

Two- and Three-body hyperon-nucleon interactions.

Are we missing something?

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Collaborators:

- D. Lonardoni (Trento)
- P. Armani (Trento)
- S. Gandolfi (L.A.N.L.)
- G. Co' (Lecce)



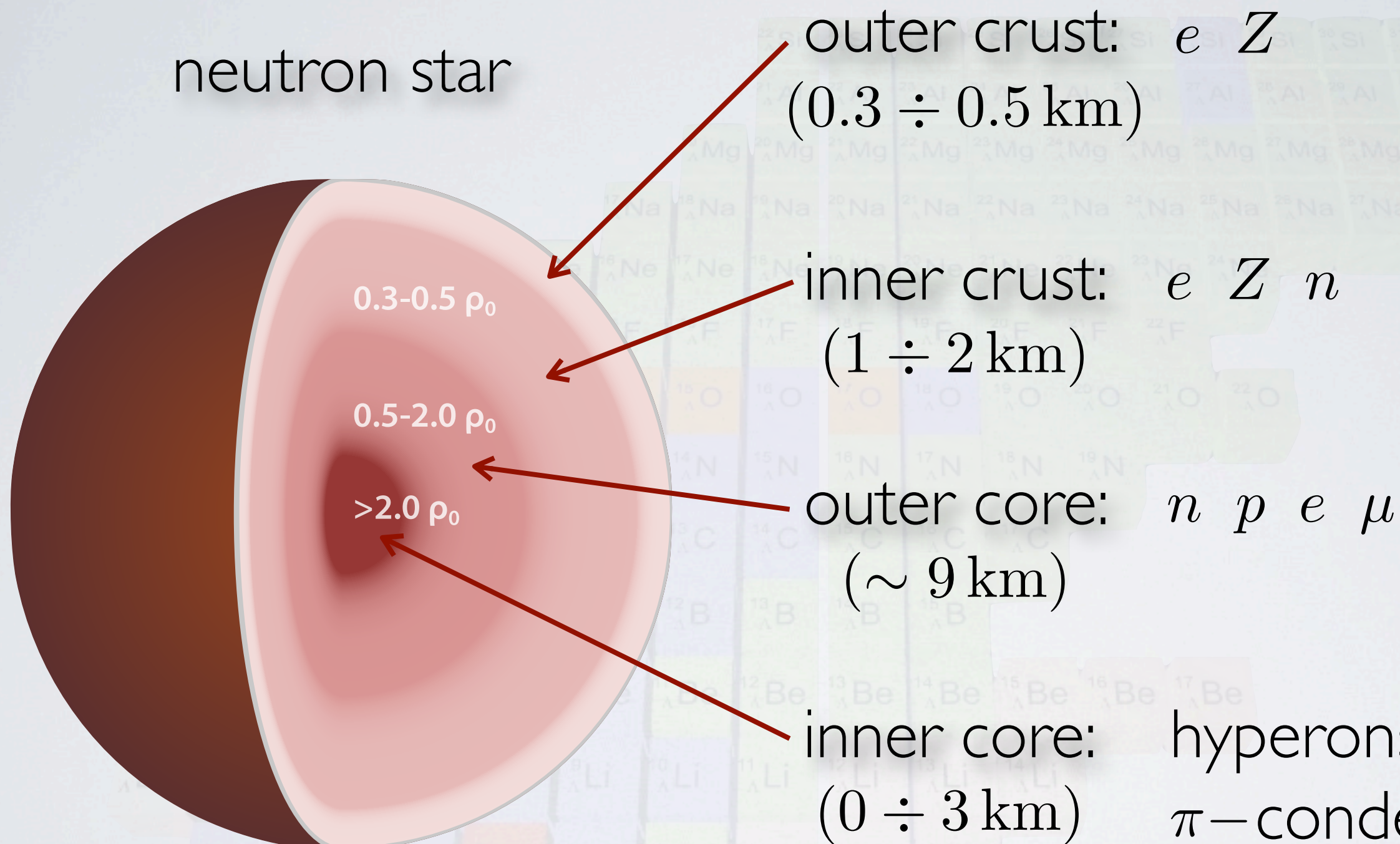
Outline

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1. On hyperons in nuclei & nuclear matter
 - astrophysical interest
 - experimental status
 - hyperon-nucleon interactions
2. The idea & the method
 - AFDMC for nuclei & hypernuclei
3. Results
4. Conclusions

On hyperons in nuclei & nuclear matter

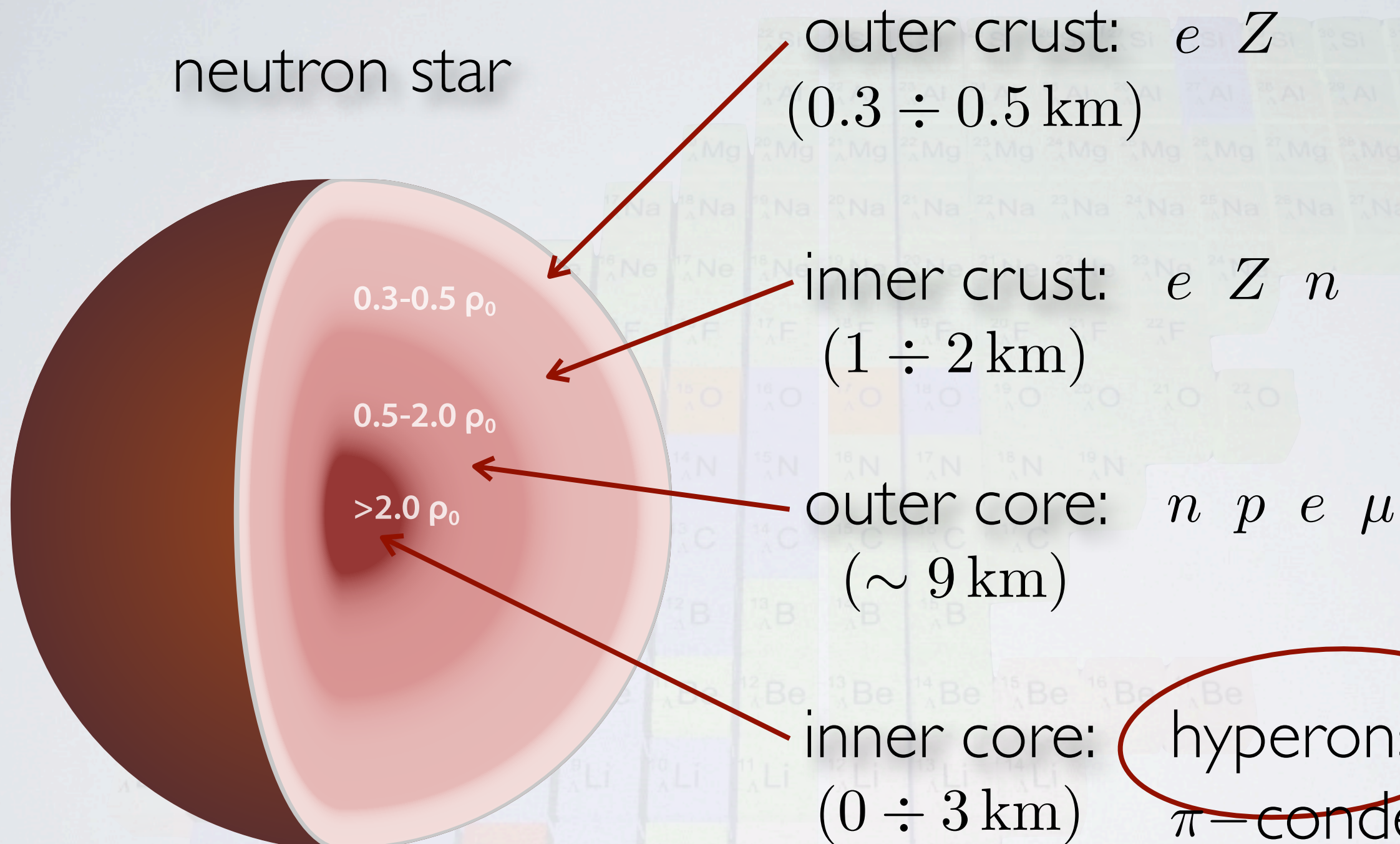
neutron star



$R \sim 10$ km
 $M \sim 1.4 M_{\odot}$

On hyperons in nuclei & nuclear matter

neutron star



$R \sim 10$ km
 $M \sim 1.4 M_{\odot}$

inner core:
($0 \div 3$ km)

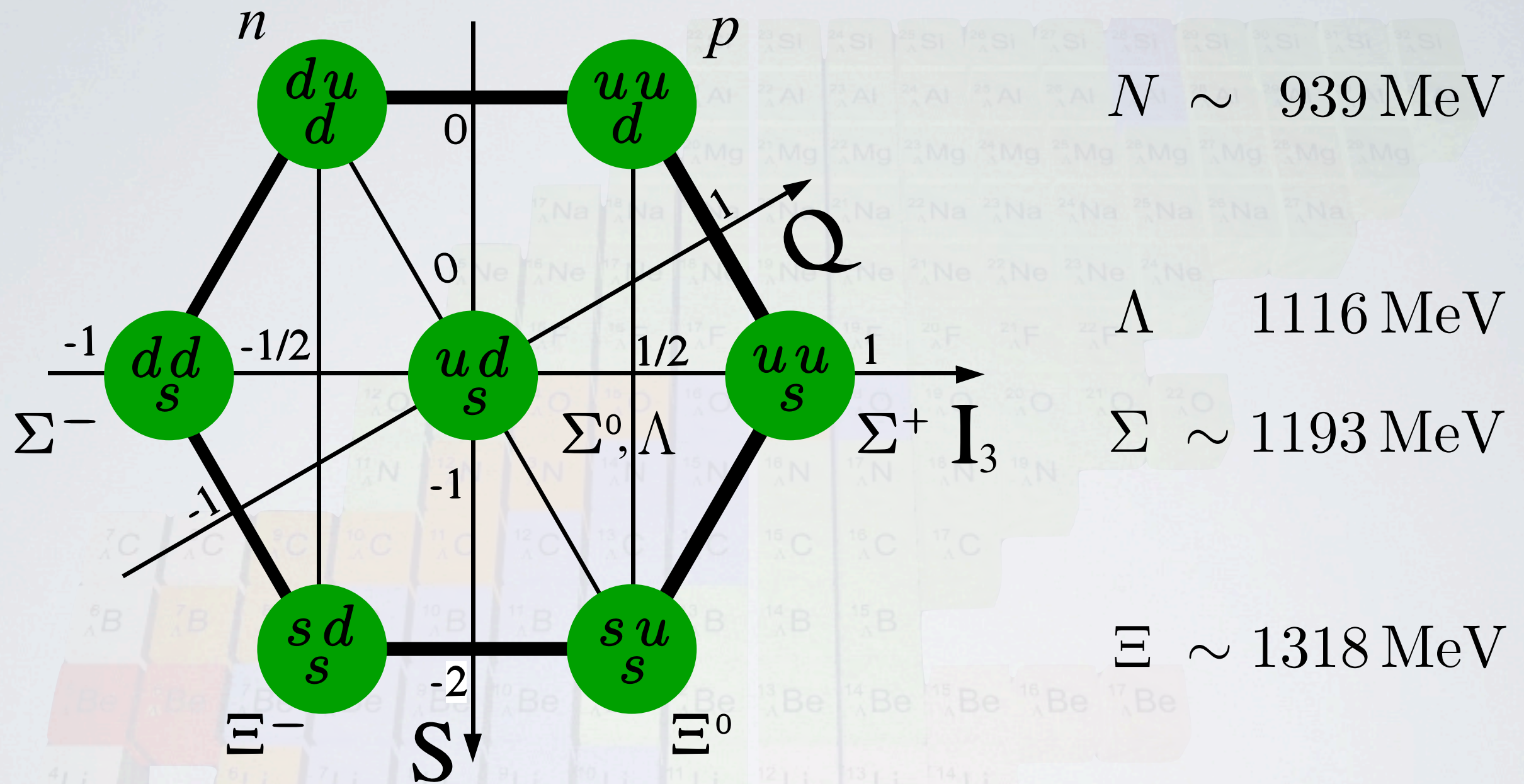
hyperons ?

π -condensate ?

K -condensate ?

quark plasma ?

On hyperons in nuclei & nuclear matter

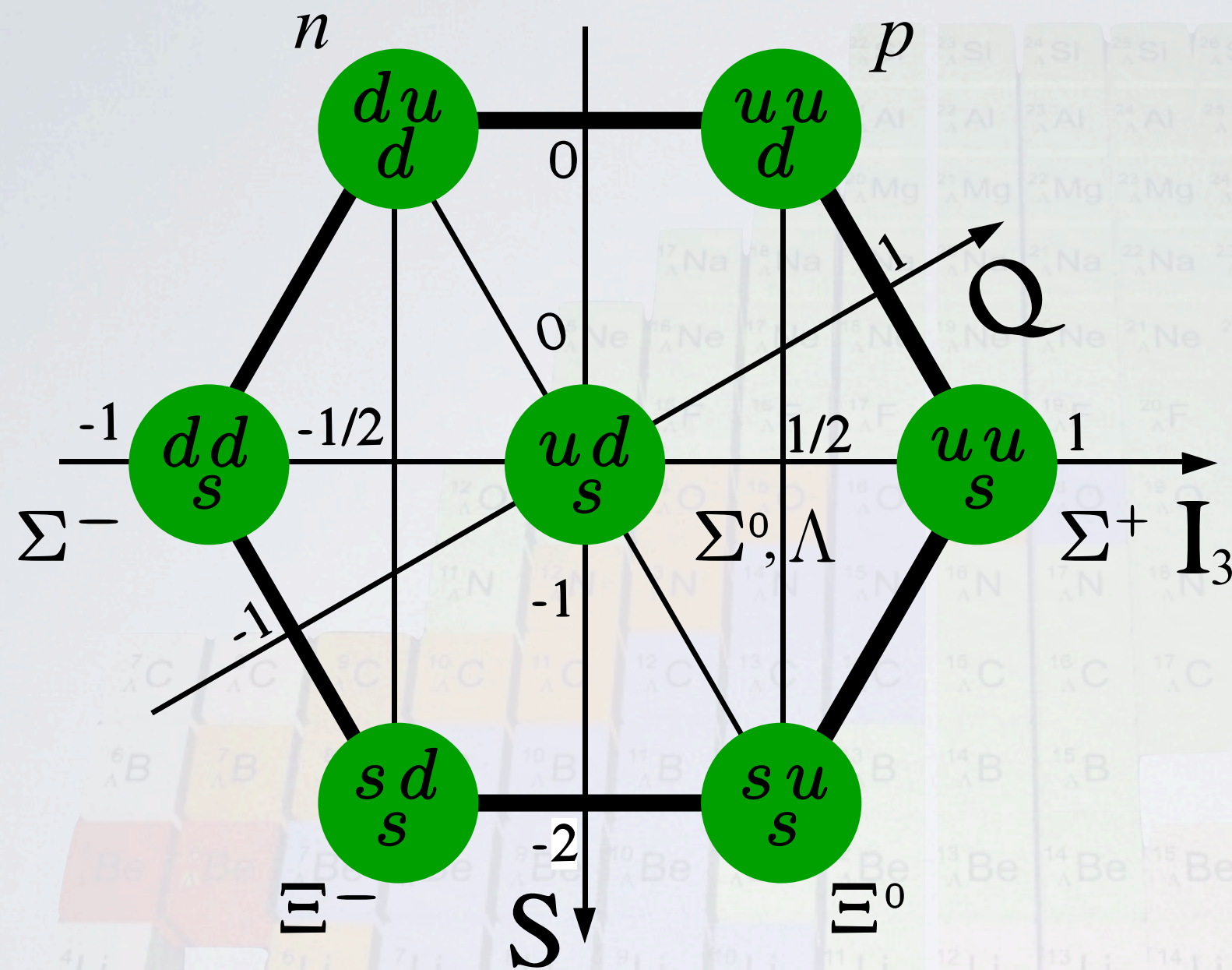


single Λ
hypernucleus

double Λ
hypernucleus



On hyperons in nuclei & nuclear matter



$$N \sim 939 \text{ MeV}$$

$$\Lambda \quad 1116 \text{ MeV}$$

$$\Sigma \sim 1193 \text{ MeV}$$

$$\Xi \sim 1318 \text{ MeV}$$



single Λ
hypernucleus

double Λ
hypernucleus



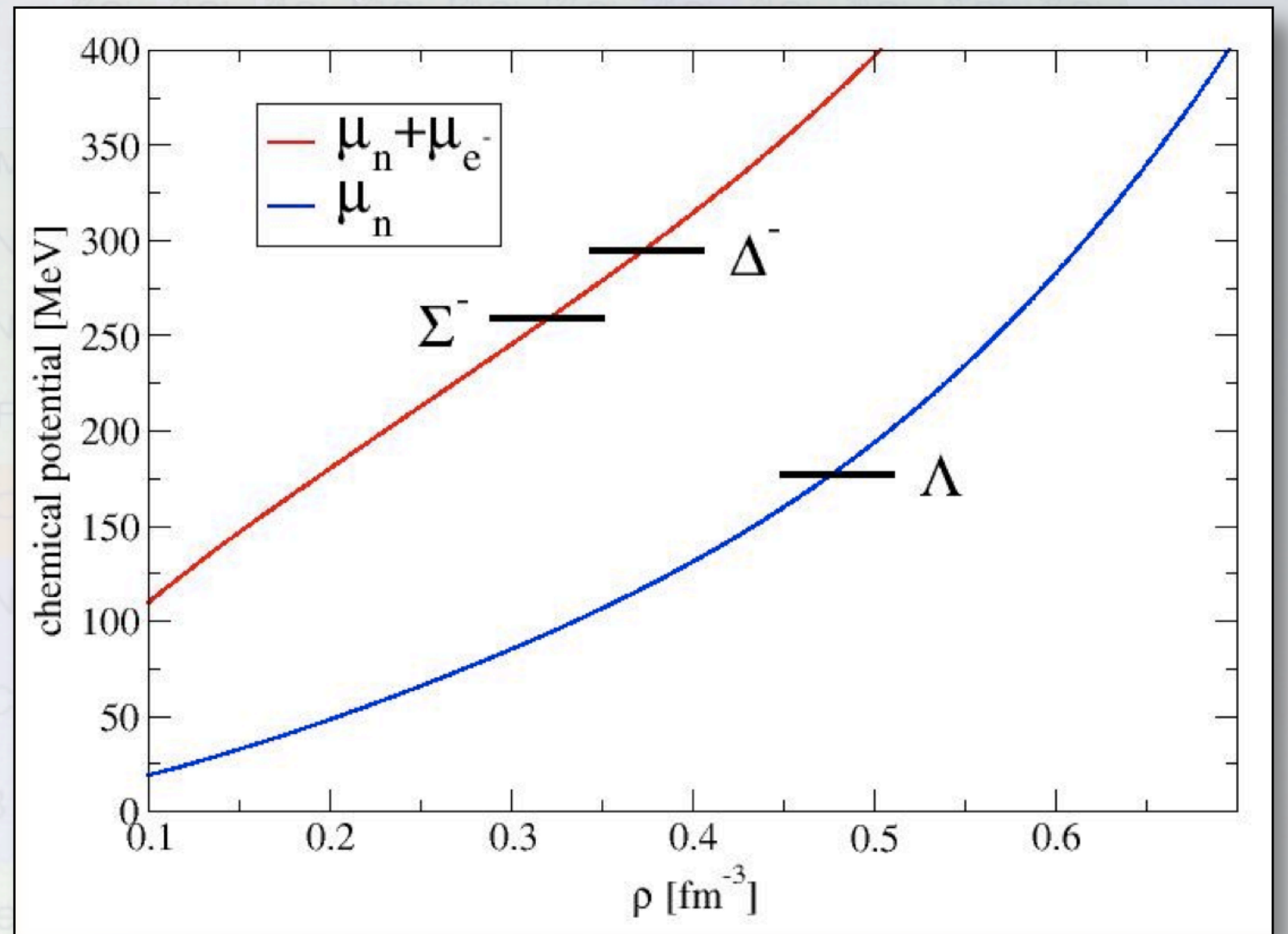
Neutron stars: appearance of hyperons

β equilibrium:

$$Q = -1 : \mu_{Y^-} = \mu_n + \mu_e$$

$$Q = 0 : \mu_{Y^0} = \mu_n$$

$$Q = +1 : \mu_{Y^+} = \mu_n - \mu_e$$



higher mass $\longrightarrow$ lower kinetic energy



softening of the EOS

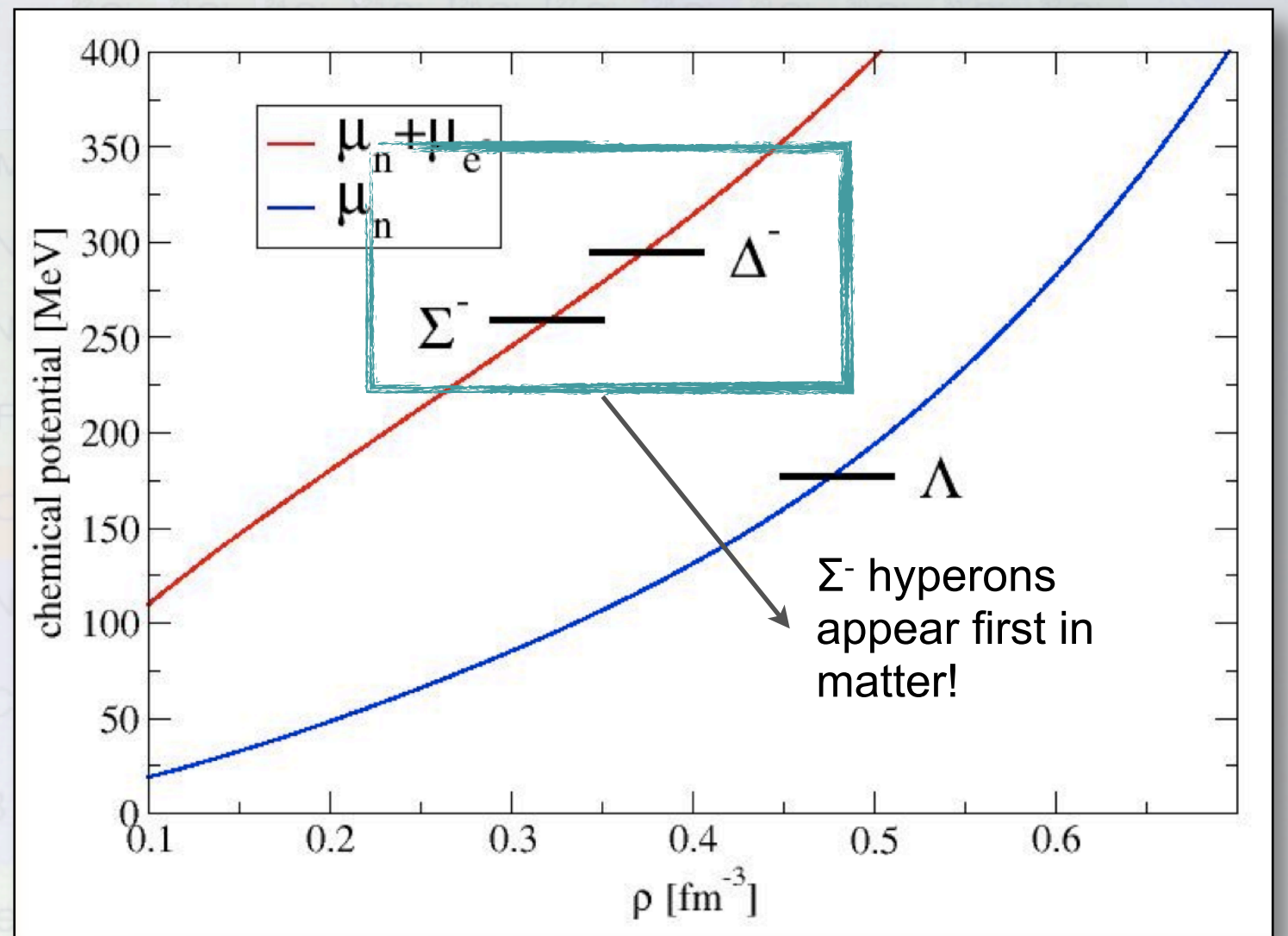
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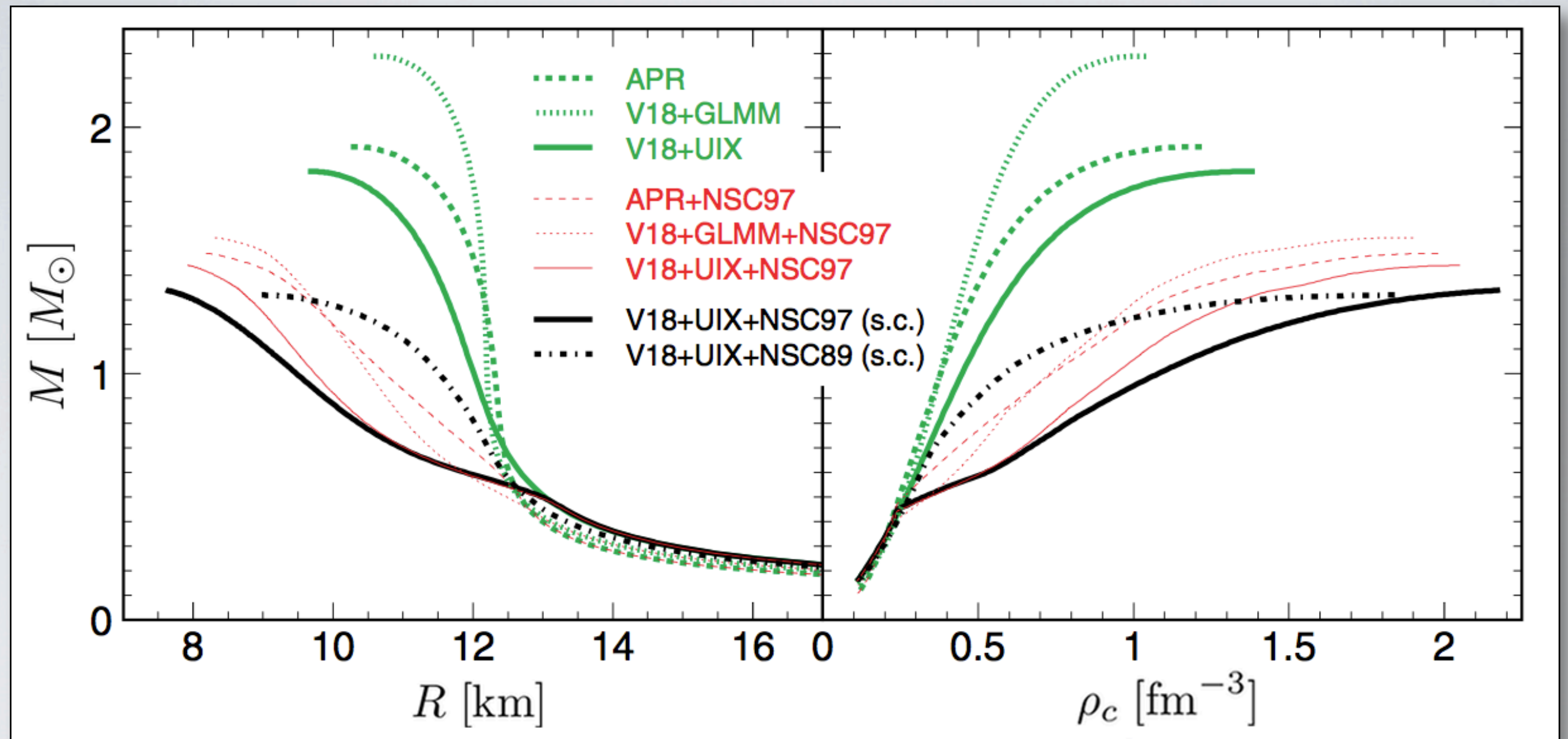


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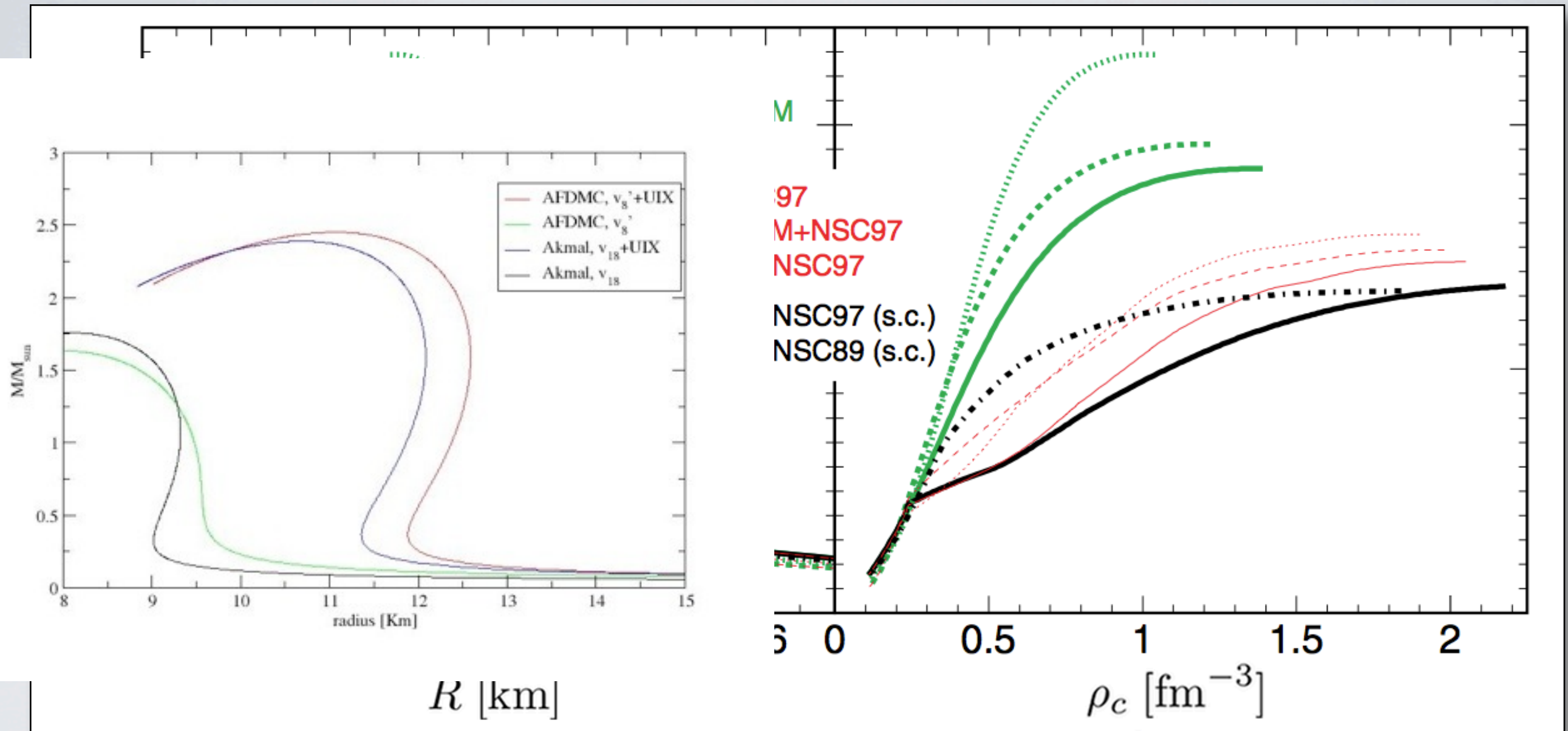
Neutron stars: maximum mass



H. -J. Schulze, A. Polls, A. Ramos and I. Vidaña. Maximum mass of neutron stars.
Phys. Rev. C, 73(5): 058801, May 2006

M_{max} not sensitive to the details of nuclear interaction

Neutron stars: maximum mass

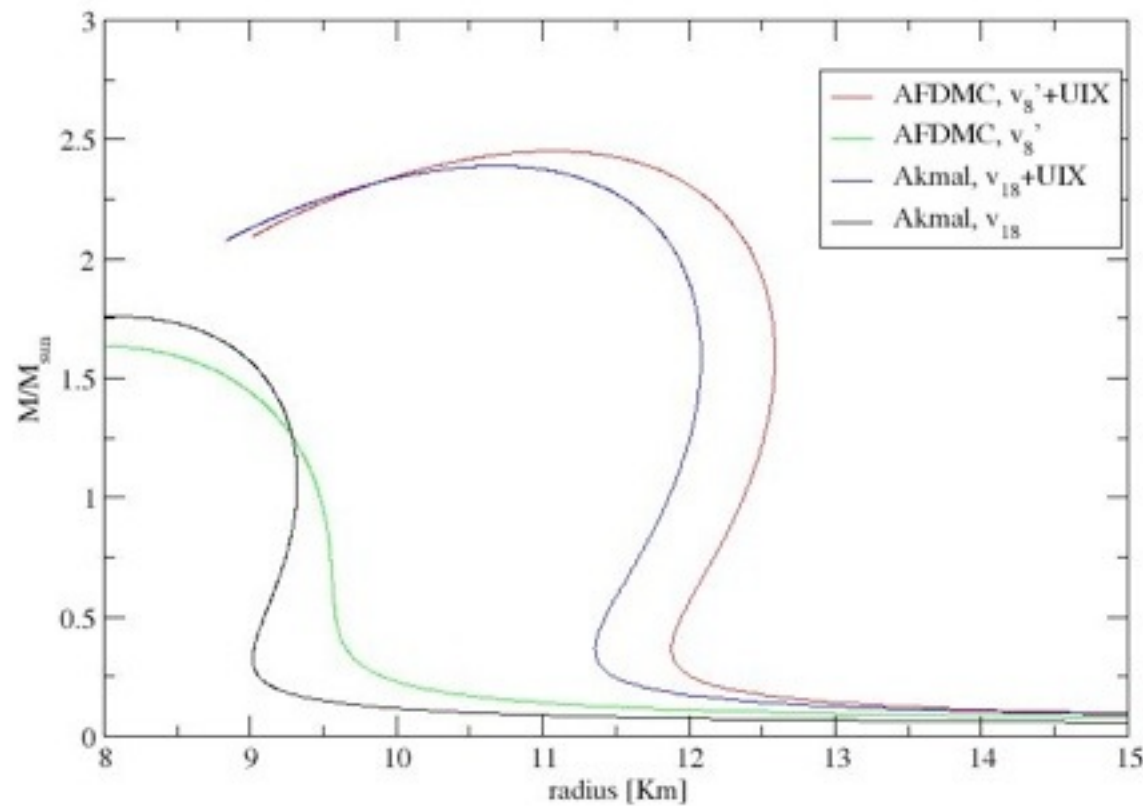


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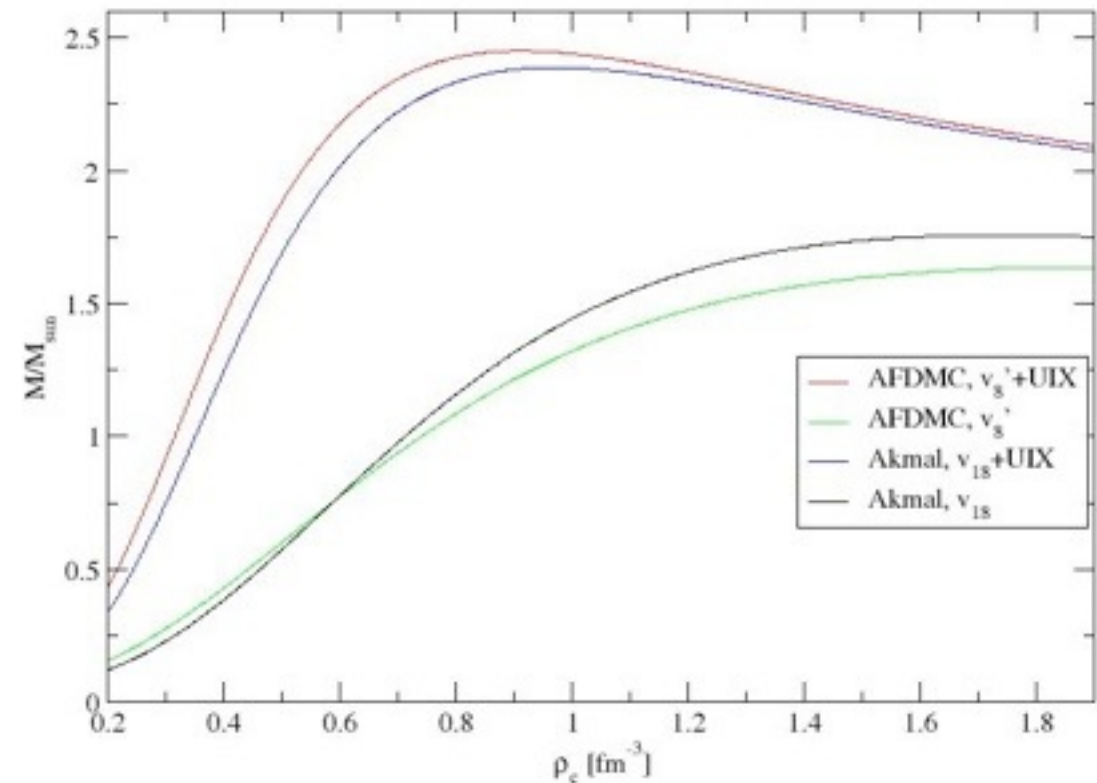
M_{max} not sensitive to the details of nuclear interaction

Neutron stars: maximum mass

S. Gandolfi, A. Yu. Illarionov, K. E. Schmidt, F. P. , and S. Fantoni
Phys. Rev. C **79**, 054005 (2009)



R [km]

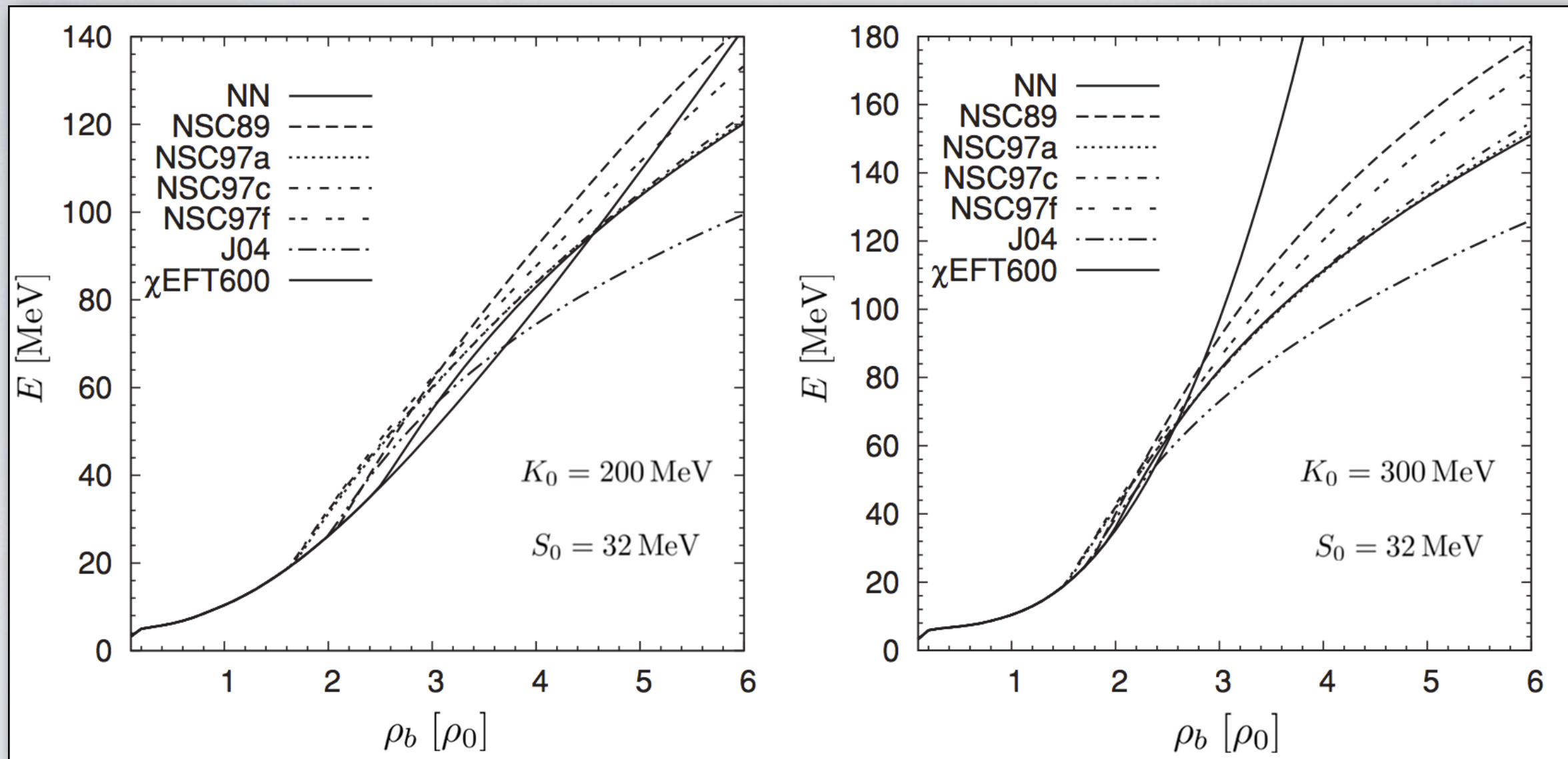


ρ_c [fm $^{-3}$]

H. -J. Schulze, A. Polls, A. Ramos and I. Vidaña. Maximum mass of neutron stars.
Phys. Rev. C, 73(5): 058801, May 2006

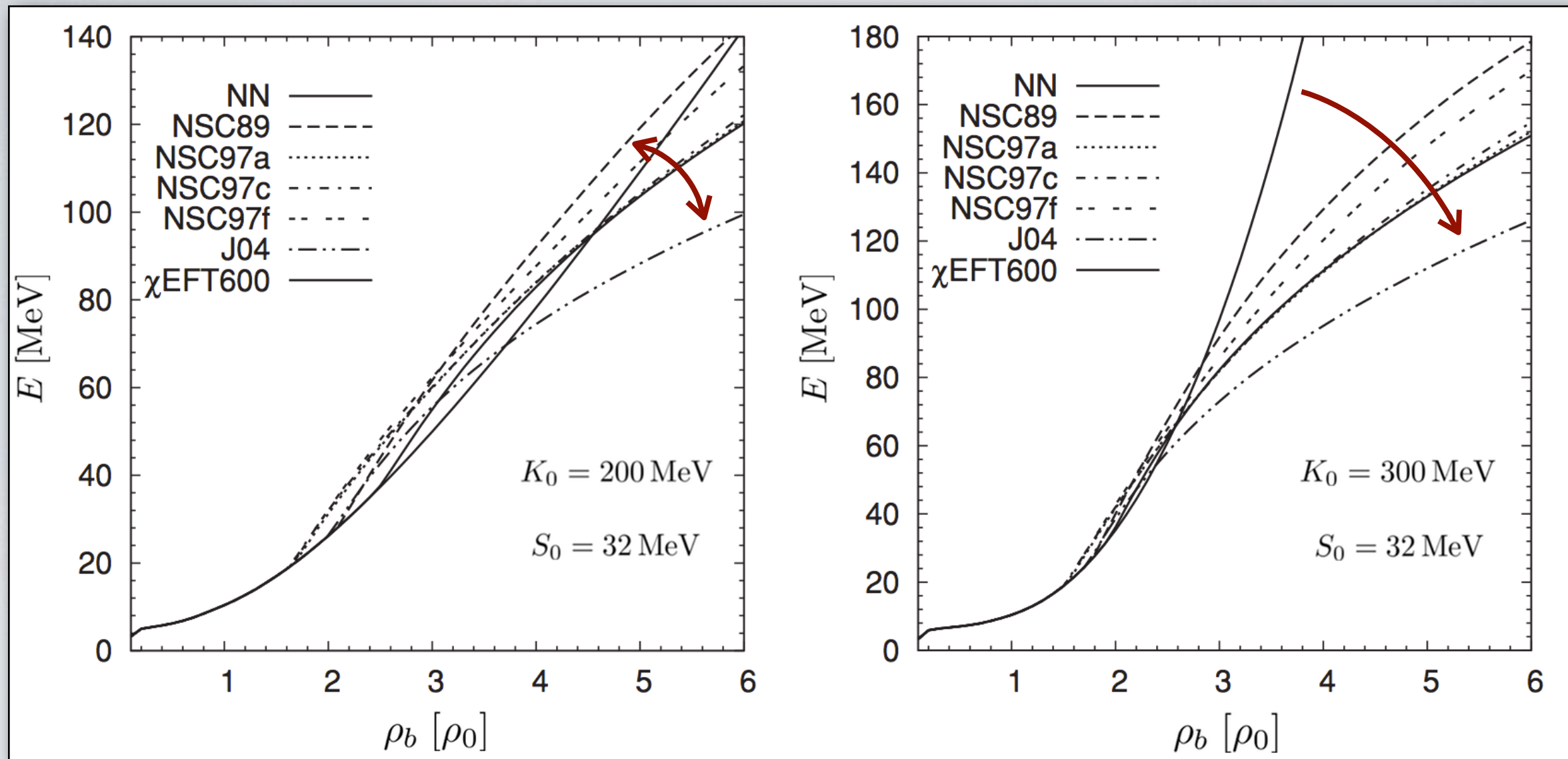
M_{max} not sensitive to the details of nuclear interaction

On hyperons in nuclei & nuclear matter



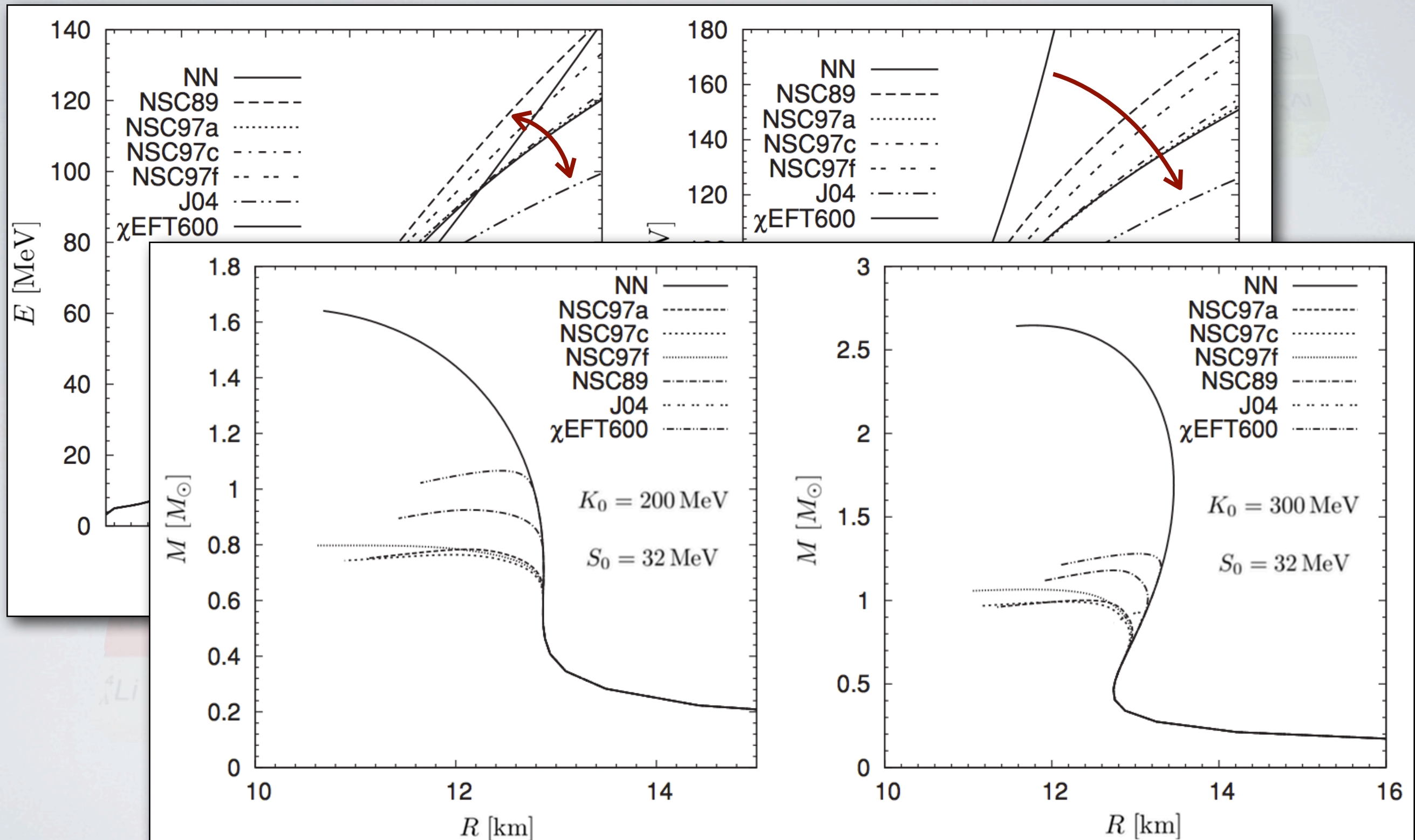
H. Ćapo, B.-J. Schaefer, and J. Wambach. Appearance of hyperons in neutron stars.
Phys. Rev. C, 81 (3): 035803, Mar 2010

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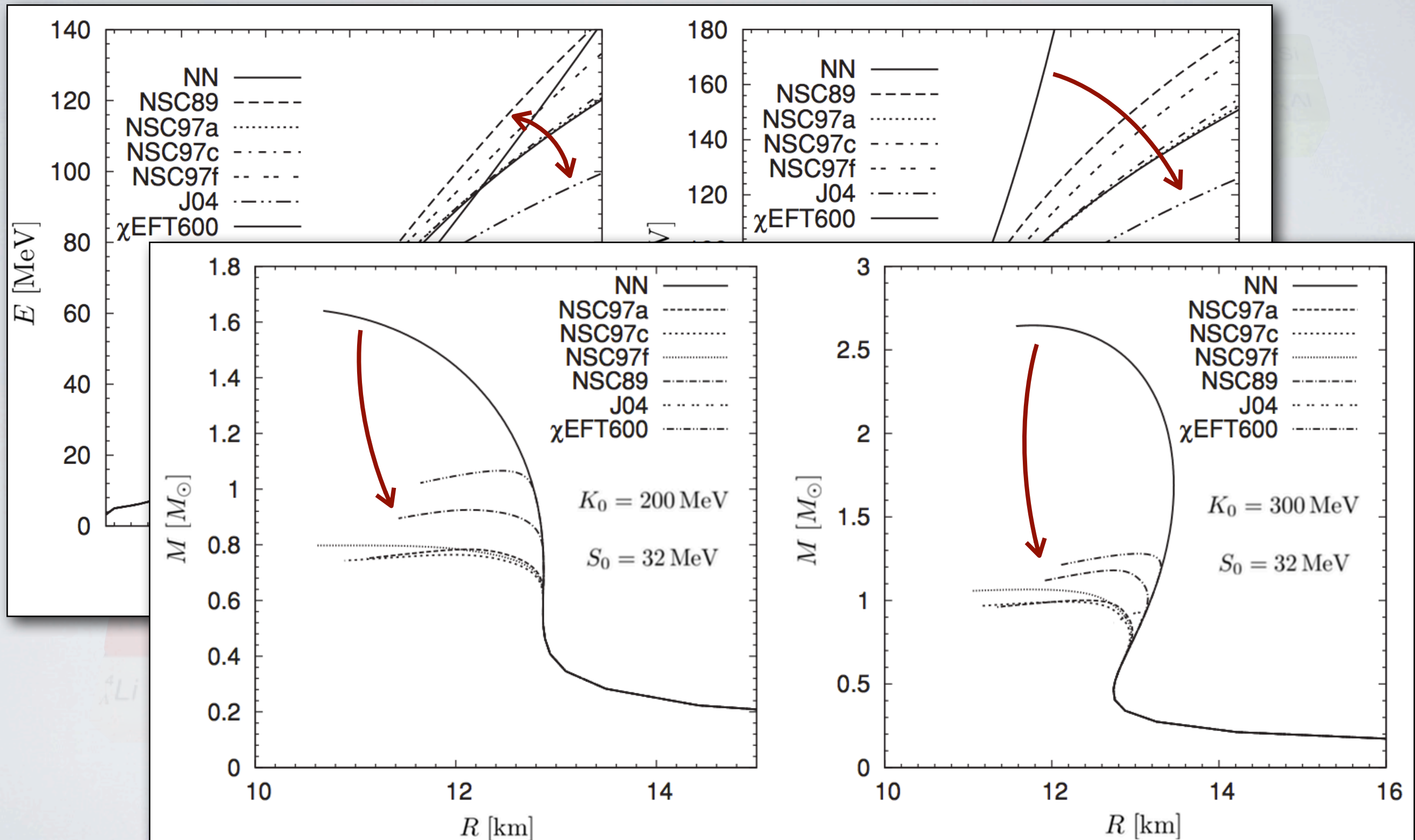
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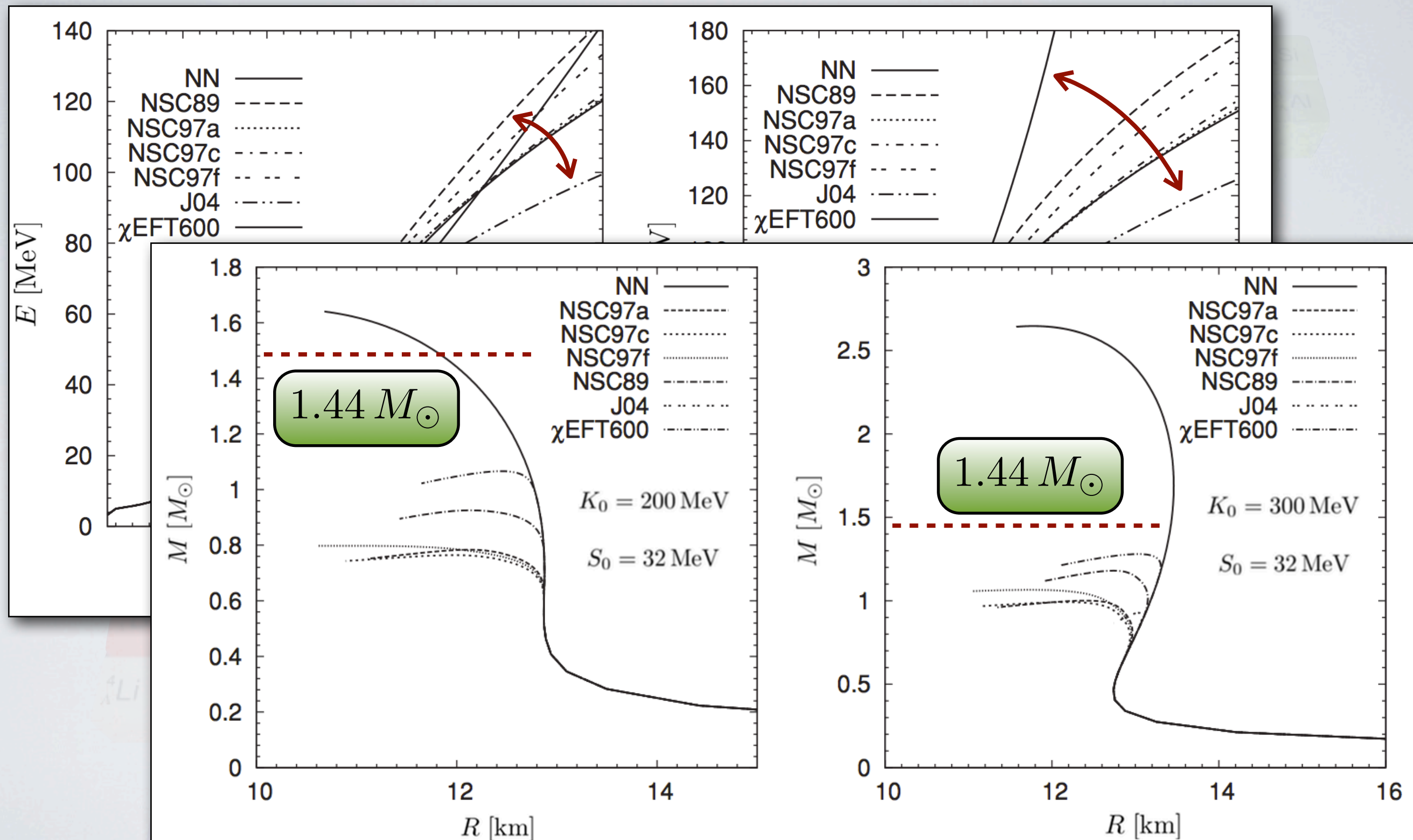
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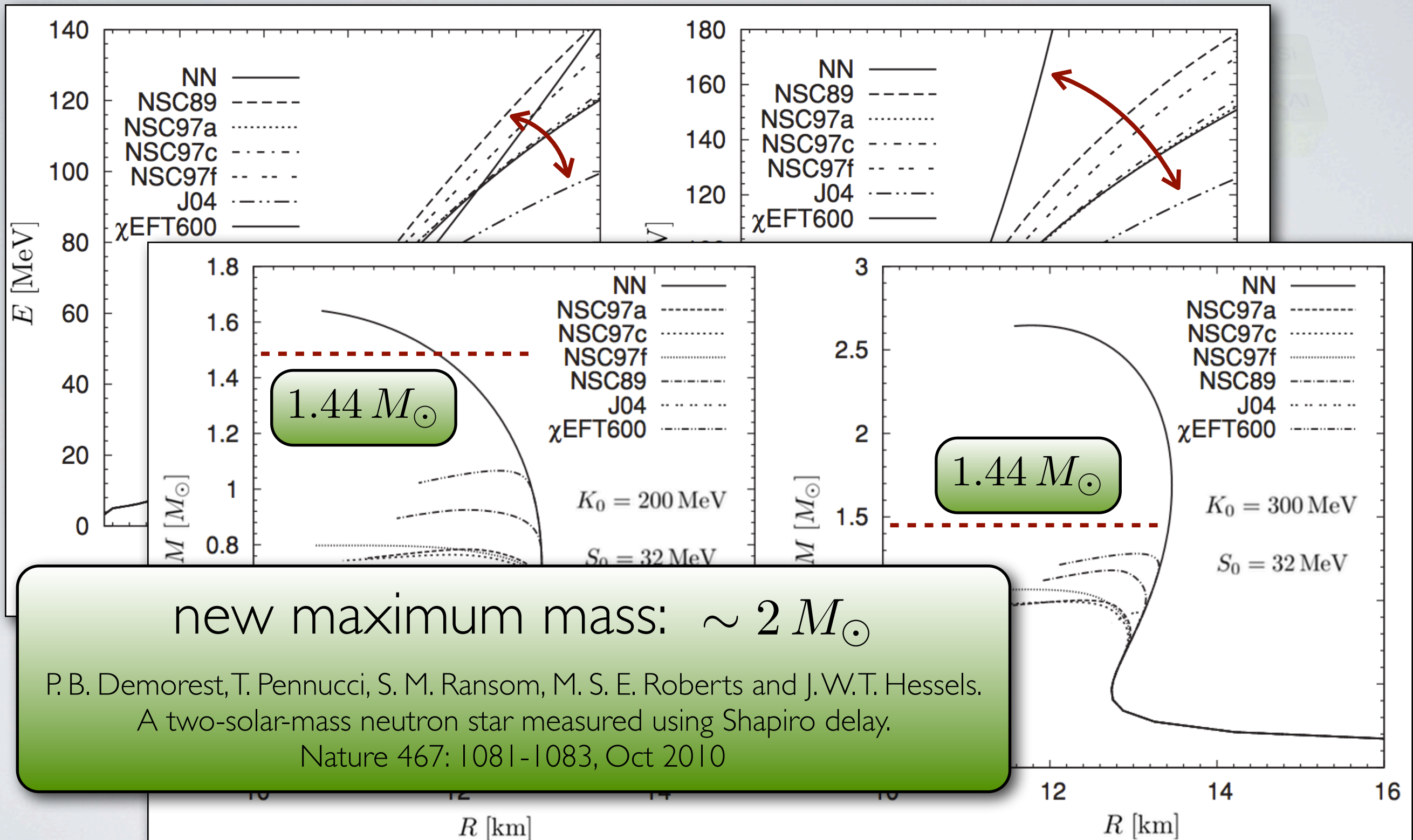
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Phys. Rev. C, 81 (3): 035803, Mar 2010

On hyperons in nuclei & nuclear matter



new maximum mass: $\sim 2 M_\odot$

P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts and J. W. T. Hessels.

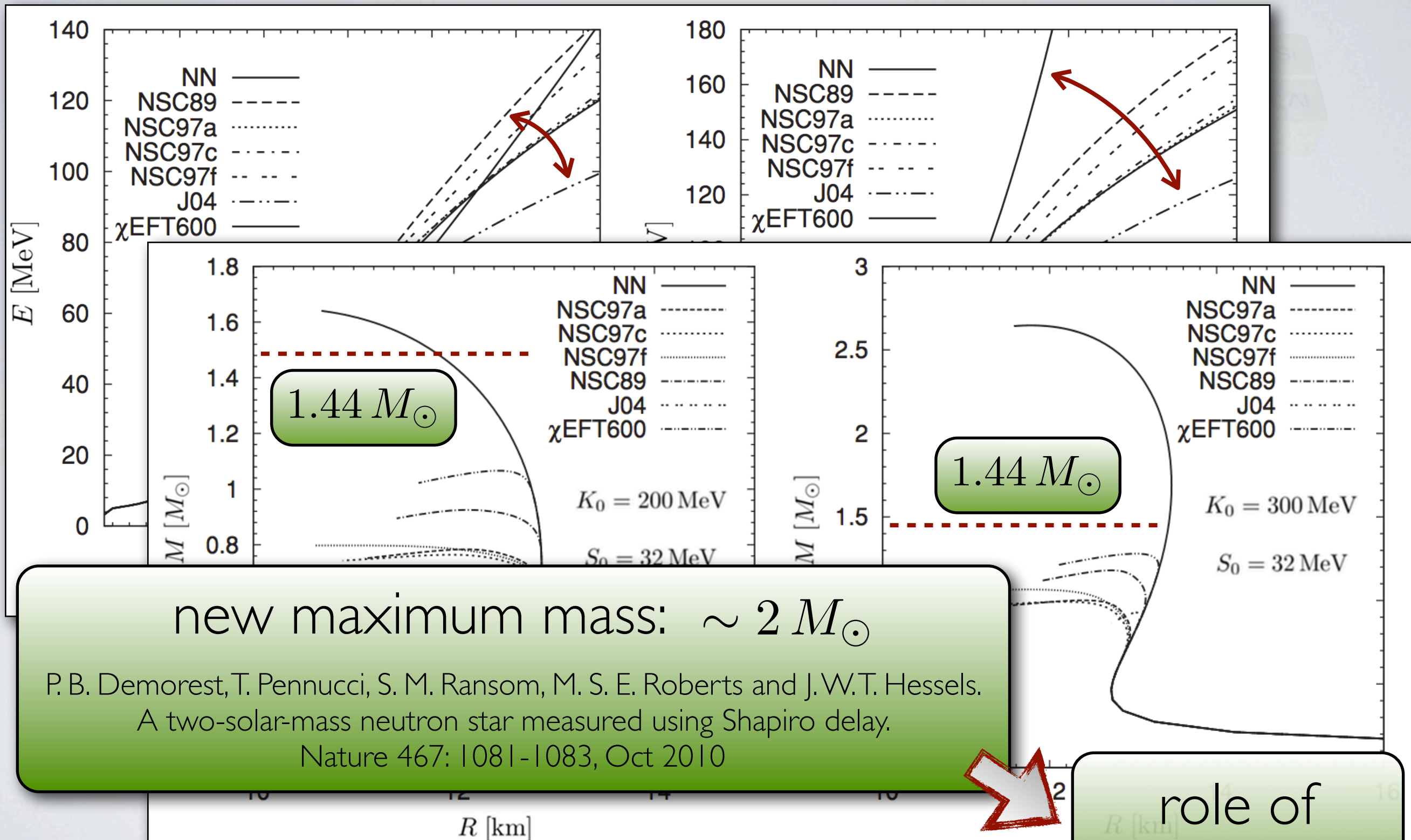
A two-solar-mass neutron star measured using Shapiro delay.

Nature 467: 1081-1083, Oct 2010

H. Dapo, B.-J. Schaefer, and J. Wambach. Appearance of hyperons in neutron stars.

Phys. Rev. C, 81 (3): 035803, Mar 2010

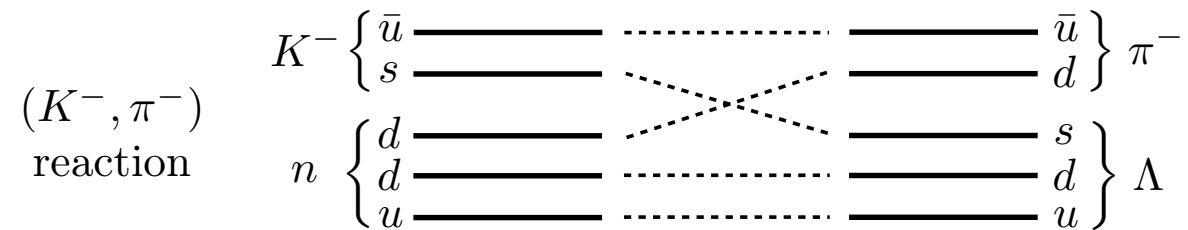
On hyperons in nuclei & nuclear matter



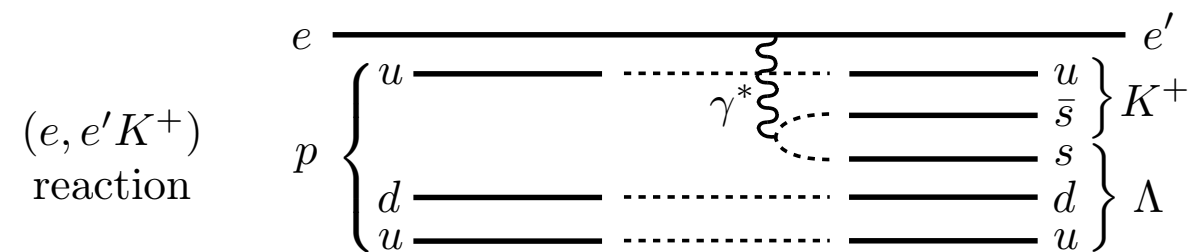
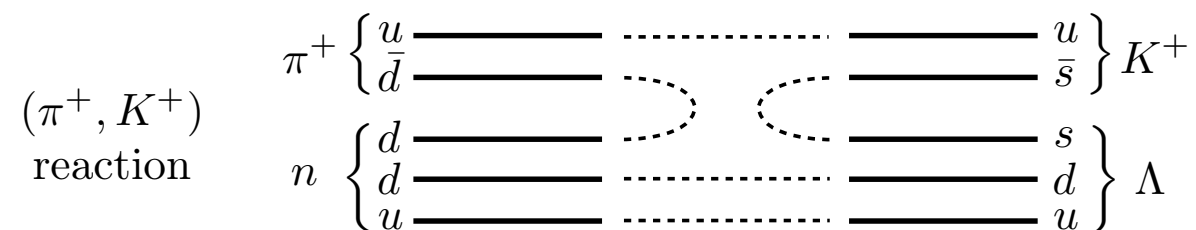
H. Dapo, B.-J. Schaefer, and J. Wambach. Appearance of hyperons in neutron stars.
Phys. Rev. C, 81 (3): 035803, Mar 2010

On hyperons in nuclei & nuclear matter

Strangeness exchange reaction



Associated production reaction

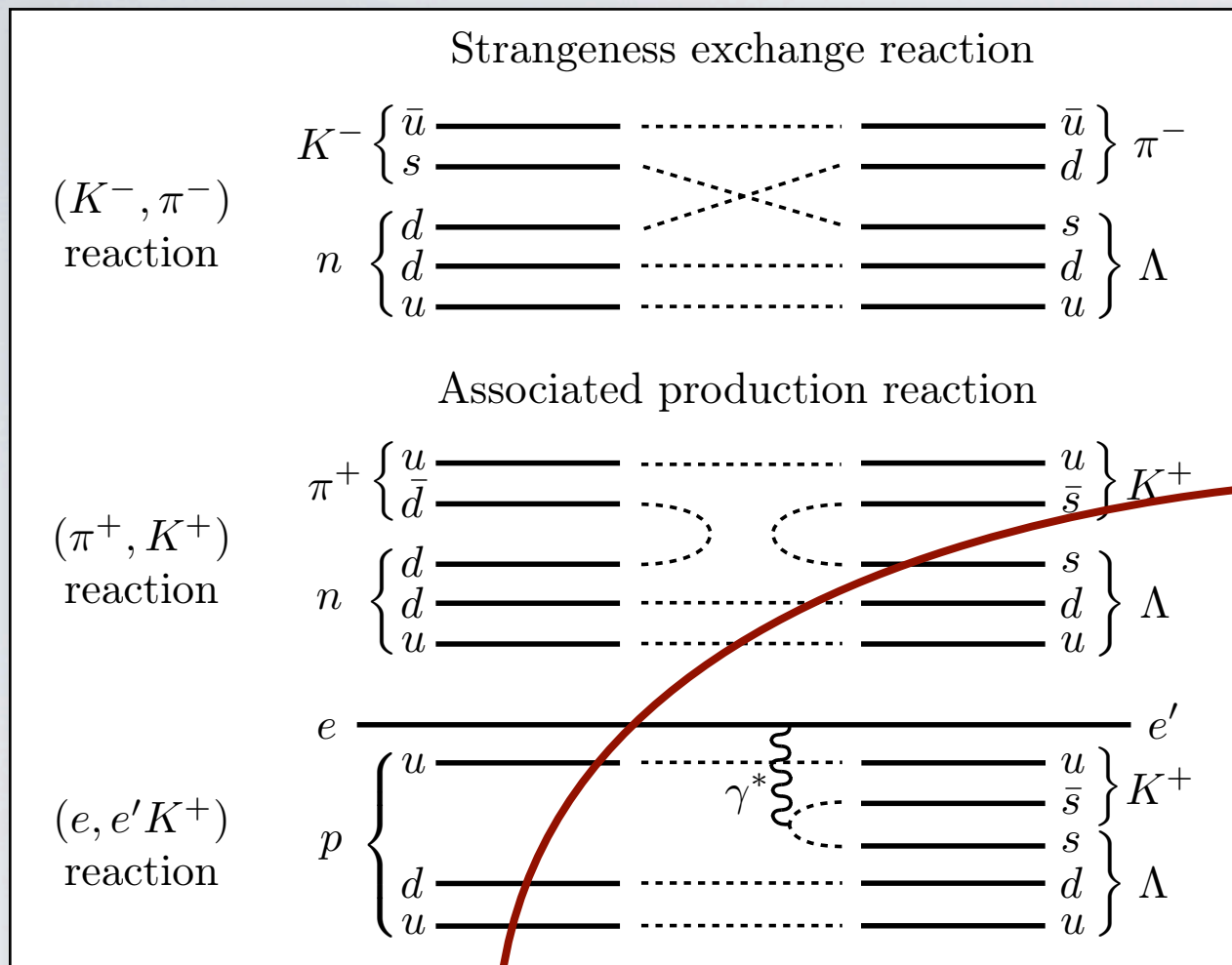


$$^A_Z(K^-, \pi^-)^A_\Lambda Z$$

$$^A_Z(\pi^+, K^+)^A_\Lambda Z$$

$$^A_Z(e, e' K^+)^A_\Lambda [Z - 1]$$

On hyperons in nuclei & nuclear matter



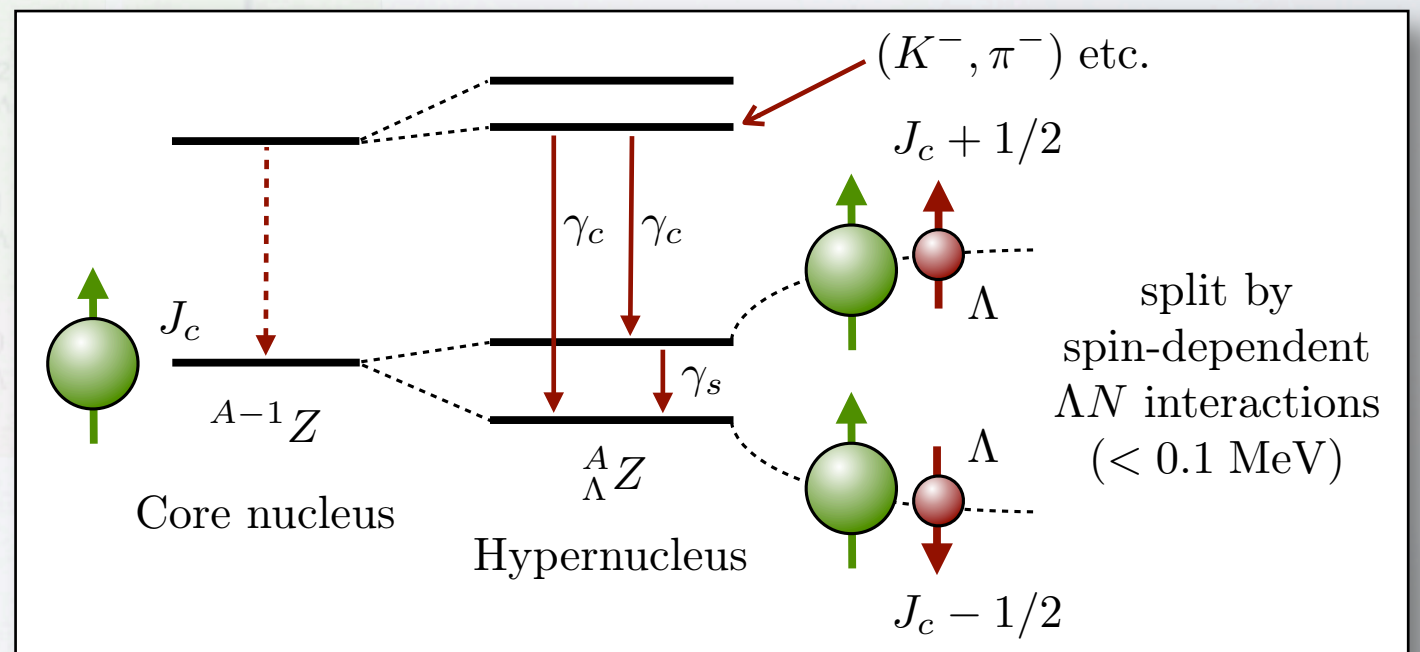
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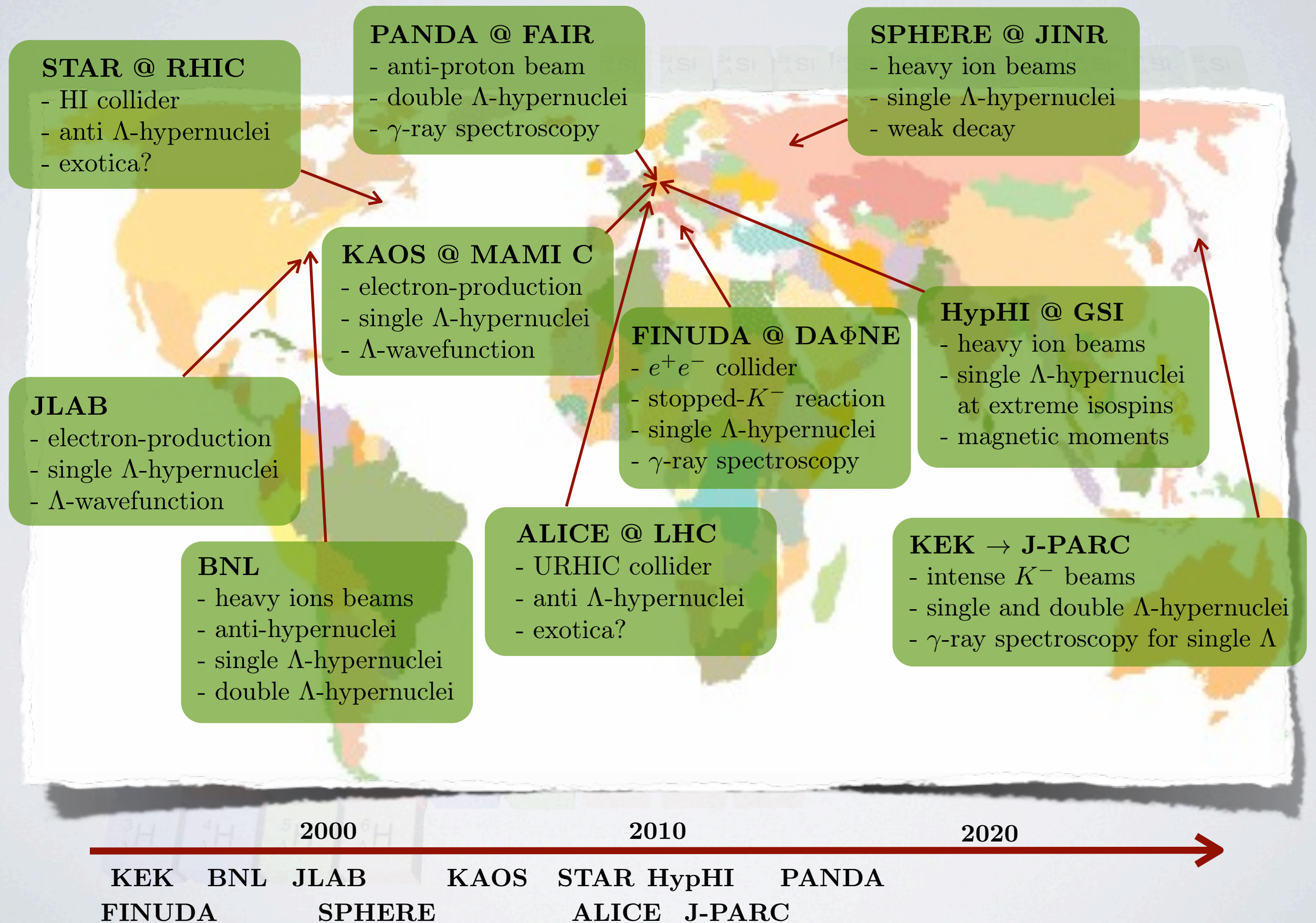
$$^A_Z(e, e' K^+)^A_\Lambda [Z - 1]$$

hypernuclear
fine structure

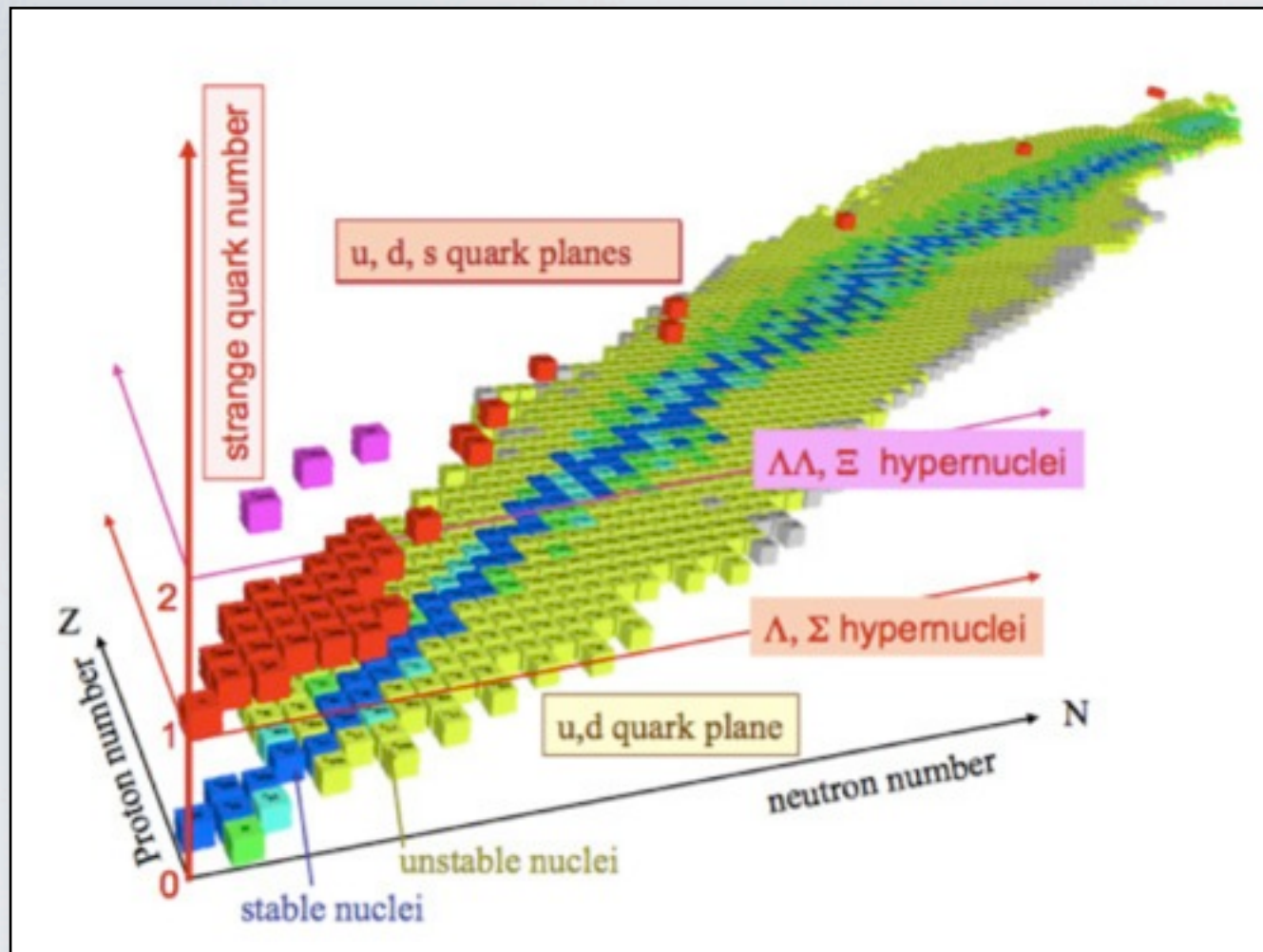
γ -ray spectroscopy



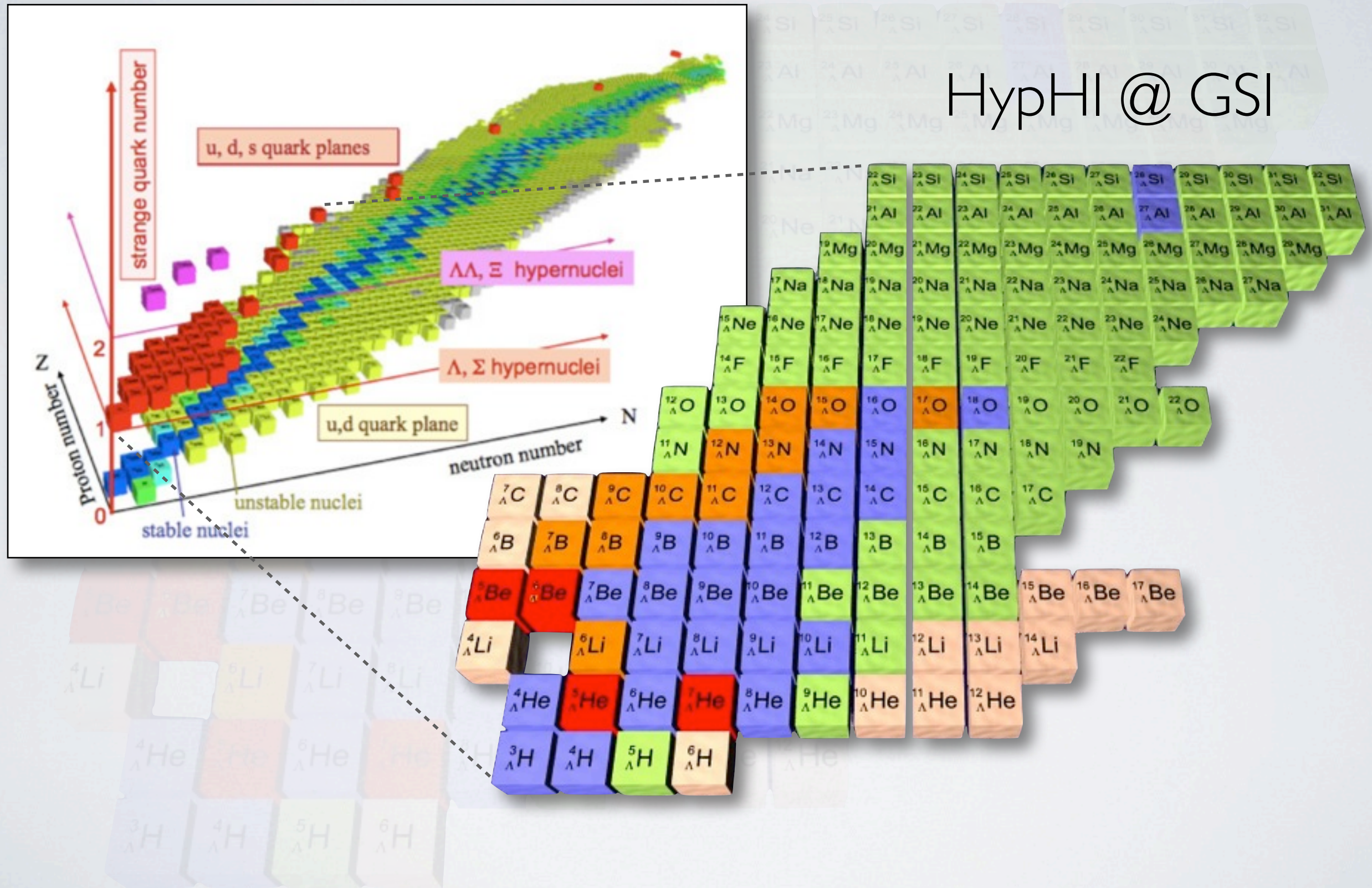
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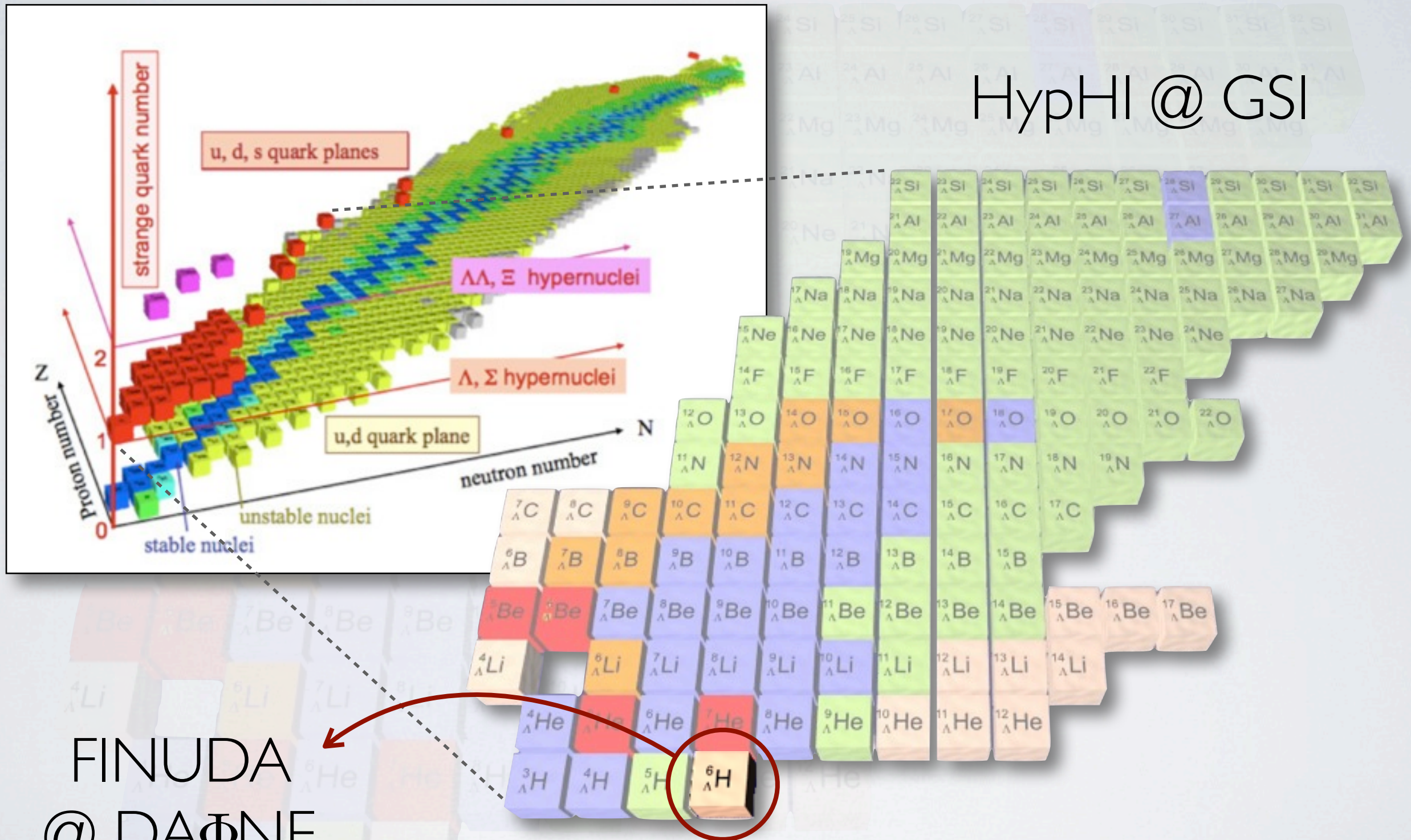
On hyperons in nuclei & nuclear matter



On hyperons in nuclei & nuclear matter



On hyperons in nuclei & nuclear matter



FINUDA
@ DAΦNE

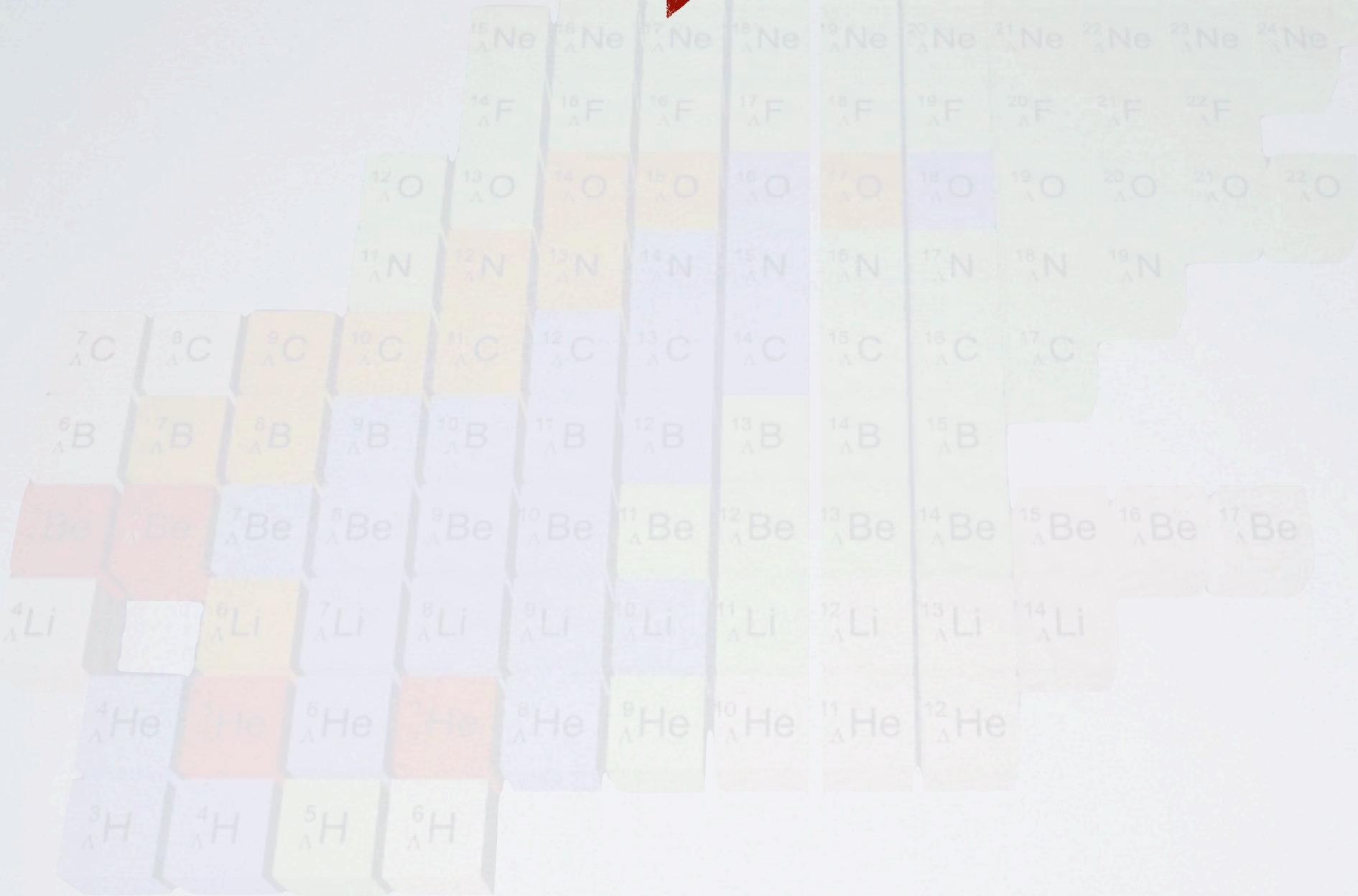
M. Agnello, L. Benussi et al. Evidence for Heavy Hyperhydrogen ${}^6_{\Lambda}\text{H}$. Phys. Rev. Lett. 108(4):1-5, Jan 2012

On hyperons in nuclei & nuclear matter

lack of data



interaction $\Lambda - N$



On hyperons in nuclei & nuclear matter

lack of data



interaction $\Lambda - N$

- Phenomen.
 - phenomenological nature
 - many-body effects hidden
 - used in shell-model calculations
- Nijmegen & Jülich
 - one boson exchange model
 - all the hyperons, only 2-body terms
 - several parametrizations
- χ -EFT
 - derived from chiral EFT
 - 2-body and 3-body terms
- Usmani
 - diagrammatic contributions, due to pion exchange
 - 2-body ΛN and 3-body $\Lambda N N$ terms

On hyperons in nuclei & nuclear matter

lack of data

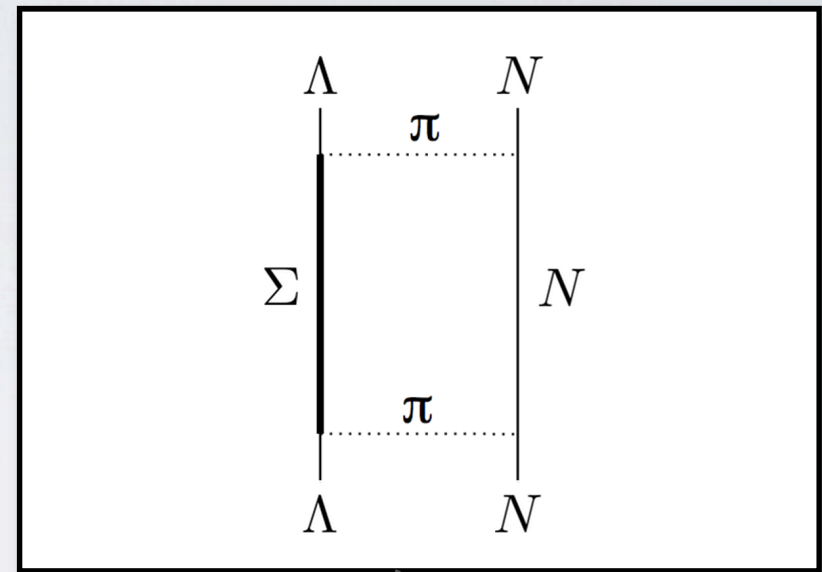
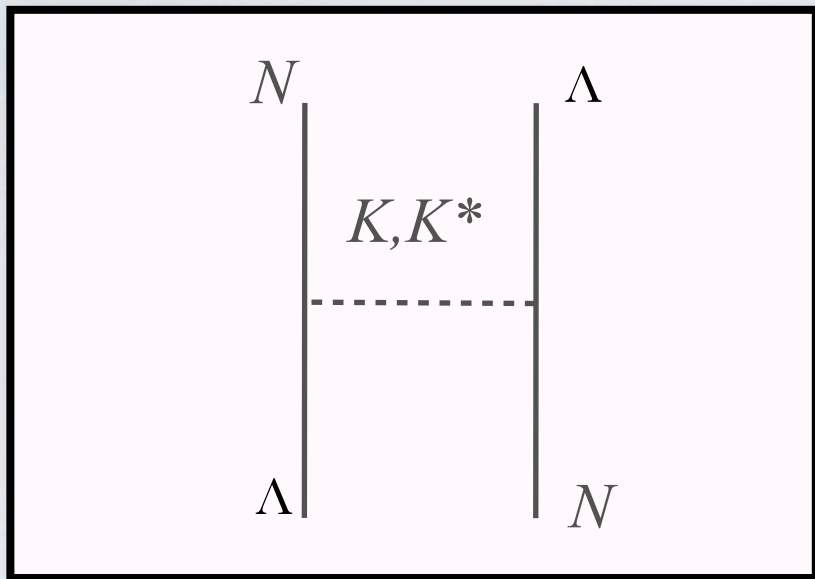


interaction $\Lambda - N$

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On hyperons in nuclei & nuclear matter

Usmani potential: 2-body ΛN



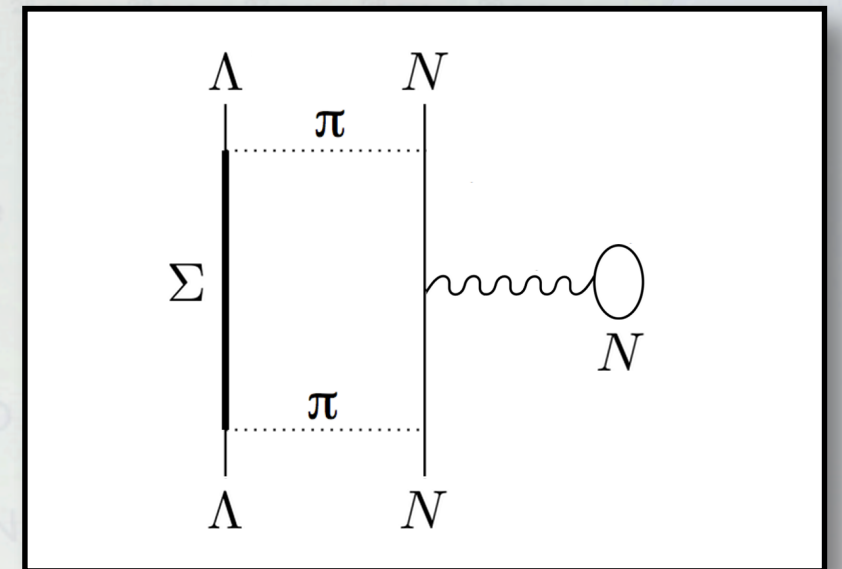
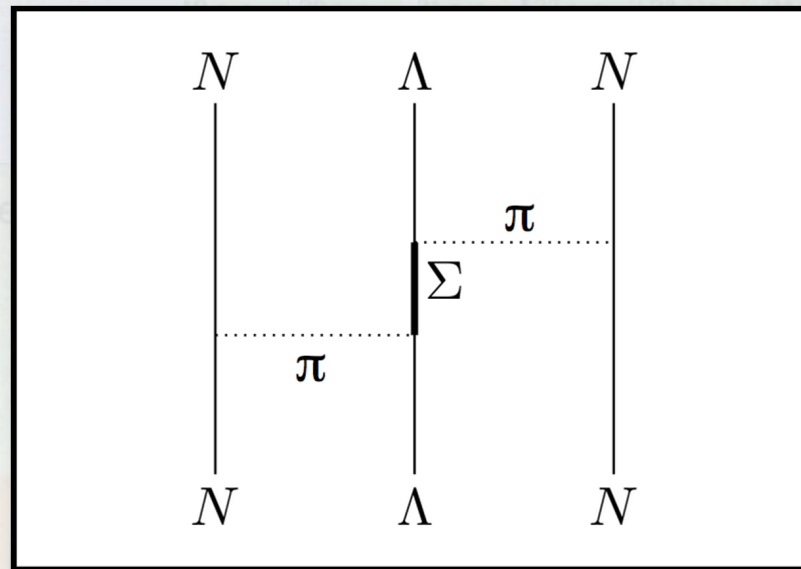
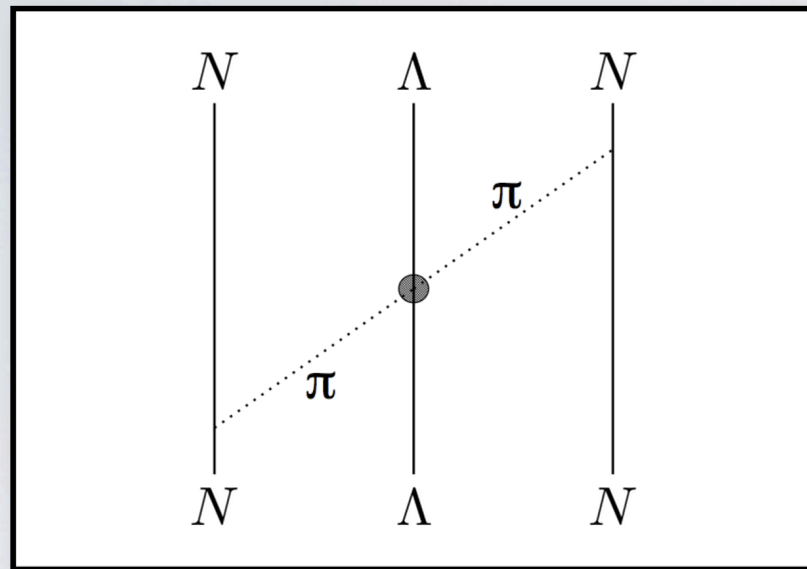
$$V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$$

$\varepsilon \sim 0.1 \div 0.3$

$$\begin{cases} v_0(r) &= v_c(r) - v_{2\pi}(r) \\ v_c(r) &= W_c \left[1 + e^{\frac{r-\bar{r}}{a}} \right]^{-1} \\ v_{2\pi}(r) &= \bar{v} T_\pi^2(m_\pi r) \end{cases} \quad \begin{cases} v_\sigma &= v_s - v_t \\ \bar{v} &= \frac{1}{4} (v_s + 3v_t) \end{cases}$$

On hyperons in nuclei & nuclear matter

Usmani potential: 3-body ΛNN

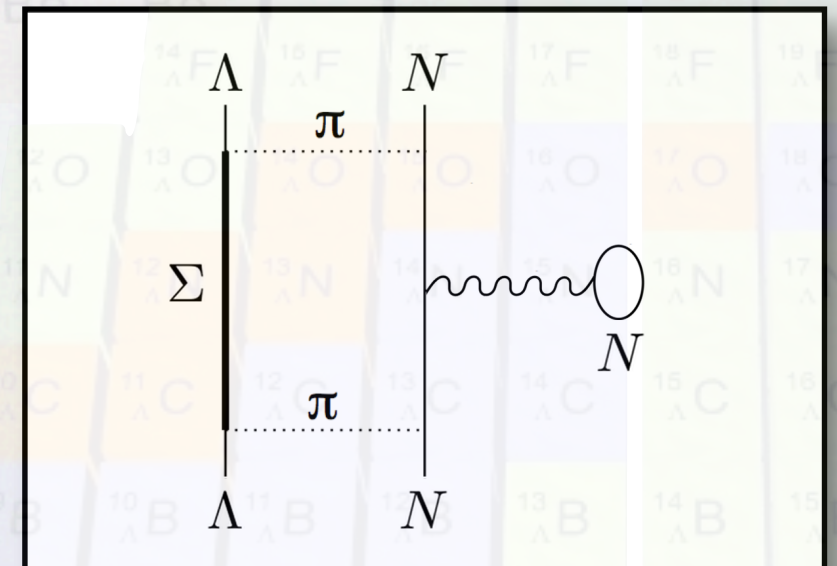
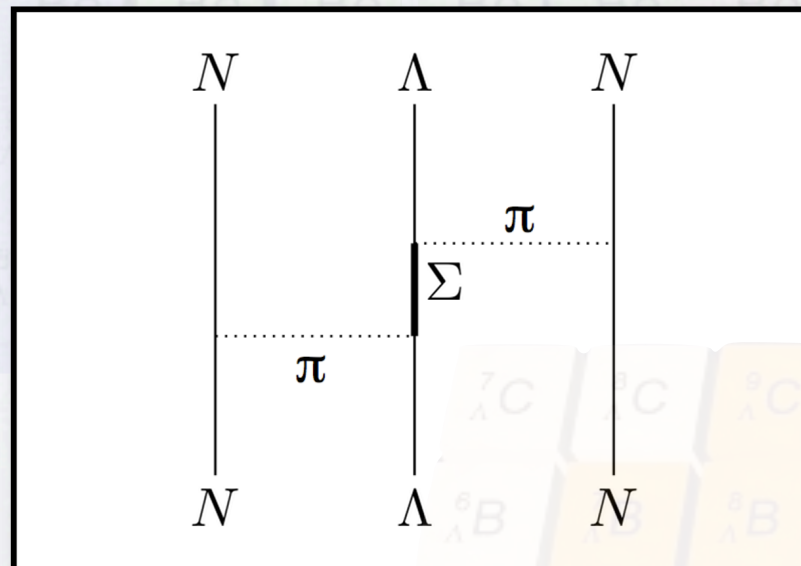
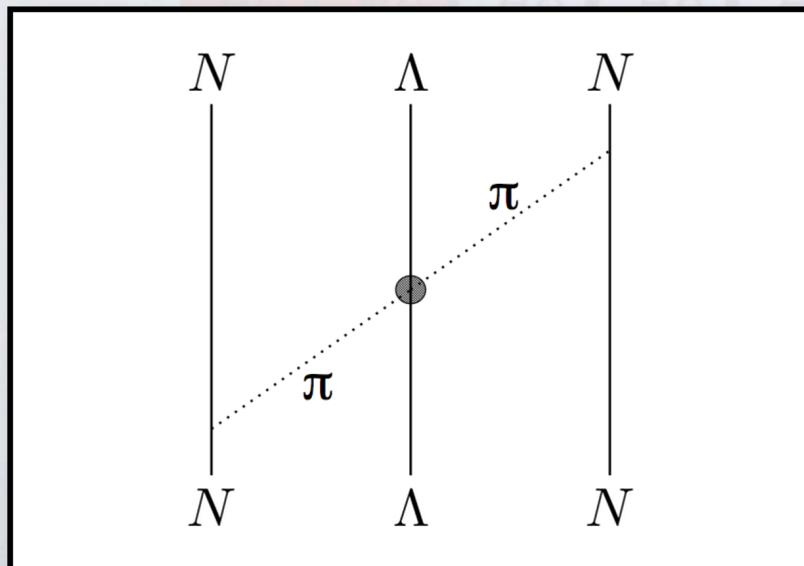
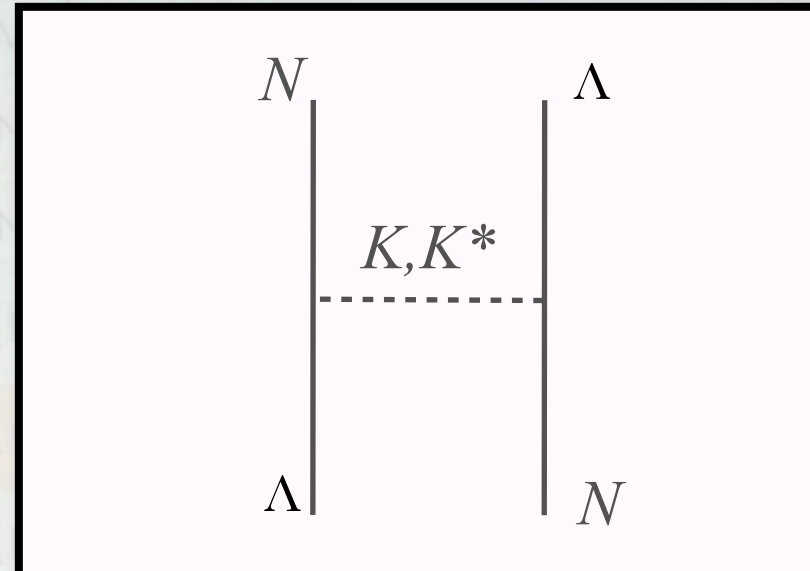
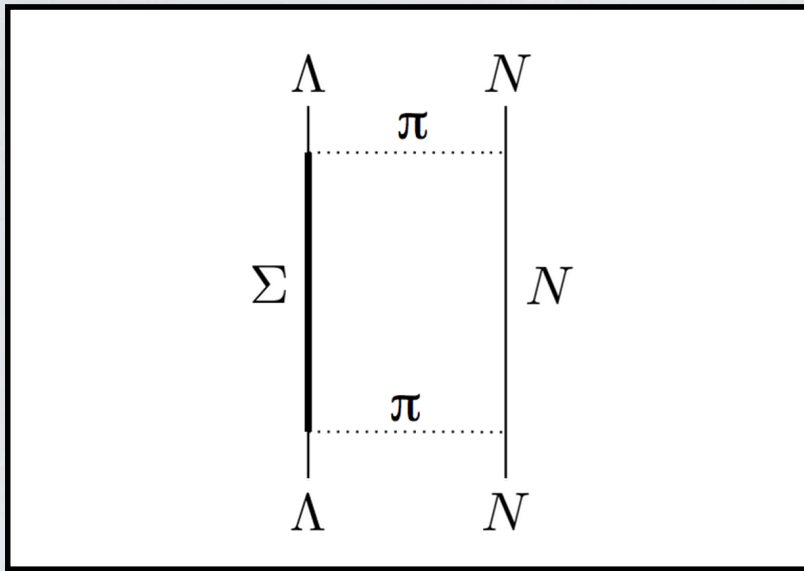


$$V_{\Lambda ij} = V_{\Lambda ij}^{2\pi} + V_{\Lambda ij}^D$$

$$\begin{cases} V_{\Lambda ij}^{2\pi} = C_{2\pi}^{SW} \mathcal{O}_{\Lambda ij}^{2\pi, SW} + C_{2\pi}^{PW} \mathcal{O}_{\Lambda ij}^{2\pi, PW} \\ V_{\Lambda ij}^D = W^D T_{\pi}^2(m_{\pi} r_{\Lambda i}) T_{\pi}^2(m_{\pi} r_{\Lambda j}) \left[1 + \frac{1}{6} \boldsymbol{\sigma}_{\Lambda} \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right] \end{cases}$$

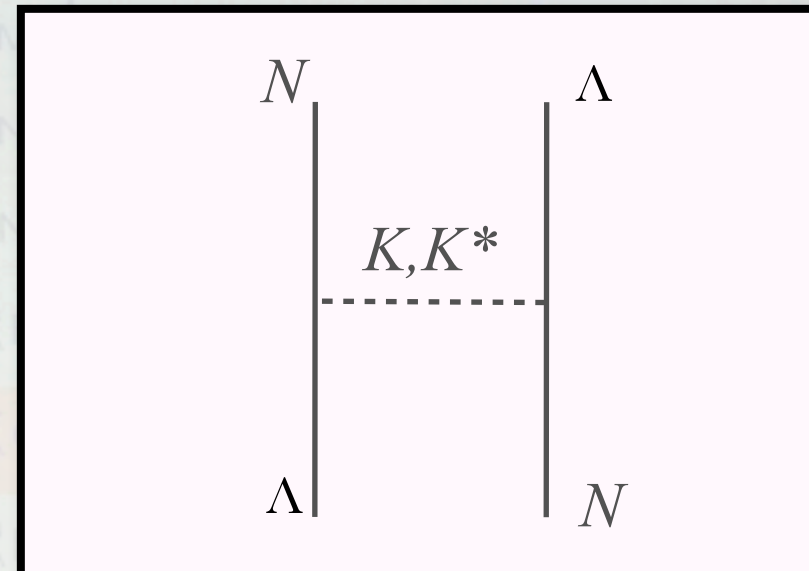
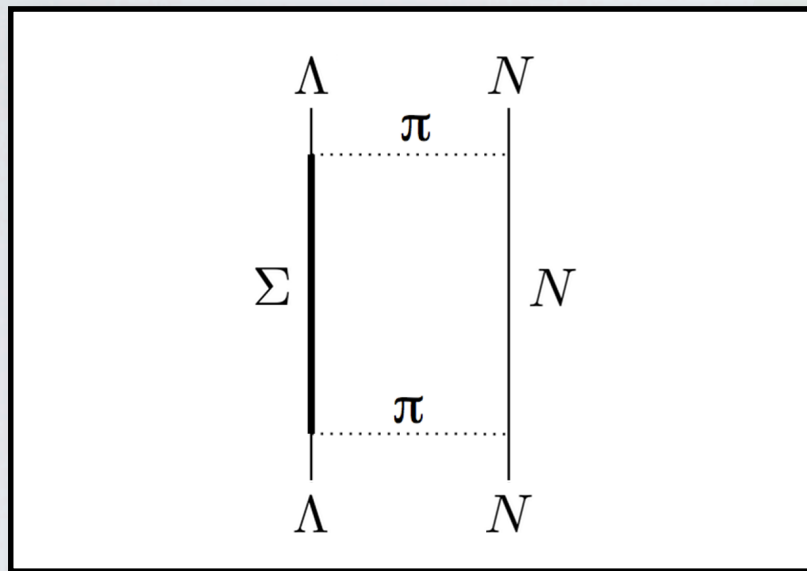
On hyperons in nuclei & nuclear matter

Usmani potential: 2-body ΛN + 3-body $\Lambda N N$



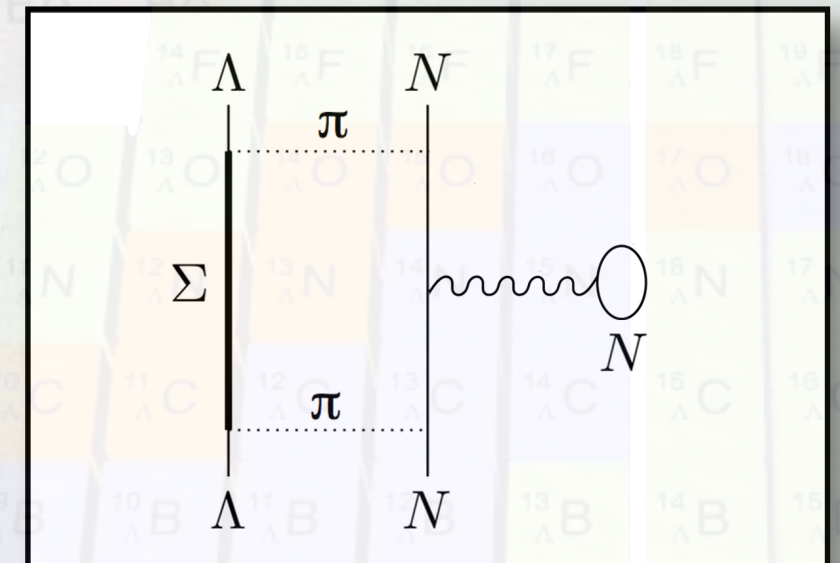
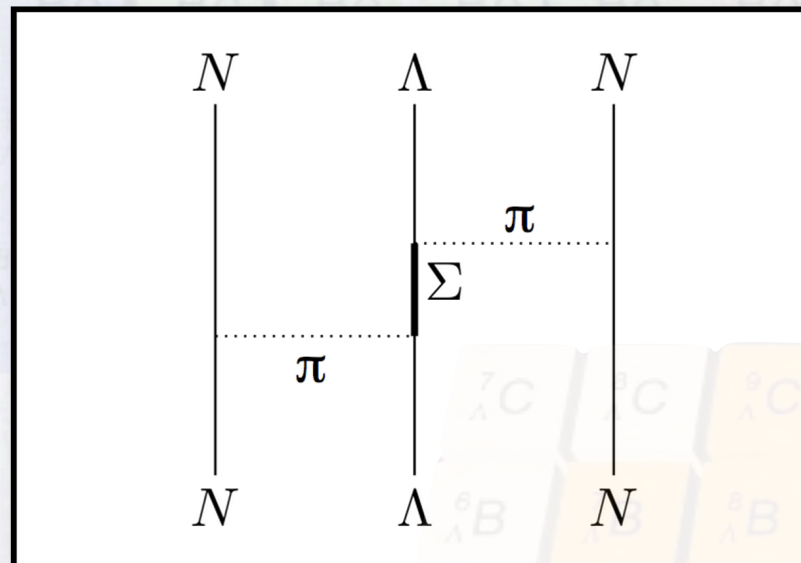
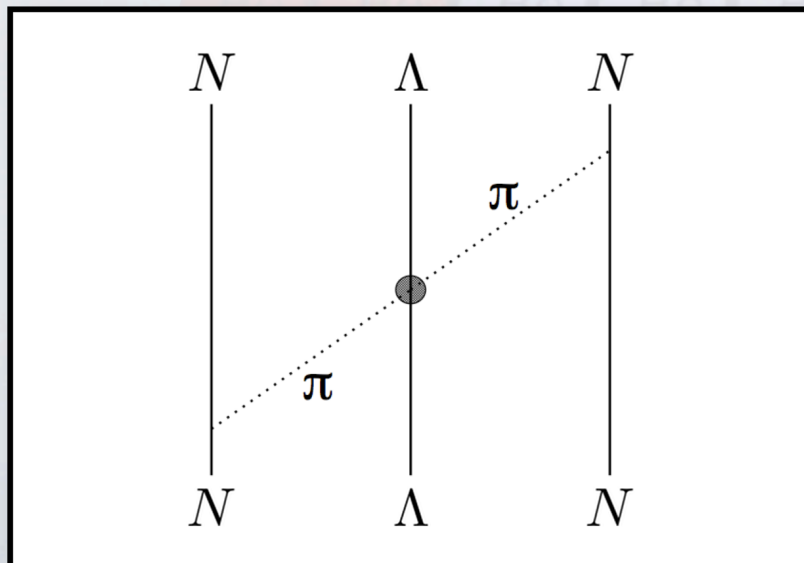
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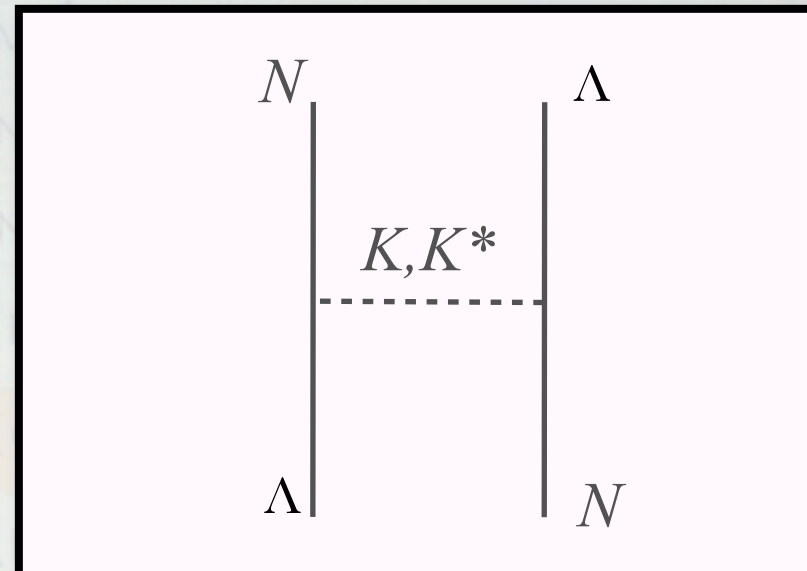
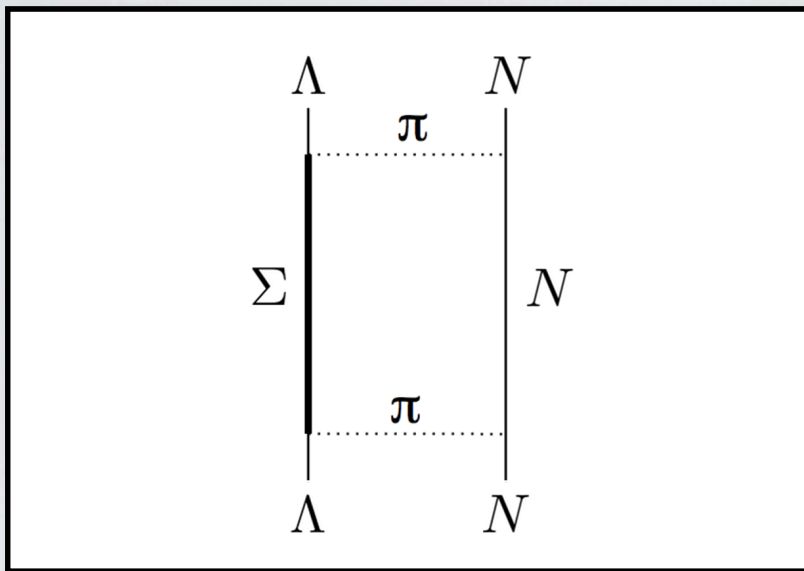
2 & 3-body: 2π exchange

$$\Delta m(\Sigma, \Lambda) < \Delta m(\Delta, N)$$



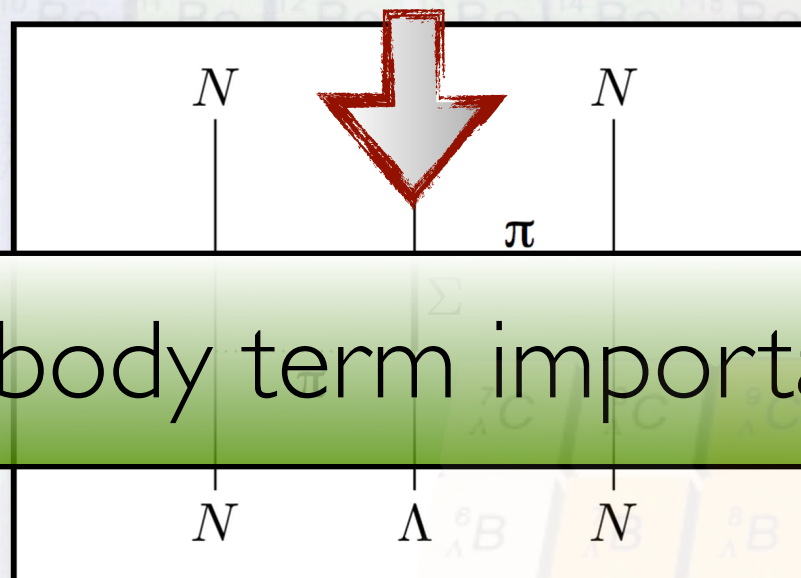
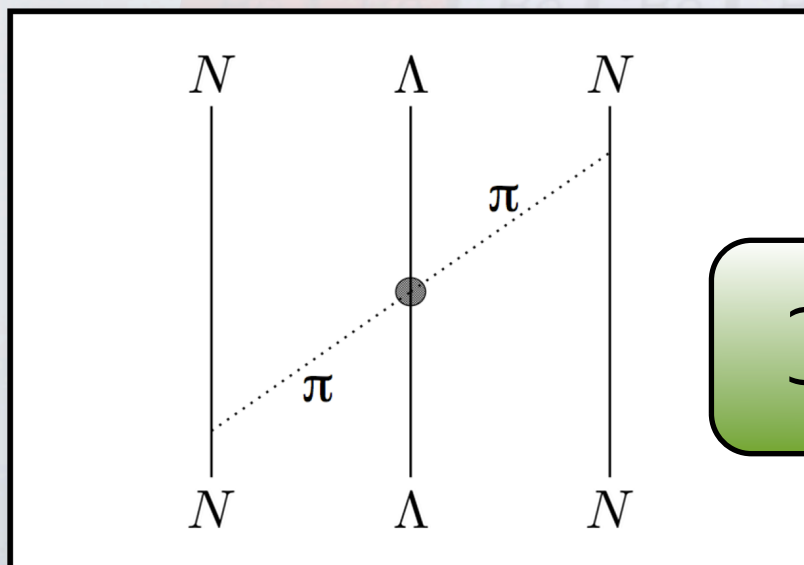
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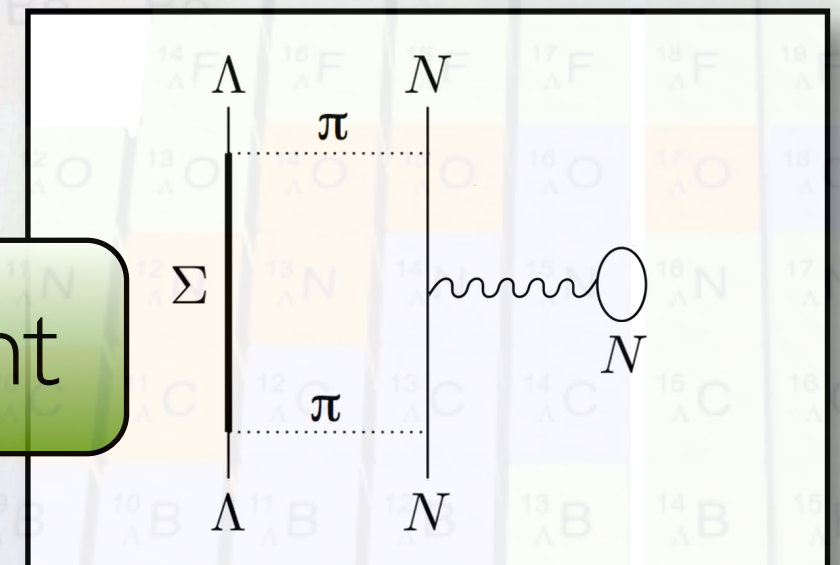


2 & 3-body: 2π exchange

$$\Delta m(\Sigma, \Lambda) < \Delta m(\Delta, N)$$



3-body term important



Usmani hypernuclear potential

Constant	Value	Unit
$m_{\pi^{\pm}}$	139.57018(35)	MeV
m_{π^0}	134.9766 (6)	MeV
W_c	2137	MeV
r_0	0.5	fm
a	0.2	fm
ε	$0.1 \div 0.38$	—
$\bar{v}$	6.15(5)	MeV
v_s	6.33, 6.28, 6.23	MeV
v_t	6.09, 6.04, 5.99	MeV
v_σ	0.24	MeV
c	2.0	fm^{-2}
W^D	$0.01 \div 0.05$	MeV
C^P	$0.4 \div 2.0$	MeV
C^S	~ 1.5	MeV

$$V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r) \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N$$

$$P_x = (-1)^l$$

$$v_0(r) = v_c(r) - v_{2\pi}(r)$$

$$v_c(r) = W_c \left[1 + e^{\frac{r-\bar{r}}{a}} \right]^{-1}$$

$$v_{2\pi} = \bar{v} T_\pi^2(m_\pi r)$$

$$\bar{v} = \frac{1}{4}(v_s + 3v_t)$$

$$v_\sigma = v_s - v_t$$

$$T_\pi(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{x} \xi_T(r)$$

$$\xi_Y(r) = \xi_T^{1/2}(r) = \left(1 - e^{-cr^2} \right)$$

$$V_{\Lambda ij} = V_{\Lambda ij}^D + V_{\Lambda ij}^{2\pi}$$

$$V_{\Lambda ij}^D = W^D T_\pi^2(m_\pi r_{\Lambda i}) T_\pi^2(m_\pi r_{\Lambda j}) \left[1 + \frac{1}{6} \boldsymbol{\sigma}_\Lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right]$$

$$V_{\Lambda ij}^{2\pi} = V_{\Lambda ij}^P + V_{\Lambda ij}^S$$

$$V_{\Lambda ij}^P = - \left(\frac{1}{6} C^P \right) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \{X_{i\Lambda}, X_{\Lambda j}\}$$

$$V_{\Lambda ij}^S = C^S Z(m_\pi r_{i\Lambda}) Z(m_\pi r_{j\Lambda}) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{i\Lambda} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{j\Lambda} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$X_{\Lambda i} = (\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i) Y_\pi(m_\pi r_{\Lambda i}) + S_{\Lambda i} T_\pi(m_\pi r_{\Lambda i})$$

$$Z(x) = \frac{x}{3} [Y_\pi(x) - T_\pi(x)]$$

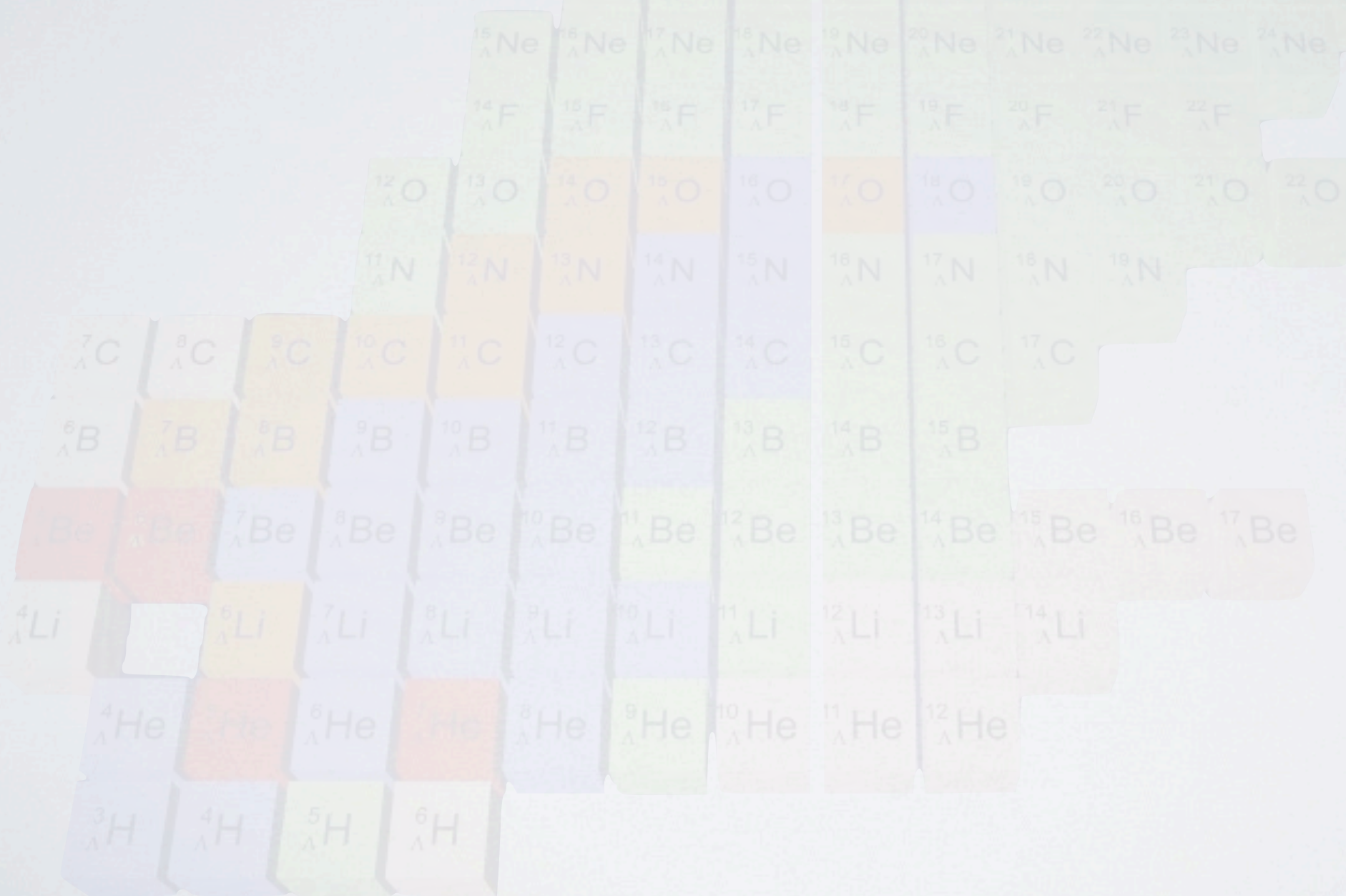
$$Y_\pi(x) = \frac{e^{-x}}{x} \xi_Y(x)$$

$$S_{\Lambda i} = 3(\boldsymbol{\sigma}_\Lambda \cdot \hat{\mathbf{r}}_{\Lambda i})(\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{\Lambda i}) - \boldsymbol{\sigma}_\Lambda \boldsymbol{\sigma}_i$$

How we do it

Solve the many-body Hamiltonian $\rightarrow$ ab-initio method

Auxiliary Field Diffusion Monte Carlo (AFDMC)



How we do it

Solve the many-body Hamiltonian $\rightarrow$ ab-initio method

Auxiliary Field Diffusion Monte Carlo (AFDMC)

$$\tau = \frac{it}{\hbar} \rightarrow -\frac{\partial}{\partial \tau} \psi(\mathbf{R}, \tau) = \mathcal{H} \psi(\mathbf{R}, \tau)$$

How we do it

Solve the many-body Hamiltonian $\rightarrow$ ab-initio method

Auxiliary Field Diffusion Monte Carlo (AFDMC)

$$\tau = \frac{it}{\hbar} \rightarrow -\frac{\partial}{\partial \tau} \psi(\mathbf{R}, \tau) = \mathcal{H} \psi(\mathbf{R}, \tau)$$



$$\mathcal{D} = \hbar^2 / 2m$$

$$\psi(\mathbf{R}, \tau + d\tau) \simeq \int \left(\frac{1}{4\pi\mathcal{D}d\tau} \right)^{\frac{3N}{2}} e^{-\frac{(\mathbf{R}-\mathbf{R}')^2}{4\mathcal{D}d\tau}} e^{-\left(\frac{\hat{V}(\mathbf{R}') + \hat{V}(\mathbf{R})}{2} - E_0 \right) d\tau} \psi(\mathbf{R}', \tau) d\mathbf{R}'$$

How we do it

Solve the many-body Hamiltonian $\rightarrow$ ab-initio method

Auxiliary Field Diffusion Monte Carlo (AFDMC)

$$\tau = \frac{it}{\hbar} \rightarrow -\frac{\partial}{\partial \tau} \psi(\mathbf{R}, \tau) = \mathcal{H} \psi(\mathbf{R}, \tau)$$



$$\mathcal{D} = \hbar^2 / 2m$$

$$\psi(\mathbf{R}, \tau + d\tau) \simeq \int \underbrace{\left(\frac{1}{4\pi\mathcal{D}d\tau} \right)^{\frac{3N}{2}} e^{-\frac{(\mathbf{R}-\mathbf{R}')^2}{4\mathcal{D}d\tau}}}_{G_0(\mathbf{R} \leftarrow \mathbf{R}', d\tau)} e^{-\left(\frac{\hat{V}(\mathbf{R}') + \hat{V}(\mathbf{R})}{2} - E_0 \right) d\tau} \underbrace{\psi(\mathbf{R}', \tau) d\mathbf{R}'}_{G_{\hat{V}}(\mathbf{R} \leftarrow \mathbf{R}', d\tau)}$$

How we do it

Solve the many-body Hamiltonian $\rightarrow$ ab-initio method

Auxiliary Field Diffusion Monte Carlo (AFDMC)

$$\tau = \frac{it}{\hbar} \rightarrow -\frac{\partial}{\partial \tau} \psi(\mathbf{R}, \tau) = \mathcal{H} \psi(\mathbf{R}, \tau)$$



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$$\psi(\mathbf{R}, \tau + d\tau) \simeq \int \left(\frac{1}{4\pi\mathcal{D}d\tau} \right)^{\frac{3N}{2}} e^{-\frac{(\mathbf{R}-\mathbf{R}')^2}{4\mathcal{D}d\tau}} e^{-\left(\frac{\hat{V}(\mathbf{R}') + \hat{V}(\mathbf{R})}{2} - E_0 \right) d\tau} \psi(\mathbf{R}', \tau) d\mathbf{R}'$$

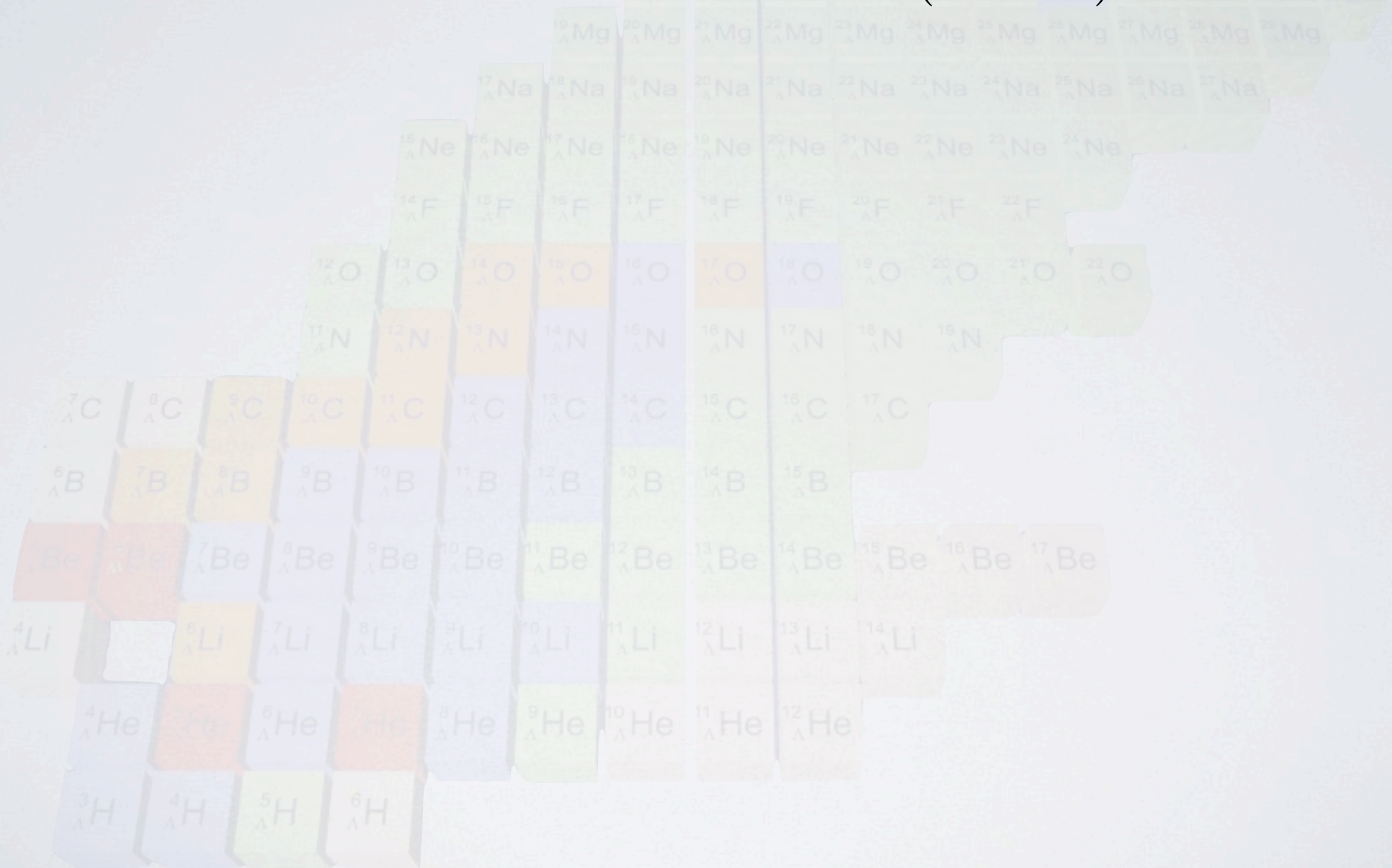
$$G_0(\mathbf{R} \leftarrow \mathbf{R}', d\tau)$$

$$G_{\hat{V}}(\mathbf{R} \leftarrow \mathbf{R}', d\tau)$$

problem with nuclear potentials

How we do it

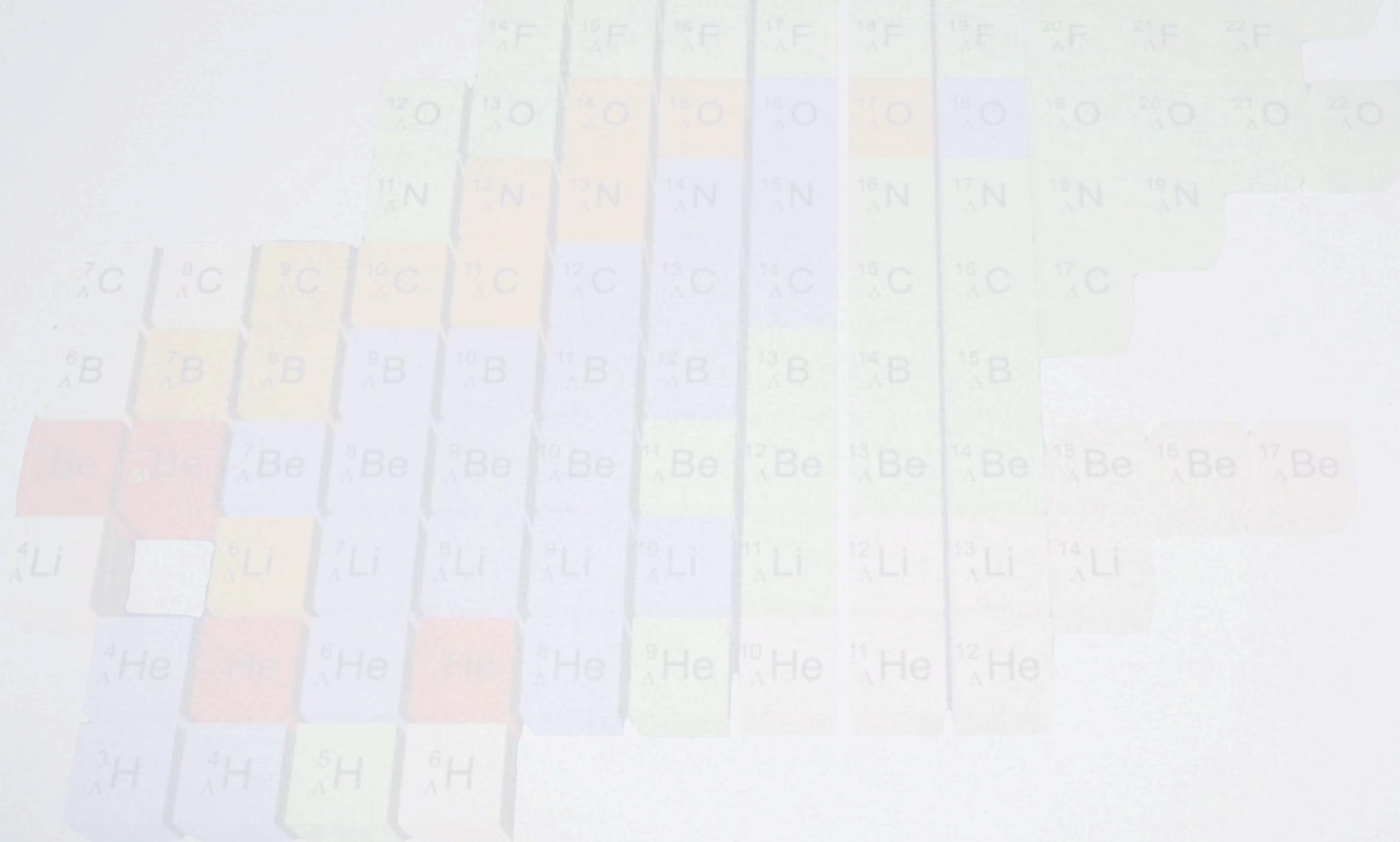
$$\mathcal{P} \sim e^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} \quad \Rightarrow \quad \psi \sim \sum \frac{A!}{Z!(A-Z)!} 4^A \text{ terms}$$



How we do it

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high computational cost $\rightarrow$ GFMC: $A \leq 12$



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Idea: Hubbard-Stratonovich transformation

$$e^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int dx e^{-\frac{x^2}{2} + \sqrt{-\lambda d\tau} x \mathcal{O}}$$

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auxiliary field

rotation over spin-isospin configurations

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auxiliary field

rotation over spin-isospin configurations

$\Rightarrow$ computational cost: $\sim A! \rightarrow \sim A^3$

Auxiliary Field Diffusion Monte Carlo

$$\text{AV6: } \mathcal{O}_{ij}^{p=1,6} = \underbrace{\{1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}\}}_{V_{SD} + V_{SI}} \otimes \{1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

$$V_{SD} = \frac{1}{2} \sum_{n=1}^A \sum_{\alpha=1}^3 \lambda_n^{(\tau)} \mathcal{O}_{n\alpha}^{(\tau)2} \quad \mathcal{A}_{i,j}^{(\tau)} : A \times A$$

$$+ \frac{1}{2} \sum_{n=1}^{3A} \lambda_n^{(\sigma)} \mathcal{O}_n^{(\sigma)2} \quad \mathcal{A}_{i\alpha,j\beta}^{(\sigma)} : 3A \times 3A$$

$$+ \frac{1}{2} \sum_{n=1}^{3A} \sum_{\alpha=1}^3 \lambda_n^{(\sigma\tau)} \mathcal{O}_{n\alpha}^{(\sigma\tau)2} \quad \mathcal{A}_{i\alpha,j\beta}^{(\sigma\tau)} : 3A \times 3A$$

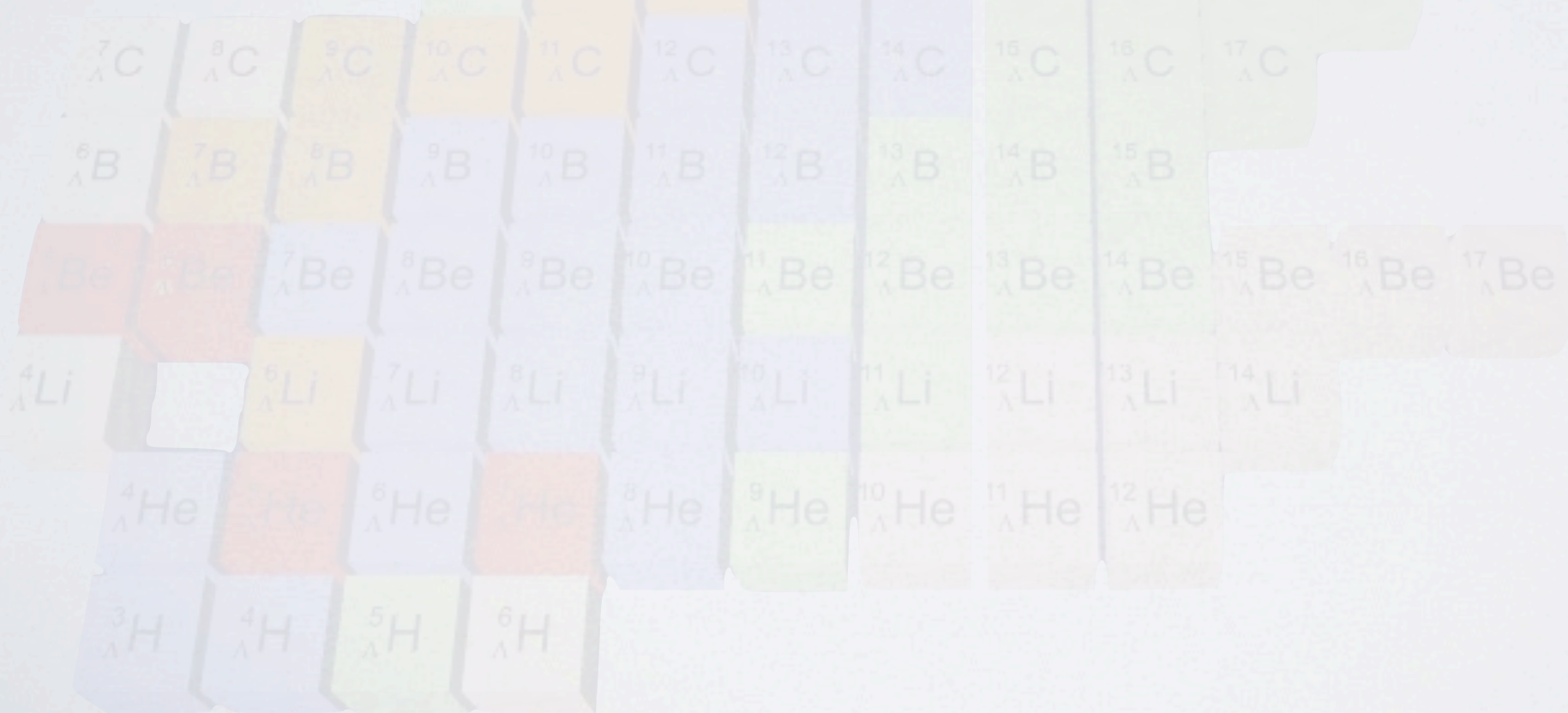
good for Hubbard-Stratonovich $\rightarrow 15A$ auxiliary fields

rotation of spin-isospin states

How we do it

AFDMC: from nuclei to hypernuclei

- modify the wave function
- add the kinetic and potential terms for the Λ particle
- take care of center of mass corrections
- modify the HS propagator & spin-isospin rotations



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ground state energy for nuclei & hypernuclei

- experiments
- Usmani VMC

$$B_{\Lambda} = BE_{nuc} - BE_{hyp}$$

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➡ ground state energy for nuclei & hypernuclei

- experiments
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$$B_{\Lambda} = BE_{nuc} - BE_{hyp}$$

Hyp.: nuclear effects
cancel at most



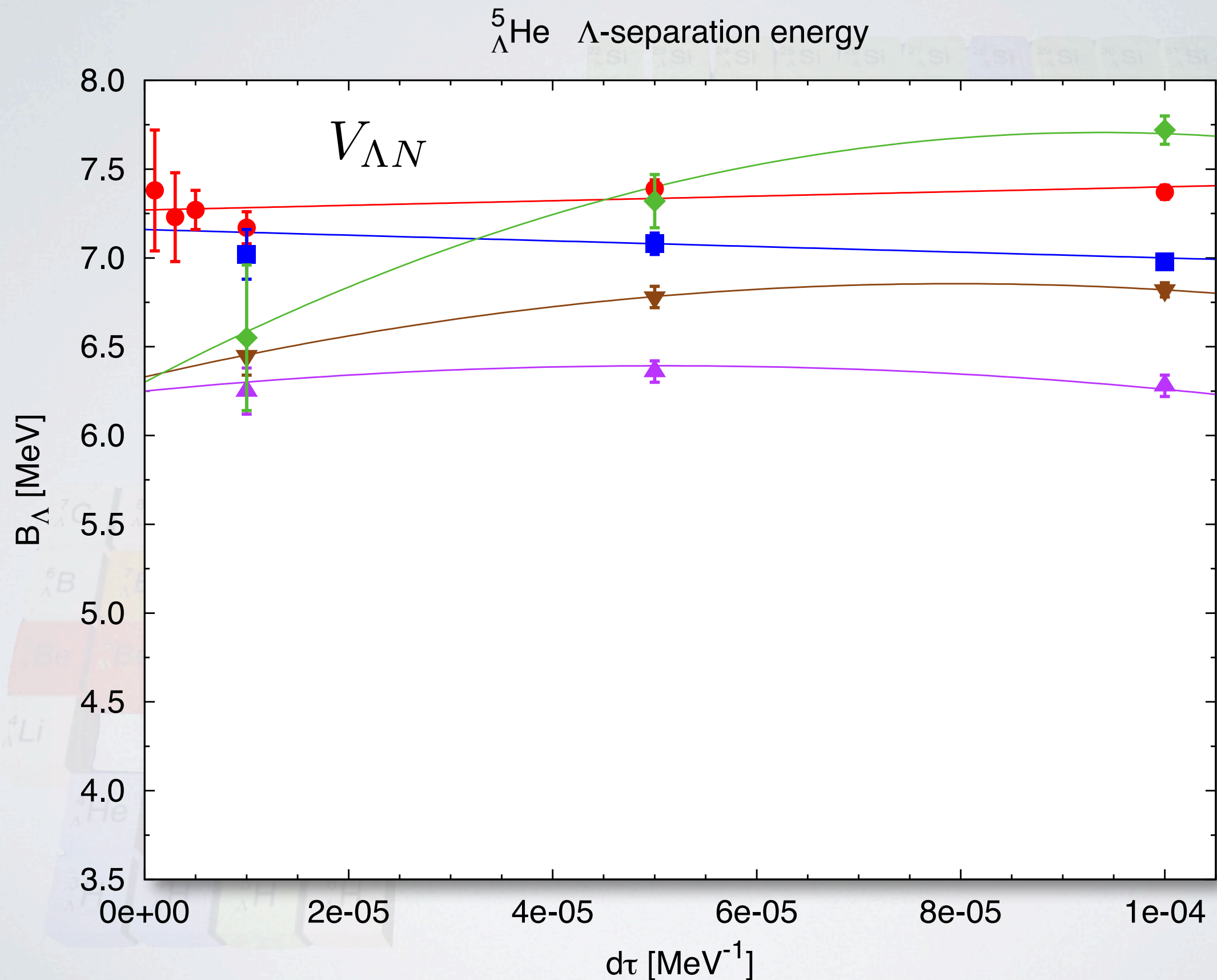
information about the
hyperon-nucleon interaction

Preliminary results

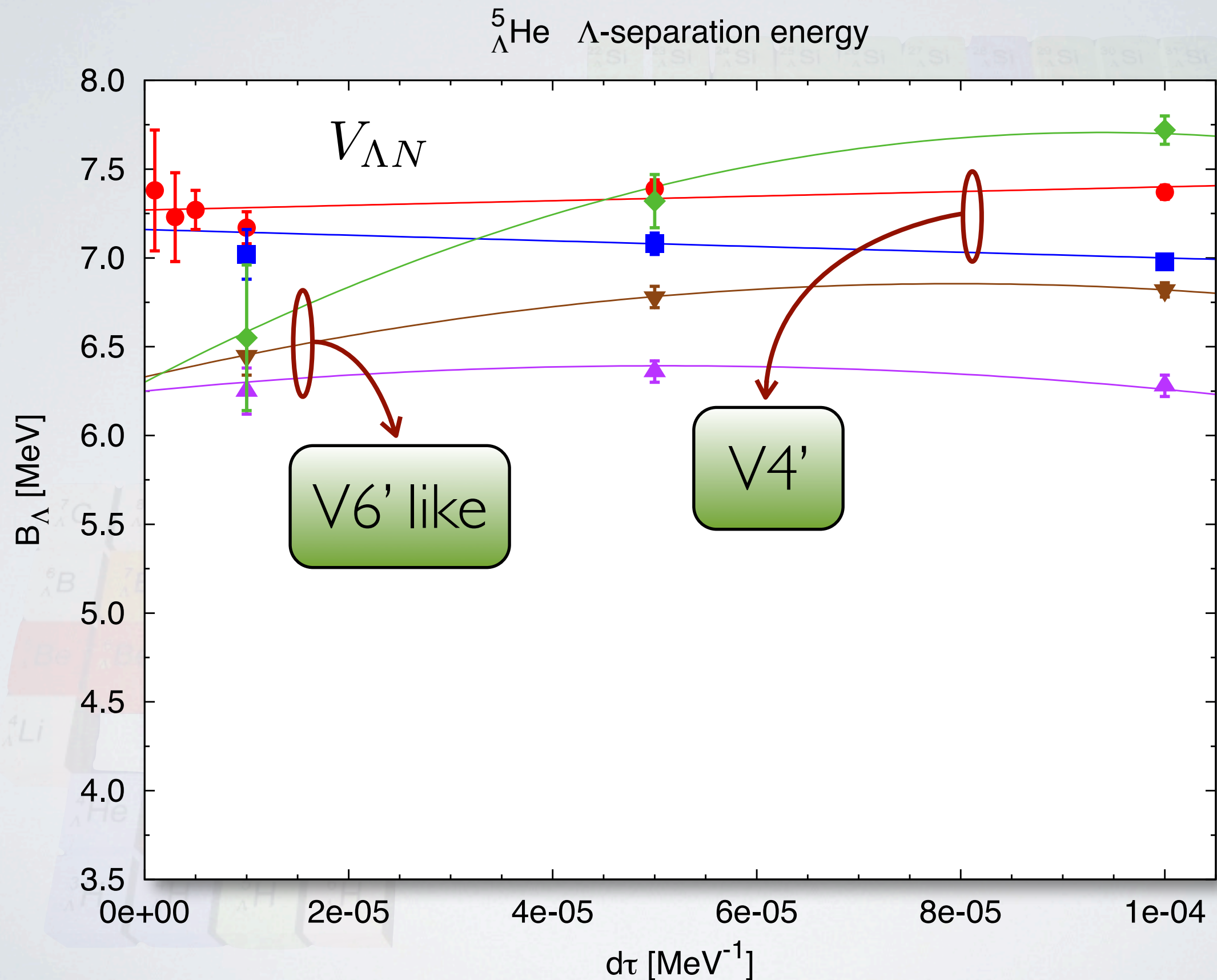
AFDMC for nuclei & hypernuclei

- nuclear potentials
 - Argonne V4' & V6'
 - Argonne V1', V2' & V8'(6), Minnesota (test)
- hypernuclear potentials
 - Usmani 2-body & 3-body ($\varepsilon = 0$ and $C_{2\pi}^{SW} = 0$)
- wave function
 - Skyrme & HF single particle orbitals for SD
- observables
 - nucl. & hyp. binding energy
 - Λ separation energy
 - Λ & N single particle density

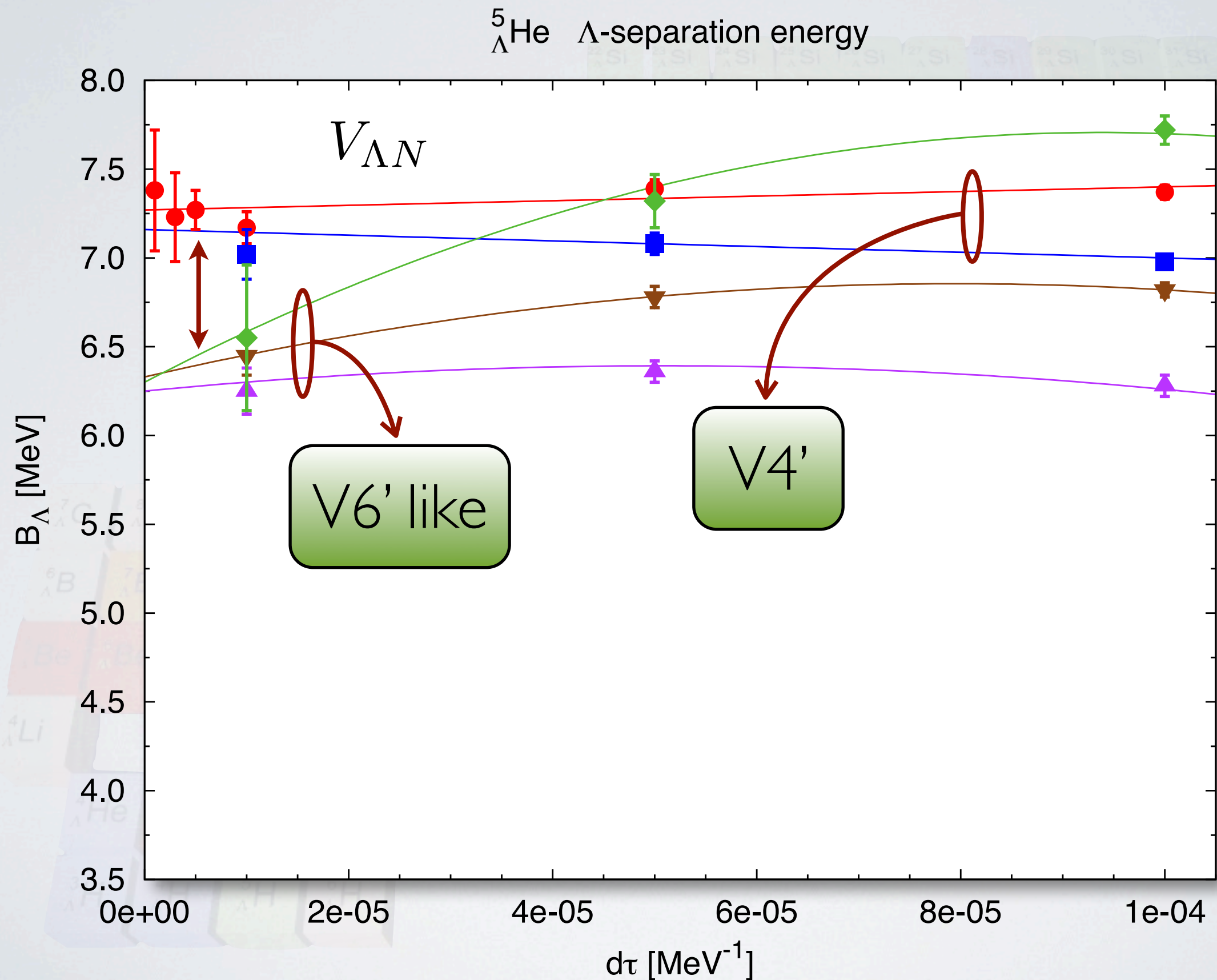
Preliminary results



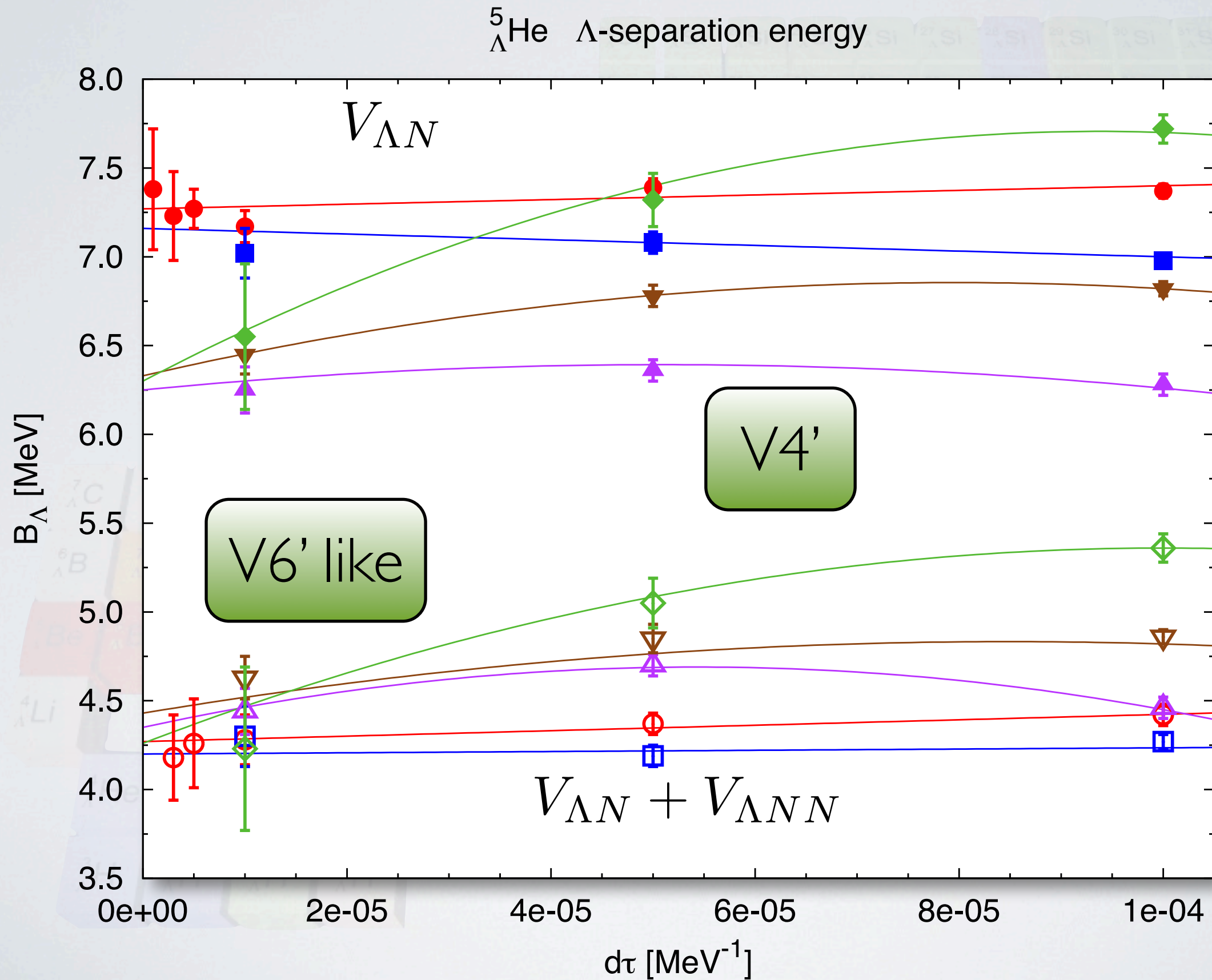
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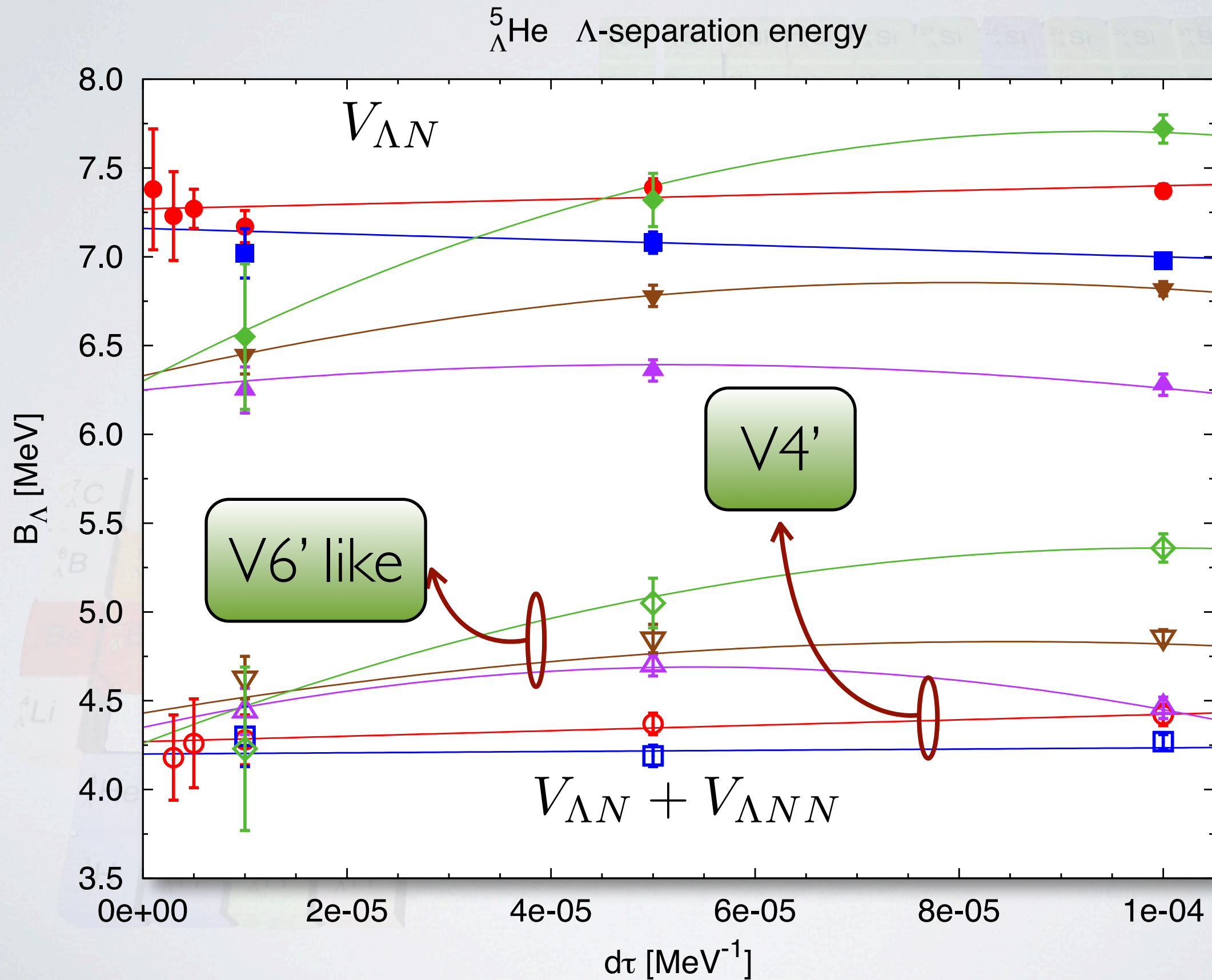
Preliminary results



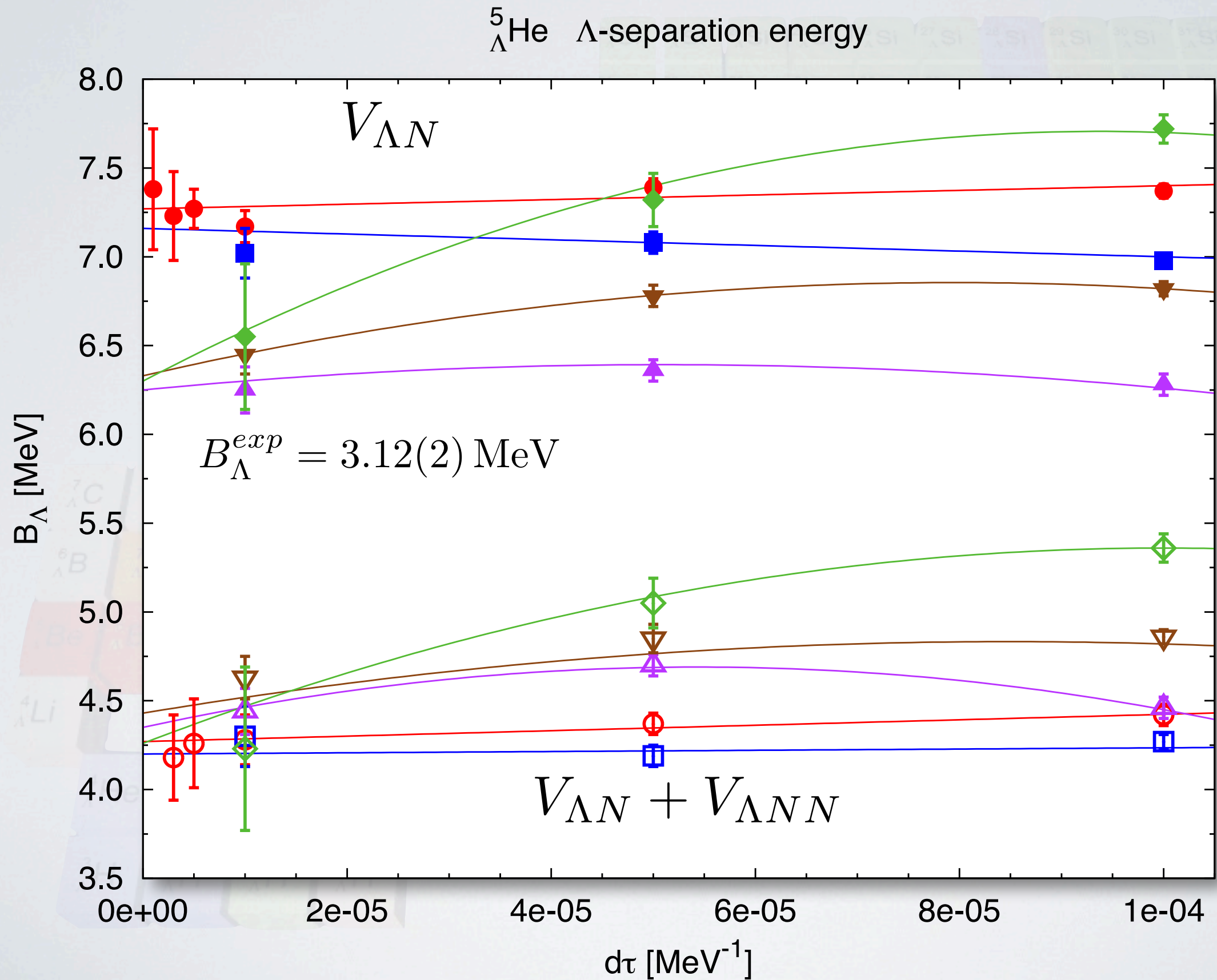
Preliminary results



Preliminary results

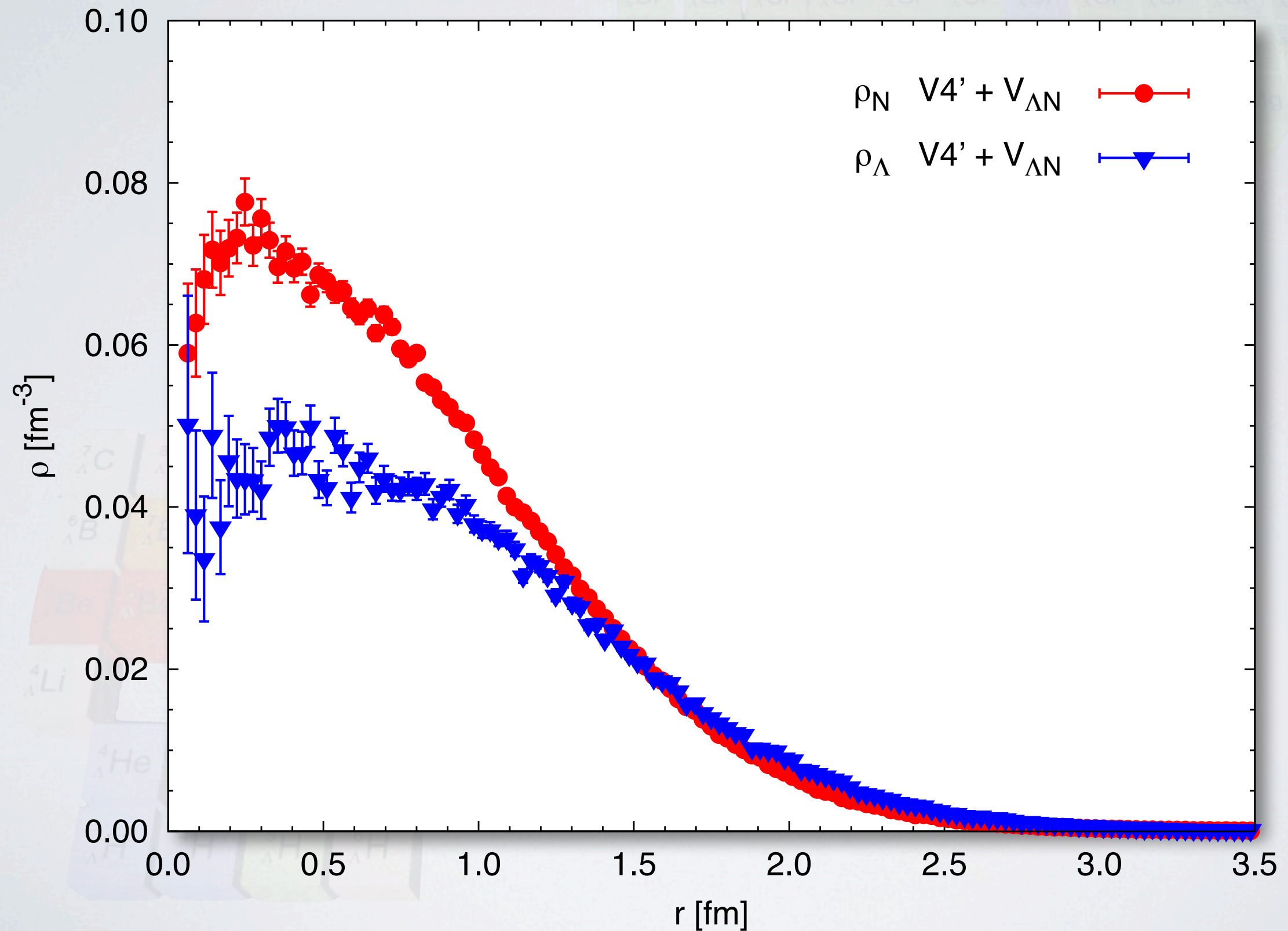


Preliminary results



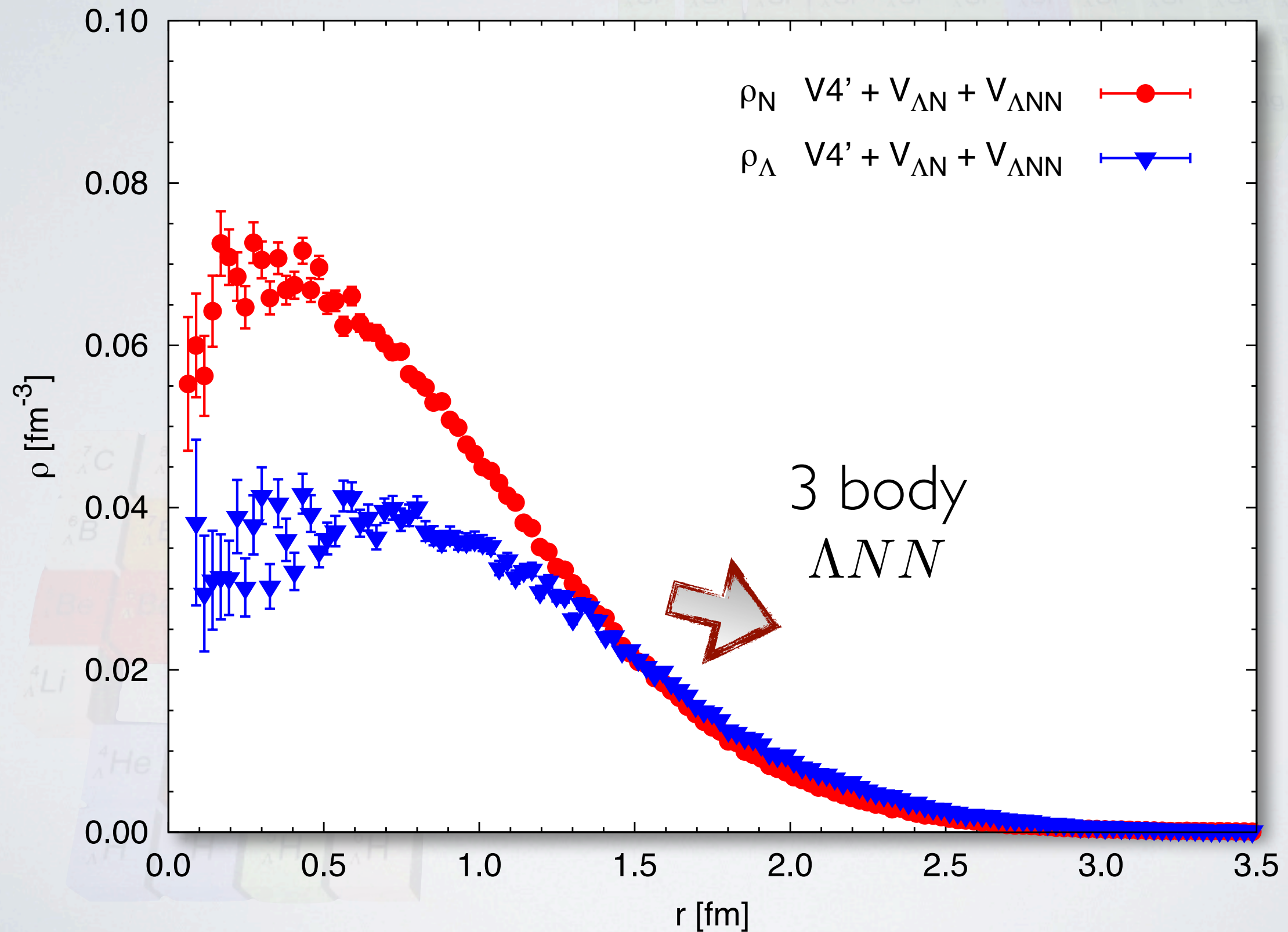
Preliminary results

${}^5_{\Lambda}\text{He}$ single particle density



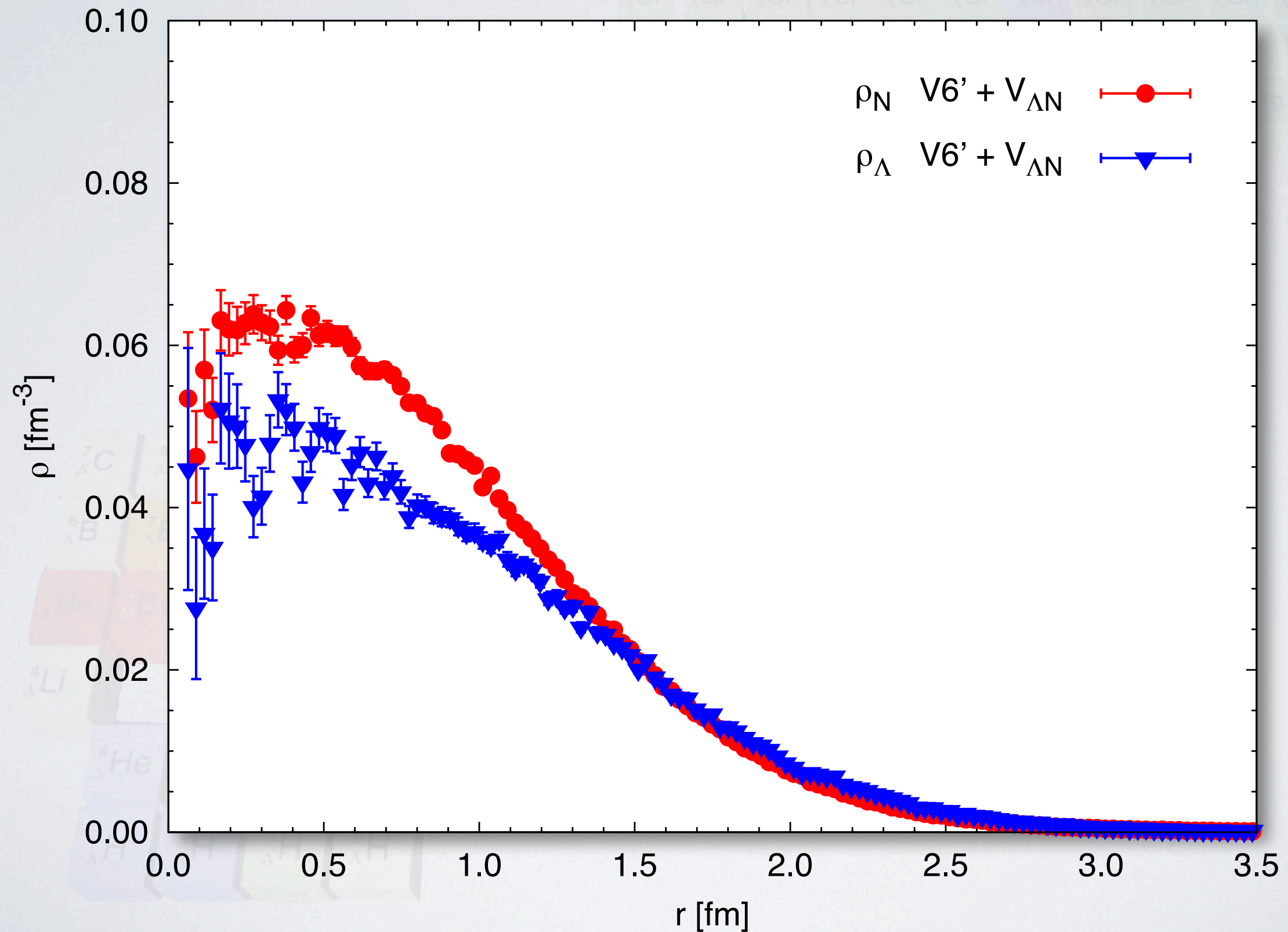
Preliminary results

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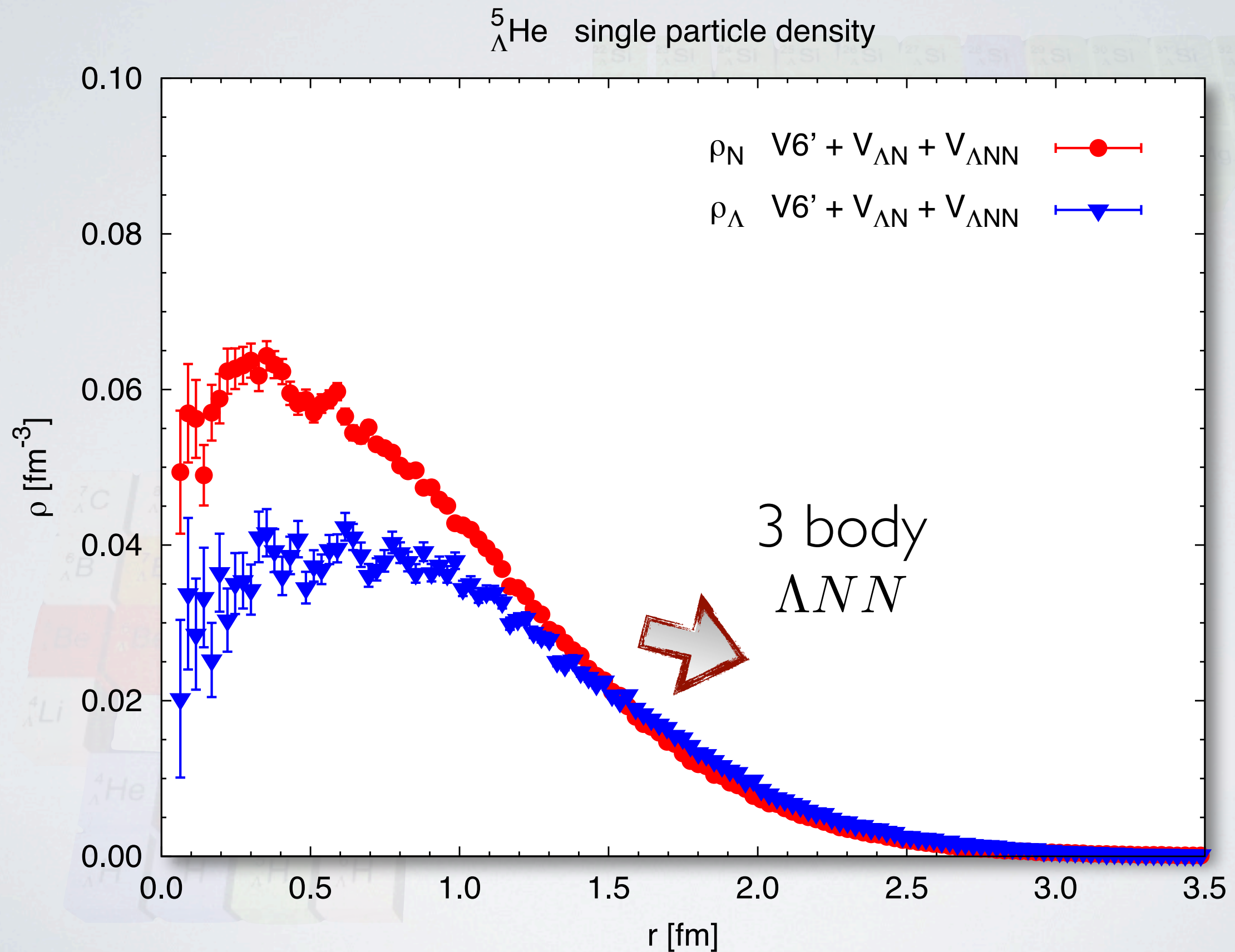


Preliminary results

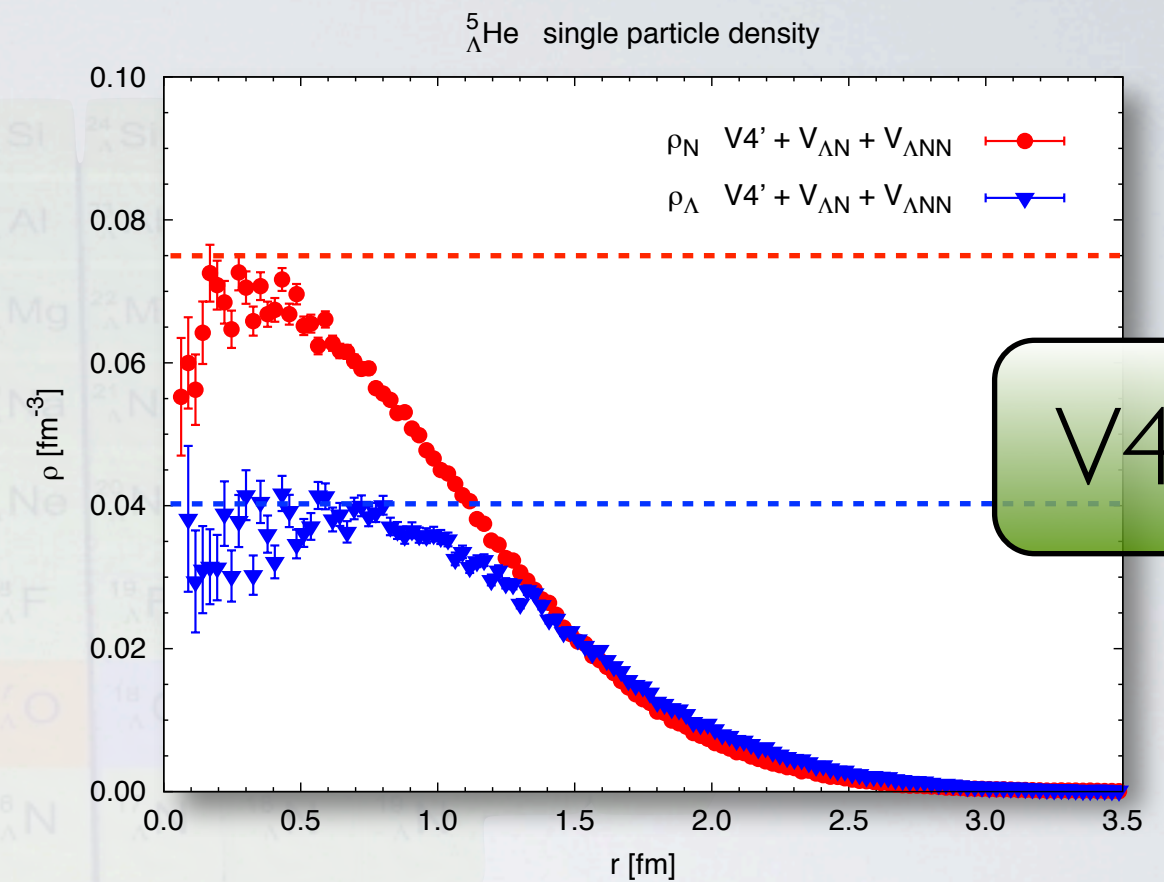
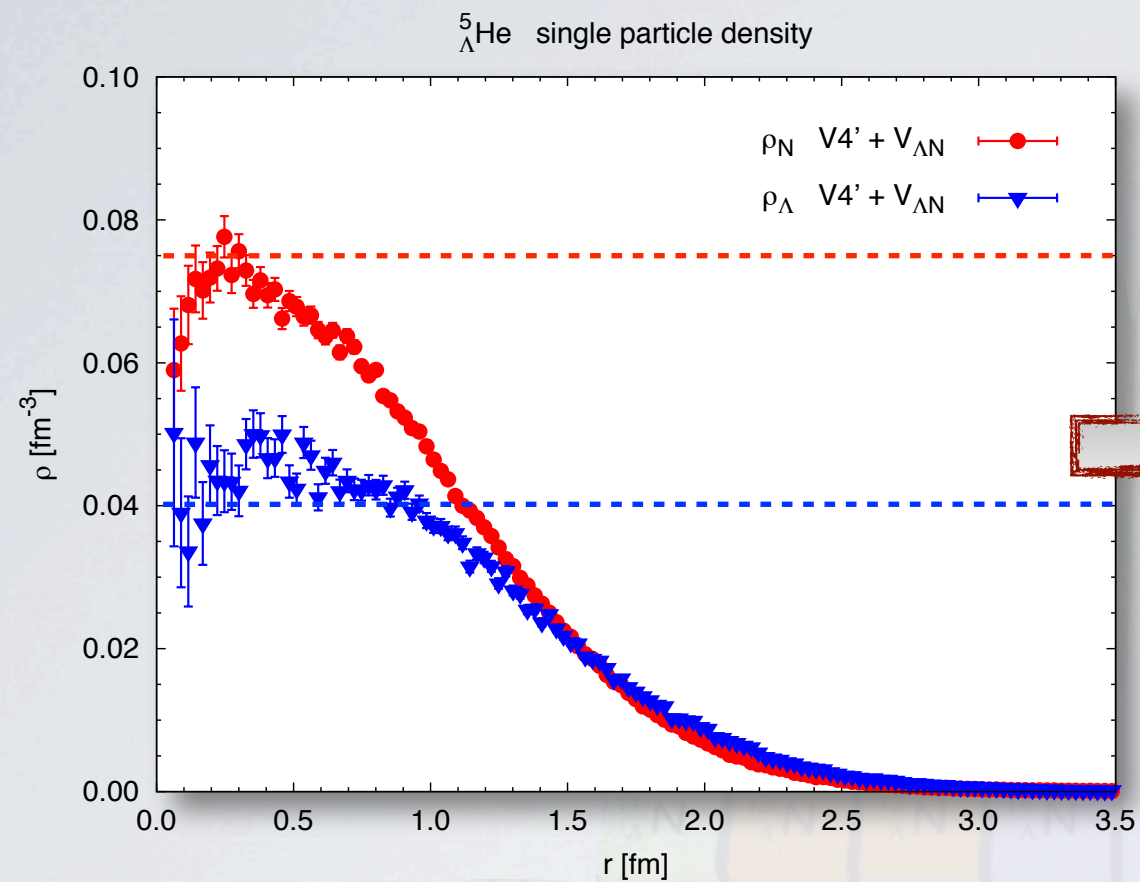
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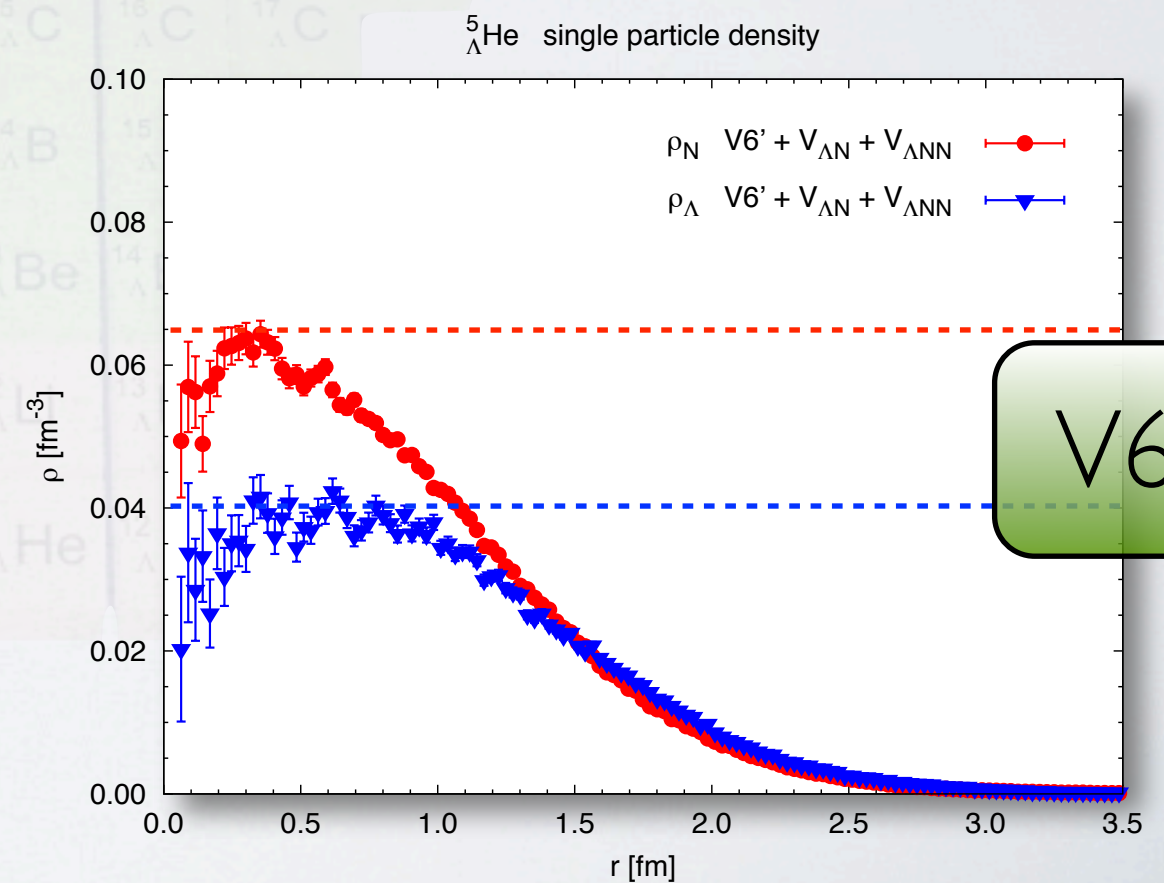
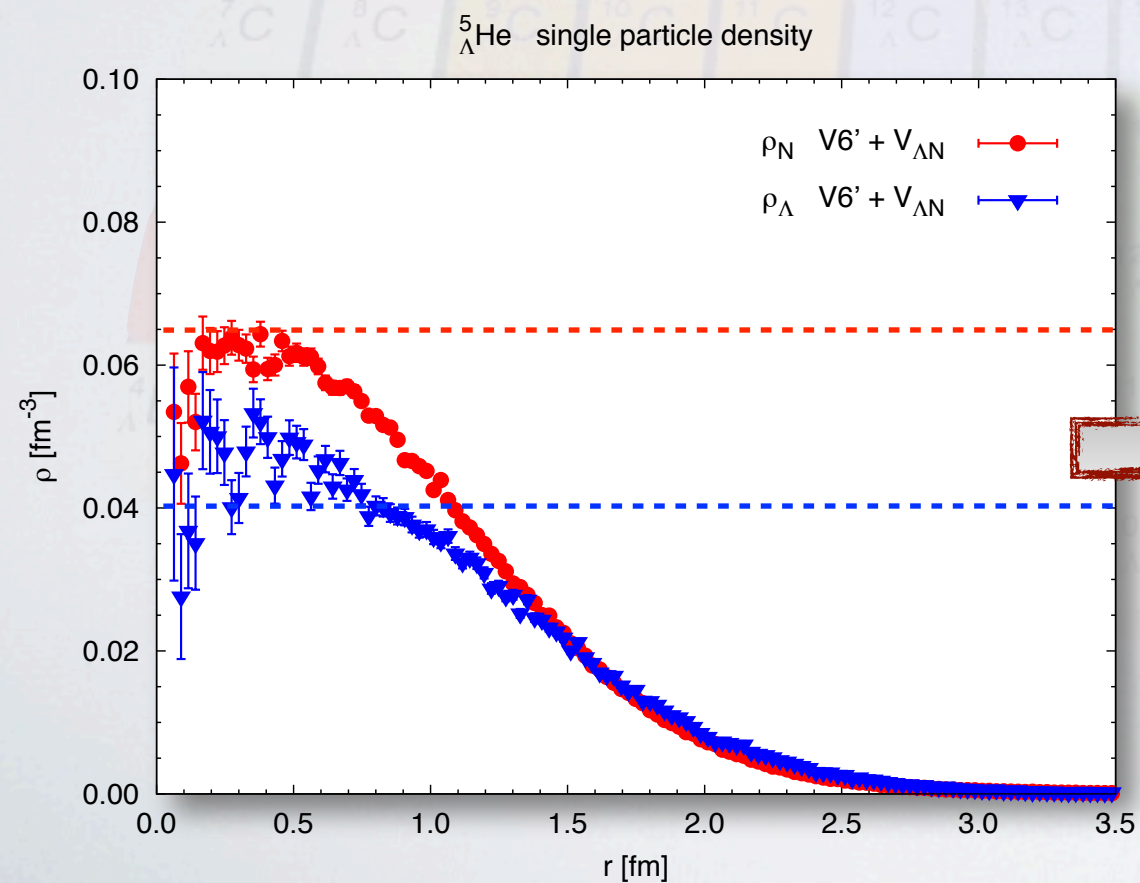
Preliminary results



Preliminary results



V4'



V6'

Results

	$BE_{no\Lambda}$	BE^{2B}	BE^{2B+3B}	B_{Λ}^{2B}	B_{Λ}^{2B+3B}	B_{Λ}^{exp}
${}^4_{\Lambda}\text{H}$	-8.64(2)	-11.22(5)	-10.27(6)	2.58(5)	1.63(6)	2.04(4)
${}^4_{\Lambda}\text{He}$	-8.63(2)	-11.16(3)	-10.18(4)	2.53(4)	1.55(4)	2.39(3)
${}^5_{\Lambda}\text{He}$	-32.56(2)	-39.54(4)	-36.77(3)	6.98(4)	4.21(4)	3.12(2)
${}^6_{\Lambda}\text{H}$	-2.11(5)	-6.43(6)	-5.18(6)	4.32(8)	3.07(8)	4(1)
${}^6_{\Lambda}\text{He}$	-28.05(6)	-36.72(6)	-33.69(5)	8.67(8)	5.64(8)	4.2(1)
${}^7_{\Lambda}\text{He}$	-29.62(3)	-40.07(3)	-34.17(4)	10.45(4)	4.55(5)	5.23(*)
${}^{13}_{\Lambda}\text{C}$	-78.35(8)	-105.66(9)	-93.23(8)	27.31(11)	14.88(10)	11.7(1)
${}^{16}_{\Lambda}\text{O}$	-147.9(2)	-188.5(3)	-164.3(2)	40.6(3)	16.4(3)	12.5(4)
${}^{17}_{\Lambda}\text{O}$	-180.7(2)	-224.7(3)	-197.0(2)	44.0(4)	16.3(3)	13.59(*)

Results

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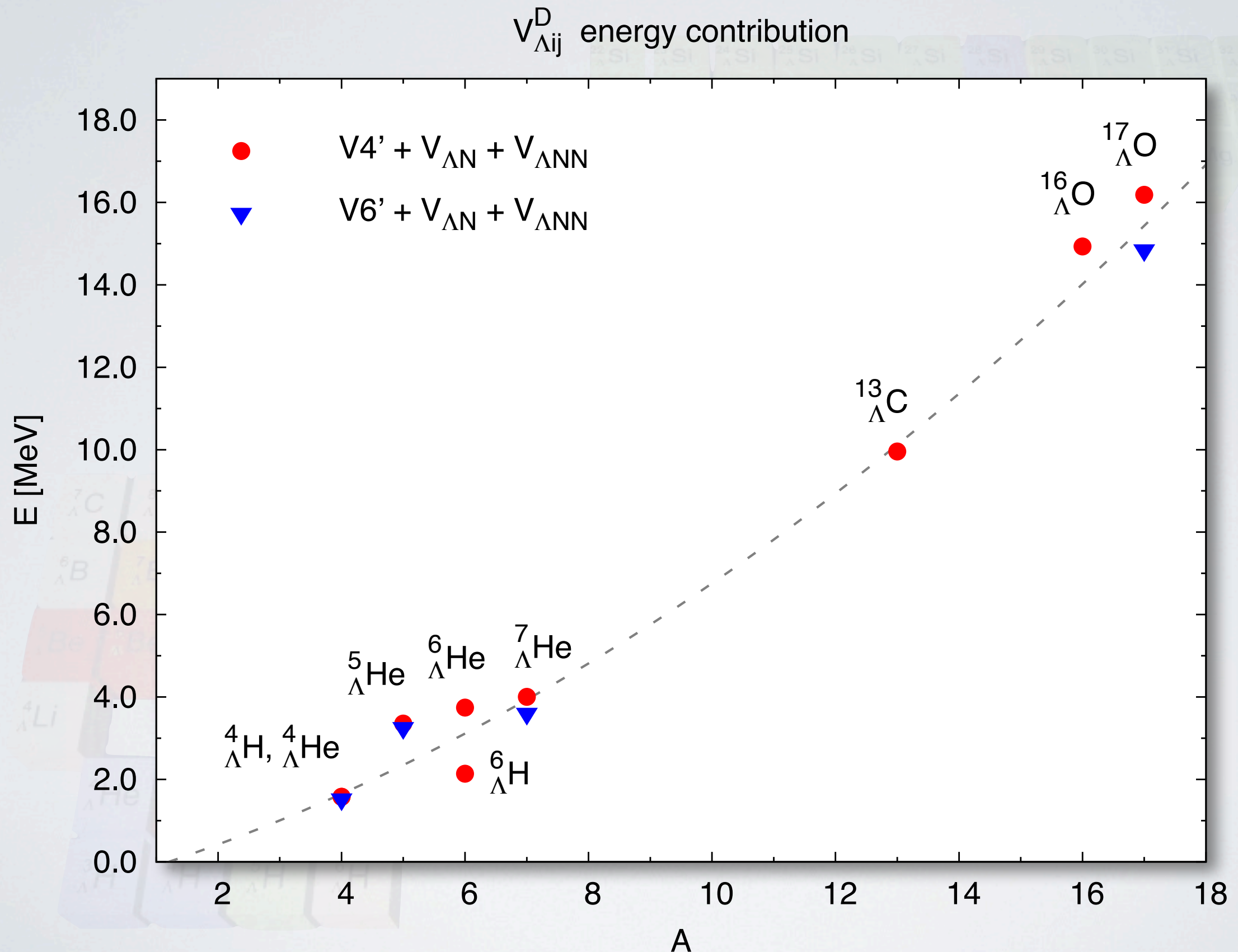
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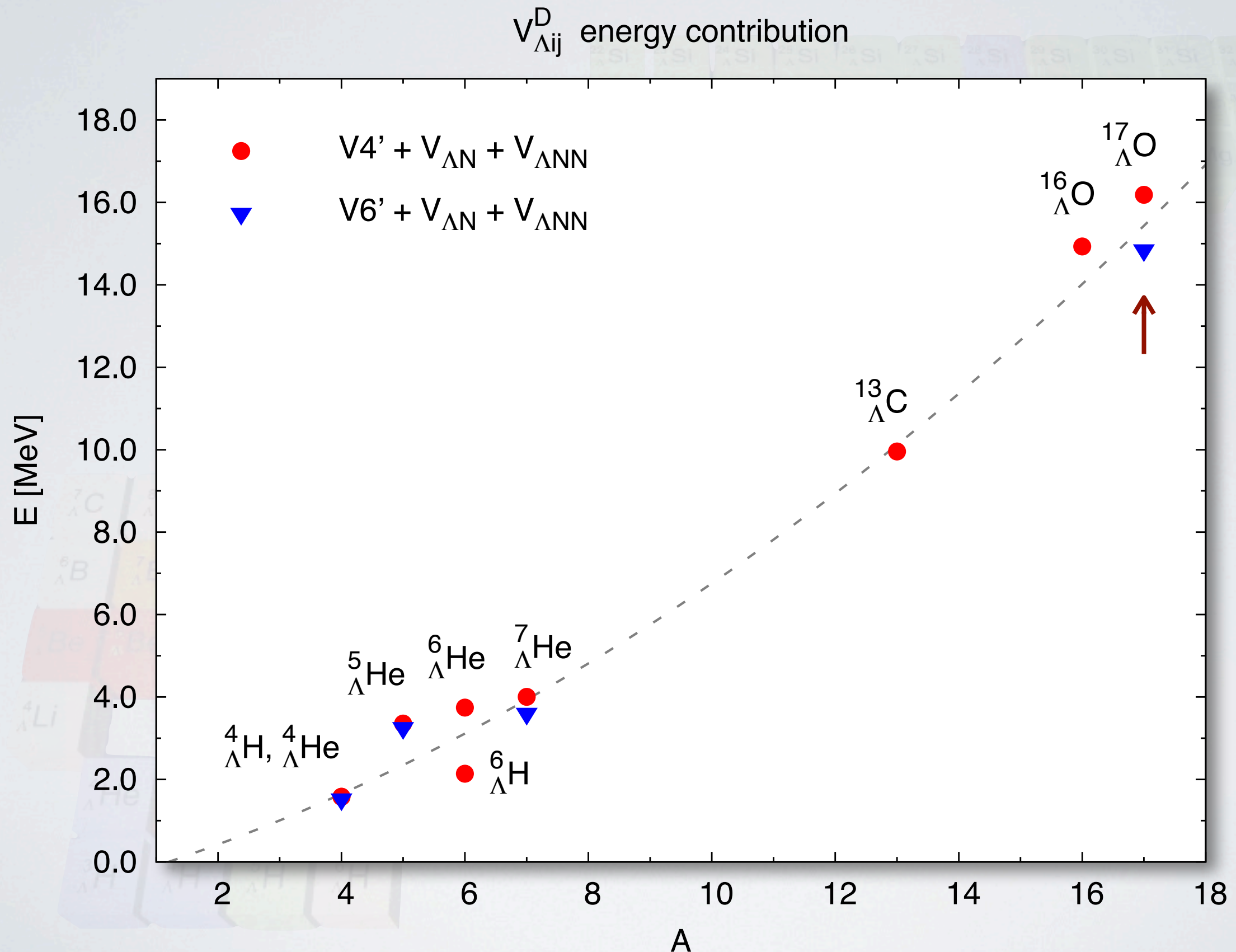
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BE_{nucl}^{exp} [MeV]	BE_{hyp}^{exp} [MeV]
${}^3\text{H}$ -8.482	${}^4_{\Lambda}\text{H}$ -10.522
${}^3\text{He}$ -7.718	${}^4_{\Lambda}\text{He}$ -10.108
${}^4\text{He}$ -28.296	${}^5_{\Lambda}\text{He}$ -31.416
${}^5\text{H}$ -2.744	${}^6_{\Lambda}\text{H}$ -6.744
${}^5\text{He}$ -27.406	${}^6_{\Lambda}\text{He}$ -31.606
${}^6\text{He}$ -29.269	${}^7_{\Lambda}\text{He}$ -34.499
${}^{12}\text{C}$ -92.162	${}^{13}_{\Lambda}\text{C}$ -103.862
${}^{15}\text{O}$ -111.956	${}^{16}_{\Lambda}\text{O}$ -124.456
${}^{16}\text{O}$ -127.619	${}^{17}_{\Lambda}\text{O}$ -141.209

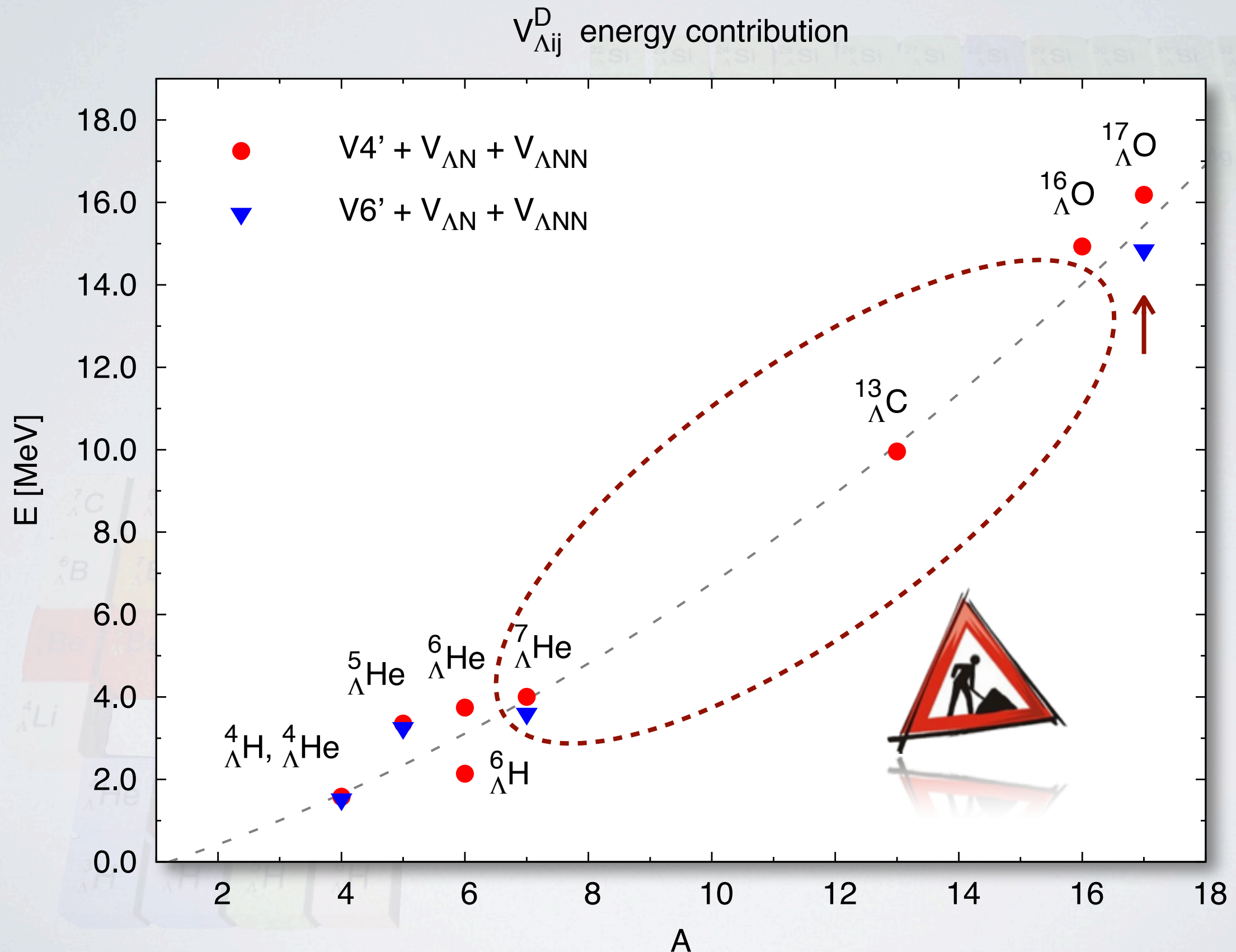
Preliminary results



Preliminary results



Preliminary results



Preliminary results

W^D	$C_{2\pi}^{SW}$	$C_{2\pi}^{PW}$	$B_{\Lambda} \ ^5_{\Lambda}\text{He}$	$B_{\Lambda} \ ^{17}_{\Lambda}\text{O}$
0.013	0.0	0.60	4.78(5)	
0.015	0.0	0.60	4.27(4)	16.4(3)
0.017	0.0	0.60	4.03(5)	
0.025	0.0	0.60	3.15(5)	10.3(5)
0.012	0.0	0.75	5.19(7)	
0.013	0.0	0.75	4.98(5)	
		B_{Λ}^{exp}	3.12(2)	13.59(*)

Preliminary results

old

W^D	$C_{2\pi}^{SW}$	$C_{2\pi}^{PW}$	$B_{\Lambda} \ ^5_{\Lambda}\text{He}$	$B_{\Lambda} \ ^{17}_{\Lambda}\text{O}$
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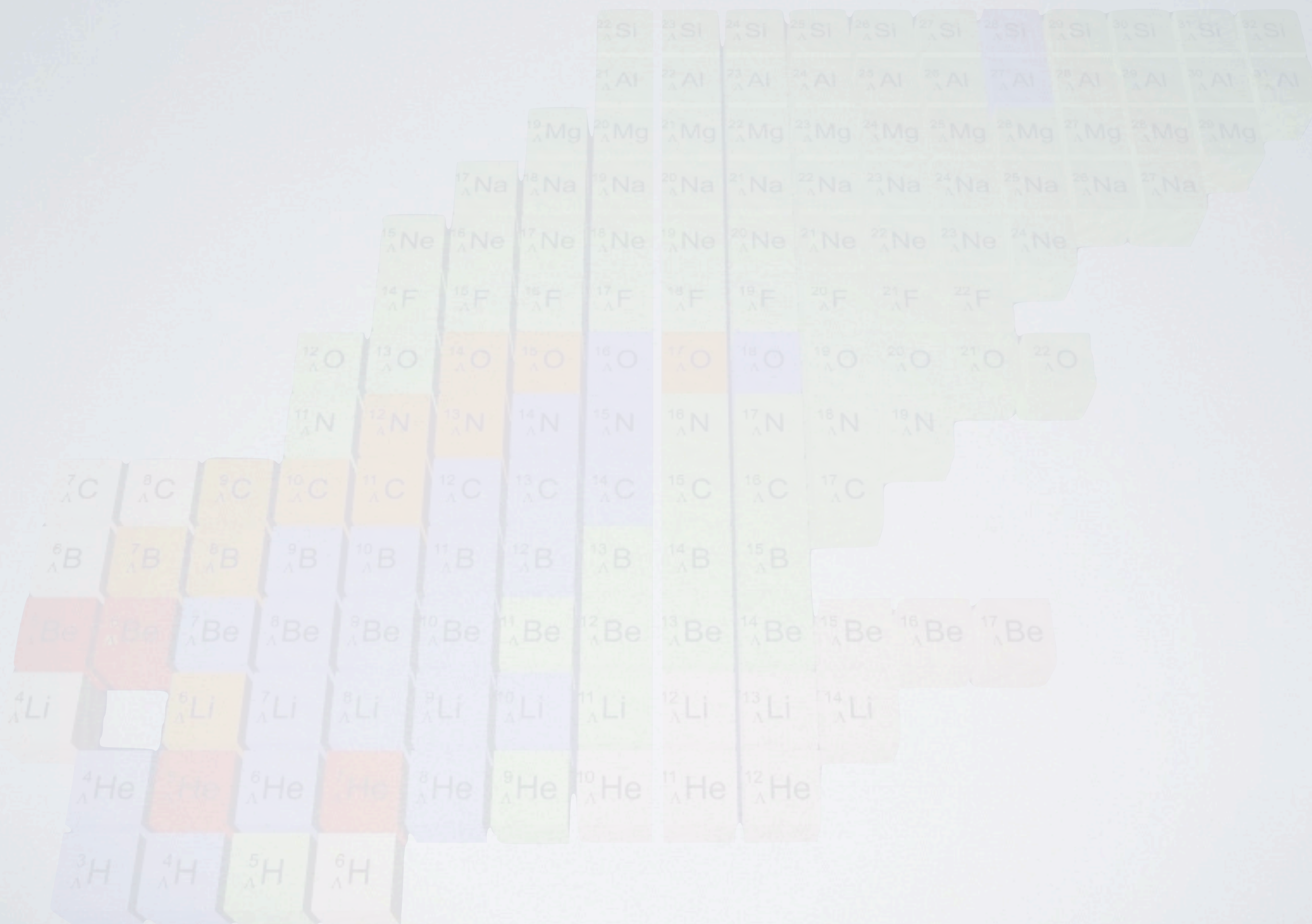
Preliminary results

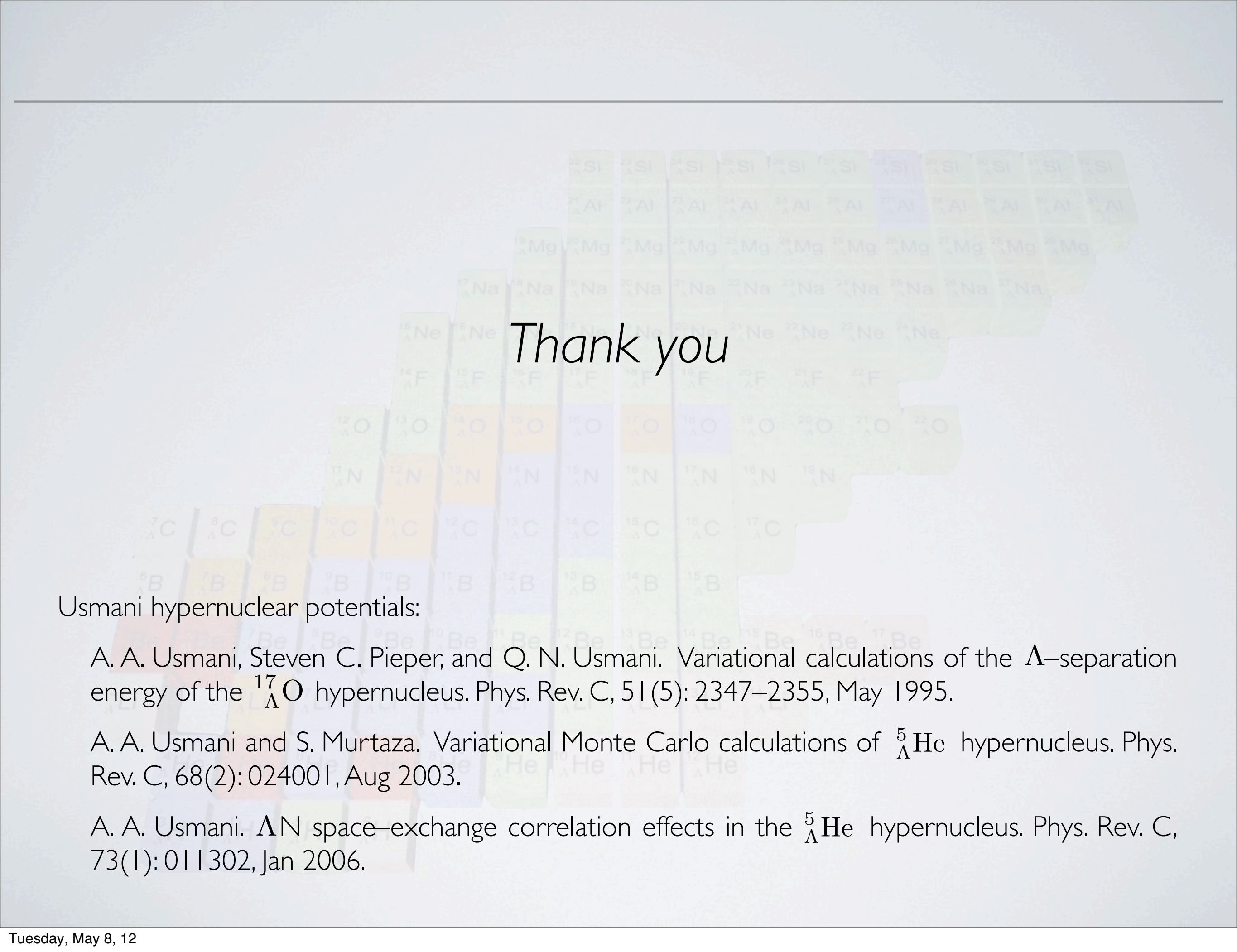
	W^D	$C_{2\pi}^{SW}$	$C_{2\pi}^{PW}$	$B_{\Lambda} \ ^5_{\Lambda}\text{He}$	$B_{\Lambda} \ ^{17}_{\Lambda}\text{O}$
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new	0.025	0.0	0.60	3.15(5)	10.3(5)
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	0.013	0.0	0.75	4.98(5)	
			B_{Λ}^{exp}	3.12(2)	13.59(*)

fine tuning of the parameters needed

Conclusions and perspectives

- AFDMC algorithm can be used to study hypernuclear systems: information about the ΛN interaction
- the 3-body ΛNN interaction is fundamental for the computation of the hyperon separation energy, but a fine tuning of the parameters is needed
- the 3-body ΛNN interaction is repulsive in hypernuclei: extrapolating to nuclear matter we might be lead to expect qualitatively different predictions of the NS properties
- the inclusion of a 2-body $\Lambda\Lambda$ interaction in the AFDMC algorithm is also possible: deeper investigation of hypernuclear potentials Λ





Thank you

Usmani hypernuclear potentials:

A. A. Usmani, Steven C. Pieper, and Q. N. Usmani. Variational calculations of the Λ -separation energy of the $^{17}_{\Lambda}\text{O}$ hypernucleus. Phys. Rev. C, 51(5): 2347–2355, May 1995.

A. A. Usmani and S. Murtaza. Variational Monte Carlo calculations of $^5_{\Lambda}\text{He}$ hypernucleus. Phys. Rev. C, 68(2): 024001, Aug 2003.

A. A. Usmani. Λ N space-exchange correlation effects in the $^5_{\Lambda}\text{He}$ hypernucleus. Phys. Rev. C, 73(1): 011302, Jan 2006.

Usmani hypernuclear potential

Constant	Value	Unit
$m_{\pi^{\pm}}$	139.57018(35)	MeV
m_{π^0}	134.9766 (6)	MeV
W_c	2137	MeV
r_0	0.5	fm
a	0.2	fm
ε	$0.1 \div 0.38$	—
$\bar{v}$	6.15(5)	MeV
v_s	6.33, 6.28, 6.23	MeV
v_t	6.09, 6.04, 5.99	MeV
v_σ	0.24	MeV
c	2.0	fm^{-2}
W^D	$0.01 \div 0.05$	MeV
C^P	$0.4 \div 2.0$	MeV
C^S	~ 1.5	MeV

Diffusion Monte Carlo

DMC = projection method

$$\tau = \frac{it}{\hbar}$$



$$-\frac{\partial}{\partial \tau} \psi(\mathbf{R}, \tau) = \mathcal{H} \psi(\mathbf{R}, \tau)$$

$$\begin{aligned} \psi(\mathbf{R}, \tau) &= e^{-(\mathcal{H}-E_0)\tau} \psi(\mathbf{R}, 0) & \psi(\mathbf{R}, 0) &= \sum_{n=0}^{\infty} c_n \varphi_n(\mathbf{R}) \\ &= \sum_{n=0}^{\infty} c_n e^{-(E_n-E_0)\tau} \varphi_n(\mathbf{R}) \end{aligned}$$



$$\lim_{\tau \rightarrow \infty} \psi(\mathbf{R}, \tau) = c_0 \varphi_0(\mathbf{R})$$

Diffusion Monte Carlo

$$\begin{aligned}\psi(\mathbf{R}, \tau + d\tau) &= \langle \mathbf{R} | \psi(\tau + d\tau) \rangle \\ &= \int \underbrace{\langle \mathbf{R} | e^{-(\mathcal{H} - E_0)d\tau} | \mathbf{R}' \rangle}_{G(\mathbf{R}, \mathbf{R}', d\tau)} \underbrace{\langle \mathbf{R}' | \psi(\tau) \rangle}_{\text{walkers}} d\mathbf{R}'\end{aligned}$$

kinetic term potential term

