Breakup Reactions and Spectroscopic Factors: a Theoretical Viewpoint

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Breakup reaction

Breakup used to study exotic nuclear structures
e.g. halo nuclei:

- large matter radius
- small $S_{\rm n}$ or $S_{\rm 2n}$

$\Rightarrow$ seen as dense core with neutron halo

Short lived $\Rightarrow$ studied through reactions like breakup:
halo dissociates from core by interaction with target

Information sought through reactions:

- Binding energy (e.g. ^{19}C)
- lj of halo neutron(s) (e.g. ^{31}Ne)
- SF

Introduction

Reaction models rely on single-particle model of a two-body projectile (core c + fragment f):

$[T_r + V(r) - \epsilon]\phi_{nlj}(r) = 0$,

with $\int_0^\infty |\phi_{nlj}(r)|^2 dr = 1$

In reality, there is admixture of configurations:

$$^{A}Y(J^\pi) = \ ^{A-1}X(J^\pi_c) \otimes f(lj) + \ldots$$

The overlap wave function is

$$\psi_{lj}(r) = \langle ^{A-1}X(J^\pi_c)|a_{lj}(r)|^{A}Y(J^\pi)\rangle$$

Spectroscopic Factor: $\mathcal{S}_{lj} = \int_0^\infty |\psi_{lj}(r)|^2 dr$

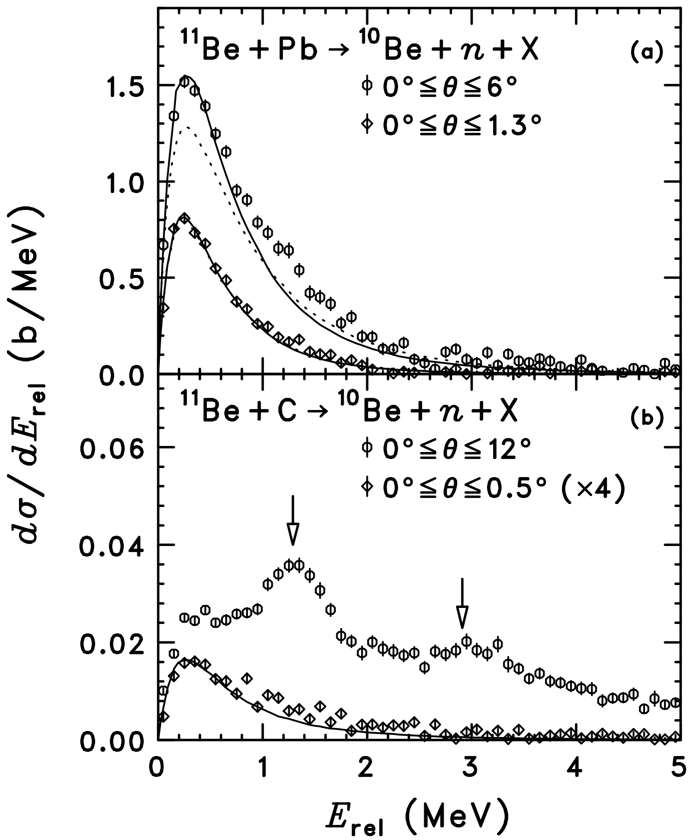
Single-particle approximation $\equiv \psi_{lj} = \sqrt{\mathcal{S}_{lj}}\phi_{nlj}$

$\Rightarrow$ usual idea: $\mathcal{S}_{lj} = \sigma^{exp}_{\rm bu}/\sigma^{th}_{\rm bu}$

^{11}Be+Pb→^{10}Be+n+Pb @69AMeV

Experiment:

[Fukuda *et al.* PRC 70, 054606 (2004)]

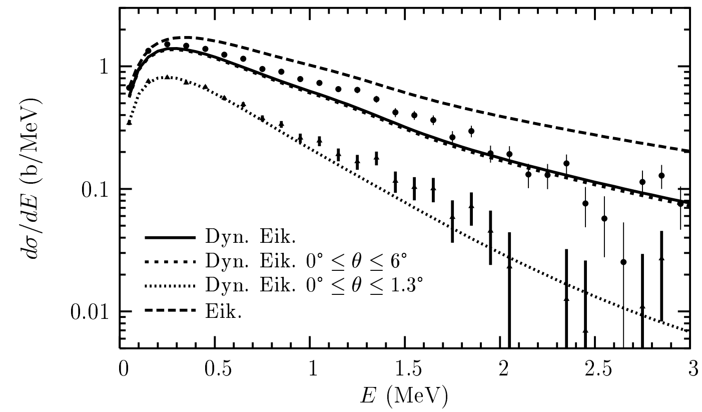


They get $\mathcal{S}_{s1/2} = 0.72$

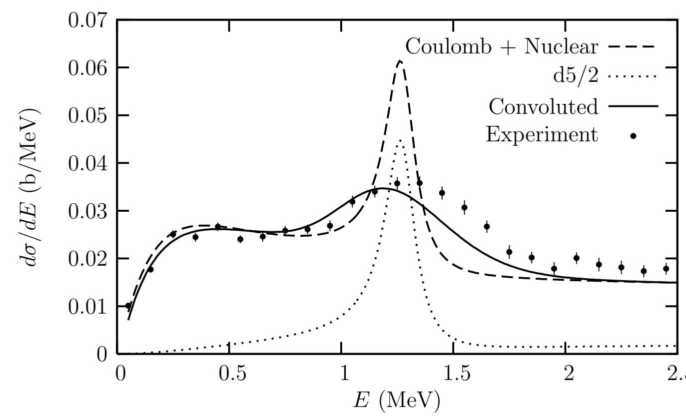
for ^{10}Be$(0^+)\otimes$n$(2s_{1/2})$

(our) Theory:

[Goldstein *et al.* PRC 73, 024602 (2006)]



[PC *et al.* PRC 70, 064605 (2004)]



With $\mathcal{S}_{s1/2}$=1

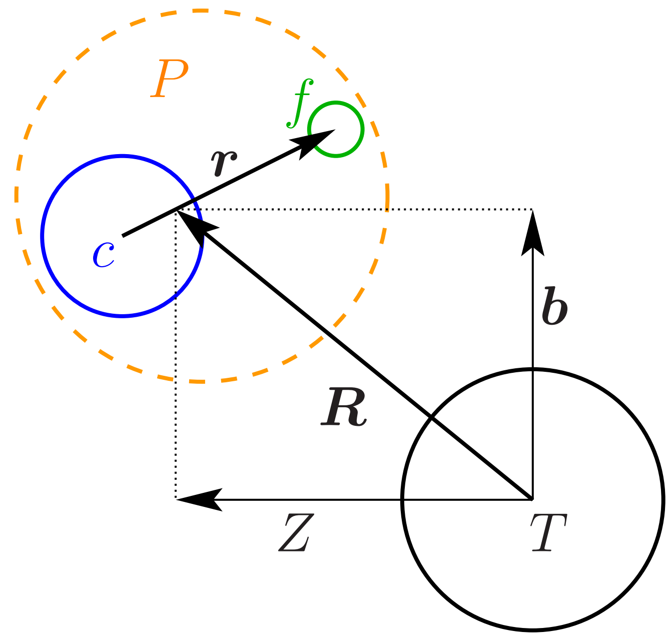
Outline

- Breakup models: CDCC, Time-Dependent,
  Dynamical Eikonal Approximation
- What do we probe in breakup ?
  - Peripherality of breakup reactions (ANC vs SF)
  - Description of the continuum
  - Projectile-target interaction (V_{PT})
- Influence of couplings upon halo wave function
  Can we get SF from ANC?
- Ratio of angular distributions:
  a new way to remove V_{PT} dependence
- Conclusion

Framework

Projectile (P) modelled as a two-body system:
core (c)+loosely bound fragment (f) described by

$$H_0 = T_r + V_{cf}(\boldsymbol{r})$$



V_{cf} adjusted to reproduce bound state Φ_0 and resonances

Target T seen as structureless particle

P-T interaction simulated by optical potentials
$\Rightarrow$ breakup reduces to three-body scattering problem:

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{R},\boldsymbol{r}) = E_T\Psi(\boldsymbol{R},\boldsymbol{r})$$

with initial condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\Phi_0(\boldsymbol{r})$

CDCC

Solve the three-body scattering problem:

$$[T_R + H_0 + V_{cT} + V_{fT}]\,\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

by expanding Ψ on eigenstates of H_0

$\Psi(\boldsymbol{r},\boldsymbol{R}) = \sum_i \chi_i(\boldsymbol{R})\Phi_i(\boldsymbol{r})$ with $H_0\Phi_i = \epsilon_i\Phi_i$

Leads to set of coupled-channel equations (hence CC)

$$[T_R + \epsilon_i + V_{ii}]\,\chi_i + \sum_{j\neq i} V_{ij}\chi_j = E_T\chi_i,$$

with $V_{ij} = \langle\Phi_i|V_{cT} + V_{fT}|\Phi_j\rangle$

The continuum has to be discretised (hence CD)

[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]

Fully quantal approximation

No approx. on P-T motion, no restriction on energy

But expensive computationally (at high energies)

Time-dependent model

P-T motion described by classical trajectory $\boldsymbol{R}(t)$

[Esbensen, Bertsch and Bertulani, NPA 581, 107 (1995)]

[Typel and Wolter, Z. Naturforsch. A54, 63 (1999)]

P structure described quantum-mechanically by H_0

Time-dependent potentials simulate P-T interaction

Leads to the resolution of time-dependent Schrödinger equation (TD)

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},\boldsymbol{b},t) = [H_0 + V_{cT}(t) + V_{fT}(t)]\Psi(\boldsymbol{r},\boldsymbol{b},t)$$

Solved for each $\boldsymbol{b}$ with initial condition $\Psi \underset{t\to-\infty}{\longrightarrow} \Phi_0$

Many programs have been written to solve TD

Lacks quantum interferences between trajectories

Dynamical Eikonal Approximation

Three-body scattering problem:

$$[T_R + H_0 + V_{cT} + V_{fT}]\,\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0$

Eikonal approximation: factorise $\Psi = e^{iKZ}\widehat{\Psi}$

$$T_R\Psi = e^{iKZ}[T_R + vP_Z + \frac{\mu_{PT}}{2}v^2]\widehat{\Psi}$$

Neglecting T_R vs P_Z and using $E_T = \frac{1}{2}\mu_{PT}v^2 + \epsilon_0$

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi} \underset{Z\to-\infty}{\longrightarrow} \Phi_0(\boldsymbol{r})$

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

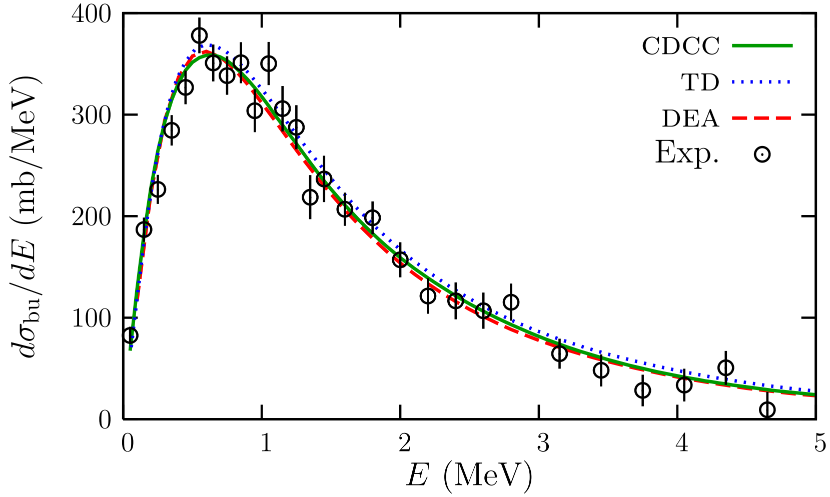
Same equation as TD with straight line trajectories

^{15}C + Pb @ $68A$MeV

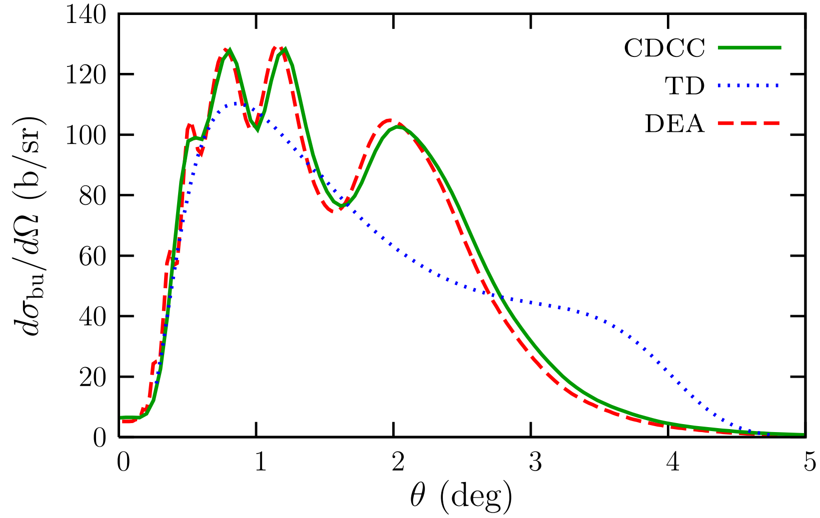
Comparison of CDCC, TD, and DEA

[PC, Esbensen, and Nunes, PRC 85, 044604 (2012)]

$d\sigma_{\rm bu}/dE$



$d\sigma_{\rm bu}/d\Omega$



All models agree

Data: [Nakamura *et al.*

PRC 79, 035805 (2009)]

DEA agrees with CDCC

TD reproduces trend

but lacks oscillations

ANC vs SF

Is $\mathcal{S}_{lj} = \sigma^{exp}_{\rm bu} / \sigma^{th}_{\rm bu}$?

Is breakup really sensitive to SF ?
i.e. do we probe the whole overlap wave function ?

Isn't breakup peripheral?
i.e. sensitive only to asymptotics ?

$$\psi_{lj}(r) \underset{r\to\infty}{\longrightarrow} \mathcal{C}_{lj}\, e^{-\kappa r}$$

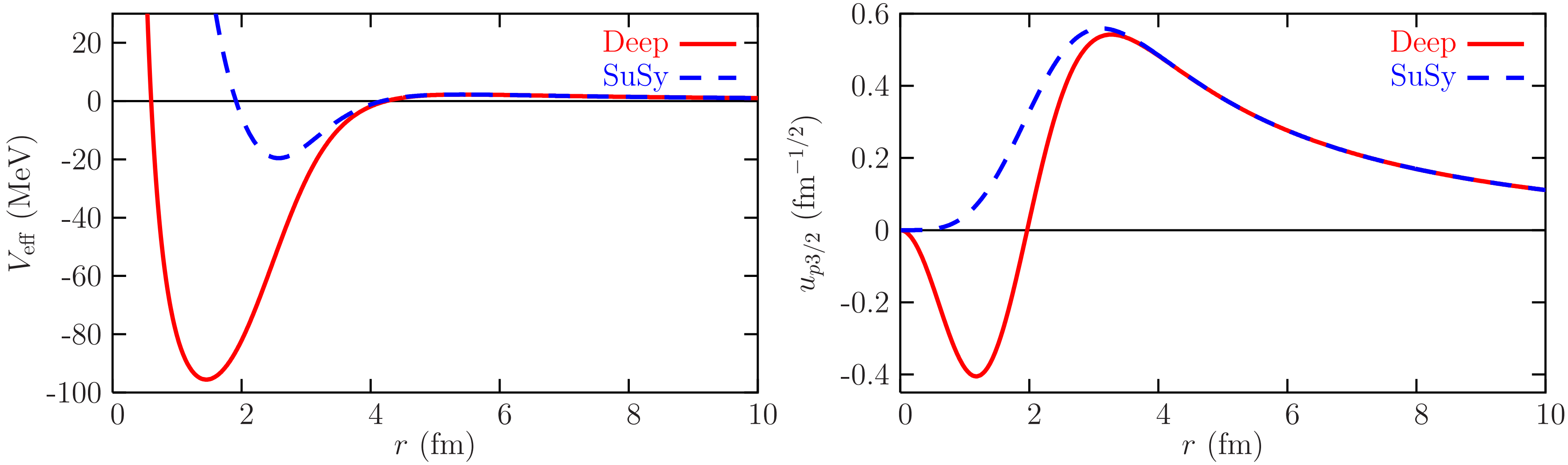
Asymptotic Normalisation Coefficient: $\mathcal{C}_{lj}$

Test this with two descriptions of projectile
with different interiors but same asymptotics.

[PC and Nunes, PRC 75, 054609 (2007)]

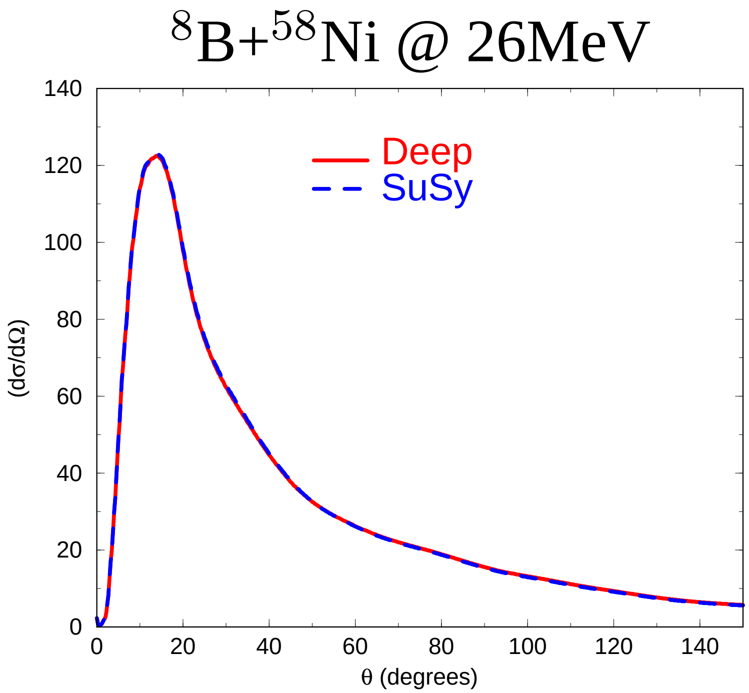
SuSy transformations

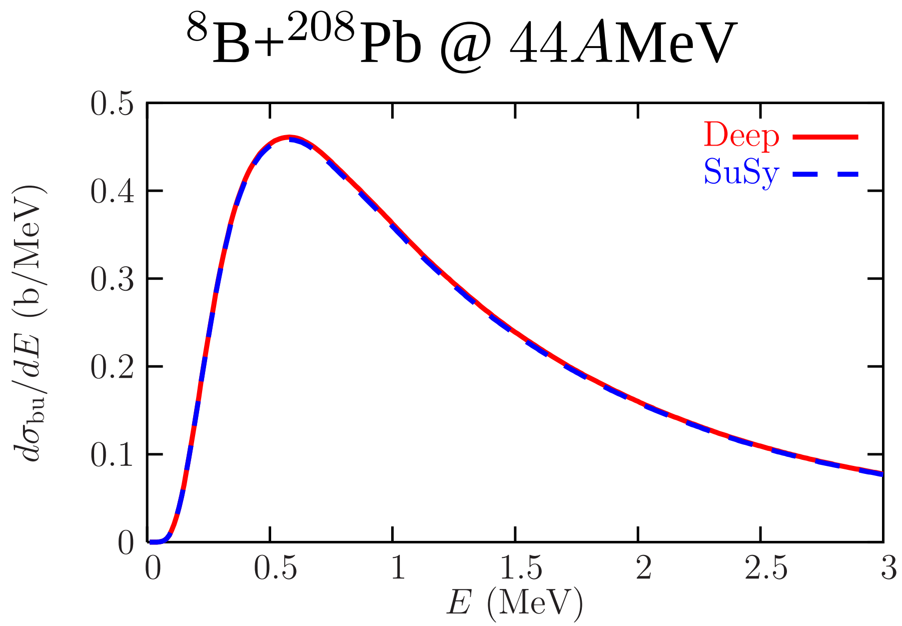
Use 2 V_{cf} with different interior but same asymptotics obtained by SuSy transfo. [D. Baye PRL 58, 2738 (1987)]



- Deep potential ⇒ spurious deep bound state ⇒ node in physical bound state
- Remove deep state by SuSy ⇒ remove node but keep same asymptotics (ANC and phase shift)
- Analyse difference in $\sigma_{\rm bu}^{th}$ between deep vs SuSy

Peripherality of breakup reactions





No difference between deep and SuSy potentials

at low and intermediate energies, on light and heavy targets,
for energy and angular distributions

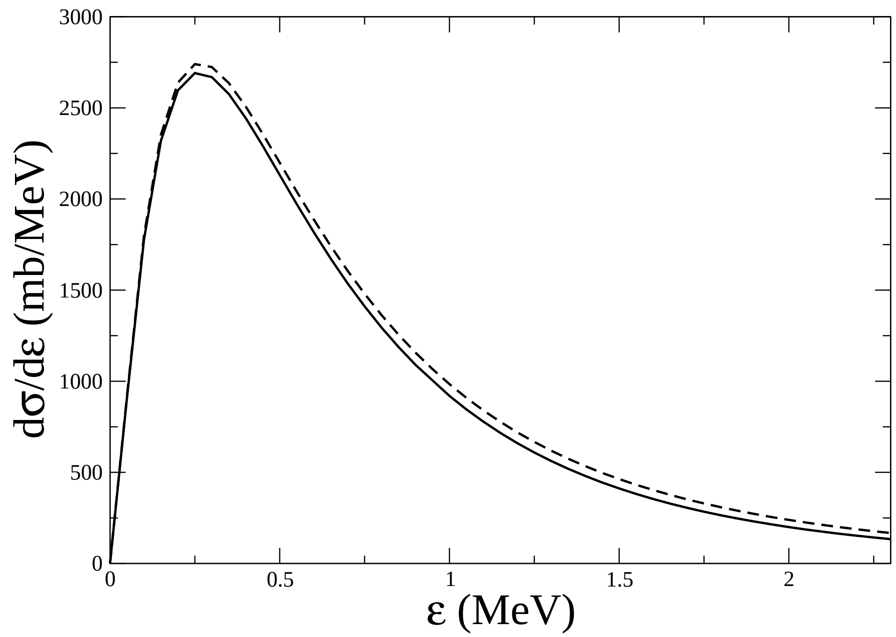
⇒ breakup probes only ANC

⇒ SF extracted from measurements are questionable?

[PC, Nunes, PRC 75, 054609 (2007)]

Similar study

Garcia-Camacho et al. NPA 776, 118 (2006)



Using either single particle wave function (solid) or its asymptotic expansion (dashed)

⇒ same conclusion with SF≠1

Asymptotic version

ψ_{lj} and ϕ_{nlj} exhibit same asymptotics:

$$\psi_{lj}(r) \underset{r\to\infty}{\longrightarrow} \mathcal{C}_{lj}\, e^{-\kappa r} \qquad \phi_{nlj}(r) \underset{r\to\infty}{\longrightarrow} b_{nlj}\, e^{-\kappa r}$$

$\Rightarrow$ Asymptotic version of the single-particle approx.:

$$\psi_{lj} \underset{r\to\infty}{\longrightarrow} \frac{\mathcal{C}_{lj}}{b_{nlj}} \phi_{nlj} \Rightarrow \mathcal{S}_{lj} = \frac{\mathcal{C}_{lj}^2}{b_{nlj}^2}$$

Since ANC accessible to breakup reactions,
can we still extract SF from reaction data?

What effects of couplings between configurations ?

- ψ_{lj} compared to ϕ_{nlj}
- SF $\mathcal{S}_{lj}$
- ANC $\mathcal{C}_{lj}$

c-f system with couplings

We use a model where core can be in different states $\Phi_i(\xi)$ described as levels of deformed rotor

$$\Psi^{J^\pi} = \sum_i \psi_i(r)\mathcal{Y}_i(\Omega)\Phi_i(\xi)$$

The c-f Hamiltonian reads [Nunes NPA 596, 171 (1996)]

$$H_0 = H_c + T_r + V_{cf}(\boldsymbol{r},\beta,\xi)$$

with $V_{cf}(\boldsymbol{r},\beta,\xi) = V_0\left[1+\exp\left(\frac{r-R_0[1+\beta Y_2^0(\Omega)]}{a}\right)\right]^{-1}$

$\Rightarrow$ set of coupled equations

$$[T_r + V_{ii}(r) + E_i - \epsilon]\psi_i(r) = -\sum_{i'\neq i} V_{ii'}(r)\psi_{i'}(r),$$

with $V_{ii'}(r) = \langle\Phi_i(\xi)\mathcal{Y}_i(\Omega)|V_{cf}(\boldsymbol{r},\beta,\xi)|\Phi_{i'}(\xi)\mathcal{Y}_{i'}(\Omega)\rangle$

We analyse the validity of single-particle approx. for one-neutron halo nucleus ^{11}Be

[PC, Danielewicz, Nunes, PRC 82, 054612 (2010)]

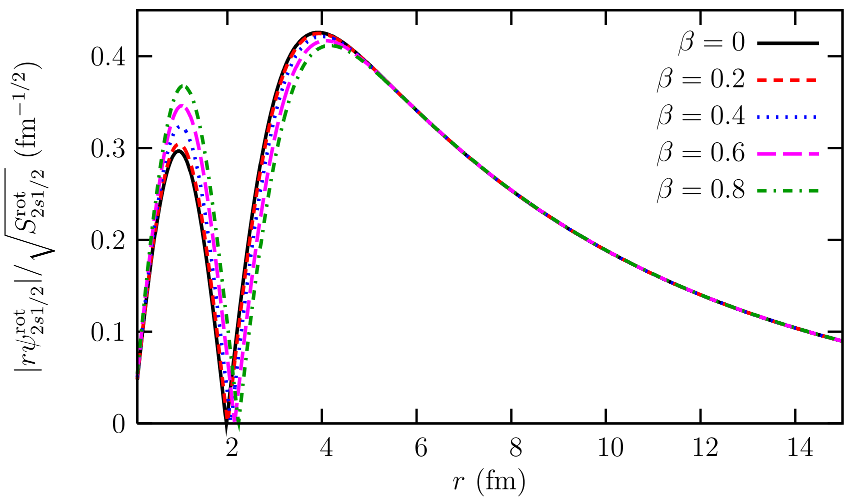
Influence of coupling (ψ vs. ϕ)

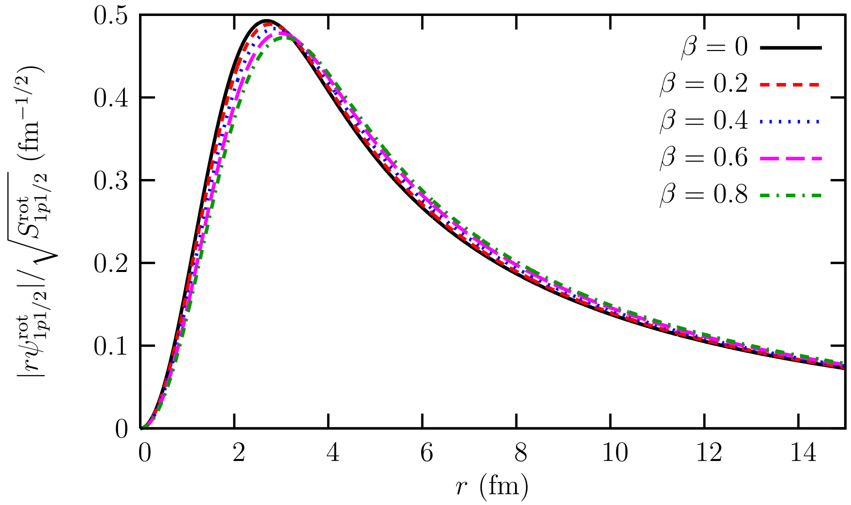
^{11}Be $\equiv$ ^{10}Be+n has two bound states

- $\varepsilon_{1/2^+} = -0.504$ MeV
- $\varepsilon_{1/2^-} = -0.184$ MeV

$$\Psi^{1/2^+} = \psi_{s1/2}\Phi_{0^+} + \psi_{d3/2}\Phi_{2^+} + \psi_{d5/2}\Phi_{2^+}$$

$$\Psi^{1/2^-} = \psi_{p1/2}\Phi_{0^+} + \psi_{p3/2}\Phi_{2^+} + \psi_{f5/2}\Phi_{2^+}$$





$\Rightarrow$ single-particle approx. fails: $\psi_{lj}(r) \neq \sqrt{\mathcal{S}_{lj}}\phi_{nlj}(r)$

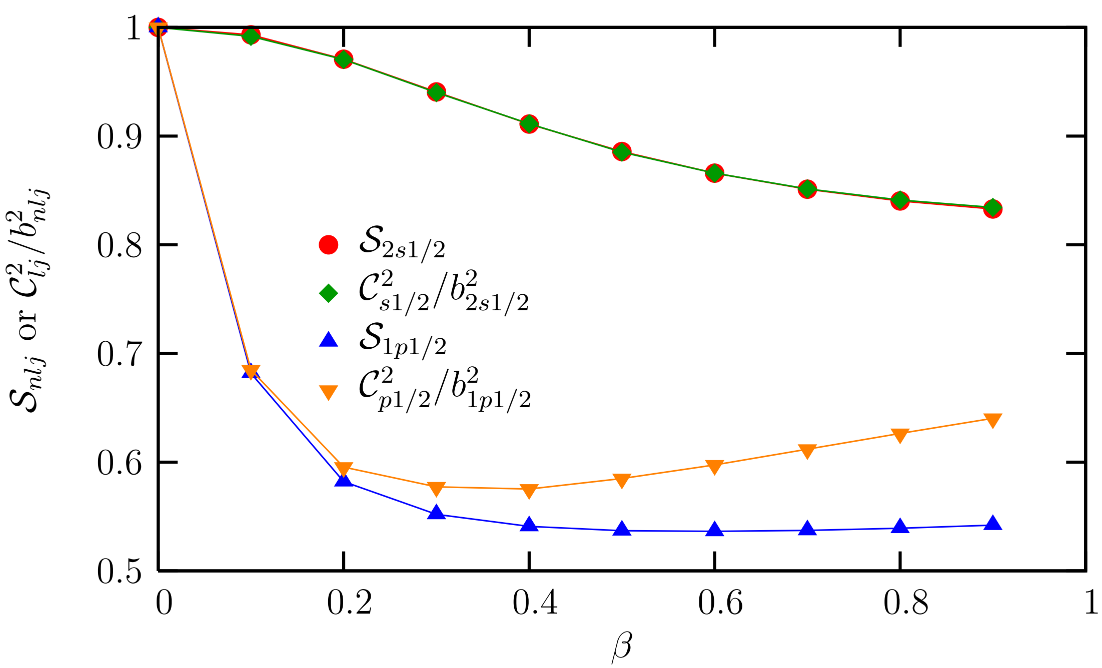
But, for the ground state, $\psi_{s_{1/2}} \xrightarrow[r\to\infty]{} \sqrt{\mathcal{S}_{s_{1/2}}}\phi_{2s_{1/2}} \quad \forall\beta$

Comparing $\mathcal{S}$ and $\mathcal{C}^2/b^2$

We find $\psi_{s_{1/2}} \xrightarrow[r\to\infty]{} \sqrt{\mathcal{S}_{s_{1/2}}}\phi_{2s_{1/2}} \quad \forall\beta$

$\Rightarrow$ Asymptotic version of single particle approx.?

i.e. is $\mathcal{C}^2_{lj}/b^2_{nlj}$ a good approx. of $\mathcal{S}_{lj}$?

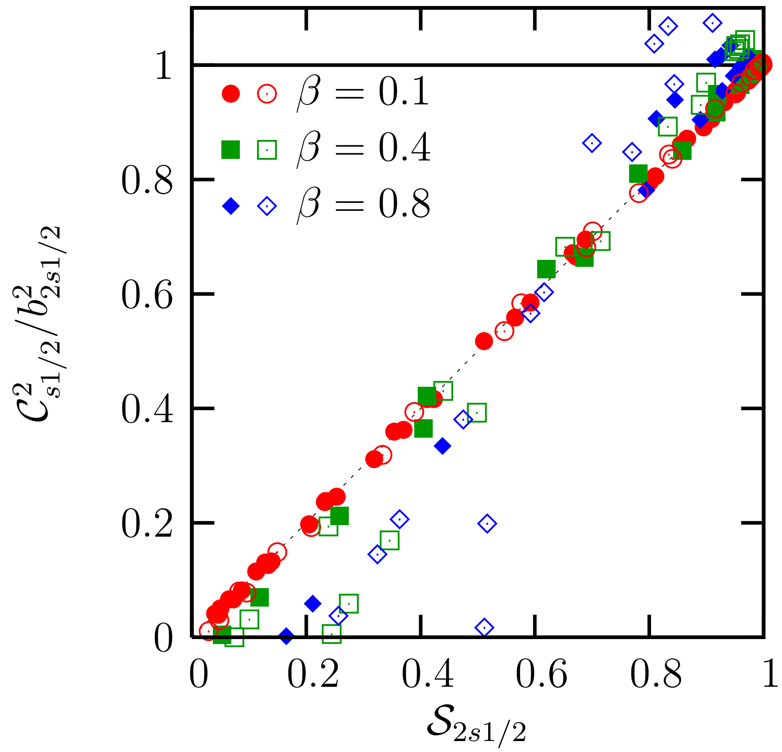


- g.s.: Small admixture, approx. OK
- e.s.: Large admixture, approx. fails $\beta > 0.2$

$\Rightarrow$ Approx. breaks at large admixture and/or coupling?

Exploring the model

To understand this, we push the model to its limits



- General trend validates $\mathcal{S} \sim \mathcal{C}^2/b^2$
- Very large admixture obtained even for small β
- Approx. breaks down at large couplings for large admixtures

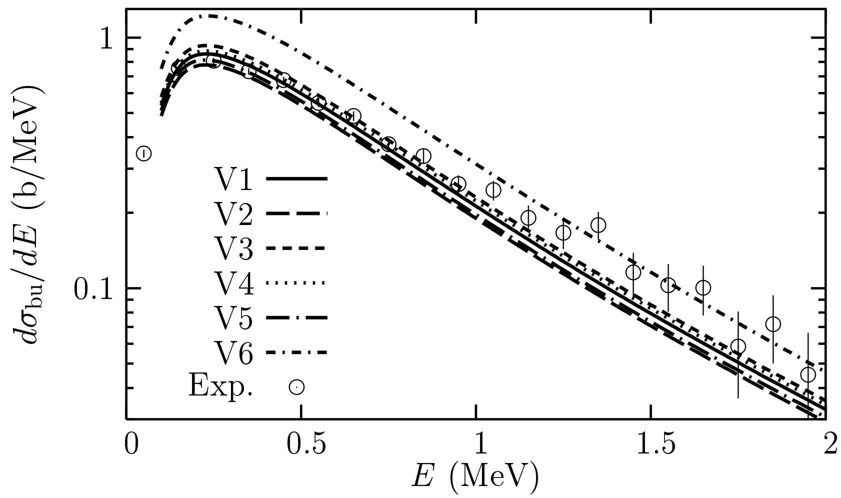
$\Rightarrow \mathcal{S} \sim \mathcal{C}^2/b^2$ for small coupling strength β and/or when component is dominant (i.e. large $\mathcal{S}$)
i.e. when coupling term in equations is small

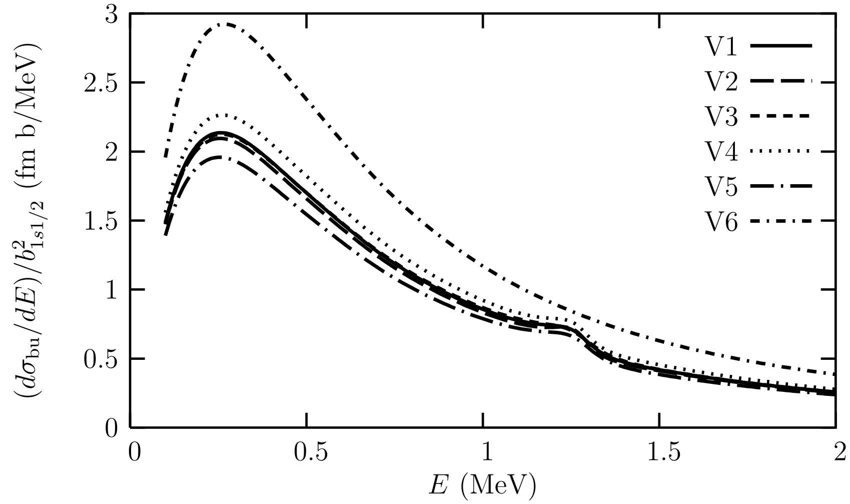
Sensitivity to the c-f continuum

Is breakup sensitive only to bound-state properties?

Influence of c-f continuum

^{11}Be on Pb @ $69A$MeV





[PC, Nunes, PRC 73, 014615 (2006)]

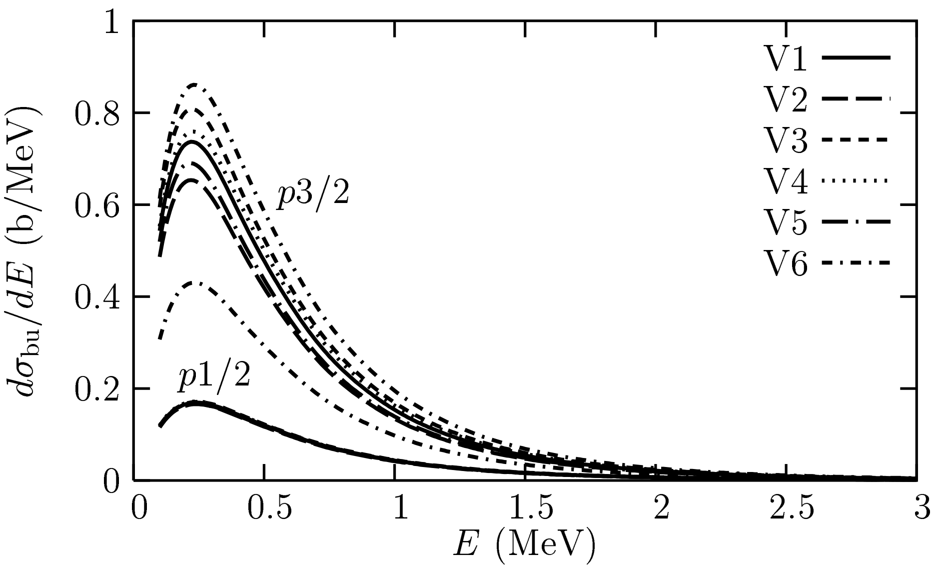
Sensitivity to continuum of projectile

I can get what you want for SF… (PC 2006)

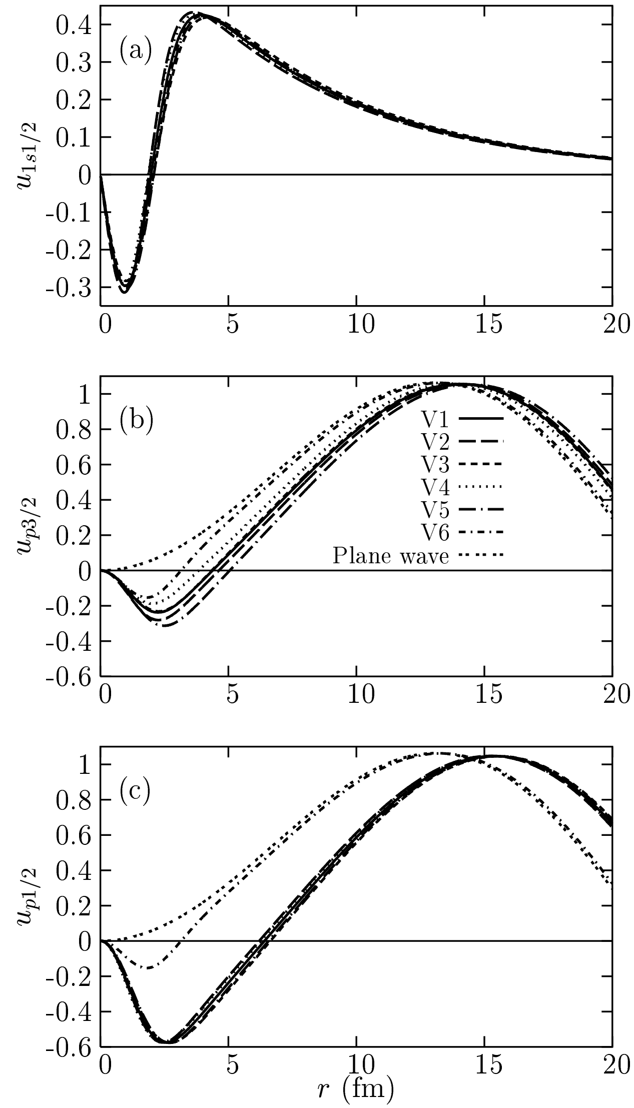
Role of continuum

Where does it come from?

p-wave contributions

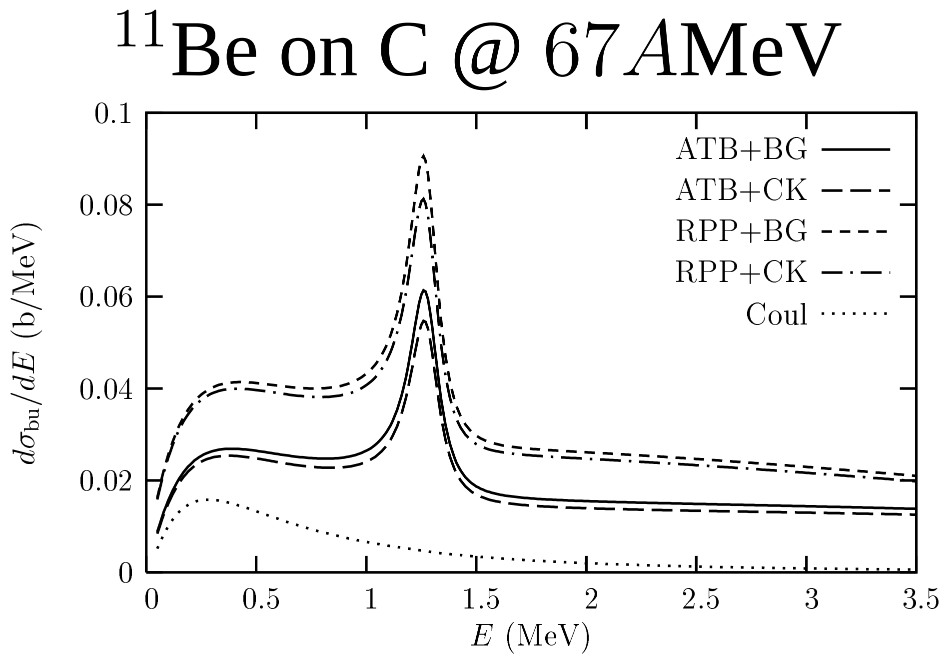


Wave functions



$\Rightarrow$ need contraints on c-f continuum e.g. from microscopic structure calculations

Influence of V_{PT}



Sensitivity to P-T optical potentials

NB: Coulomb breakup less sensitive to V_{PT}

$\Rightarrow$ phenomenological inputs not free from uncertainty

$\Rightarrow$ cautious when extracting SF/ANC from data

Can we remove/reduce the sensitivity to V_{PT}?

Maybe using the Ratio technique…

Recoil Excitation and Breakup

Assumes [R. Johnson *et al.* PRL 79, 2771 (1997)]

- adiabatic approximation
- $V_{\mathrm{n}T} = 0$

$\Rightarrow$ excitation and breakup due to recoil of the core

Elastic scattering: $\frac{d\sigma_{\mathrm{el}}}{d\Omega} = |F_{00}|^2 (\frac{d\sigma}{d\Omega})_{\mathrm{pt}}$

$F_{00} = \int |\Phi_0|^2 e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} d\boldsymbol{r}$ $\qquad \boldsymbol{Q} \propto (\boldsymbol{K} - \boldsymbol{K}')$

$\Rightarrow$ scattering of compound nucleus $\equiv$

form factor $\times$ scattering of pointlike nucleus

Similarly for breakup: $\frac{d\sigma_{\mathrm{bu}}}{dE d\Omega} = |F_{E0}|^2 (\frac{d\sigma}{d\Omega})_{\mathrm{pt}}$

$$|F_{E0}|^2 = \sum_{ljm} \left| \int \Phi_{ljm}(E) \Phi_0 e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} d\boldsymbol{r} \right|^2$$

$\Rightarrow$ explains similarities in angular distributions

provides the idea for the ratio technique…

Ratio technique

$$d\sigma_{\rm bu}/d\sigma_{\rm el} = |F_{E0}(\boldsymbol{Q})|^2/|F_{00}(\boldsymbol{Q})|^2$$

- completely independent of reaction process
  not affected by V_{PT}; i.e. the same for all targets
- probes only projectile structure
- no need to normalise exp. cross sections

Test this using Dynamical Eikonal Approximation,
[B. Baye, P.C., G. Goldstein, PRL 95, 082502 (2005)]

- without adiabatic approximation
- including V_{nT}

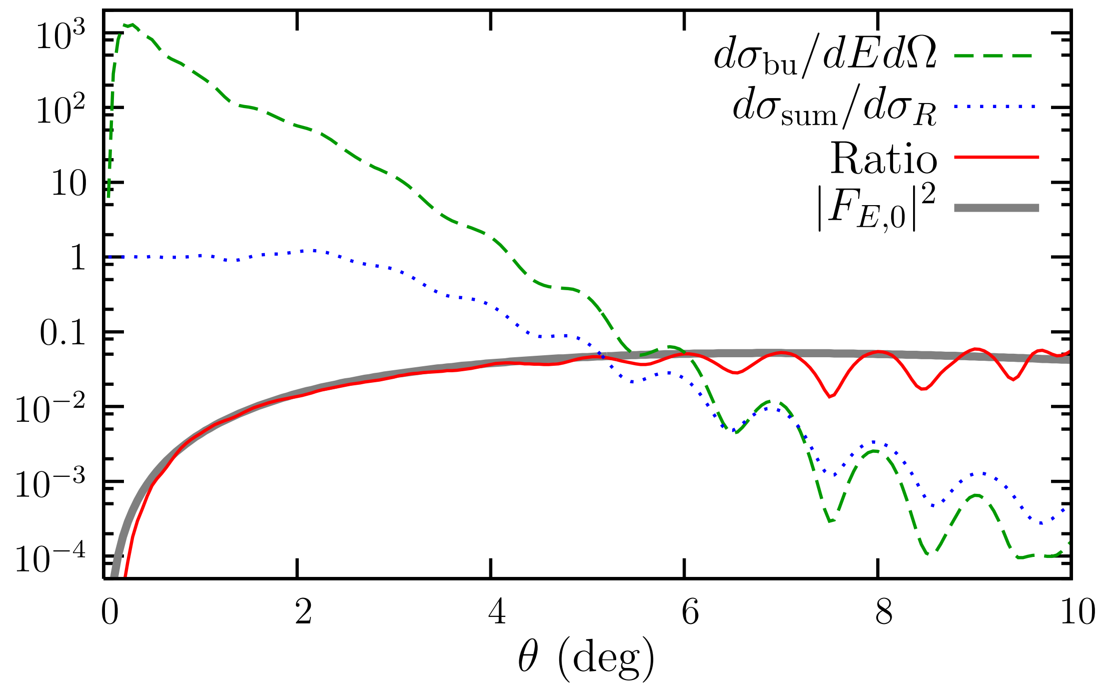
Alternative: $d\sigma_{\rm bu}/d\sigma_{\rm sum} = |F_{E0}|^2$

$$= \sum_{ljm} \left| \int \Phi_{ljm}(E)\Phi_0 e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} d\boldsymbol{r} \right|^2$$

with $\frac{d\sigma_{\rm sum}}{d\Omega} = \frac{d\sigma_{\rm el}}{d\Omega} + \frac{d\sigma_{\rm inel}}{d\Omega} + \int \frac{d\sigma_{\rm bu}}{dEd\Omega}dE$

Testing with DEA

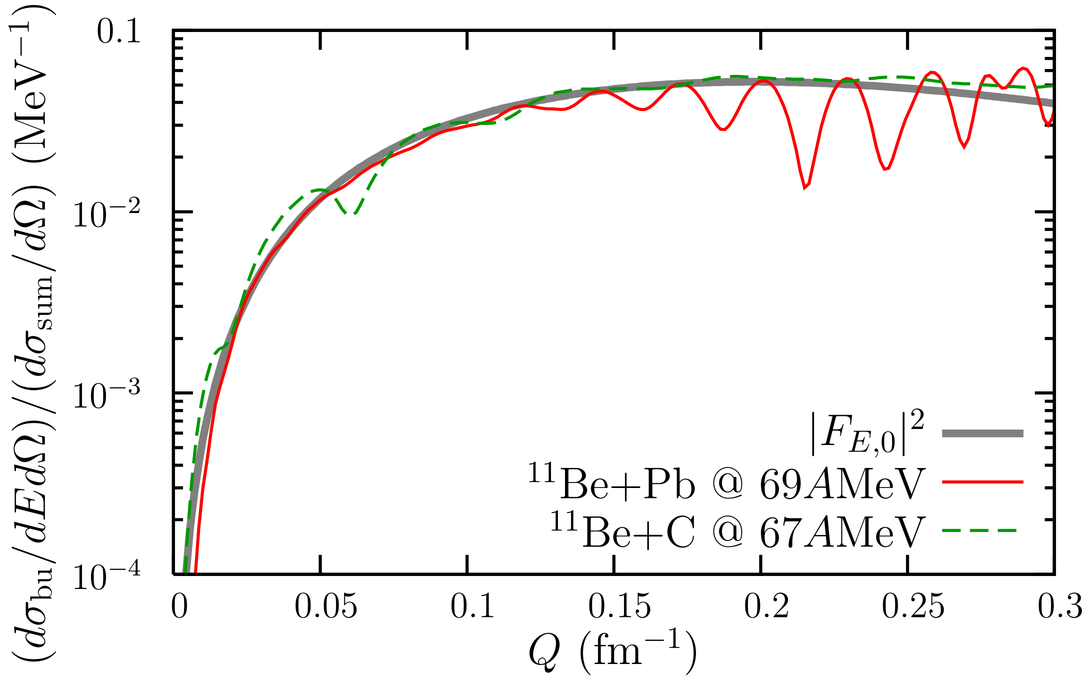
^{11}Be+Pb @ 69AMeV [P. C., R. Johnson, F. Nunes, PLB 705, 112 (2011)]



- removes most of the angular dependence
- REB predicts ratio $= |F_{E0}|^2$ confirmed by DEA calculations

$\Rightarrow$ probe structure with little dependence on reaction

(In)sensitivity to V_{PT}



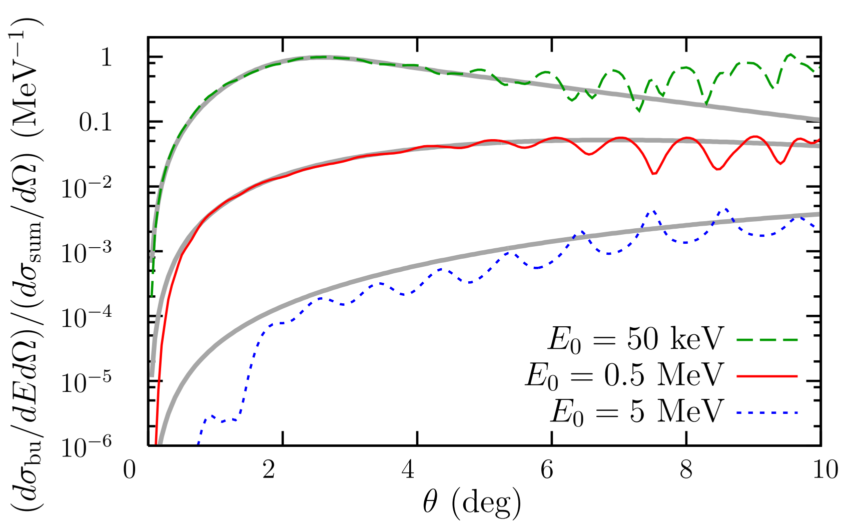
Similar for Coulomb and nuclear dominated collisions

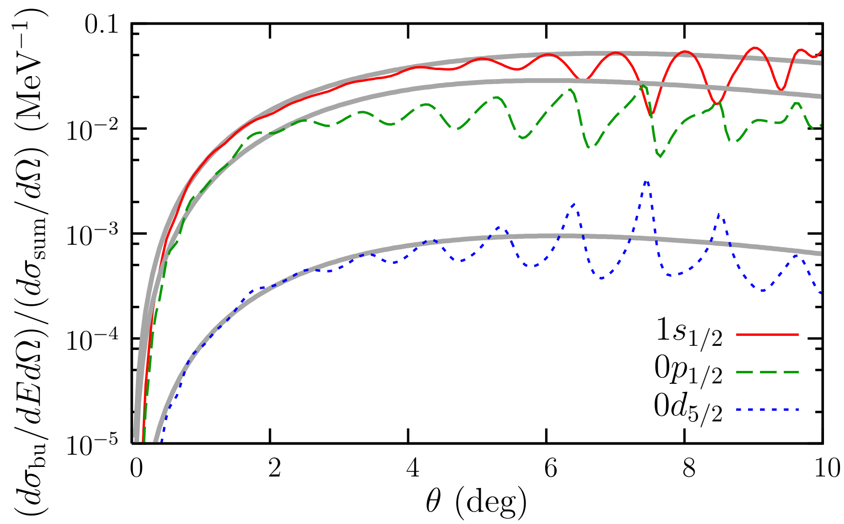
$\Rightarrow$ nearly independent of the reaction process

Sensitivity to projectile description

Study sensitivity to

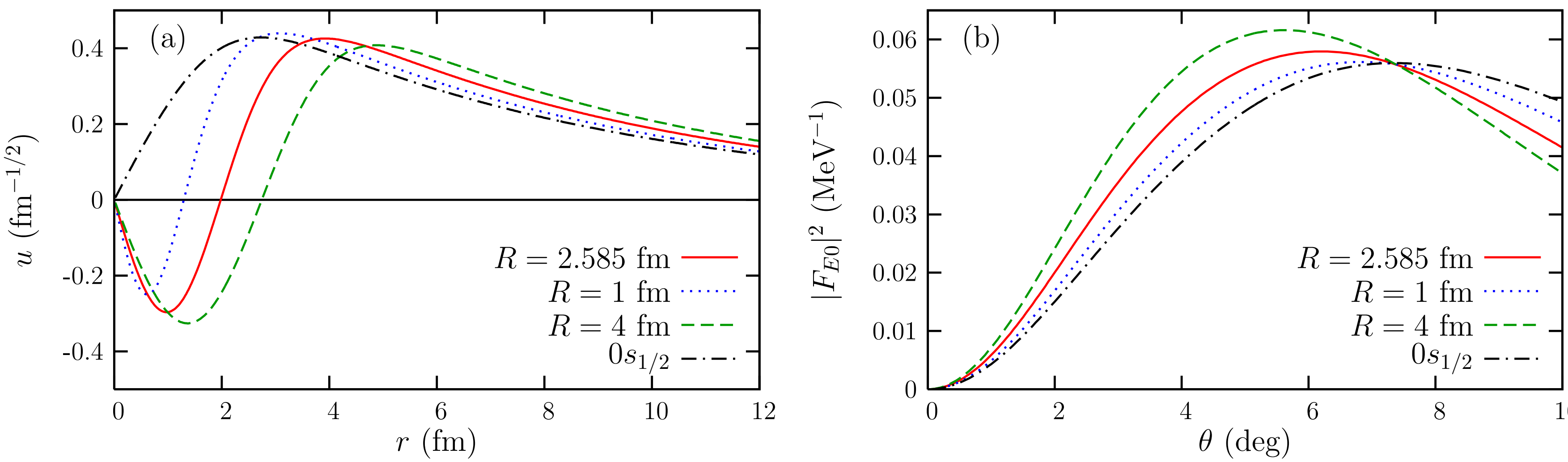
binding energy | bound-state orbital





- Sensitive to both binding energy and orbital in both shape and magnitude
- Works better for loosely-bound projectile (adiabatic approximation ?)

Sensitivity to radial wave function



- Changes in $|F_{E0}|^2$ similar to those in u_{lj}
- Forward angles probe asymptotics of u_{lj}
- Large angles probe the interior of u_{lj}
  may be difficult to distinguish experimentally

$\Rightarrow$ Ratio scans radial wave function

$\Rightarrow$ maybe can get SF

Conclusion and outlook

Good understanding of reaction process

Breakup models agree with each other (@$70A$MeV)

SF extracted from $\sigma_{\rm bu}^{exp}/\sigma_{\rm bu}^{th}$ BUT:

- Probes only ANC
  but maybe link with SF?
- Sensitive to description of continuum
  to be constrained by structure models?
- Sensitive to V_{PT}
  can be reduced using ratio

Next step: improve projectile description

- core excitation, e.g. XCDCC
- microscopic description

Thanks to my collaborators

Filomena Nunes — NSCL

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Mahir Hussein — Universidade de São Paulo

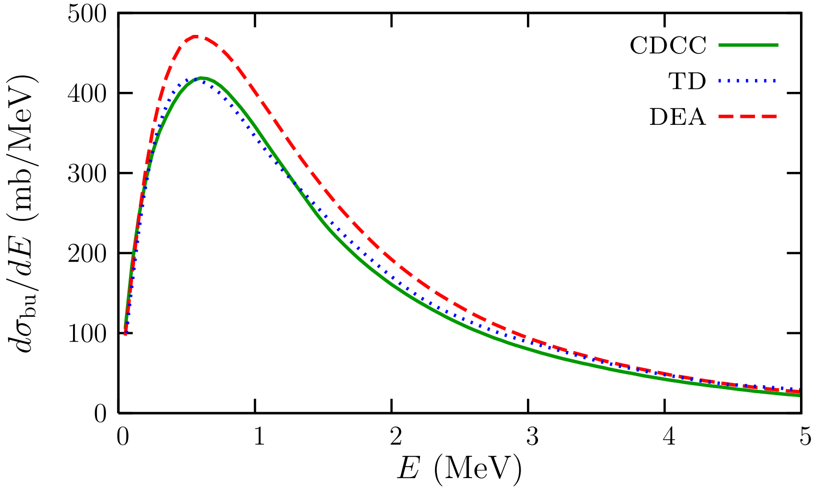
Ron Johnson — University of Surrey

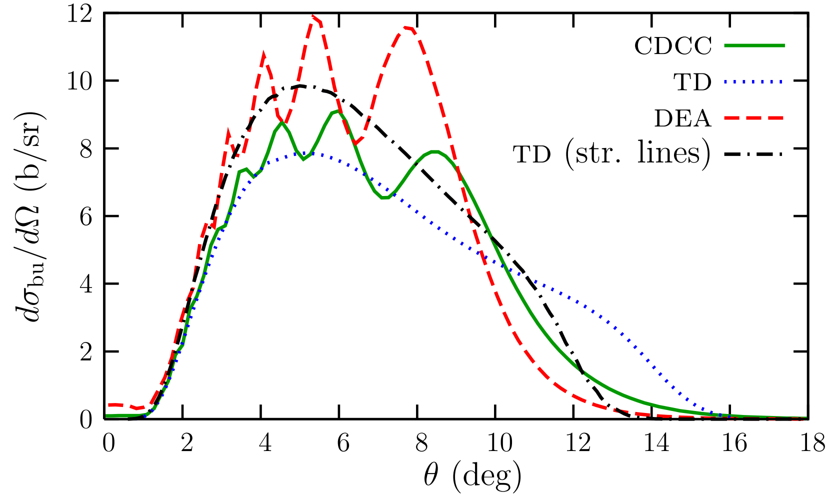
Henning Esbensen — Argonne National Laboratory

Ian Thompson

^{15}C + Pb @ $20A$MeV

$d\sigma_{\rm bu}/dE$ $d\sigma_{\rm bu}/d\Omega$





TD $\equiv$ CDCC
DEA too high

TD gives trend of CDCC
(lacks oscillations)
DEA peaks too early

DEA$\neq$CDCC due to Coulomb deflection
(TD straight lines)