Chiral three-body forces: From neutron matter to neutron stars

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In collaboration with:
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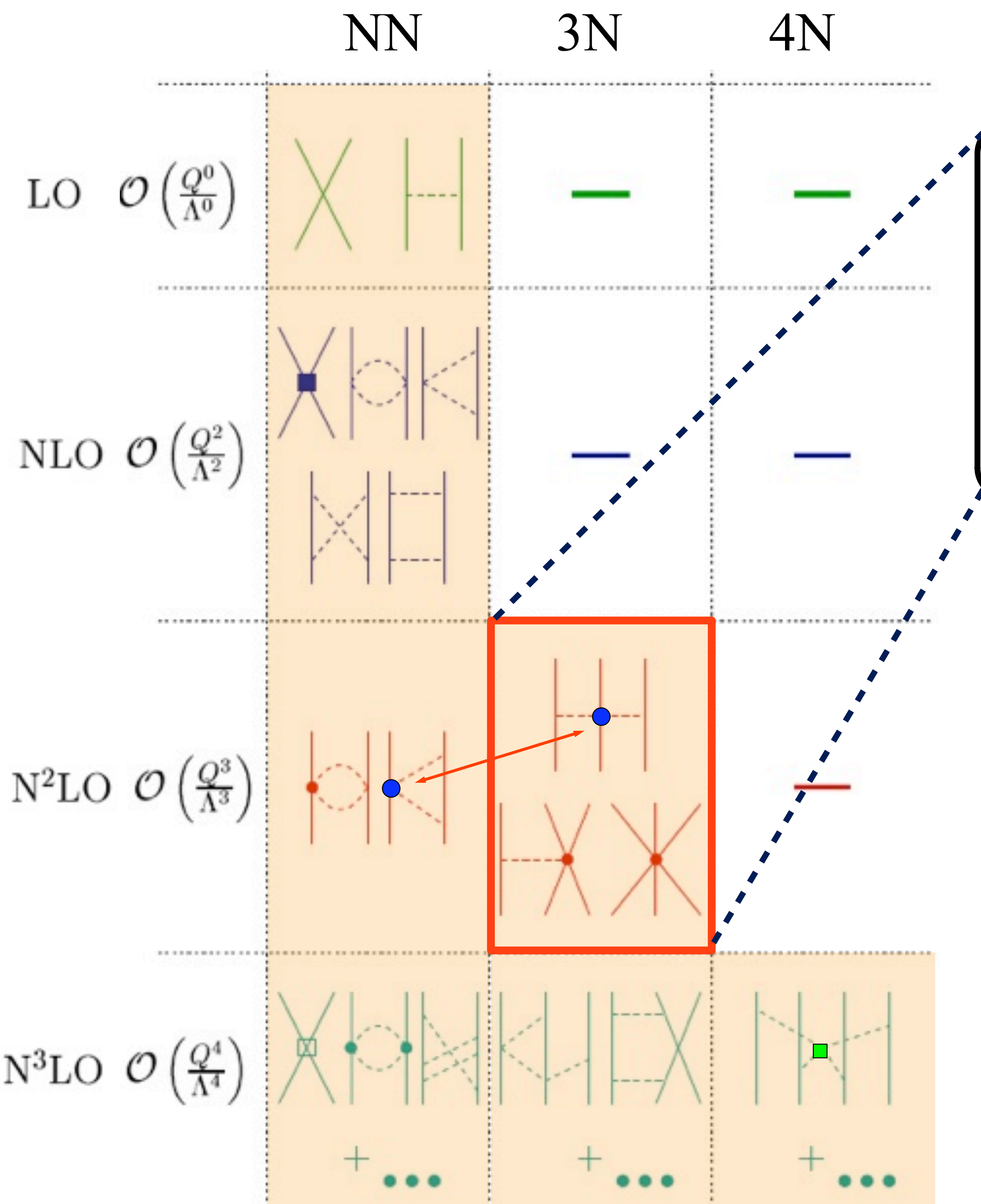
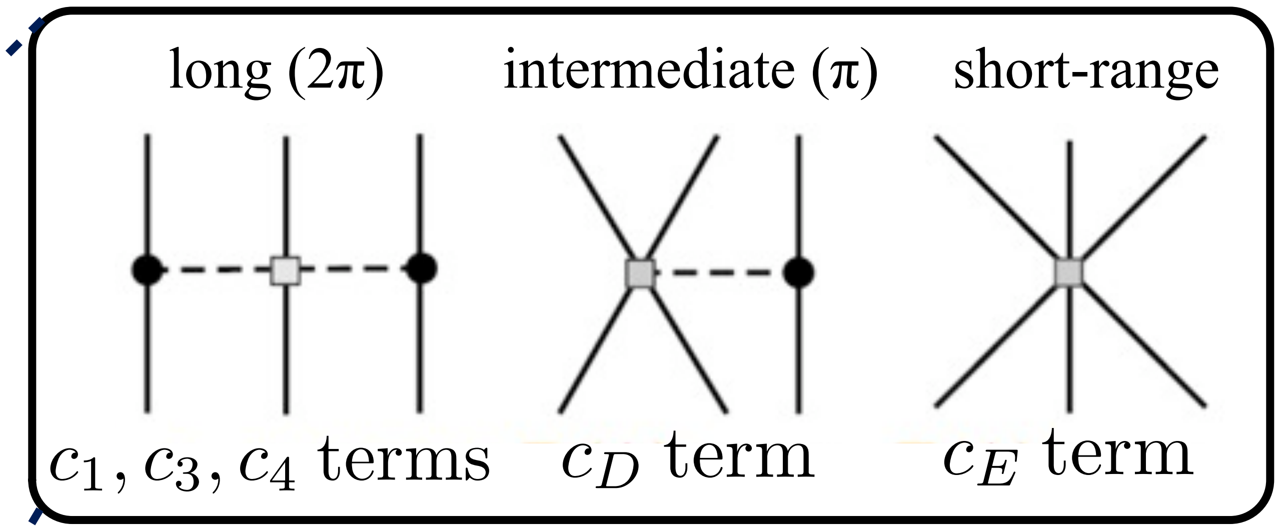
EMMI program
The Extreme Matter Physics of Nuclei:
From Universal Properties to Neutron-Rich Extremes

Darmstadt, April 20, 2012

THE OHIO STATE UNIVERSITY

Chiral EFT for nuclear forces, leading order 3N forces





large uncertainties in coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\ , \quad c_3 = -4.7^{+1.5}_{-1.0}\ , \quad c_4 = 3.5^{+0.5}_{-0.2}$$

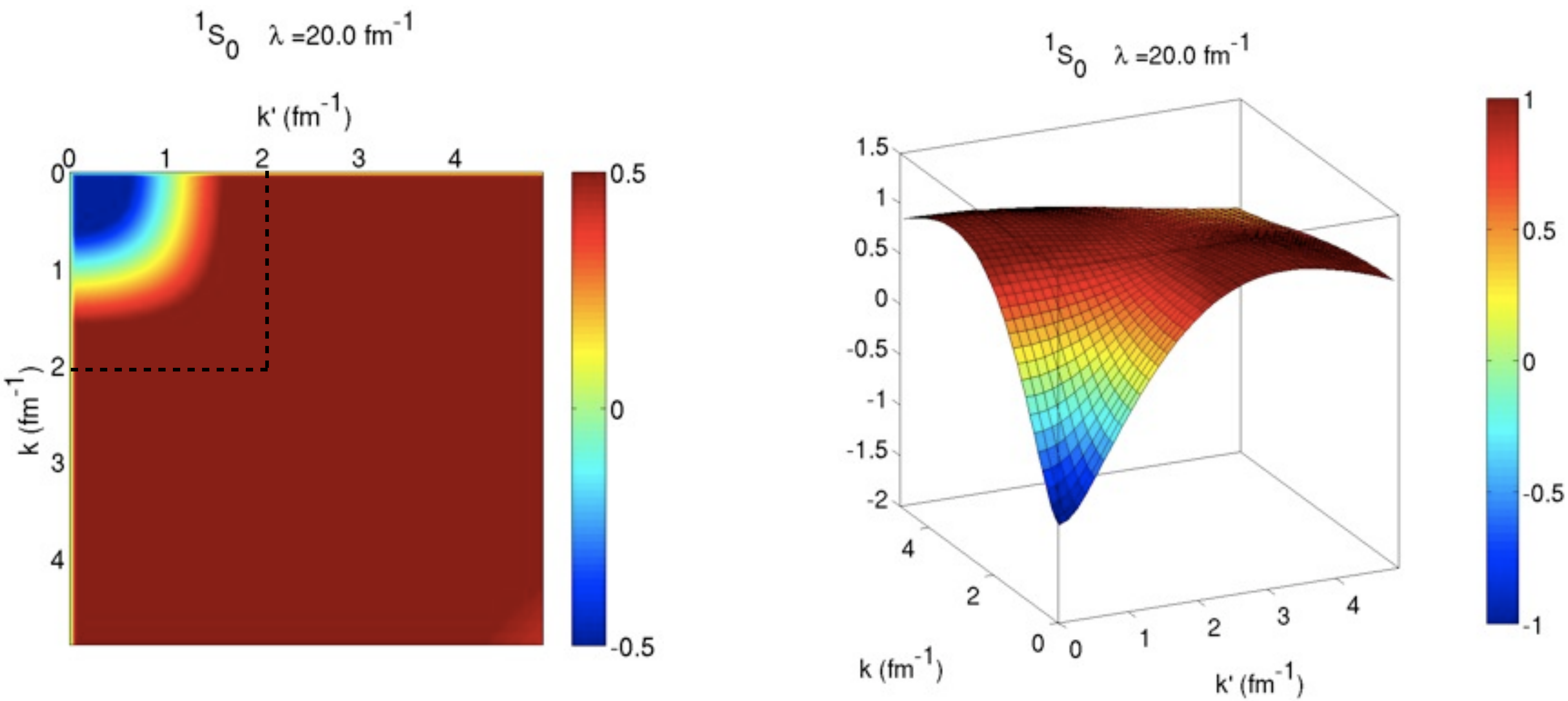
lead to theoretical uncertainties in many-body observables

Low-momentum interactions:
The (Similarity) Renormalization Group

- goal: generate unitary transformation of "hard" Hamiltonian

$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

- basic idea: change resolution in small steps: $\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$



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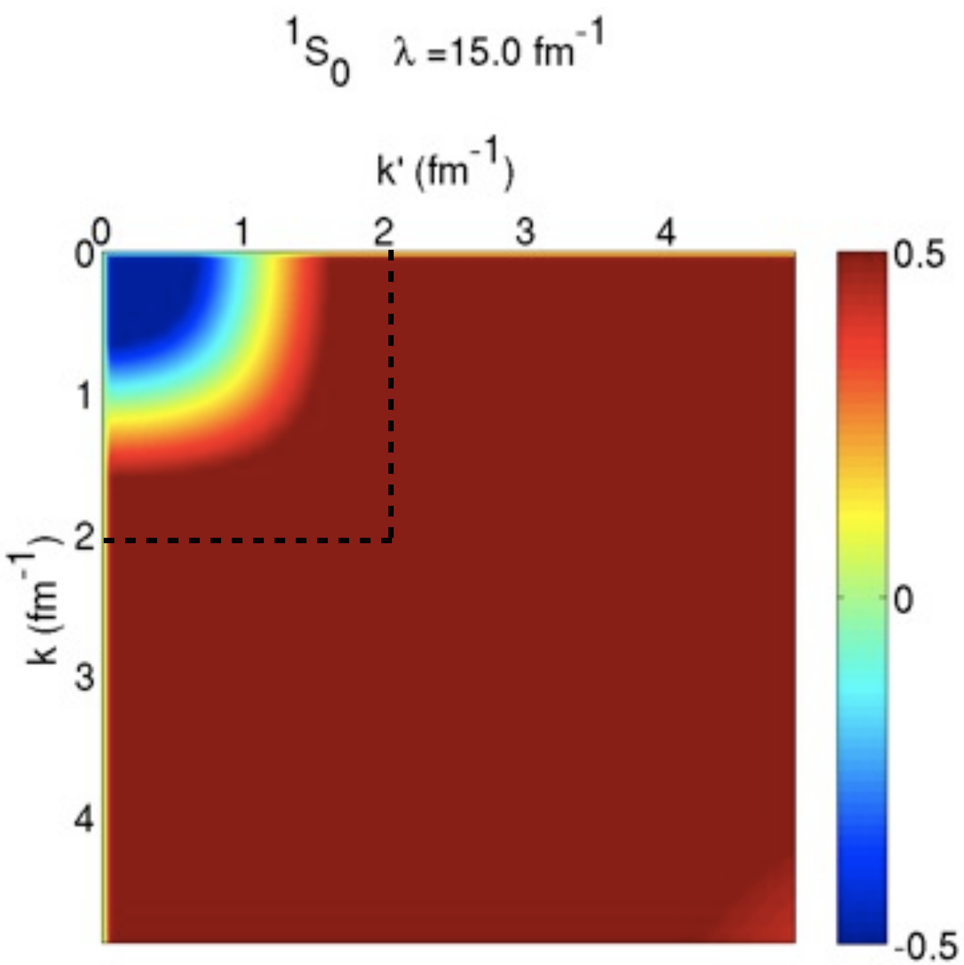

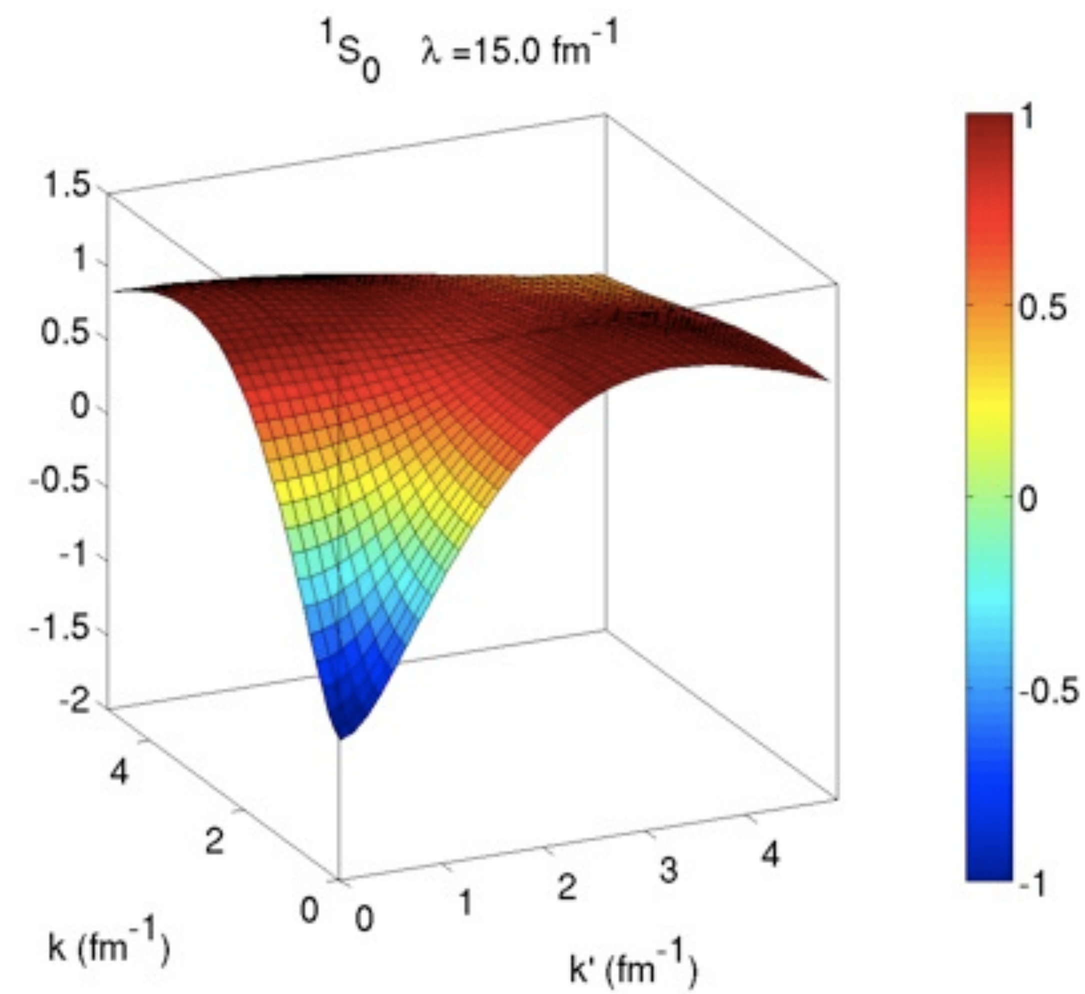

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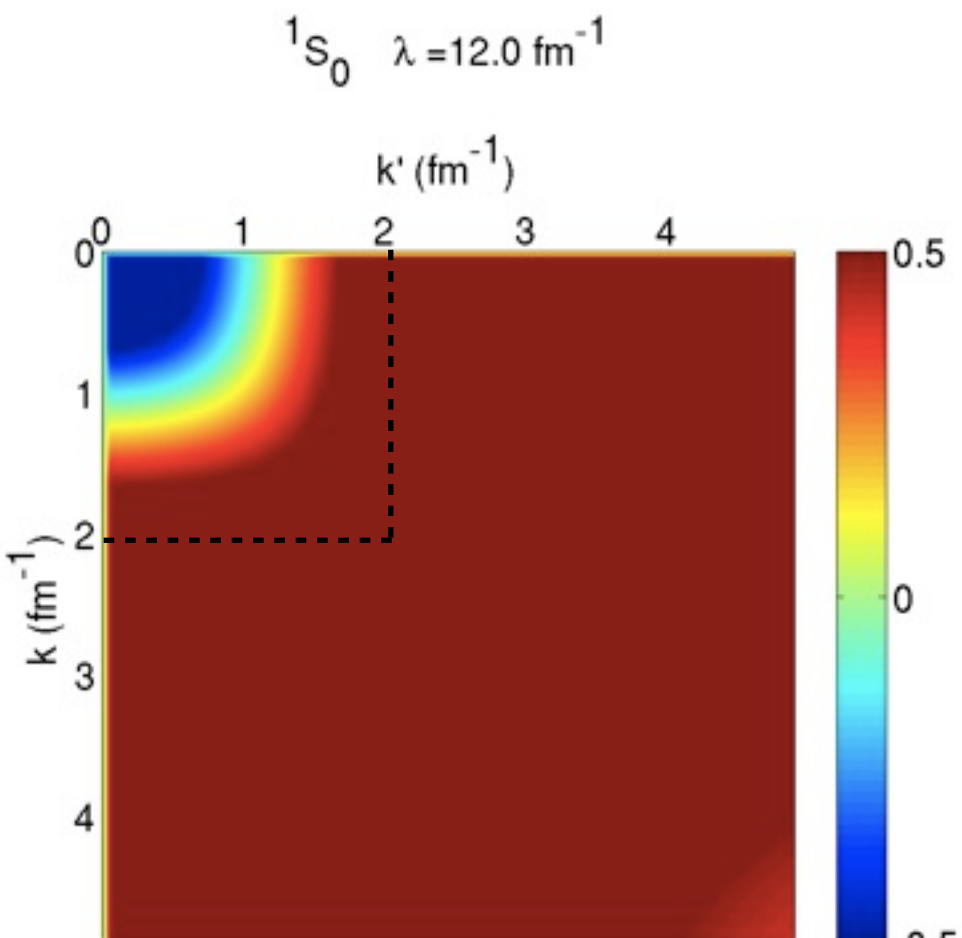

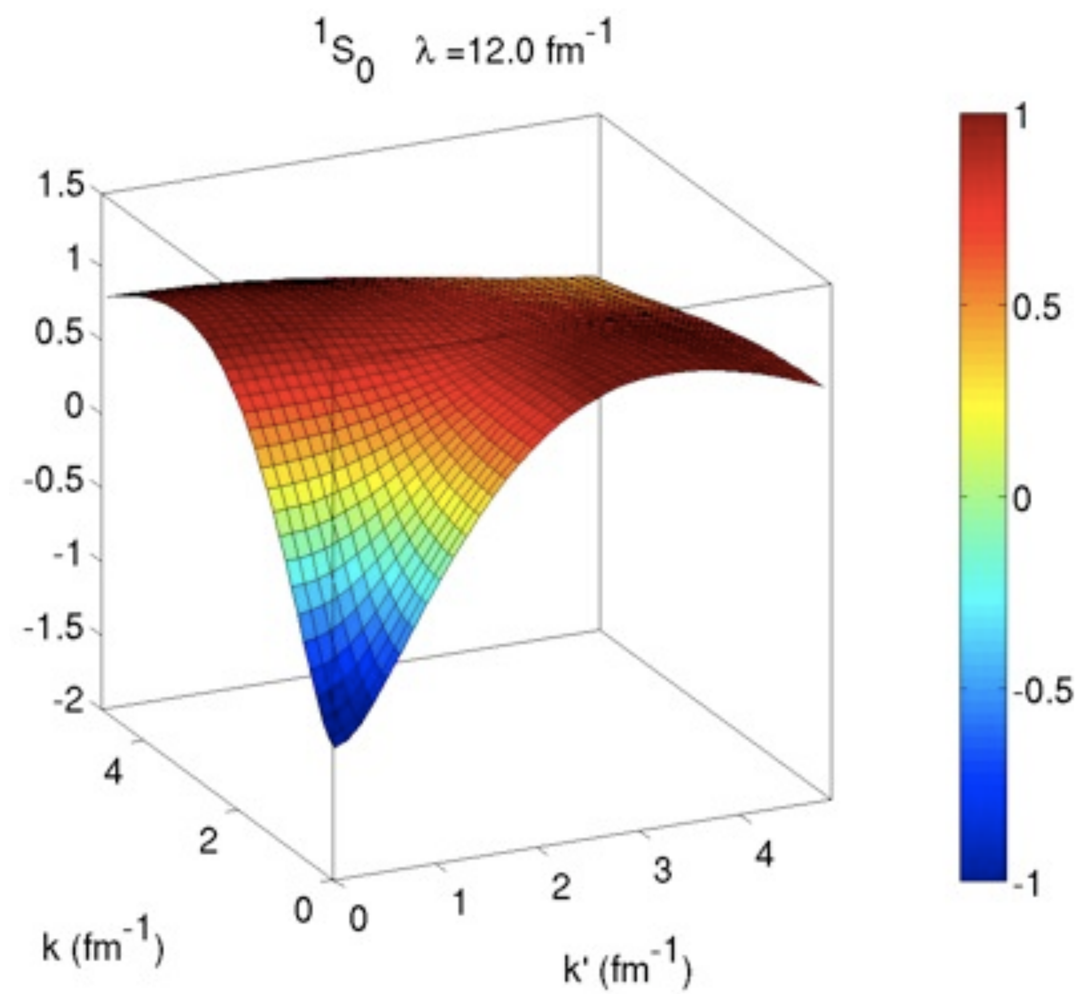

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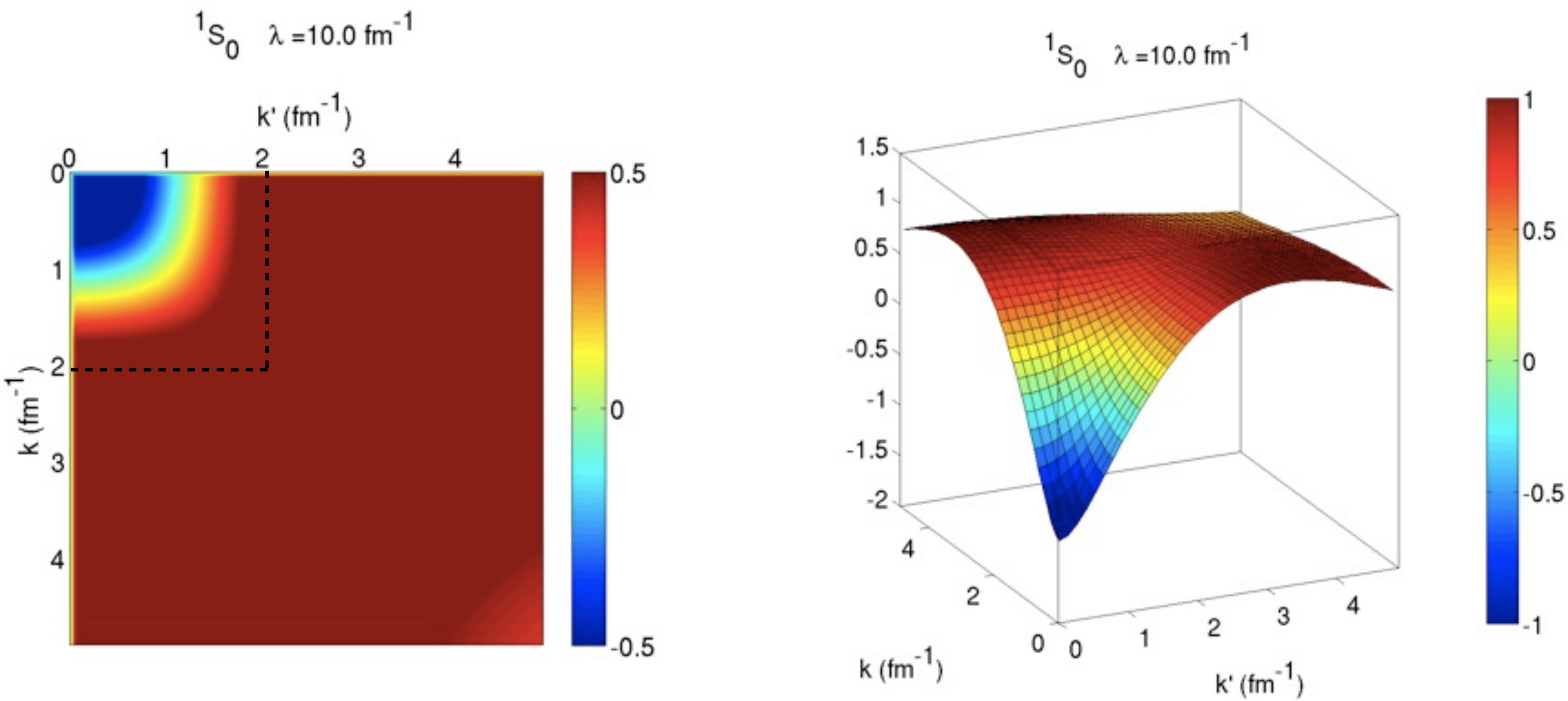

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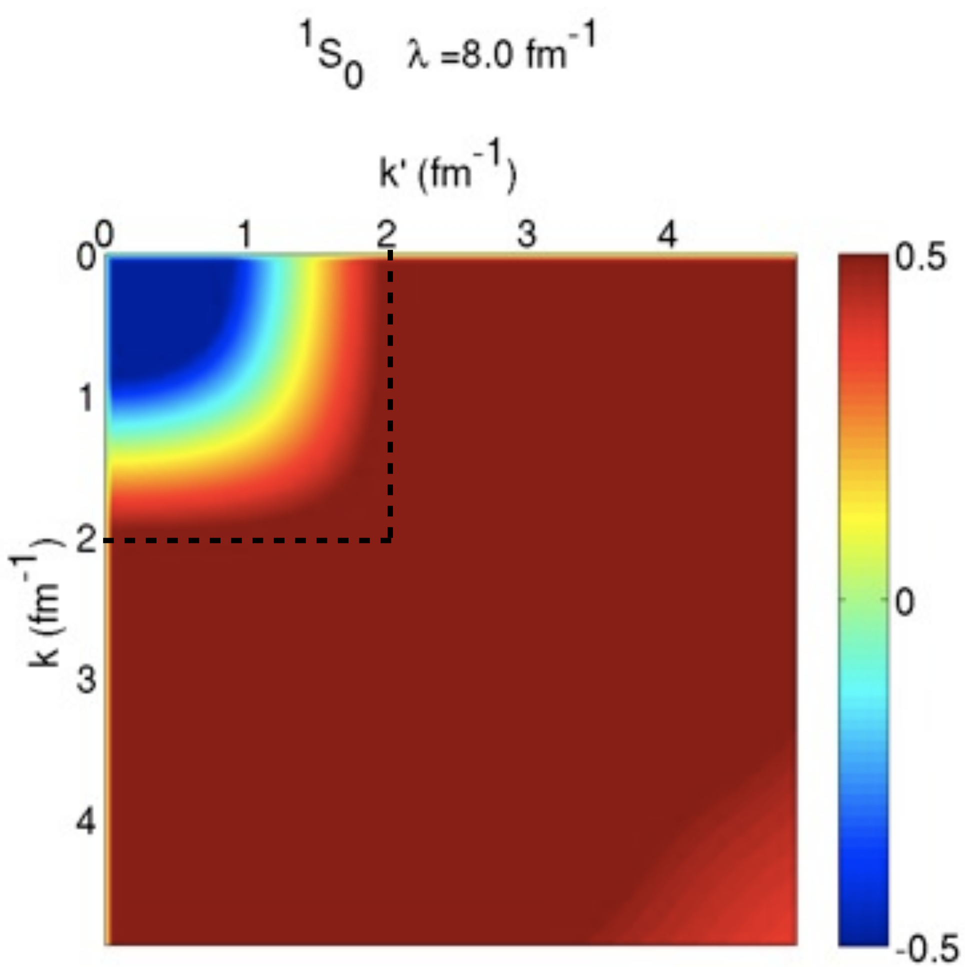

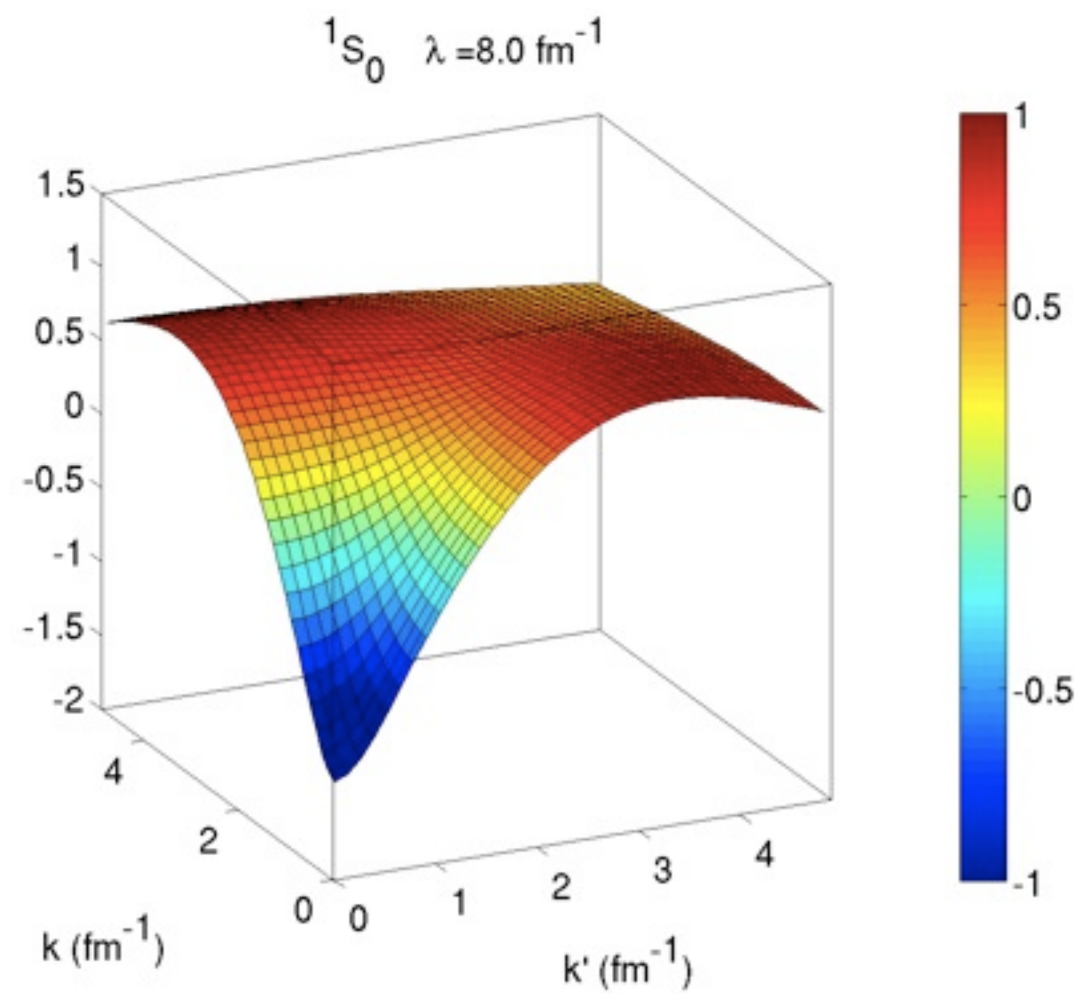

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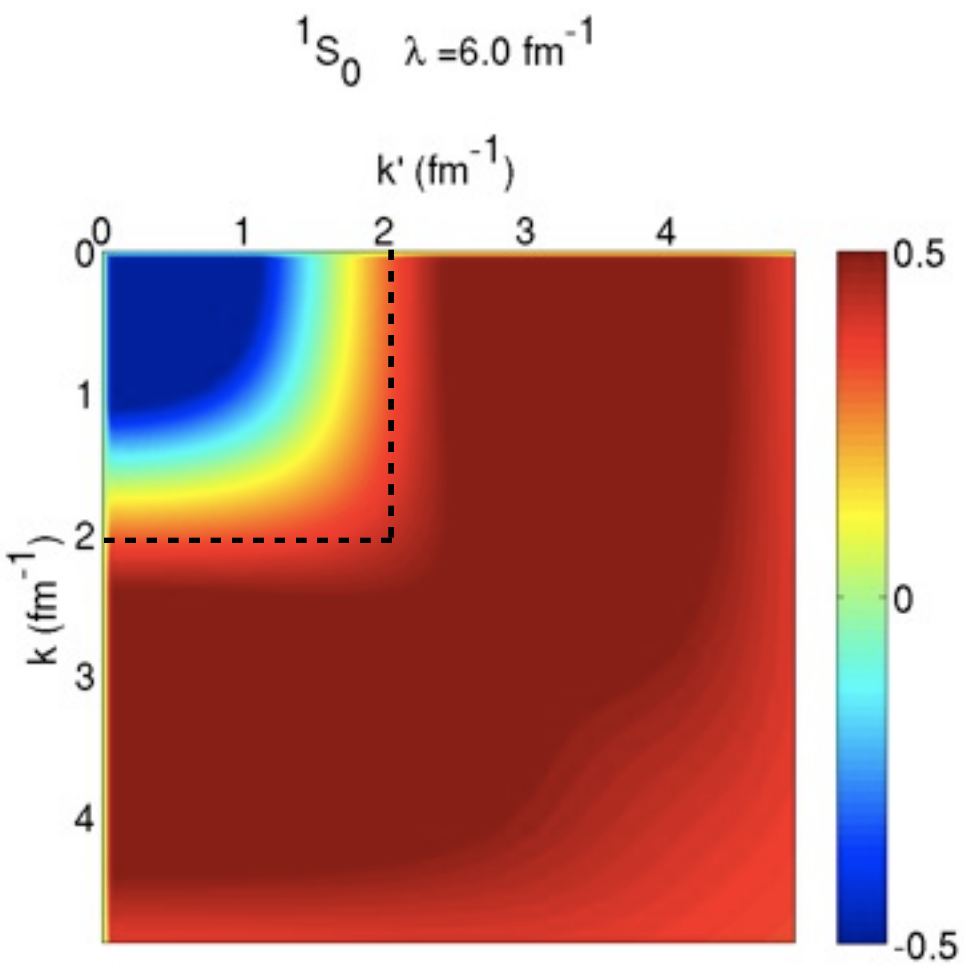

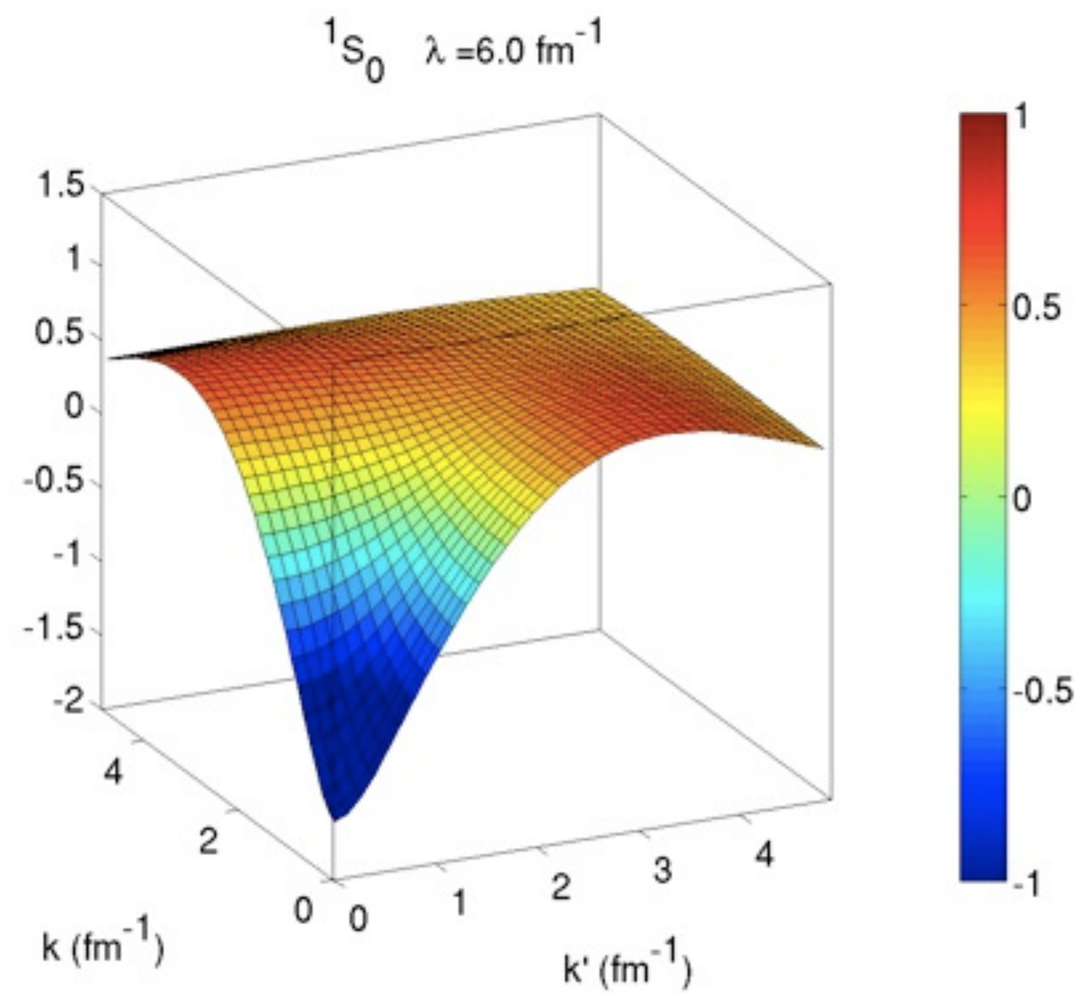

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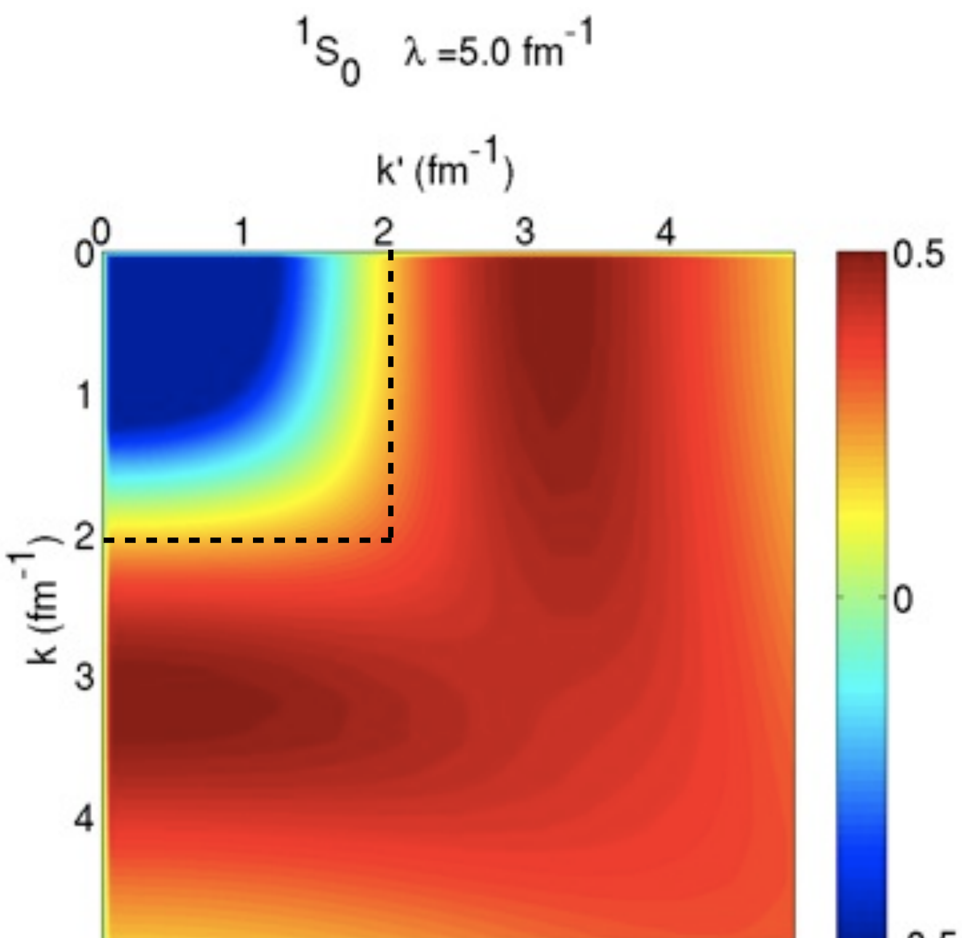

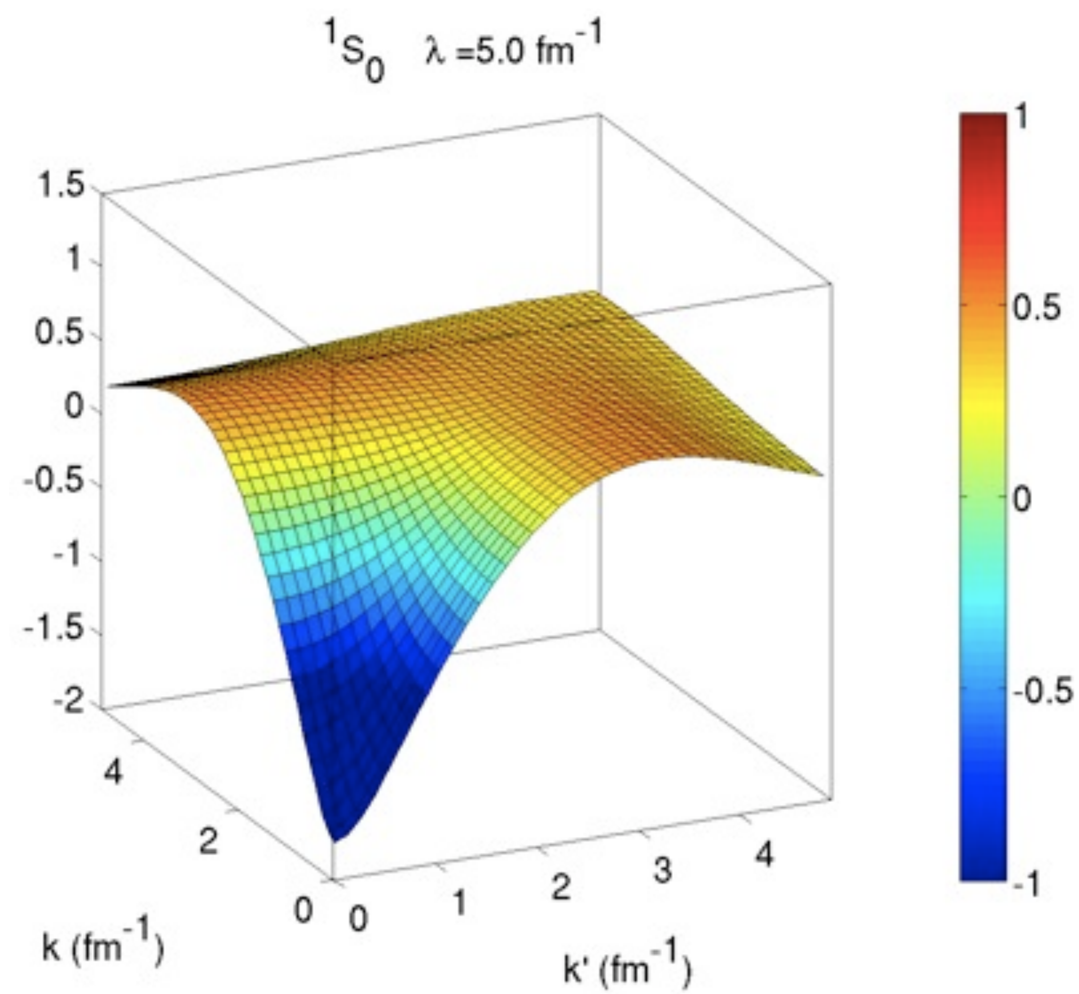

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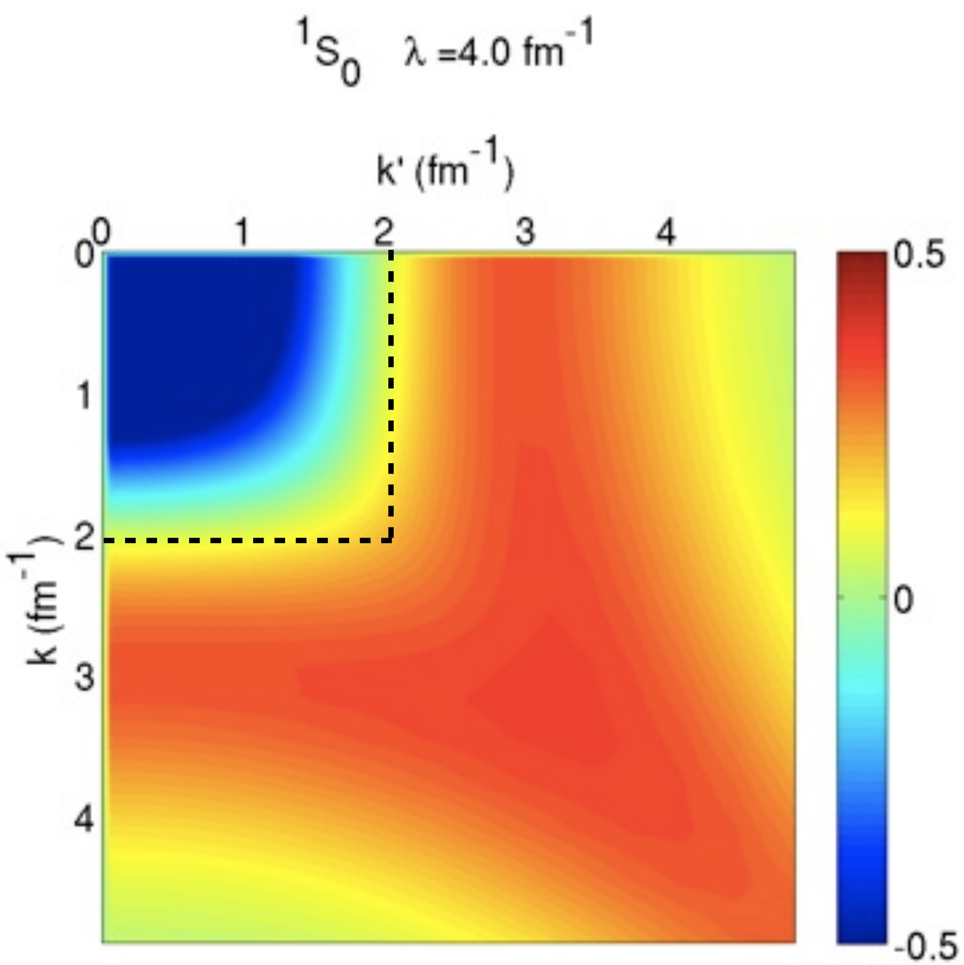

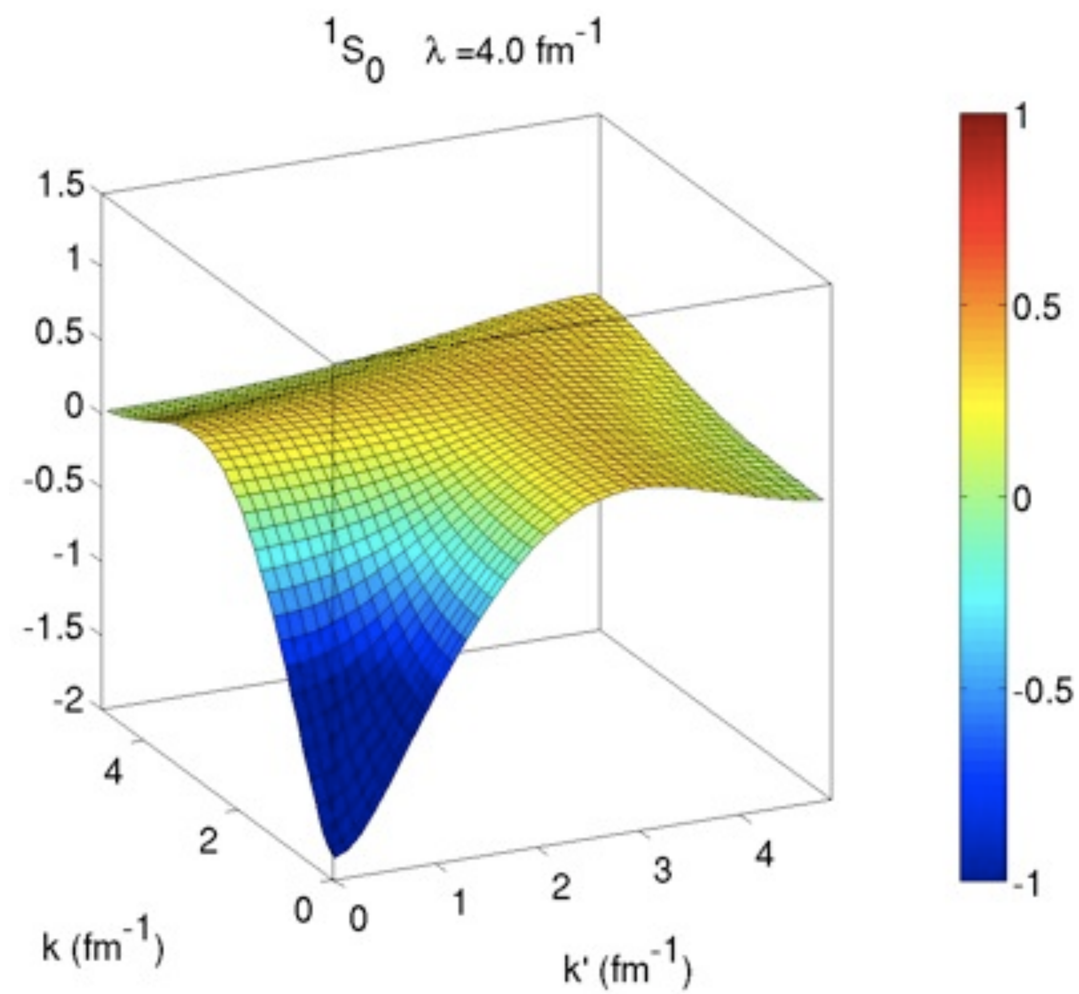

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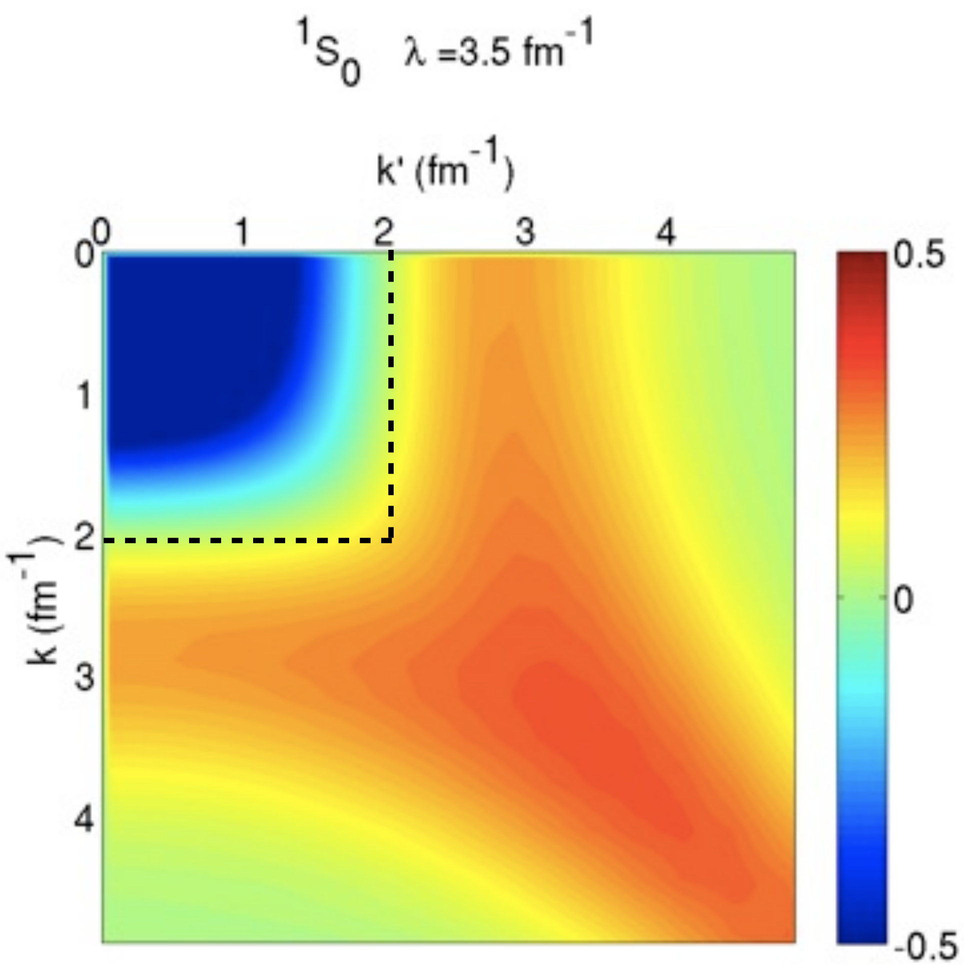

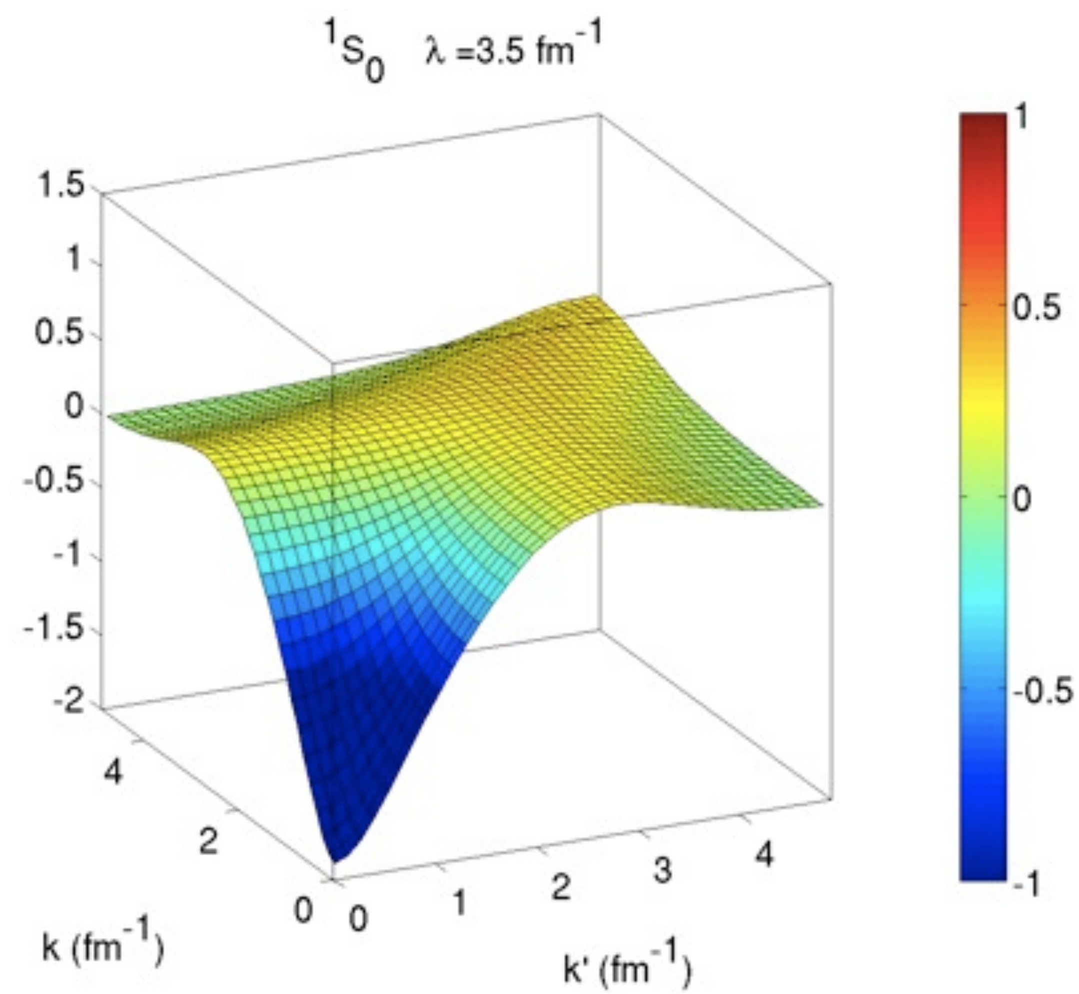

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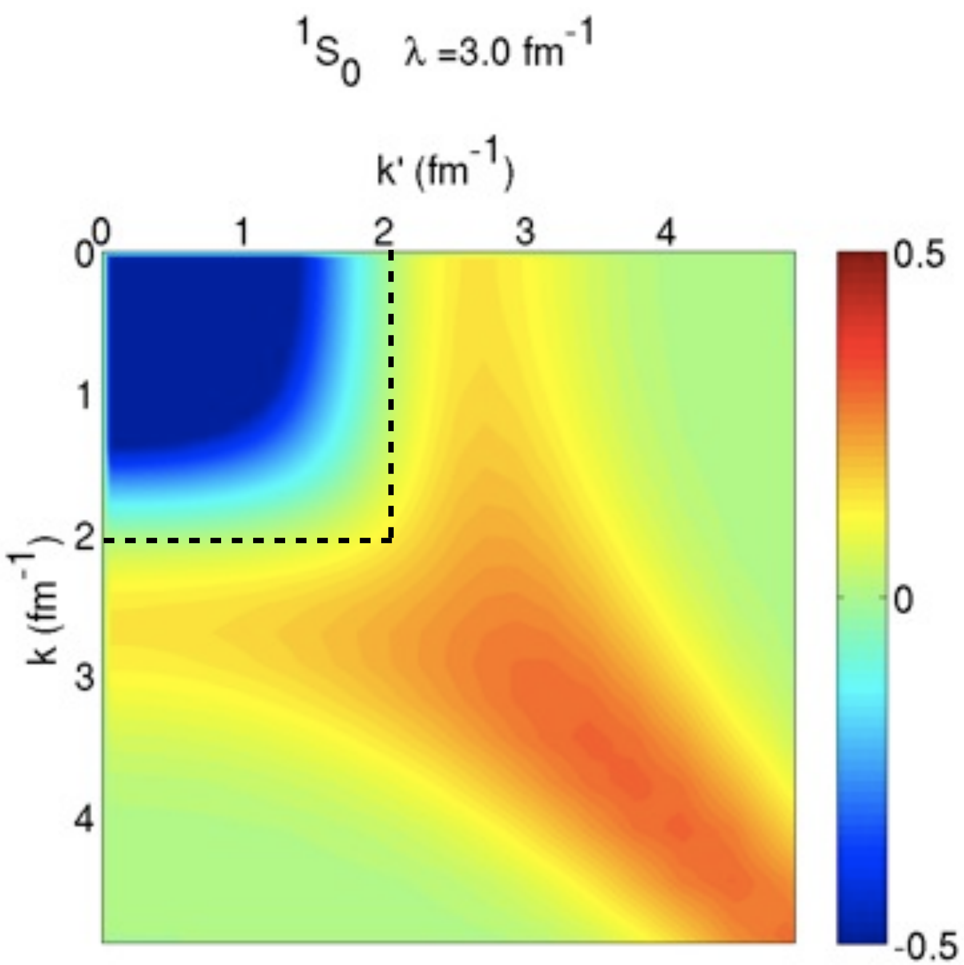

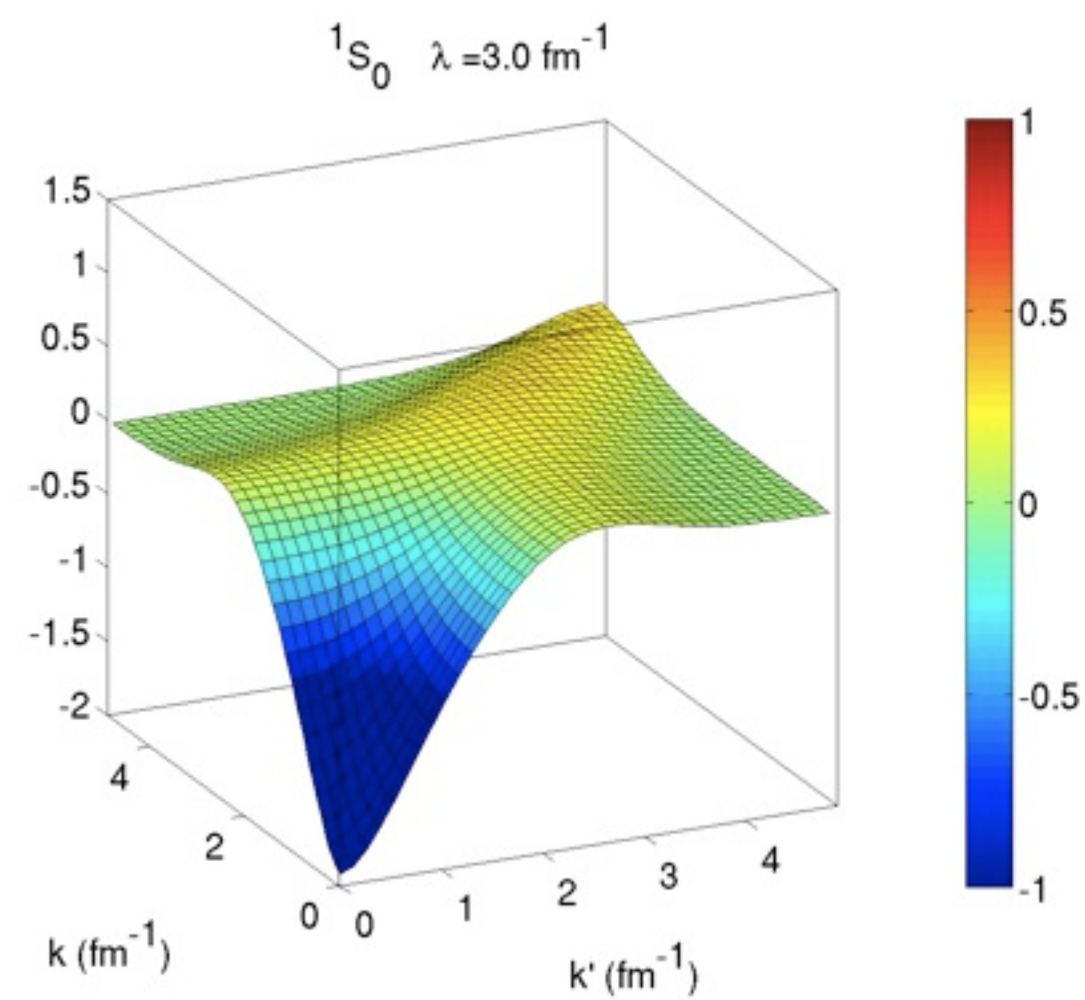

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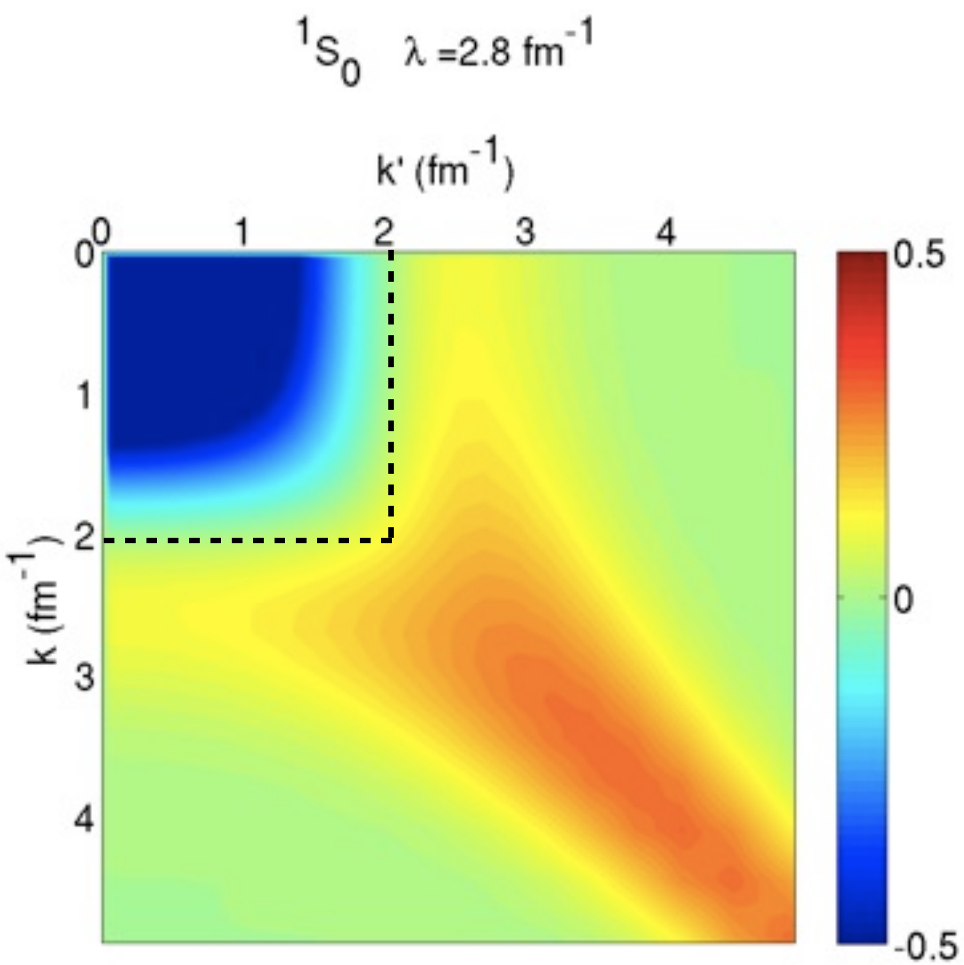

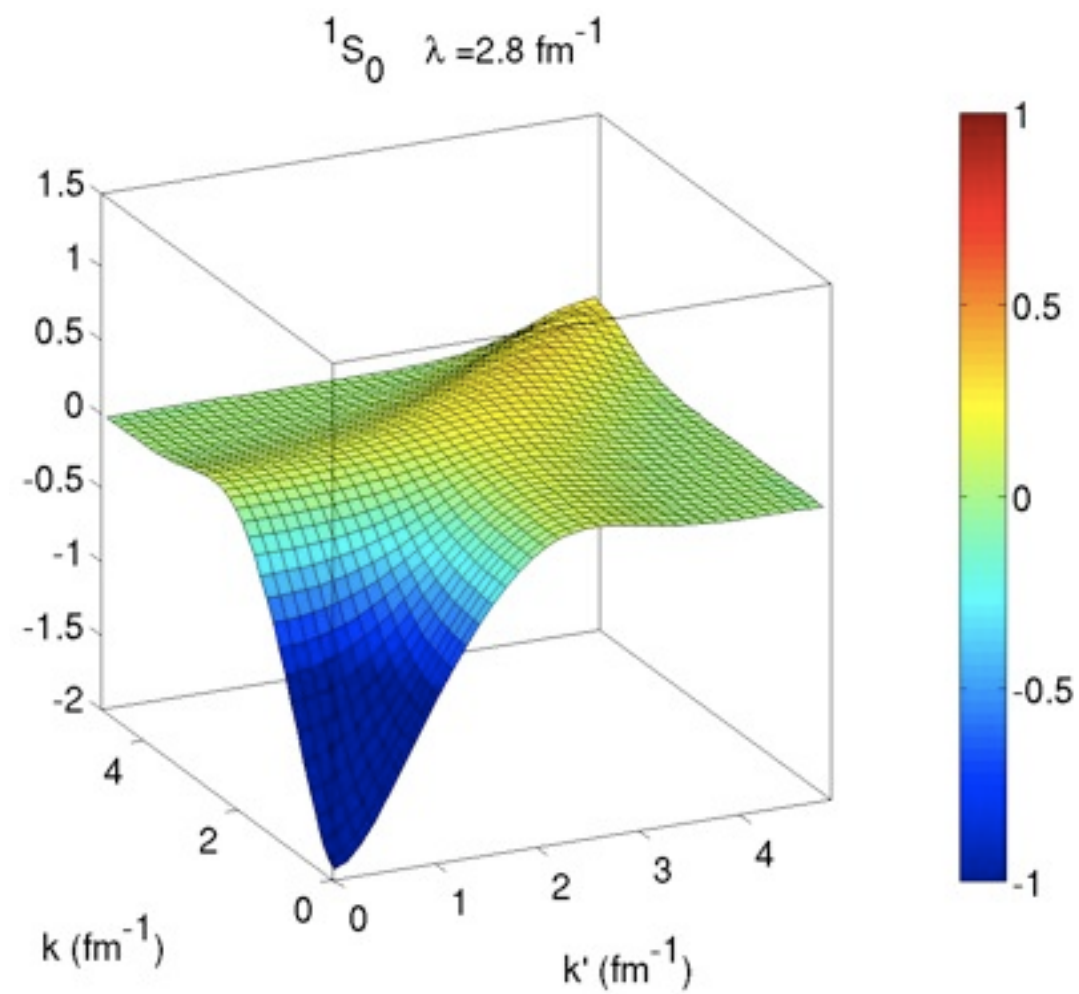

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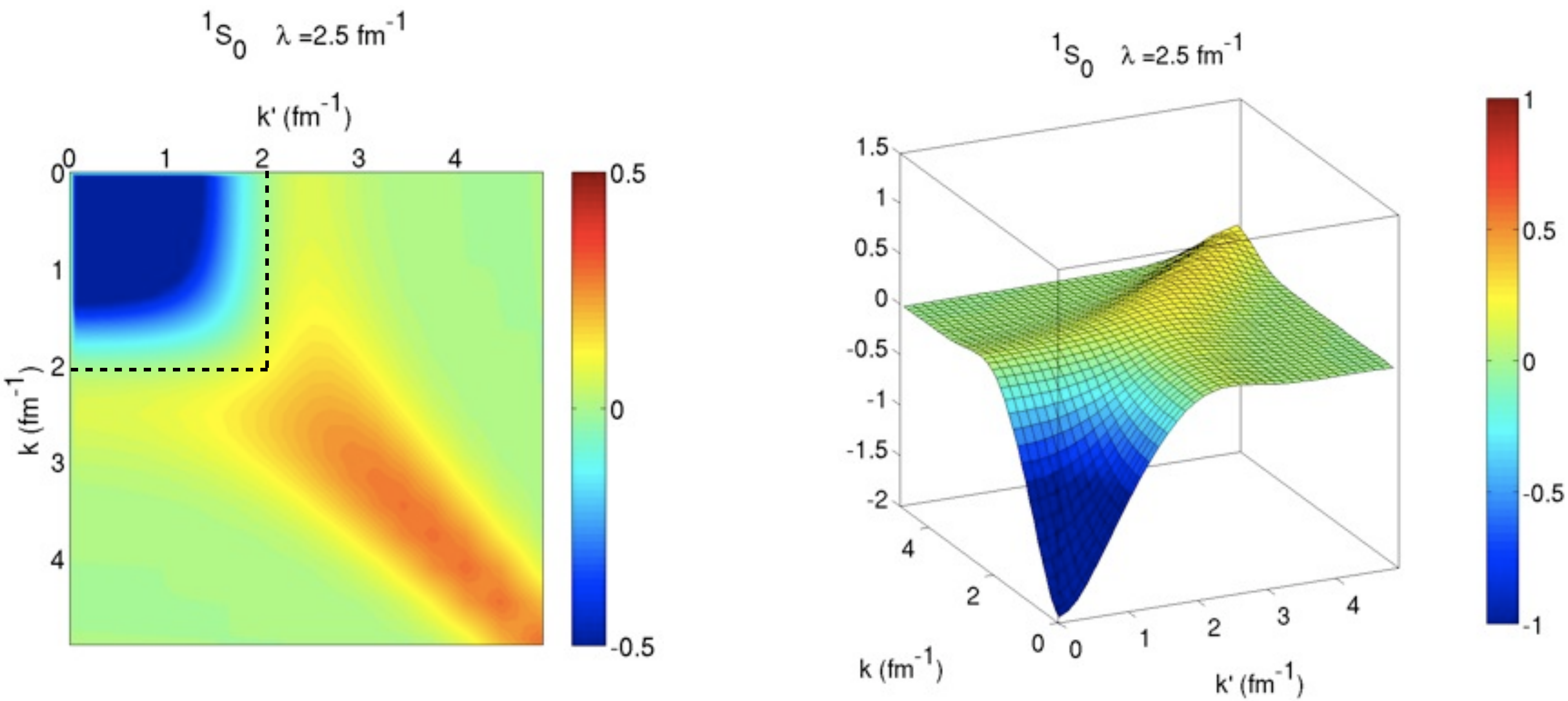

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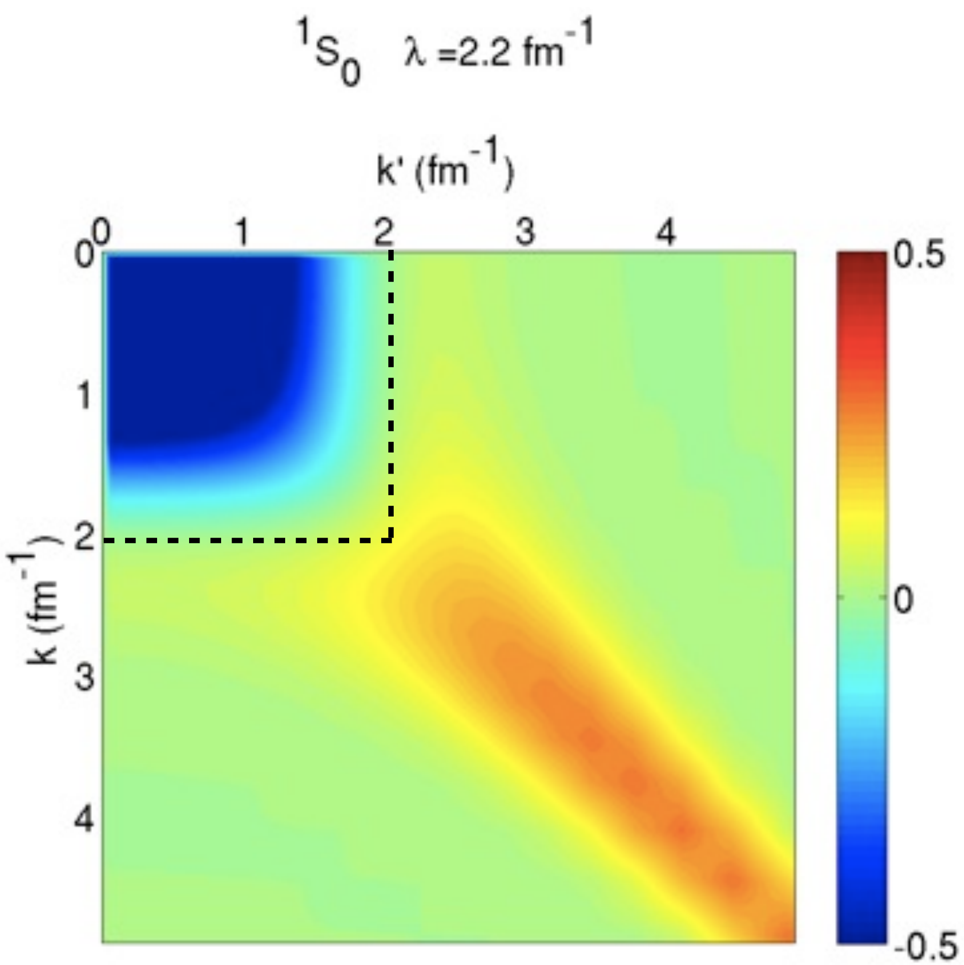

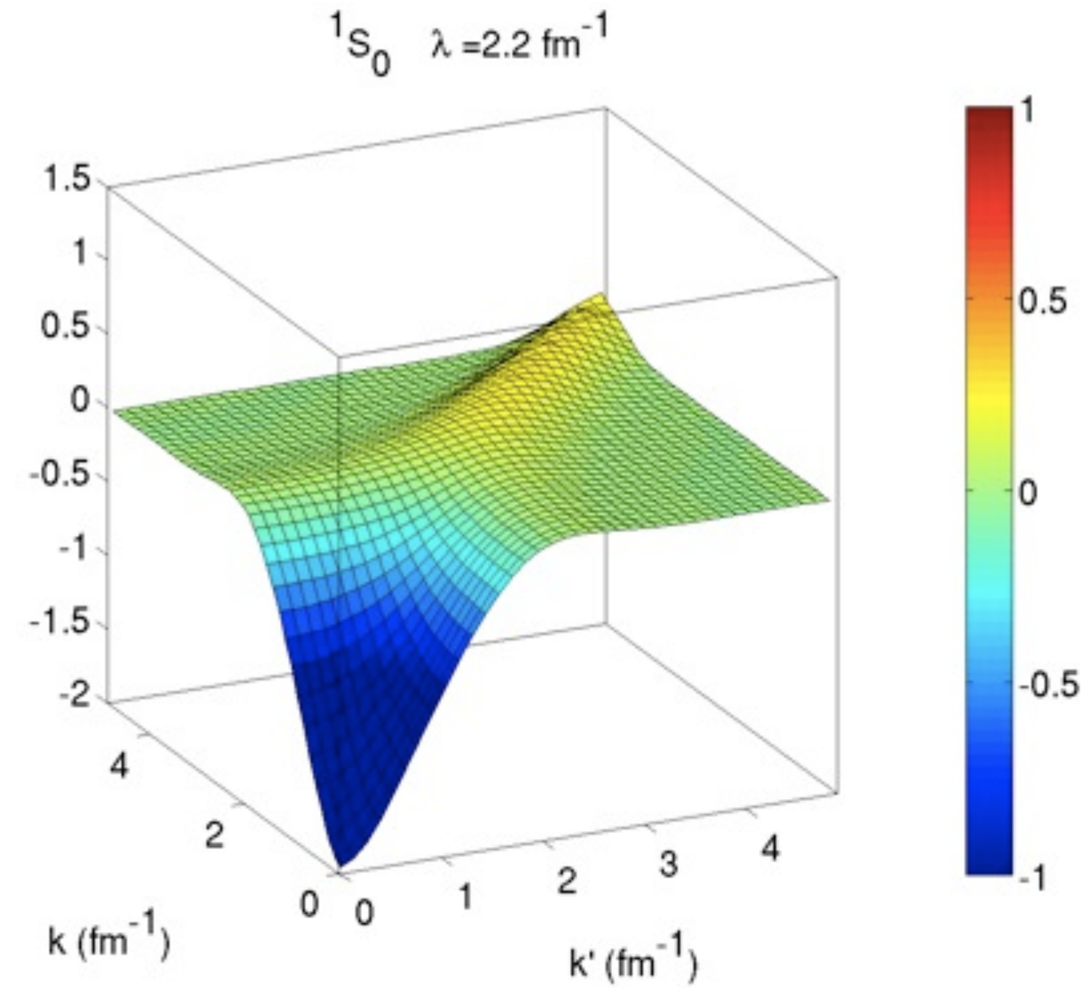

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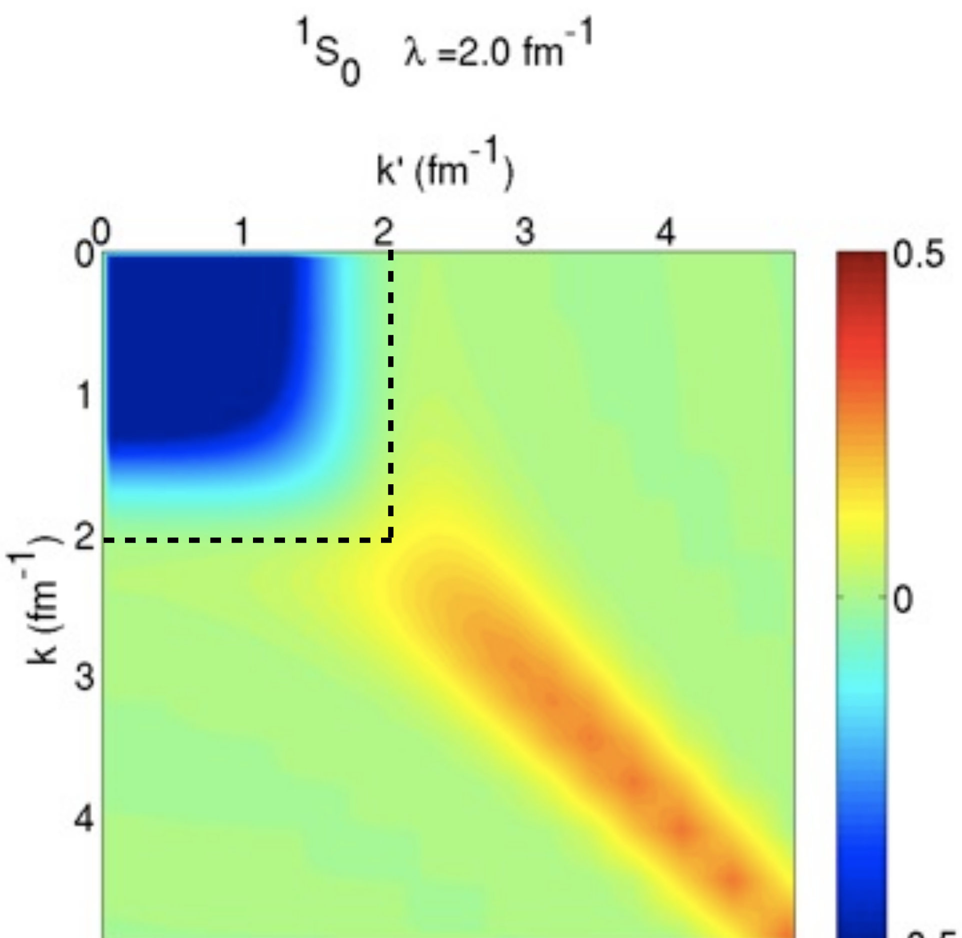
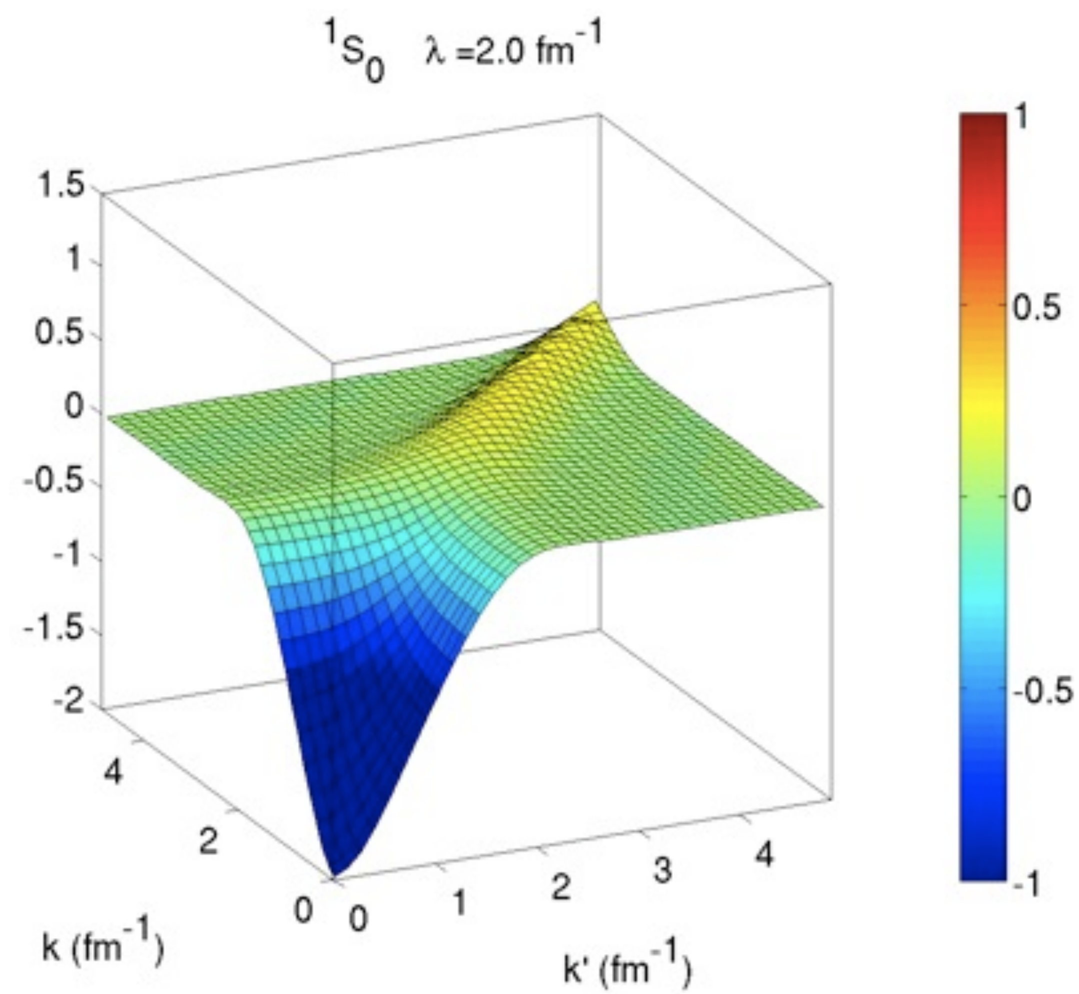
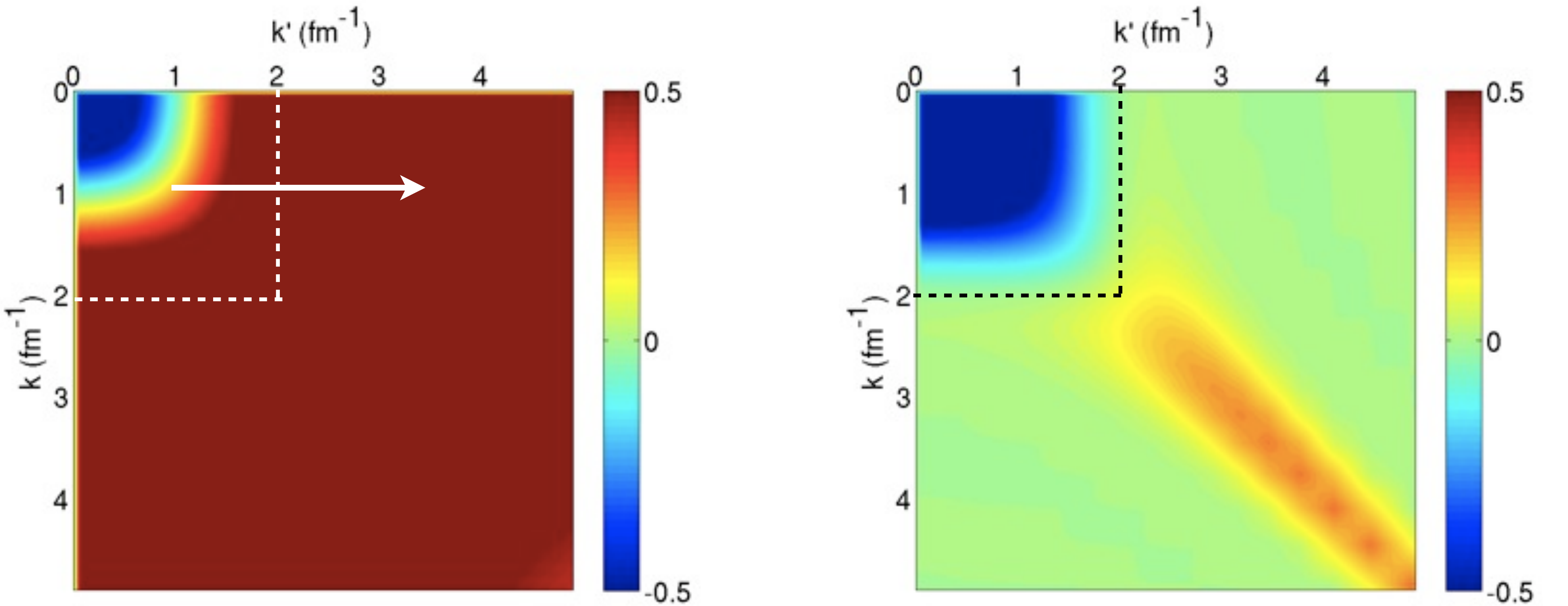




Changing the resolution: The (Similarity) Renormalization Group



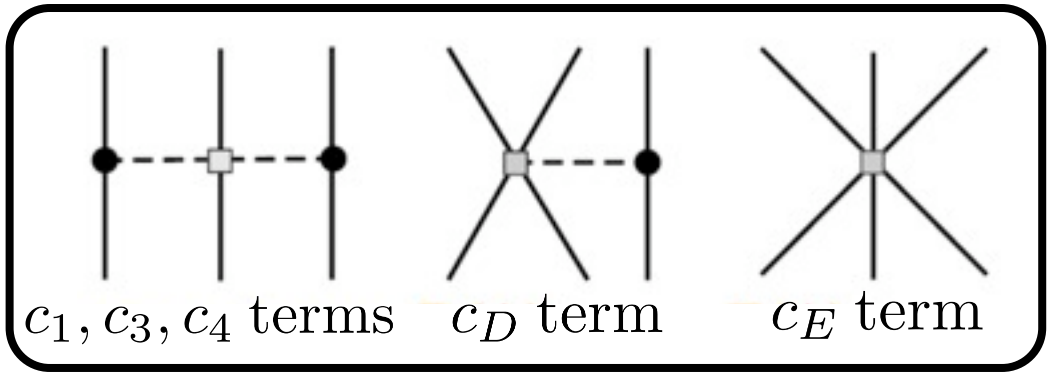
- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

RG evolution of 3N interactions

- So far:

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:



$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

$\longrightarrow$ coupling constants of natural size

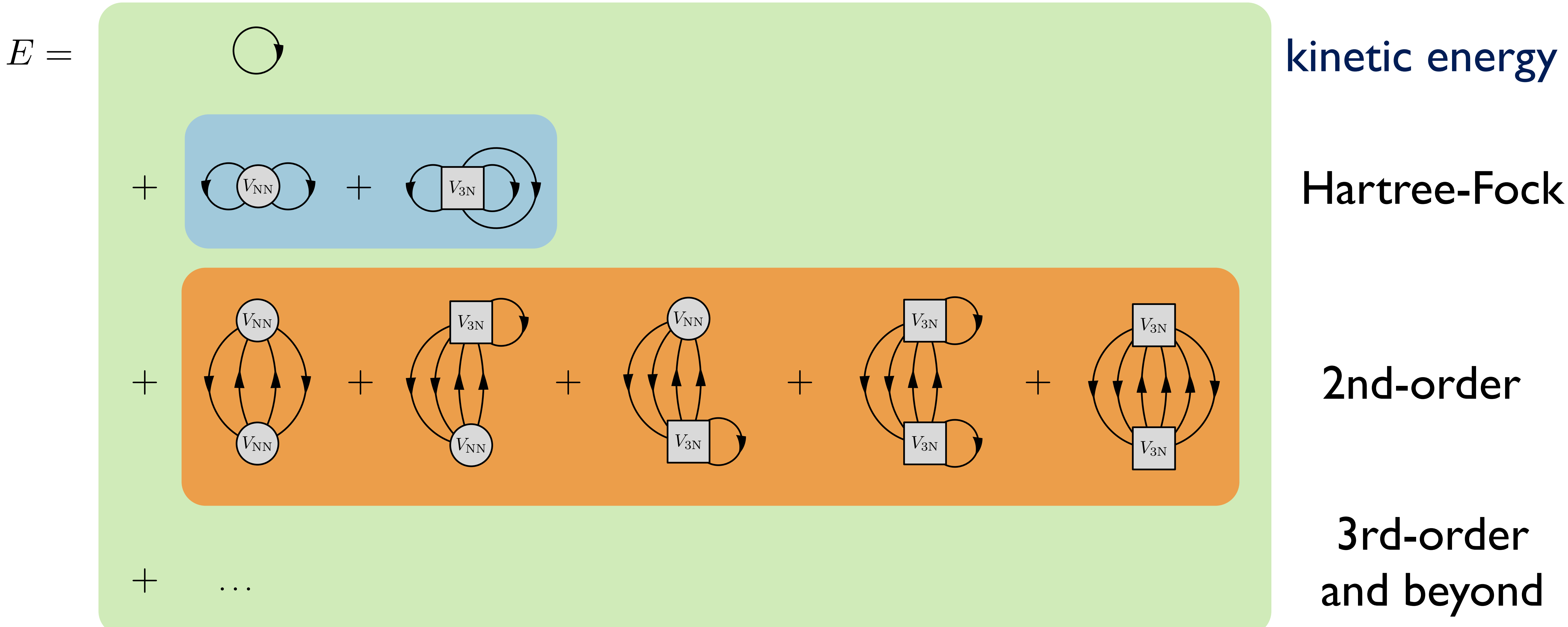
- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution

- Ideal case: evolve 3NF consistently with NN to lower resolution using the RG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - problems in heavier nuclei
  - not suitable for infinite systems

Equation of state: Many-body perturbation theory

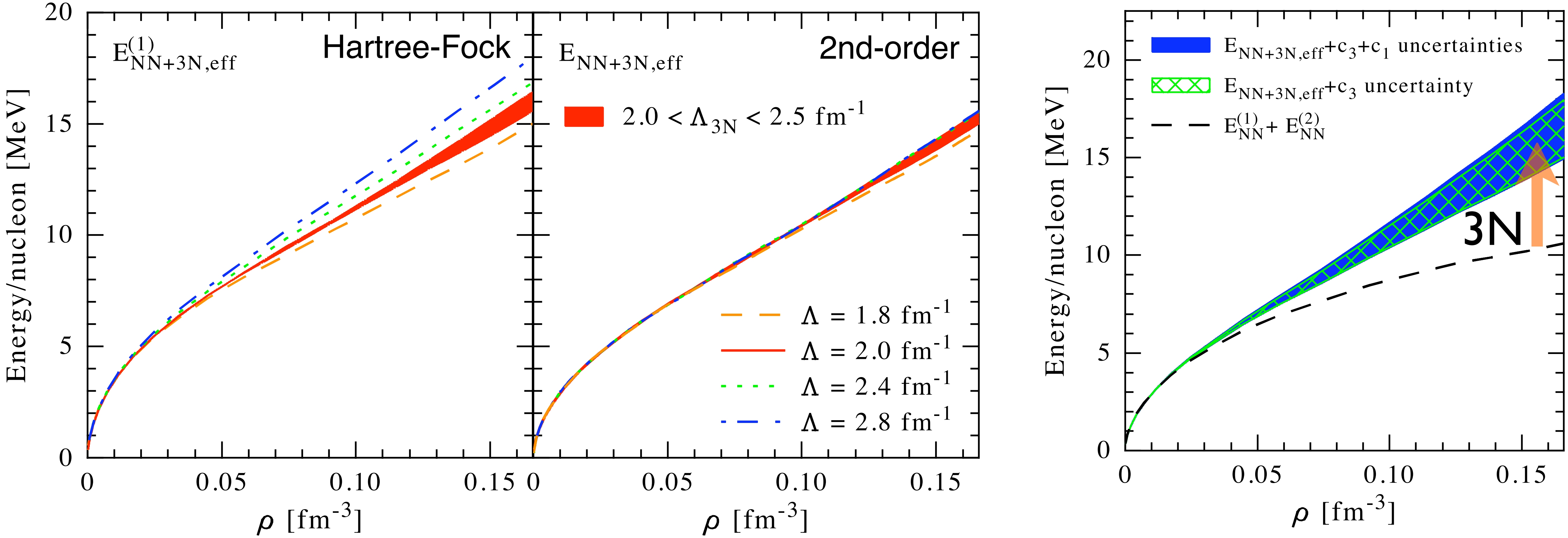
central quantity of interest: energy per particle E/N

$$H(\lambda) = T + V_{\mathrm{NN}}(\lambda) + V_{3\mathrm{N}}(\lambda) + \ldots$$



- "hard" interactions require non-perturbative summation of diagrams
- with low-resolution interactions much more perturbative
- inclusion of 3N interaction contributions crucial
- use chiral interactions as initial input for RG evolution

Equation of state of pure neutron matter



KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

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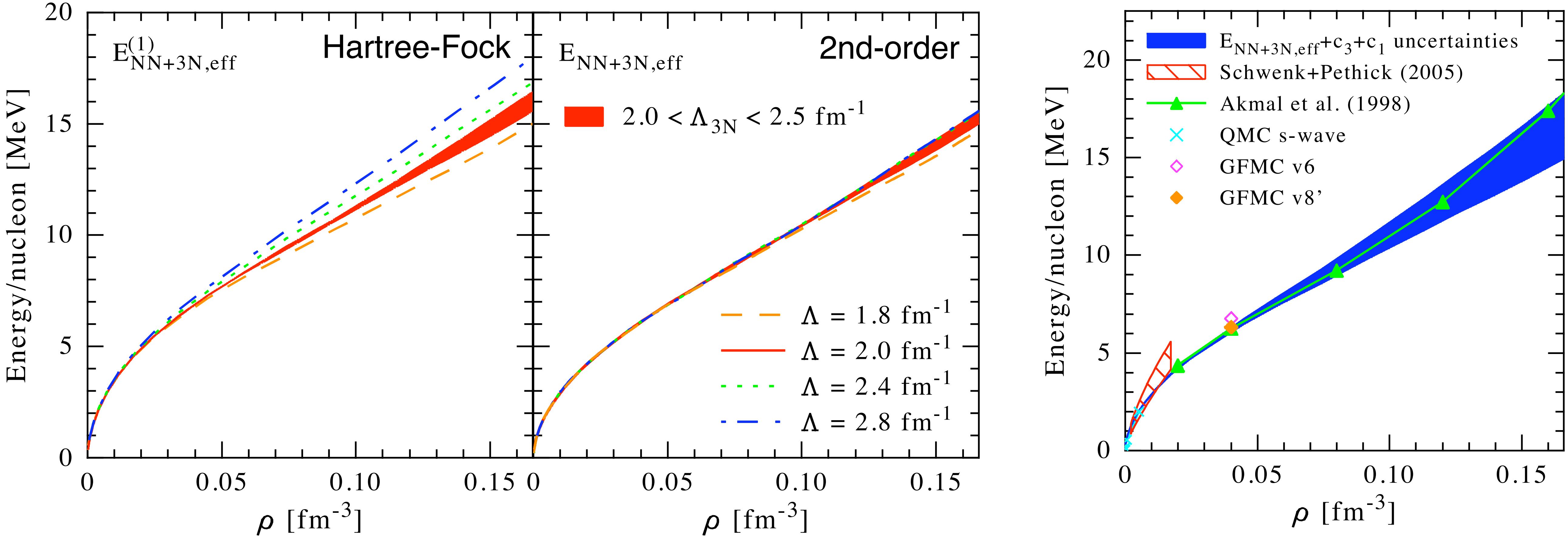

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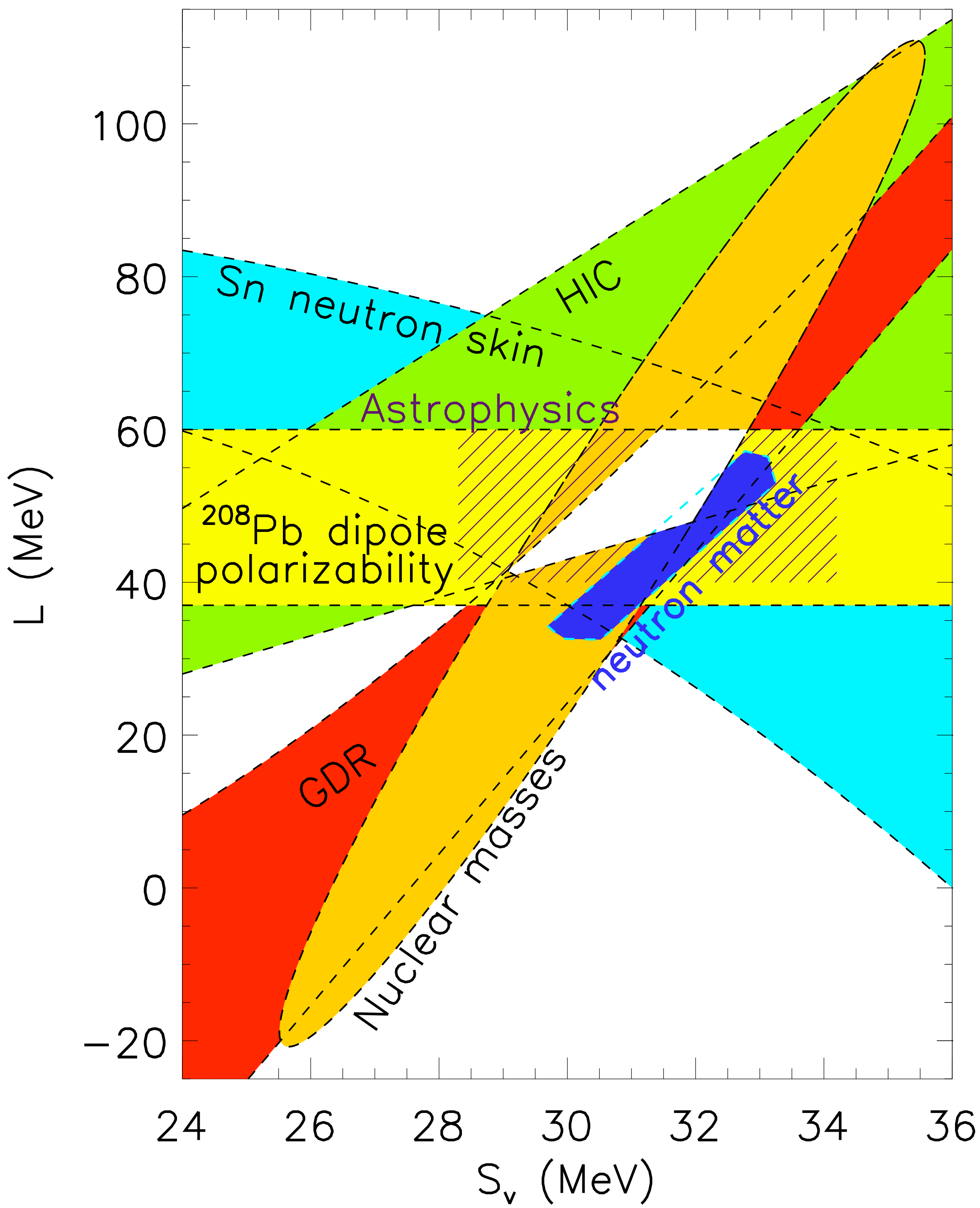
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

Symmetry energy constraints



extend EOS to finite proton fractions x

and extract symmetry energy parameters

$$S_v = \left. \frac{\partial^2 E/N}{\partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

$$L = \frac{3}{8} \left. \frac{\partial^3 E/N}{\partial \rho \partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

KH, Lattimer, Pethick and Schwenk, in preparation

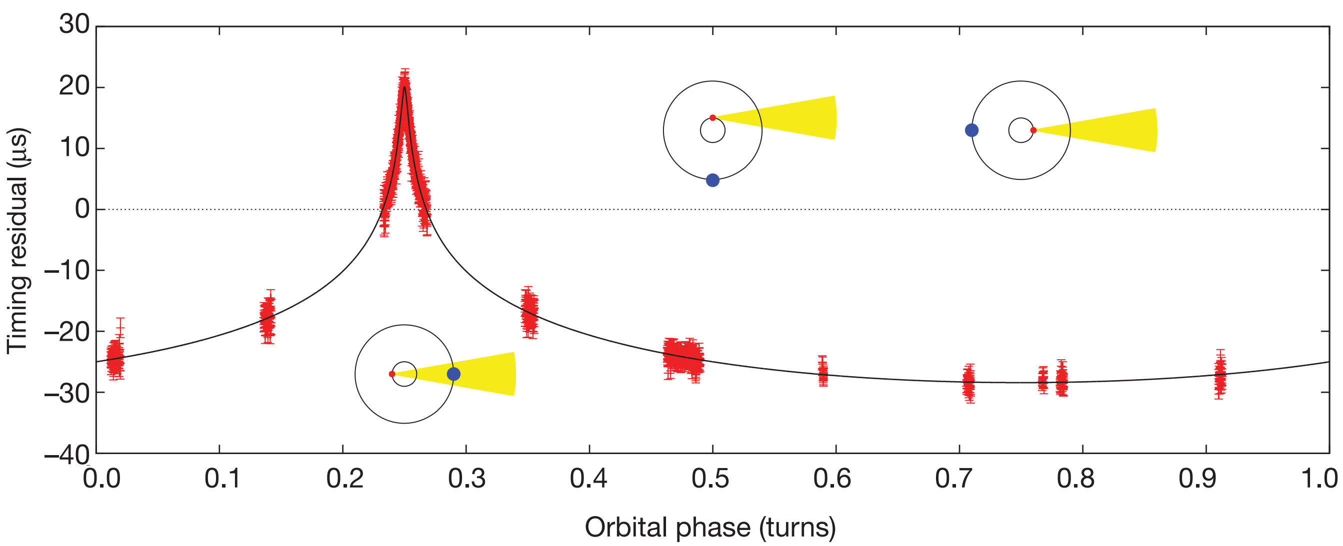
symmetry energy parameters consistent with other constraints

Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

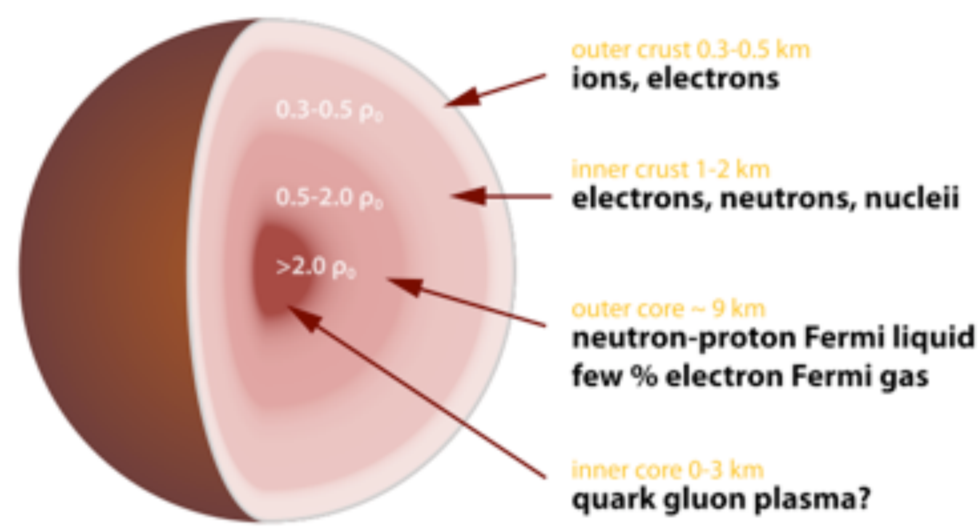


Demorest et al., Nature 467, 1081 (2010)

$$M_{\rm max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$

Credit: NASA/Dana Berry

Calculation of neutron star properties requires EOS up to high densities.



Strategy:

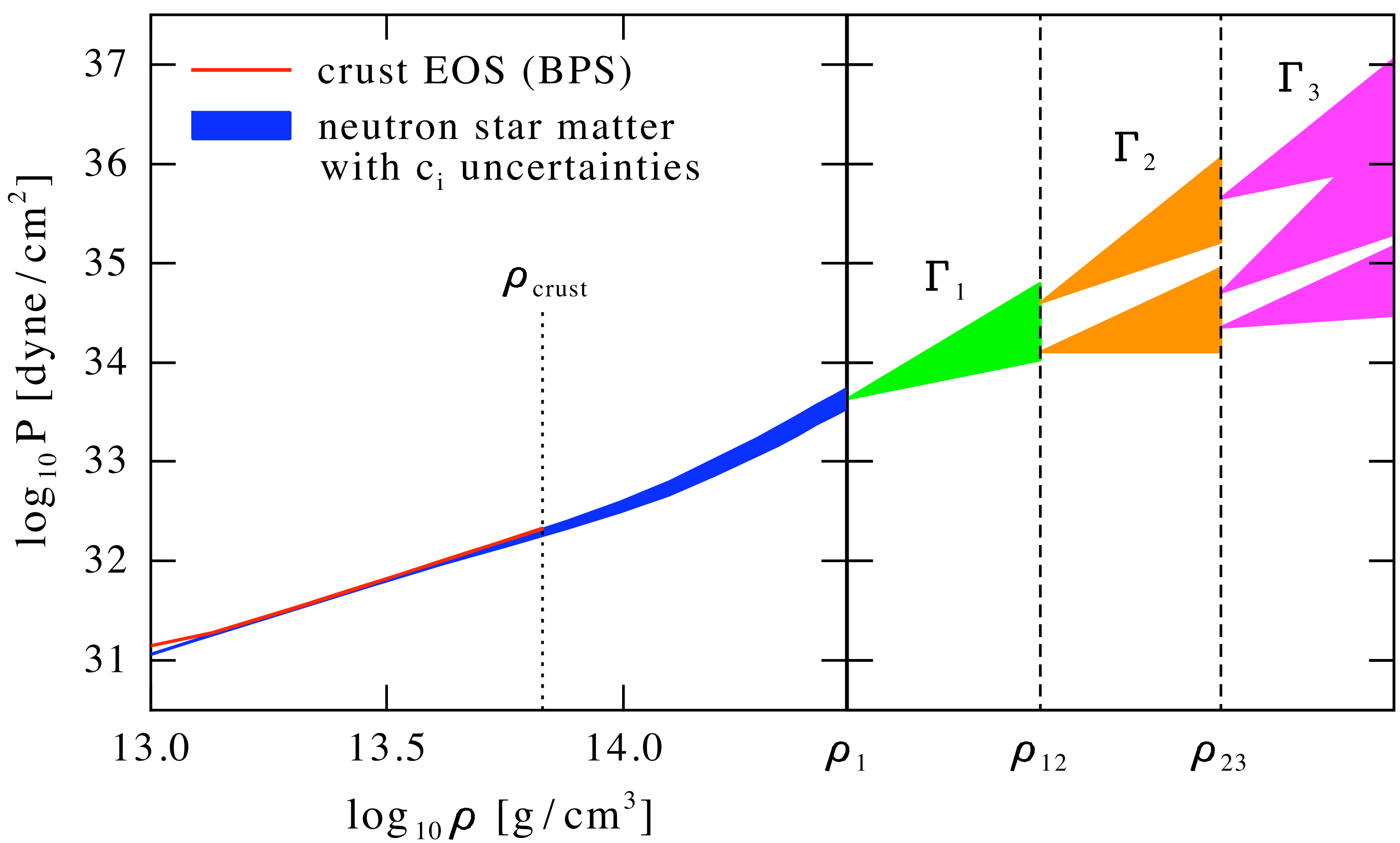
Use observations to constrain the high-density part of the nuclear EOS.

Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!



see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

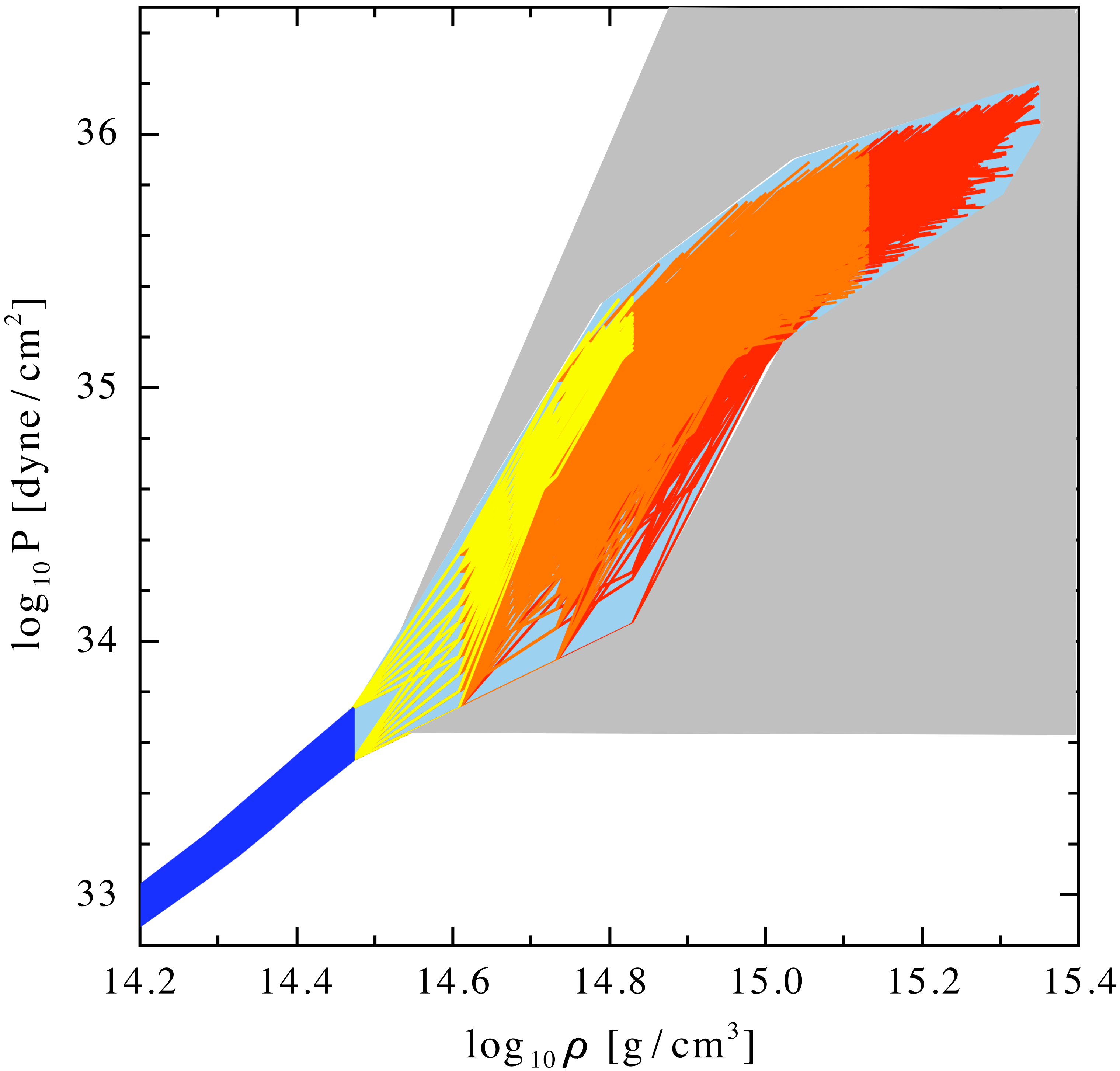
use the constraints:

recent NS observation

$$M_{\max} > 1.97\, M_\odot$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$



significant reduction of possible equations of state

Constraints on the nuclear equation of state

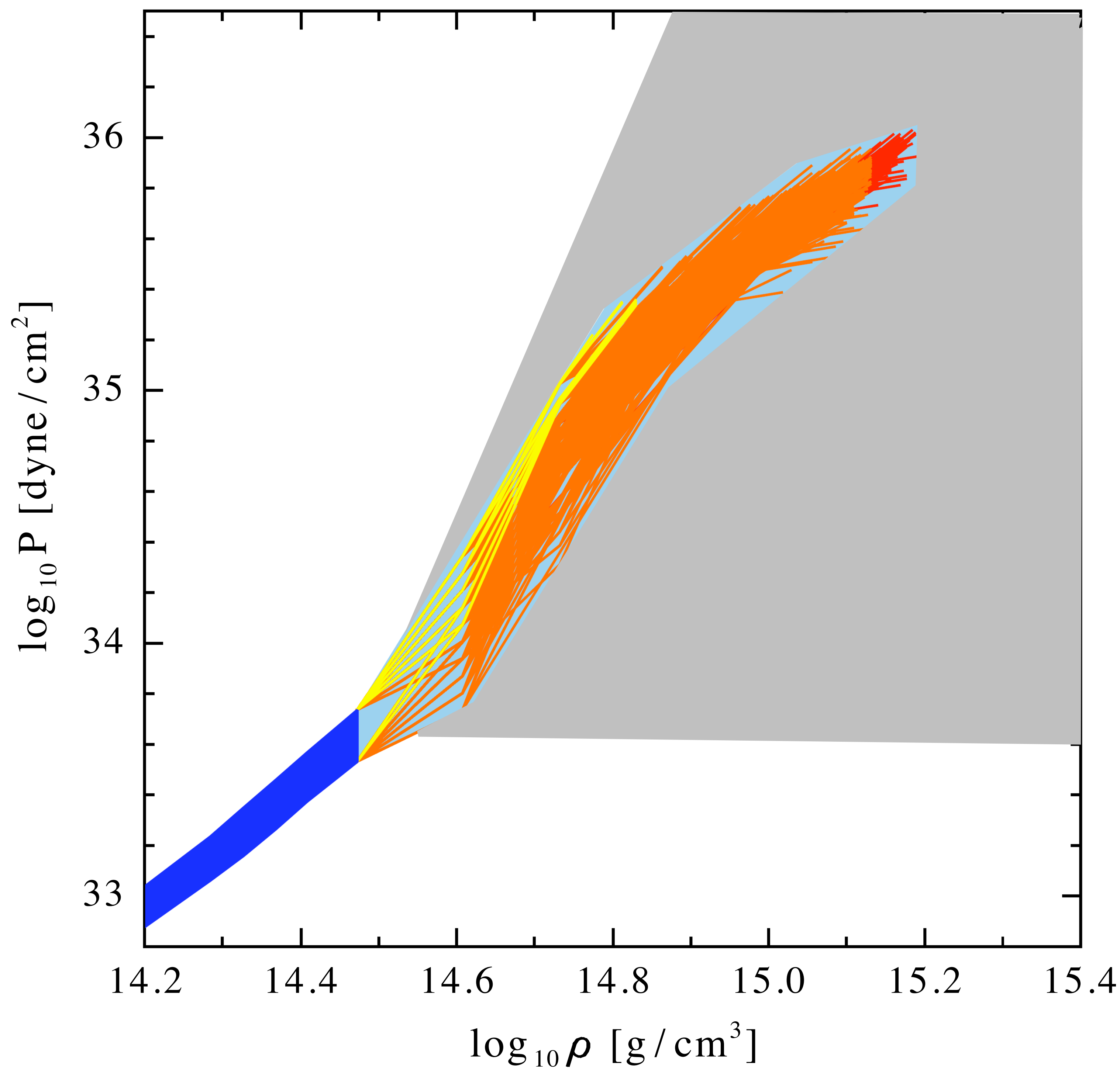
use the constraints:

NS mass

$$M_{\max} > 2.4\,M_\odot$$

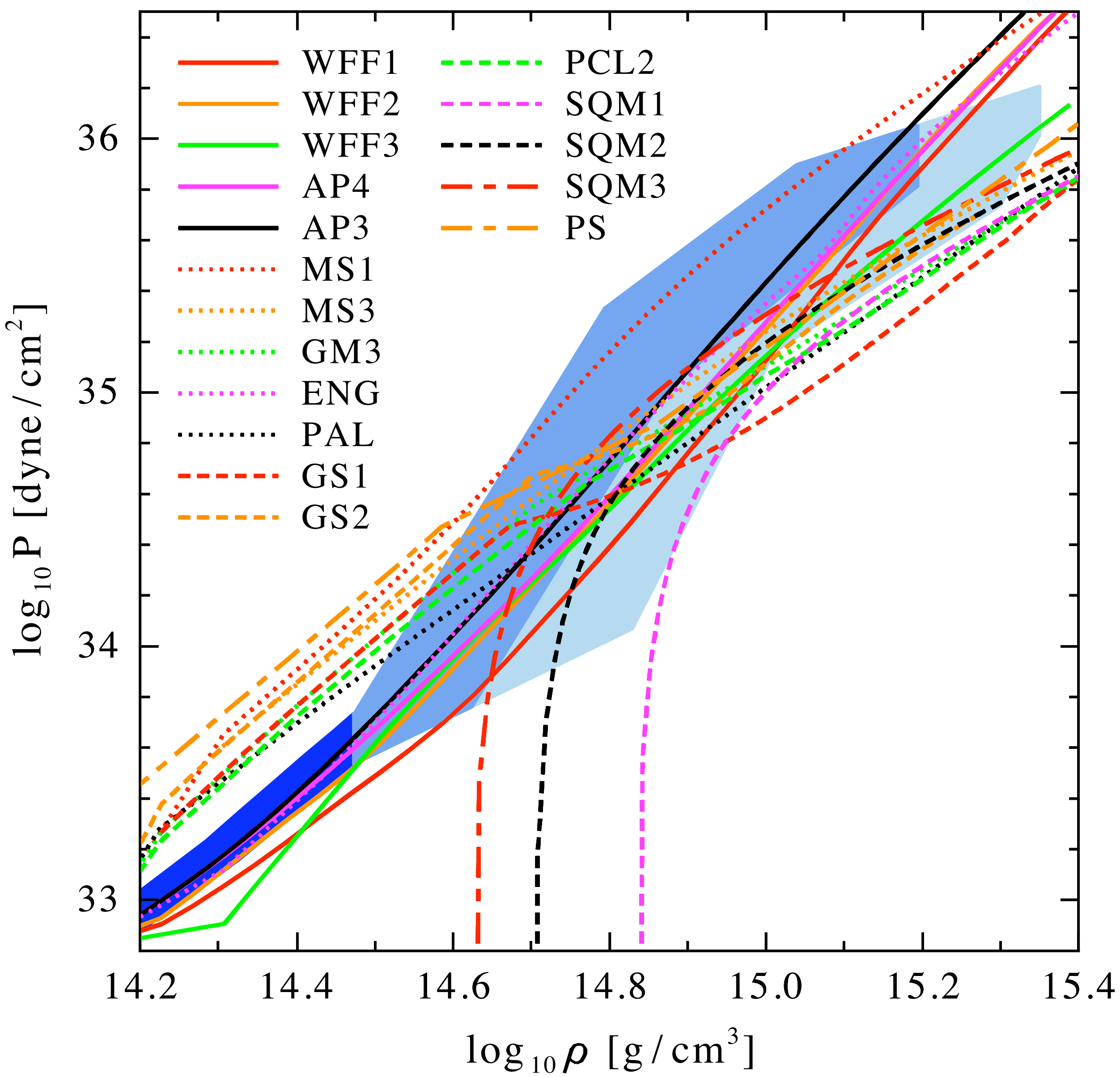
causality

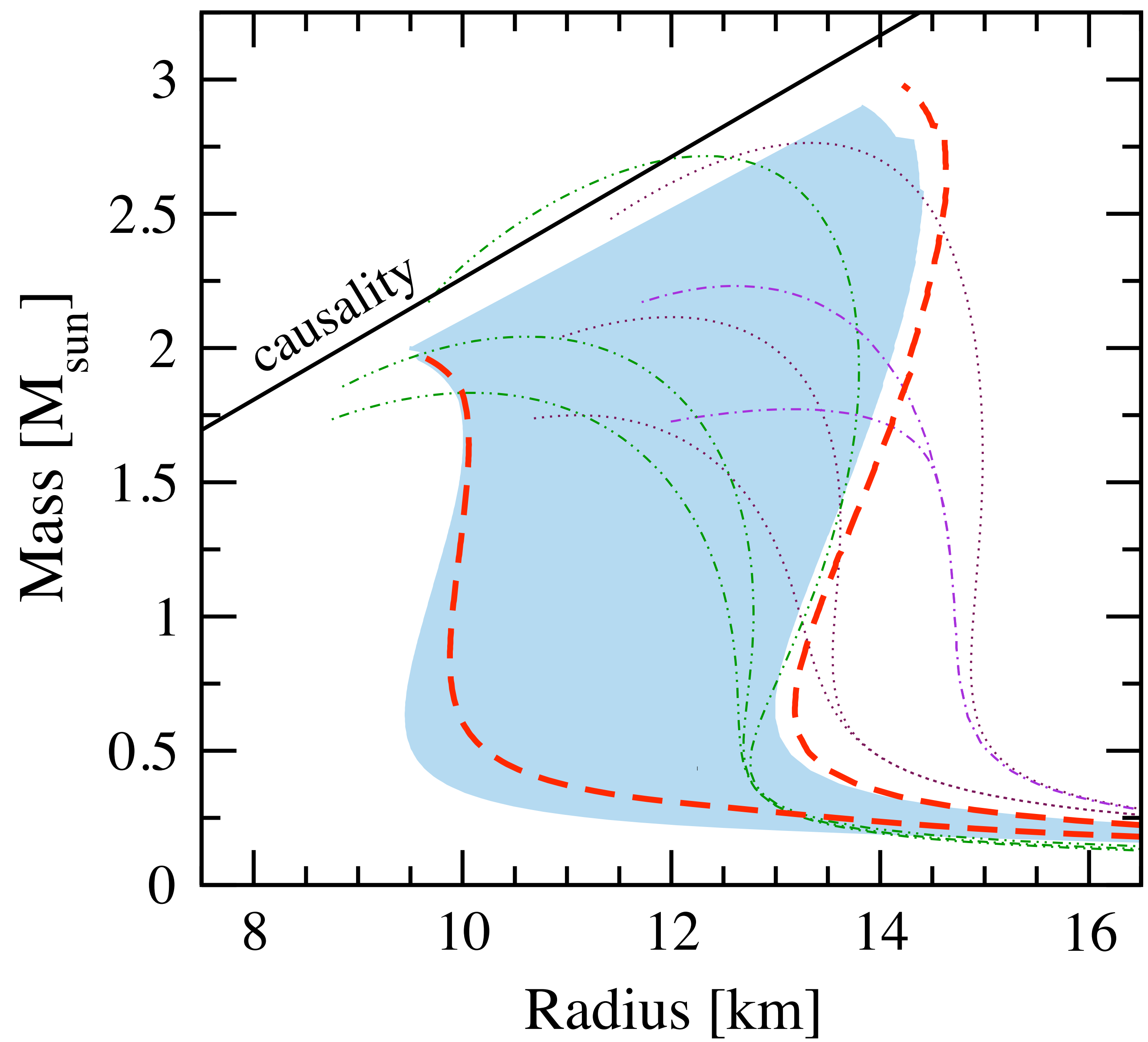
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$



increased $M_{\max}$ systematically leads to stronger constraints

Constraints on neutron star radii

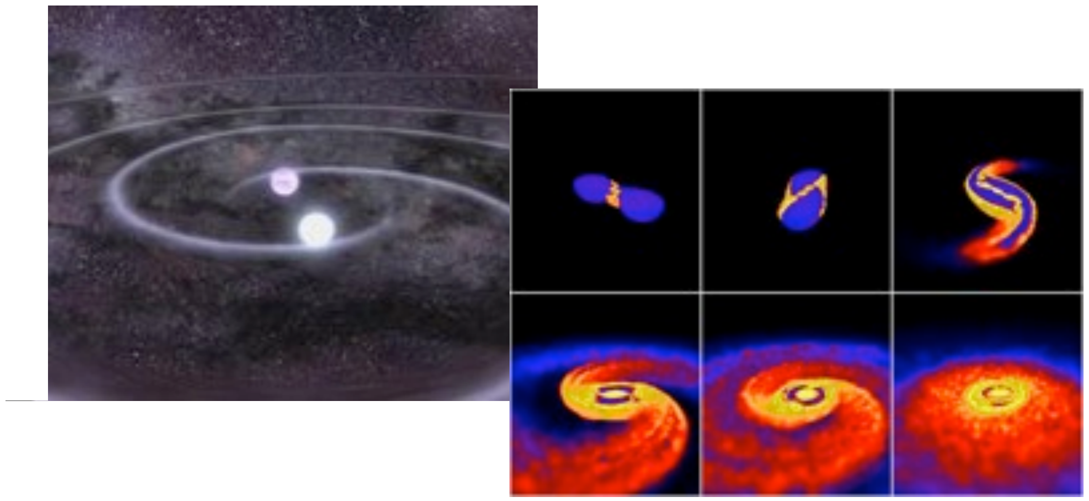


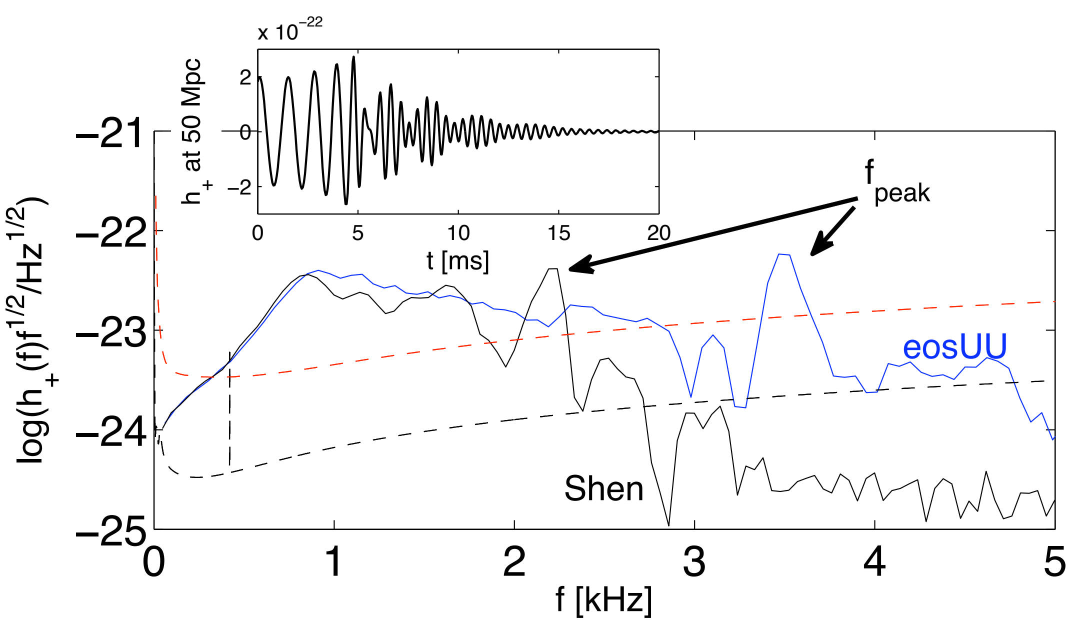


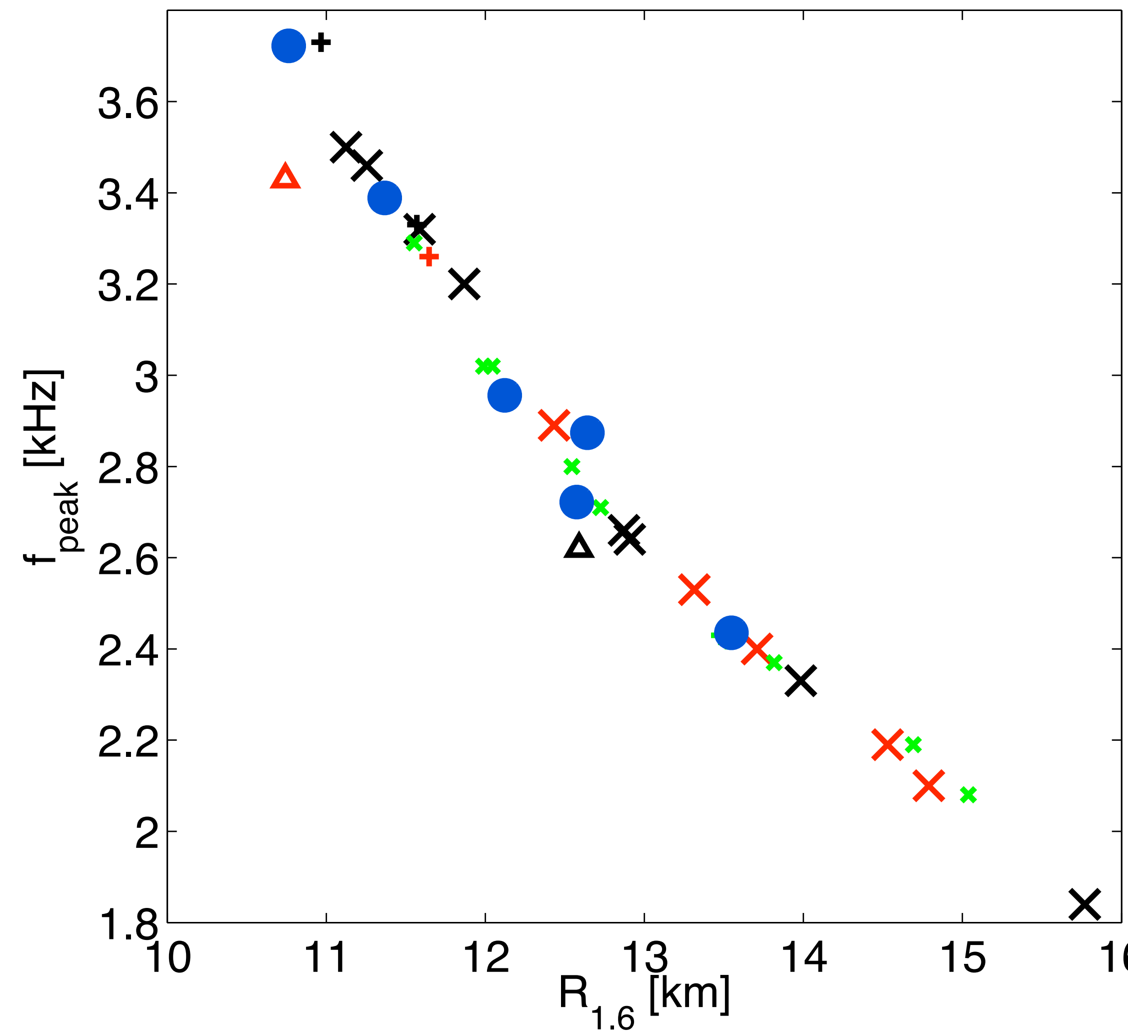
KH, Lattimer, Pethick, Schwenk, in preparation
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint for typical $1.4\,M_\odot$ neutron star: $9.8 - 13.4\,\mathrm{km}$

Gravitational wave signals from neutron star binary mergers



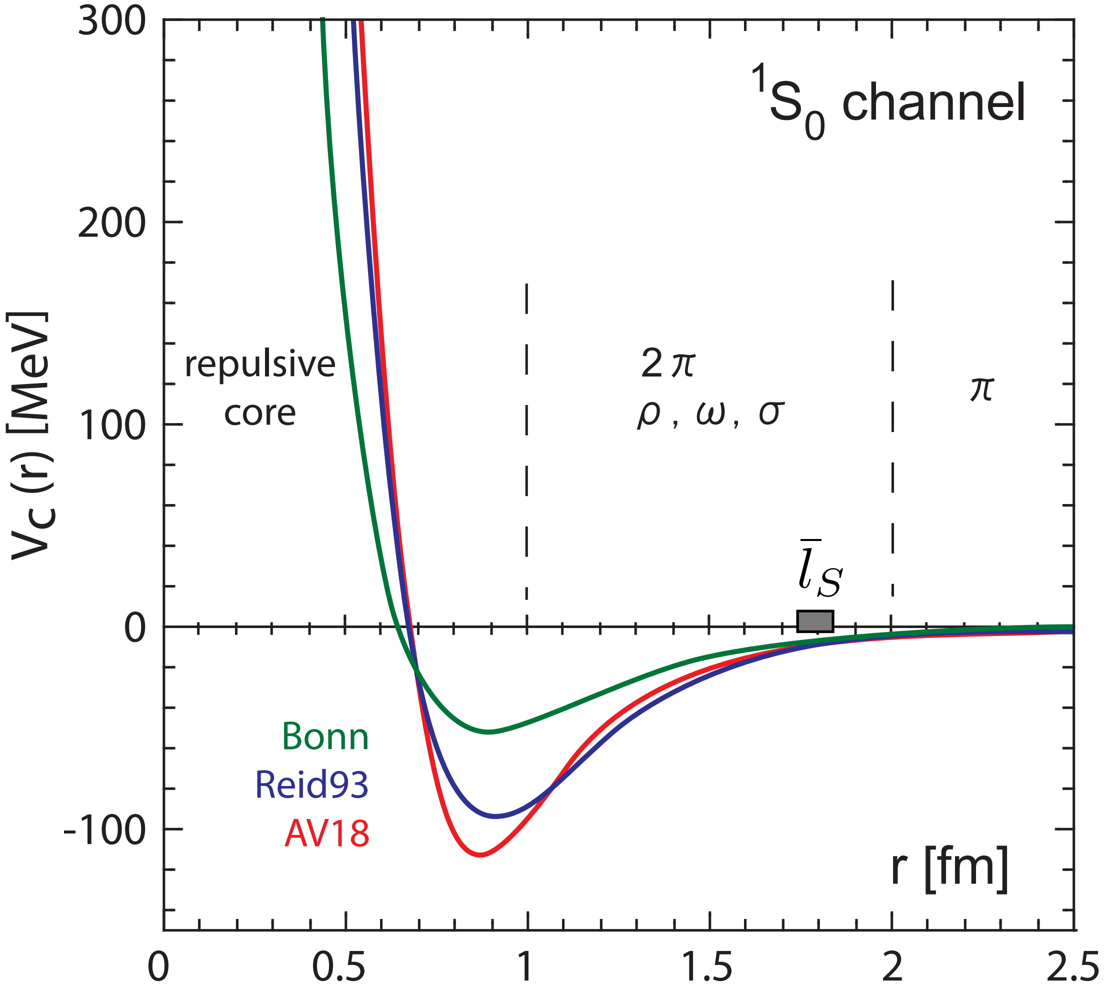


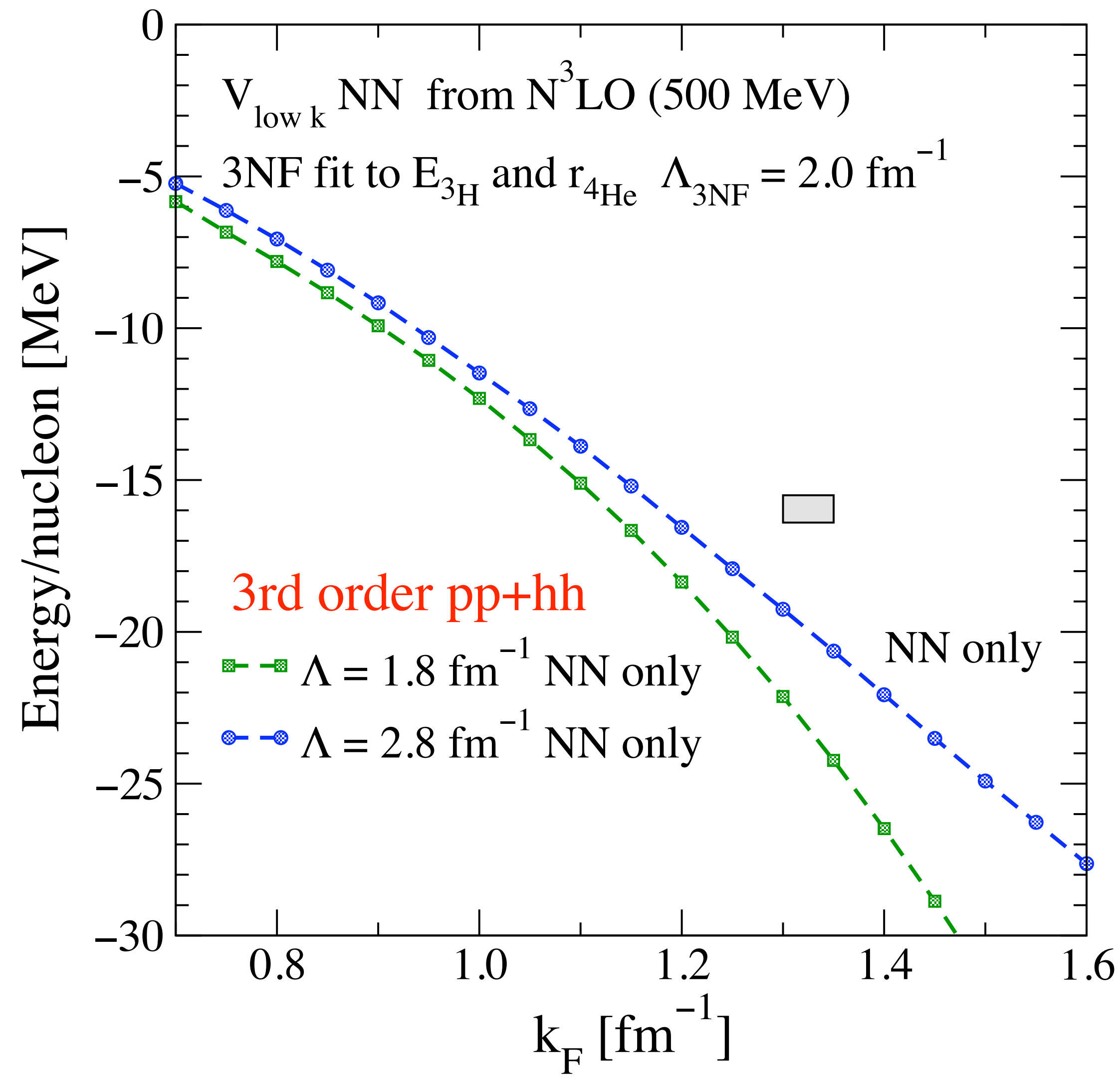


Bauswein and Janka PRL 108, 011101 (2012),
Bauswein, Janka, KH, Schwenk arXiv:1204.1888

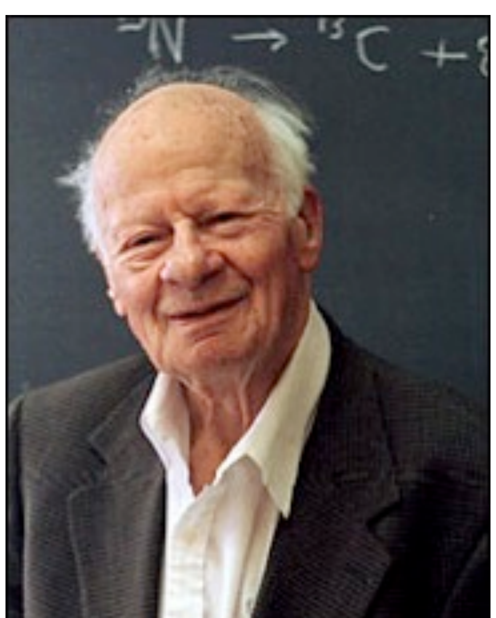
- high-density part of nuclear EOS only loosely constrained
- simulations of NS binary mergers show strong correlation between between f_{peak} of the GW spectrum and the raduis of a NS
- measuring f_{peak} is key step for constraining EOS systematically at large ρ

Equation of state of symmetric nuclear matter, nuclear saturation





KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)



"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

Hans Bethe (1971)

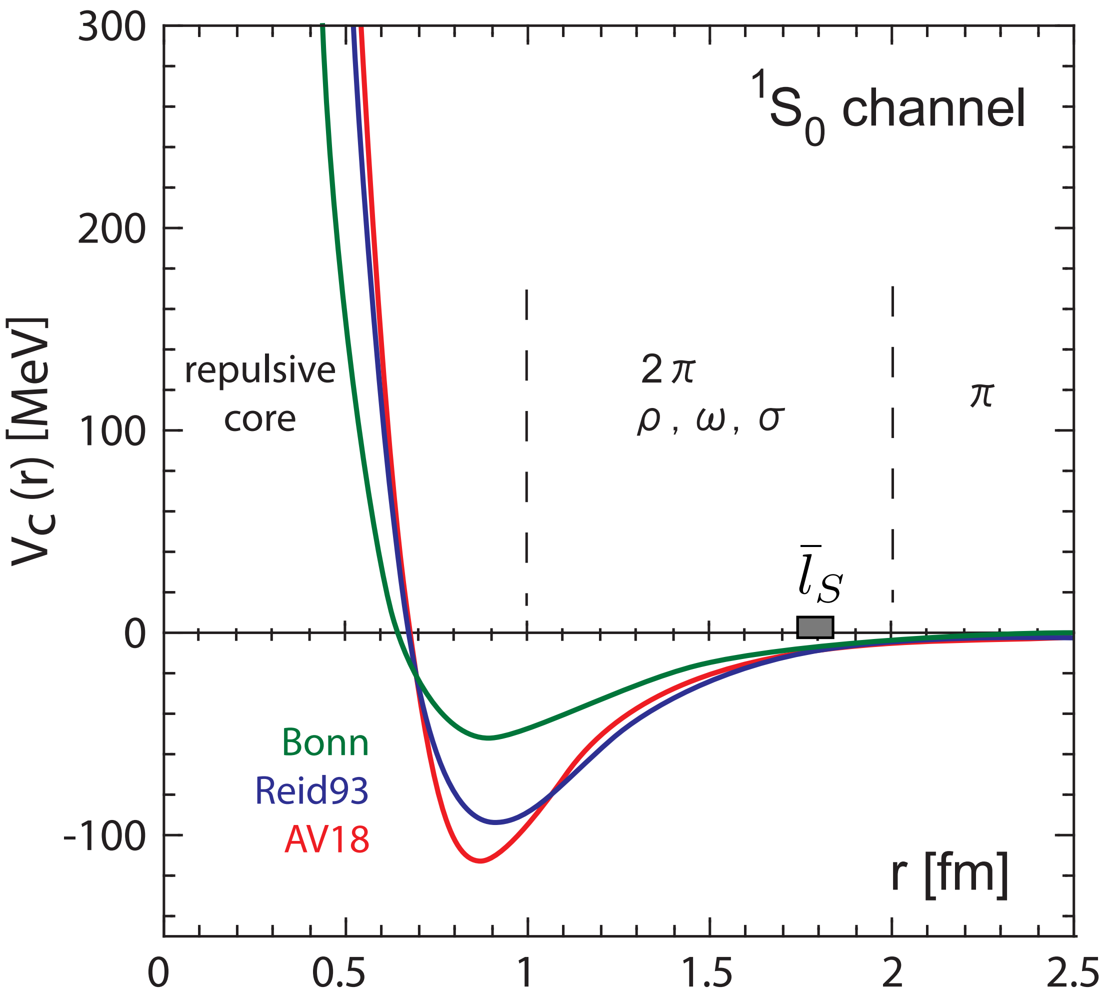
empirical nuclear saturation properties

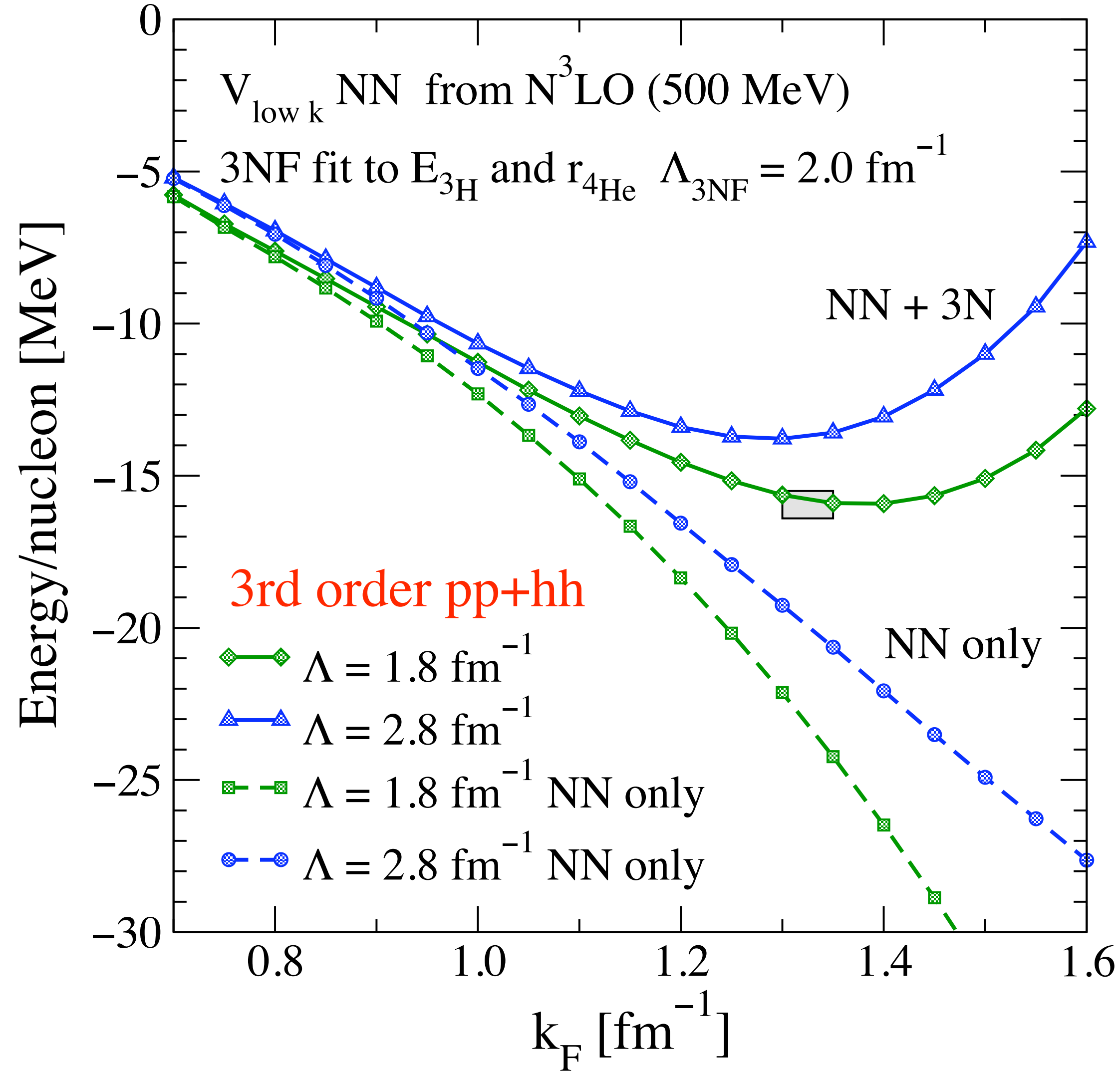
$$n_S \sim 0.16\,\mathrm{fm}^{-3}$$

$$E_{\mathrm{binding}}/N \sim -16\,\mathrm{MeV}$$

$$\bar{l}_S \sim 1.8\,\mathrm{fm}$$

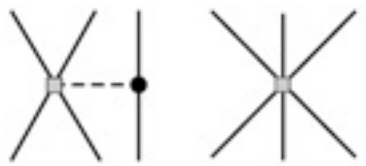
Equation of state of symmetric nuclear matter, Nuclear saturation



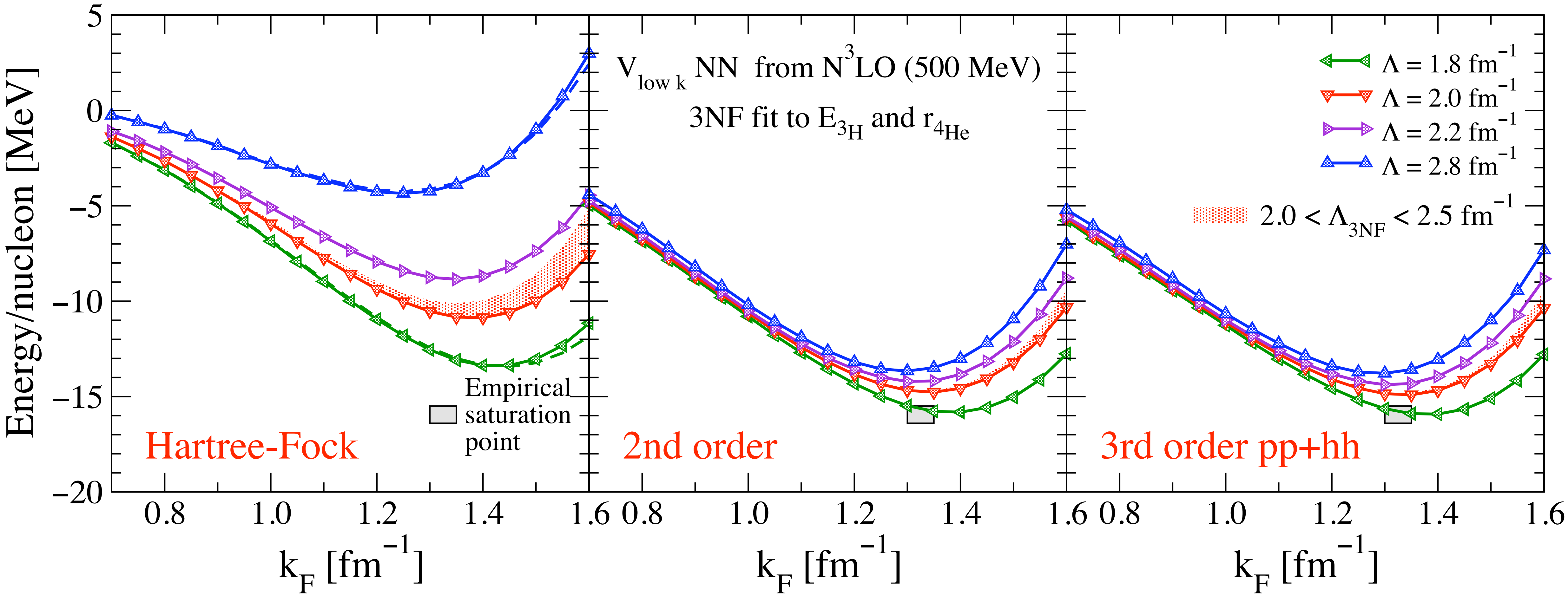


KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

- nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions
- 3N forces are essential! 3N interactions fitted to $^3\mathrm{H}$ and $^4\mathrm{He}$ properties



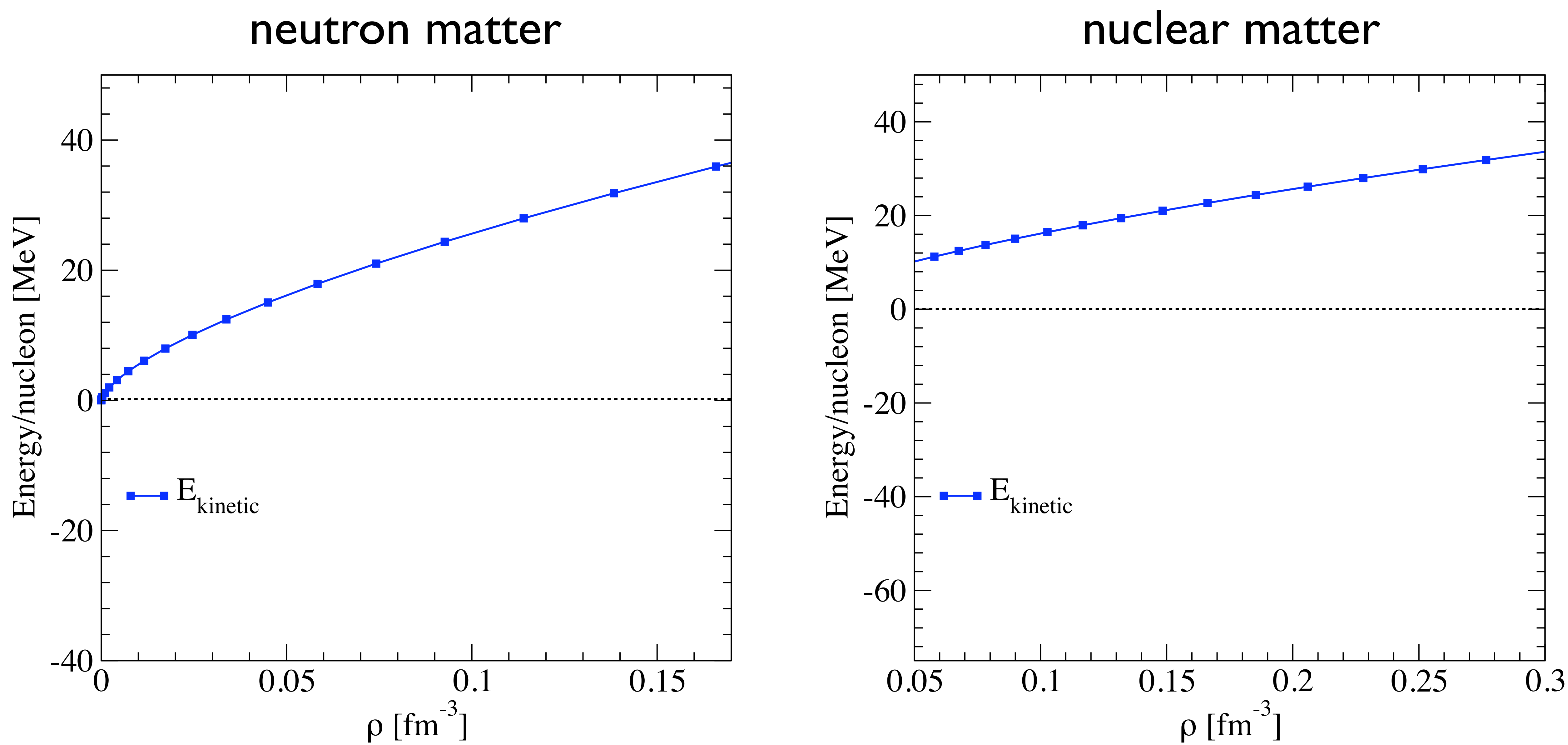
Equation of state of symmetric nuclear matter, Nuclear saturation



KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

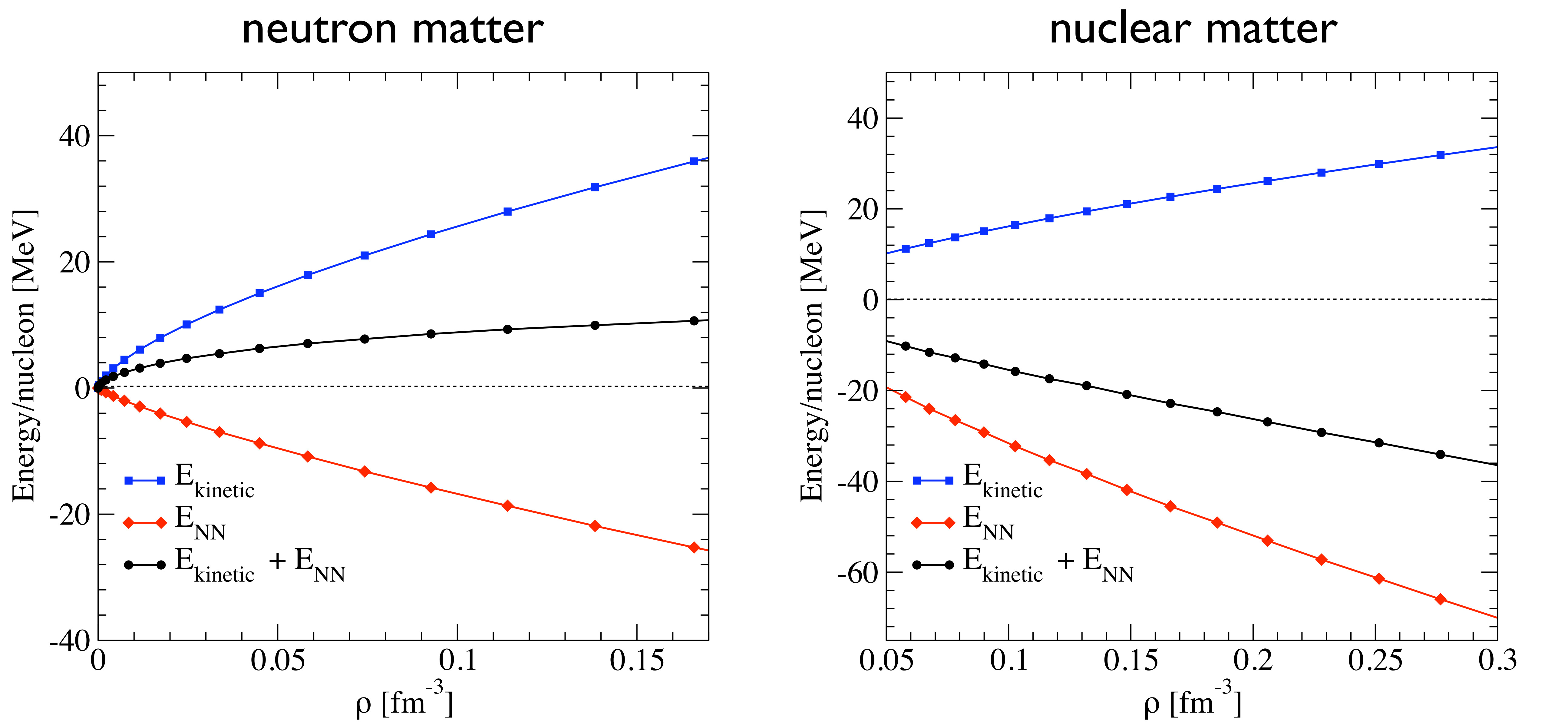
- saturation point consistent with experiment, without free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

Hierarchy of many-body contributions



- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

Hierarchy of many-body contributions



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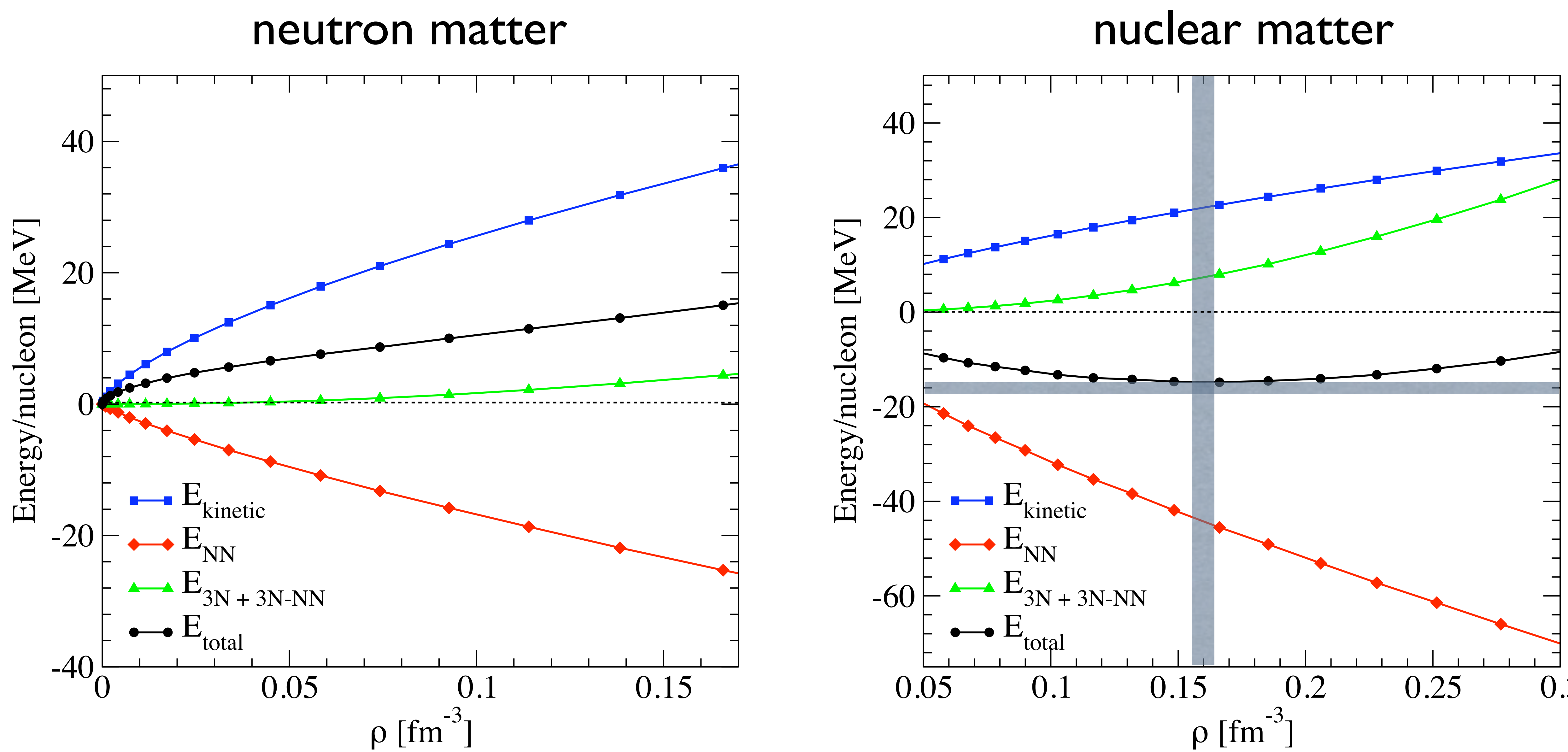
Hierarchy of many-body contributions



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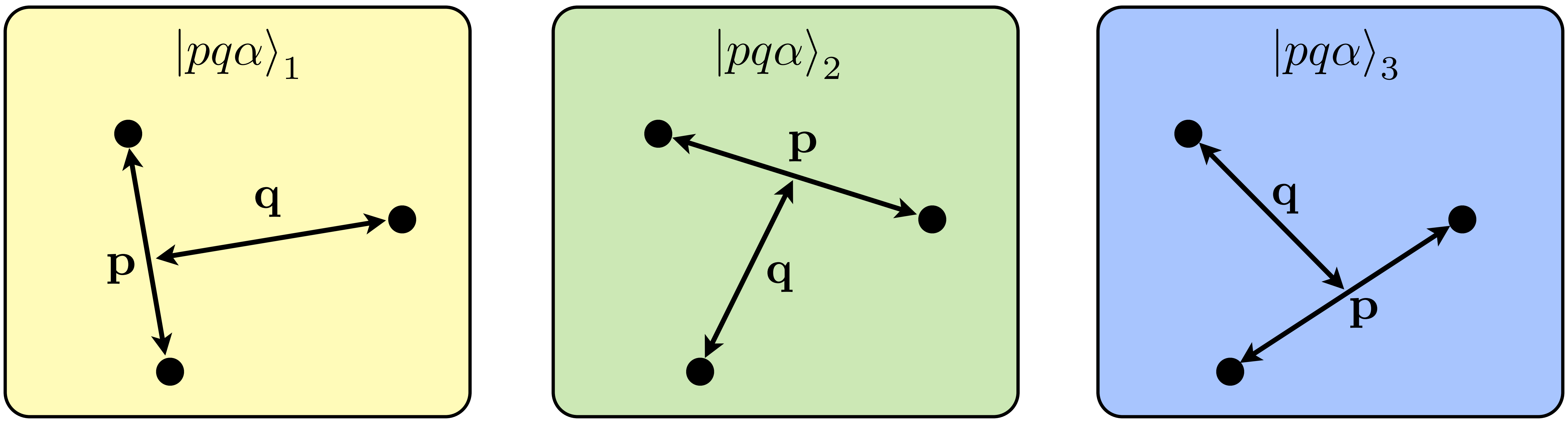
RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



Faddeev bound state equations:

$$|\psi_i\rangle = G_0\left[2t_i P + (1 + t_i G_0)\, V_{3N}^i (1 + 2P)\right]|\psi_i\rangle$$

$${}_i\langle pq\alpha|P|p'q'\alpha'\rangle_i = {}_i\langle pq\alpha|p'q'\alpha'\rangle_j$$

SRG flow equations of NN and 3N forces in Faddeev basis

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]$$

$$H = T + V_{12} + V_{13} + V_{23} + V_{123}$$

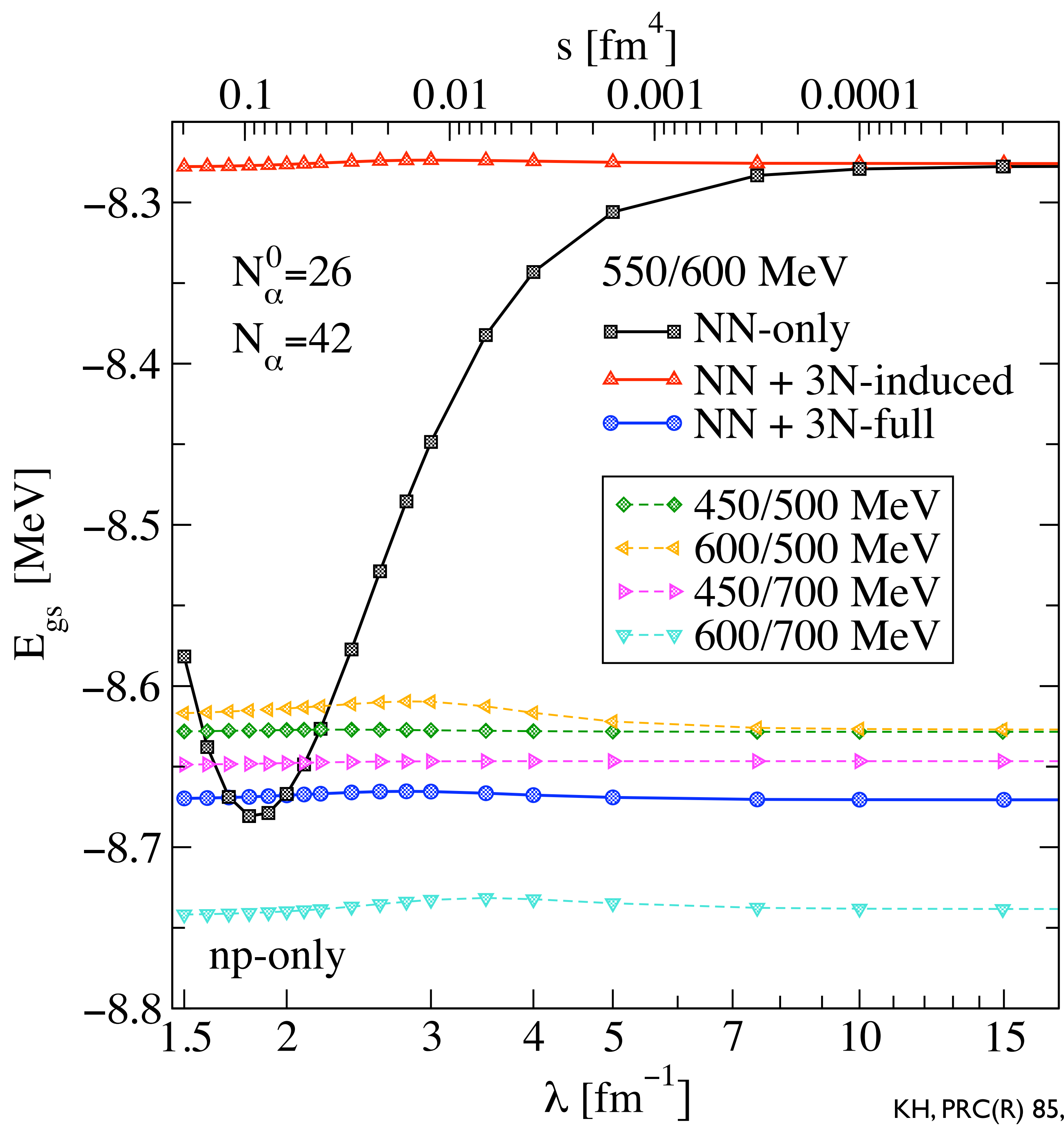
- spectators correspond to delta functions, matrix representation of H_s ill-defined
- solution: explicit separation of NN and 3N flow equations

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= [[T_{ij}, V_{ij}], T_{ij} + V_{ij}], \\
\frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}] \\
&+ [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}] \\
&+ [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}] \\
&+ [[T_{\rm rel}, V_{123}], H_s]
\end{aligned}$$

- only connected terms remain in $\frac{dV_{123}}{ds}$, 'dangerous' delta functions cancel

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

RG evolution of 3N interactions in momentum space



KH, PRC(R) 85, 021002 (2012)

First implementation:

Invariance of E_{gs}^{3H} within $16\,\mathrm{keV}$ for consistent chiral interactions at $\mathrm{N^2LO}$

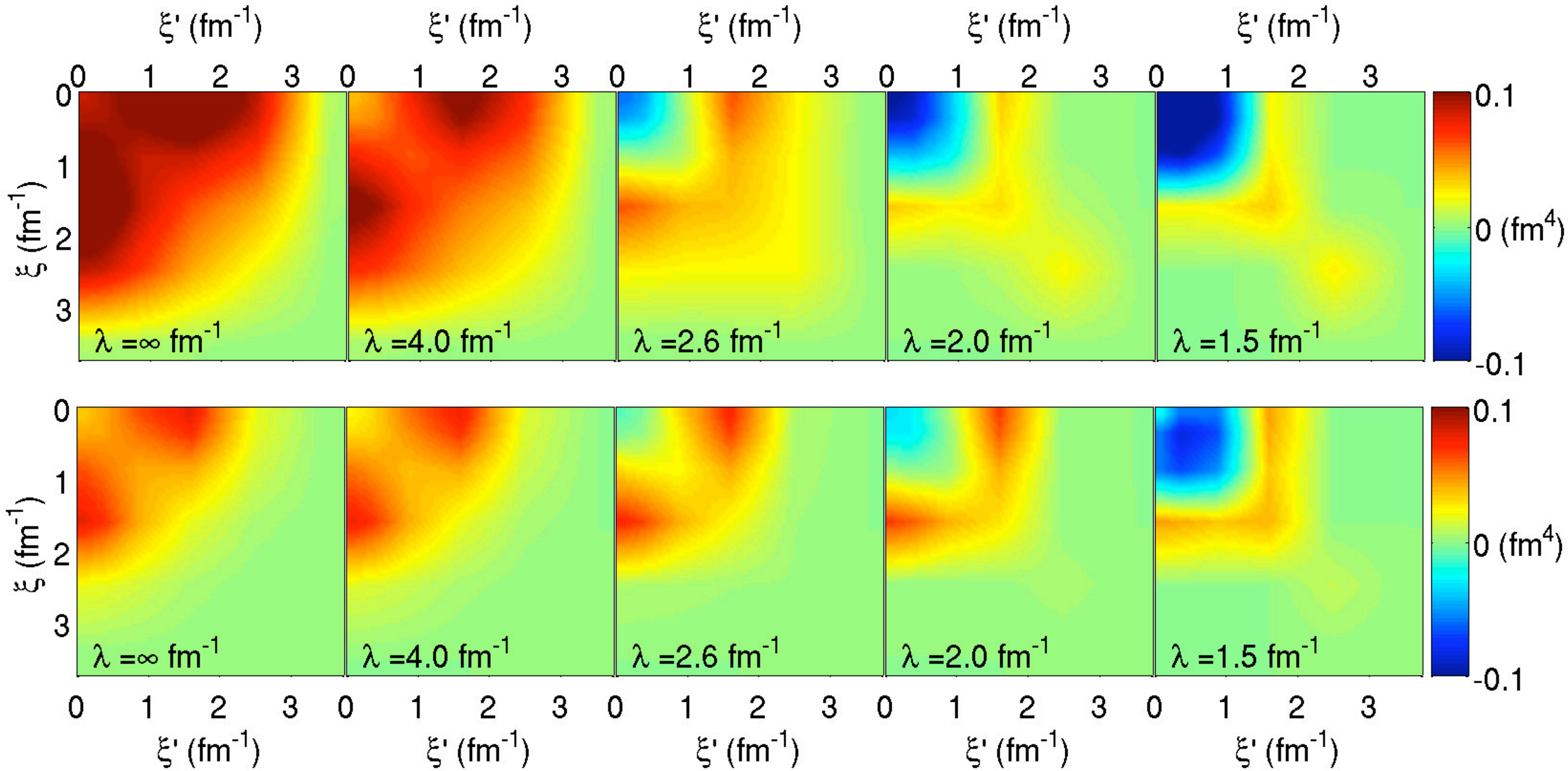
Decoupling of matrix elements

$\Lambda/\tilde{\Lambda}$

550/600 MeV

450/500 MeV

$\theta = \frac{\pi}{12}$



KH, PRC(R) 85, 021002 (2012)

hyperradius: $\xi^2 = p^2 + \frac{3}{4} q^2$

hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

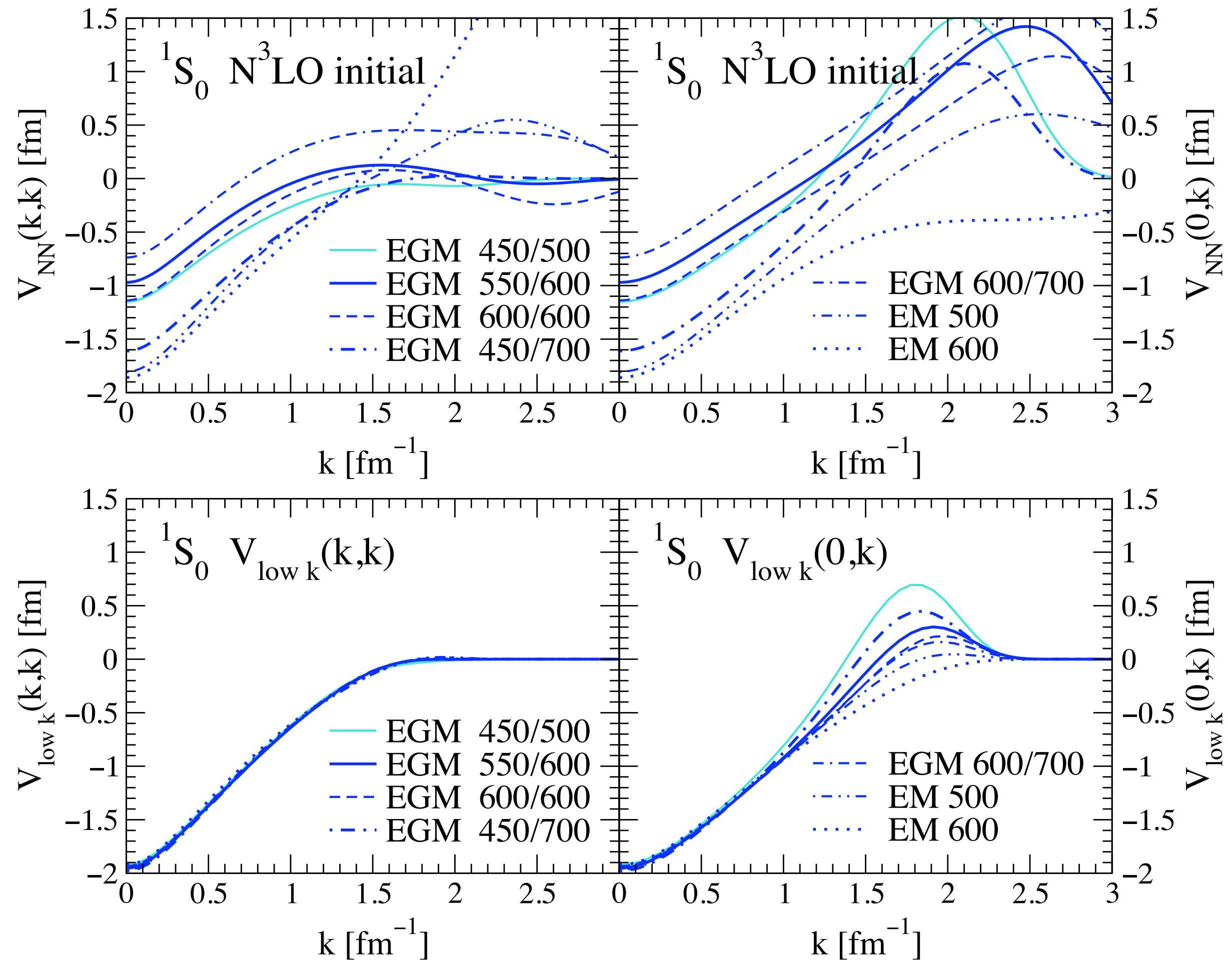
same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

phase-shift equivalence

common long-range physics

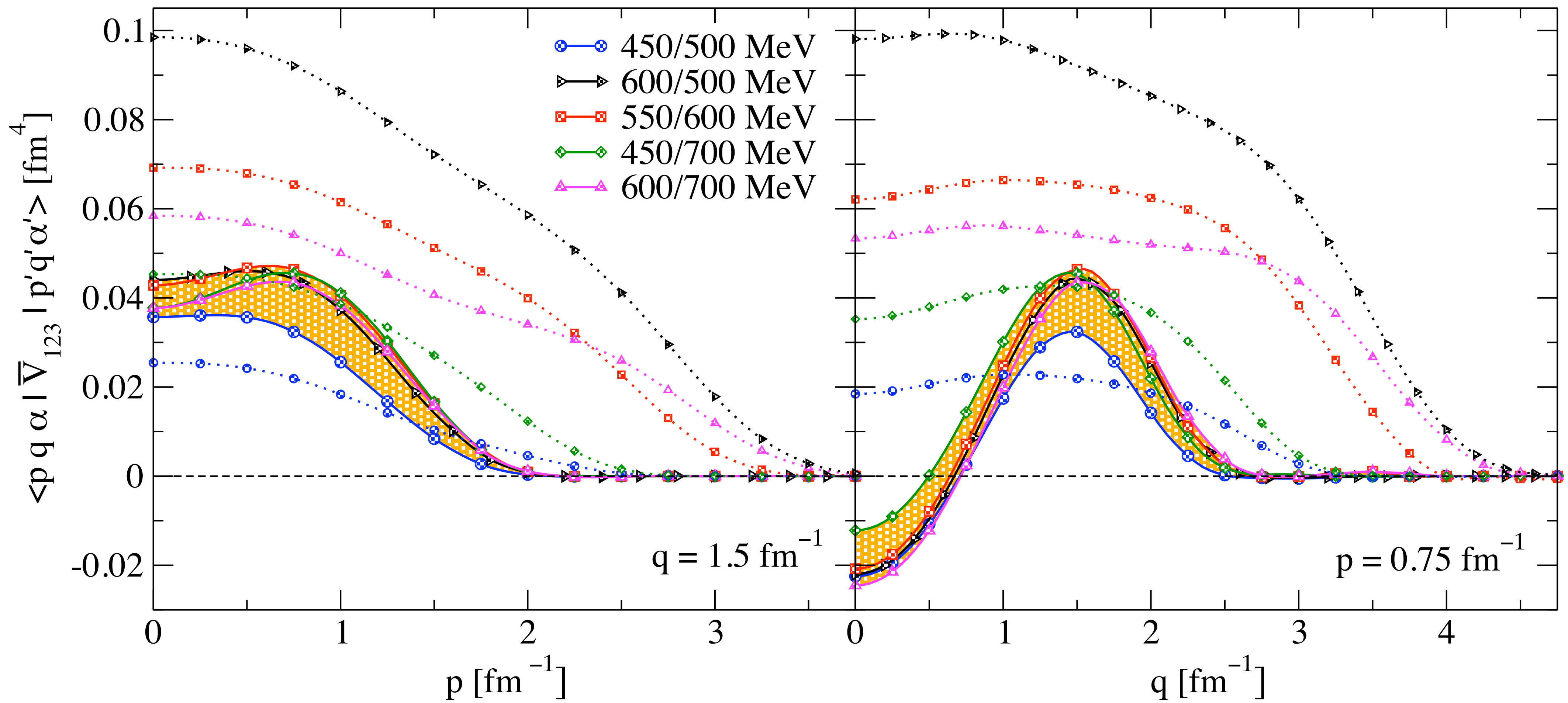
(approximate) universality of low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

Universality in 3N interactions at low resolution



KH, PRC(R) 85, 021002 (2012)

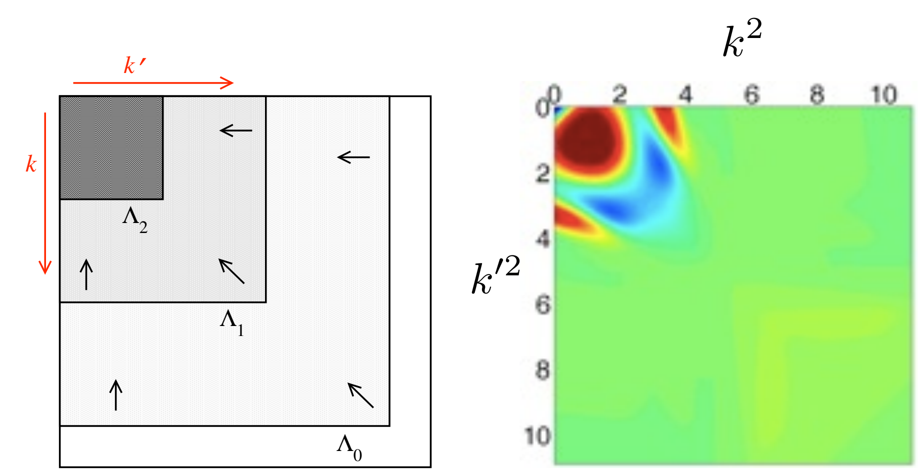
- remarkably reduced model dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$,

matrix elements with significant phase space well constrained at low resolution

- new momentum structures induced at low resolution
- study based on $\mathrm{N^2LO}$ chiral interactions, improved universality at $\mathrm{N^3LO}$?

Current/future directions

- application to infinite systems
  - equation of state
  - systematic study of induced many-body contributions
  - include initial N3LO 3N interactions (see also next talk!)
- transformation of evolved interactions to oscillator basis
  - application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)
- study of alternative generators
  - different decoupling patterns (e.g. $V_{\text{low k}}$)
  - improved efficiency of evolution
  - suppression of many-body forces



Anderson et al., PRC 77, 037001 (2008)

- explicit calculation of unitary 3N transformation
  - RG evolution of operators
  - study of correlations in nuclear systems $\longrightarrow$ factorization

Summary

- low-resolution interactions allow simpler calculations for nuclear systems
- observables invariant under resolution changes, interpretation of results can change!
- chiral EFT provides systematic framework for constructing nuclear Hamiltonians
- 3N interactions are essential at low resolution
- nucleonic matter equation of state based on low-resolution interactions consistent with empirical constraints
- constraints for the nuclear equation of state and structure of neutron stars

Outlook

- RG evolution of three-nucleon interactions: microscopic study of light nuclei and nucleonic matter using chiral nuclear interactions at low resolution
- RG evolution of operators: nuclear scaling and correlations in nuclear systems