FLUID MECHANICS GROUP AT uc3m

WORKSHOP ON HIGH ENERGY DENSITY PHYSICS OPPORTUNITIES AT FAIR

• UNIV. POLITÉCNICA DE MADRID • NOVEMBER 18TH, 2022 •

SHOCK WAVES DYNAMICS FOR ARBITRARY EQUATIONS OF STATE.
A THEORETICAL DESCRIPTION

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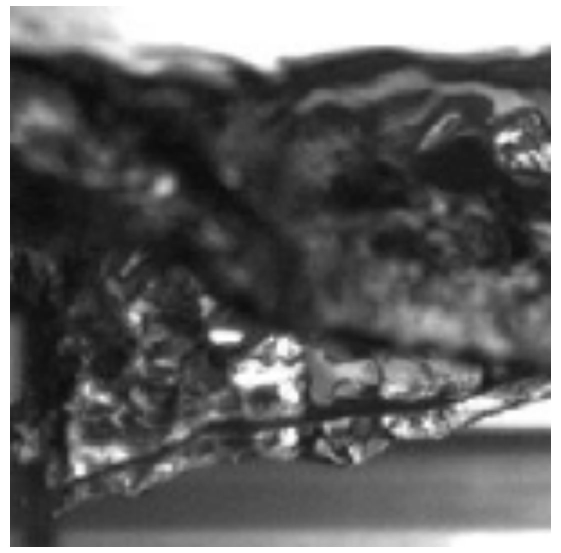
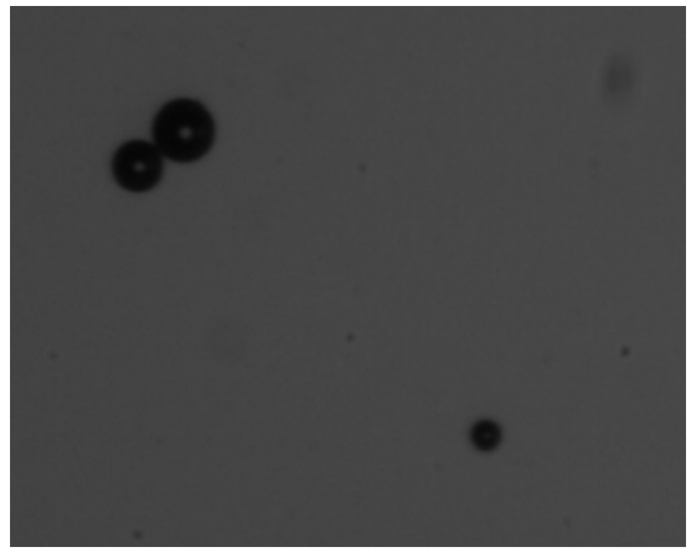
Fernando García-Rubio Laboratory for Laser Energetics at **University of Rochester**
Andrés Calvo-Rivera Fluid Mechanics Group at **Univ. Carlos III de Madrid**
Alexander L. Velikovich Plasma Physics Division at **US Naval Research Laboratory**

Multidisciplinary group that works on great variety of topics:

3 Full professors • 5 Associate professors • 4 Assistant professors • 6 PhD students



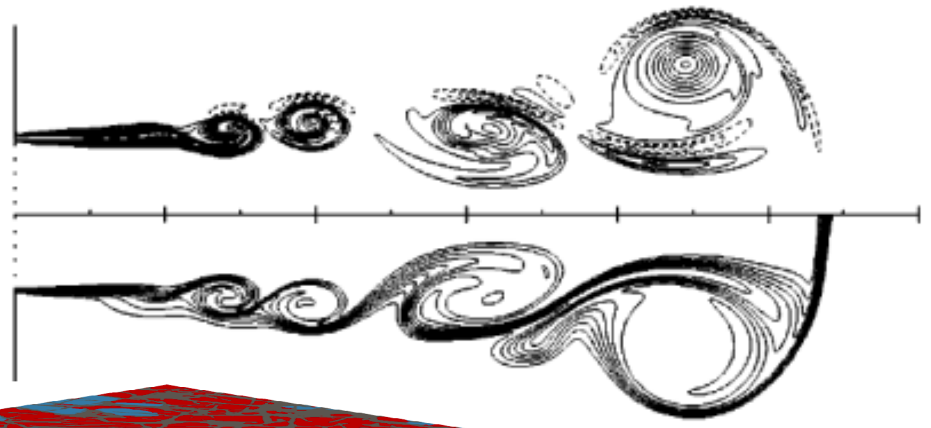


Multiphase Flows

Turbulent multiphase flows and plunging waves

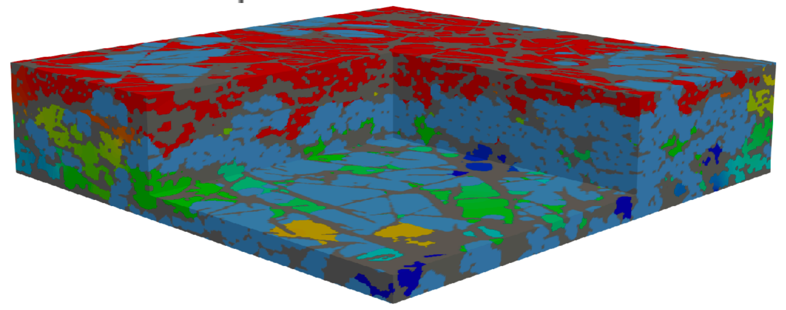
Biomedical Fluid Dynamics

Experimental, numerical and theoretical studies on ultrasound-microbubble interactions



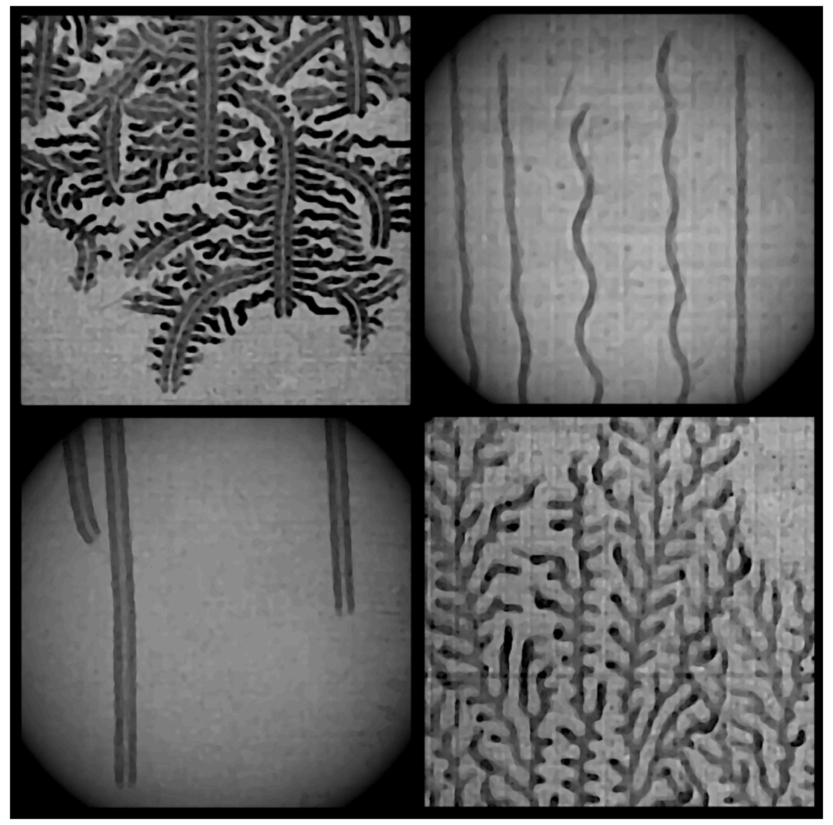
Jets & Wakes

Structure and stability properties of jets and wakes in technologically relevant contexts



Microfluidics

Microfluidics and Transport in Porous Media: PEM Fuel Cells and Flow Batteries

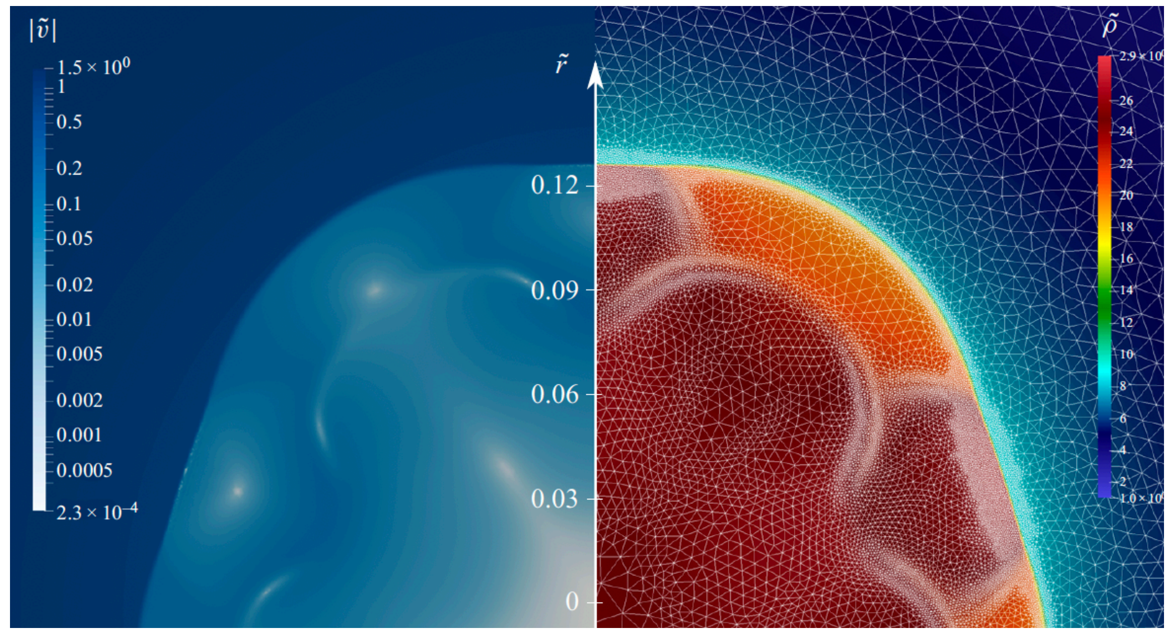


Sustainable Combustion

Reactive flows: hydrogen combustion, ignition, chemical kinetics.

Compressible flows with application to hypersonics and HEDP

Shock waves, reactive supersonic fronts, and magnetized flows



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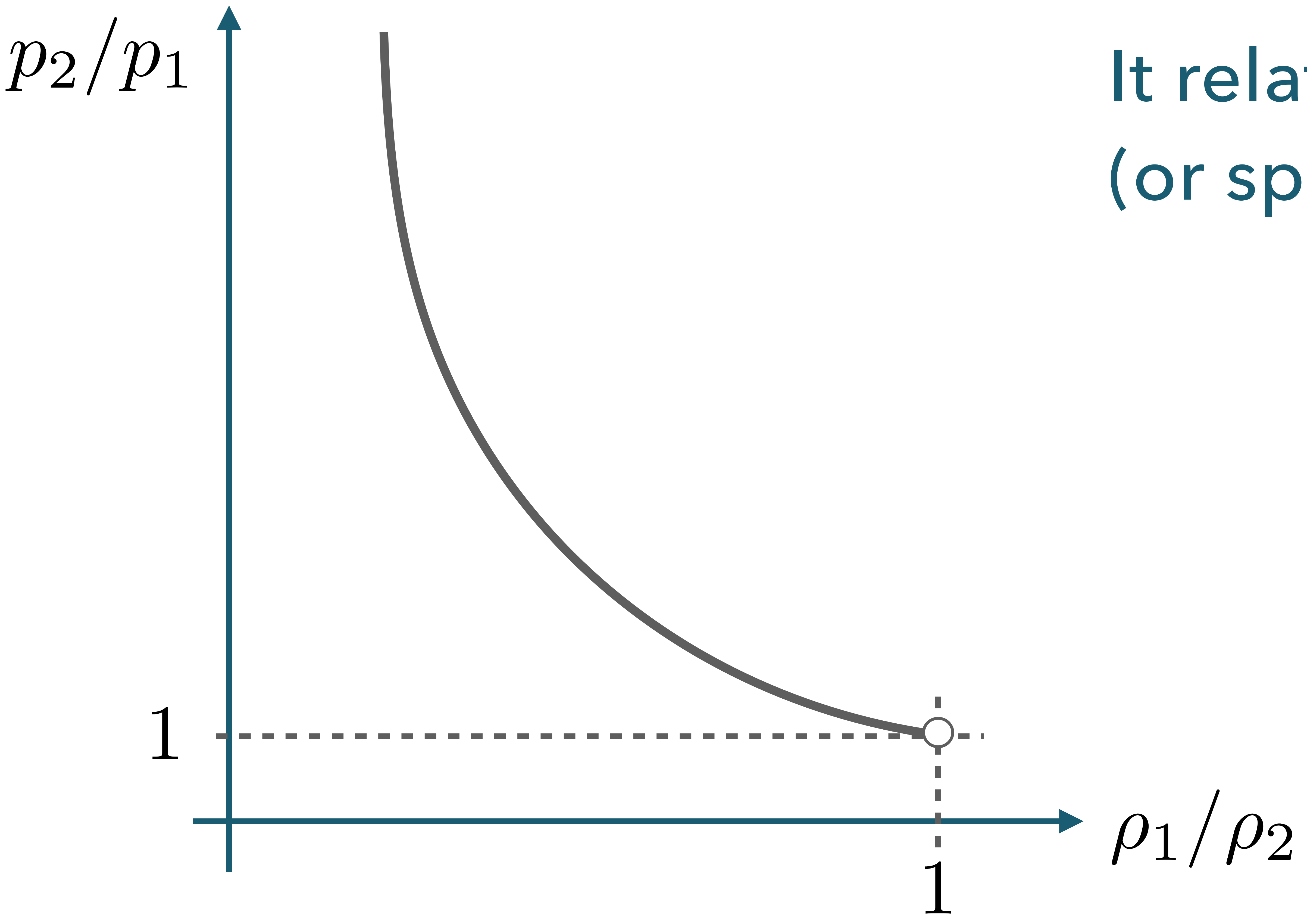
DETONATIONS

HYPERSONIC SHOCKS

MAGNETIZED SHOCKS

What is the DK instability?

Let's begin with the well-known Rankine-Hugoniot curve



It relates the pressure jump with the density (or specific volume) across the shock.

$$\left.\frac{p_2}{p_1}\right|_{\mathrm{RH}} = \mathrm{fun}(\rho_2/\rho_1, \mathrm{EoS\ parameters})$$

Each point corresponds to a given shock intensity (Mach number), for some given preshock conditions.

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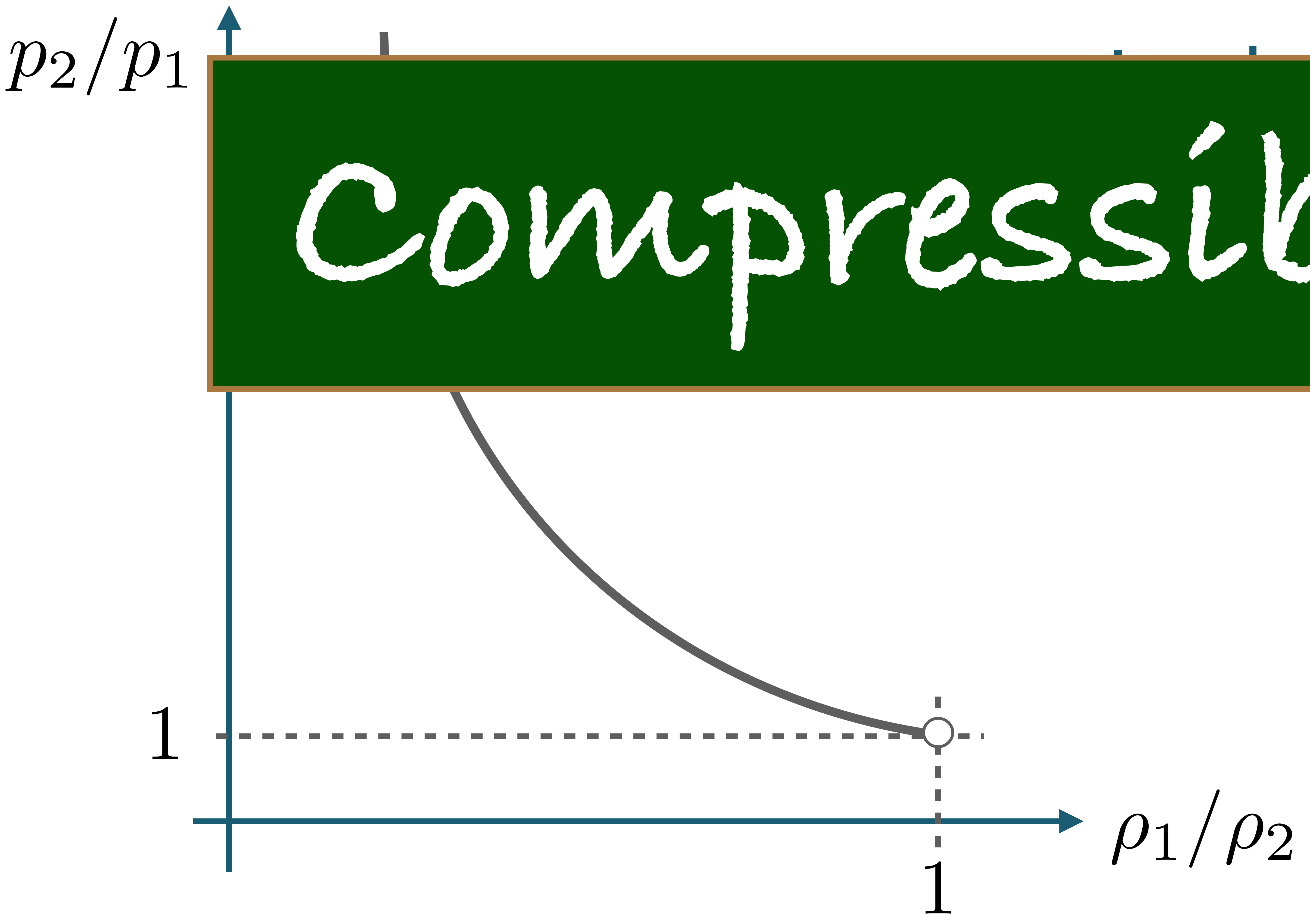

Compressible flow 101

...ensity

$p_1 \big|_{\mathrm{RH}}$ ...ers)

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A. Calvo-Rivera C. Huete (speaker)
F. García-Rubio A. L. Velikovich
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U.S. NAVAL RESEARCH LABORATORY

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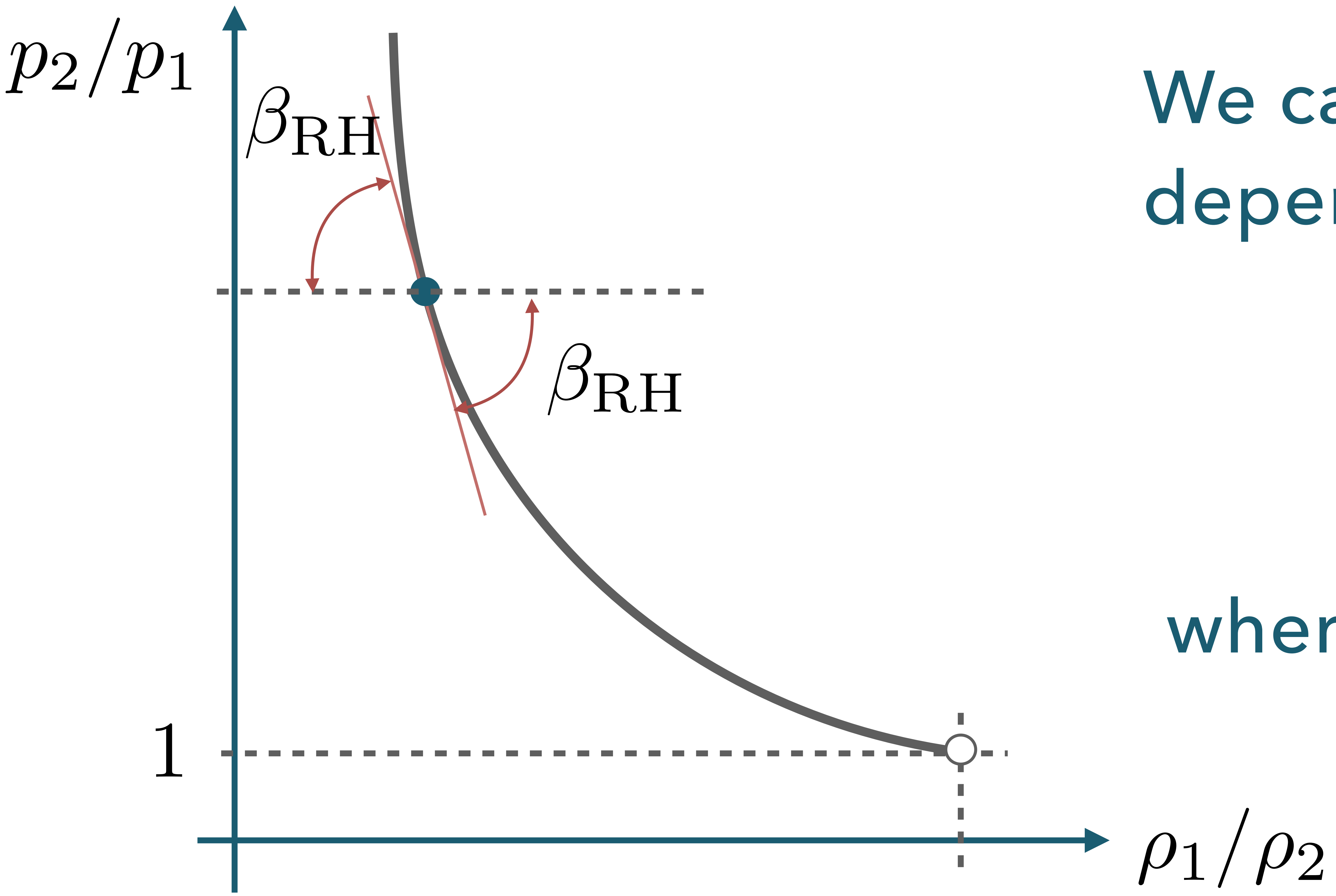

We can define the associated slope that depends on the selected Mach number

$$\tan \beta_{\mathrm{RH}} = \frac{\mathrm{d}(p_2/p_1)}{\mathrm{d}(\rho_1/\rho_2)}$$

where $\left.\frac{p_2}{p_1}\right|_{\mathrm{RH}} = \mathrm{fun}(\rho_2/\rho_1, \mathrm{EoS\ parameters})$

What is the DK instability?

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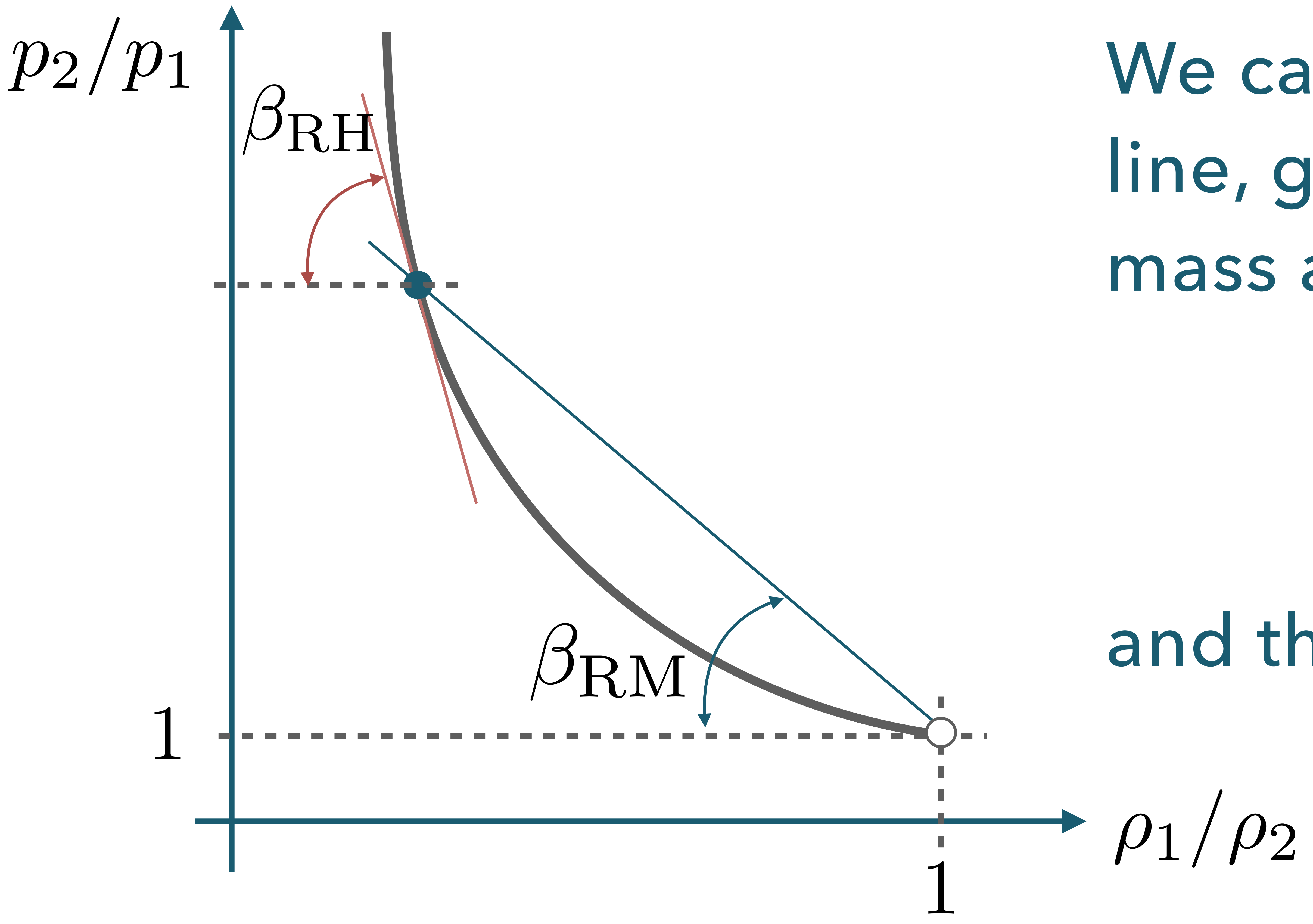


We can also define the Rayleigh-Michelson line, given by direct combination of the mass and momentum conservation eqs.

$$\frac{p_2}{p_1} = 1 + \frac{\rho_1 a_1^2}{p_1} M_1^2 \left(1 - \frac{\rho_1}{\rho_2}\right)$$

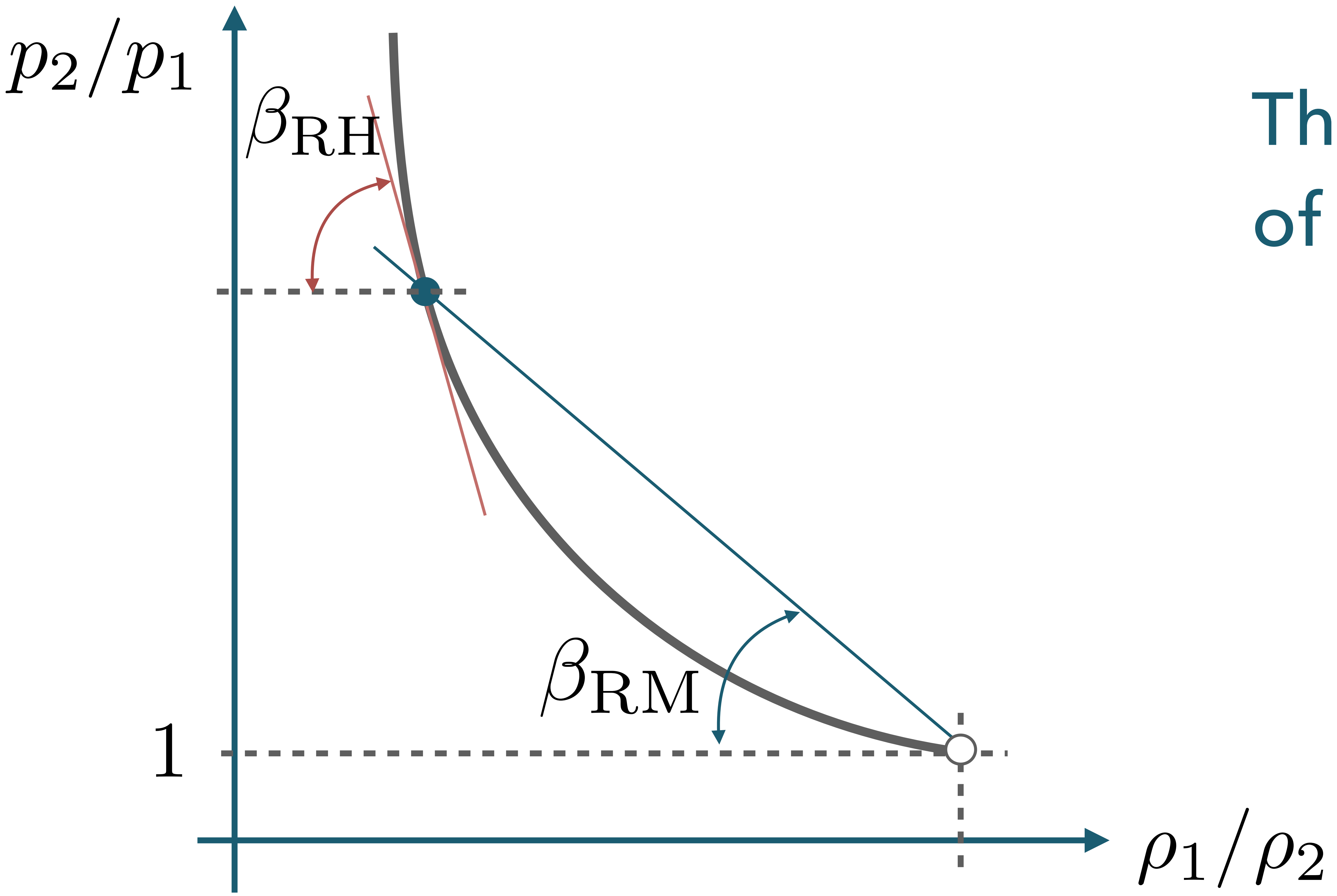
and the associated slope

$$\tan \beta_{\mathrm{RM}} = \frac{\rho_1 a_1^2}{p_1} M_1^2$$

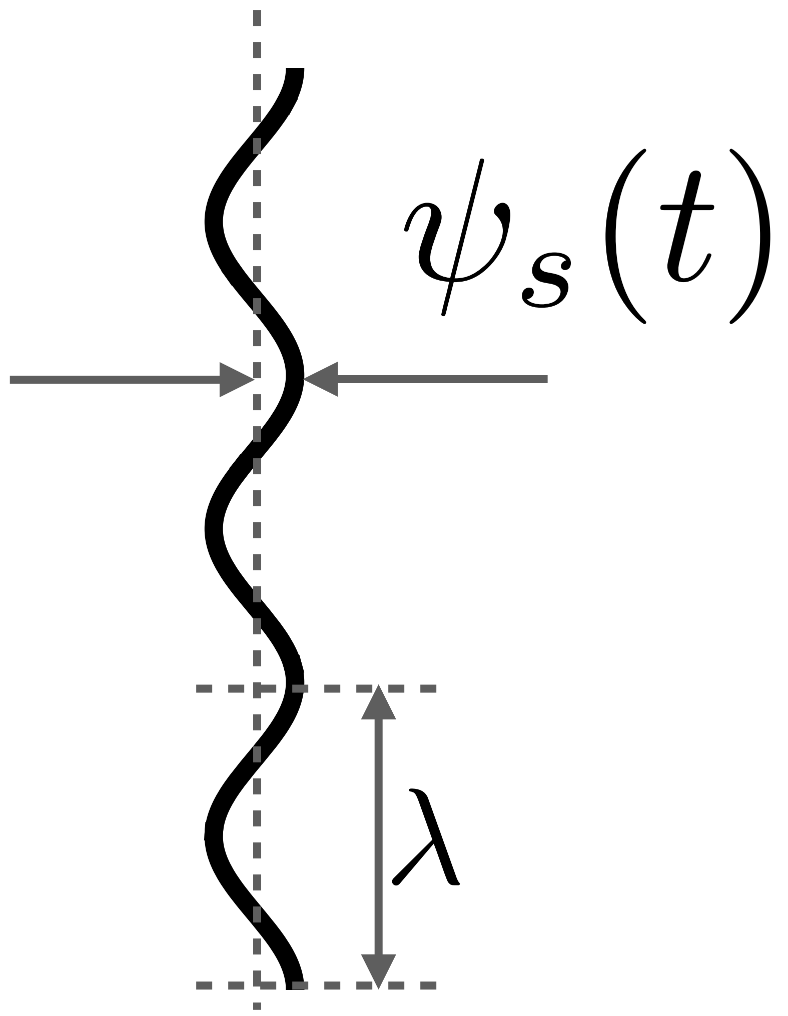
What is the DK instability?

The DK parameter is defined as

$$h = \frac{p_2 - p_1}{V_1 - V_2}\left(\frac{\mathrm{d}p}{\mathrm{d}V}\right)^{-1} = -\frac{\tan\beta_{\mathrm{RM}}}{\tan\beta_{\mathrm{RH}}}$$



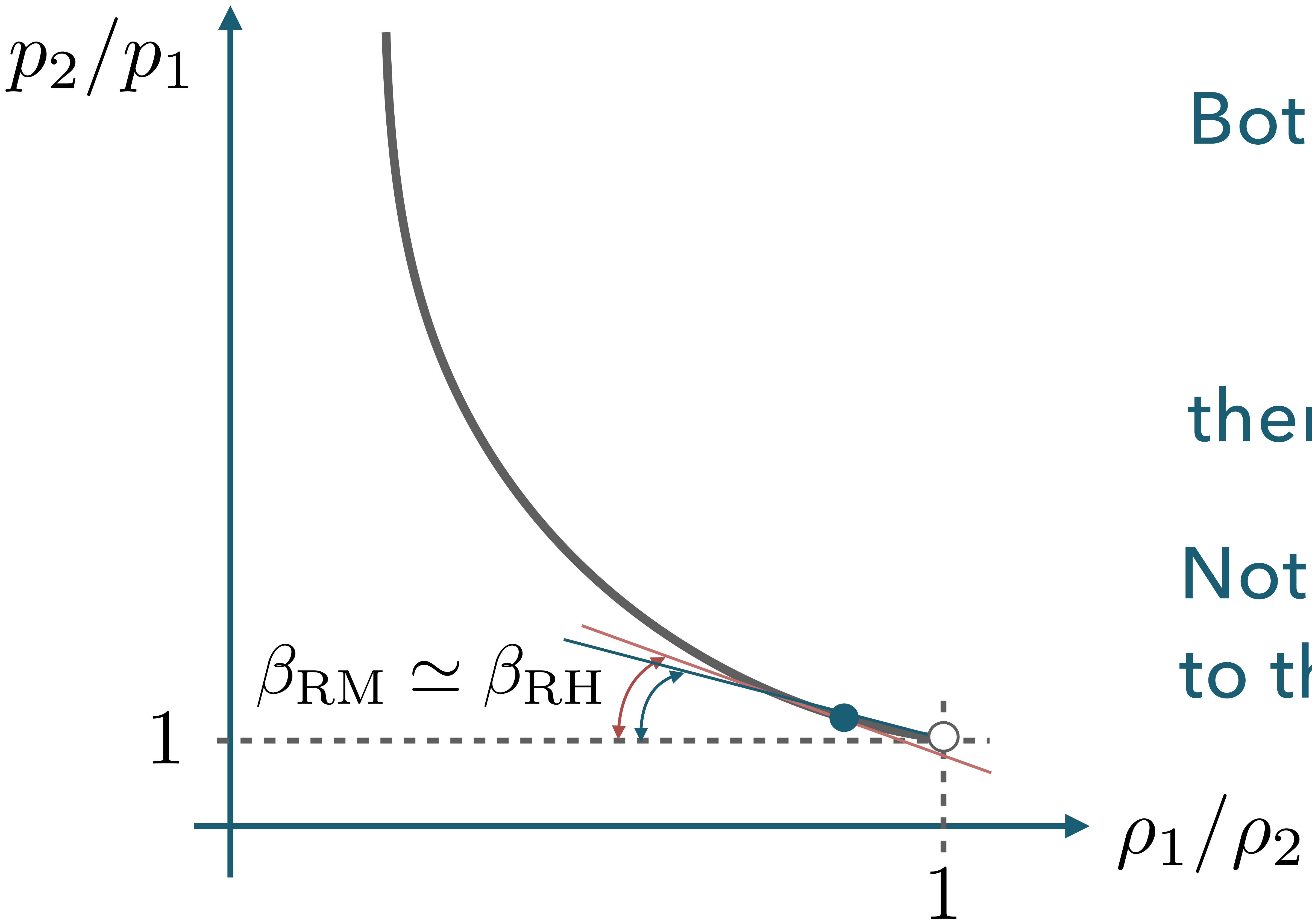
This parameter is pivotal in the evolution of a weakly perturbed shock front.



Non-equilibrium induces small flow perturbations of the order $\psi_{s0}/\lambda \ll 1$.

What is the DK instability?

The DK parameter for very weak shocks:



Both RM-line and RH-slope are the same,

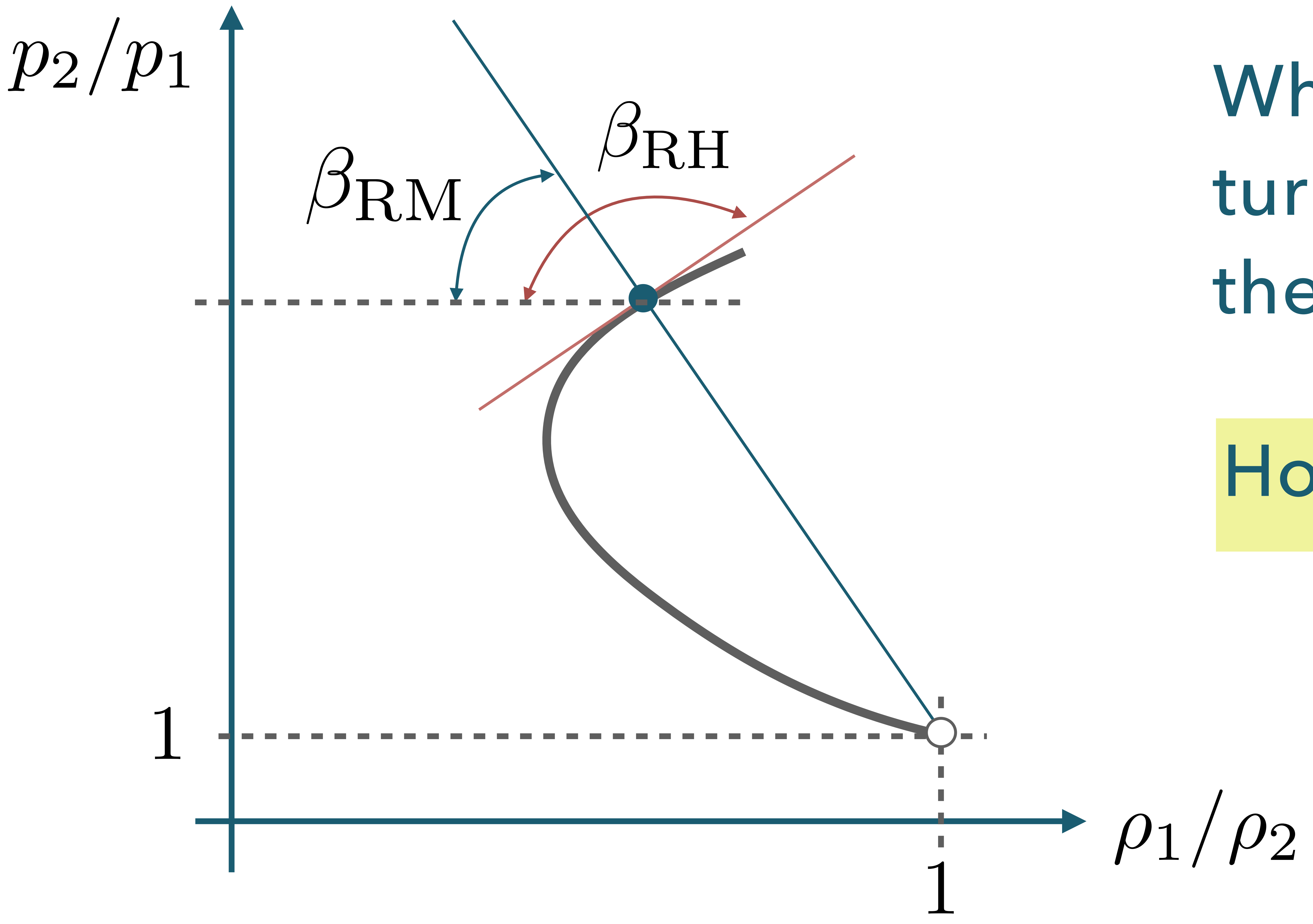
$$\tan\beta_{\mathrm{RM}} \simeq \tan\beta_{\mathrm{RM}} = \frac{\rho_1 a_1^2}{p_1}$$

thereby giving $h = -1$

Note that this slope corresponds to the isentropic relationship: $\left.\frac{\mathrm{d}p}{\mathrm{d}\rho}\right|_1 = a_1^2$

What is the DK instability?

The DK parameter for very strong shocks is harder to model.



When the RH curve exhibits a turning point, $\tan\beta_{\mathrm{RH}} < 0$, the DH parameter turns positive: $h > 0$

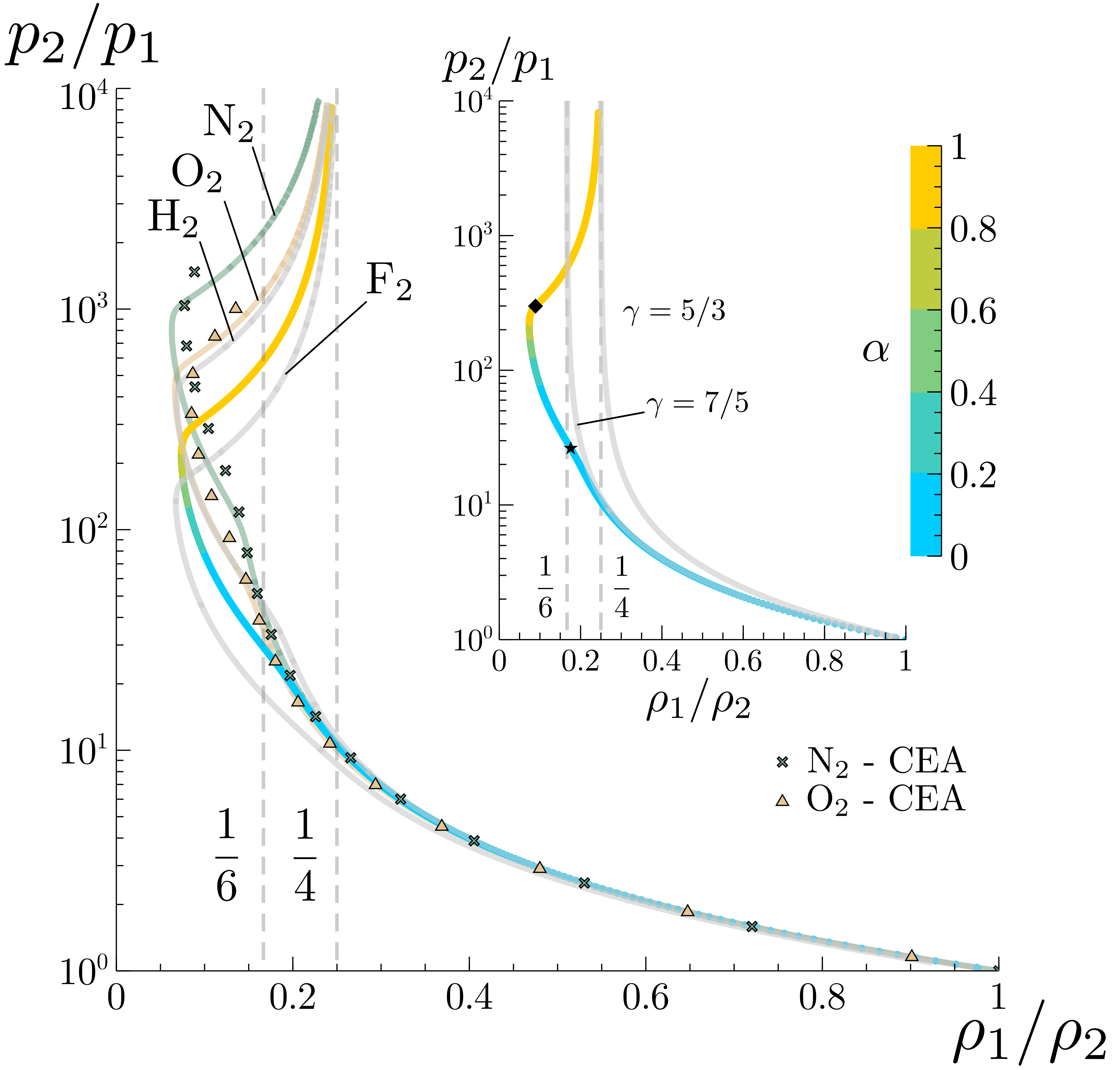
How does h affect the shock dynamics?

What is the DK instability?

The DK parameter for strong shocks is harder to model.

Effects like molecular dissociation, ionization, and radiation may shape the RH curve to exhibit a reversion.

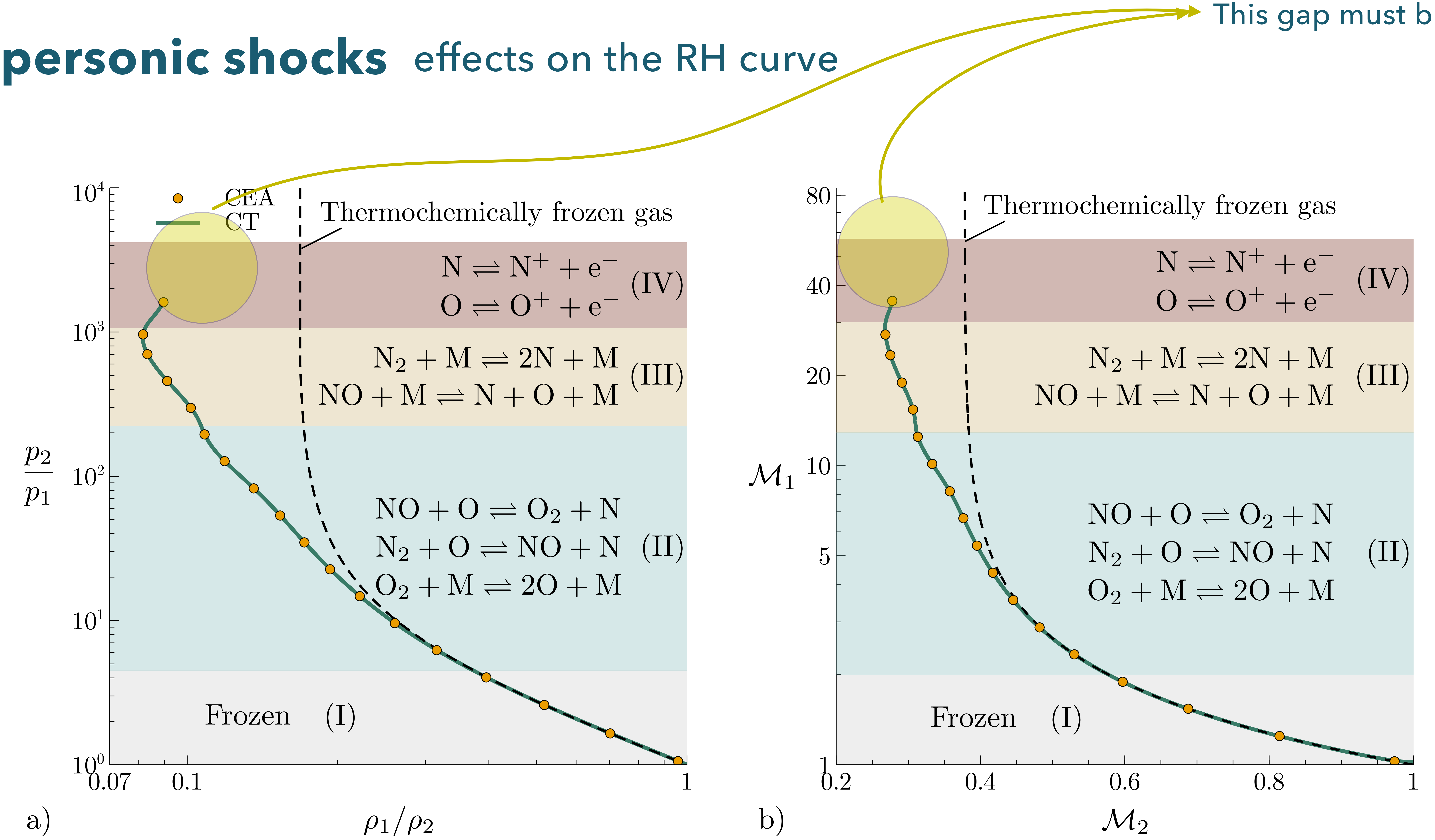
Example of RH curves for different diatomic gases



C.Huete et al., Physics of Fluids 33 (8), 086111

Hypersonic shocks effects on the RH curve

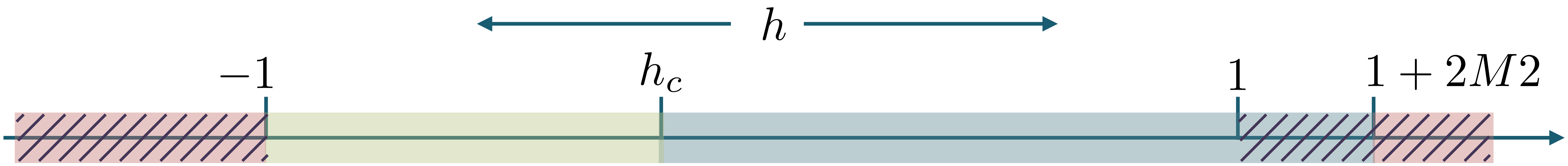
This gap must be filled



What is the DK instability?

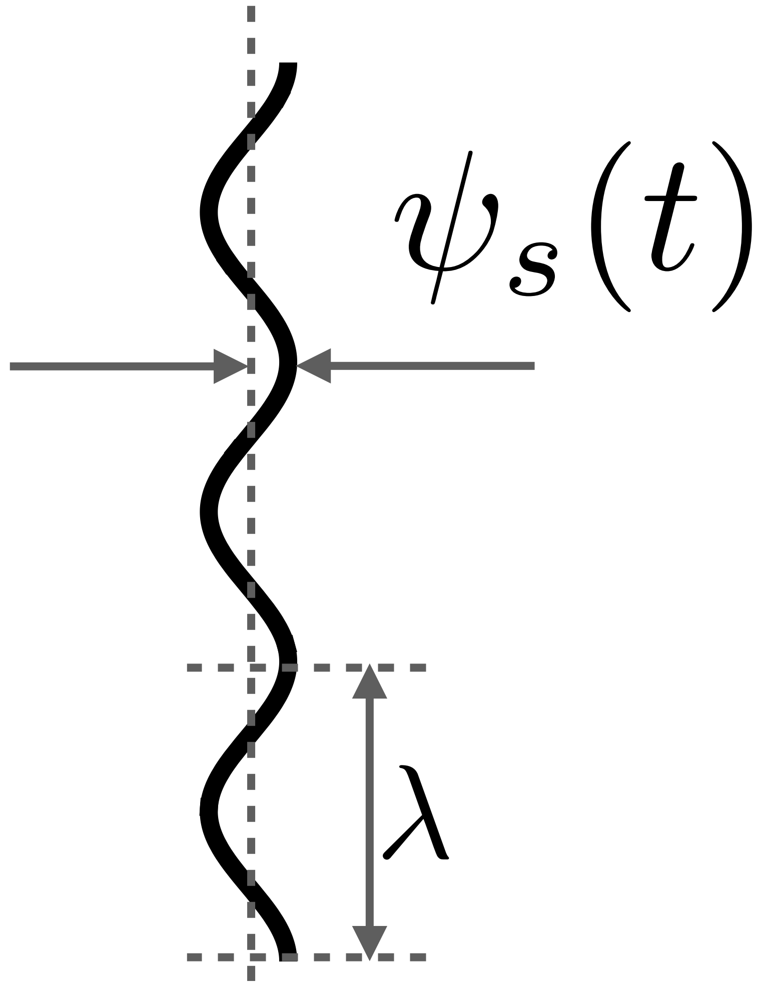
Normal-mode stability analysis of an isolated planar shock front expressed stability conditions via the DK parameter

$$\psi_s(t) \sim e^{\pm i\omega t}$$



its critical value is $$h_c = \frac{1 - M_2^2\,(1+R)}{1 - M_2^2\,(1-R)}$$

where M_2 is the post-shock Mach number, and R is the shock density compression.



[1] L. D. Landau and E. M. Lifshitz, Fluid Mechanics (Pergamon Press, 1987).

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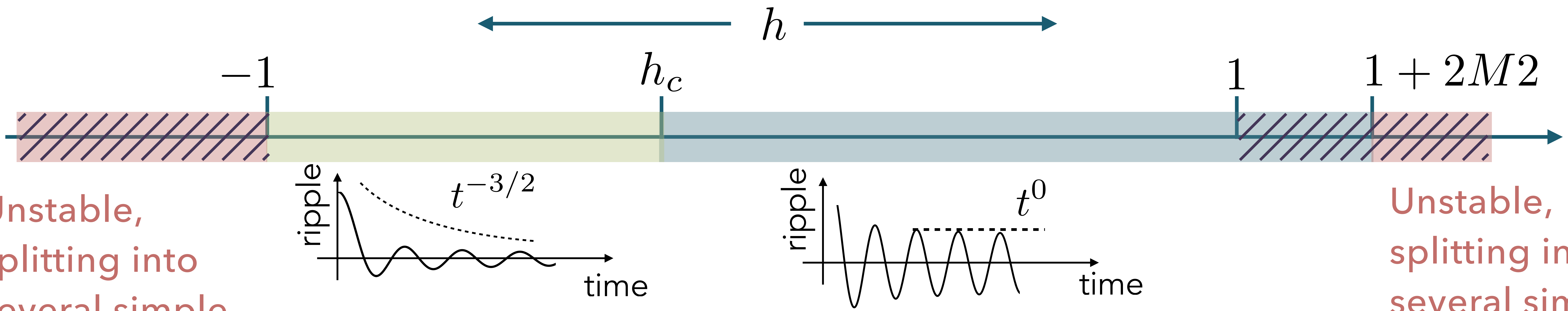

Unstable, splitting into several simple waves

Super-stable, like ideal gas EoS

Neutrally stable, non-decaying oscillations of the shock ripples, spontaneous acoustic emission (SAE) downstream

Unstable, splitting into several simple waves

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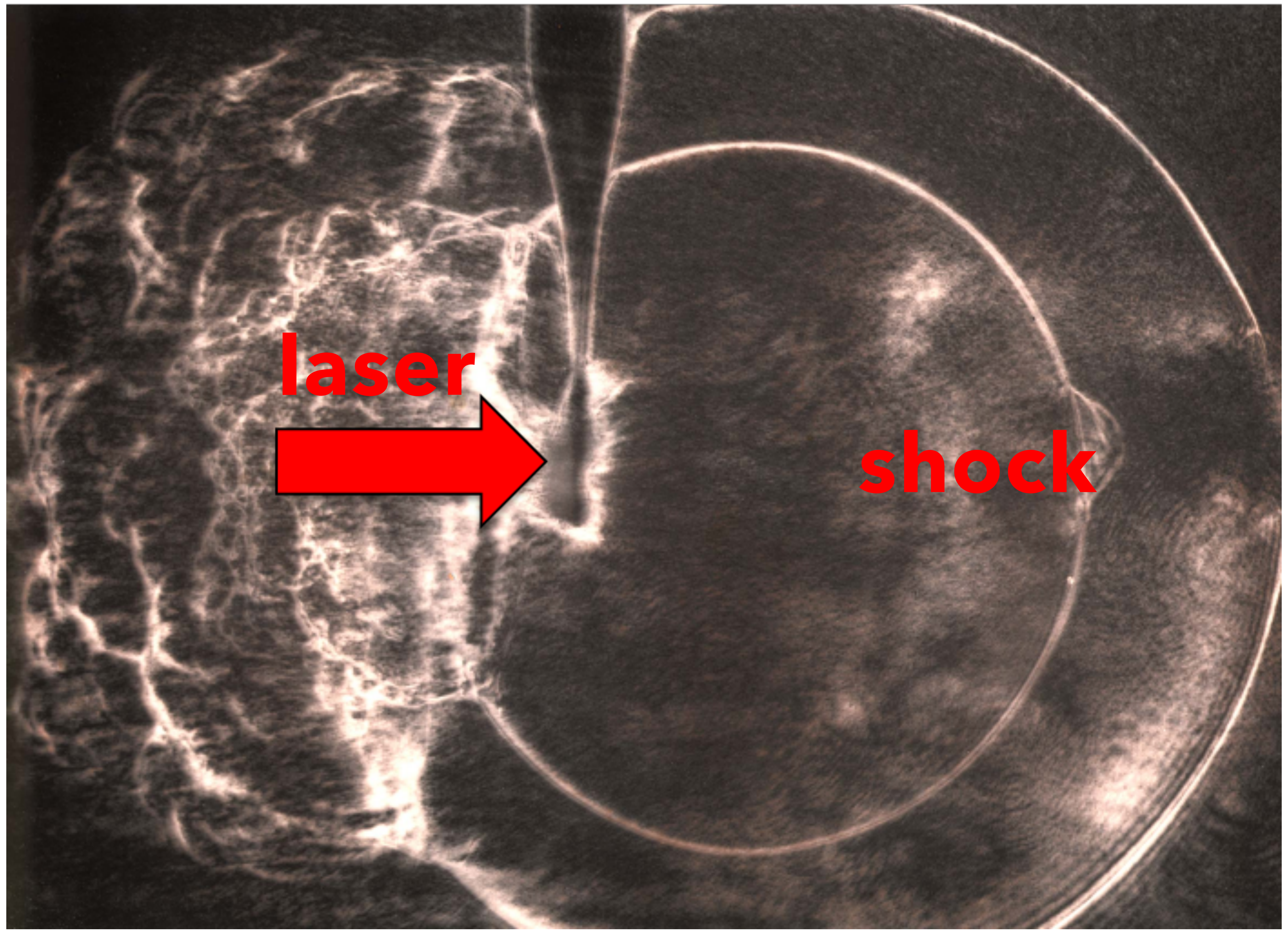
What is the DK instability?

Shock fronts in ideal gases are super-stable, much smoother than the flows driving them.



Schlieren images of stable blast waves in a low-density nitrogen produced by Jacob Grun with the PHAROS laser at NRL.

Examples in non-magnetized materials

With the development of realistic wide-range EoS models, many examples have been found theoretically, starting from Aleksey Bushman's 1976 work on copper, including:

–water (Kuznetsov and Davydova, 1988)
–shock-ionized inert gases (Mond *et al.*, 1994, 1997)
–magnesium (Lomonosov et al., 2000; Konyukhov et al., 2009)
–van der Waals fluid (Bates and Montgomery, 2000)
–metals at Gbar-Tbar shock pressures (Das et al. 2011; Wetta et al., 2018)
–shocked materials undergoing exothermic reactions (Huete et al., 2019, 2020)

What can be expected with a B field?

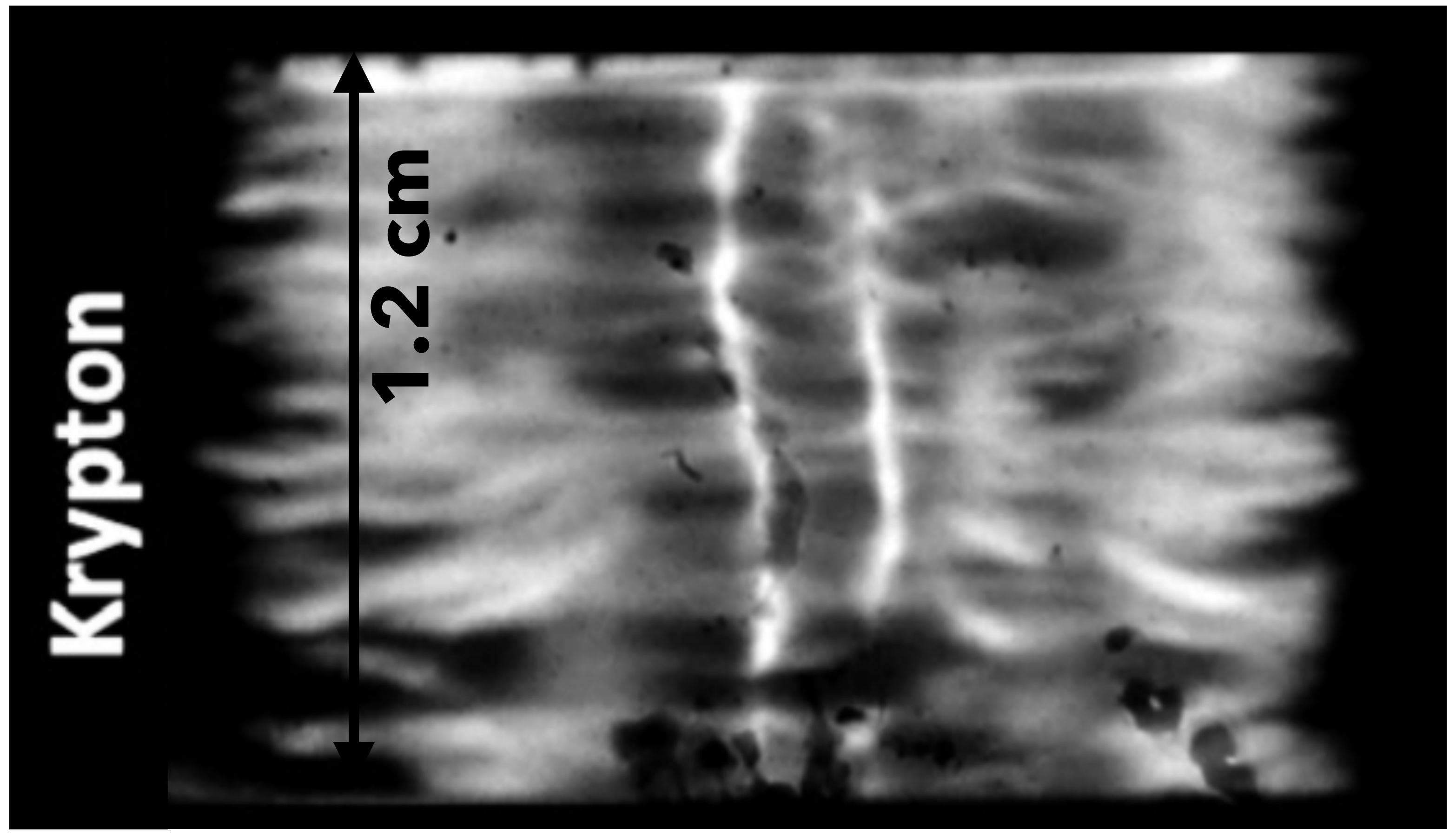


Figure from Rahman *et al.* 2019

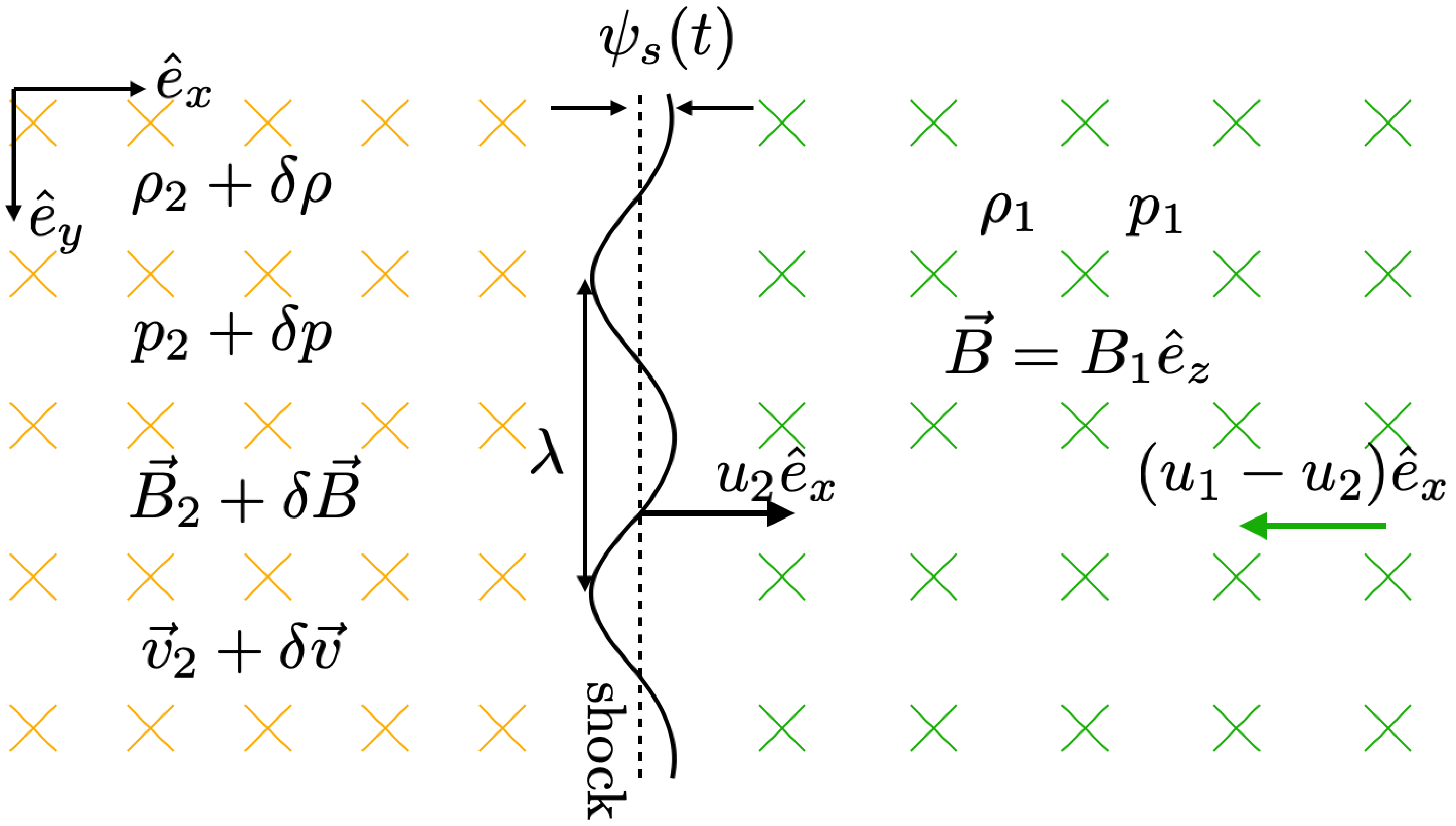
Theoretical efforts:

– Gardner and Kruskal, Phys. Fluids (1964):

Shock is stable to exponentially unstable perturbations in an ideal gas if $\gamma < 3$.

- Trakhinin, Comm. Math Physics (2003)

Let's address the simplest case first, where magnetic field does not bend



Ideal MHD equations

$$\frac{\partial \delta\rho}{\partial t} + \rho_2 \nabla \cdot \delta\vec{v} = 0$$

$$\rho_2 \frac{\partial \delta\vec{v}}{\partial t} + \nabla \delta p - \frac{1}{4\pi}\left(\nabla \times \delta\vec{B}\right) \times \vec{B}_2 = 0$$

$$\frac{\partial \delta p}{\partial t} - c_{T2}^2 \frac{\partial \delta\rho}{\partial t} = 0$$

$$\frac{\partial \delta\vec{B}}{\partial t} - \nabla \times \left(\delta\vec{v} \times \vec{B}_2\right) = 0$$

Let's address the simplest case first, where magnetic field does not bend

$$\frac{\partial^2 \delta\vec{v}}{\partial t^2} = \left(c_{T2}^2 + \frac{B_2^2}{4\pi\rho_2}\right)\left(\nabla^2\delta\vec{v} + \nabla\times\nabla\times\delta\vec{v}\right)$$

c_{F2}^2 only fast magnetosonic waves exist

$$\frac{\partial^2 \delta p^*}{\partial t^2} = c_{F2}^2 \nabla^2 \delta p^*$$

where

$$\delta p^* = \delta p + \frac{B_2}{4\pi}\delta B$$

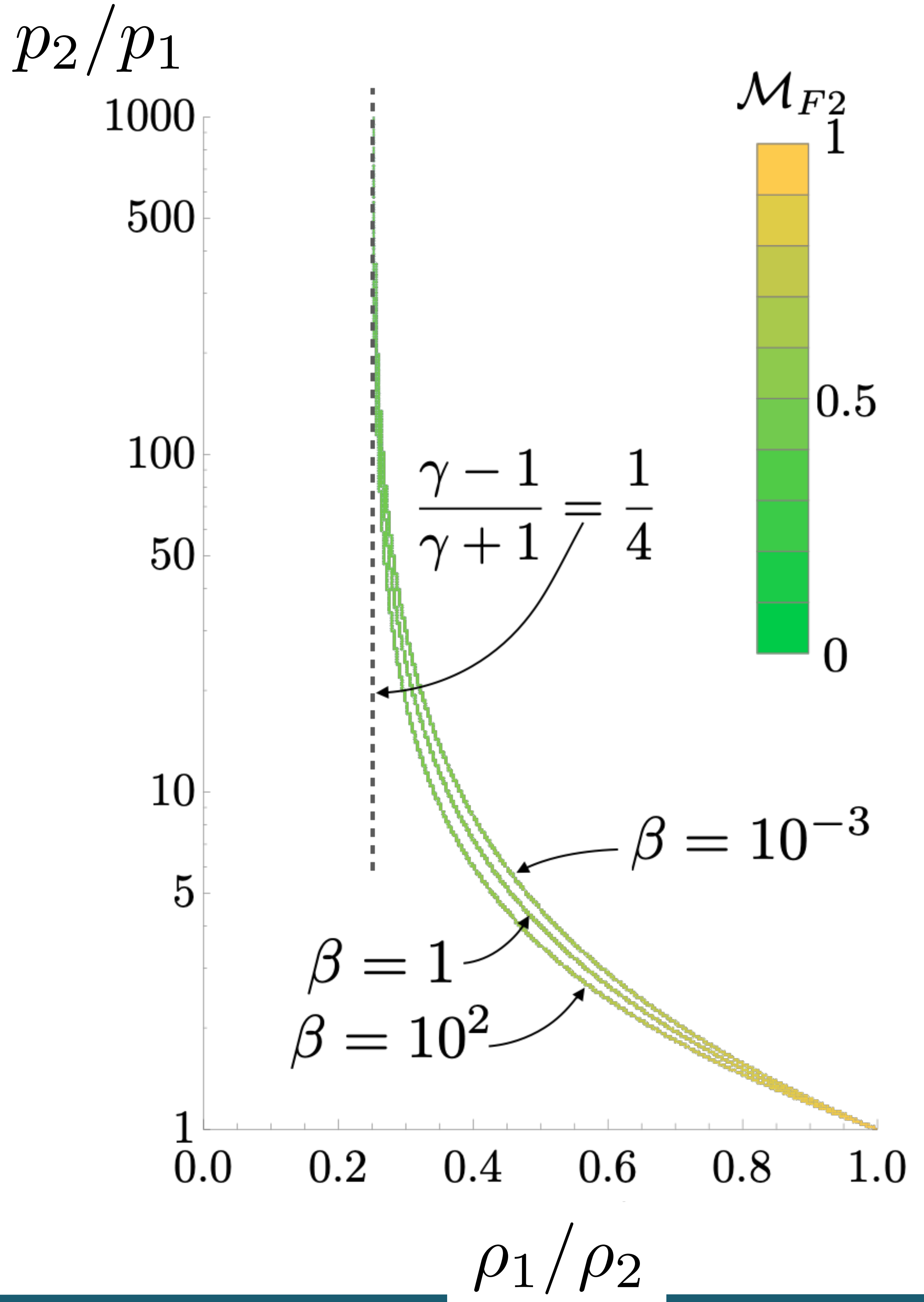
that includes the redefined DK parameter

$$h^* = \frac{p_2^* - p_1^*}{V_1 - V_2}\left(\frac{\mathrm{d}p_2^*}{\mathrm{d}V_2}\right)_H^{-1}$$

and

$$p_2^* = p_2 + B_2^2/(8\pi)$$

Let's address the simplest case first, where magnetic field does not bend



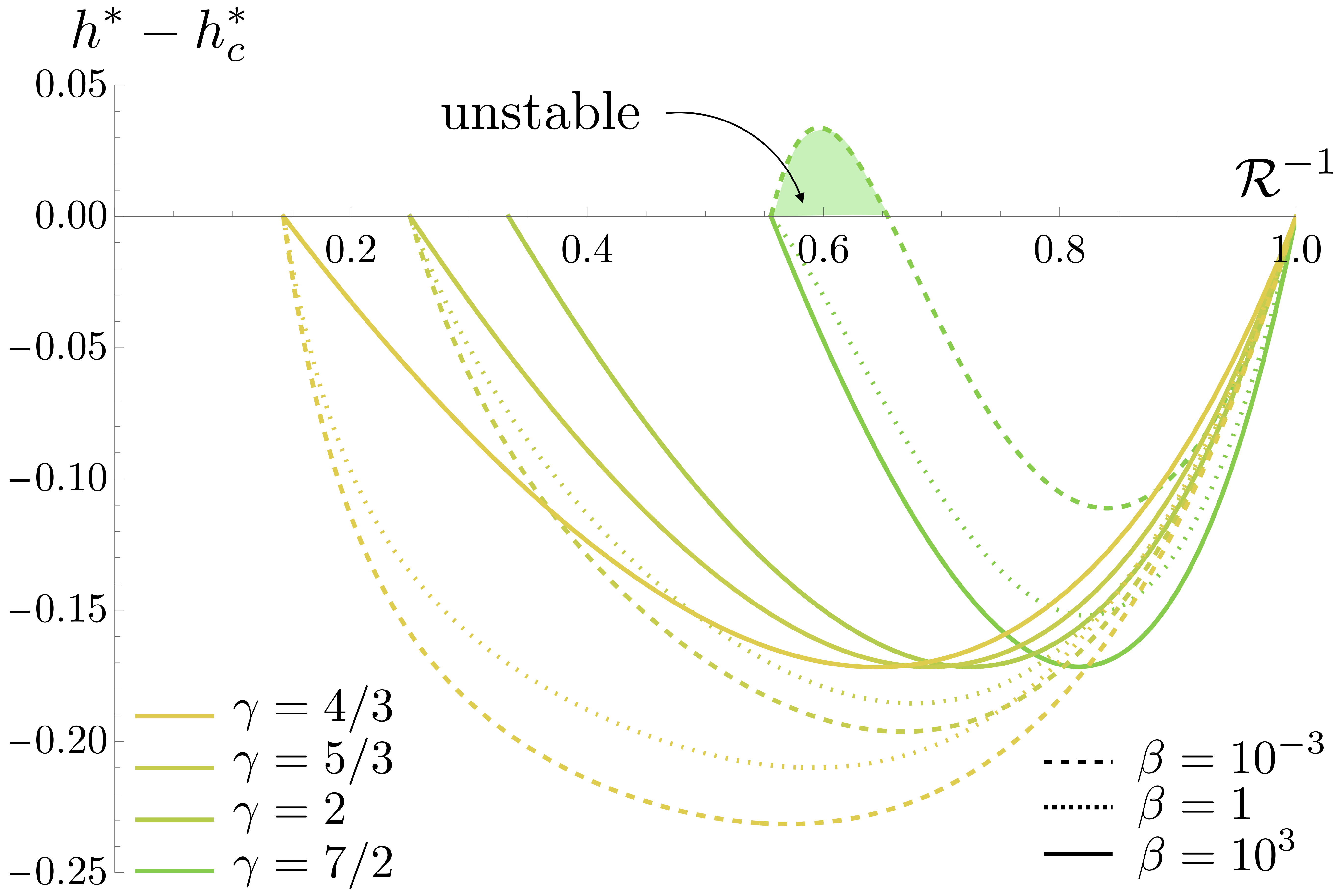
Apparently, the influence of the magnetic field, assessed by the parameter $\beta = 8\pi p_1/B_1^2$, is little.

But a closer look has to be made on $h^* - h_c^*$!

where the critical parameter must be redefined with the fast magnetosonic post-shock Mach number

$$h_c^* = \frac{1 - M_{F2}^2\,(R+1)}{1 - M_{F2}^2\,(R-1)} \qquad R = \frac{\rho_2}{\rho_1}$$

For an ideal gas



For an ideal gas, the magnetic field stabilizes the shock for

$$\gamma < 1 + \sqrt{2}$$

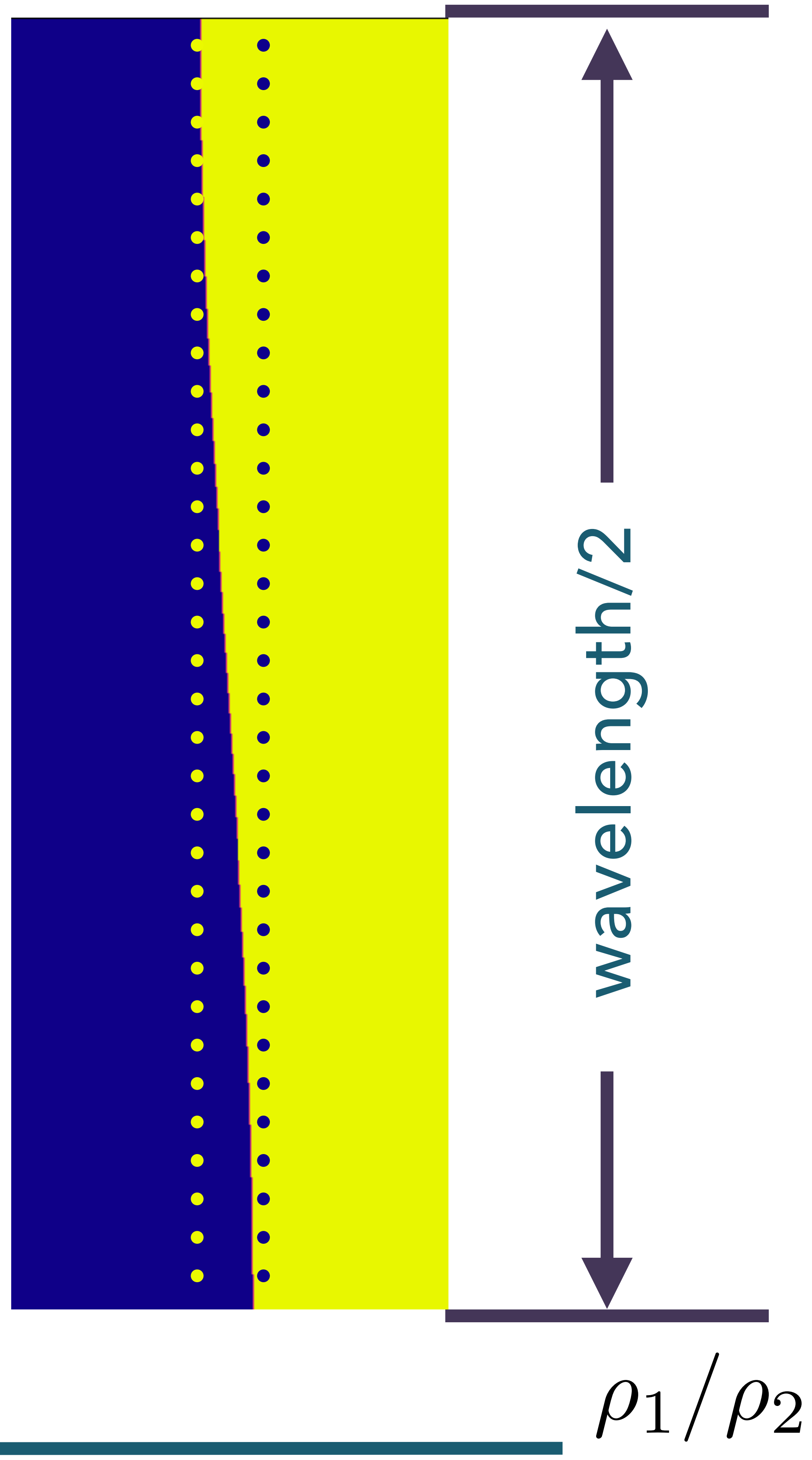
but it can **trigger** the DK instability for

$$\gamma > 1 + \sqrt{2}$$

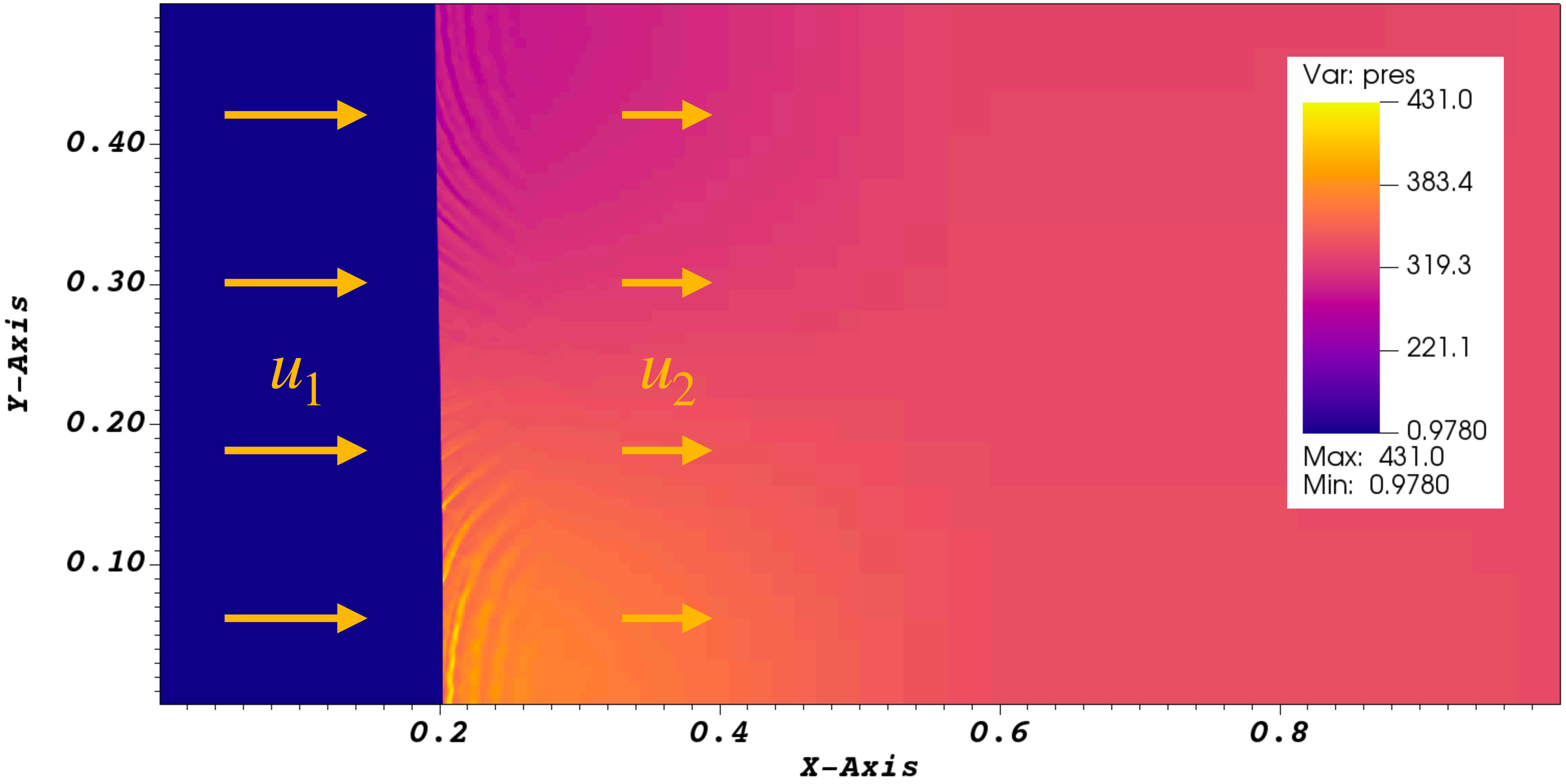
Numerical simulations

UNIVERSITY of ROCHESTER · FLASH CENTER

Weak shock perturbation



The shock front is fixed with the reference frame



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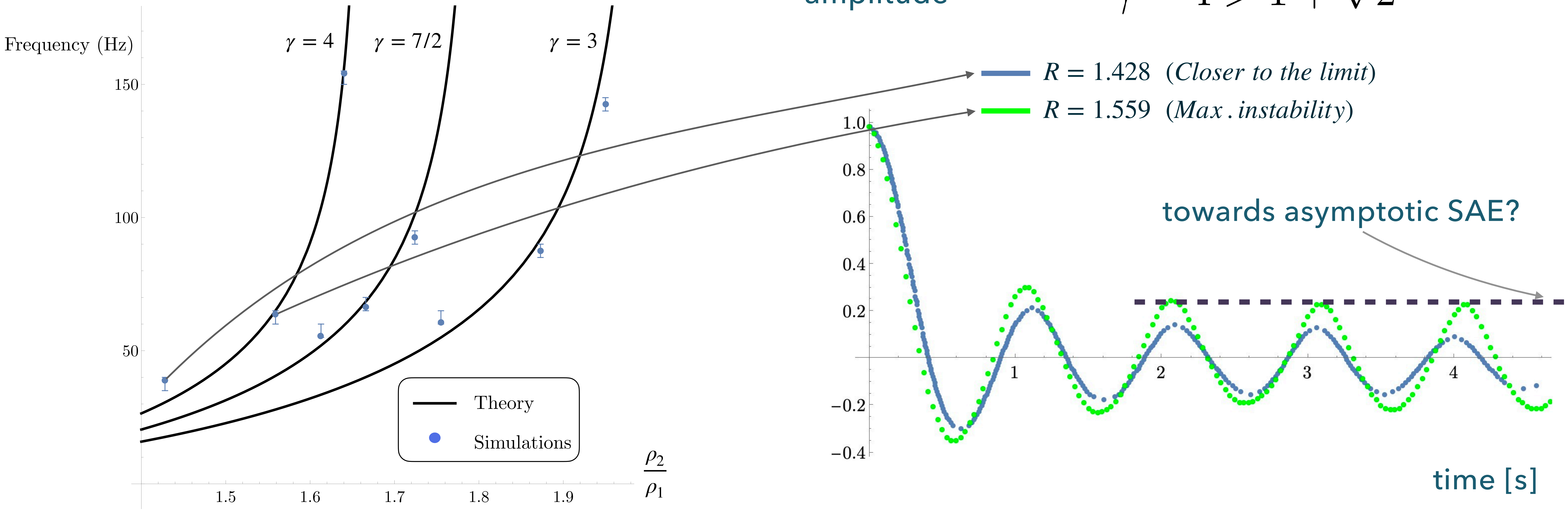
Numerical simulations

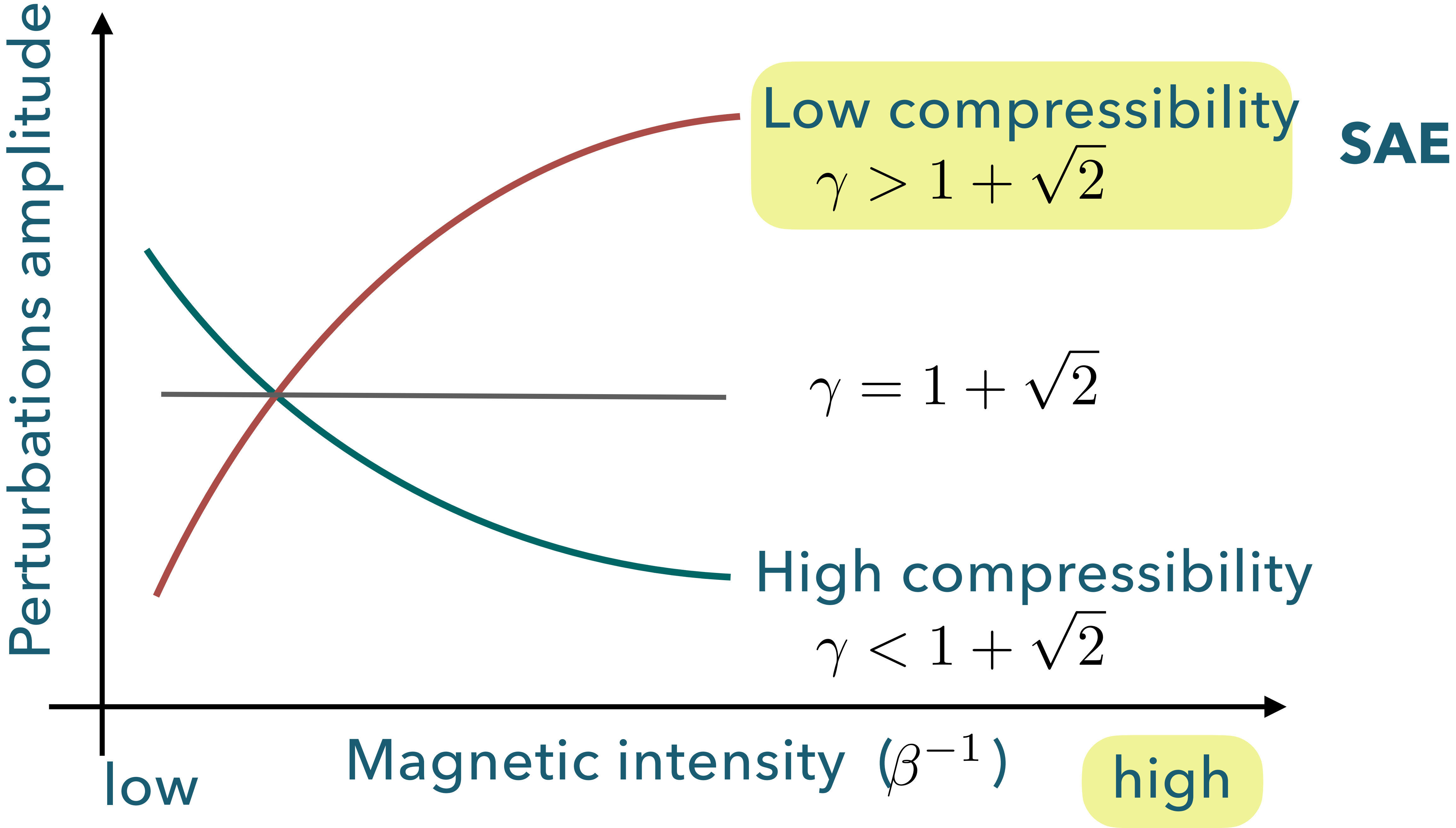
Simulations for ideal gas EoS

shock perturbation amplitude

$$\gamma = 4 > 1 + \sqrt{2}$$



Conclusions



✓ The D'yakov-Kontorovich instability can be found in a **magnetized fluid.**

✓ The role of a perpendicular magnetic field can be **stabilizer** or **destabilizer**.

✓ The analysis has ben extended and applied to **arbitrary EoS.**

✓ Numerical simulations with **FLASH code confirms** the effect of the magnetic field.

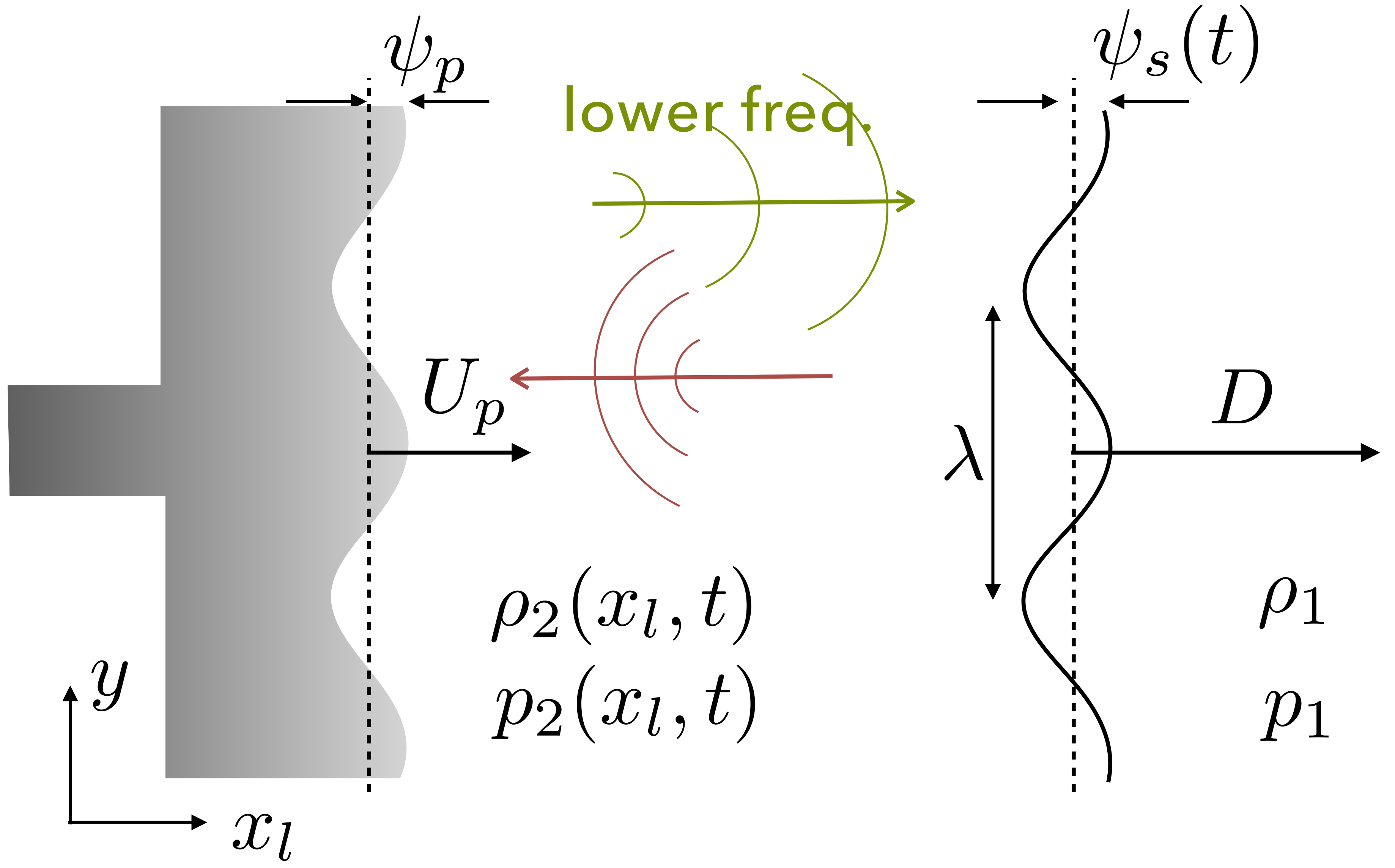
! Further analysis must be done to address the general problem that includes an **arbitrary magnetic field orientation** (three modes: fast, slow and Alfven modes)

ρ_1/ρ_2

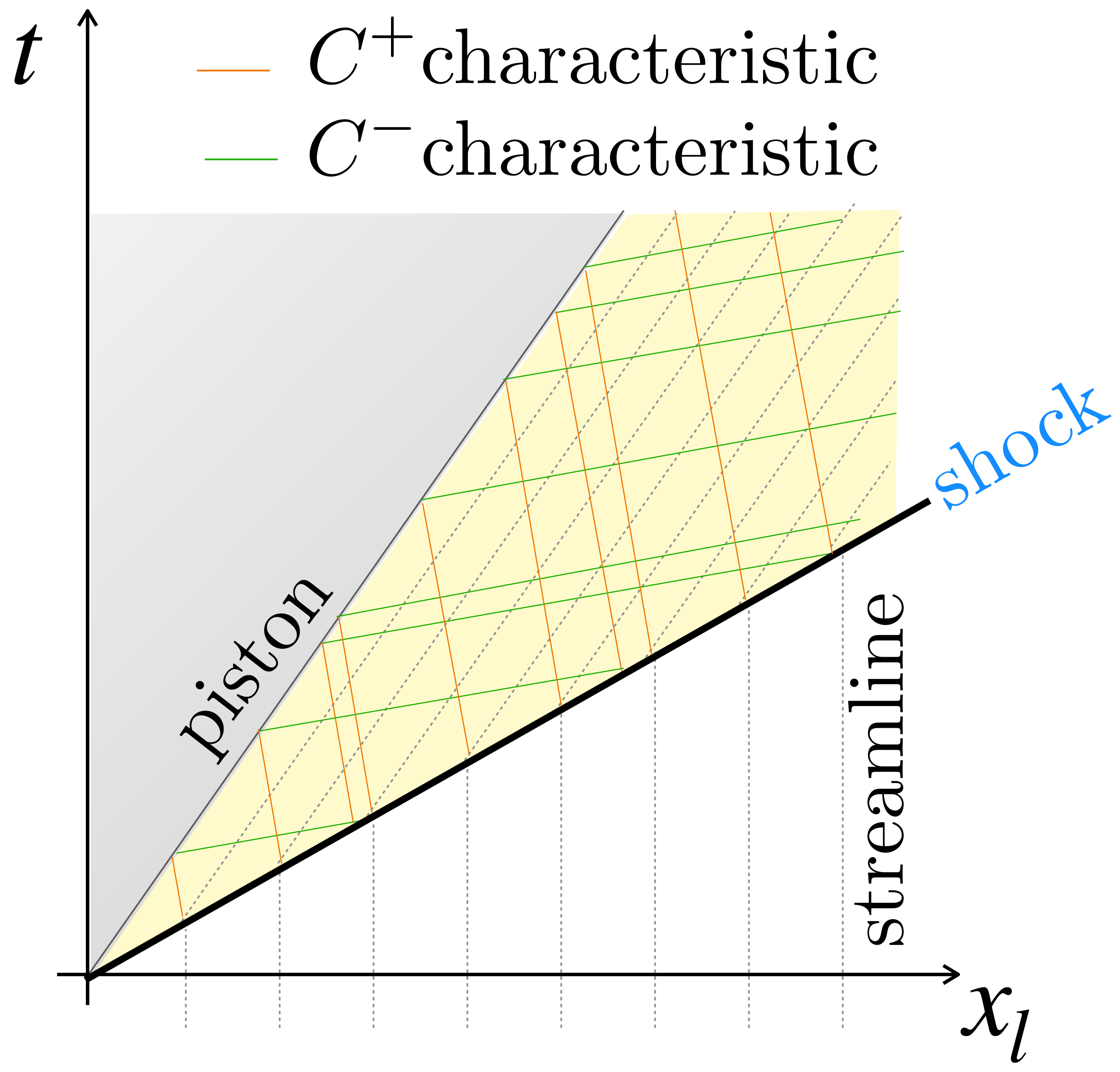
Yet this is not the end of the story:

A shock wave needs a supporting mechanism to move steadily.

When $h > h_c$, the always oscillating shock radiates non-decaying sonic waves, which “bounces” at the supporting mechanism (say a piston), then running upstream and catching up the shock from behind.



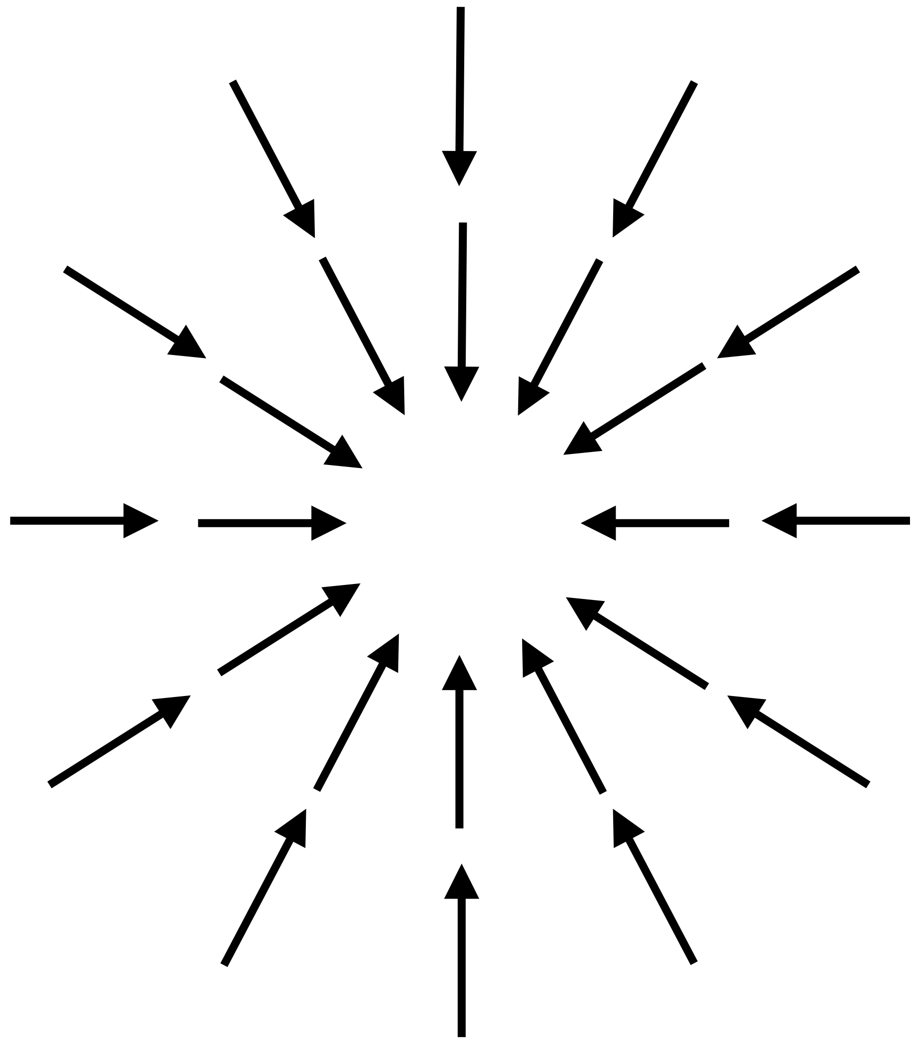
How does the needed supporting mechanism affect the shock stability?



The Noh problem

A initial mass is set to move inwards at uniform speed.

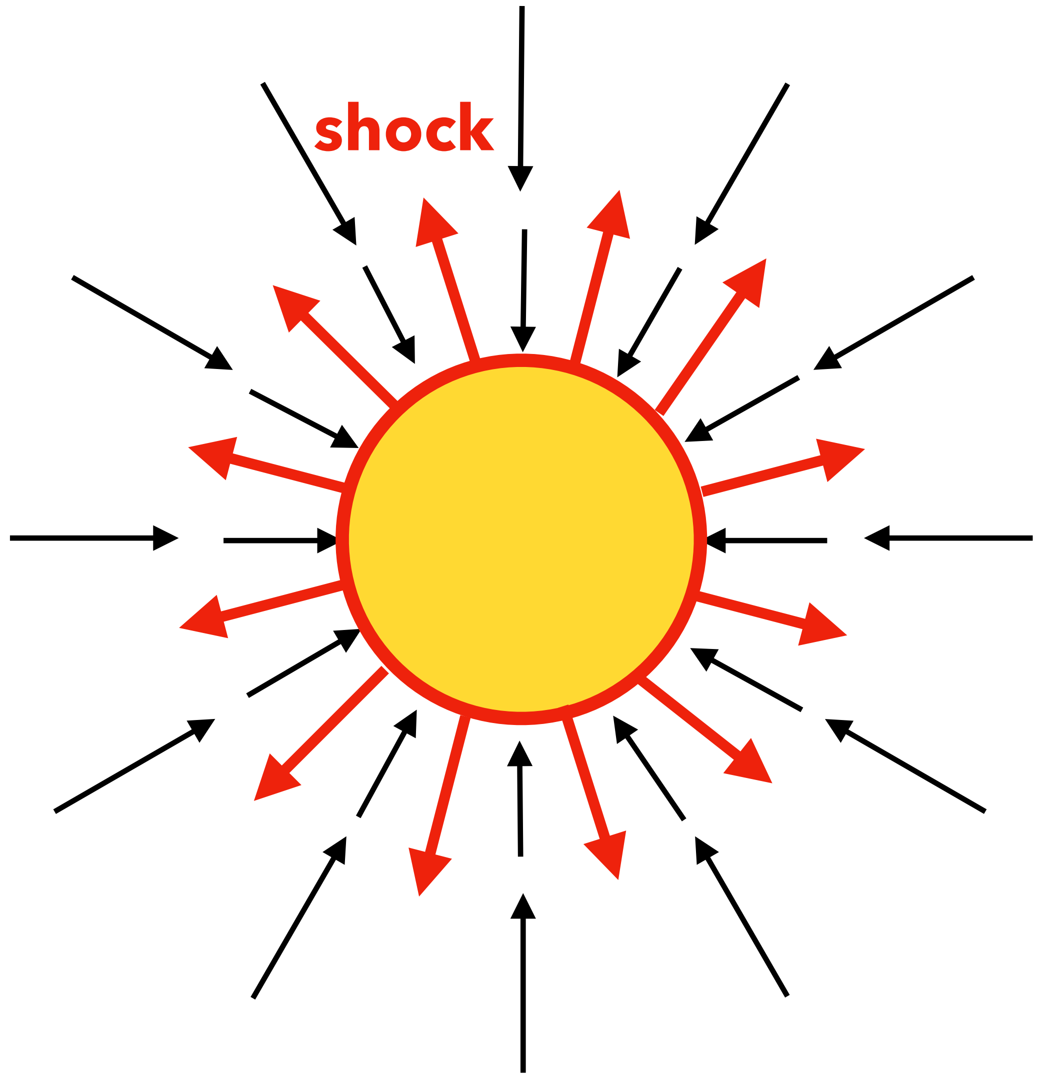
$$\rho_1(r, t=0) = \rho_0$$
$$p_1(r, t=0) = p_0$$
$$\boldsymbol{v}_1(r, t=0) = -v_0 \boldsymbol{e}_r$$



The singular condition at the center translates into the formation of a shock wave that moves outwards.

The lack of scales suggests the use of the self-similar variable to describe the upstream flow.

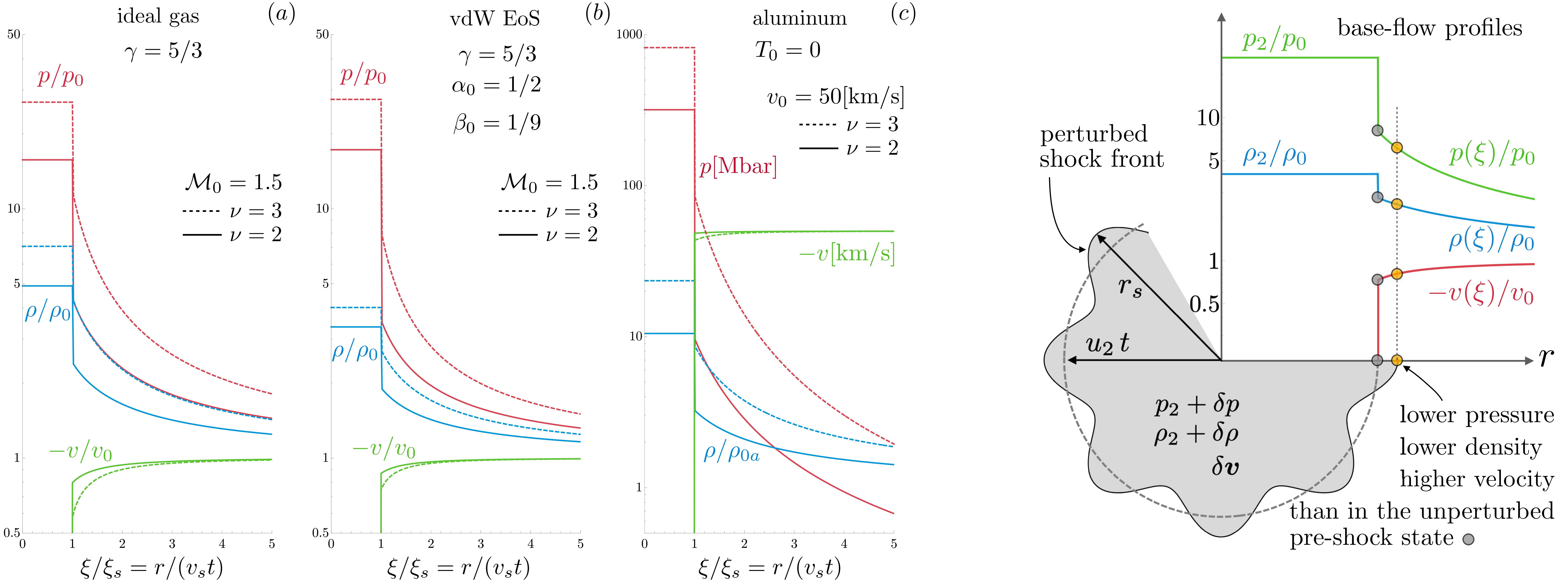
$$\xi = \frac{r}{v_0 t}$$



The base flow is the Noh solution generalized for an arbitrary EoS and shock strength.

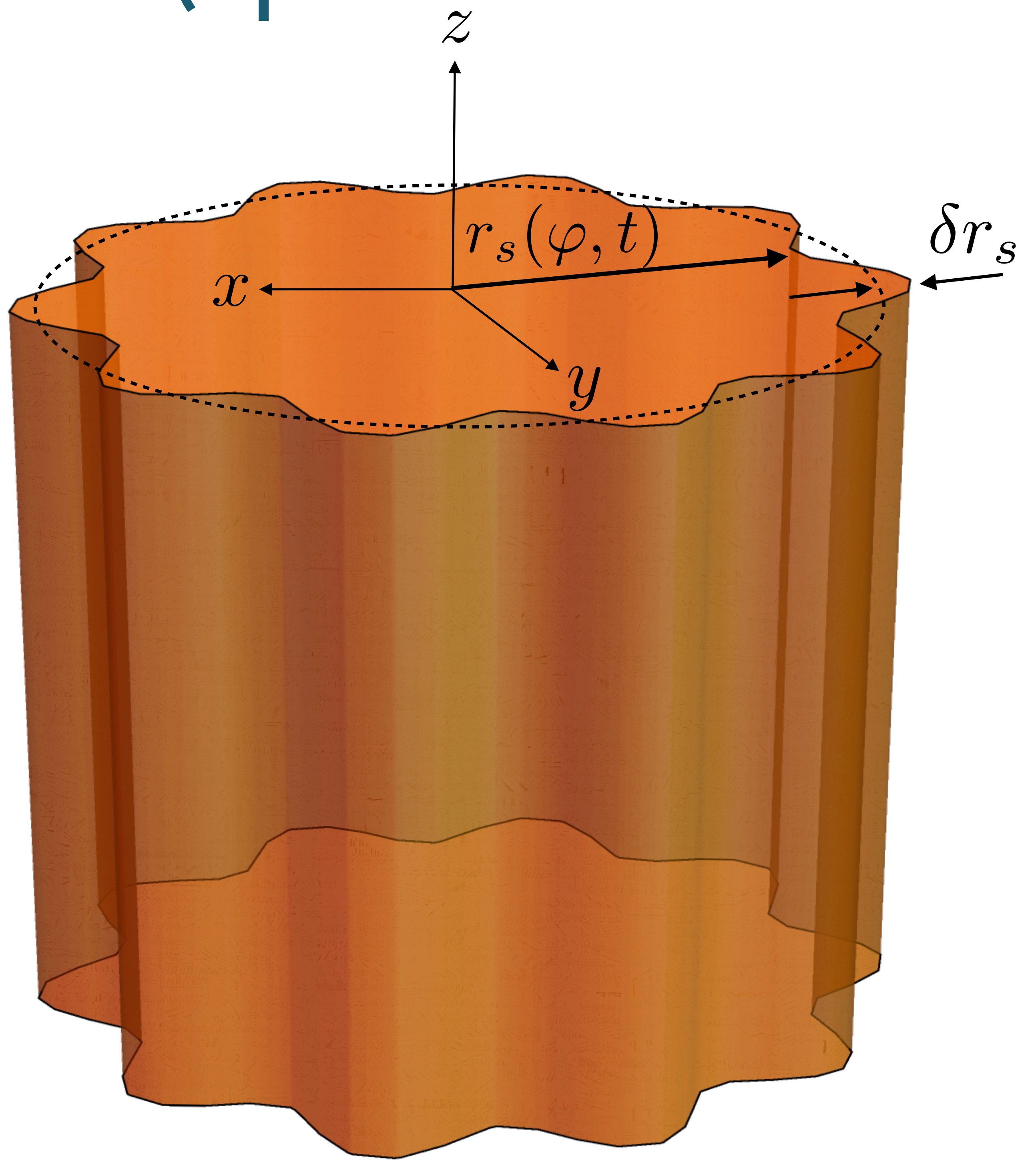
The Noh problem

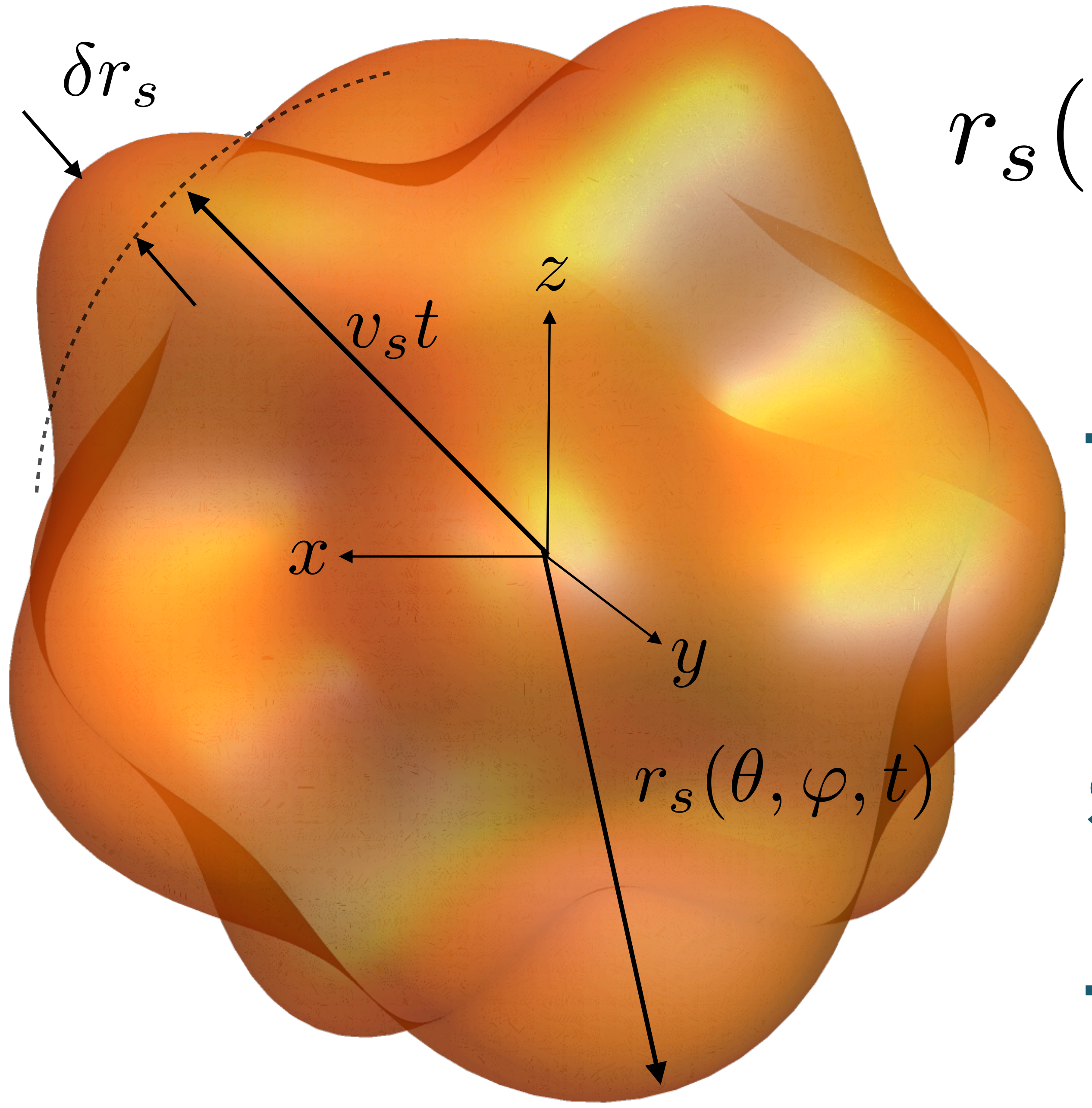
The shock moves steadily and downstream properties are uniform and steady.



The Noh problem: stability analysis

A linear- perturbation analysis is performed in terms of Legendre polynomials (spherical harmonics) $Y_l^m(\theta,\phi) = P_l^m(\cos\theta)e^{im\phi}$ in the form:





spherical

$$r_s(\theta,\phi,t) = \dot{r}_s t\left[1 + \epsilon\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma} Y_l^m(\theta,\phi)\right]$$

The eigenvalue is the complex power index, σ. Stability is determined by the sign of $\mathrm{Re}\{\sigma\}$.

The center or axis of symmetry plays the role of a rigid piston.

The Noh problem: stability analysis

The resolution of the problem provides the dispersion relationship

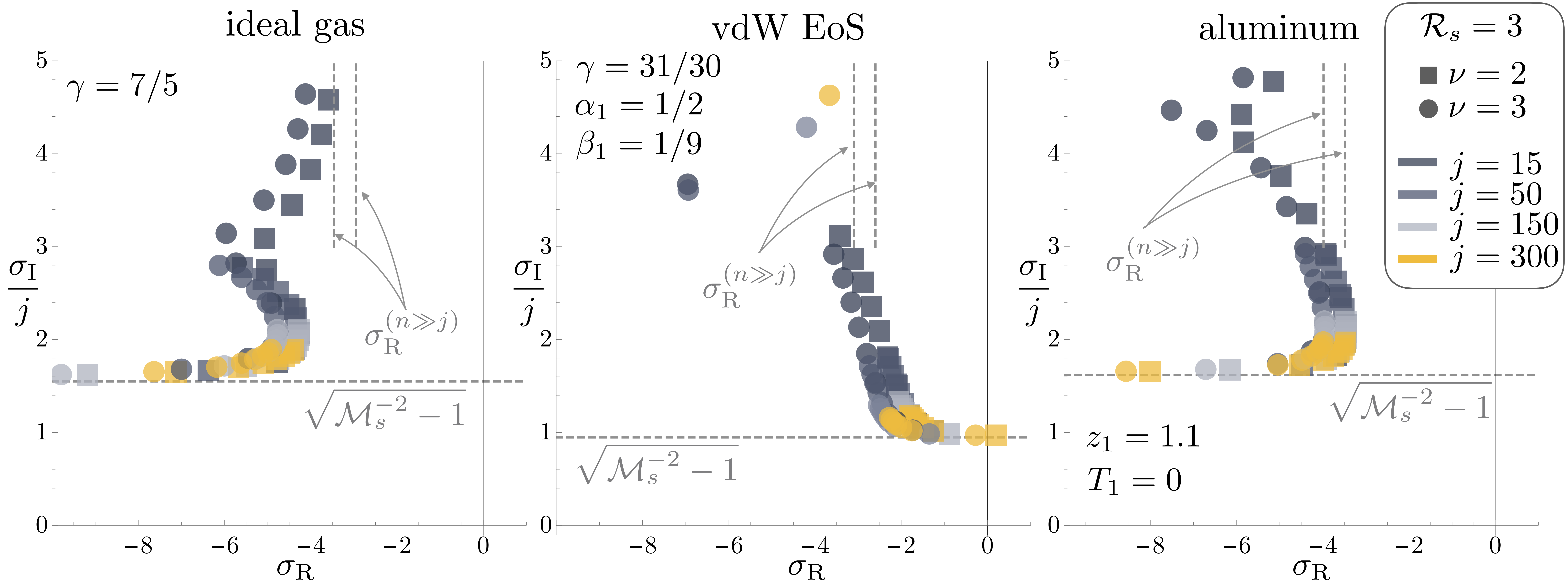
$$\left\{(\sigma+\nu-1)\left[R(\nu-1)-\sigma-\nu\right]+Rj(j+\nu-2)\right\}(1+h)F_{1s}^{+}$$
$$+\left[2(\sigma+\nu)-R(\nu-1)\,(1+h_1)\right](\sigma+\nu+j-1)F_{1s}^{-}=0.$$

That includes the Gauss hypergeometric functions

$$F_{1s}^{\pm}={}_2F_1\left(\frac{j-\sigma}{2},\frac{j\pm 1-\sigma}{2};j+\frac{\nu}{2};M_2^2\right)$$

with the parameters R, M_2, h and h_1 associated to the shock properties, and the parameters ν (cylindrical/spherical) and j (mode number).

The Noh problem: stability analysis



We can predict the stability threshold for steady expanding shocks

Summary

The isolated-shock assumption of the classic DK normal-mode Fourier analysis is formally possible but possibly inadequate for the whole unstable range they discovered.

One implicitly assumes that the shock traveled infinitely far from the piston supporting it without distortion – which is equivalent to assuming stability.

We have to solve an initial-value problem – Laplace, not Fourier!

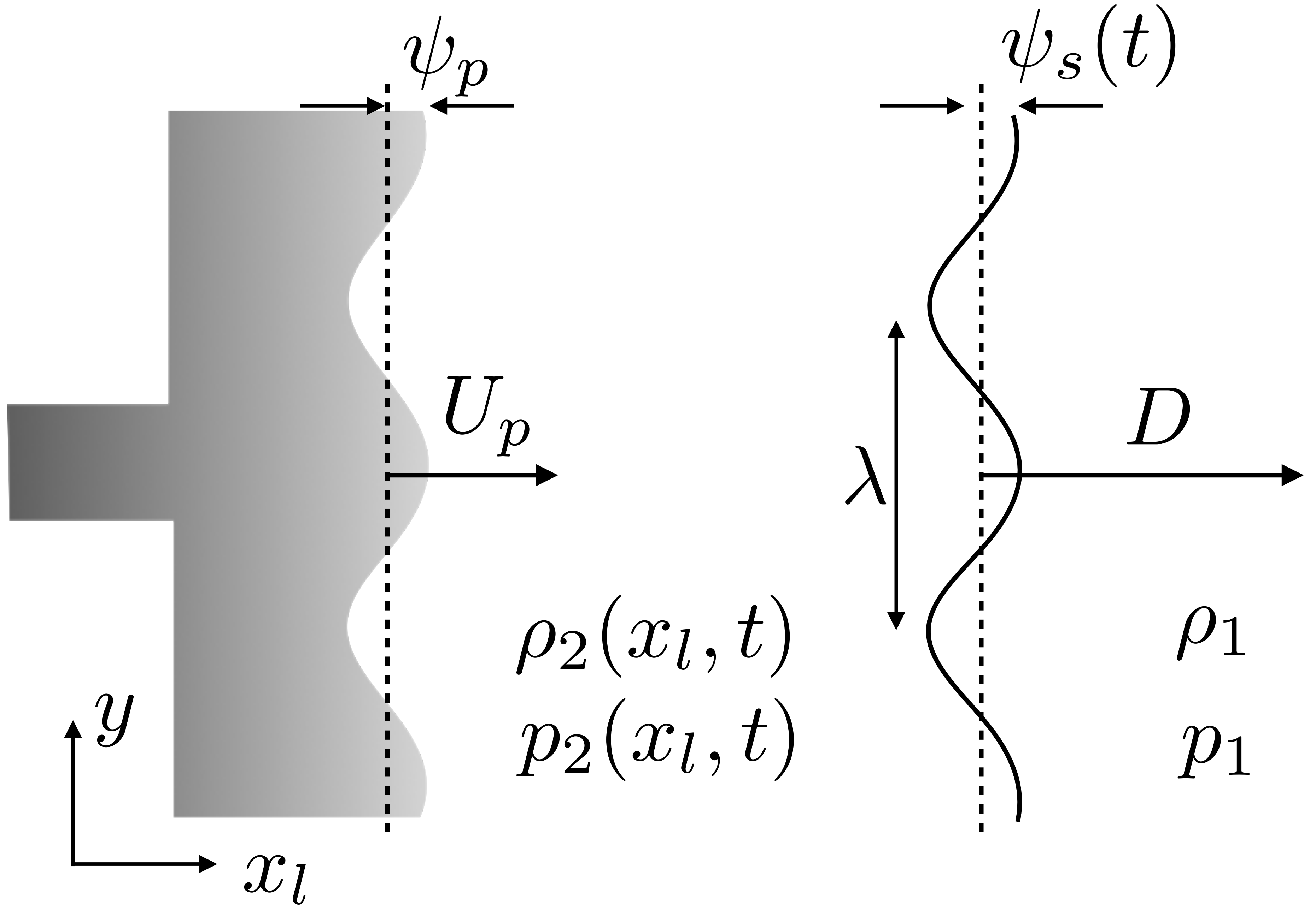
For the Noh problem (coherent supporting mechanism), being in the DK range implies that the shock is literally unstable, with a growth rate that depends on shock properties.

The proper stability analysis for a planar shock-piston flow with an arbitrary EoS needs to be done. The controversies that still exist in the literature must be finally resolved. **We are working on it.**

Piston-driven shocks

No consensus in the literature about the DK instability of a piston-supported shock for a planar geometry.

Range of h	Perturbation behavior	Source
$1 < h < 1 + 2M_2$	Growth (power-law)	[1, 2]
$h_c < h < 1 - 2M_2^2$	Non-decaying	[3]
$h_c < h < 1 + 2M_2$	Linear growth	[4]



[1] G. R. Fowles and G. W. Swan, Phys. Rev. Lett. 30, 1023 (1973).
[2] N. M. Kuznetsov, Sov. Phys. Dokl. 29, 532 (1984).
[3] J. G. Wouchuk and J. L. Cavada, Phys. Rev. E 70, 046303 (2004).
[4] J. W. Bates, Phys. Rev. E 91, 013014 (2015).

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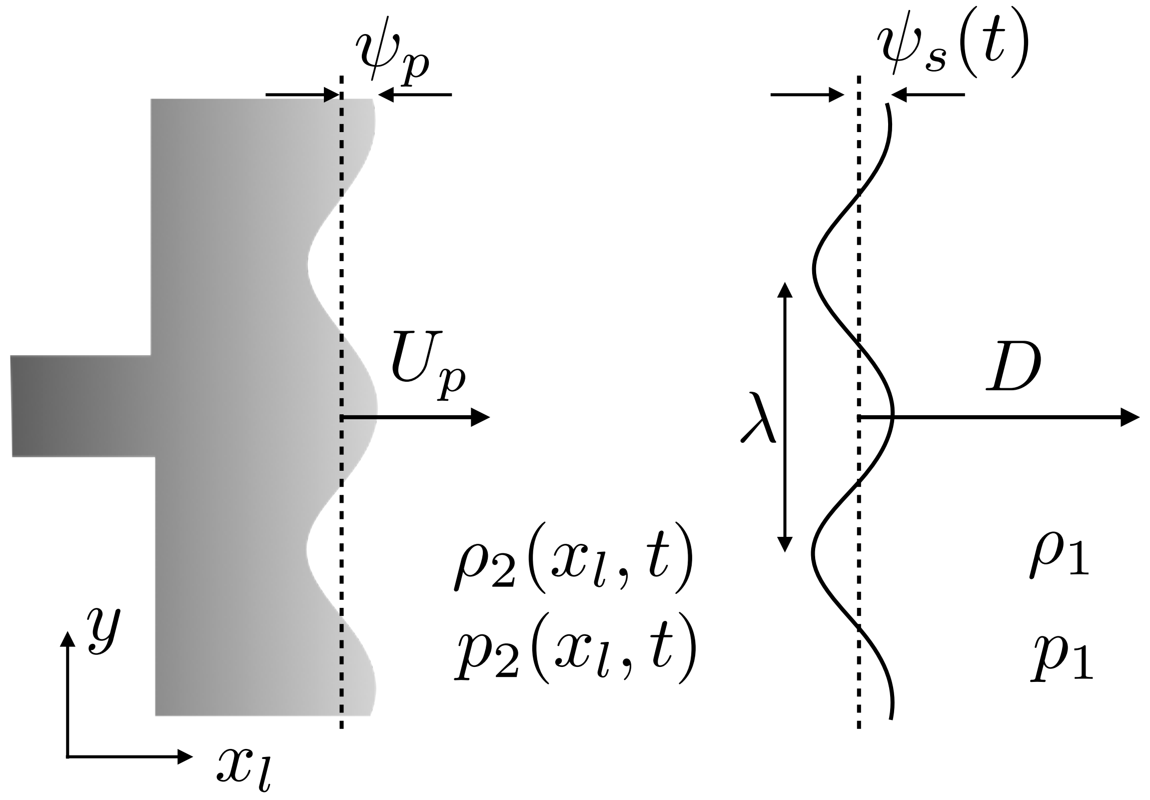
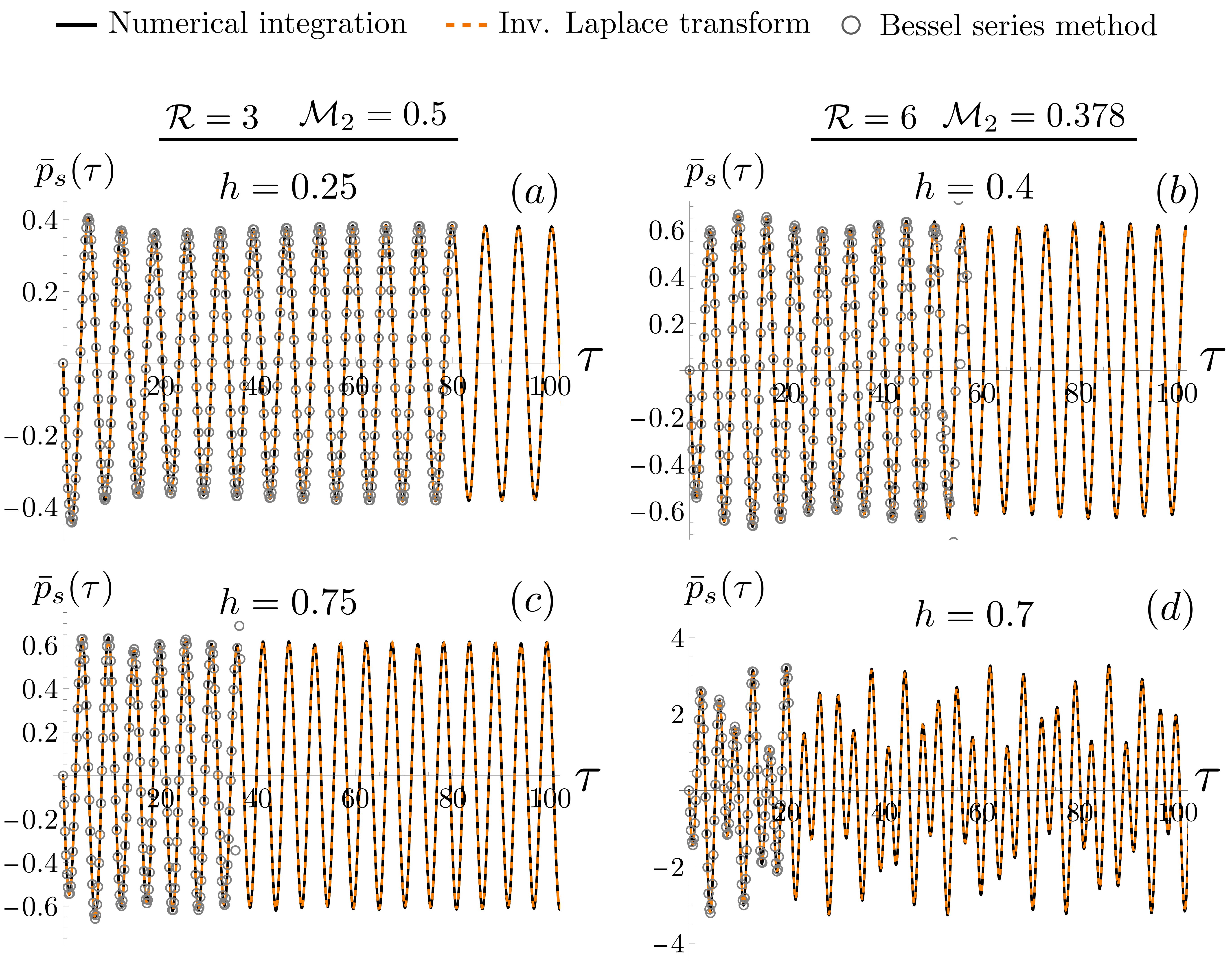


$$\bar{p}_1^\infty = \frac{\sigma_d}{\sqrt{1+4\sigma_c(\sigma_c-\sigma_b)}}\left\{\frac{1}{s_1} + \frac{1}{s_1\cosh\chi_s + \sqrt{s_1^2-1}\sinh\chi_s} + \right.$$

$$+\left[\sqrt{s_1^2-1}\cosh\chi_s + s_1\sinh\chi_s - \sigma_b\left(s_1\cosh\chi_s+\sqrt{s_1^2-1}\sinh\chi_s\right) + \right.$$

$$\left.\left. + \sigma_c\left(s_1\cosh\chi_s+\sqrt{s_1^2-1}\sinh\chi_s\right)^{-1}\right]\Pi_s\left[q=\ln(\mathrm{i}(s_1+\sqrt{s_1^2-1})+2\chi_s\right]\right\}$$



A. Calvo-Rivera
C. Huete (speaker)
F. García-Rubio
A. L. Velikovich

Global summary

The stability of steady shocks has been resolved in a great variety of conditions:

- influence of high-temperature effects
- influence of the equation of state
- influence of magnetic fields (only perp.)
- influence of the supporting mechanism:
  - isolated / Noh / piston-driven

Future works include a generalization of the **magnetic field** and the **supporting mechanism** to non-cannonical conditions