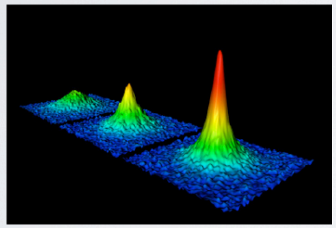
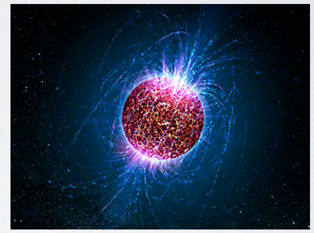
Comparison and Contrast:
Cold Atoms and Dilute Neutron Matter

J. Carlson - LANL

Fermi Condensates



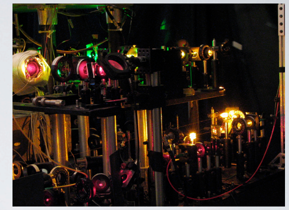
C. Regal et al. PRL 2004



Introduction

Homogeneous Matter:

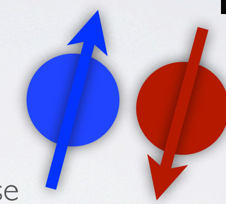
- Equation of State (T=0)
- Contact
- Gap



MIT Optical Trap

Linear Response:

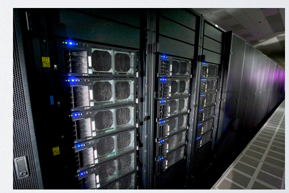
- RF Response
- Spin Response
- Density Response



Inhomogeneous (trapped) Matter

- Local Density approximation and Density functional Theory

Future Outlook



roadrunner

Interactions:

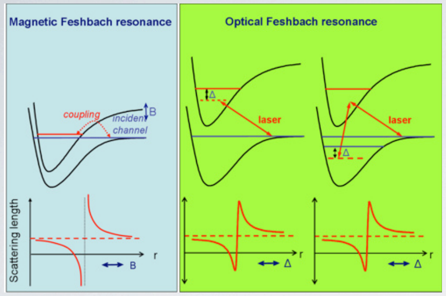


Diagram from Innsbruck

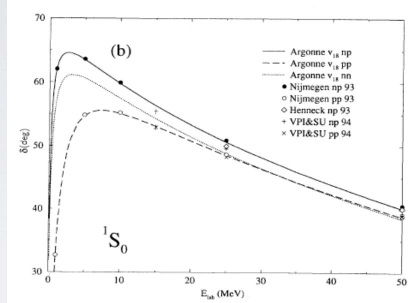
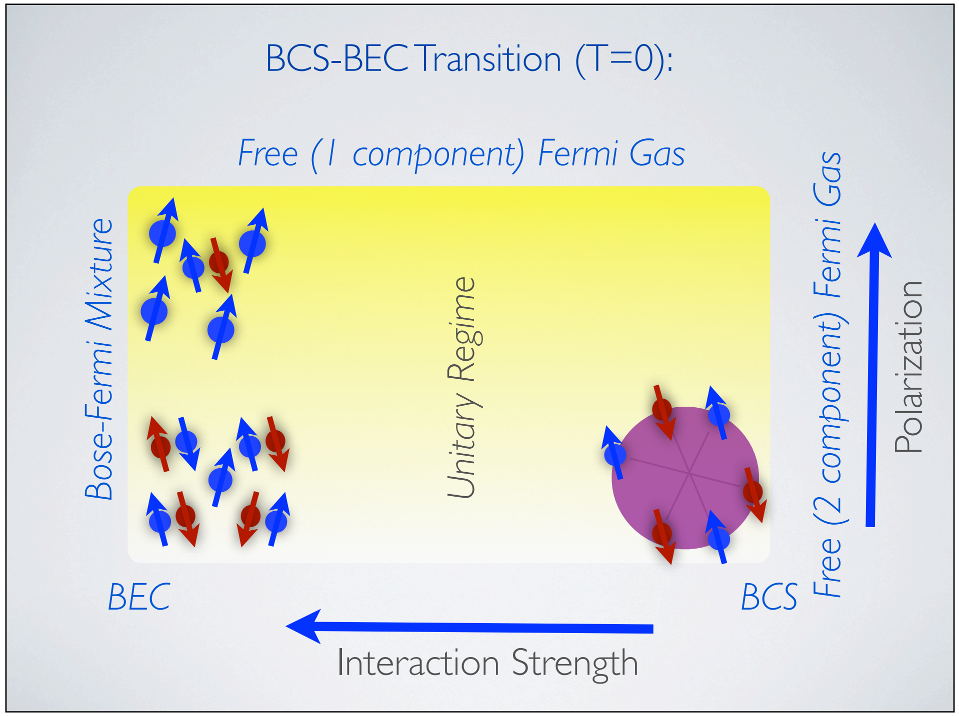
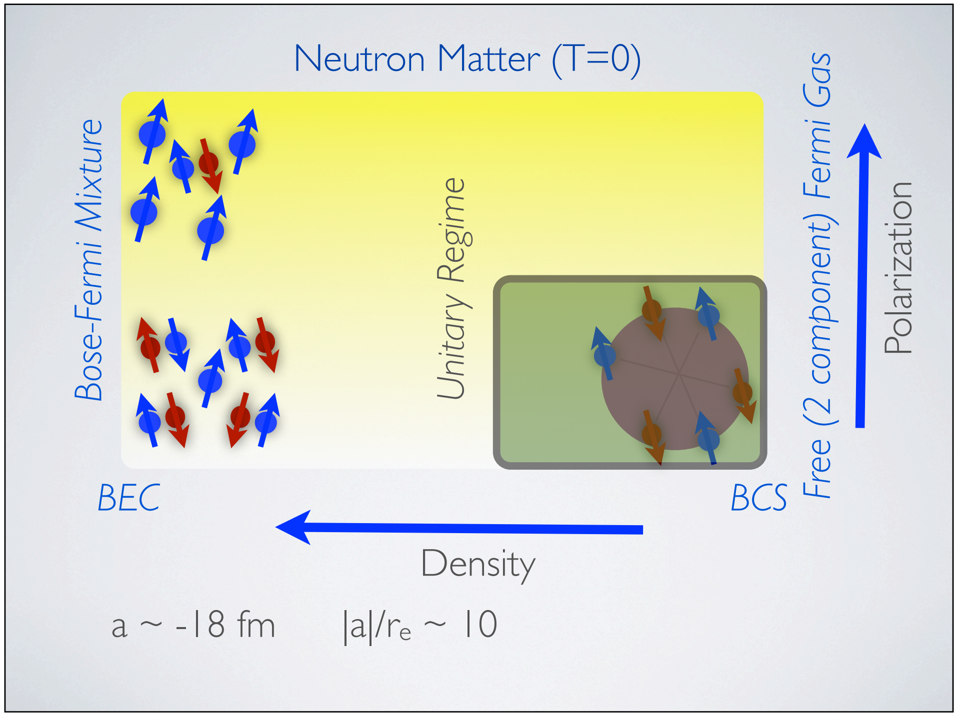


FIG. 2. Phase shifts in the $^1S_0$ channel for *np*, *nn*, and *pp* scattering, compared to various partial-wave phase-shift analyses.

Fermions: ^{6}Li, ^{40}K
Density ~ 1/ μm^3
Temperature ~ 200 nK ~ 0.1 Ef

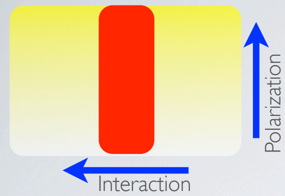
NN phase shifts ($^1S_0$)
Density ~ 1 / (10 fm)^3
T ~ 0


BCS-BEC Transition (T=0):
Free (1 component) Fermi Gas
Bose-Fermi Mixture
Unitary Regime
Free (2 component) Fermi Gas
Polarization
BEC
BCS
Interaction Strength


Neutron Matter (T=0)
Bose-Fermi Mixture
Unitary Regime
Free (2 component) Fermi Gas
Polarization
BEC
BCS
Density
a ~ -18 fm
|a|/re ~ 10

Cold Fermi Atoms: Physics Interests

- (nearly) Free Fermions
- (nearly) Free Bosons
- 'Universality' and the BCS-BEC transition
- Polarons
- Efimov States
- Superfluid Fermions (s-, p-, d-wave,... pairing)
- Exotic Polarized Superfluids (FFLO, breached pair,...)
- PseudoGap States
- Itinerant Ferromagnetism
- `Perfect' Fluids
- Reduced Dimensionality
- More than pairing (3-,4-body condensates, ...)
- Bose, Fermi Hubbard Models,
- .....



Unitarity

Unitarity = limit of 0 pair binding

a  = ∞

All quantities multiples of Fermi Gas at same ρ

At zero polarization, expect strong pairing

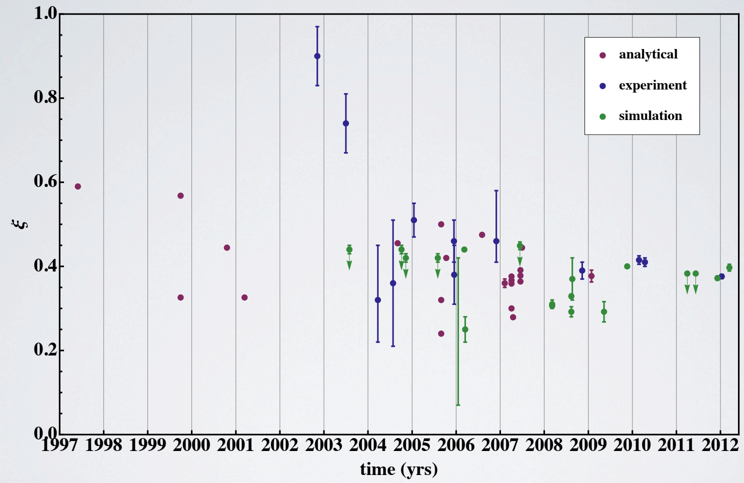
$$E = \xi \; E_{FG} = \xi \; \frac{3}{5} \; \frac{\hbar^2 k_F^2}{2m}$$

$$\Delta \;=\; \delta \; \frac{\hbar^2 k_F^2}{2m}$$

$$T_c \;=\; t \; \frac{\hbar^2 k_F^2}{2m}$$

Values of ξ, δ, t are independent of ρ

ξ: Experiments, Analytic, and Computational



Endres, Kaplan, Lee and Nicholson, arXiv:1203.3169 (2012)

Zero Temperature Simulations

Quantum Monte Carlo:

$$\Psi \ = \ \exp[-H\tau] \ \Psi_0$$
$$\Psi_0 \ = \ \Psi_{BCS} = \prod_k \ [v_k/u_k] \ a^\dagger_\uparrow(k) a^\dagger_\downarrow(-k) \ |0\rangle$$

Branching Random Walk: DMC coordinate space
AFMC orbitals

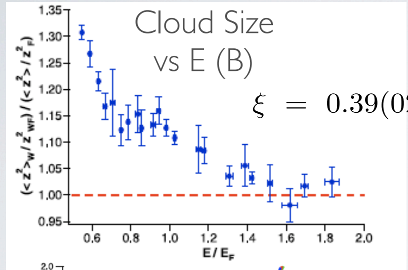
Computational requirements from workstation (Total Energy)
to largest supercomputers
(exotic superfluids)

AFMC exact for unpolarized systems (finite lattices)
good for finite temperature

DMC in continuum, but fixed-node approximation
good for polarized systems

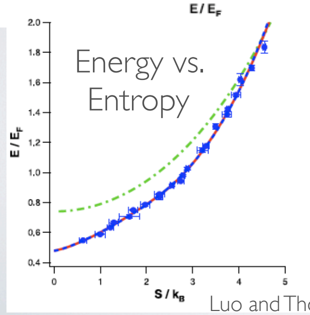
Experiments at Unitarity: # ↑ = # ↓
Cloud Size and Sound Velocity

Cloud Size vs E (B)



$$\xi \;=\; 0.39(02)$$

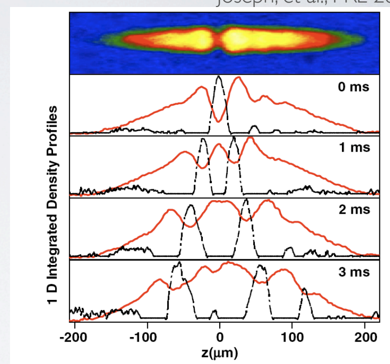
Energy vs. Entropy



Luo and Thomas, JLTP, 2009

Sound Propagation

Joseph, et al., PRL 2007

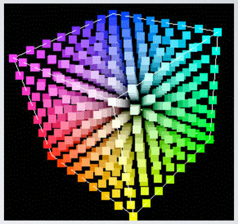


$$\frac{c_0}{v_f} \;=\; \frac{\xi^{1/4}}{\sqrt{(5)}}$$

scaling verified as ρ varied by 30

Improved Lattice (AFMC) Methods for Unitary Gas

BCS importance function
no sign problem
control of lattice size, N,
effective range



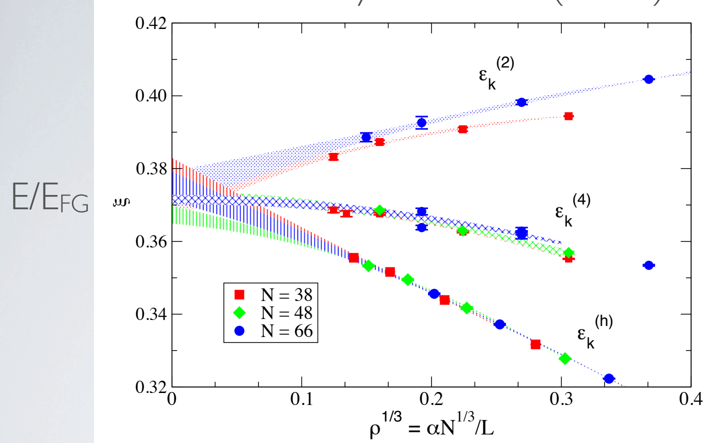
At finite (small) effective range:

$$E \,/\, E_{FG} \;=\; \xi \;+\; \mathcal{S}\; k_F\; r_e$$

ξ and $\mathcal{S}$ are universal parameters

Can measure neutron matter EOS (including effective range corrections) in cold atoms

Unitary Fermi Gas (lattice)



$\xi = 0.372(0.005)$

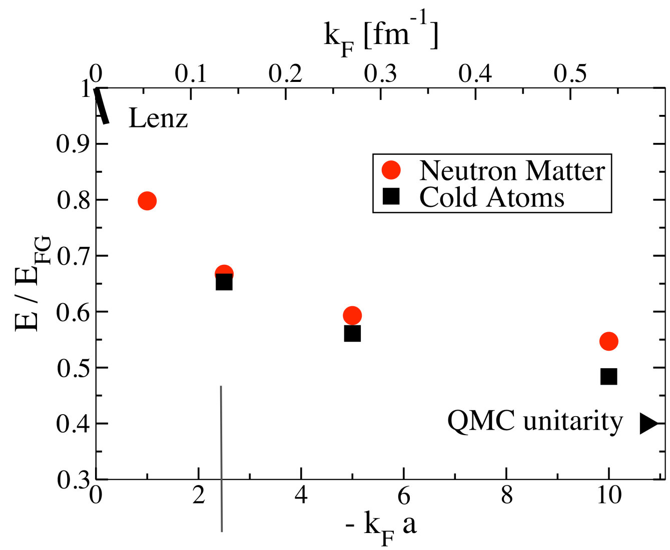
$\xi = 0.376(0.005)$ from experiment (MIT)

arXiv:1110.3309 (Hu, et al)

up to 27^3 lattice, 66 particles

K.E. Schmidt, S. Zhang, S. Gandolfi, JC, PRA 2011

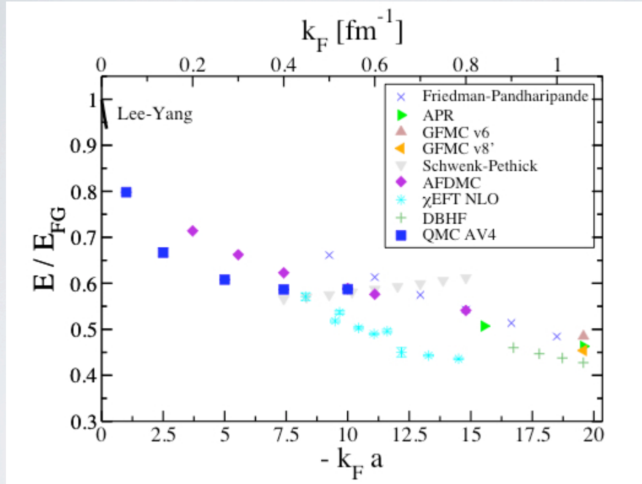
Equation of State: Cold Atoms vs. Neutron Matter



$k_F\, r_e \sim 0.3$

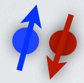
A. Gezerlis, J. C., 2008,2010

Neutron Matter EOS



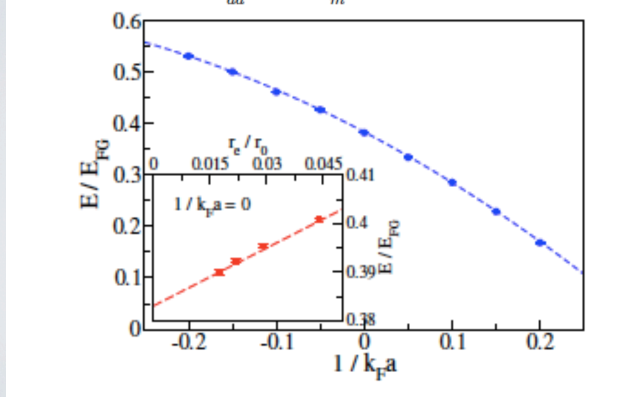
Neutron Matter EOS strongly constrained
at low-moderate densities

Contact from the EOS

Probability of  at same point

Tan, Annals of Phys. 2008

$$\frac{dE}{da^{-1}} = -\frac{\hbar^2 2\pi n A^2}{m} \to C = 8\pi^2 n^2 A^2.$$

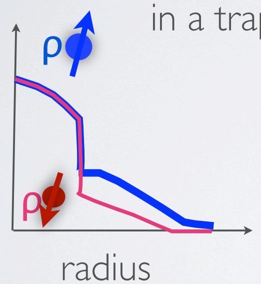


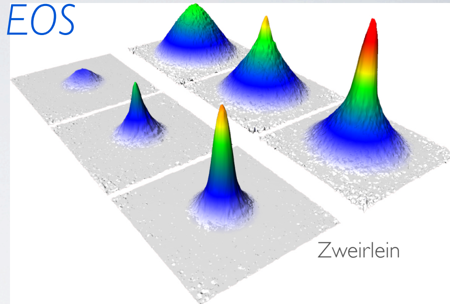
$$\frac{C}{k_F^4} = \frac{2\zeta}{5\pi} = 0.1147(3)$$

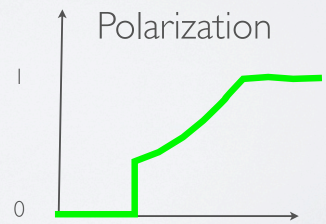
Gandolfi, et al, PRA 2011

Pairing Gap at Unitarity
from the polarized EOS

Spin up, down densities
in a trap



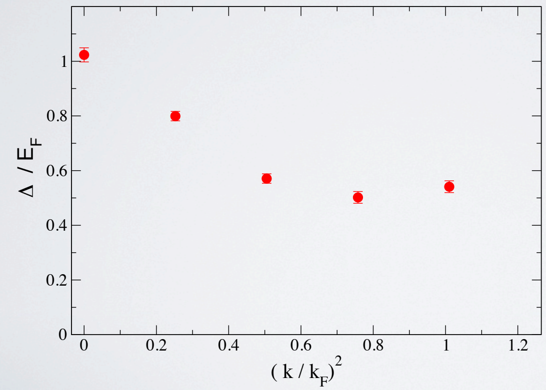




Quasiparticle Dispersion in cold Atoms

Add one to fully-paired system

Energy cost for an unpaired particle: **μ + Δ**



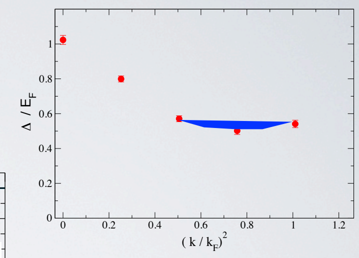
$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

$$\delta = 0.50\ (03)$$

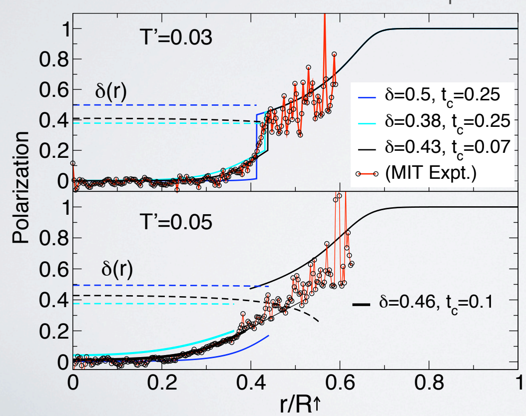
$$(k_{min}/k_f)^2 = 0.80(10)$$

JC and Reddy, PRL 2005

Pairing Gap at Unitarity - Experiment



Polarization in a trap



Largest Δ/E_f !

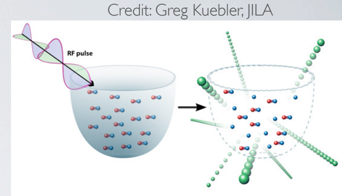
$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

$$\delta = 0.45(05)$$

JC and Reddy, PRL 2007
analyzing MIT data

Beyond the Equation of State: Structure and Dynamics

Credit: Greg Kuebler, JILA



Linear Response:

$$S(k,\omega) = \langle 0|O^{\dagger}(k)|f\rangle\langle f|O(k)|0\rangle\delta(\omega - (E_f - E_0))$$

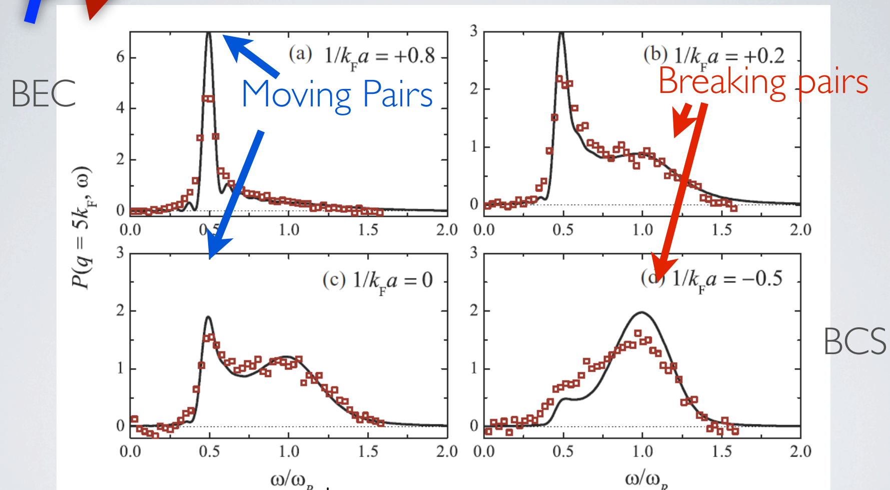
RF response (q=0): Flip atom to a new HF state

Spin response (large q): Flip atom between states and give 'kick' of momentum q

Density response (large q): Give atom `kick' of momentum q

Response sensitive to pairing gap, `contact', and more

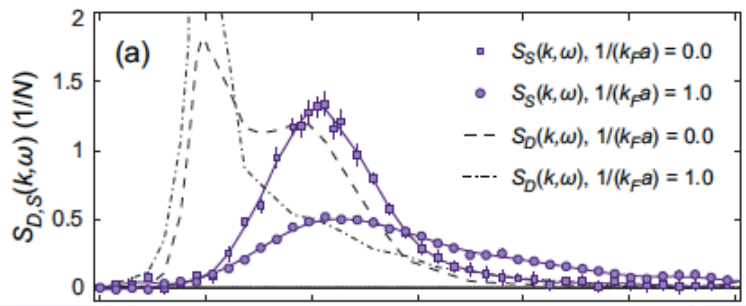
Density Response: Bragg Spectroscopy at large momentum transfer



$$S(k,\omega) = \langle 0|O^\dagger(k)|f\rangle\langle f|O(k)|0\rangle\delta(\omega - (E_f - E_0))$$

$$O = \sum_i \exp[ik \cdot r]$$

Spin Response
from Bragg Spectroscopy

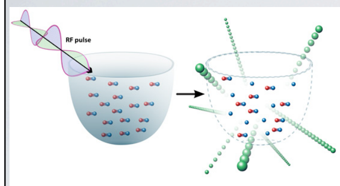


S. Hoinka, M. Lingham, M. Delehaye, and C. J. Vale, arXiv 1203.4657

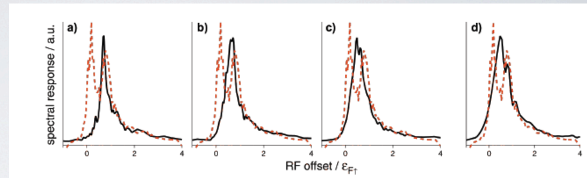
Large momentum transfer: sensitive to contact, gap,...

Spin response in neutron matter critical for neutrinos
low q important, depends upon L.S, tensor interactions

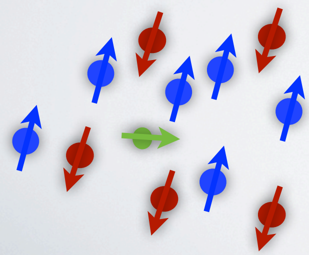
Pairing Gap at Unitarity - Experiment
RF response

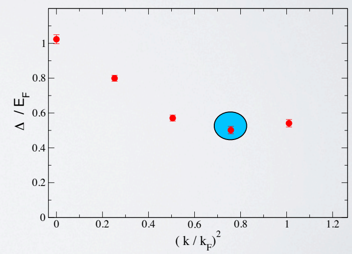


Credit: Greg Kuebler, JILA



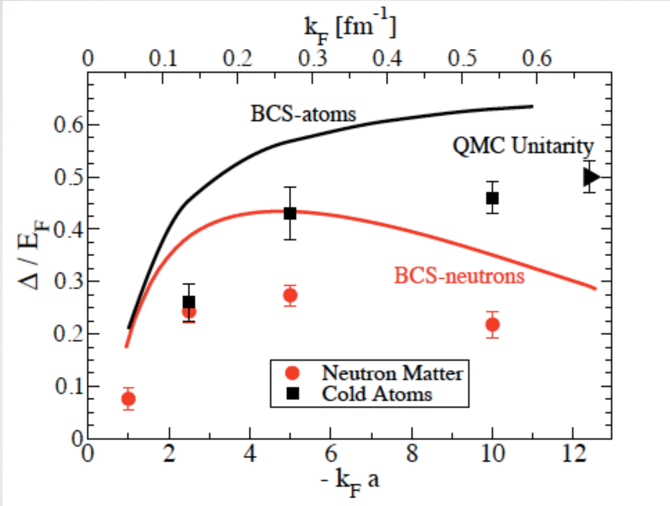
Shift of response of paired vs. unpaired atoms





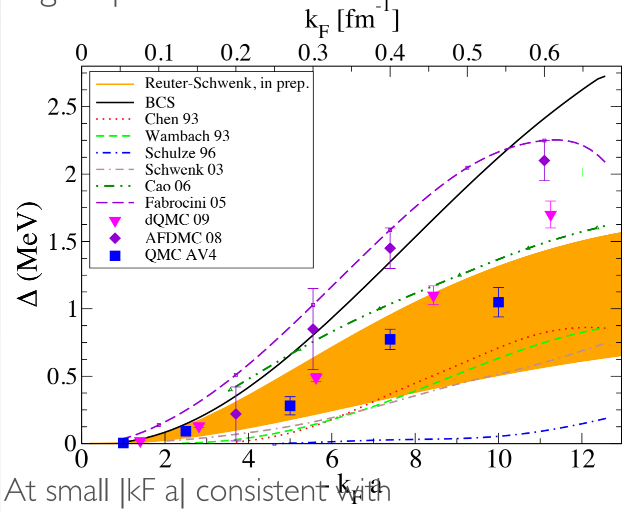
Shin, Ketterle, ... 2008

Neutron Matter and Cold Atom Pairing Gap



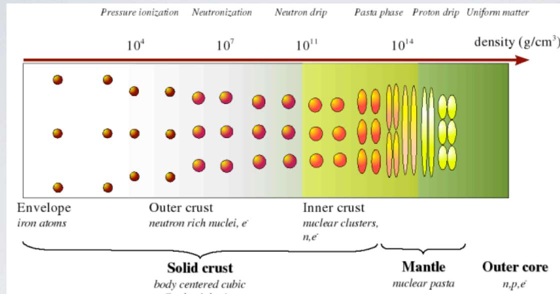
Transition from weak pairing to near unitarity

Pairing Gap: Cold Atoms and Neutron Matter

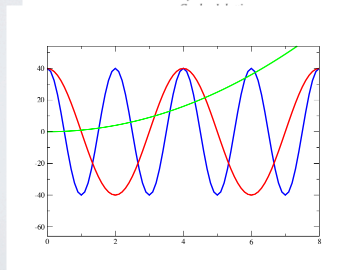


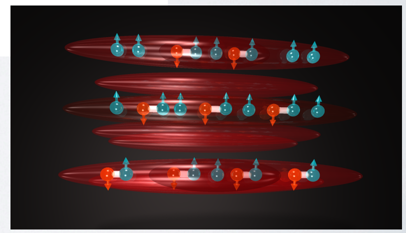
At small |kF a| consistent with
Gorkov polarization suppression of BCS
At large densities consistent with
cold atom results

Inhomogeneous Matter



Neutron Star Matter (Chamel)



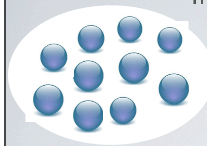


external potentials

Atoms: nearly 1D

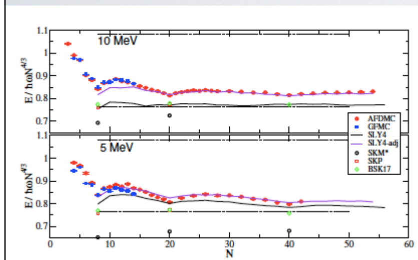
Hulet

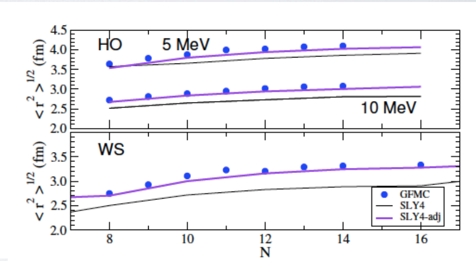
Inhomogeneous Neutron Matter



N = 6 to 50 Neutrons
Harmonic Oscillator and
Wood-Saxon external wells

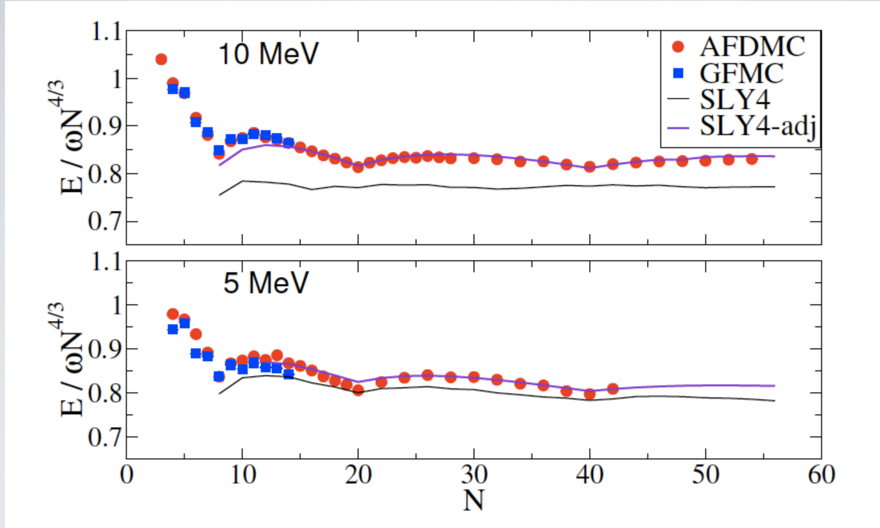
UNEDF SCIDAC





Gandolfi, et al, PRL 2011

Explore very large isospin limit of the density functional.
Examine gradient, spin-orbit, and pairing terms



Repulsive gradient terms required to fit neutron drops
also smaller spin-orbit, pairing interactions

Improved Density Functionals
Neutron Drops, Masses, Fission,...

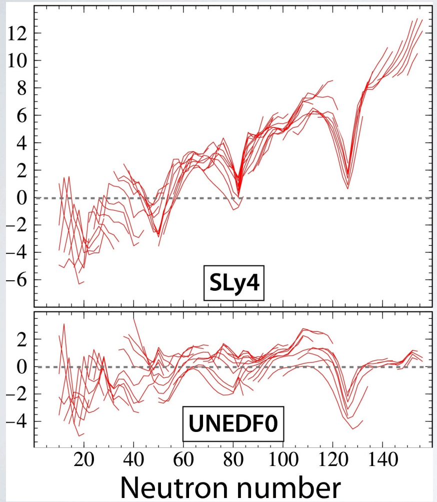
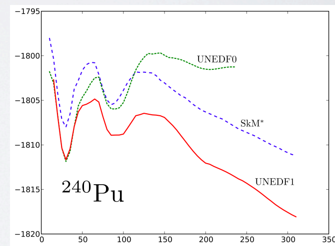
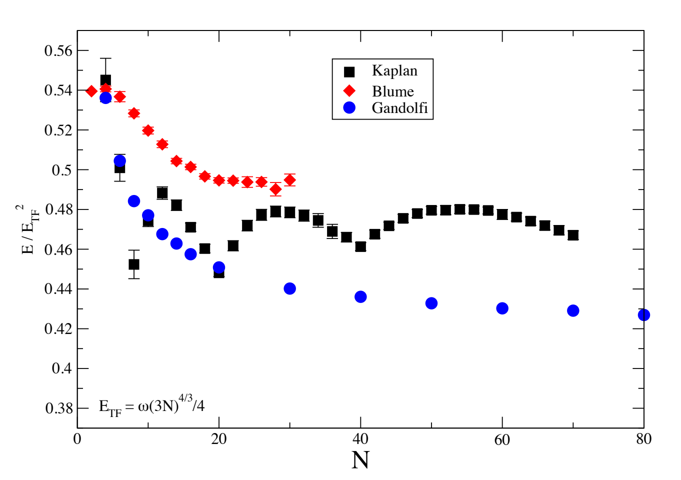


TABLE VIII: Binding energy and fission first barrier height for ^{240}Pu in units of MeV for SLy4, SkM*, UNEDF0, and UNEDF1. These are compared to the experimental value of [48].

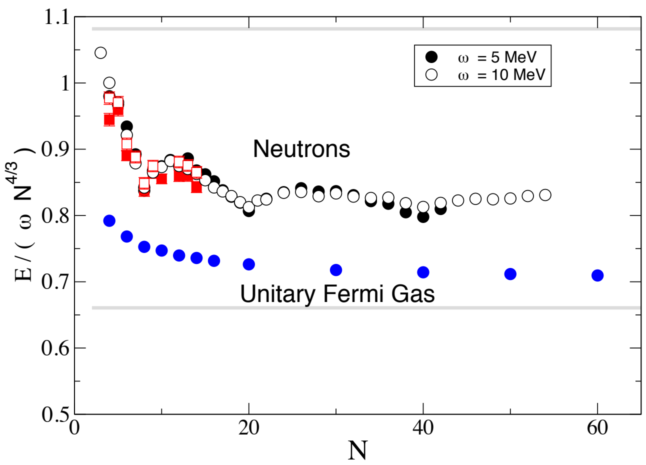
Functional	Binding Energy	First Barrier Height
SLy4	1801.5	11.9
SkM*	1804.3	9.4
UNEDF0	1811.8	9.6
UNEDF1	1811.8	6.8
Exp	1813.5	6.1



Results for Trapped Fermions at Unitarity



Trapped Neutrons and Unitary Fermi Gas



neutrons: Gandolfi, Pieper, JC, PRL 2011
atoms: Gandolfi, Gezerlis, Forbes, preliminary

Effective range impacts: EOS, shell structure, pairing gaps

Future

Transition from 3D to 2D in cold atom systems

Pairing in inhomogeneous systems in strong interaction regime

Spin/Density response at small/moderate q

Spin response in neutron matter

Additional response: viscosity,...

Low-energy excitations