Cold atoms and neutrons: variations on a theme

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Thanks to my collaborators

- Joe Carlson (LANL)
- Michael Forbes (INT)
- Stefano Gandolfi (LANL)
- Kai Hebeler (OSU)
- Thomas Lesinski (INT)
- Achim Schwenk (Darmstadt)
- Rishi Sharma (TRIUMF)
- Dam Thanh Son (INT)

This talk

Fields
- Neutron-star physics
- Cold atoms

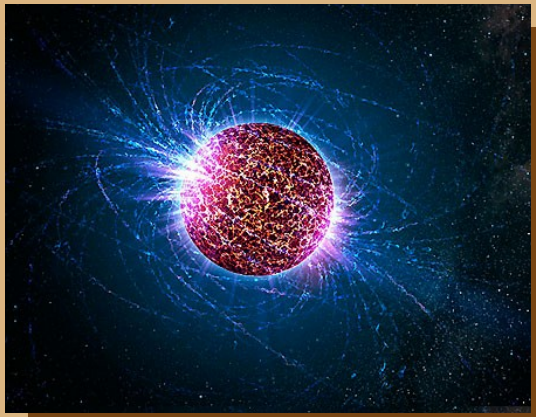
Problems
- Balanced systems
- Polarized systems / polaron
- Quantum Boltzmann particles

Methods
- Quantum Monte Carlo (QMC)
- Density Functional Theory (DFT)

Motivation: Fields

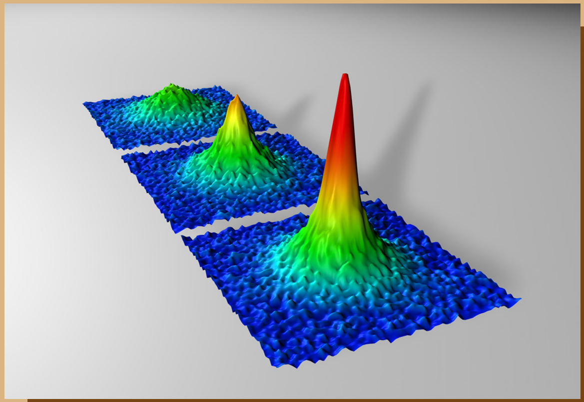
Neutron stars

- MeV scale
- $O(10^{40})$ neutrons



Cold atoms

- peV scale
- $O(10^5)$ atoms



- Very similar E/E_{FG} and Δ/E_F
- Intermediate to strong coupling

Reminder: $E_{FG} = 3/5 N E_F$, $E_F = \hbar^2 k_F^2/2m$, $\rho = g k_F^3/6\pi^2$

Motivation: Problems

Weak coupling

- $k_F a \to 0$
- Studied for decades
- Experimentally difficult
- Pairing exponentially small
- Analytically known

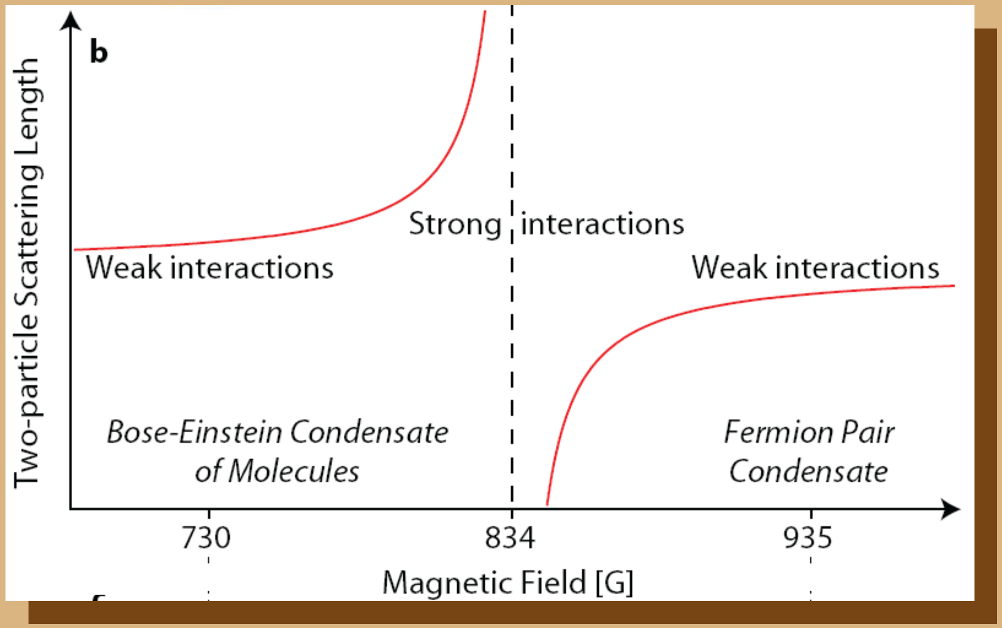
Strong Coupling

- $k_F a \to \infty$
- More recent (2000s)
- Experimentally probed
- Pairing significant
- Non-perturbative

Connection:

Using “Feshbach” resonances one can tune the coupling

Credit: Thesis of Martin Zwierlein



Motivation: Methods

Quantum Monte Carlo

- Microscopic
- Computationally demanding (3N particle coordinates + spins)
- Limited to small N

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H}-E_T)\tau}\Psi_V$$
$$\to \alpha_0 e^{-(E_0-E_T)\tau}\Psi_0$$

Density Functional Theory

- More phenomenological
- Easier (orbitals → density → energy density)
- Can do larger N

$$E = \int d^3r \, \{\mathcal{E}[\rho(\mathbf{r})] + \rho(\mathbf{r})V_{\text{ext}}(\mathbf{r})\}$$

Research Strategies

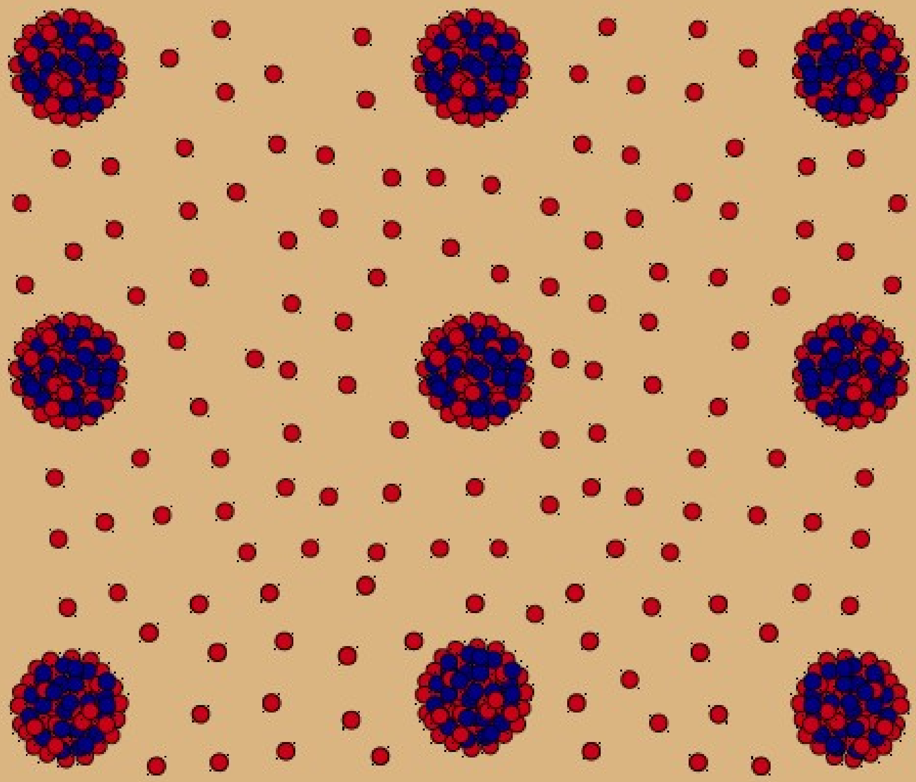
i) Use QMC as a benchmark with which to compare DFT results

ii) Constrain DFT with QMC, then use DFT to make predictions

Neutron star inner crust

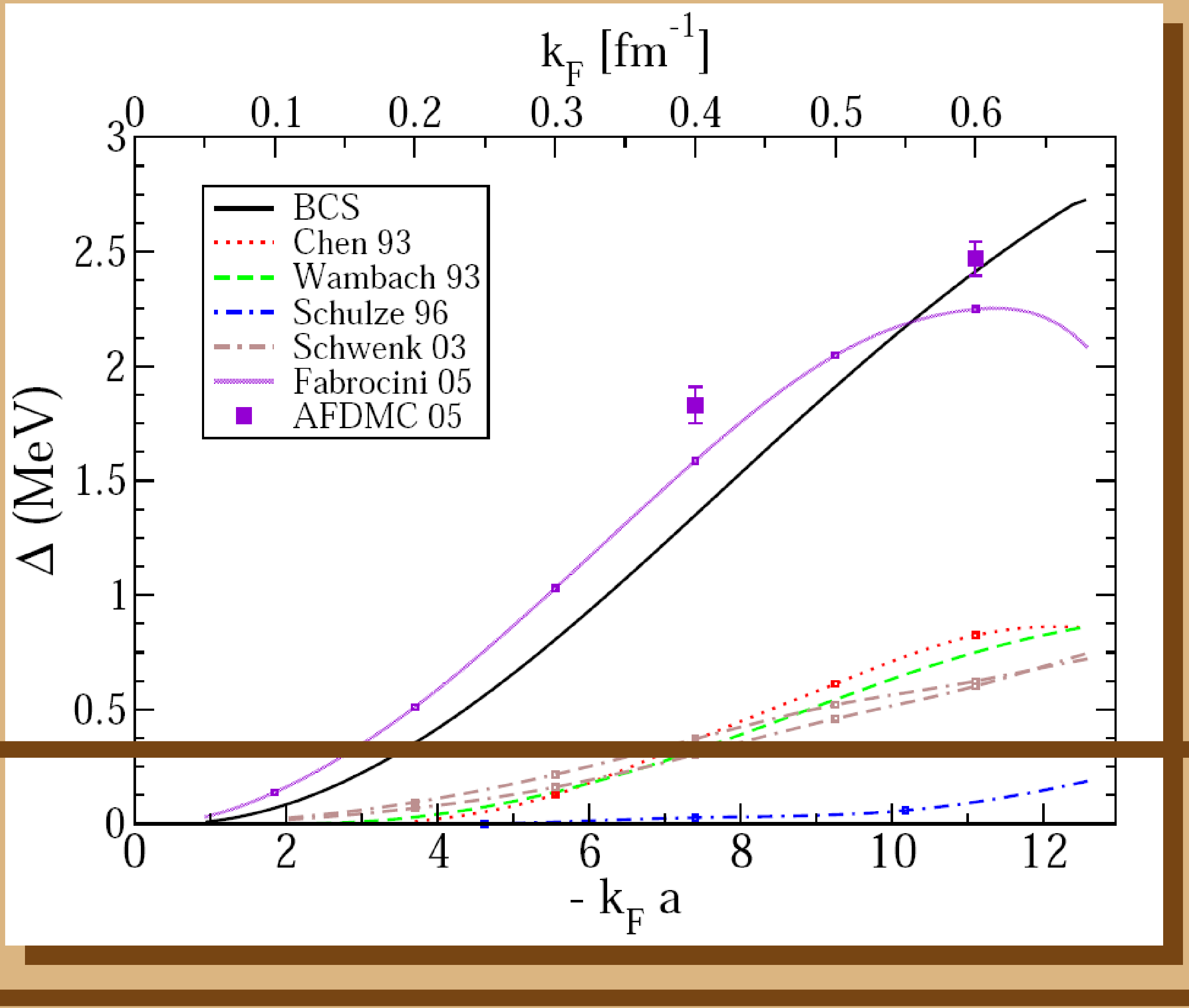
Low-density neutron matter in ion lattice

- Pairing significant: singlet in matter, possibly triplet in nuclei
- Nuclear toy problem of physical relevance



$^1S_0$ neutron matter pairing gap

No experiment → no consensus



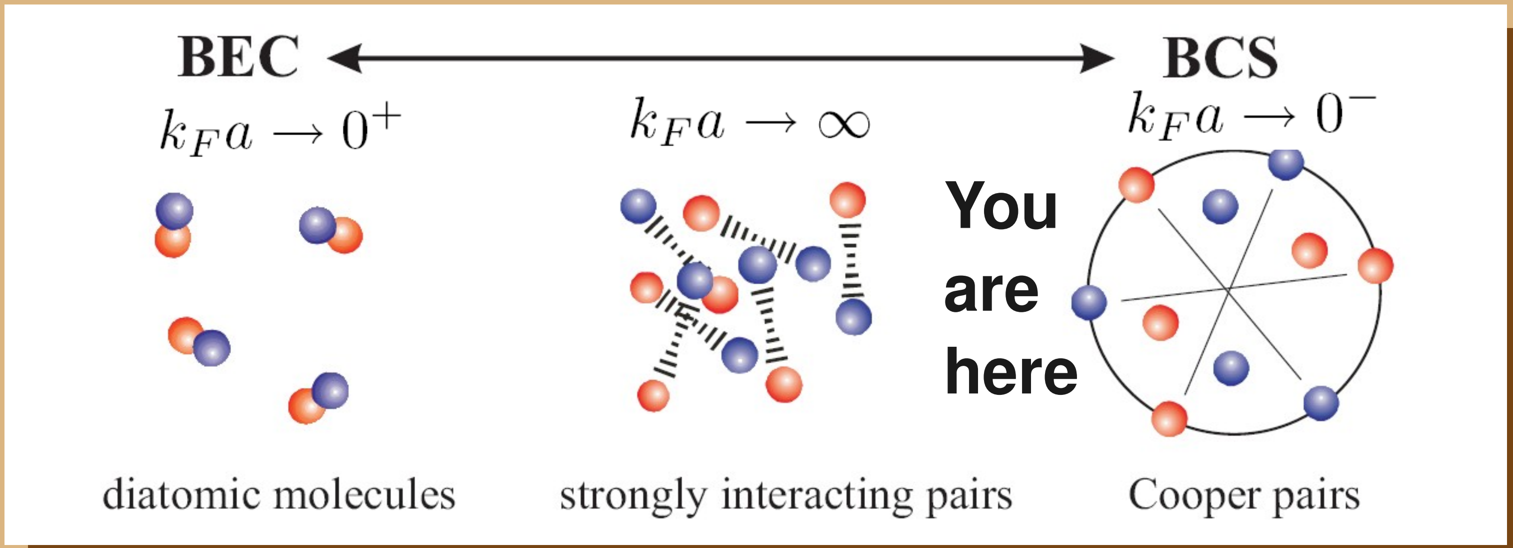
Cold atoms to the rescue

Theoretical **many-body problem formulated by George Bertsch more than 10 years ago:**

"What is the ground-state energy of a gas of spin-1/2 particles with infinite scattering length, zero range interaction?"

$$E = \xi E_{FG} \quad E_{FG} = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m}$$

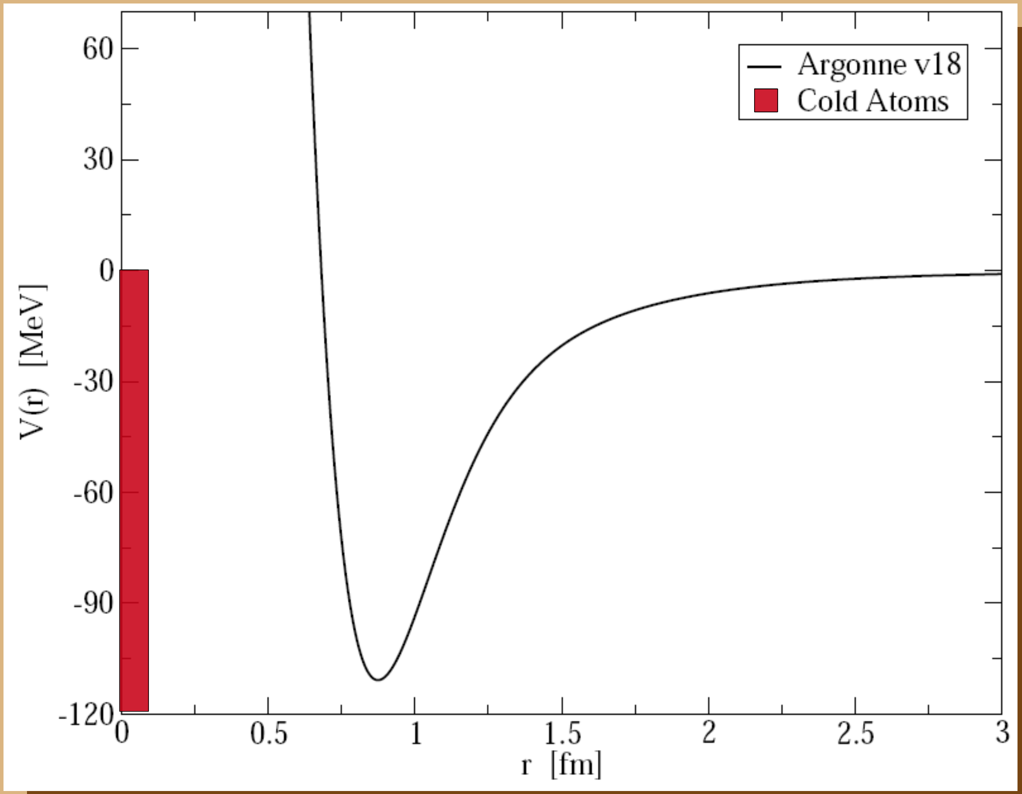
Now within *direct* experimental reach!



Credit: Ph.D. Thesis of Cindy Regal

Hamiltonian: unity in diversity

$$\mathcal{H} = -\frac{\hbar^2}{2m}\sum_{k=1}^{N}\nabla_k^2 + \sum_{i<j'} v(r_{ij'})$$



Neutron matter

$^1S_0$ channel of AV18 – later AV4

a = -18.5 fm, r_e = 2.7 fm

Cold atoms

modified Pöschl-Teller potential

a = tunable, r_e =tunable/infinitesimal

What do we know for sure?

Weak Coupling

Equation of state: $$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} k_F a + \frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a)^2$$

Pairing gap: $$\frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}} \Delta_{\mathrm{BCS}}$$

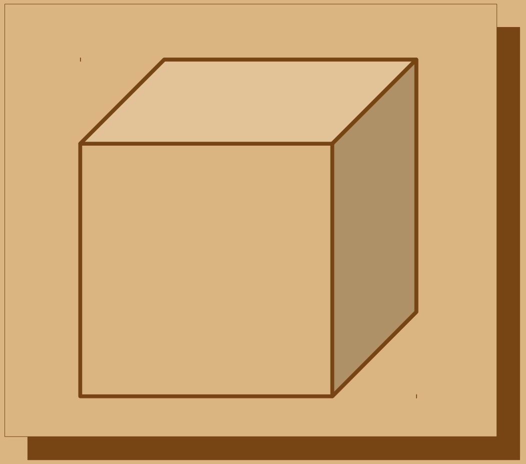
Strong Coupling

Mean-field BCS is easy but unreliable:

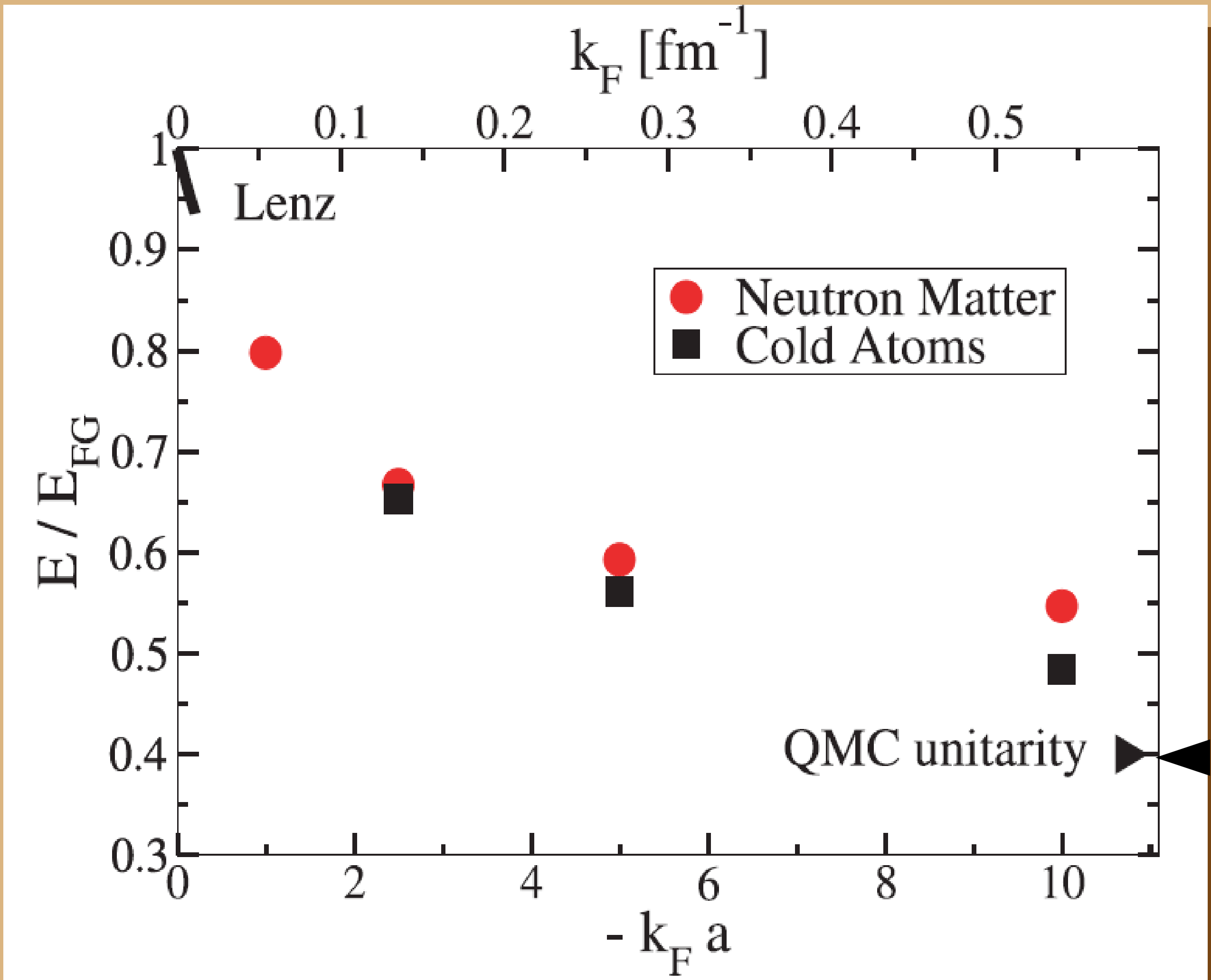
$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k} | V | \mathbf{k}' \rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}$$

Ab initio GFMC is difficult but accurate:

$$\Psi_V = \prod_{i<j} f(r_{ij}) \; \mathcal{A}[\prod \phi(r_{ij})]$$



Equations of state: results



- Results identical at low density
- Range important at high density
- Duke and ENS experiments at unitarity (current QMC and MIT experiment are lower)

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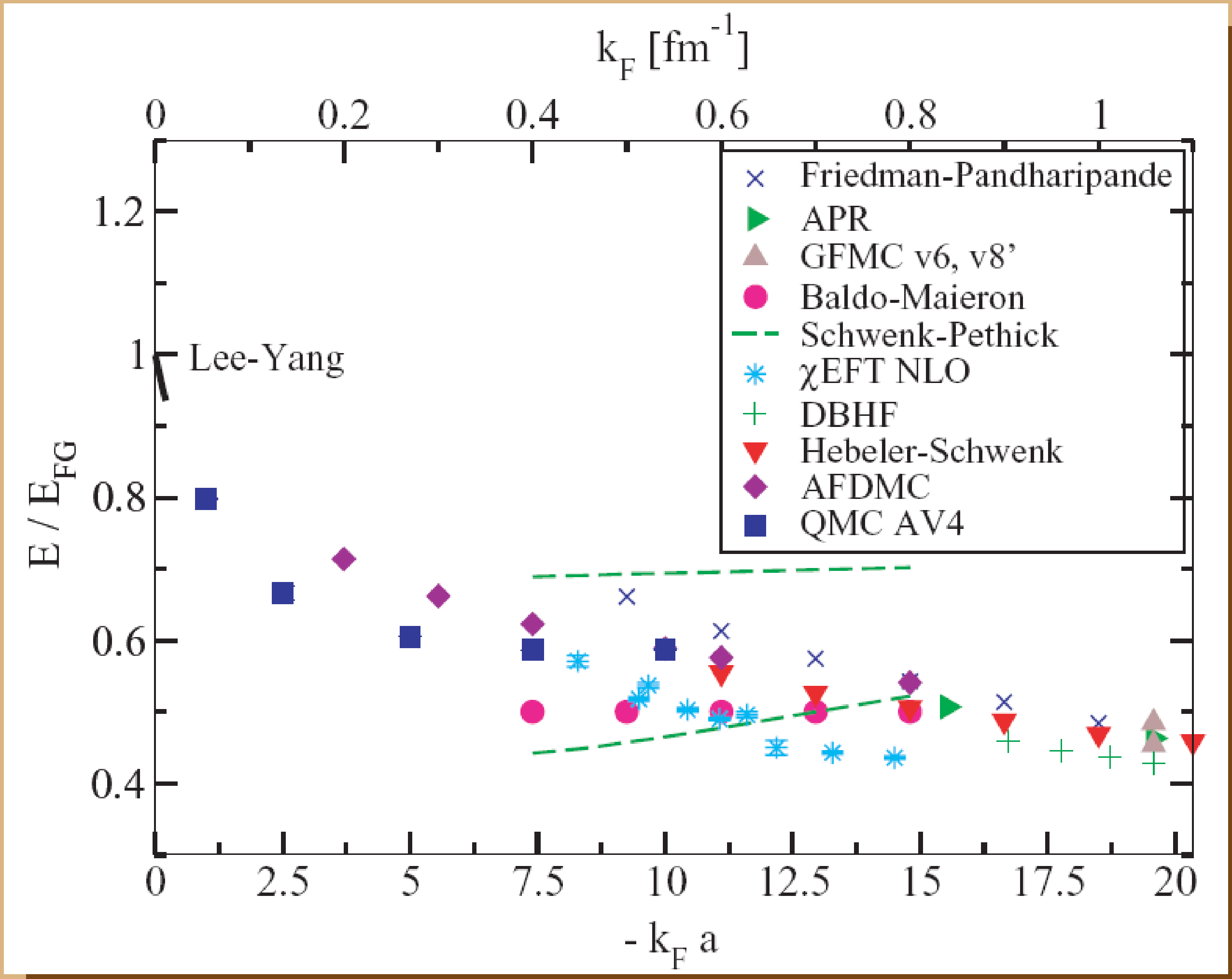
A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)

Equations of state: comparison



- Lowest densities on the market; agreement with Lee-Yang trend
- At higher densities all calculations are in qualitative agreement

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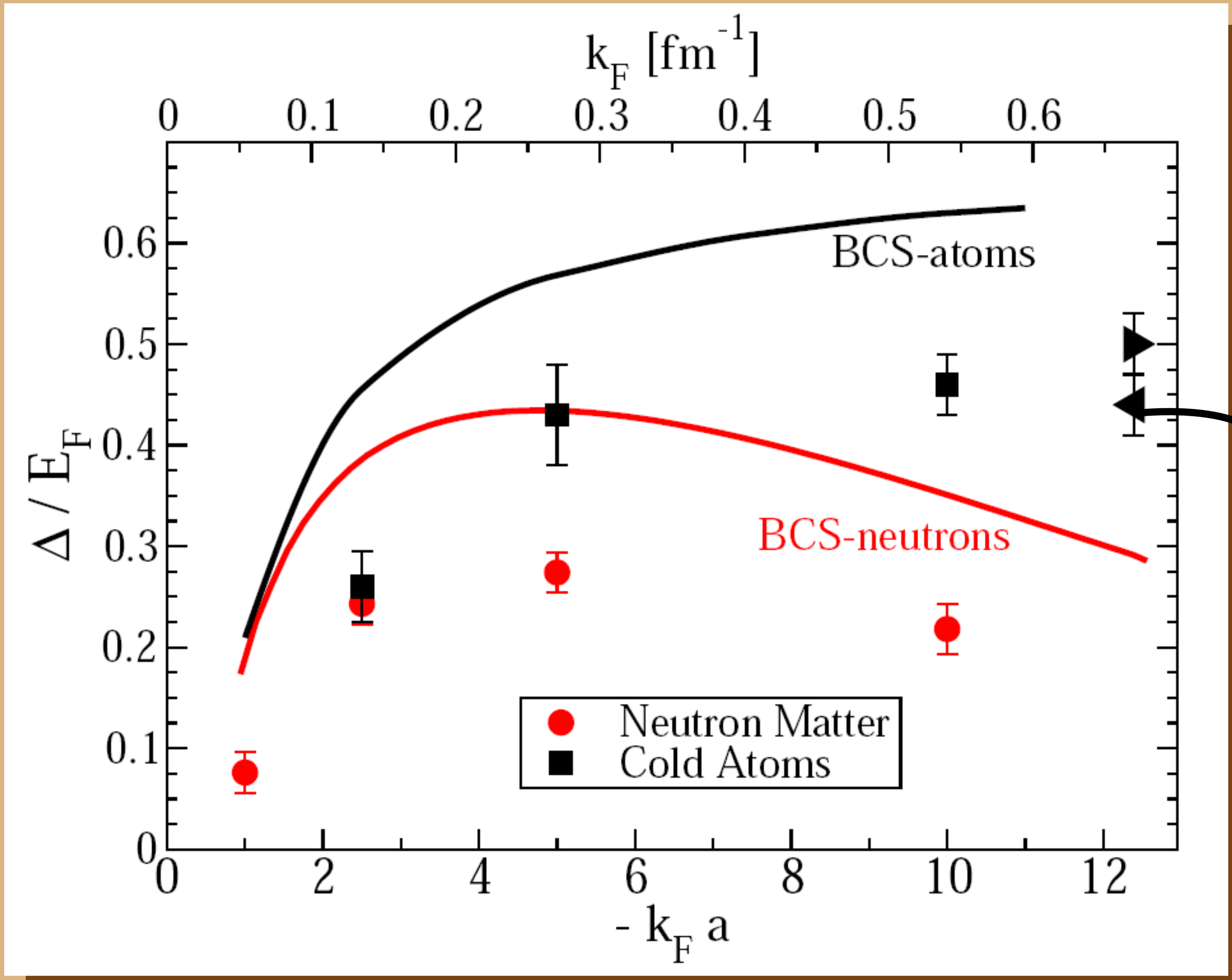
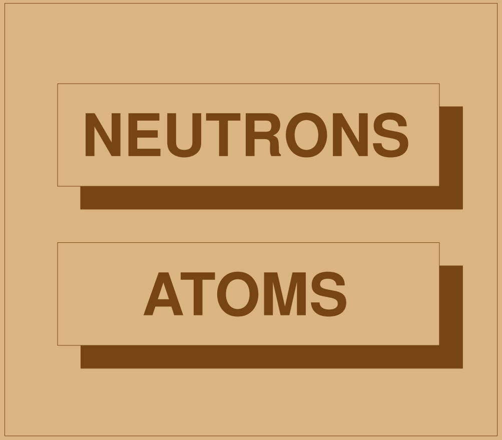
A. Gezerlis and J. Carlson, Phys. Rev. C **81**, 025803 (2010)

Pairing gaps: results



A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)

- Results identical at low density
- Range important at high density
- Two independent MIT experiments at unitarity



The DFT connection

Microscopic constraints for Skyrme functionals

Use gaps shown above:

N. Chamel, S. Goriely, and J. M. Pearson, Nucl. Phys. A **812**, 27 (2008).

Likewise for equation of state:

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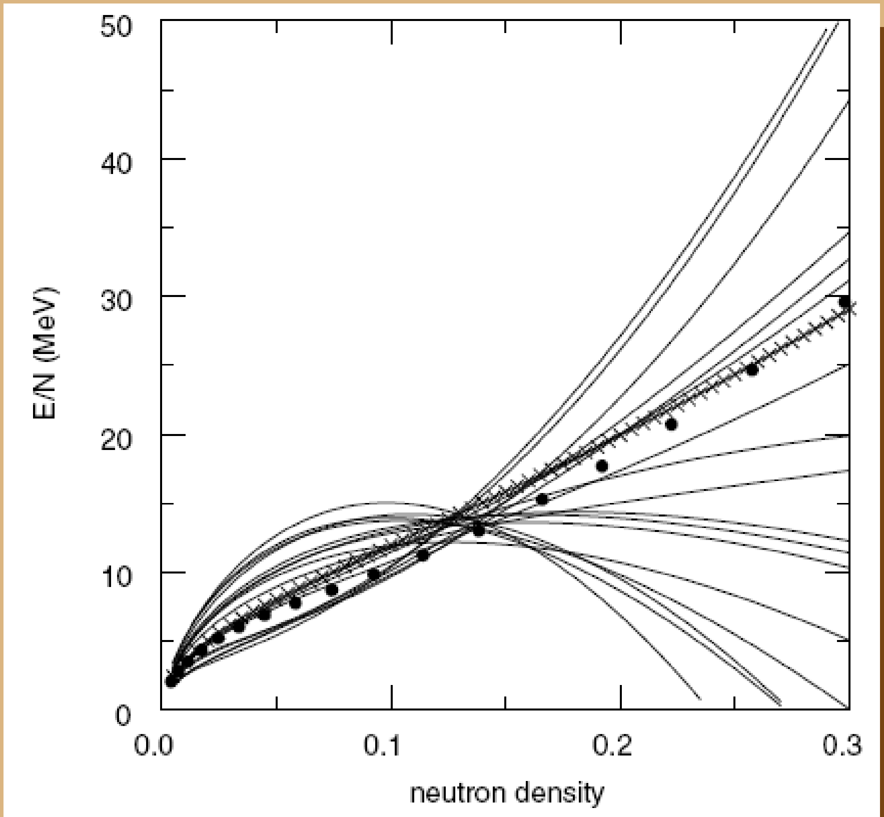


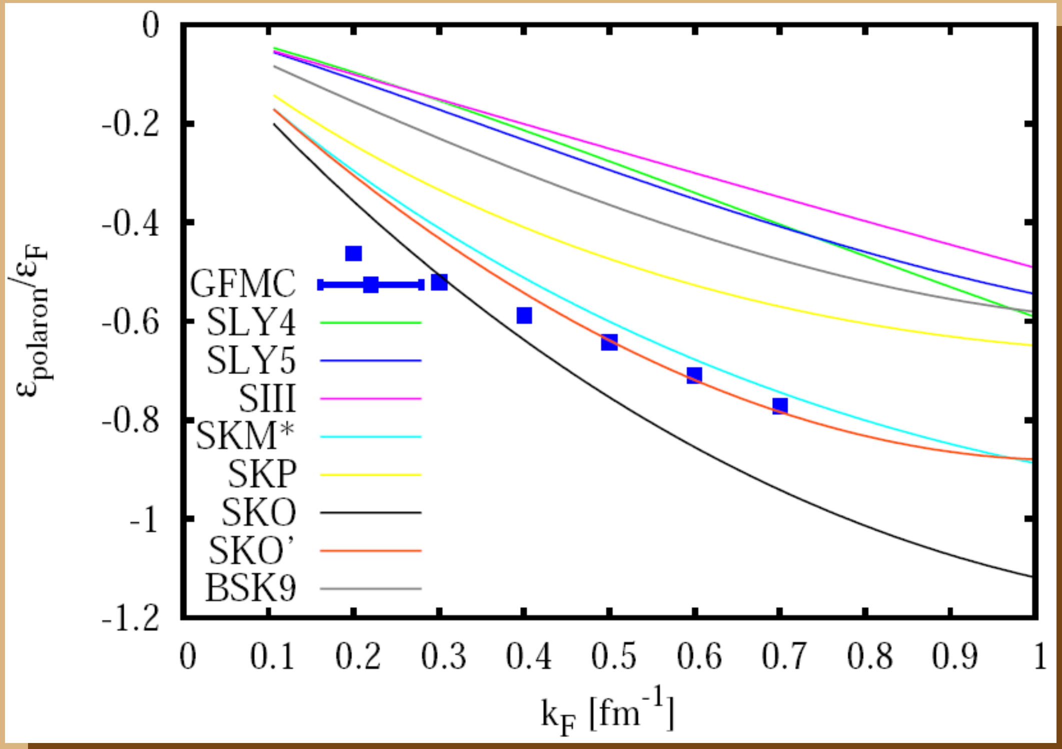
B. A. Brown, Phys. Rev. Lett. **85**, 5296 (2000).

The neutron polaron

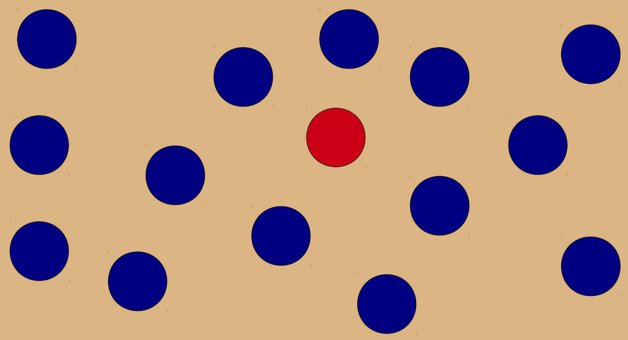
New constraint: extreme case of one impurity

Landau-Pomeranchuk $x = N_\downarrow / N_\uparrow$

$$E = \frac{3}{5} N_\uparrow E_{F\uparrow} \left(1 - Ax + \frac{m}{m*} x^{5/3} + F x^2\right)$$



M. M. Forbes, A. Gezerlis, K. Hebeler,
T. Lesinski, A. Schwenk, *in preparation* (2012)



- Microscopic benchmark
- New functional needed

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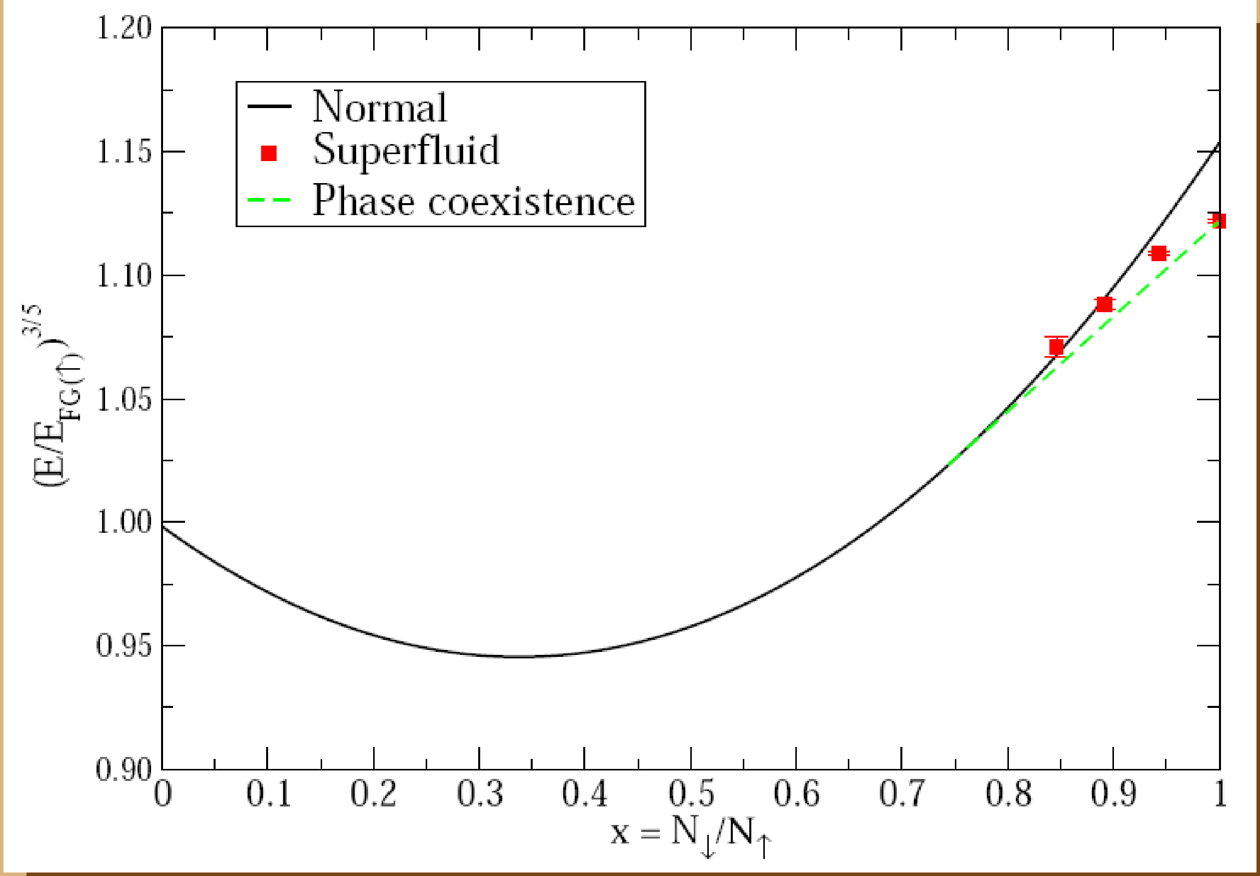
General case: phase separation



- Polaron is near x=0
- For most concentrations normal gas is favored
- Near balanced gas phase separation is lower in energy than homogeneous superfluid

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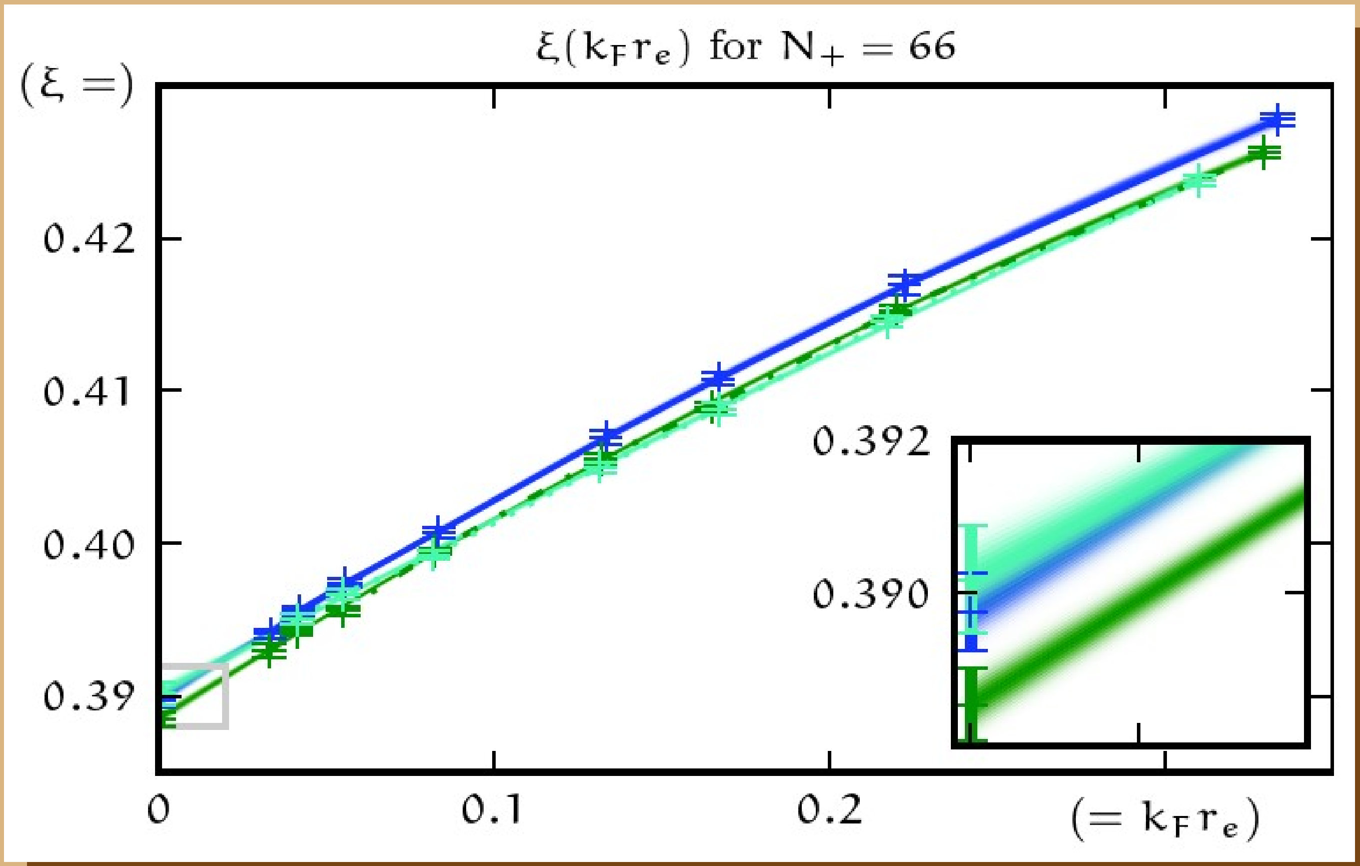
A. Gezerlis and R. Sharma, Phys. Rev. C **85**, 015806 (2012)

A. Gezerlis, Phys. Rev. C **83**, 065801 (2011)

Back to atoms: new QMC results

Carefully extapolated to zero effective range



We have:

- analyzed dependence on particle number for the first time
- used new potential with repulsive core
- carefully extrapolated to zero range

M. M. Forbes, S. Gandolfi, A. Gezerlis,
to be submitted to Phys. Rev. A (2012)

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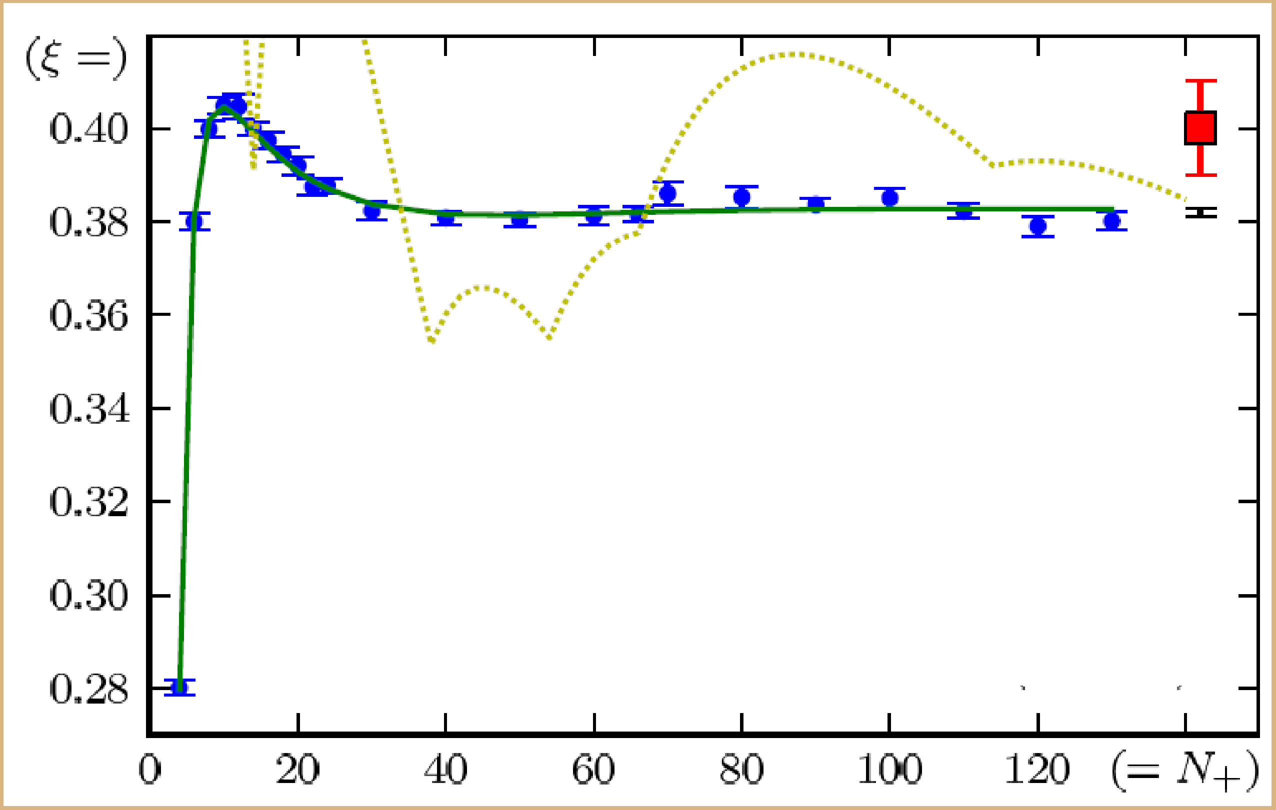
New DFT results

QMC + SLDA in a box



- No gradient corrections to be accounted for
- Allows us to reach the thermodynamic limit
- No pairing term in the functional leads to large shell corrections

M. M. Forbes, S. Gandolfi, A. Gezerlis,
Phys. Rev. Lett. **106**, 235303 (2011)

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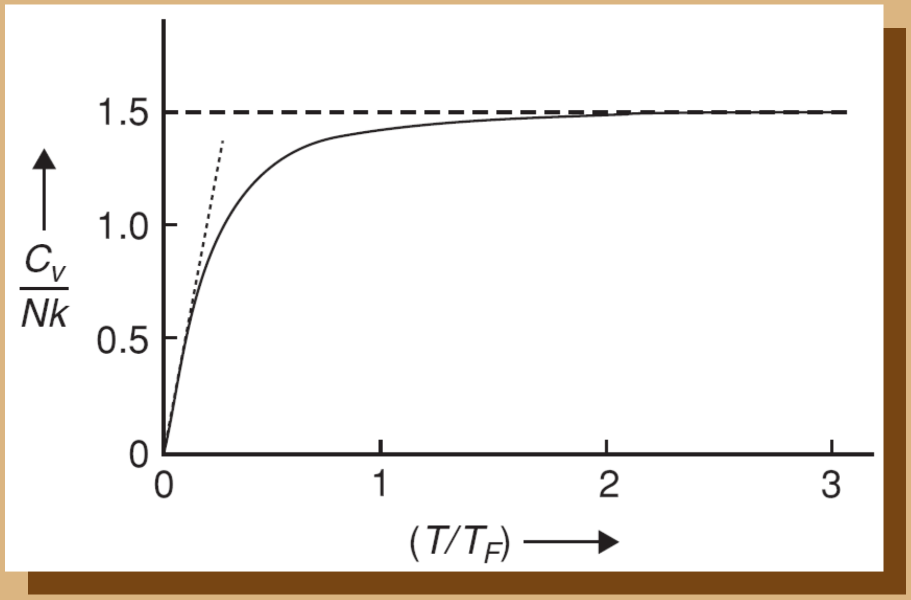
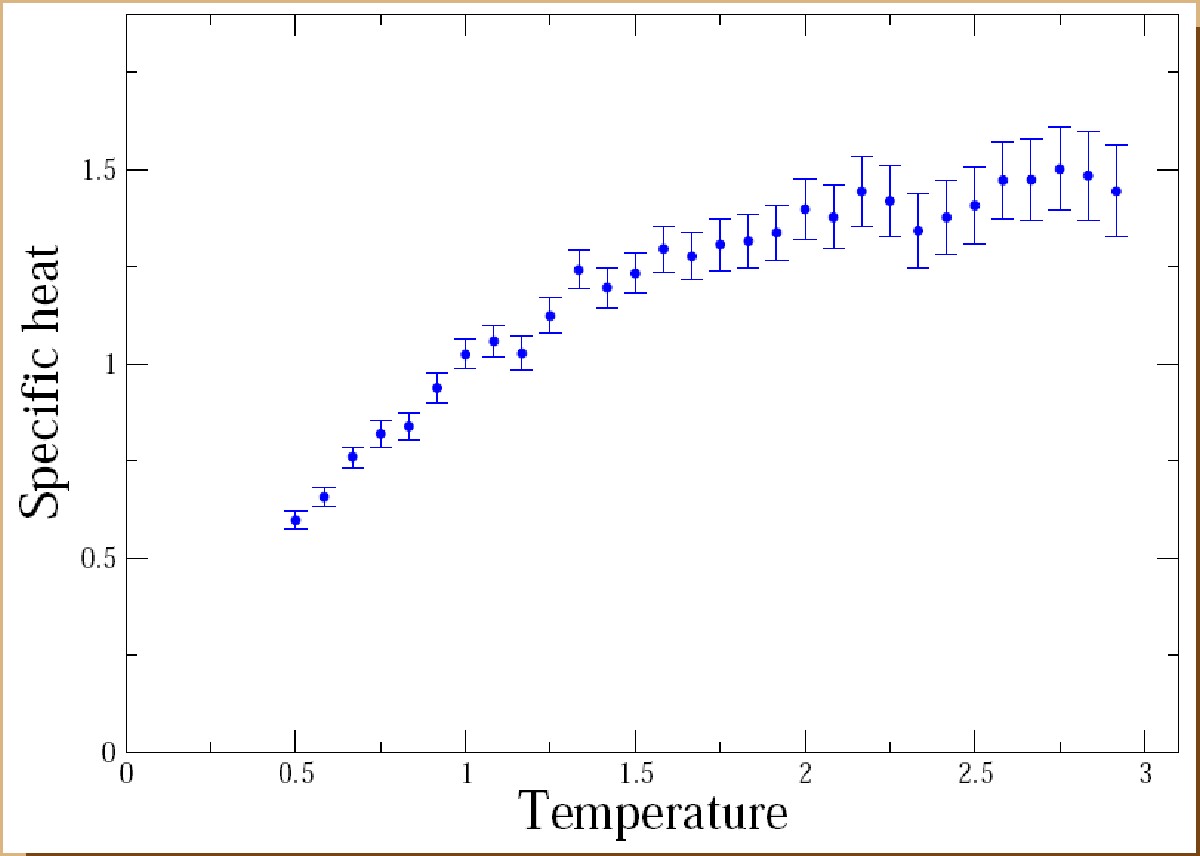
Specific heat near zero temperature

- Classically $$C_V = \frac{3}{2} N k_B$$
- Ideal Bose gas $C_V \propto T^{3/2}$ (experimentally $C_V \propto T^3$)
- Ideal Fermi gas $C_V \propto T$



“Large N_c” limit in cold atoms

Quantum Boltzmann particles: hard spheres



A. Gezerlis, J. Carlson, and D. T. Son,
In preparation (2012)

- Relevant to many-hyperfine-state experiments
- (Exact) Path Integral Monte Carlo
- Classical limit recovered

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Conclusions

- Cold-atom experiments can constrain nuclear theory
- Neutron matter calculations impact both neutron-star phenomenology and heavy nuclei fits
- Phenomenology (DFT) and *ab initio* (QMC) are mutually beneficial

The Present Future

- EFT for QMC
- Highly asymmetric nuclear matter
- 3-species fermionic systems: Efimov physics
- Particles in an external potential: static response