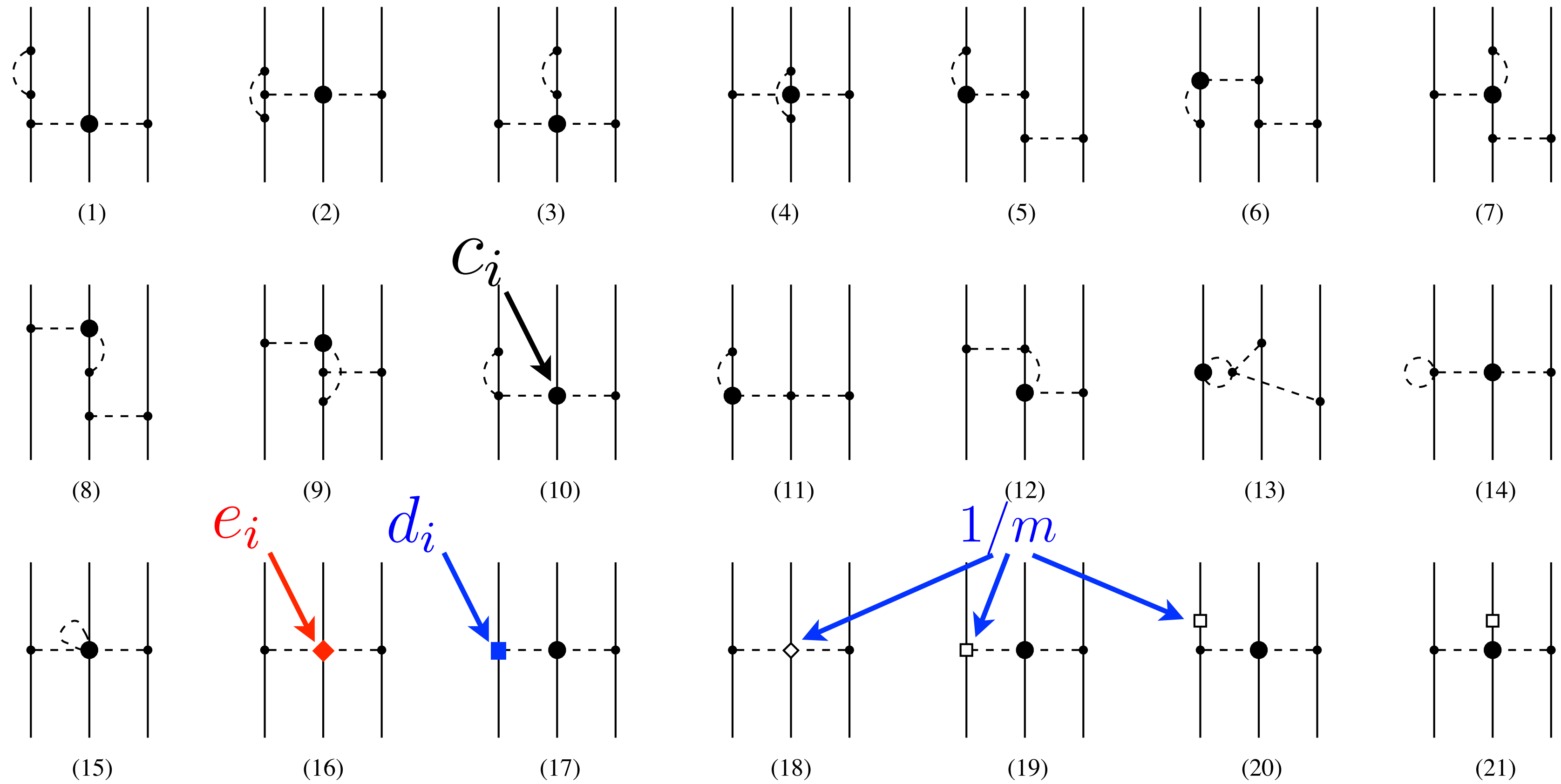
  g_A, F_π, M_π, C_T
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF



Three-nucleon forces

- Three-nucleon forces at N^4LO

Two pion exchange part *Epelbaum, Gasparyan, H.K., ´12*



c_i´s LECs from $\mathcal{L}_{\pi N}^{(2)}$, d_i´s LECs from $\mathcal{L}_{\pi N}^{(3)}$, e_i´s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to πN - scattering data

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

Two-pion-exchange at N^{4}LO

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big) \,,$$

$$\begin{aligned}
\mathcal{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6} \Big[M_\pi^2 q_2^2 \big(F_\pi^2 \left(2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3 \right) \\
&+ g_A (144 c_1 - 53 c_2 - 90 c_3) \big) + M_\pi^4 \left(F_\pi^2 \left(4608\pi^2 \bar{d}_{18} (2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38}) \right) \right. \\
&+ \left. g_A \left(72 \left(64\pi^2 \bar{l}_3 + 1 \right) c_1 - 24 c_2 - 36 c_3 \right) \right) + q_2^4 \left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2 g_A (5c_2 + 18 c_3) \right) \Big] \\
&- \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \left(M_\pi^2 + 2q_2^2 \right) \left(4M_\pi^2 (6c_1 - c_2 - 3c_3) + q_2^2 (-c_2 - 6c_3) \right) \\
\mathcal{B}^{(5)}(q_2) &= -\frac{g_A}{2304\pi^2 F_\pi^6} \Big[M_\pi^2 \left(F_\pi^2 \left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37}) \right) + 108 g_A^3 c_4 + 24 g_A c_4 \right) \\
&+ q_2^2 \left(5 g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A \right) \Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2) \left(4M_\pi^2 + q_2^2 \right)
\end{aligned}$$

Some LECs can be absorbed by shifting c_i´s

$$\begin{aligned}
c_1 &\to c_1 - 2M_\pi^2 \left(\bar{e}_{22} - 4\bar{e}_{38} - \frac{\bar{l}_3 c_1}{F_\pi^2} \right) , \\
c_3 &\to c_3 + 4M_\pi^2 \left(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36} + 2\frac{\bar{l}_3 c_1}{F_\pi^2} \right) , \\
c_4 &\to c_4 + 4M_\pi^2 (2\bar{e}_{21} - \bar{e}_{37}) \,,
\end{aligned}$$

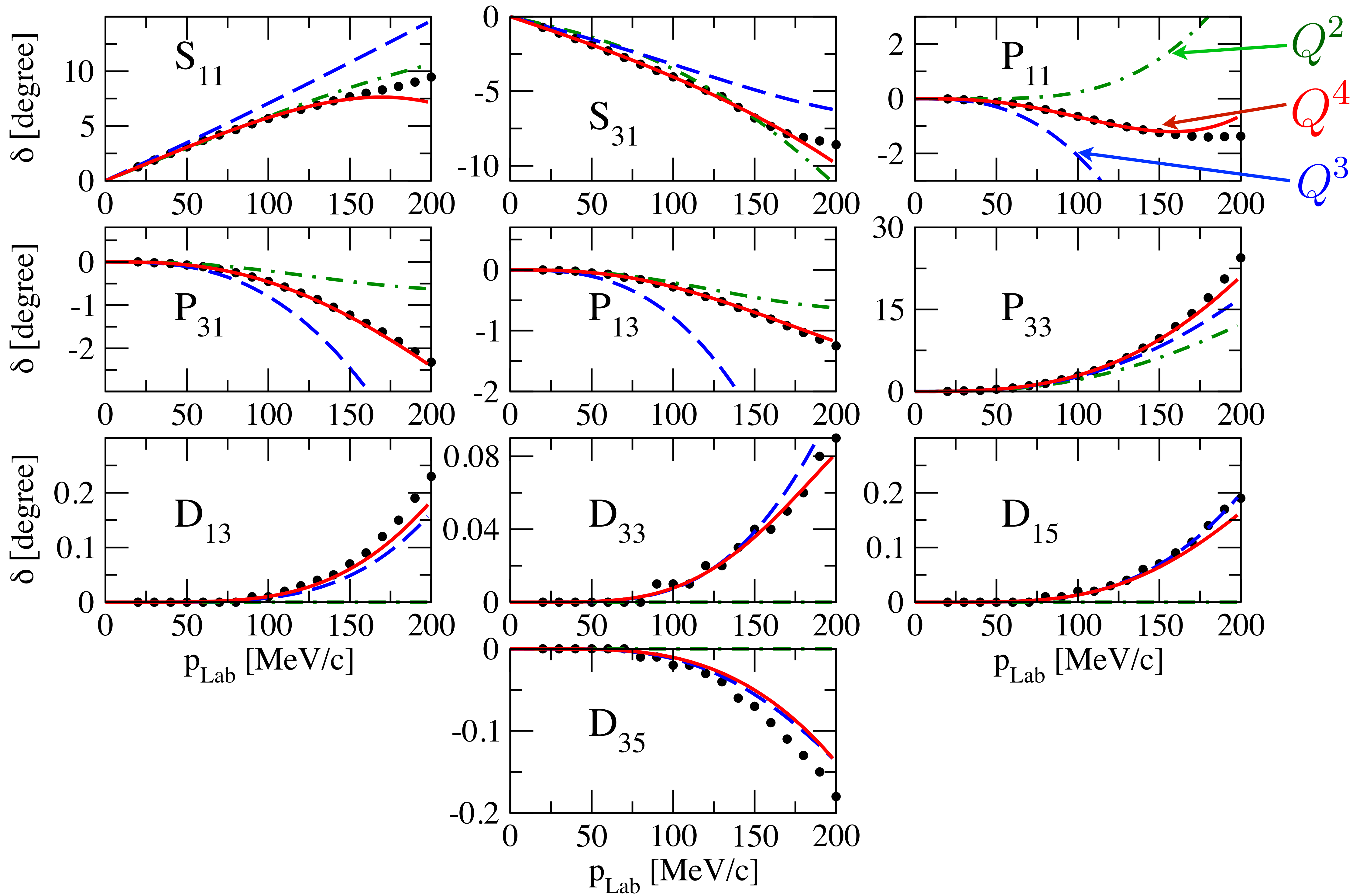
$$L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q} \log \frac{\sqrt{q^2 + 4M_\pi^2} + q}{2M_\pi}$$

- No d_i dependence of TPE-contr. besides d_{18}
- Pion-nucleon scattering does strongly depend on d_i´s

$$g_{\pi NN} = \frac{g_A \, m}{F_\pi} \left(1 - \frac{2M_\pi^2 \, \bar{d}_{18}}{g_A} \right)$$ ⟸ Violation of Goldberger-Treiman relation

KH-Fit to pion-nucleon scattering

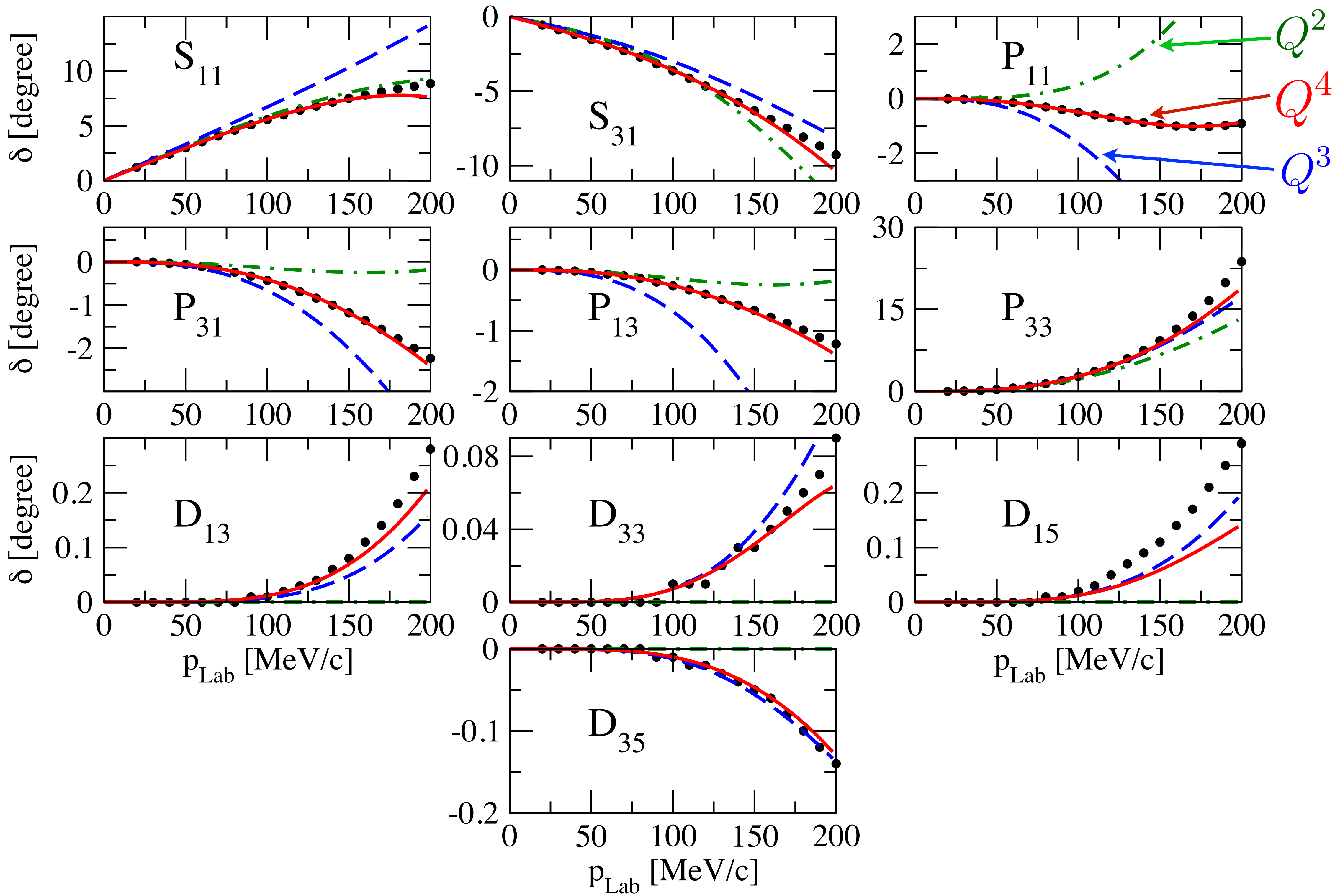
KH-PWA: R. Koch Nucl. Phys. A 448 (1986) 707



Data fitted for $p_{Lab} < 150\,\mathrm{MeV}$

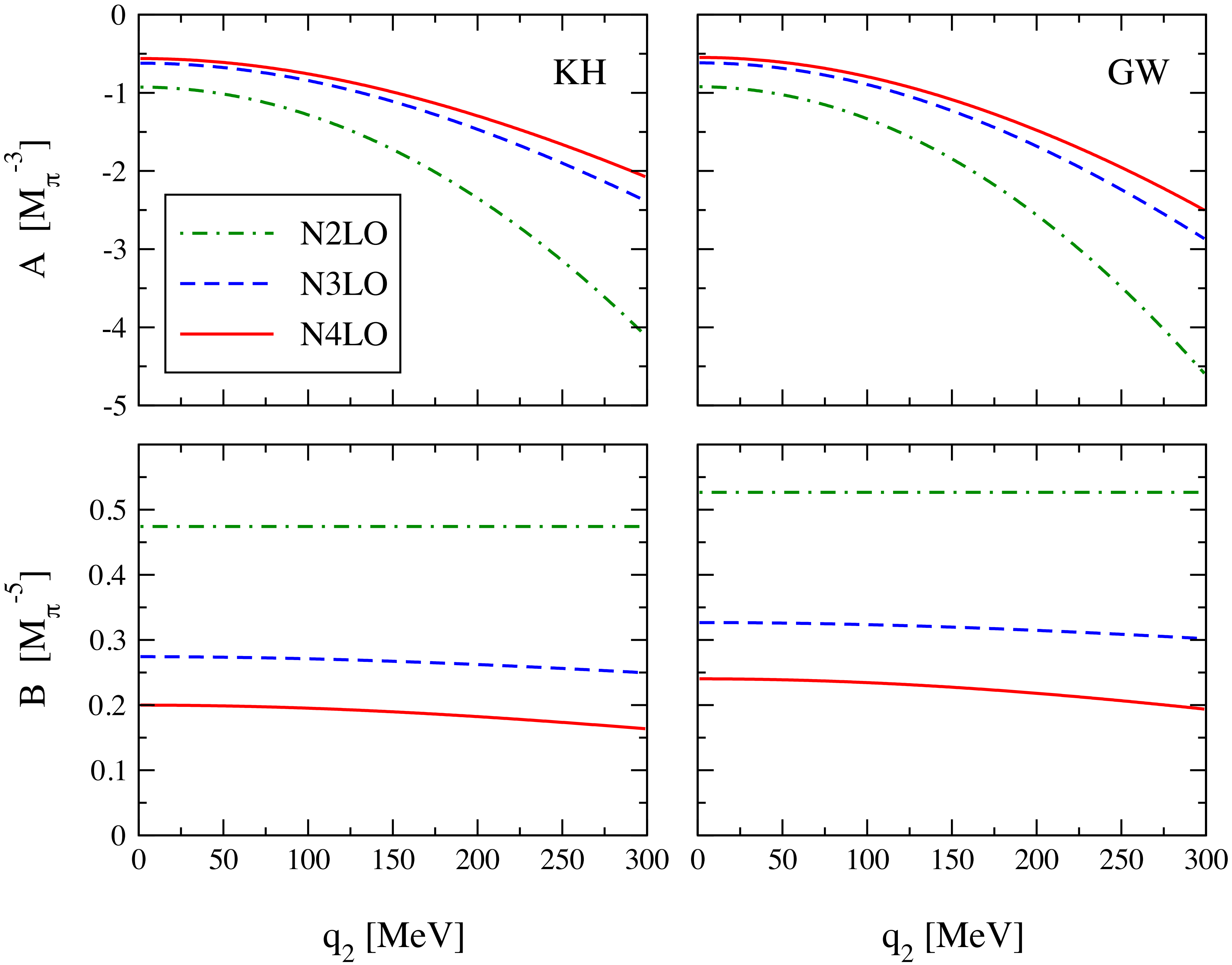
GW-Fit to pion-nucleon scattering

GW-PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205



Data fitted for $p_{\mathrm{Lab}} < 150\,\mathrm{MeV}$

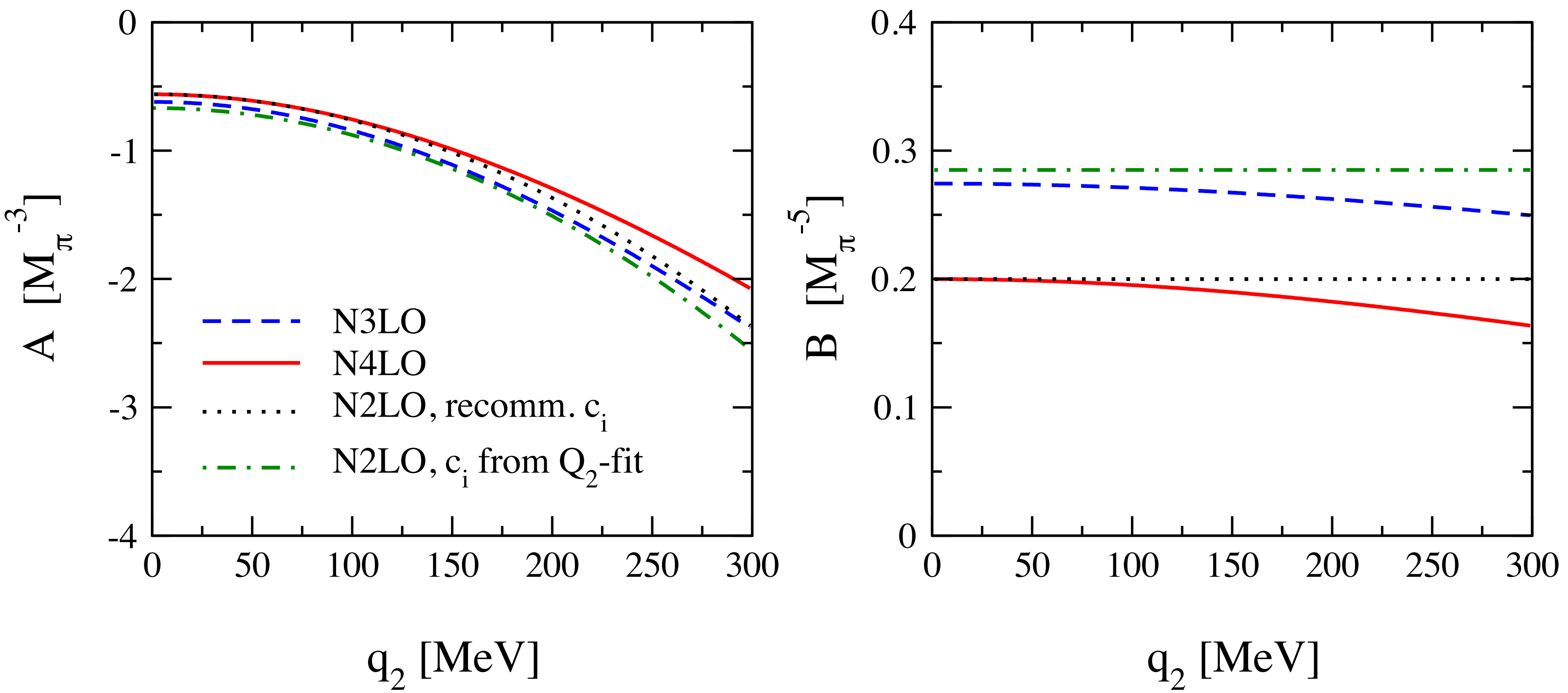
Two-pion-exchange at N^4LO



	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
GW-fit	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
KH-fit	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

- No dependence on d_i´s
- e_i´s are of natural size
- Good convergence of TPE 3NF

Working with N^2LO 3NF



Recommended c_i´s by working with N^2LO 3NF

$$c_1^{\rm KH} = -0.37\,{\rm GeV}^{-1}, \quad c_3^{\rm KH} = -2.71\,{\rm GeV}^{-1}, \quad c_4^{\rm KH} = 1.41\,{\rm GeV}^{-1},$$
$$c_1^{\rm GW} = -0.73\,{\rm GeV}^{-1}, \quad c_3^{\rm GW} = -3.38\,{\rm GeV}^{-1}, \quad c_4^{\rm GW} = 1.69\,{\rm GeV}^{-1}.$$

- With these parameters we get at $q_2 = 0$ the value and slope of N^4LO result
- c_i´s fitted to pion-nucleon Q^2(KH-fit) lead to slightly different results for B-function

$$c_1 = -0.25\,{\rm GeV}^{-1}, \quad c_2 = 2.02\,{\rm GeV}^{-1}, \quad c_3 = -2.80\,{\rm GeV}^{-1}, \quad c_4 = 2.01\,{\rm GeV}^{-1}.$$

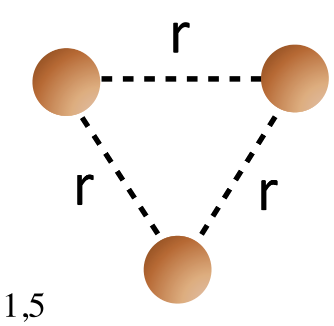
Two-pion-exchange up to N^{4}LO

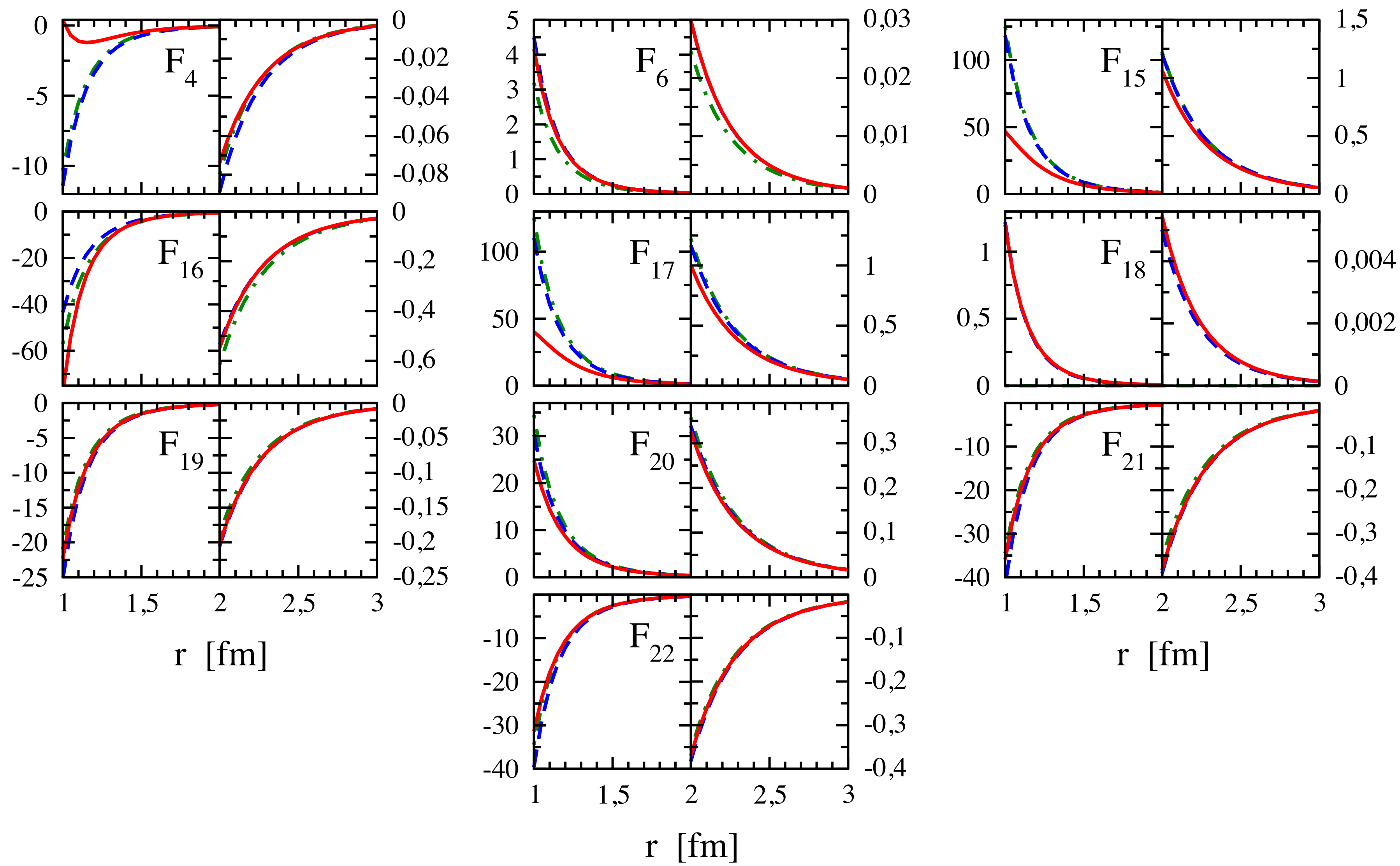
Epelbaum, Gasparyan, H.K., in preparation

Most general, local 3NF involves 89 operators, can be generated (by perm.) from 22 structures

$$V_{3N}^{\rm loc.} = \sum_{i=1}^{22} \mathcal{G}_i \, F_i(r_{12}, r_{23}, r_{31}) + 5 \, \text{perm.} \quad \text{with} \quad \mathcal{G}_1 = 1 \,, \quad \mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \,, \quad \mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3 , \quad \ldots$$

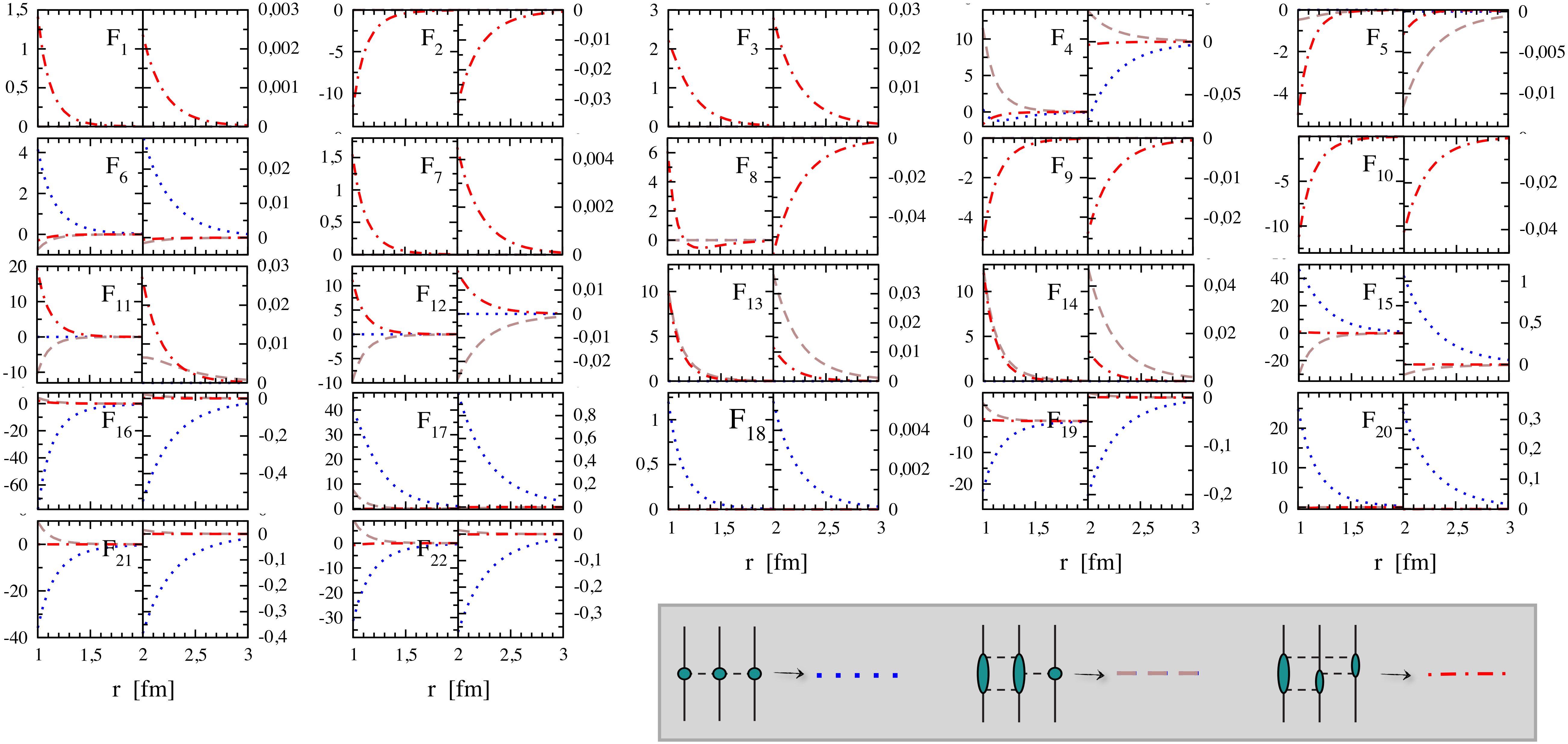
Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration





Complete long-range 3NF up to N^{4}LO

Epelbaum, Gasparyan, H.K., in preparation



- Predictions based entirely on chiral symmetry + input from πN, benchmarks for lattice-QCD
- Implications for Nd, light nuclei & nuclear matter? *(work in progress ...)*
- $2\pi - 1\pi$ and ring-topology: already converged? ChPT with explicit Δ´s *(work in progress ...)*

Summary

- Chiral nuclear forces are analyzed up to N^3LO
- Long-range part of chiral three-nucleon forces is analyzed up to N^4LO
- In general there are 89 spin-isospin structures in local 3NF's built out of 22 + perm.
- Two-pion-exchange part dominates 3NF but does not fill all 22 structures
- With two-pion-one-pion-exchange and ring diagrams all 22 structures are filled

Outlook

- Partial wave decomposition of N^3LO three-nucleon forces
- Complete study of 3NF and 4NF up to N^4LO with explicit delta-isobar
- Implementations in Nd, light nuclei & nuclear matter