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HARMONIC EFT: TRAPPED ATOMS AND LIBERATED NUCLEONS

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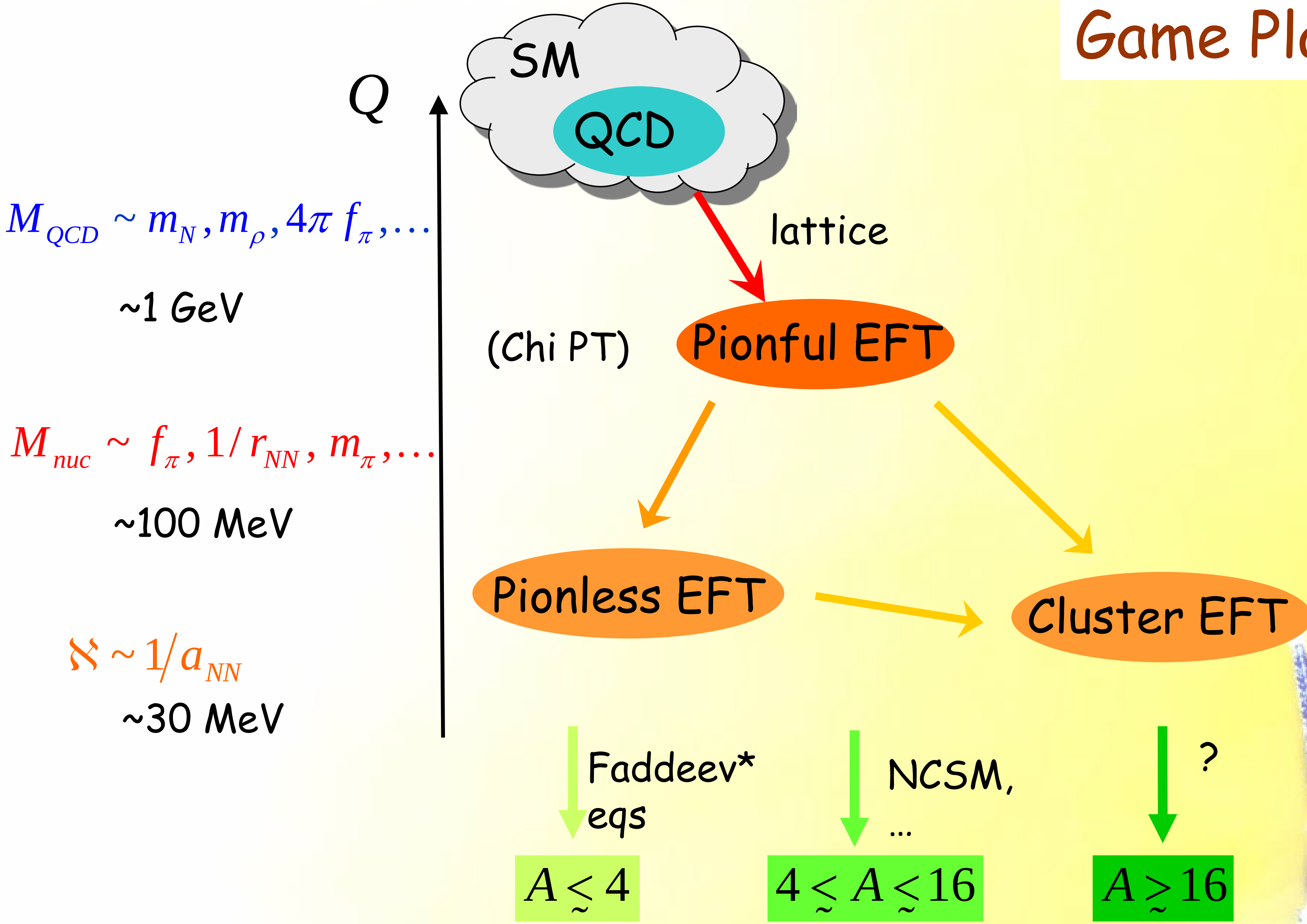
Outline

- Why
- (Pionless) Effective Field Theory
- Life in the Harmonic Box
- Trapped Fermions
- Liberated Nucleons
- Conclusion & Outlook

Game Plan



Q
SM
QCD
lattice
$M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots$
~1 GeV
(Chi PT)
Pionful EFT
$M_{nuc} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$
~100 MeV
Pionless EFT
Cluster EFT
$\aleph \sim 1/a_{NN}$
~30 MeV
Faddeev* eqs
NCSM, …
?
$A \lesssim 4$
$4 \lesssim A \lesssim 16$
$A \gtrsim 16$

Facts of Life

- there is *always** an underlying theory

  all interactions among low-energy d.o.f.s allowed by symmetries

- there is *always** a "model space"

  renormalization-group invariance to tame arbitrary UV cutoff

$$Q \sim m \ll M \begin{cases} T = T^{(\infty)}(Q) \sim \underbrace{N(M)}_{\text{normalization}} \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_i \underbrace{\tilde{c}_{\nu,i}(\Lambda)}_{\text{parameters}} \left[\frac{Q}{M}\right]^{\nu} \underbrace{F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right)}_{\text{non-analytic, from loops}} \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

"power counting"

truncate … $T = T^{(\bar{\nu})}\left\{1 + \mathcal{O}\left(\underbrace{\frac{Q}{M}, \frac{Q}{\Lambda}}_{\text{there are always* such errors}}\right)\right\}$ ⇒ want… $\Lambda \gtrsim M$

there are *always** such errors

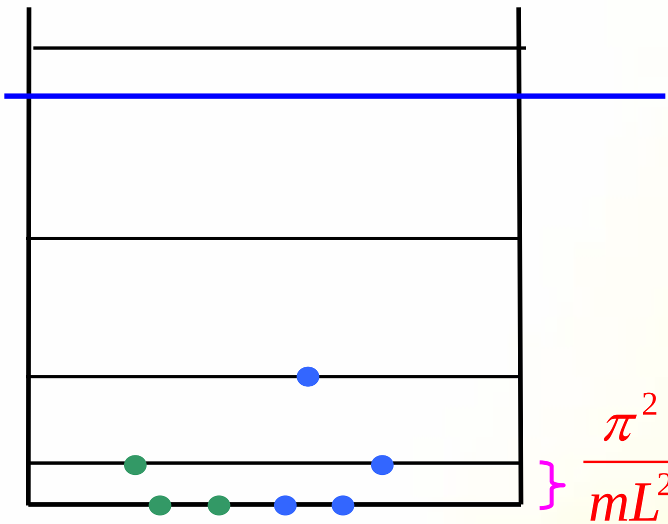
* except, *maybe,* at the Planck scale

$A \gtrsim 4$

As A grows, given computational power limits
number of accessible one-nucleon states

$\Rightarrow$ IR cutoff in addition to UV cutoff

Lattice Box

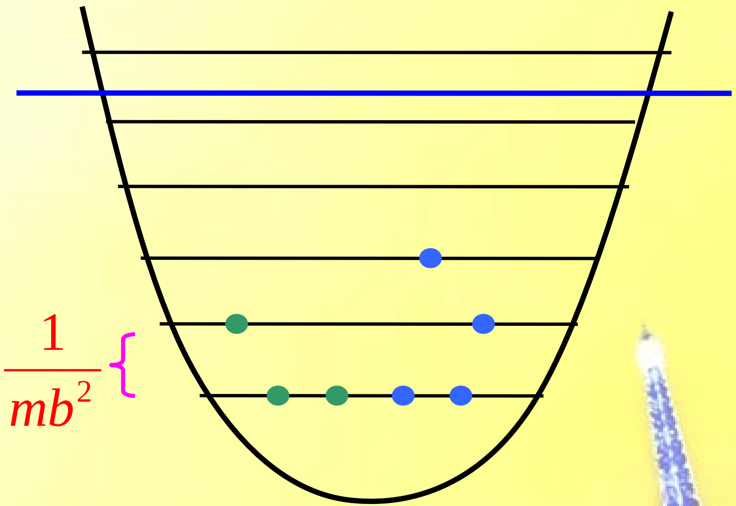


$$\frac{N^2\pi^2}{mL^2}$$

$$\frac{\pi^2}{mL^2}$$

$$L = Na$$

Harmonic-Oscillator Box
"No-Core Shell Model"



$$\frac{N_{\max}}{mb^2}$$

$$\frac{1}{mb^2}$$

$$b = \sqrt{2/m\omega}$$

nuclear matter

few nucleons

Mueller *et al* '99

Lee *et al* '05

...

finite nuclei

few atoms

Stetcu *et al* '06

...

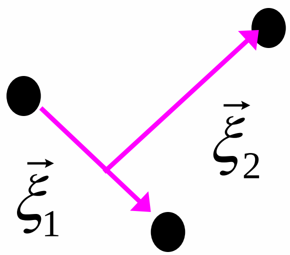
Stetcu *et al* '07

...

single particle $$\phi_{nl(s)j}\left(\vec{r}\right) = N_{nl} b^{-3/2} \left(\frac{r}{b}\right)^{l} \exp\left(-r^2/2b^2\right) L_n^{(l+1/2)}\left(r^2/b^2\right) \left[Y_l\left(\hat{r}\right) \otimes \chi_s\right]_j$$

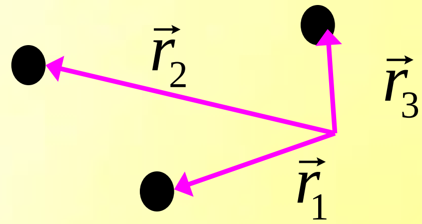
generalized Laguerre polynomial

$A \leq 4$: internal (Jacobi) coordinates



HO Basis

$A \geq 3$: Slater-determinant



$$\phi_{\{n\}}\left(\vec{\xi}_1, \vec{\xi}_2\right) = \mathcal{A}\left[\phi_{nlj}\left(\vec{\xi}_1\right)\phi_{n'l'j'}\left(\vec{\xi}_2\right)\right]_{JI}$$

code `a la

Navratil, Kamuntavicius + Barrett '00

reduced dimensions, but difficult antisymmetrization

$$\phi_{\{n\}}\left(\vec{r}_1, \vec{r}_2, \vec{r}_3\right) = \begin{vmatrix} \phi_{n_1 l_1 j_1}\left(\vec{r}_1\right) & \phi_{n_2 l_2 j_2}\left(\vec{r}_1\right) & \phi_{n_3 l_3 j_3}\left(\vec{r}_1\right) \\ \phi_{n_1 l_1 j_1}\left(\vec{r}_2\right) & \phi_{n_2 l_2 j_2}\left(\vec{r}_2\right) & \phi_{n_3 l_3 j_3}\left(\vec{r}_2\right) \\ \phi_{n_1 l_1 j_1}\left(\vec{r}_3\right) & \phi_{n_2 l_2 j_2}\left(\vec{r}_3\right) & \phi_{n_3 l_3 j_3}\left(\vec{r}_3\right) \end{vmatrix}$$

code: REDSTICK Ormand '05

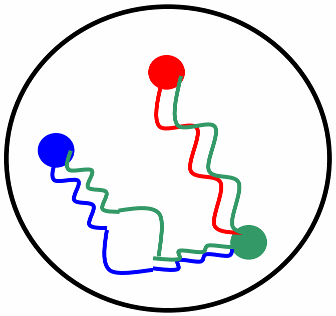
maximum number of excitations

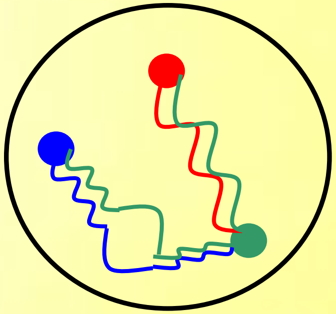
$$\psi_A\left(\vec{r}\right) = \sum_{\{n\}}^{N_{\max}} A_{\{n\}} \phi_{\{n\}}\left(\vec{r}\right)$$

Any EFT will do; for definiteness, pionless.

$r_{NN} \sim 1 \text{ fm}$

deuteron





$a_{NN} \sim 1/\aleph \cong 4.5 \text{ fm}$

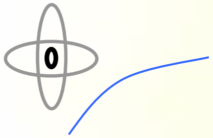
QCD: $SU(3)$ gauge theory of quarks

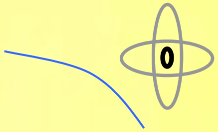
cf.

QED: $U(1)$ gauge theory of electrons and nuclei

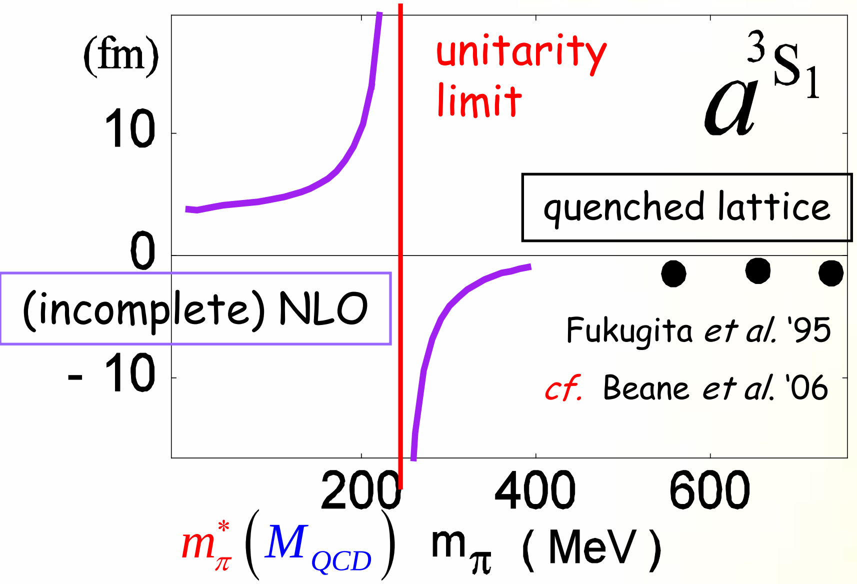
$r_{\mathrm{He4He4}} \sim 5\,\mathring{\mathrm{A}}$

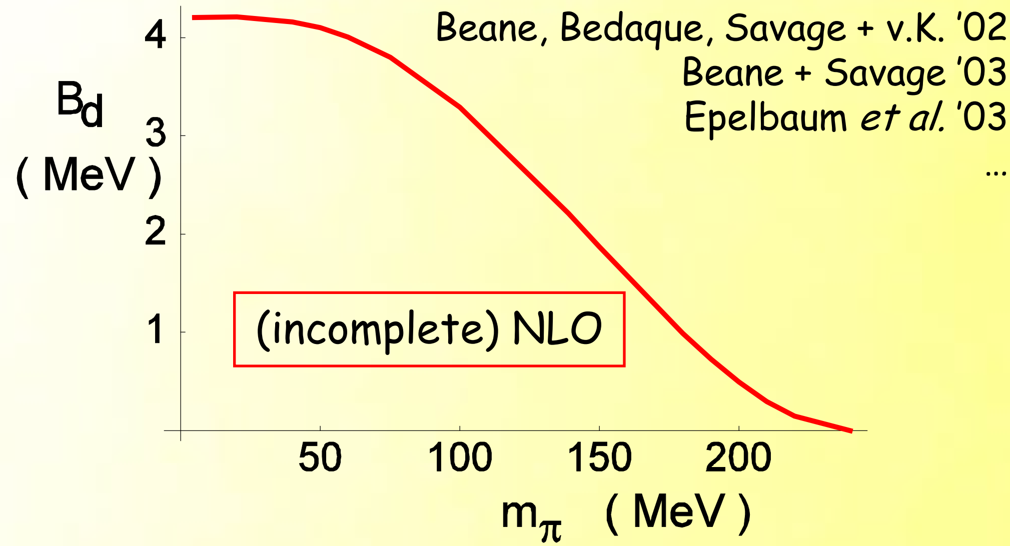
He4 dimer





$a_{\mathrm{He4He4}} \sim 1/\aleph \cong 125\,\mathring{\mathrm{A}}$

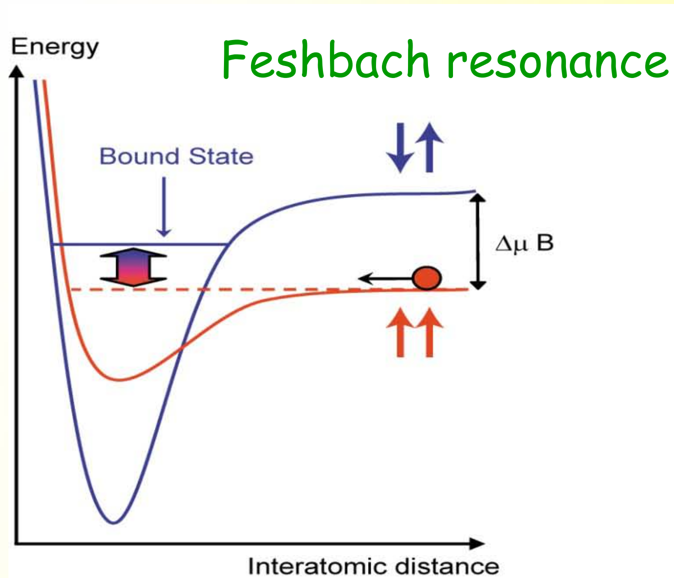


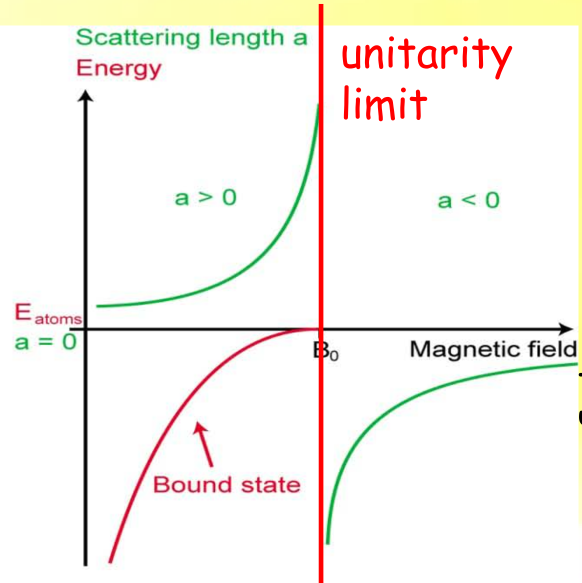


QCD near a
Feshbach resonance
in pion mass

Scale $\aleph \sim \dfrac{m_\pi - m_\pi^*}{m_\pi^*} M_{nuc}$ emerges

cf.
atoms as
magnetic field
varies





MIT webpage

$Q \sim ℵ \ll M_{nuc}$

contact EFT

- degrees of freedom: nucleons
- symmetries: Lorentz, $\cancel{P}$, $\cancel{T}$
- expansion in: $\dfrac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \cdots & \text{multipole} \end{cases}$ $\sim \dfrac{1}{3}$

Universality:
first orders apply also to atoms $M_{nuc} \to 1/l_{vdW}$ *where* $V(r) = -\dfrac{l_{vdW}^4}{2mr^6} + \ldots$

$$\mathcal{L}_{EFT} = N^+ \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + C_0\, N^+ N\, N^+ N + D_0 N^+ N\, N^+ N\, N^+ N$$
$$+ N^+ \frac{\nabla^4}{8m_N{}^3} N + C_2\, N^+ N\, N^+ \nabla^2 N + \ldots$$

v.K. '97 '99
Kaplan, Savage + Wise '98
Gegelia '98
...

(omitting spin, isospin)

two-body sector ~
effective-range expansion

$$V_{ij} = \underbrace{C_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)}_{\substack{\text{LO} \\ a_2}} - \underbrace{2C_2 \nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)}_{\substack{\text{NLO} \\ a_2,\, r_2}} + \underbrace{4C_4 \nabla^4 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)}_{\substack{\text{NNLO} \\ a_2,\, r_2}} + \dots$$

v.K. '97
Kaplan, Savage + Wise '98
Gegelia '98

$$V_{ijk} = \underbrace{D_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)}_{\substack{\text{LO} \\ a_3}} + \underbrace{D_2 \nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)}_{\substack{\text{NNLO} \\ a_3,\, r_3}} + \dots$$

Bedaque, Hammer + v.K. '99
Hammer + Mehen '00
...

$$V_{ijkl} = \underbrace{E_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)\delta^{(3)}\left(\vec{r}_k - \vec{r}_l\right)}_{\text{not LO}} + \dots$$

Platter, Hammer + Meissner '04

Untrapped nucleons

LO

$$H_A^{(0)} = \frac{1}{2m_N A}\sum_{[i<j]}\left(\vec{p}_i - \vec{p}_j\right)^2 + C_{0[0]}\sum_{[i<j]_0}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)$$

$S=0$ pairs

$$+ C_{0[1]}\sum_{[i<j]_1}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + D_0\sum_{[i<j<k]}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)$$

$S=1$ pairs

$S=1/2$ triplets

EFT PC effectively justifies (modified) cluster approximation

$$H_A^{(0)}\psi_A^{(0)}\left(\vec{r}\right) = E_A^{(0)}\psi_A^{(0)}\left(\vec{r}\right)$$

Stetcu, Barrett +v.K., '07

parameters fitted to d, t, a ground-state energies

predicted 4He excited, 6Li ground energies

works within ~30%

but parameters proliferate: *e.g.*, at NLO two more 2-body parameters

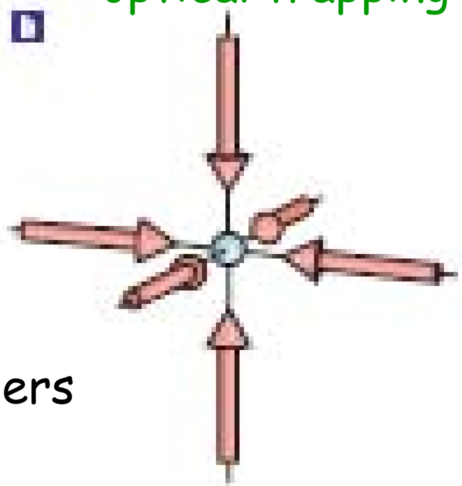
can we fit them to scattering data?

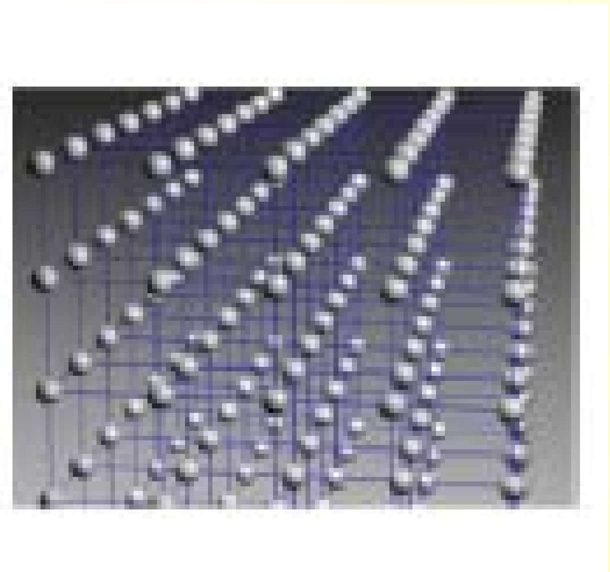
$C_{2[0]}\left(\Lambda\right),\ C_{2[1]}\left(\Lambda\right)$

Yes, trap them!

Trapped fermions

I. Bloch



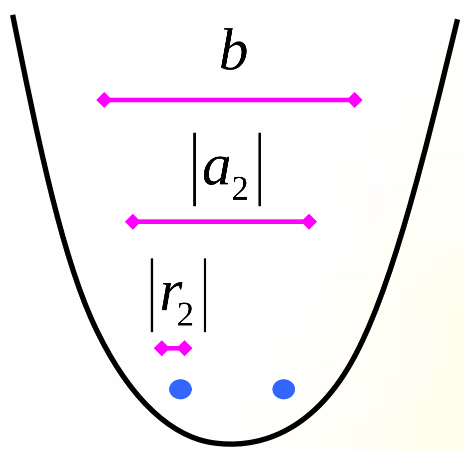


$$V(\vec{r}) \propto \alpha(\omega_L)\left|\vec{E}(\vec{r})\right|^2$$
$$\propto \sum_i \sin^2(k_L r_i)$$
$$\approx k_L^2 \vec{r}^{\,2}$$

standing waves

low-tunneling regime

(band insulator)



$$\frac{b}{|r_2|} \gg 1$$ universal behavior

$$\frac{b}{|a_2|}\begin{cases} \to \infty \\ \lesssim 1 \\ \to 0 \end{cases}$$

untrapped limit

significant trap effects

only low-energy scale given by b
some semi-analytical results known

test our method

Rotureau, Stetcu, Barrett + v.K., '09

Life in the Box

$$H_A = \frac{\omega}{2}\left\{\sum_{i=1}^{A}\left[\frac{1}{2}b^2 p_i^2 + 2\frac{r_i^2}{b^2}\right] + 2\mu_2 b^2 V\left(\left\{\vec{r}_i - \vec{r}_j\right\}\right)\right\} = H_A^{(cm)} + H_A^{(rel)}$$

two-body reduced mass $\mu_2 = m/2$

S waves only in LO

LO $$H_A^{(0)}\left|\psi_A^{(0)}\right\rangle = E_A^{(0)}\left|\psi_A^{(0)}\right\rangle$$

NLO $$E_A^{(1)} = \left\langle\psi_A^{(0)}\middle|V_A^{(1)}\middle|\psi_A^{(0)}\right\rangle$$

NNLO $$E_A^{(2)} = \left\langle\psi_A^{(0)}\middle|V_2^{(2)}\middle|\psi_A^{(0)}\right\rangle + \frac{1}{2}\left\{\left\langle\psi_A^{(0)}\middle|V_2^{(1)}\middle|\psi_A^{(1)}\right\rangle + \left\langle\psi_A^{(1)}\middle|V_2^{(1)}\middle|\psi_A^{(0)}\right\rangle\right\}$$

etc.

$A=2$

Stetcu, Barrett, Vary + v.K. '08
Rotureau, Stetcu, Barrett + v.K. '10

LO $H_2^{(0)}\left|\psi_2^{(0)}\right\rangle = E_2^{(0)}\left|\psi_2^{(0)}\right\rangle$

$$\Rightarrow \frac{2\pi b}{\mu_2 C_0^{(0)}\left(N_{2\max},\omega\right)} = -\frac{2}{\pi^{1/2}}\sum_{n=0}^{N_{2\max}/2}\frac{L_n^{(1/2)}(0)}{2n+3/2-\left(E_2^{(0)}/\omega\right)}$$

input one $\frac{E_2^{(0)}}{\omega} = \frac{E_2^{(0)}}{\omega}\left(\frac{b}{a_2}\right)$ ⇨ determine $C_0^{(0)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

e.g. lowest level

NLO $E_2^{(1)} = \left\langle\psi_2^{(0)}\right|V_2^{(1)}\left|\psi_2^{(0)}\right\rangle = \ldots$

input second level ⇨ determine $C_2^{(1)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

NNLO $E_2^{(2)} = \left\langle\psi_2^{(0)}\right|V_2^{(2)}\left|\psi_2^{(0)}\right\rangle + \frac{1}{2}\left\{\left\langle\psi_2^{(0)}\right|V_2^{(1)}\left|\psi_2^{(1)}\right\rangle + \left\langle\psi_2^{(1)}\right|V_2^{(1)}\left|\psi_2^{(0)}\right\rangle\right\} = \ldots$

input third level ⇨ determine $C_4^{(2)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

etc.

Where do levels come from?

$$N_{2\max} \to \infty$$

$$\psi_2\left(0<r\ll b\right) \propto \frac{1}{r}\left\{1-2\underbrace{\frac{\Gamma\left(\frac{3}{4}-\frac{E_2}{2\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_2}{2\omega}\right)}\frac{r}{b}}_{=\left[1-\mu a_2 r_2 E+\ldots\right]\frac{r}{a_2}}+\mathcal{O}\left(\frac{r^2}{b^2}\right)\right\}$$

$$\Rightarrow \frac{\Gamma\left(\frac{3}{4}-\frac{E_2}{2\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_2}{2\omega}\right)}=\frac{b}{2a_2}\left\{1-\frac{a_2 r_2}{b^2}\frac{E_2}{\omega}+\ldots\right\}$$

LO

NLO, NNLO

Busch *et al.* '98
Blume + Greene '02
Block + Holthaus '02
Bolda, Tiesinga + Julienne '02
...

$$\boxed{\frac{b}{a_2}\to\infty}$$

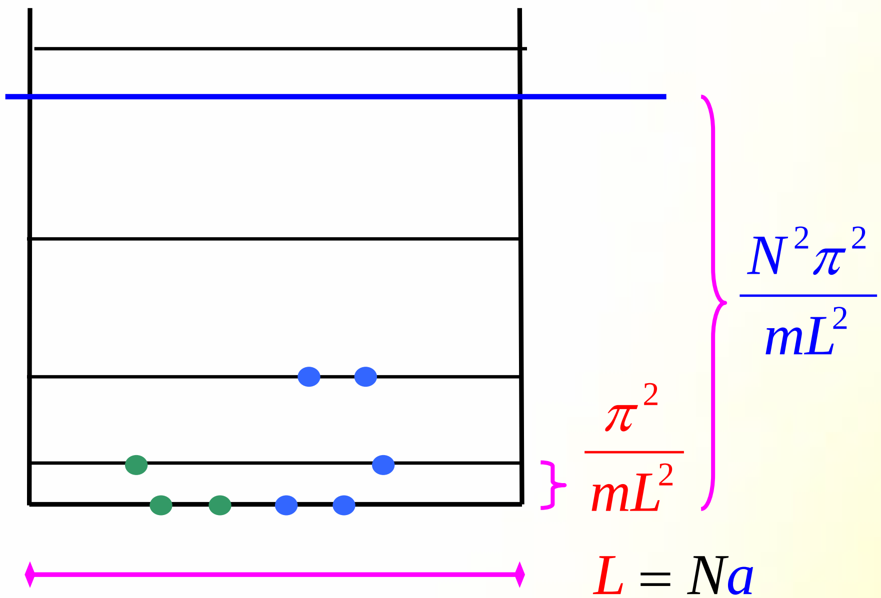
$$\begin{cases} \dfrac{E_{2,0}}{\omega}=-\dfrac{b^2}{a_2^2}+\ldots & \text{untrapped bound state} \\ \dfrac{E_{2,n}}{\omega}=-\dfrac{1}{2}+2n+\ldots \quad (n=1,2,\ldots) & \text{scattering states} \end{cases}$$

Lattice EFT

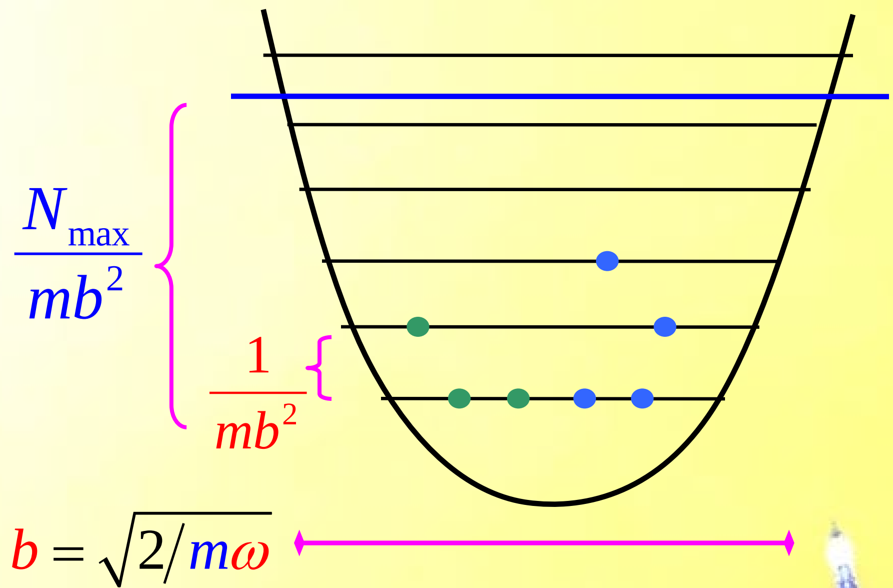
Harmonic EFT

Lattice Box

cf. Fukuda + Newton '54



Harmonic-Oscillator Box



Parameters fitted to *E* from

$$2N - 2\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<N} \frac{1}{(2\pi\mathbf{n})^2 - mEL^2} = -\frac{\sqrt{mEL^2}}{2}\cot\delta(E)$$

Luescher '91

$$\frac{\Gamma\left(\frac{3}{4} - \frac{mEb^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{mEb^2}{2}\right)} = -\frac{\sqrt{mEb^2}}{2}\cot\delta(E)$$

Busch *et al.* '98

LO RG running

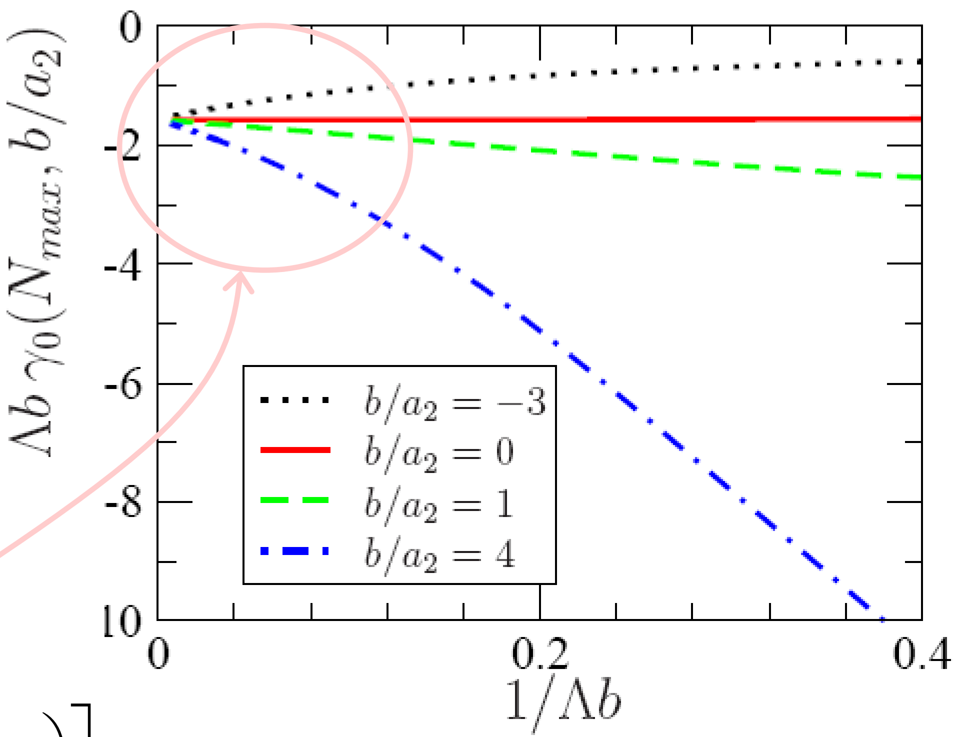
$$\Lambda b\gamma_0\left(N_{2\max}, b/a_2\right) \equiv \frac{\mu_2\Lambda}{2\pi} C_0^{(0)}\left(N_{2\max}, \omega\right)$$

$$\frac{b}{|a_2|} \to 0 \qquad \frac{E_{2,n}^{(0)}}{\omega} = \frac{1}{2} + 2n \qquad (n = 0, 1, \dots)$$

$$\Rightarrow \quad \Lambda b\,\gamma_0\left(N_{2\max}, b/a_2\right) \cong -\frac{\pi}{2}$$

$$\frac{b}{a_2} \to -\infty \qquad \frac{E_{2,n}^{(0)}}{\omega} = \frac{3}{2} + 2n \qquad (n = 0, 1, \dots)$$

$$\Rightarrow \quad \Lambda b\,\gamma_0\left(N_{2\max}, b/a_2\right) = 0$$

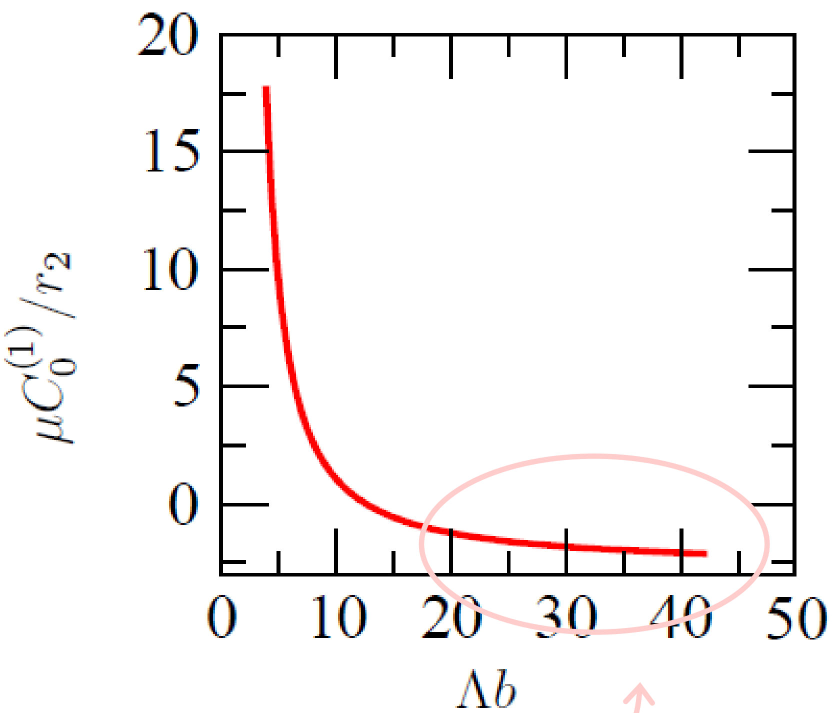


$$\Lambda b \gg 1$$

$$\Lambda b\gamma_0\left(N_{2\max}, b/a_2\right) = -\frac{\pi}{2}\left[1 + \frac{\pi b}{2a_2}\frac{1}{\Lambda b} + \mathcal{O}\left(\frac{1}{\Lambda^2 b^2}\right)\right]$$

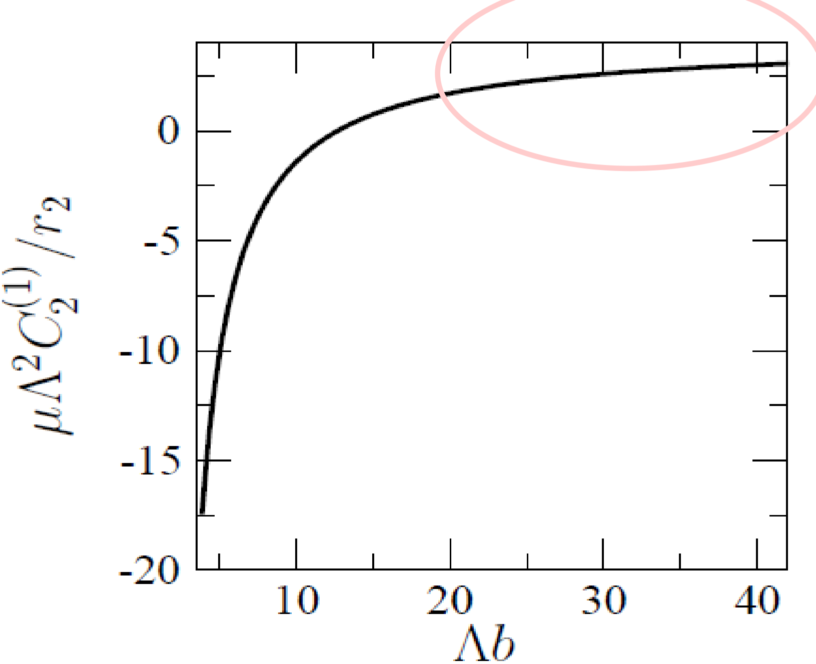
$$\frac{1}{\Lambda b} = \left[2\left(N_{2\max} + 3/2\right)\right]^{-1/2}$$

NLO RG running



$b/a_2 = 1$

$r_2/b = 0.1$



$\Lambda b \gg 1$

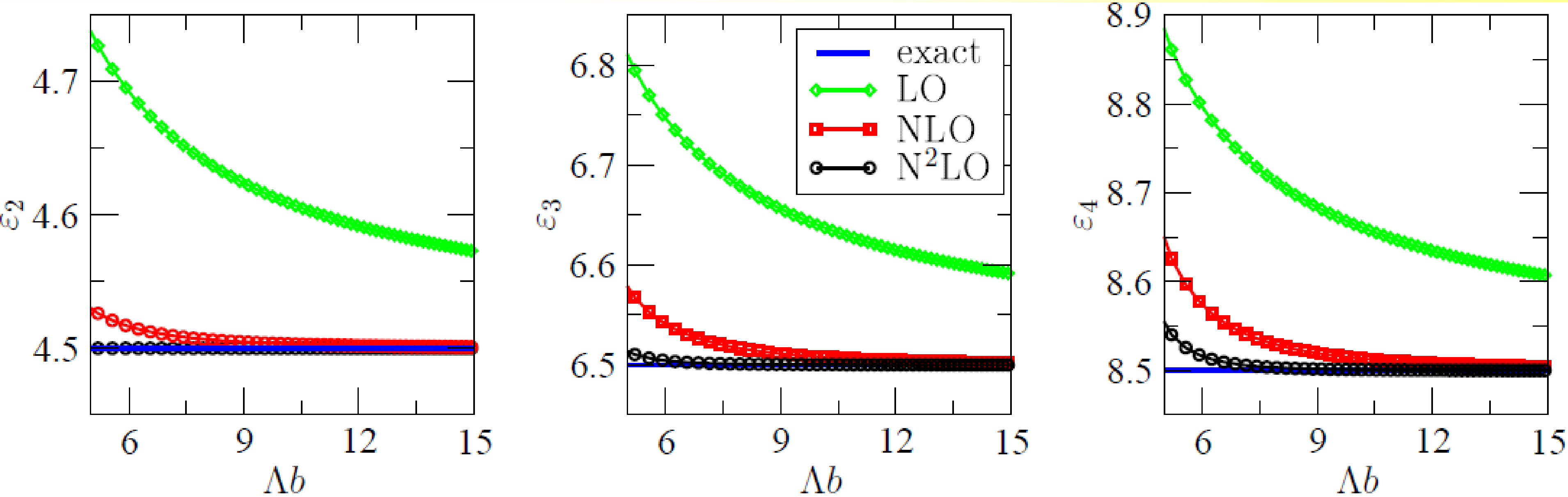
$$\frac{\mu_2}{r_2} C_0^{(1)} = -\frac{\pi^3}{12}\left[1+\mathcal{O}\left(\frac{1}{r_2\Lambda}, \frac{1}{a_2\Lambda}\right)\right]$$

$$\frac{\mu_2\Lambda^2}{r_2} C_2^{(1)} = \frac{\pi^3}{8}\left[1+\mathcal{O}\left(\frac{1}{r_2\Lambda}, \frac{1}{a_2\Lambda}\right)\right]$$

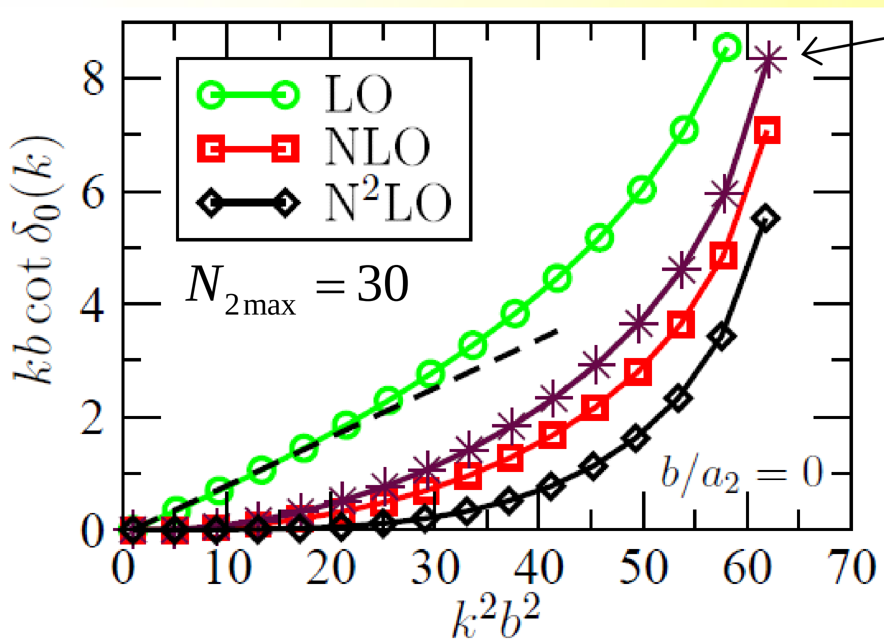
Etc.

at unitarity

Stetcu, Barrett, Vary + v.K. '08
Rotureau, Stetcu, Barrett + v.K. '10



$\varepsilon_n \equiv E_{2n}/\omega$



NNLO Hamiltonian fully diagonalized: worse than NLO!

cf. Luu *et al*. '10

Rotureau, Stetcu, Barrett, Birse + v.K. '10

$A \geq 3$ include few-body forces

$$N_{A\max} \geq N_{2\max} \begin{cases} \text{1)} \quad N_{A\max} \gg N_{2\max} \quad \Rightarrow E_A = E_A\left(N_{2\max}, \omega\right) \\ \text{2)} \quad N_{2\max} \gg 1 \end{cases}$$

$\frac{b}{a_2} \to \infty$

- lowest states: free-space bound states
  binding energy info $B_{A,0} = -E_{A,0}, \ldots$
- other states: scattering states
  phase-shift info, for example:

$$\frac{\Gamma\left(\frac{3}{4} - \frac{E_{A,n} - E_{A-1,0}}{2\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_{A,n} - E_{A-1,0}}{2\omega}\right)} = -\sqrt{\frac{E_{A,n} - E_{A-1,0}}{2\omega}} \cot \delta_{1,A-1}\left(\frac{2}{b}\sqrt{\frac{E_{A,n} - E_{A-1,0}}{2\omega}}\right)$$

S-wave phase shift for particle/lighter b.s. scattering

Trapped two-component fermions: $S = 1/2$

$$V = \sum_{[i<j]_0} \left\{ C_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) - 2C_2 \nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + 4C_4 \nabla^4 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) \right\}$$

$S = 0$ pairs

S wave only in LOs

up to NNLO

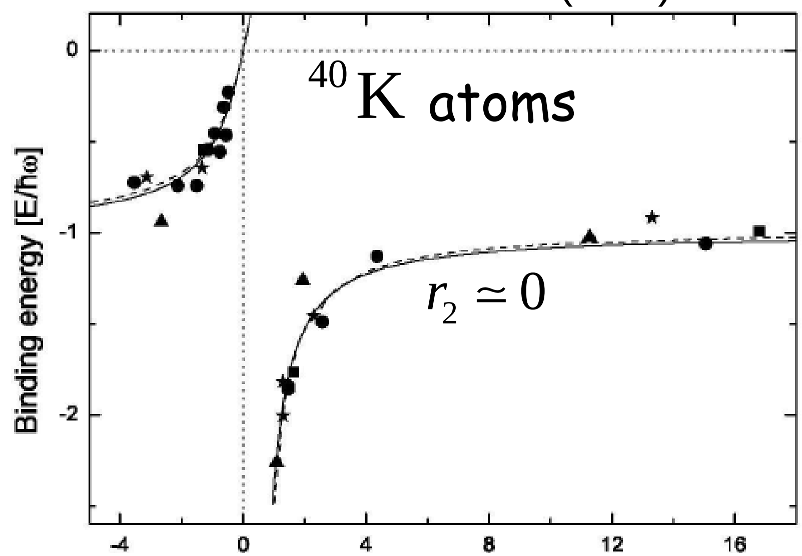
no $\begin{cases} S = 1 \text{ two-body force} \\ \text{three-body force in LOs} \end{cases}$

+ HO <u>is</u> physics

$A = 2$ fit to data *e.g.*

$A \geq 3$ no fit

Stoeferle et al (ETH) '06



$E/\omega \to E/\omega + 3/2$ $a_{ho} \to b$

$$\frac{b}{a_2} \to -\infty \quad \frac{E_A}{\omega} = \text{filling of HO shells}$$

$$\frac{b}{|a_2|} \to 0 \quad \frac{E_A}{\omega} = \varepsilon_A\left(N_{2\max}\right)$$ (independent of ω since *b* only scale)

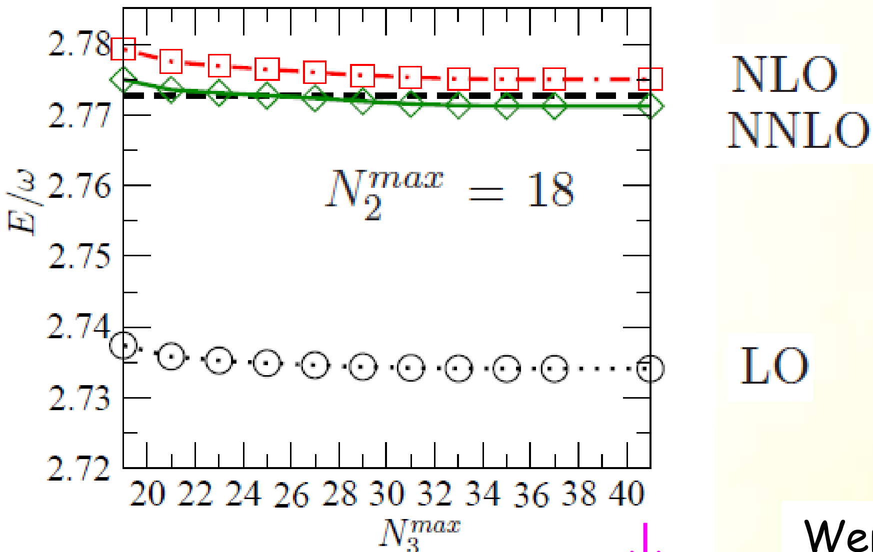
$A = 3$ at unitarity

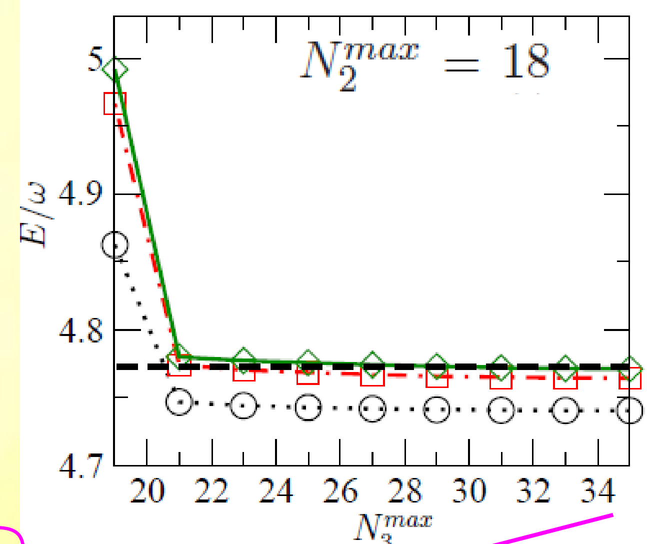
Stetcu, Barrett, Vary + v.K., '07
Rotureau, Stetcu, Barrett, Birse + v.K. '10

ground state

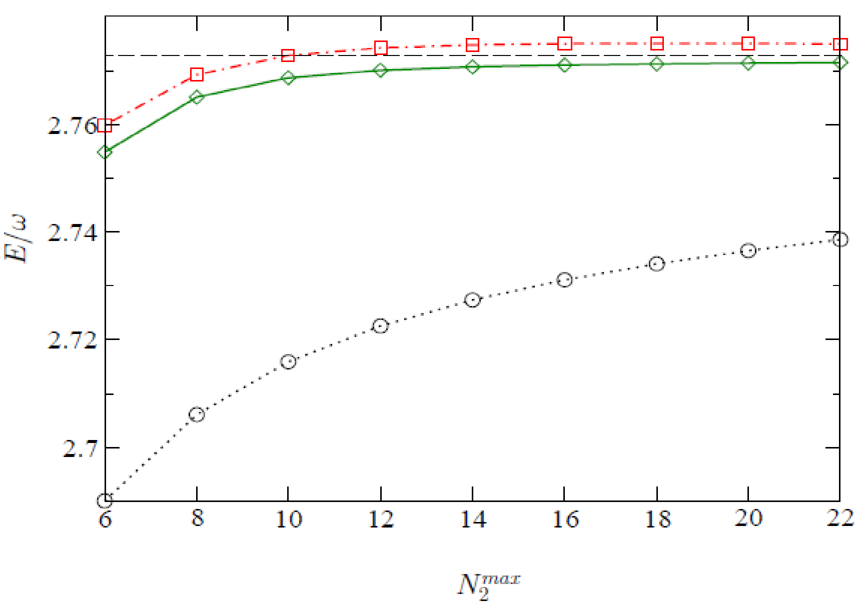
$L^\pi = 1^-$

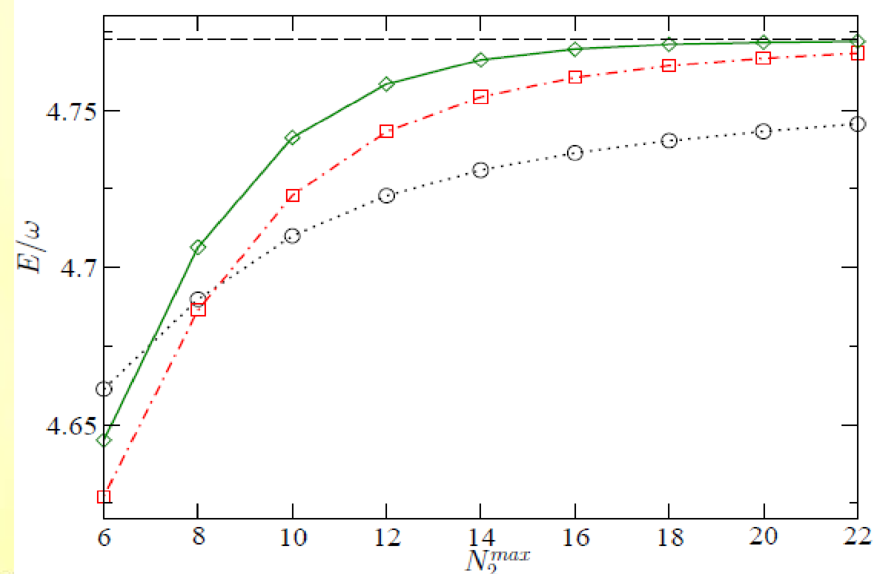
first excited state





Werner+ Castin '06





Errors at unitarity

$$\frac{E_3(N_2^{max})}{\omega} = \frac{E_3(\infty)}{\omega} + \frac{\alpha_1}{(N_2^{max}+3/2)^{1/2}} + \frac{\alpha_3}{(N_2^{max}+3/2)^{3/2}} + \frac{\alpha_5}{(N_2^{max}+3/2)^{5/2}} + \dots$$

ground state

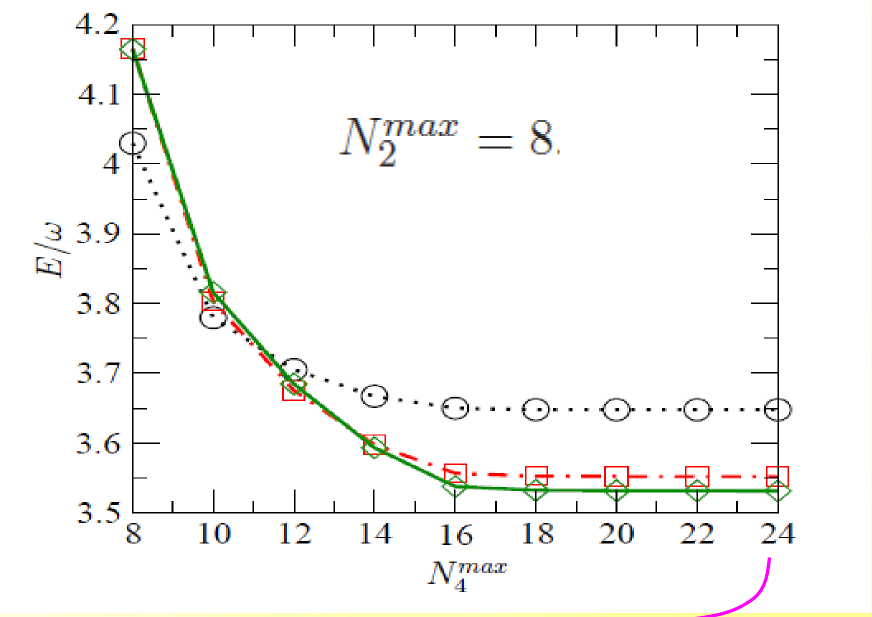
LO $\alpha_1 = -0.139$ $\alpha_3 = -0.63$

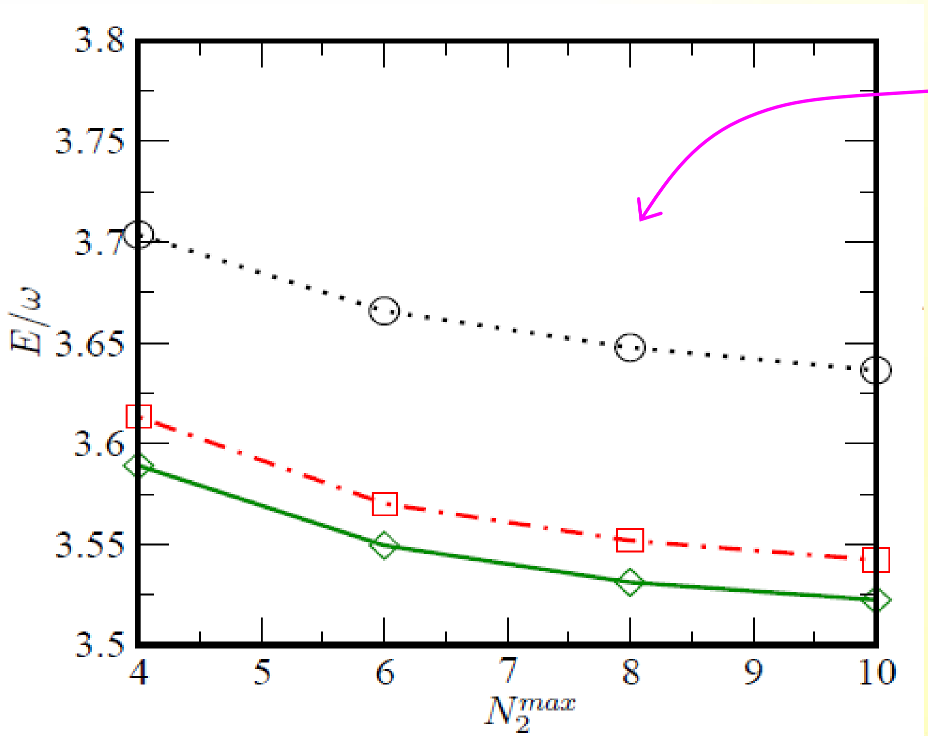
NNLO $\alpha_1 = 0$ $\alpha_3 = -0.30$ $\alpha_5 = -5.8$

Stetcu, Barrett, Vary + v.K., '07
Rotureau, Stetcu, Barrett, Birse + v.K. '10

$A=4$ at unitarity

$L^\pi = 0^+$ ground state





LO

NLO

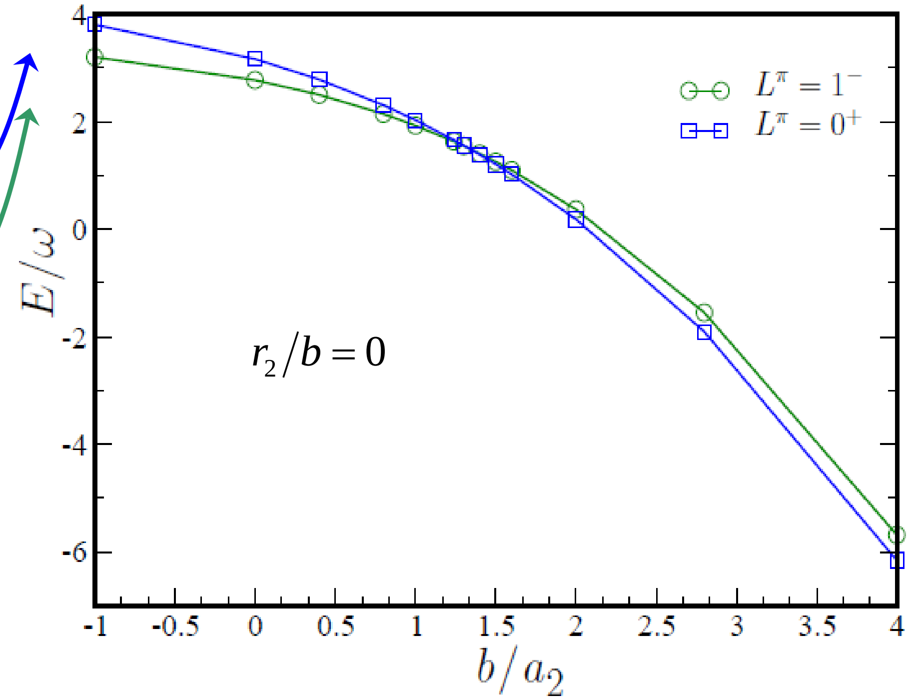
NNLO

cf.

$$\frac{E_4^{(\infty)}}{\omega} \cong \begin{cases} 3.6 \pm 0.1 & \text{Chang + Bertsch '07} \\ 3.551 \pm 0.009 & \text{von Stecher } \textit{et al.} \text{ '07} \\ 3.545 \pm 0.003 & \text{Alhassid, Bertsch + Fang '08} \end{cases}$$

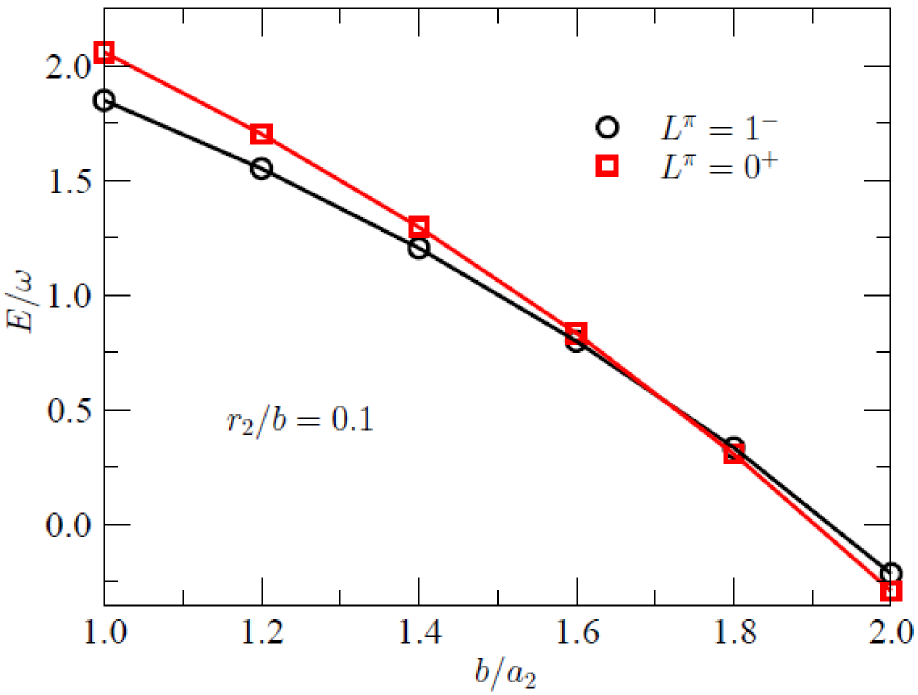
Stetcu, Barrett, Vary + v.K., '07
Kerstner + Duan '07
Rotureau, Stetcu, Barrett, Birse + v.K. '10

$$\frac{E_3}{\omega} \rightarrow \begin{cases} 5 & 1S2P \\ 4 & 2S1P \end{cases}$$



$A = 3$

inversion of g.s. parity!



$$\frac{E_3}{\omega} \approx -\frac{b^2}{a_2^2}$$

$(\text{atom+dimer})_{S\ \text{wave}}$

NNLO

cliparts.com

Rotureau, Stetcu, Barrett, Birse + v.K. '10

cf. von Stecher, Blume + Greene '08

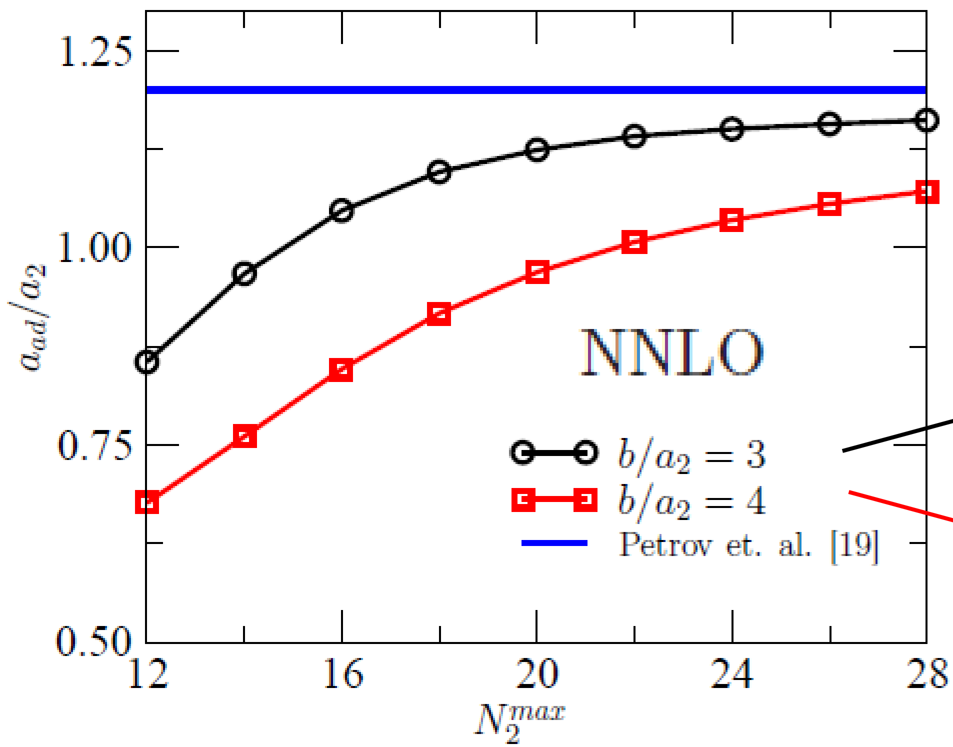
$$\frac{\Gamma(3/4-(E_{3;n}-E_{2;0})/2\omega)}{\Gamma(1/4-(E_{3;n}-E_{2;0})/2\omega)}=\frac{b'}{2a_{ad}}-\frac{r_{ad}}{2b'}\frac{E_{3;n}-E_{2;0}}{\omega}+\dots$$

3-body energy above dimer g.s.

$$b'=\frac{1}{\sqrt{\mu_{ad}\omega}}$$

use two levels, eliminate r_{ad}:

$A=3$



better precision at smaller cutoffs

better dimer inside trap

Liberated nucleons

add $\begin{cases} S=1 \text{ two-body force} \\ \text{three-body force in LOs} \end{cases}$

+ HO is <u>not</u> physics

$$V=\sum_{S=0,1}\sum_{[i<j]_S}\left\{C_{0[S]}\,\delta^{(3)}\left(\vec{r}_i-\vec{r}_j\right)-2C_{2[S]}\nabla^2\delta^{(3)}\left(\vec{r}_i-\vec{r}_j\right)\right\}$$

$$+D_0\sum_{[i<j<k]}\delta^{(3)}\left(\vec{r}_i-\vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j-\vec{r}_k\right)$$

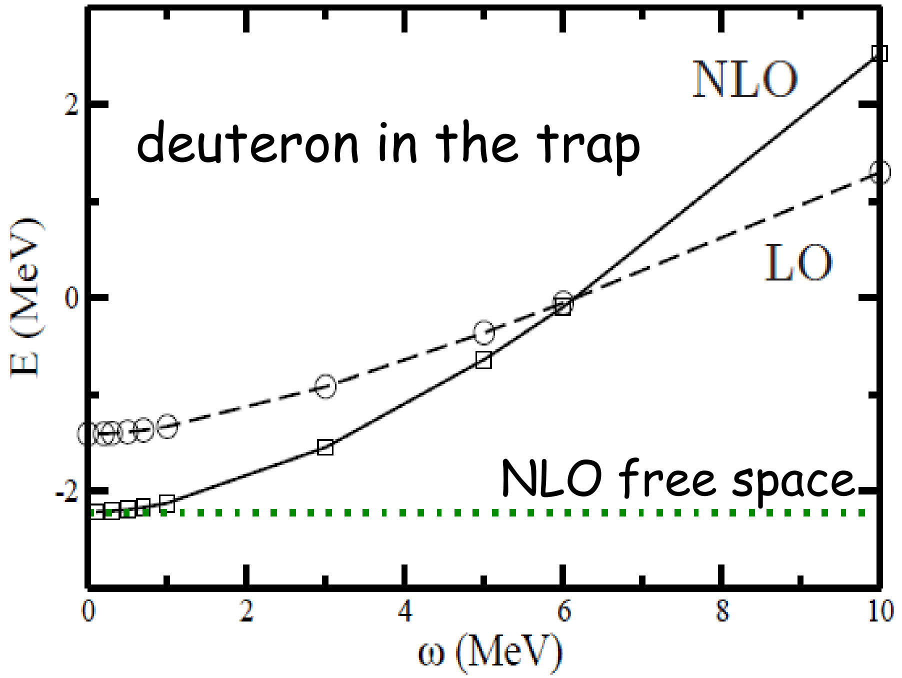
single parameter

$S=1/2$ triplets

S wave only

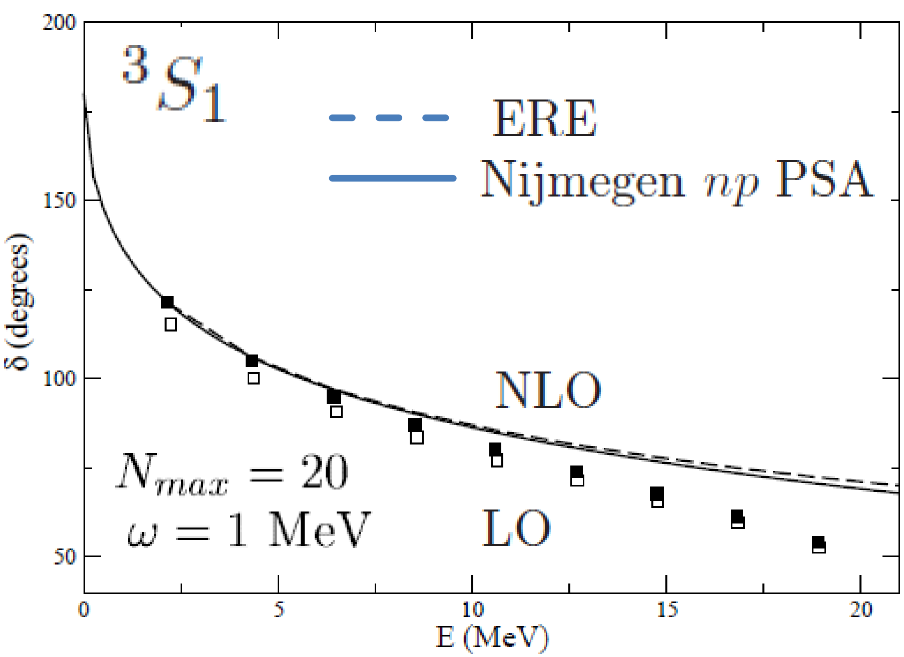
up to NLO

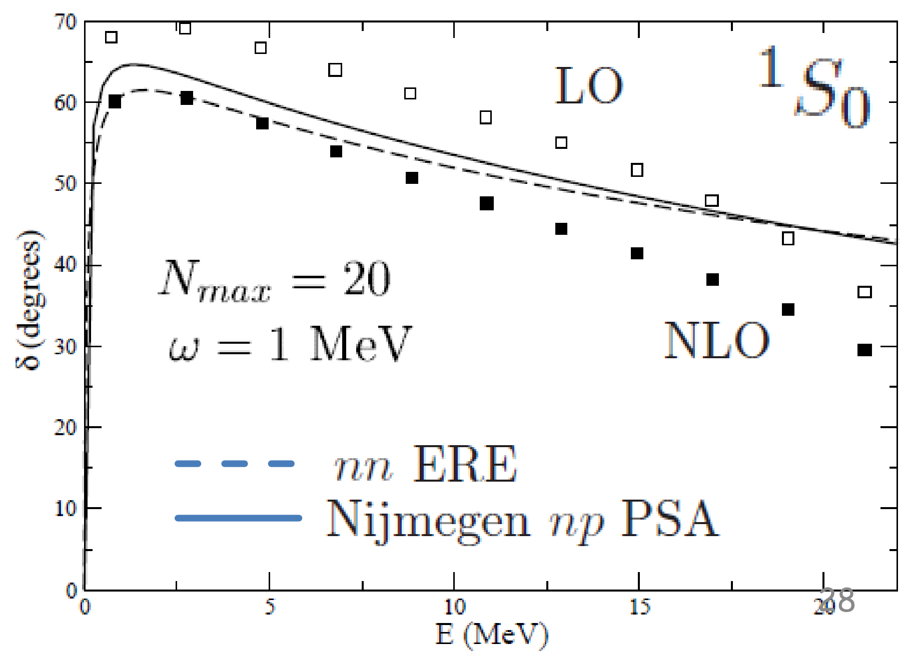
Rotureau, Stetcu, Barrett + v.K. '11



$A = 2$

NLO



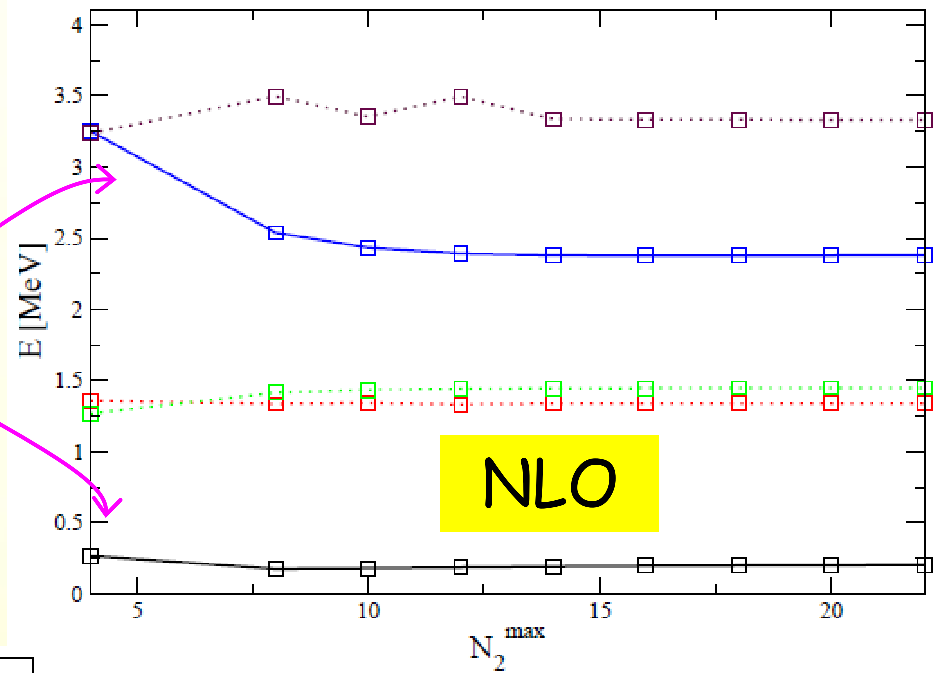


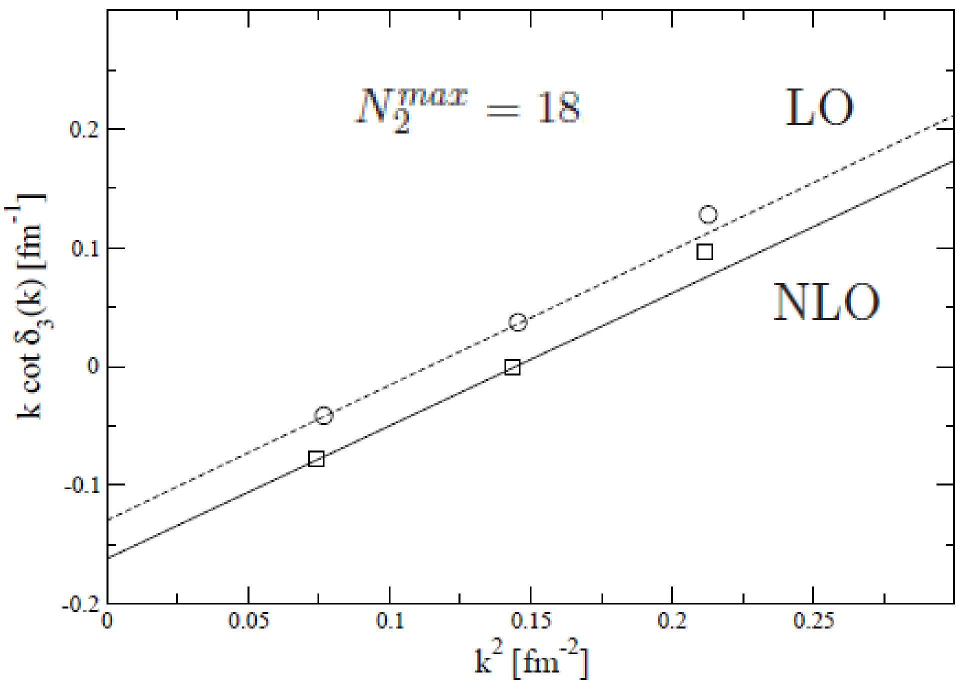
Rotureau, Stetcu, Barrett + v.K. '11

$A = 3$

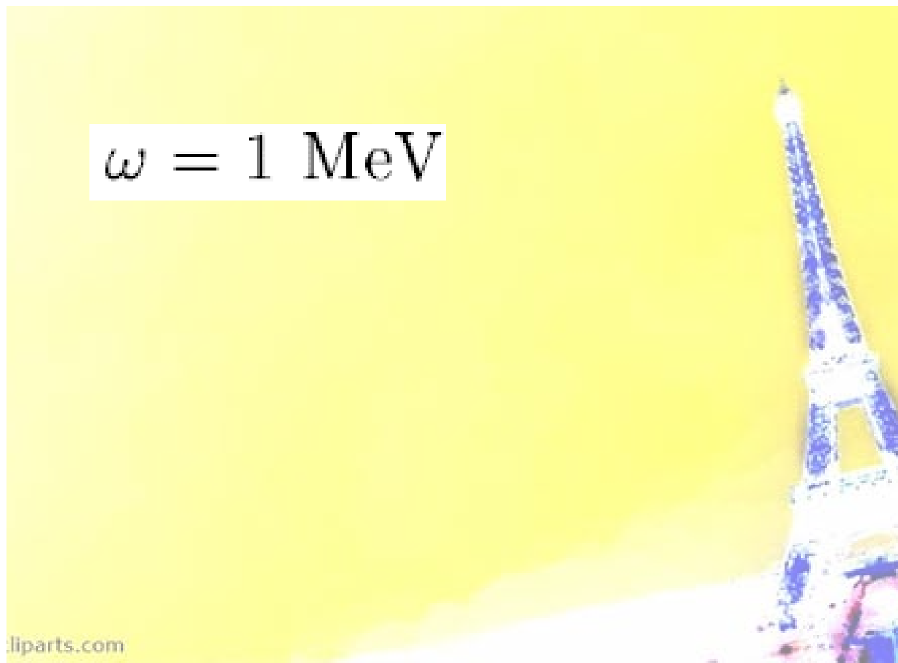
$I = 1/2,\ J^{\pi} = 3/2^{+}$

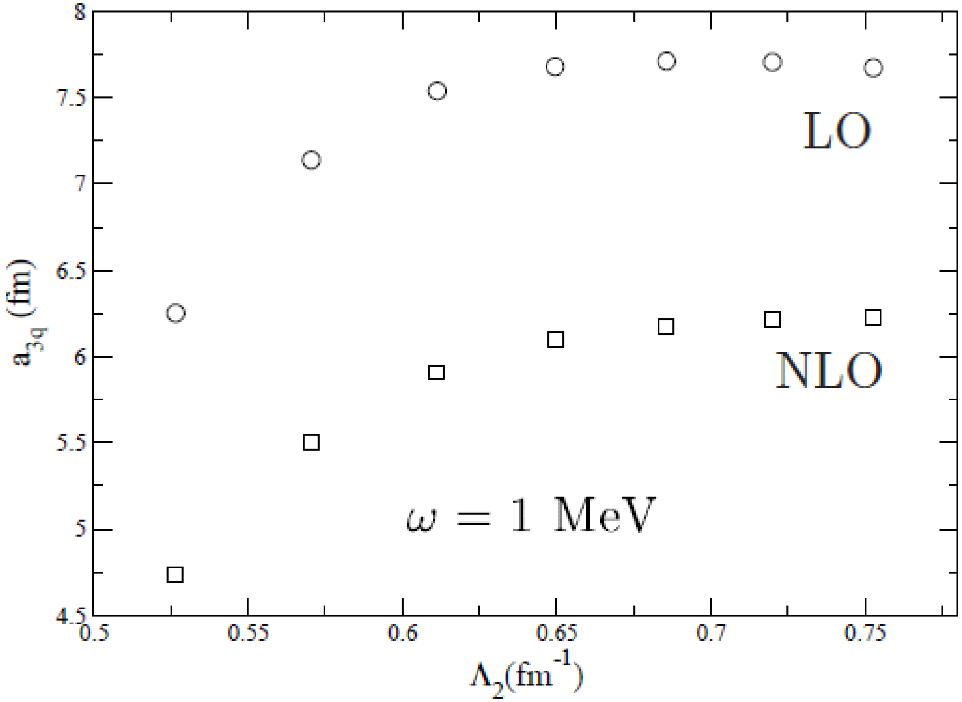
$L = 0$





$\omega = 1\ \mathrm{MeV}$





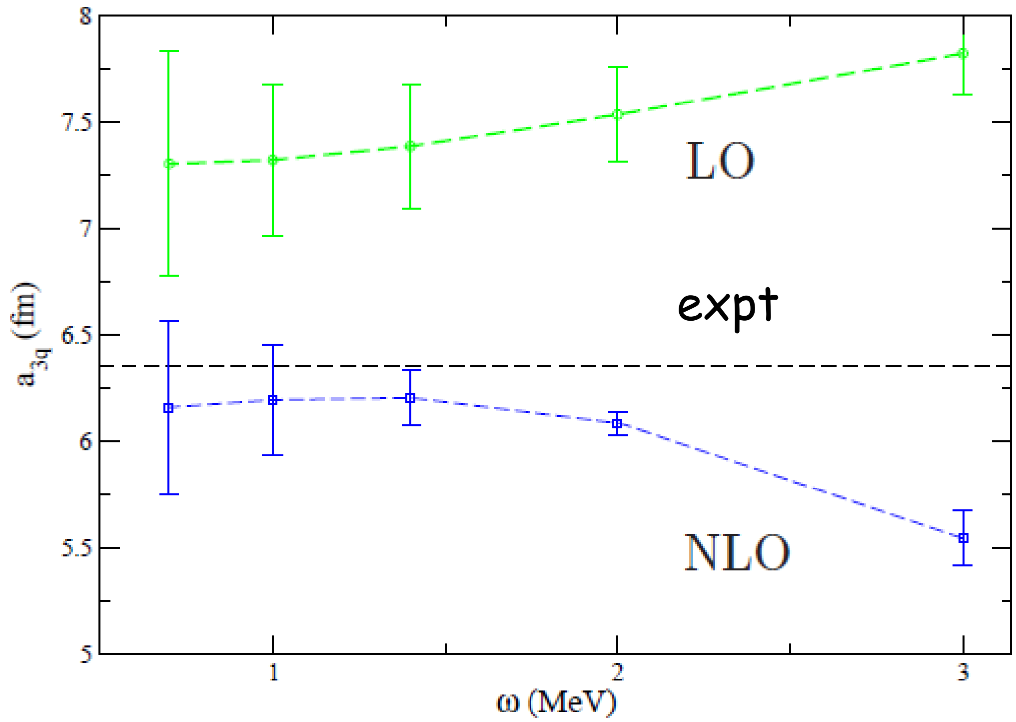
$$\frac{1}{a_{3q}} = \frac{1}{a_{3q}(\infty)} + \frac{\alpha_1}{\Lambda_2^{p_1}} + \frac{\alpha_2}{\Lambda_2^{p_2}}$$

$${}^4a_3 = 6.35 \pm 0.02 \text{ fm}$$

Dilg *et al.* '71

cf. NNLO $\quad {}^4a_3 = 6.33 \pm 0.10 \text{ fm}$

Bedaque, Hammer + v.K. '98

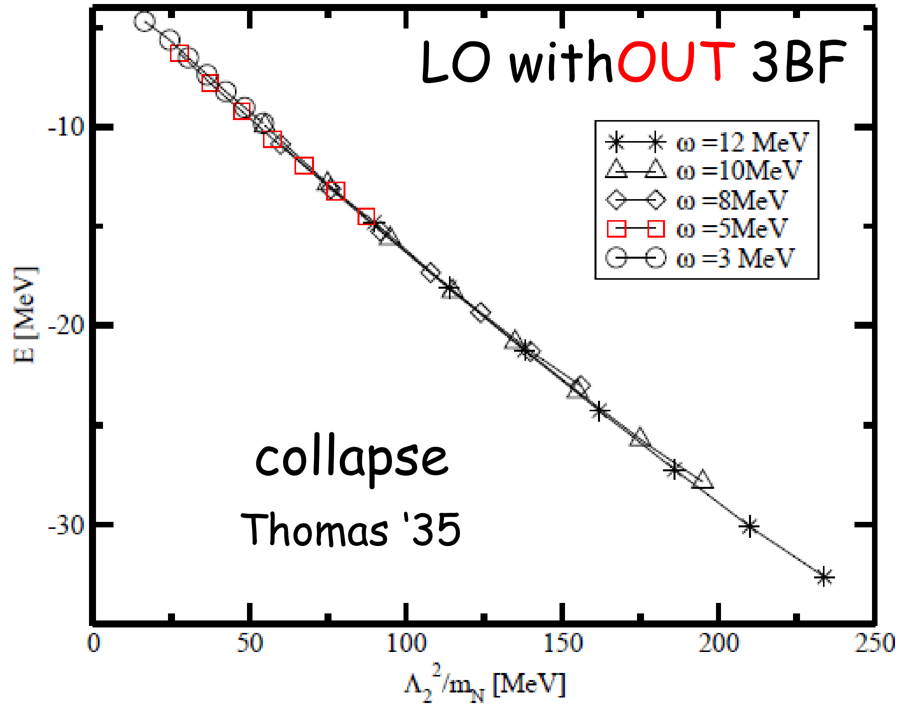


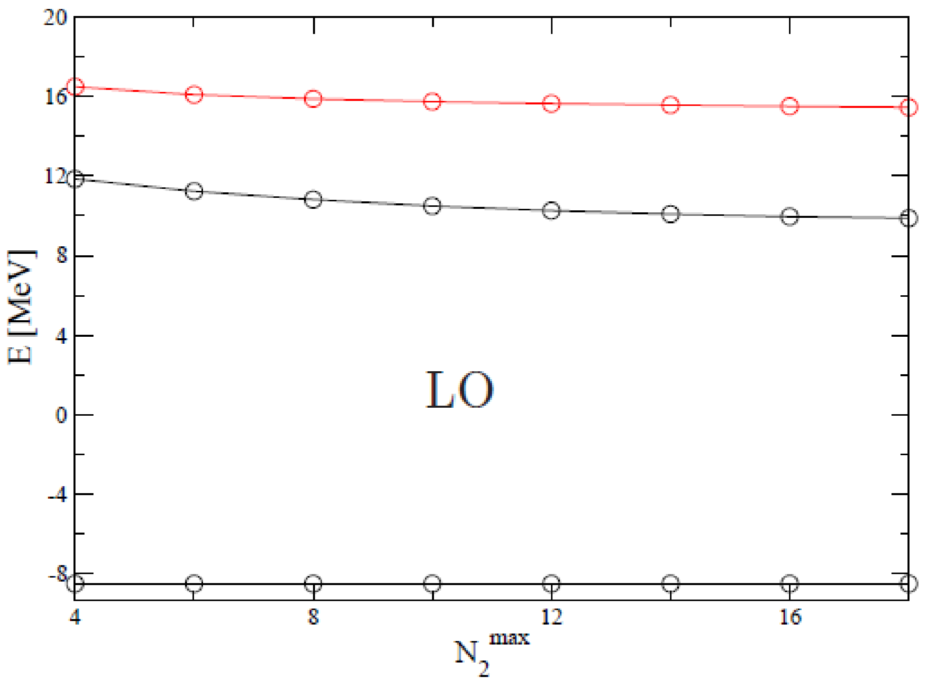
$$I = 1/2,\ J^{\pi} = 1/2^{+}$$

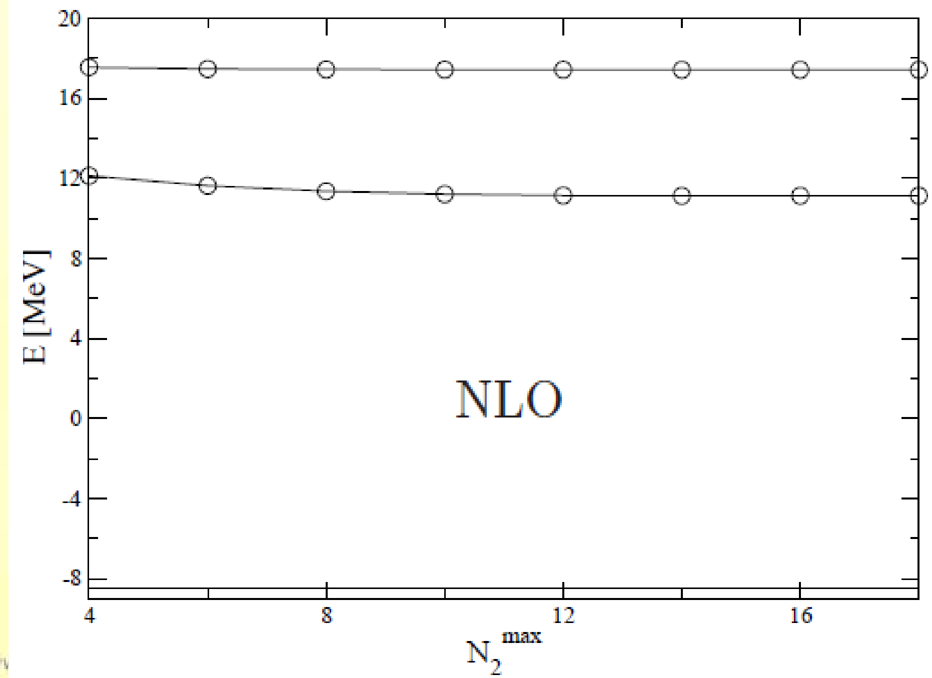
similar for bosons

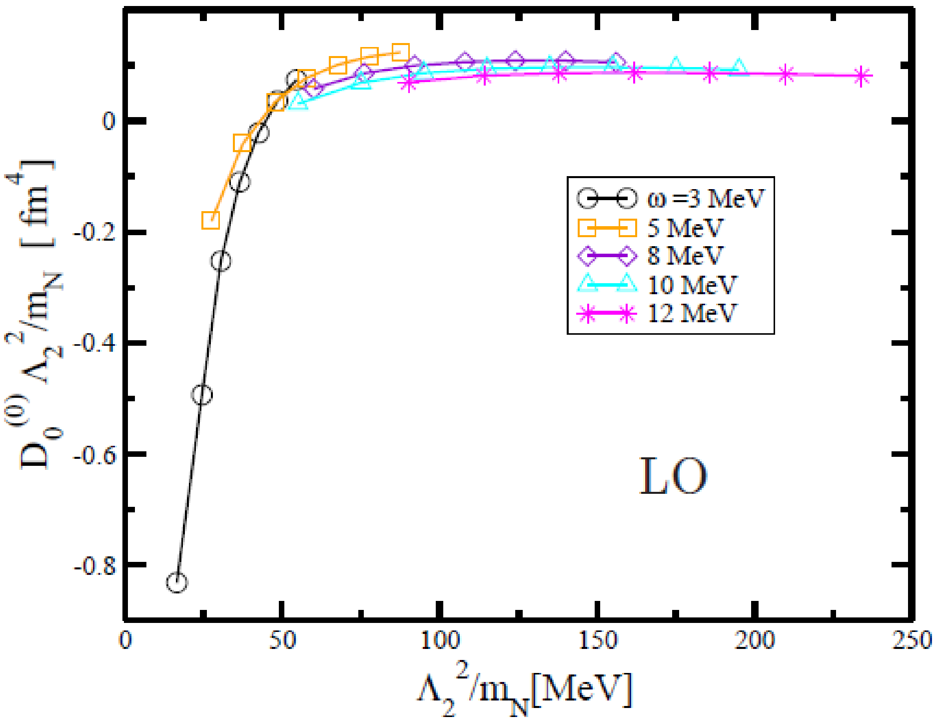
Toelle, Hammer + Metsch '10

fit 3BF to triton BE





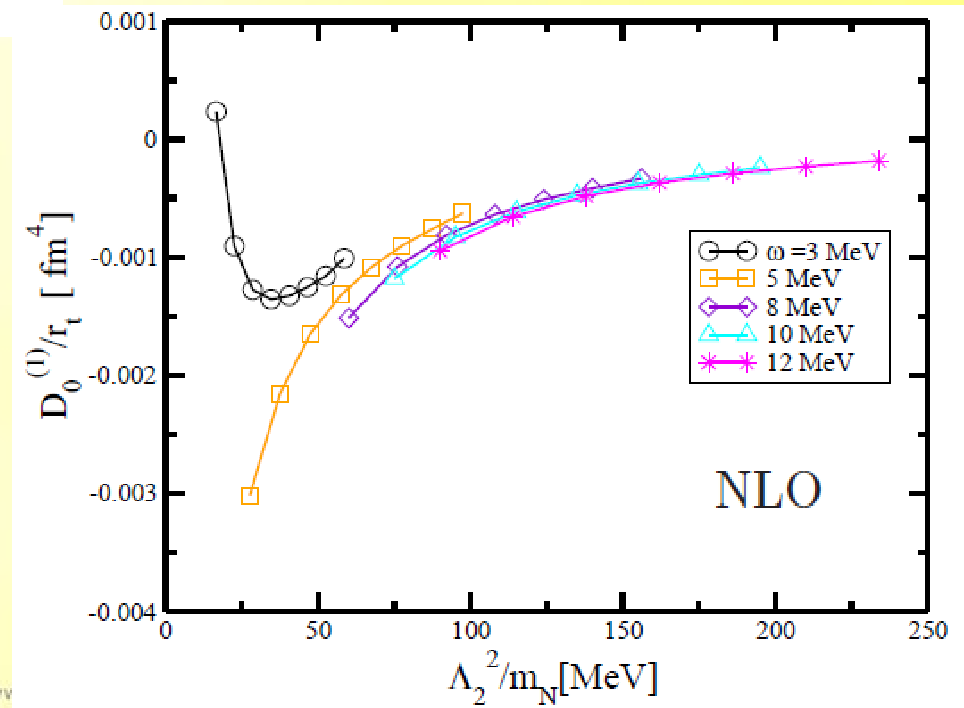




fit 3BF to triton BE

cf. limit cycle

Bedaque, Hammer + v.K. '99



Conclusion & Outlook

- ✓ EFT can be solved in HO basis with scattering input
- ✓ Nucleons with pionless EFT in HO similar to trapped atoms near a Feshbach resonance
- ✓ Convergence improves with increasing order
- ✓ Few-body binding energies and scattering parameters can be calculated
- ✓ More extensive calculations with nucleons are needed