Simulating Graphene with a Microwave Photonic Crystal

TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI 2012

- Microwave billiards and graphene
- Dirac spectrum in a photonic crystal
  - Reflection and transmission spectra
- Microwave Dirac billiard
  - Spectral properties
  - Length spectrum of classical periodic orbits
- Summary and outlook

Supported by DFG within SFB 634

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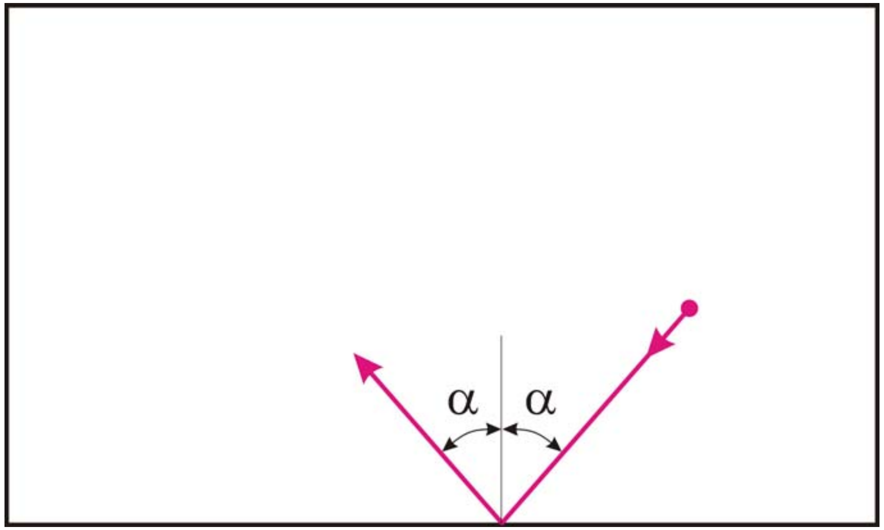
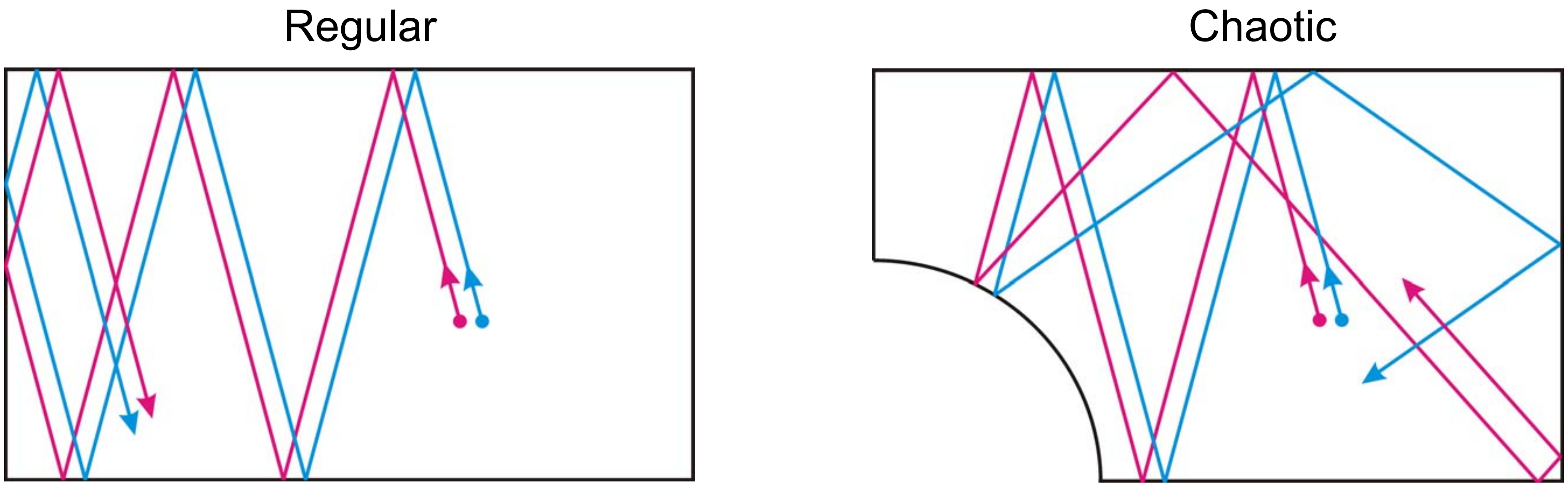
Classical Billiard



- Pointlike particle in a closed area
- Specular reflection
- Shape of boundary defines the type of dynamics

Regular and chaotic dynamics

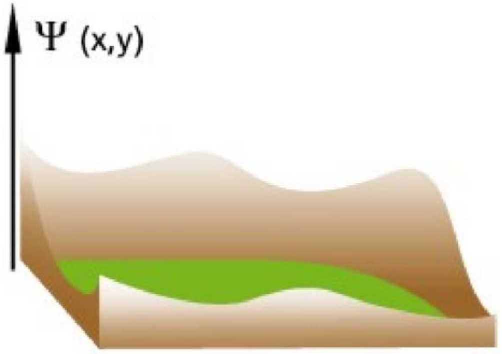


- Energy and p_x^2 are conserved
- Equations of motion are integrable
- Predictable for infinitely long times
- Billiards in quantum mechanics?

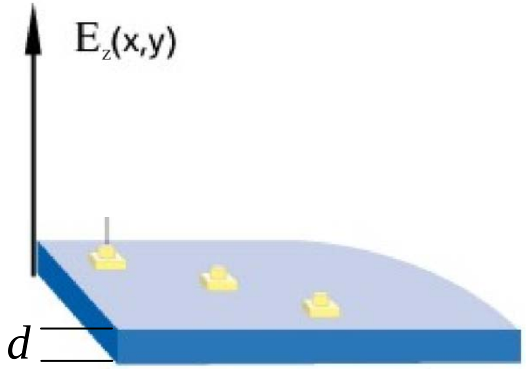
- Only energy is conserved
- Exponential divergence
- Predictable for a finite time only

Quantum Billiards and Microwave Billiards

Quantum billiard



Microwave billiard



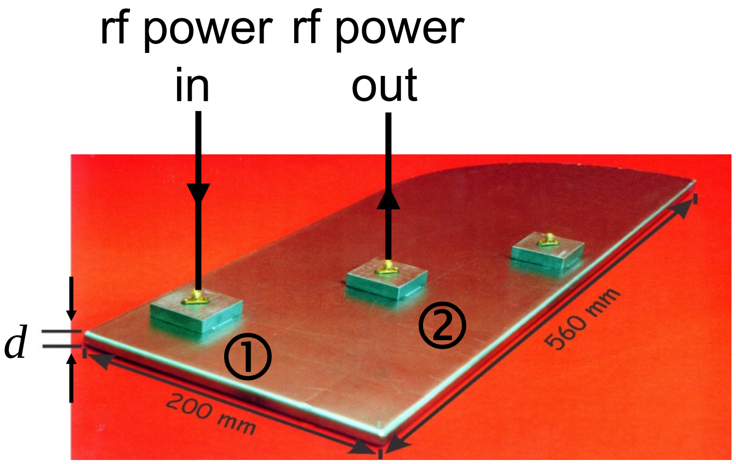
$\lambda > 2d$

$$\left(\frac{\hbar}{2m}\Delta + E\right)\Psi = 0, \;\; \Psi|_{\partial\Omega} = 0 \quad \longleftrightarrow \quad \left(\Delta + k^2\right) E_z = 0, \;\; E_z\big|_{\partial\Omega} = 0$$

eigenvalue E ↔ wave number $k = \frac{2\pi f}{c}$

eigenfunction Ψ ↔ electric field strength E_z

Measurement Principle

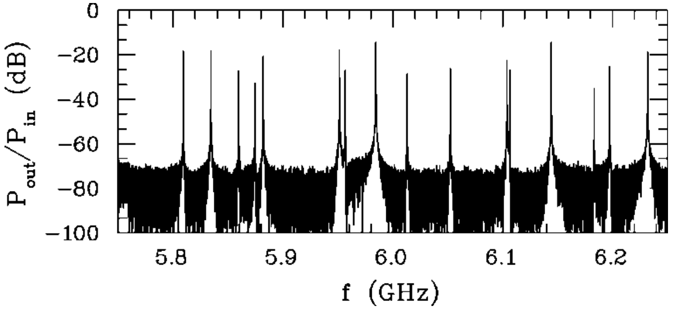


$$\longrightarrow \quad \frac{P_{out,2}}{P_{in,1}} = \left|S_{21}\right|^2$$

Resonance spectrum

Resonance density

Length spectrum



$$\longrightarrow \quad \rho(f) = \sum_{\nu} \delta(f - f_{\nu}) \quad \xrightarrow{FT} \quad \tilde{\rho}(l)$$

$$\rho(f) = \rho_{Weyl}(f) + \rho_{fluc}(f)$$

Microwave Billiards as Model Systems

- Microwave billiards serve as models for nonrelativistic quantum billiards
  - Study of universal spectral fluctuation properties
  - Generic properties of chaotic scattering systems
- Aim: relativistic quantum billiards
- Experimental realization: microwave analogon of graphene

Nobel Prize in Physics 2010

Photo: Sergeom, Wikimedia Commons

Andre Geim

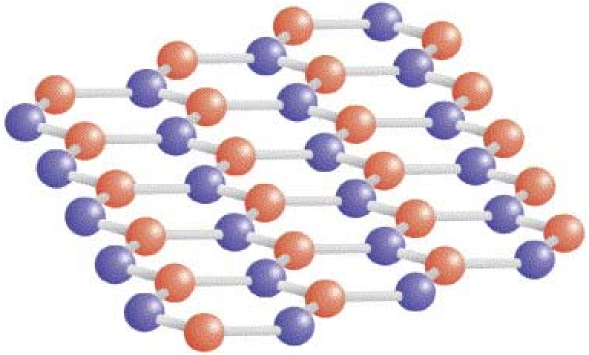
Photo: University of Manchester, UK

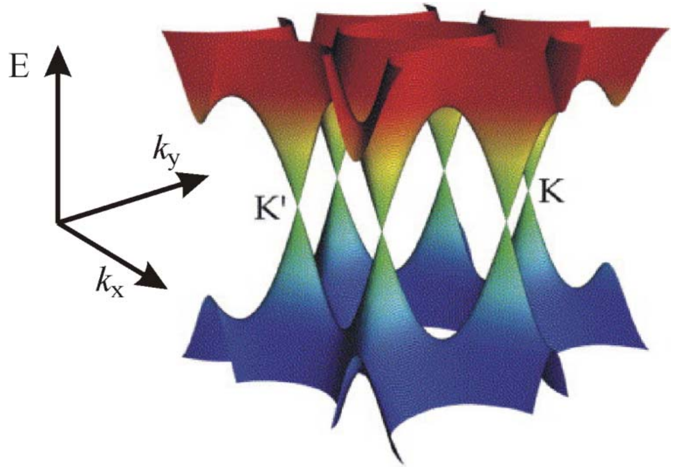
Konstantin Novoselov

The Nobel Prize in Physics 2010 was awarded jointly to Andre Geim and Konstantin Novoselov *"for groundbreaking experiments regarding the two-dimensional material graphene"*

Graphene

M. Katsnelson, Materials Today, 2007

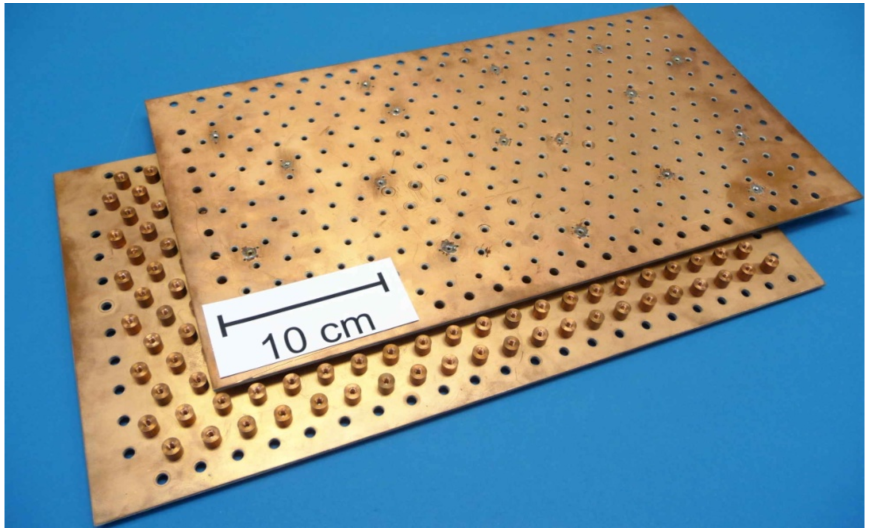




- Triangular Bravais lattice with two carbon atoms per unit cell
- Near each corner of the first hexagonal Brillouin zone the electron energy E has a conical dependence on the quasimomentum
- *„What makes graphene so attractive for research is that the spectrum closely resembles the Dirac spectrum for massless fermions."*
- Can we simulate it with electromagnetic waves?

Photonic Crystal

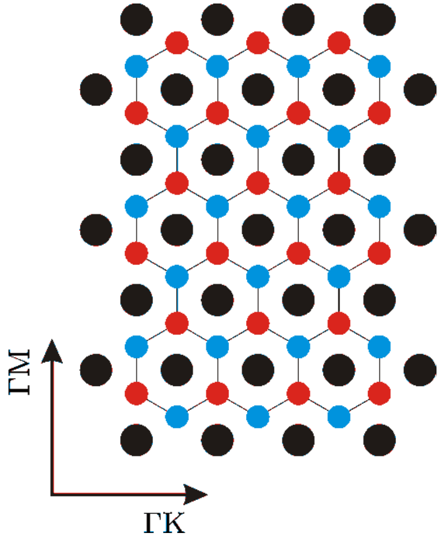
- A photonic crystal is a structure, whose electromagnetic properties vary periodically in space, e.g., an array of metallic cylinders

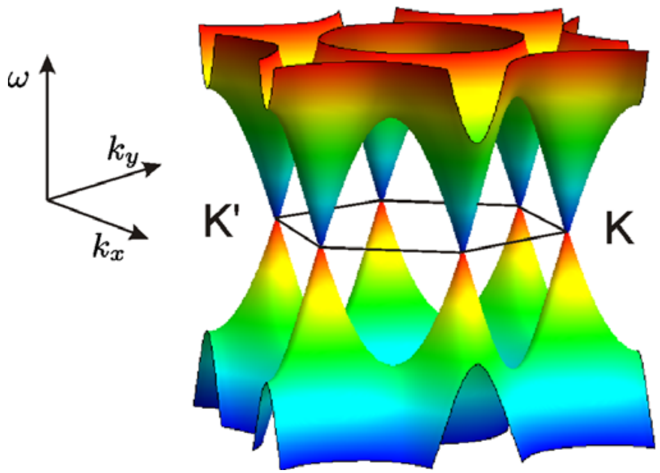


- Below a certain frequency only two dimensional TM modes exist
- Propagating modes are solutions of the 2D scalar Helmholtz equation
- Equivalent to Schrödinger equation for quantum multiple-scattering problem

Calculated Photonic Band Structure

- Dispersion relation $\omega(\vec{k})$ of a photonic crystal exhibits a band structure analogous to the electronic band structure in a solid



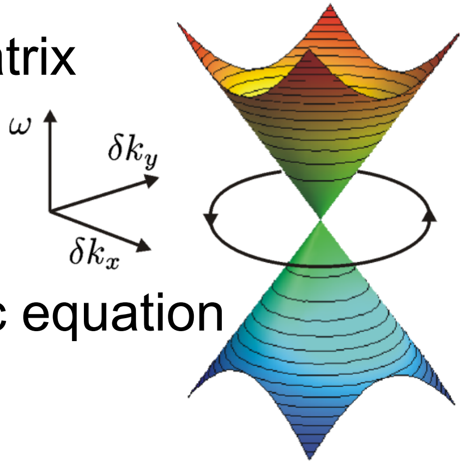


- The triangular photonic crystal possesses a Dirac spectrum
- The voids form a honeycomb lattice like atoms in graphene

Effective Hamiltonian around the Dirac Point

- Close to Dirac point the effective Hamiltonian is a 2x2 matrix

$$\hat{H}_{\text{eff}} = \omega_D \mathbb{1} + v_D \left(\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y \right)$$



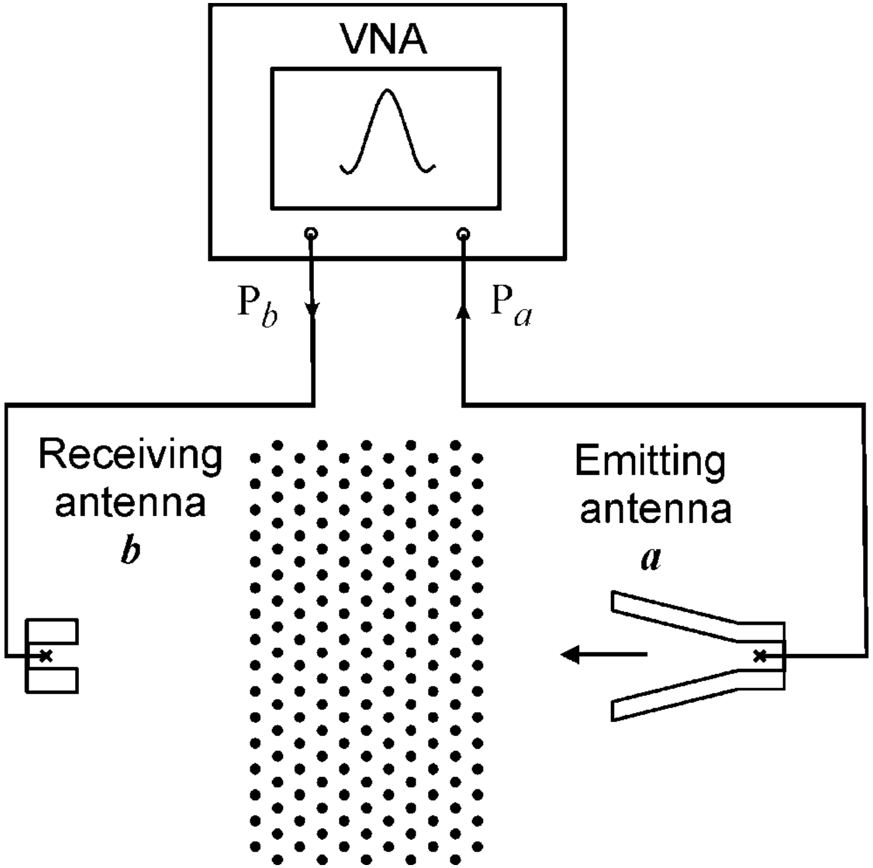
- Substitution $\delta k_x \rightarrow -i\partial_x$ and $\delta k_y \rightarrow -i\partial_y$ leads to the Dirac equation

$$\begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = i \frac{\omega - \omega_D}{v_D} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- Experimental observation of a Dirac spectrum in an open photonic crystal

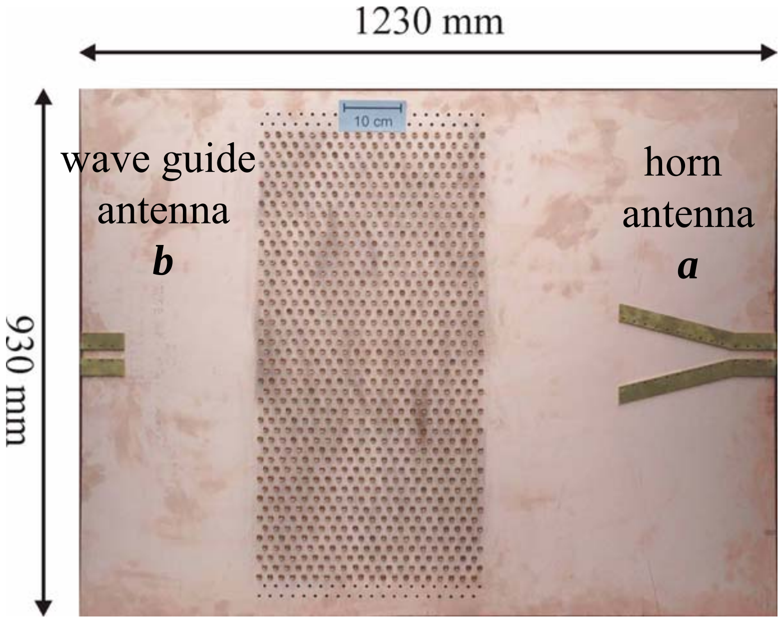
- Next: Scattering experiment

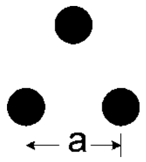
Scattering Experiment



- Horn antenna emits approximately plane waves
- VNA measures the modulus of the scattering matrix given by
  $$|S_{ba}|^2 = \frac{P_b}{P_a}$$
- Transmission: $|S_{ab}|^2, |S_{ba}|^2$
- Reflection: $|S_{aa}|^2, |S_{bb}|^2$

Experimental Realization of 2D Photonic Crystal

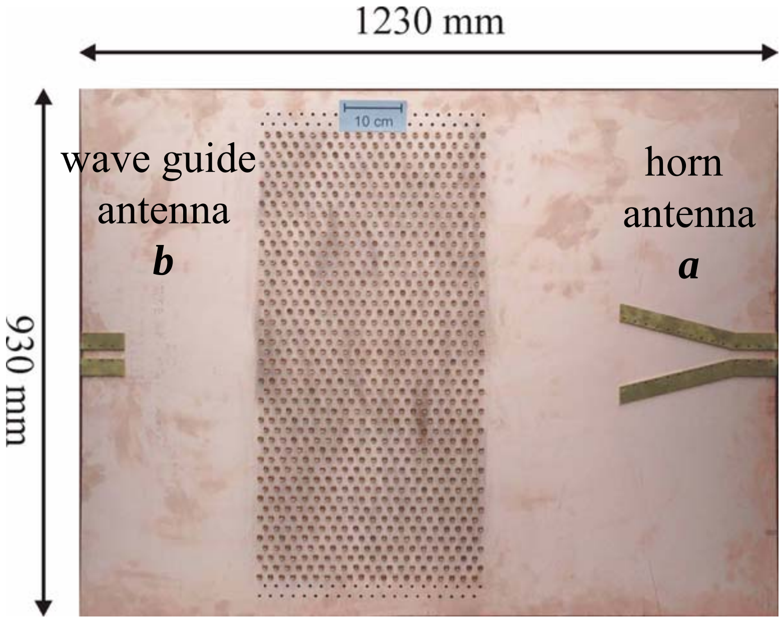


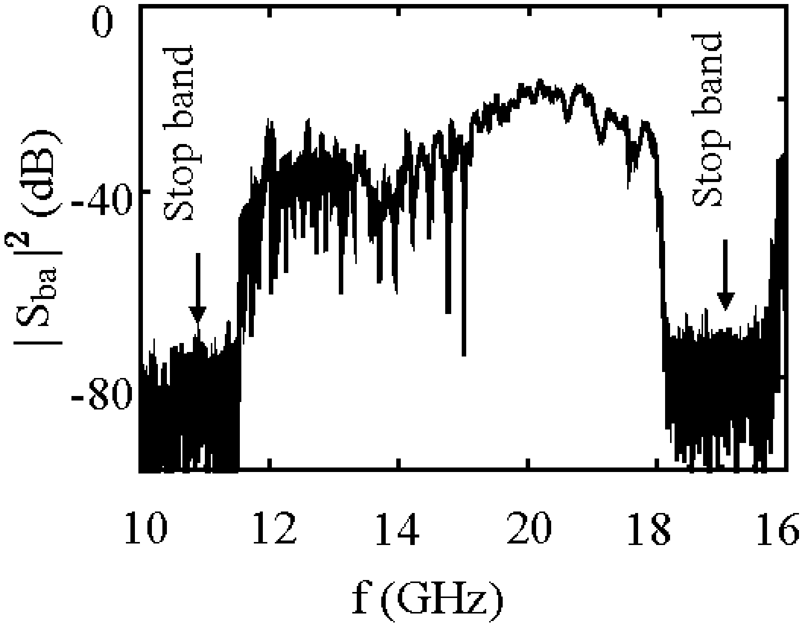
- # cylinders: 23 x 38 = 874
- Cylinder radius: R = 5 mm
- Lattice constant: a = 20 mm


- Crystal size: 400 x 900 x 8 mm
- Frequency: $f_{\max}$=19 GHz

- First step: experimental observation of the band structure

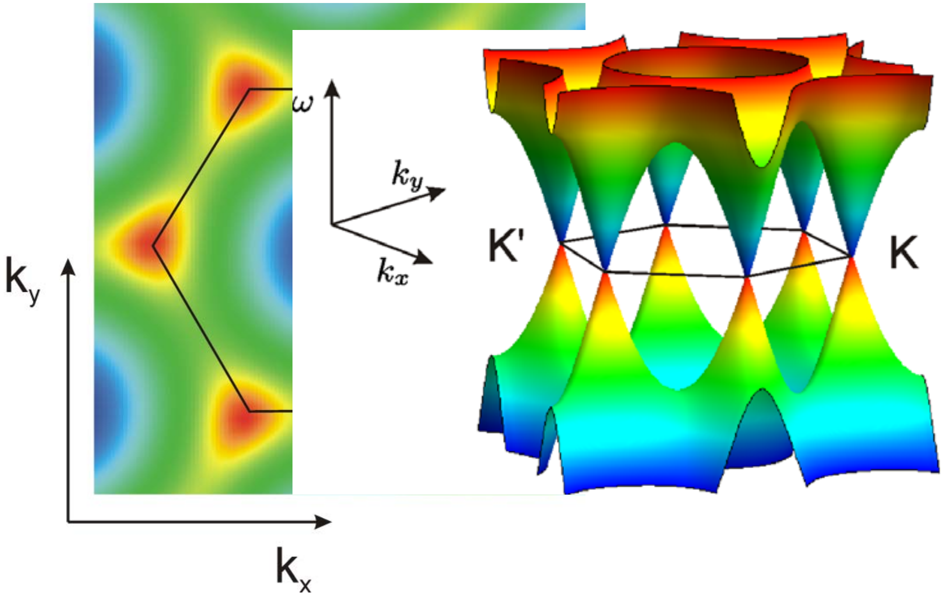
Transmission through the Photonic Crystal

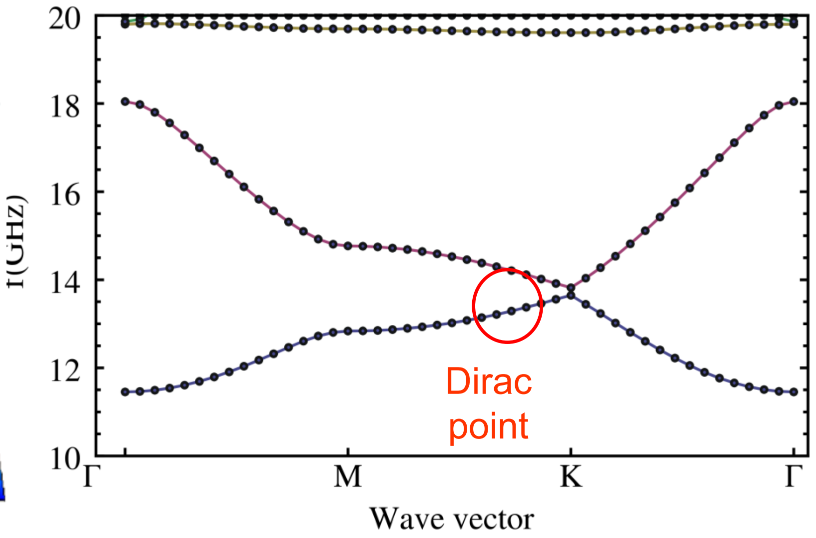




- Transmission spectrum possesses two stop bands
- Next: comparison with calculated band structure

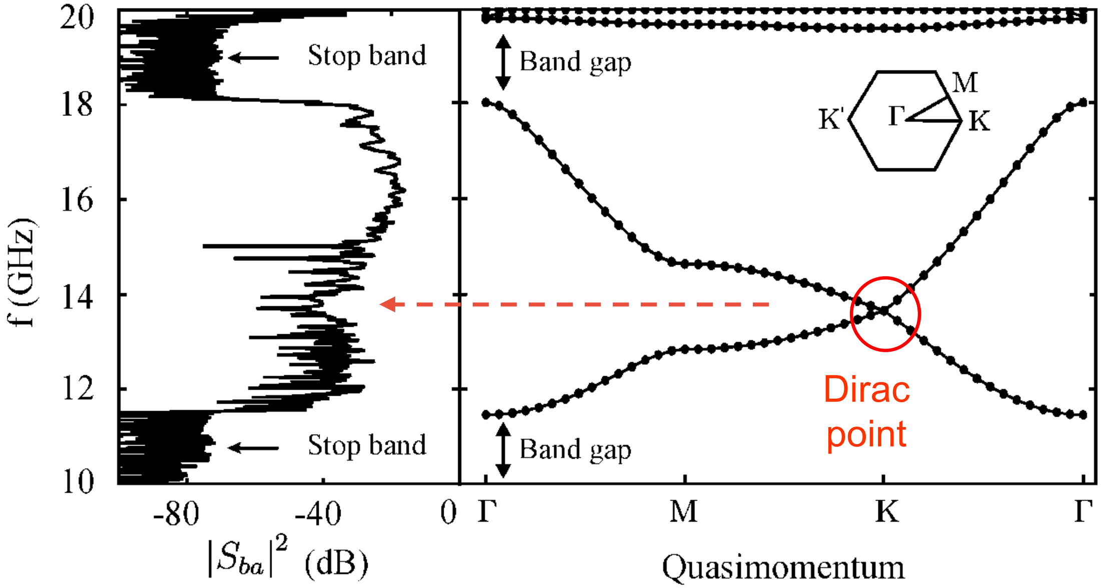
Projected Band Diagram





- The density plot of the 1st frequency band
- The projected band diagram along the path ΓMKΓ
- The 1st and 2nd frequency bands touch each other at the corners of the Brillouin zone → Dirac Point

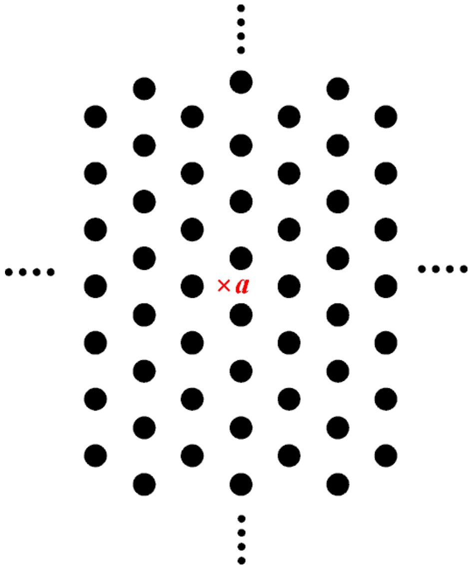
Transmission through the Photonic Crystal

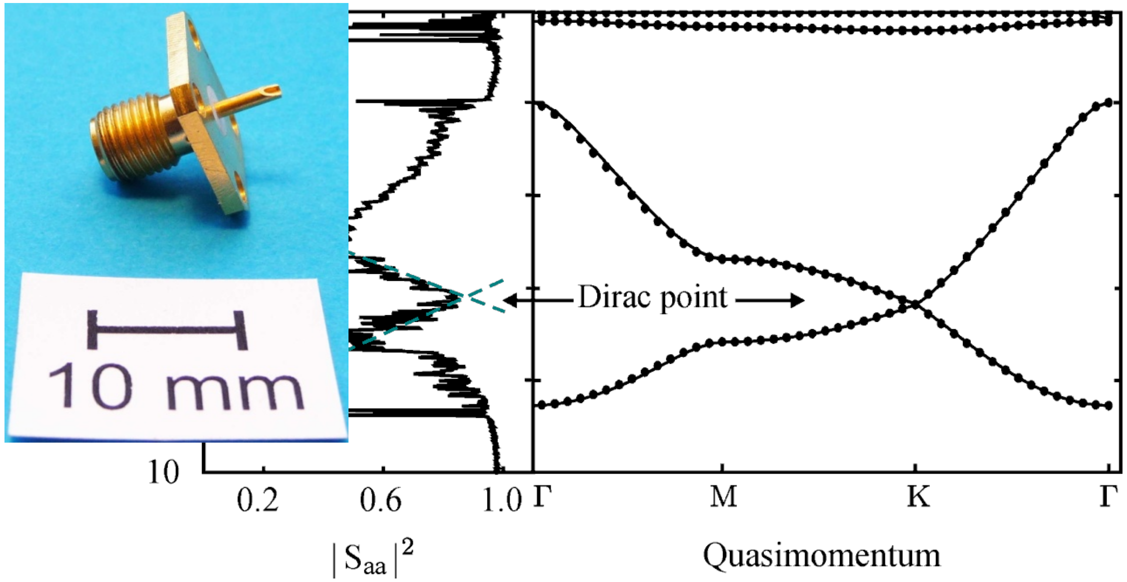


- The positions of measured stop bands coincide with the calculated ones
  → lattice parameters chosen correctly
- Dirac point is not sufficiently pronounced in the transmission spectra
  → single antenna reflection measurement

Single Antenna Reflection Spectrum

- Measurement with a wire antenna ***a*** put through a drilling in the top plate → point like field probe





- Characteristic cusp structure around the Dirac frequency
- Next: analysis of the measured spectrum

Local Density of States and Reflection Spectrum

- LDOS

$$L(\vec{r},f)\propto\int_{BZ}|\psi(\vec{k},\vec{r})|^2\frac{1}{2\pi}\delta(f-f(\vec{k}))d^2k$$

- LDOS around the Dirac point (Wallace,1947)

$$L(\vec{r}_a,f)\sim\frac{\langle|\psi(\vec{r_a})|^2\rangle}{v_D^2}|f-f_D|$$

- The scattering matrix formalism relates the reflection spectra to the local density of states (LDOS)

$$1-|S_{aa}(f)|^2\propto\ L(\vec{r}_a,f)$$

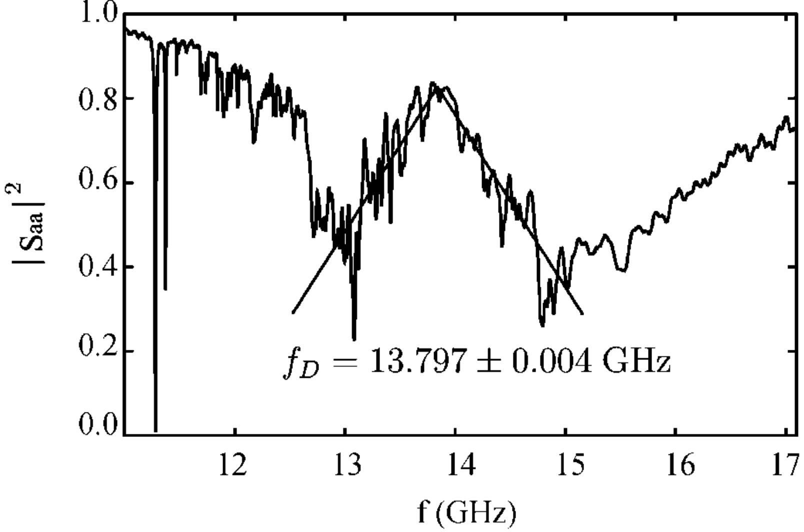
- Three-parameter fit formula $|S_{aa}(f)|^2 = \underset{\uparrow}{D} - \underset{\uparrow}{C}\,|f - \underset{\uparrow}{f_D}|$

fit parameters

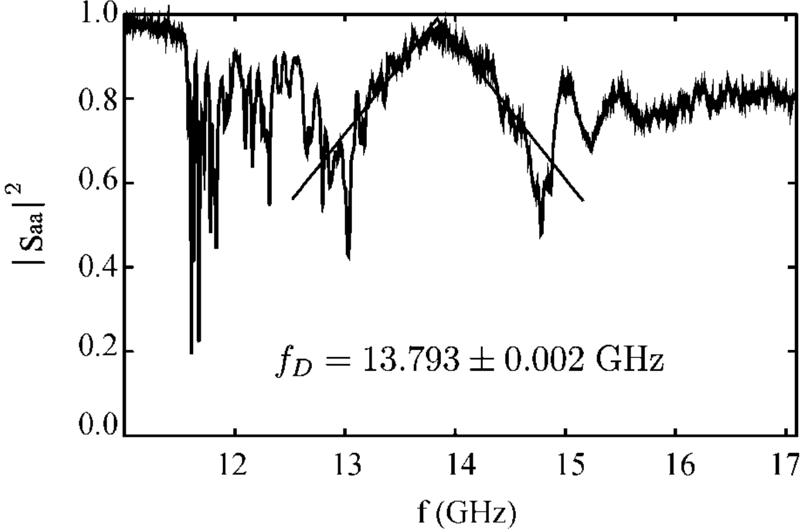
Reflection Spectra

- Description of experimental reflection spectra $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

antenna ***a*** in the
middle of the crystal

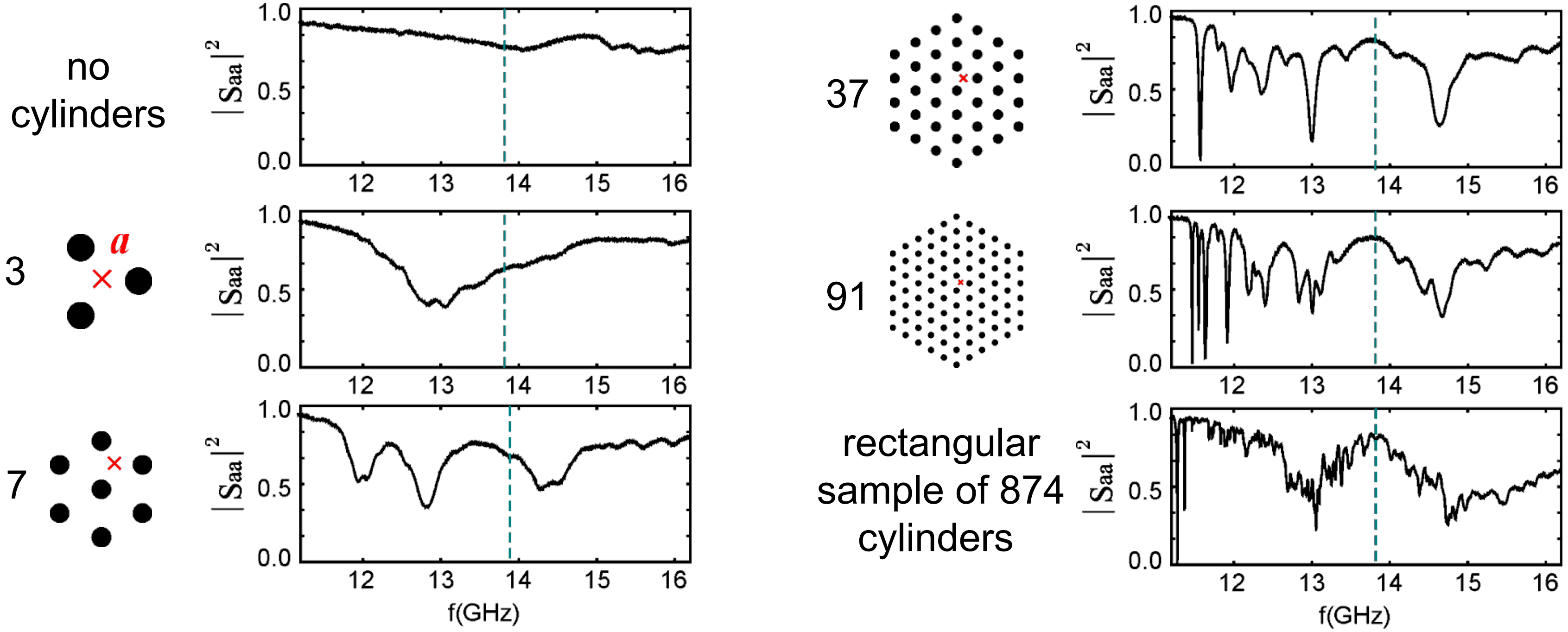


antenna ***a*** 6 rows
apart from the boundary



- Experimental Dirac frequencies agree with calculated one, f_D =13.81 GHz, within the standard error of the fit
- Oscillations around the mean intensity → origin?

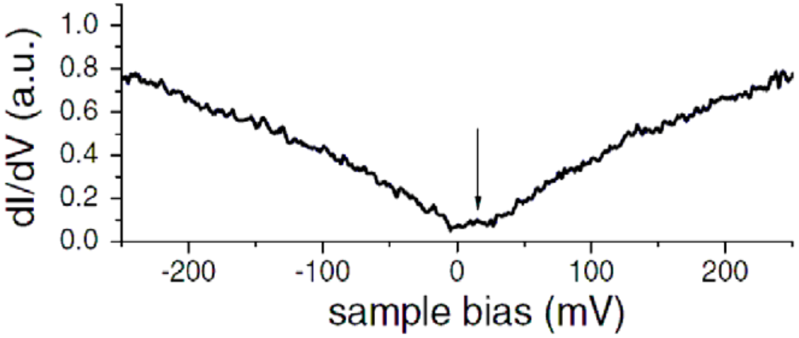
Dependence of Oscillations on Crystal Size



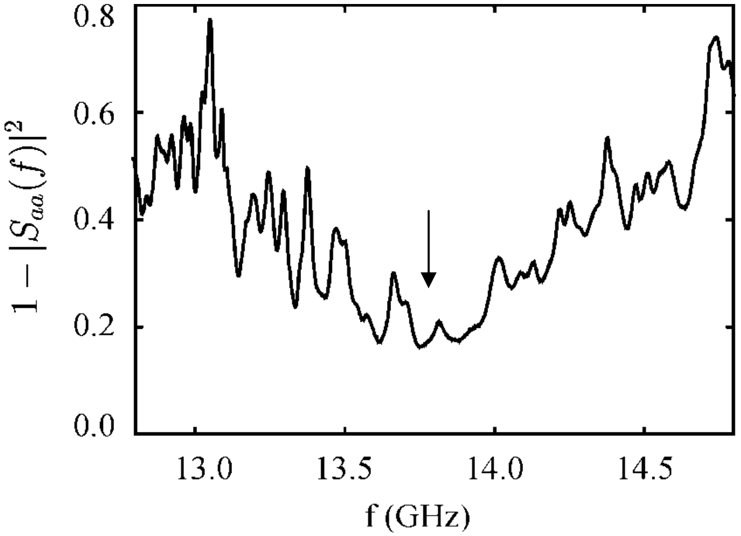
- Nature of the oscillations is a finite size effect
- Period of the oscillations is thus related to the photonic crystal size

Comparison with STM Measurements

graphene flake, Li *et al.* (2009)



photonic crystal



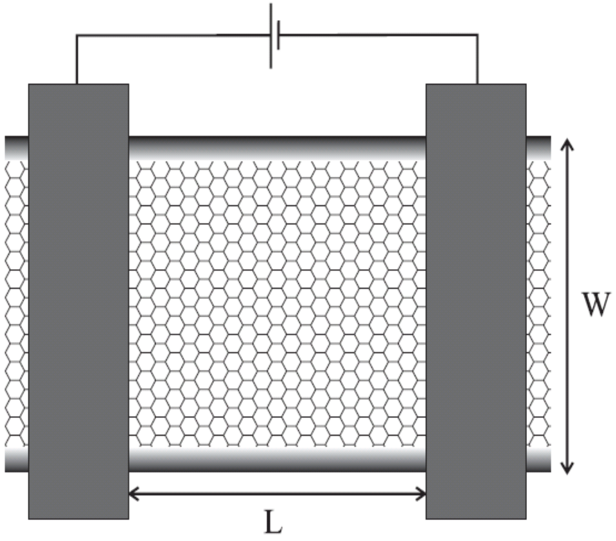
- Tunneling conductance is proportional to LDOS
- Similarity with measured reflection spectrum of the photonic crystal
- Oscillations in STM are not as pronounced due to the large sample size

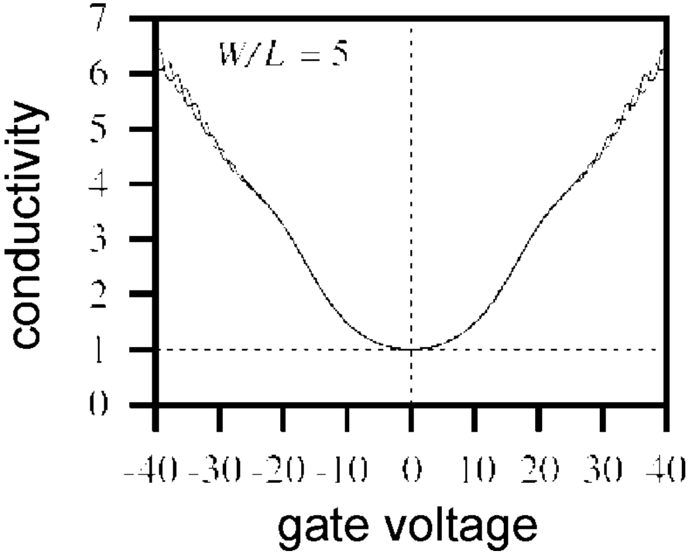
Summary I

- Connection between reflection spectra and LDOS is established
- Cusp structure in the reflection spectra is identified with the Dirac point
- Photonic crystal simulates one particle properties of graphene
- Results are published in Phys. Rev. B **82,** 014301 (2010)
- Next: transmission near the Dirac Point

Transport near the Dirac Point

J. Tworzydło, B. Trauzettel, M. Titov, A. Rycerz, and C. W. J. Beenakker, PRL **96**, 246802 (2006)

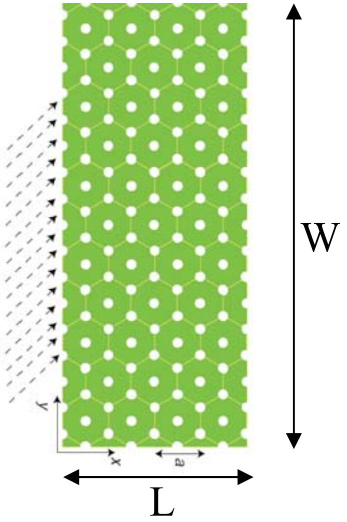


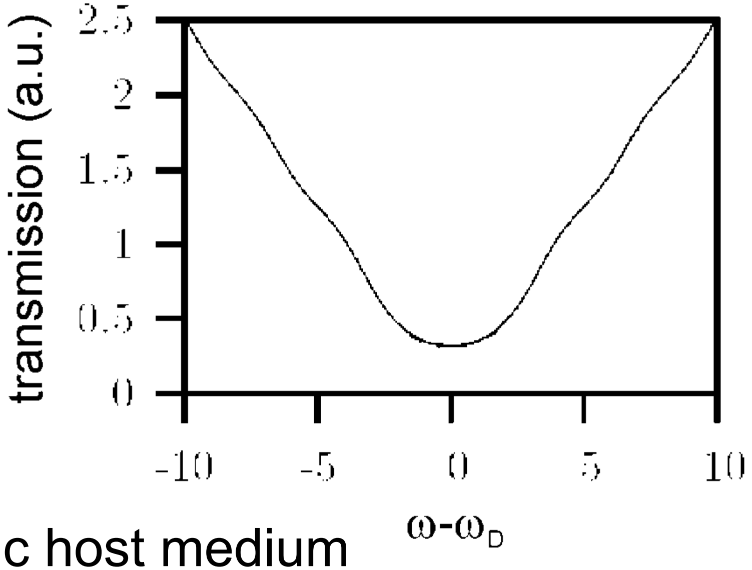


- Density of states vanishes at the Dirac point
- Finite conductivity σ of a graphene sample at Dirac point
- Conductance of a graphene ribbon scales as $G \sim \frac{e^2}{h}\frac{W}{L}$

Extremal Transmission through a Photonic Crystal

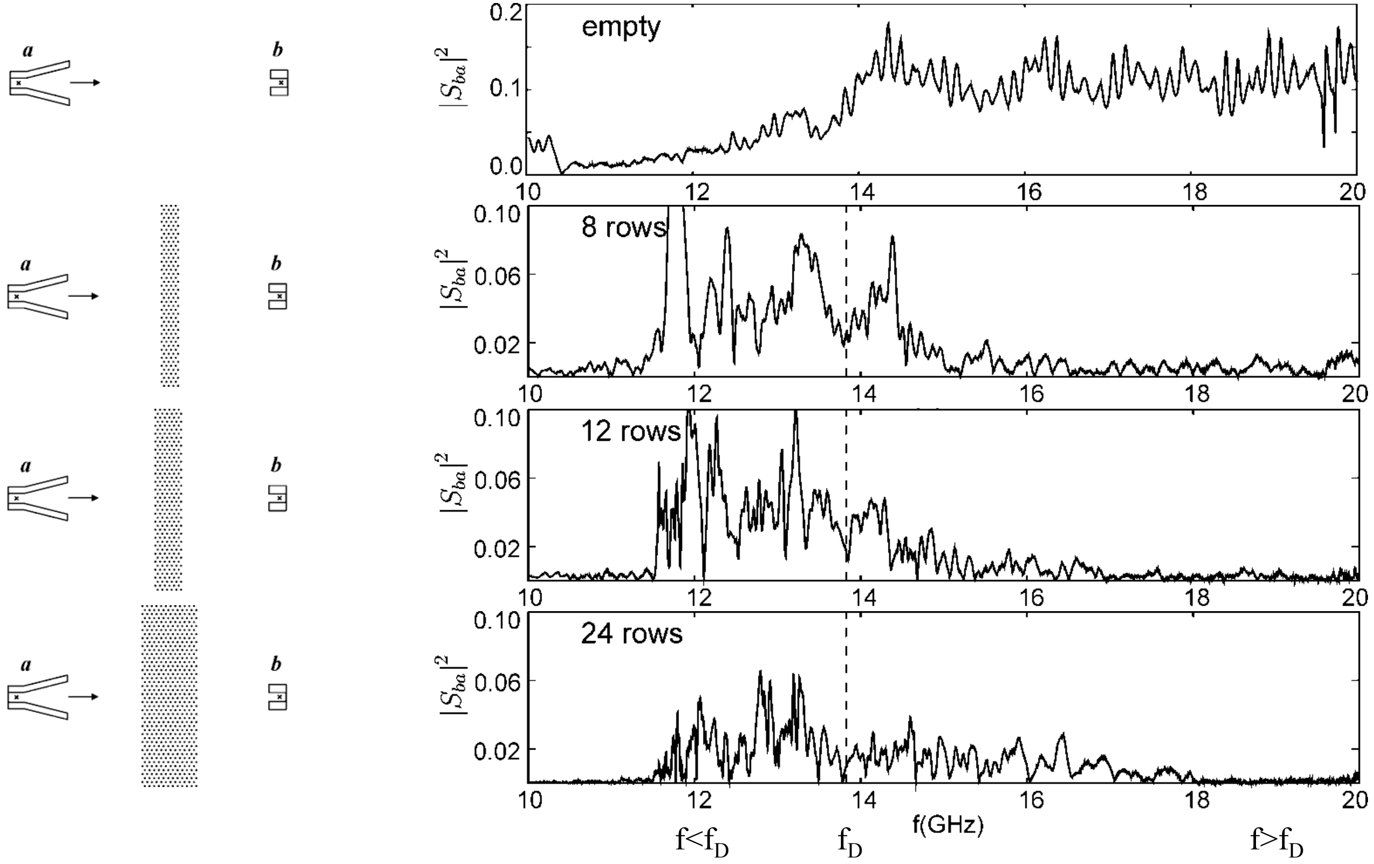
R.A. Sepkhanov, Ya.B. Bazalij and C.W.J. Beenakker (2007)





- Triangular array of holes in a dielectric host medium
- Transmission has a minimum at the Dirac frequency
- Vanishing DOS → transmission by means of evanescent modes
- Transmitted power decays as 1/L in contrast to exponential decay at a band gap

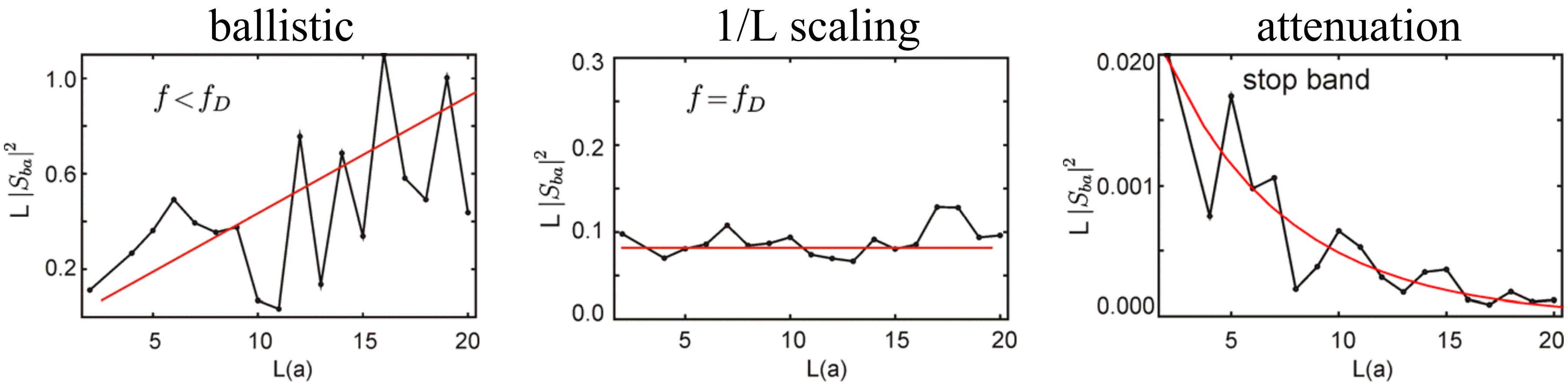
Transmission Spectra through Photonic Crystals in ΓK Direction: Some Examples



- Transmission minimum at the Dirac frequency
- Scaling of the transmission with crystal thickness?

Extremal Transmission at the Dirac Point

- Thickness L of the photonic crystal varies from 4 to 40 layers



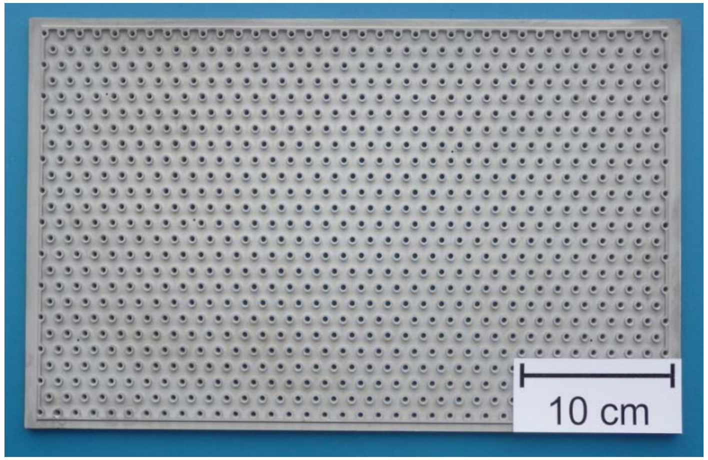
- Ballistic transport in the transmission bands
- 1/L scaling at the Dirac frequency
- Exponential attenuation in the stop bands

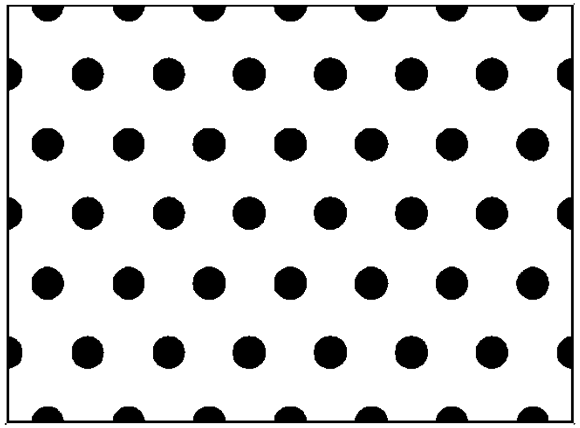
Summary II

- Transmission spectra of photonic crystals of different thickness L were measured
- Transmission spectra of the photonic crystal near the Dirac frequency show similar scaling as the conductance in graphene, i.e., 1/L behaviour
- Results are published in Phys. Rev. B **85,** 064301 (2012)
- Next: Photonic crystal in a box

Dirac Billiard

- Photonic crystal in a box: bounded area = billiard





- 888 cylinders milled out of a brass plate
- Height d = 3 mm $\rightarrow f_{max}^{2D}$ = 50 GHz for 2D system
- Lead plated $\rightarrow$ superconducting below 7.2 K $\rightarrow$ high Q value
- Boundary does not violate the translation symmetry $\rightarrow$ no edge state

Maksim and 888 Screws

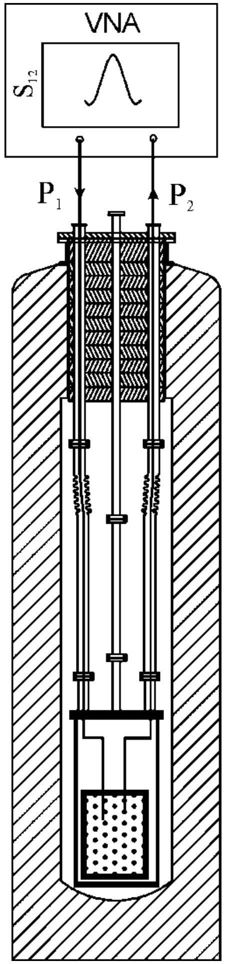
15.12.2011, 12:19h

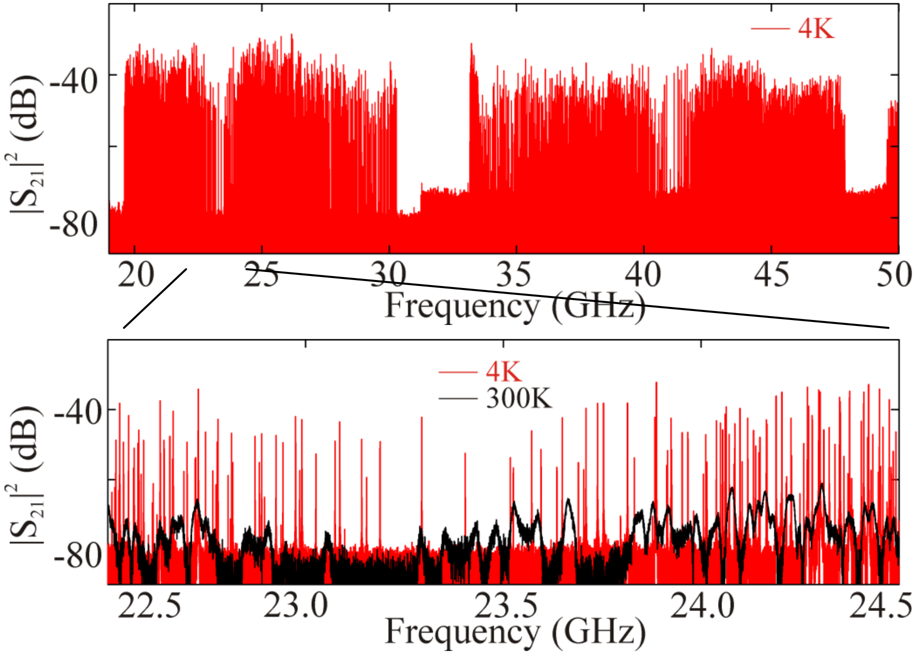
13:06h

13:27h

14:47h

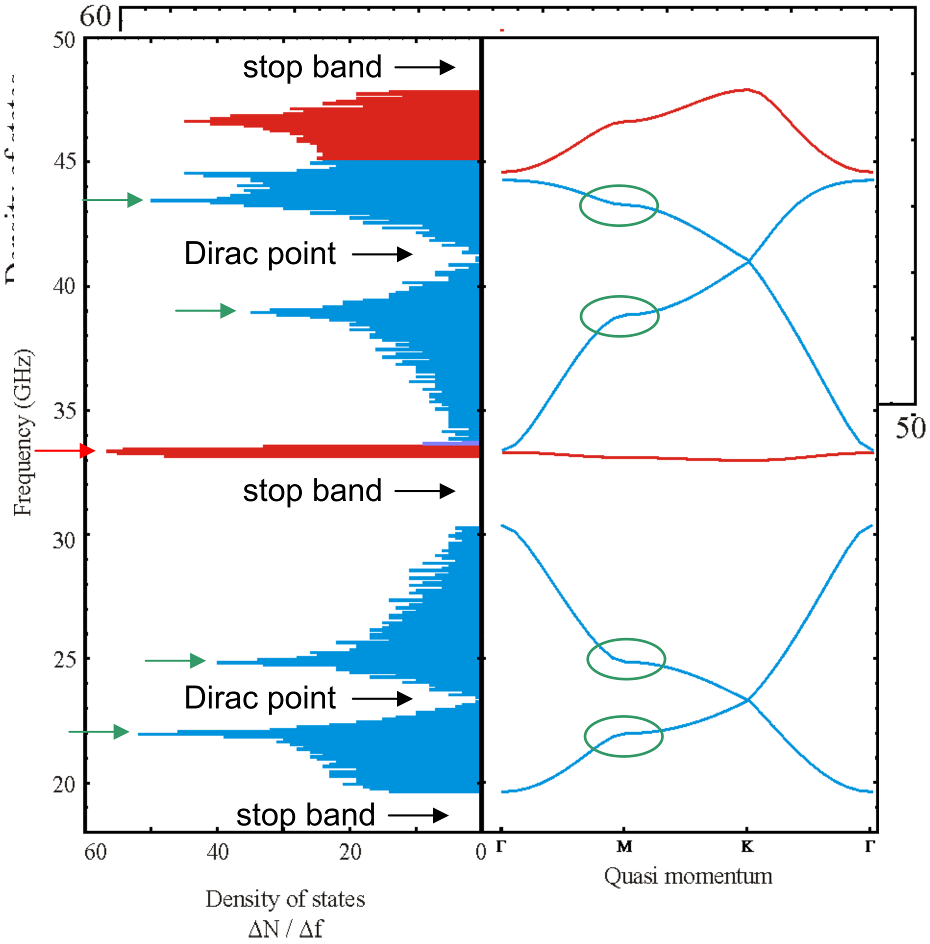
Transmission Spectrum at 4 K





- Pronounced stop bands
- Quality factors > $5 \cdot 10^5$
- $\langle\Gamma\rangle/\langle D\rangle = 10^{-3} \rightarrow$ complete spectrum
- Altogether 5000 resonances observed

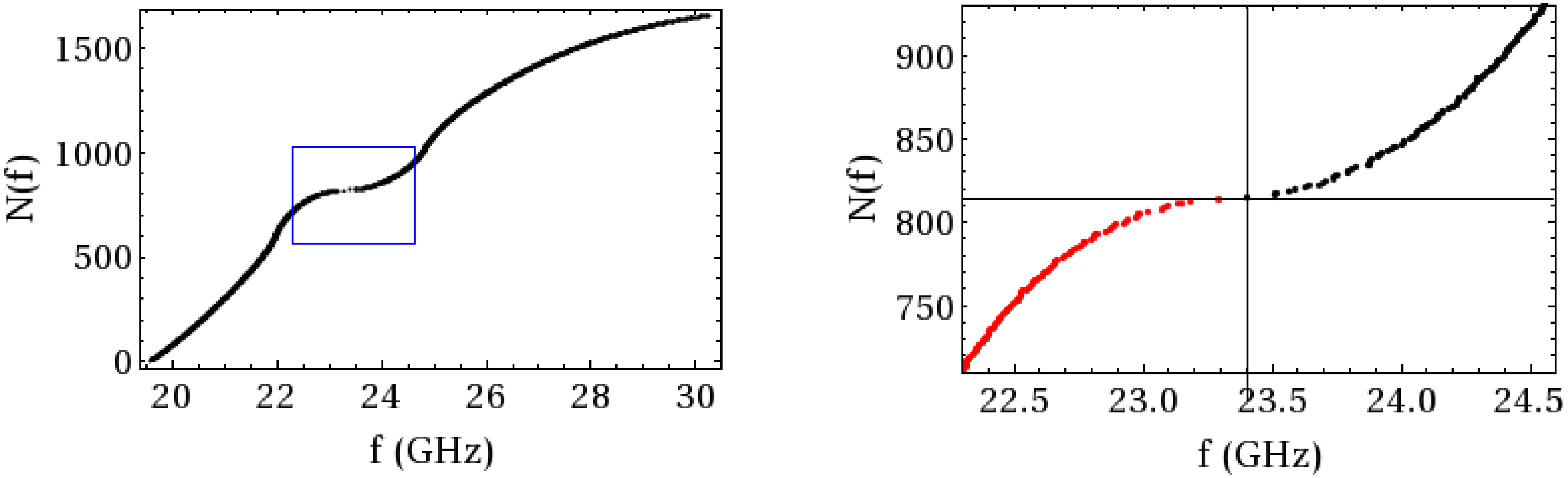
Density of States of the Measured Spectrum and the Band Structure



- Positions of stop bands are in agreement with calculation
- DOS related to slope of a band
- Dips correspond to Dirac points
- High DOS at van Hove singularities
- Flat band has very high DOS
- Qualitatively in good agreement with prediction for graphene

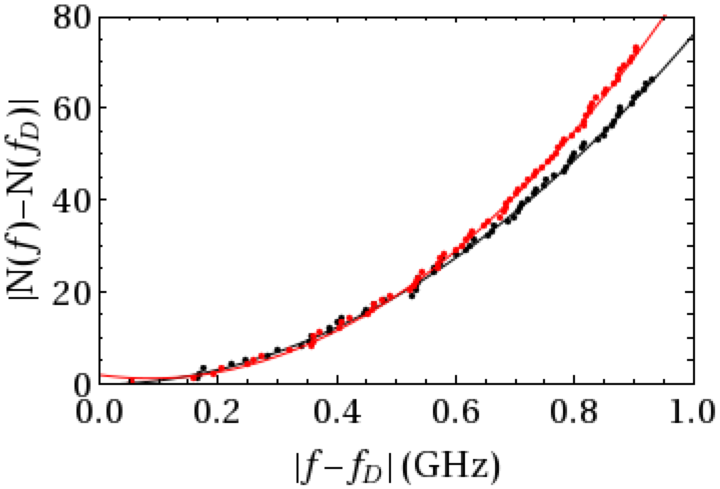
(Castro Neto *et al.*, RMP **81**,109 (2009))

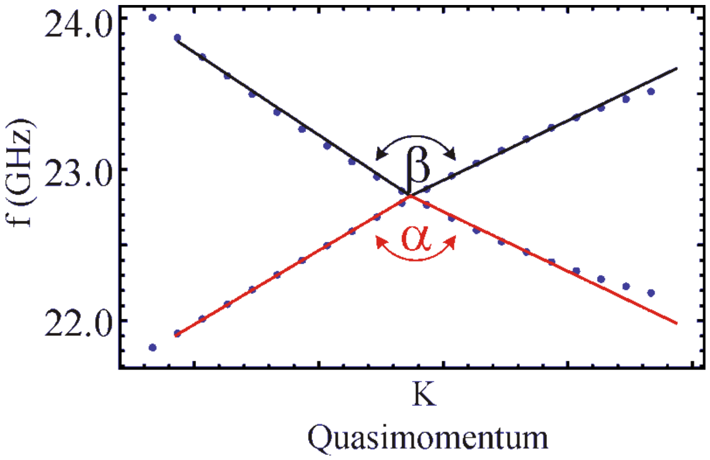
Integrated Density of States: 1^{st} and 2^{nd} Bands

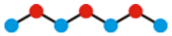


- Does not follow Weyl law for 2D resonators ($N_{Weyl}(f) = \frac{4\pi A}{c^2} f^2$)
- Small slope at the Dirac frequency → nearly vanishing DOS
- Two parabolic branches
- Approximately symmetric with respect to the Dirac frequency

Integrated DOS near Dirac Point





- Weyl law for Dirac billiard $N(k) = \frac{A}{2\pi}k^2 + \frac{U_{zz}}{\pi}k + C$ (J. Wurm *et al.*, PRB **84**, 075468 (2011))
  - U_{zz} is length of zigzag edges 
  - $k = 2\pi\frac{|f-f_D|}{v_D}$
  - group velocity v_D is a free parameter
- Same area A for two branches, but different group velocities → α≠β

  → electron-hole asymmetry like in graphene

Periodic Orbit Theory (POT)

- Description of quantum spectra in terms of classical periodic orbits

spectrum

wave numbers

spectral density

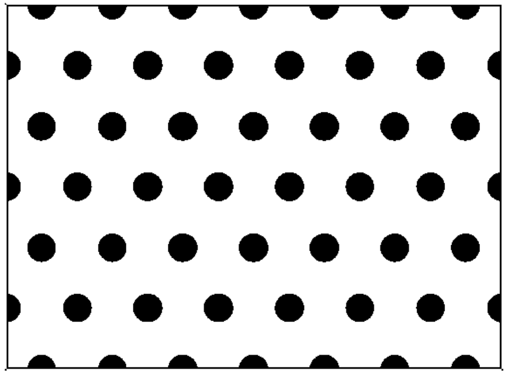
FT

length spectrum

$$\{f_i\} \longrightarrow \{k_i\} \longrightarrow \rho(k) = \sum \delta(k - k_i) \longrightarrow \tilde{\rho}(l) = \int_0^{k_{max}} dk\, e^{ikl} \rho(k)$$

Peaks at the lengths of POs

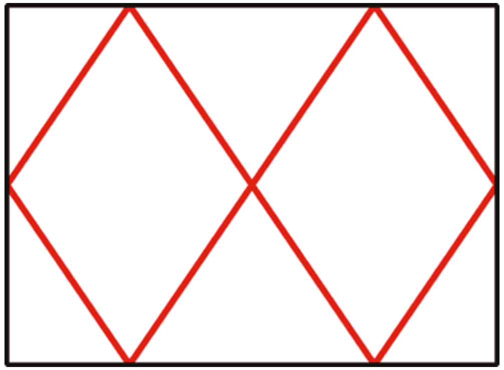
Dirac billiard



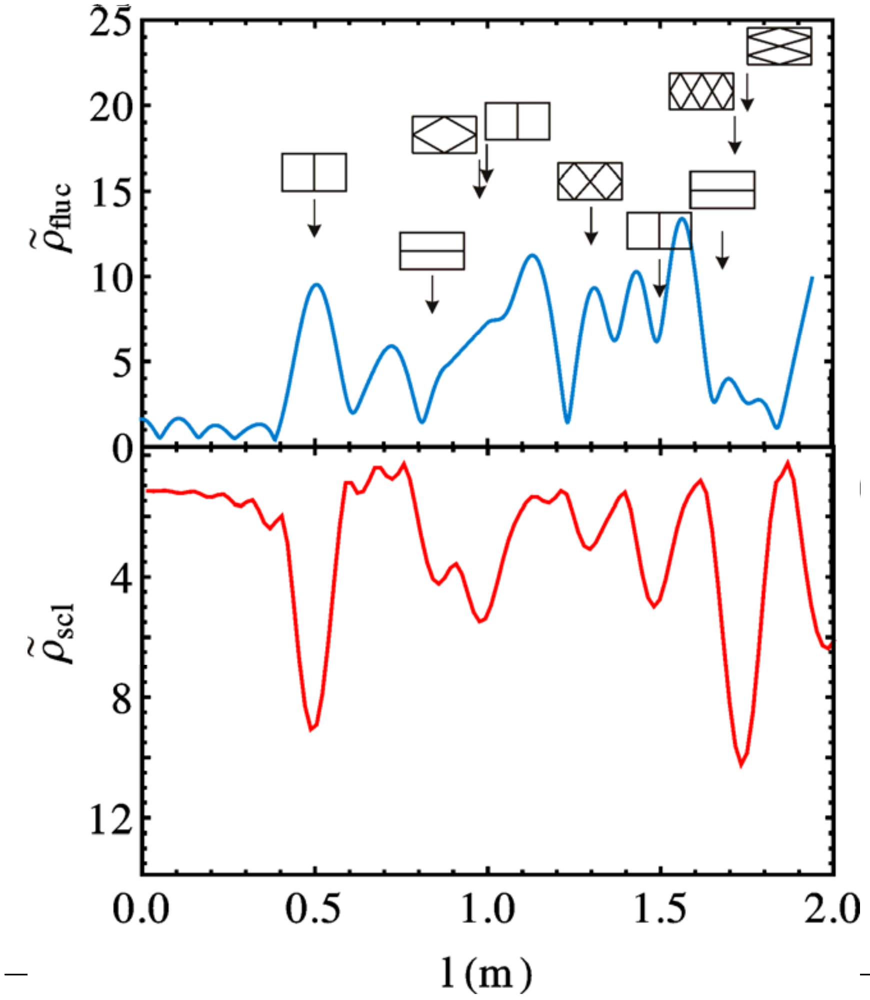
Effective description

around Dirac point

Periodic orbits



Experimental Length Spectrum

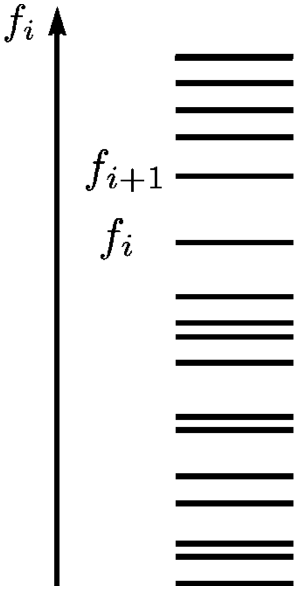


- k_{max} = 70 m^{-1}, corresponds to 80 levels
- Some peak positions deviate from the lengths of POs
- Comparison with semiclassical predictions

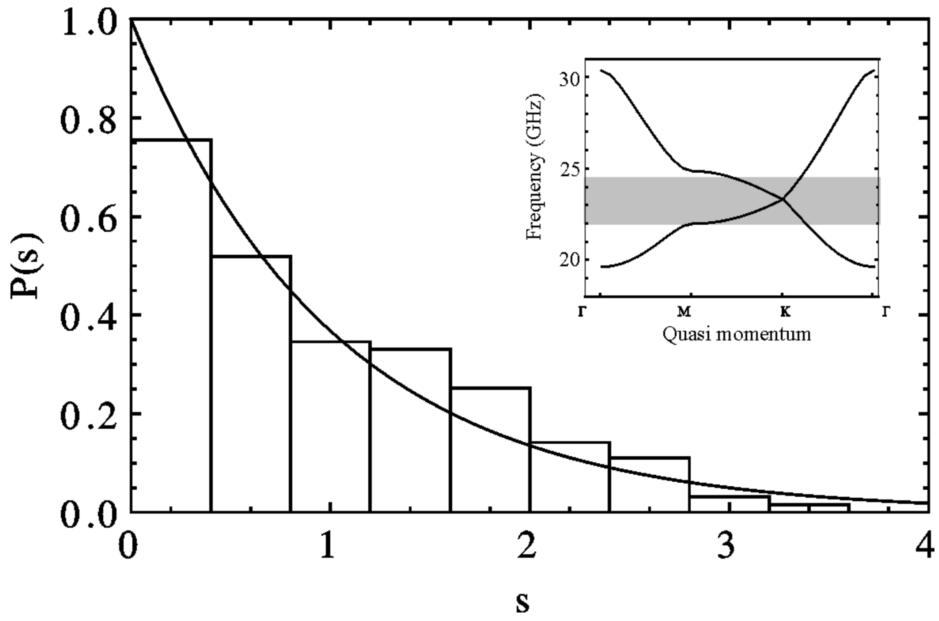
  (J. Wurm *et al.*, PRB **84**, 075468 (2011))
- Possible reasons for deviations:

  - Short sequence of levels
  - Anisotropic dispersion relation around Dirac point

Spectral Properties of a Rectangular Dirac Billiard: Nearest Neighbour Spacing Distribution

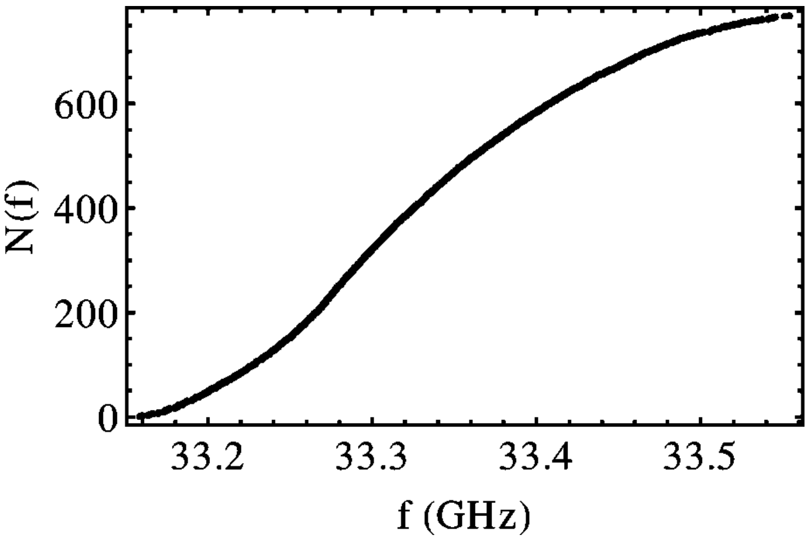


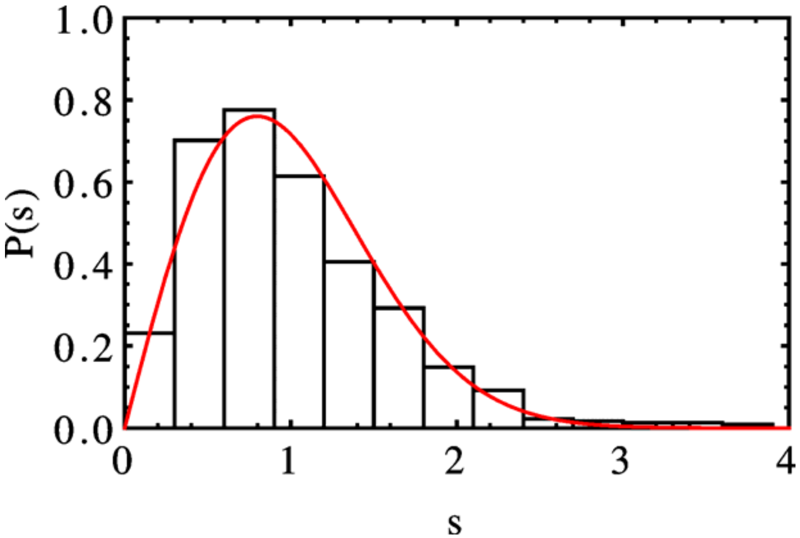
$$s_i = f_{i+1} - f_i$$



- 159 levels around Dirac point
- Rescaled resonance frequencies such that $\langle s_i \rangle = 1$
- Poisson statistics
- Similar behavior at second Dirac point

NND: 3rd Band





- Very dense spectrum $\langle\Gamma\rangle/\langle D\rangle \approx 10^{-1}$
- Rescale frequencies with a polynom of 8th order
- Seems to agree with GOE
- Missing levels?

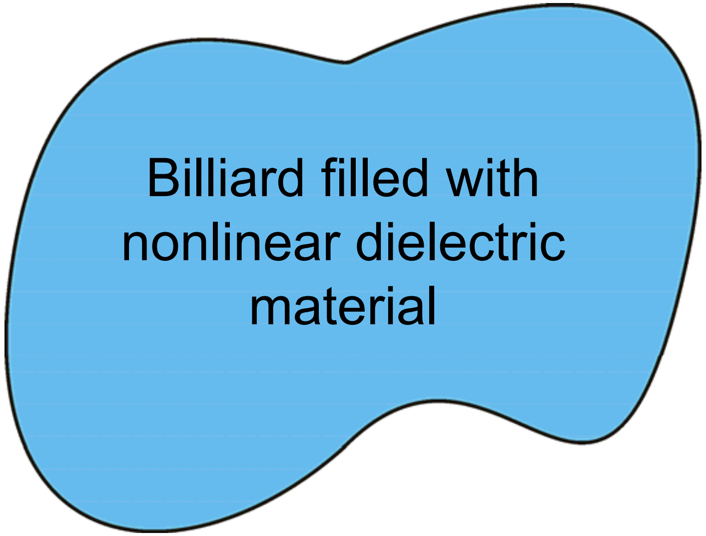
Summary III

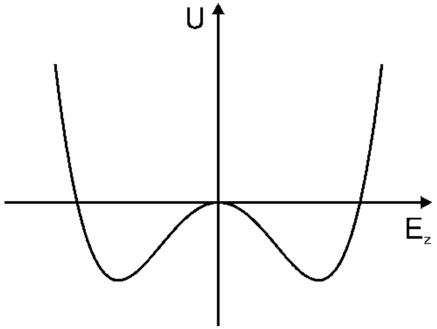
- Photonic crystal simulates one particle properties of graphene
- Realisation of superconducting microwave Dirac billiard i.e. photonic crystal in a metallic box serves as a model for a relativistic quantum billiard
- Experimental DOS reproduces the calculated photonic band structure
- Fluctuation properties of the spectrum were investigated
- Open problems: semiclassical description of the length spectrum

Outlook

in Collaboration with J.Berges, C.Fischer and L. von Smekal

- Simulation of many body effects using RF nonlinear materials



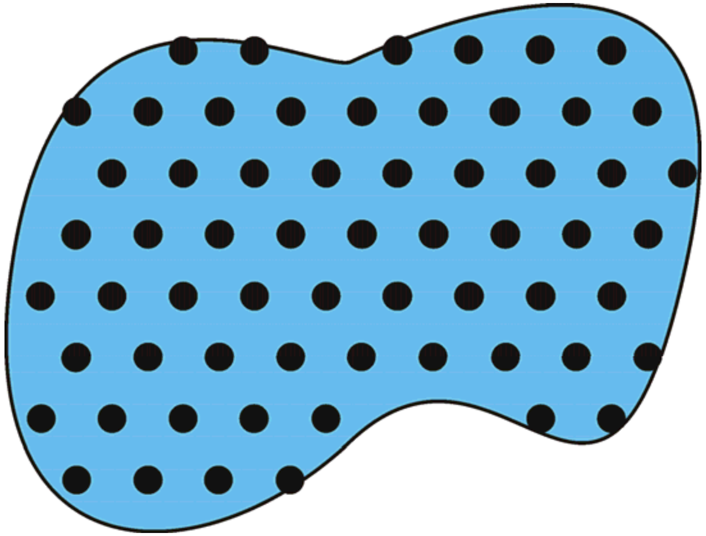


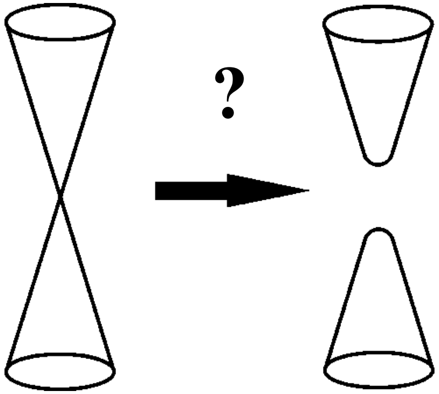
$$\Delta E_z = -k^2 E_z - \eta |E_z|^2 E_z + \gamma |E_z|^4 E_z$$

- Wave propagation described by Gross-Pitaevskii equation
- Model for interacting bosons in a hard-wall potential
- Interaction becomes observable at high RF power coupled into the resonator
- Higher-order nonlinearities produce the Mexican-hat potential

Outlook

- Photonic crystal embedded into a nonlinear medium mimics spinor fields





- Wave propagation described by a nonlinear Dirac equation
- Interacting fermions in a graphene flake $\alpha_g = \dfrac{e^2}{4\pi\varepsilon\hbar v_F}$

  → Laboratory for strongly interacting fermions
- Study of transport properties and semimetal-insulator phase transition