Duke Approach: Coupled Boltzmann Equations

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arXiv: 2004.06746

EMMI Rapid Reaction Task Force: Suppression and (re)generation of quarkonium in heavy-ion collisions at the LHC

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Dec. 12 2022

Coupled Transport Equations of Heavy Flavors

Open heavy quark antiquark

$C_{Q\bar{Q}}$: HQ elastic scattering and radiation

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

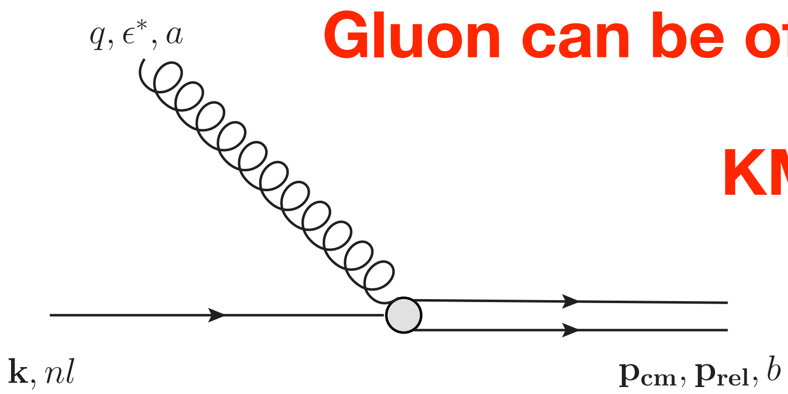
Each quarkonium state, nl = 1S, 2S,1P etc.

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$

Collision terms: dissociation and recombination

$$\mathcal{C}^{-}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \frac{g^2 T_F}{3N_c} \int \frac{d^3p_{\mathrm{cm}}}{(2\pi)^3} \frac{d^3p_{\mathrm{rel}}}{(2\pi)^3} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\mathrm{cm}} + \boldsymbol{q}) \delta\Big(E_{nl} - \frac{p^2_{\mathrm{rel}}}{M} - q_0\Big)$$
$$\times |\langle \psi_{nl} | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\mathrm{rel}}} \rangle|^2 [g_E^{++}]^{>}(q_0, \boldsymbol{q})\, f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

$$\mathcal{C}^{+}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \frac{g^2 T_F}{3N_c} \int \frac{d^3p_{\mathrm{cm}}}{(2\pi)^3} \frac{d^3p_{\mathrm{rel}}}{(2\pi)^3} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\mathrm{cm}} - \boldsymbol{q}) \delta\Big(E_{nl} - \frac{p^2_{\mathrm{rel}}}{M} + q_0\Big)$$
$$\times |\langle \psi_{nl} | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\mathrm{rel}}} \rangle|^2 [g_E^{--}]^{>}(q_0, \boldsymbol{q})\, f_{Q\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{r} = 0, \boldsymbol{p}_{\mathrm{rel}}, t)$$



Gluon can be off-shell, described by nonperturbative object $[g_E^{\pm\pm}]^{>}(q)$

KMS relation: $[g_E^{++}]^{>}(q) = e^{q_0/T}[g_E^{--}]^{>}(-q)$ —> thermalization

We used perturbative estimate of $[g_E^{\pm\pm}]^{>}(q)$

Dipole Transition Amplitudes

$$\left|\langle \psi_{nl} | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle\right|^2$$

Bound state wavefunction w/ quantum number nl

Scattering (unbound) state wavefunction w/ relative $p_{\rm rel}$

For Coulomb potential ($\alpha_s = 0.36$)

$$|\langle \Psi_{\boldsymbol{p}_{\rm rel}} | \boldsymbol{r} | \psi_{1S} \rangle|^2 = \frac{2^9 \pi^2 \eta a_B^7 p_{\rm rel}^2 (1+\eta^2)(2+\eta a_B p_{\rm rel})^2}{(1+a_B^2 p_{\rm rel}^2)^6 (e^{2\pi\eta}-1)} e^{4\eta \arctan{(a_B p_{\rm rel})}}$$

$$|\langle \Psi_{\boldsymbol{p}_{\rm rel}} | \boldsymbol{r} | \psi_{2S} \rangle|^2 = \frac{2^{18} \pi^2 \eta a_B^7 p_{\rm rel}^2 (1+\eta^2)}{(1+4a_B^2 p_{\rm rel}^2)^8 (e^{2\pi\eta}-1)} e^{4\eta \arctan{(2a_B p_{\rm rel})}} \left(-4 - 9\eta a_B p_{\rm rel} + 8 a_B^2 p_{\rm rel}^2 - 4\eta^2 a_B^2 p_{\rm rel}^2 + 4\eta a_B^3 p_{\rm rel}^3\right)^2$$

$$a_B = \frac{2}{\alpha_s C_F M}$$

$$\eta = \frac{\alpha_s M}{4 N_c p_{\rm rel}}$$

$$|\langle \Psi_{\boldsymbol{p}_{\rm rel}} | \boldsymbol{r} | \psi_{1P} \rangle|^2 = \frac{2^{16} \pi^2 \eta a_B^5}{9(1+4a_B^2 p_{\rm rel}^2)^8 (e^{2\pi\eta}-1)} e^{4\eta \arctan{(2a_B p_{\rm rel})}} \Big[\Big(8\eta(\eta^2-2)a_B^3 p_{\rm rel}^3 + 12(2\eta^2-1)a_B^2 p_{\rm rel}^2 + 18\eta a_B p_{\rm rel} + 3\Big)^2 + 32 a_B^4 p_{\rm rel}^4 (3+2\eta a_B p_{\rm rel})^2 (\eta^4 + 5\eta^2 + 4)\Big]$$

Correlated Unbound Heavy Quark Pair

Open heavy quark antiquark

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$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^+_{Q\bar{Q}} + \mathcal{C}^-_{Q\bar{Q}}$$

Each quarkonium state, nl = 1S, 2S,1P etc.

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^+_{nls} - \mathcal{C}^-_{nls}$$

$$f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) \neq f_Q(\boldsymbol{x}_Q, \boldsymbol{p}_Q, t) f_{\bar{Q}}(\boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t)$$

Can handle both correlated and uncorrelated recombination

$$\mathcal{C}_{Q\bar{Q}} = \mathcal{C}_Q + \mathcal{C}_{\bar{Q}}$$

Each independently interact with medium:
- **(1) Potential between pair screened**
- **(2) Potential depends on color, average over**
- **(3) Imaginary part provides a time scale for potential effect**

Can be improved by including potential effect on l.h.s.

We use "Lido" for open heavy flavor transport: elastic + radiation

W.Ke, Y.Xu, S.A.Bass, PRC 98, 064901 (2018)

We can use either Langevin or Boltzmann approach for open heavy quarks

Setup

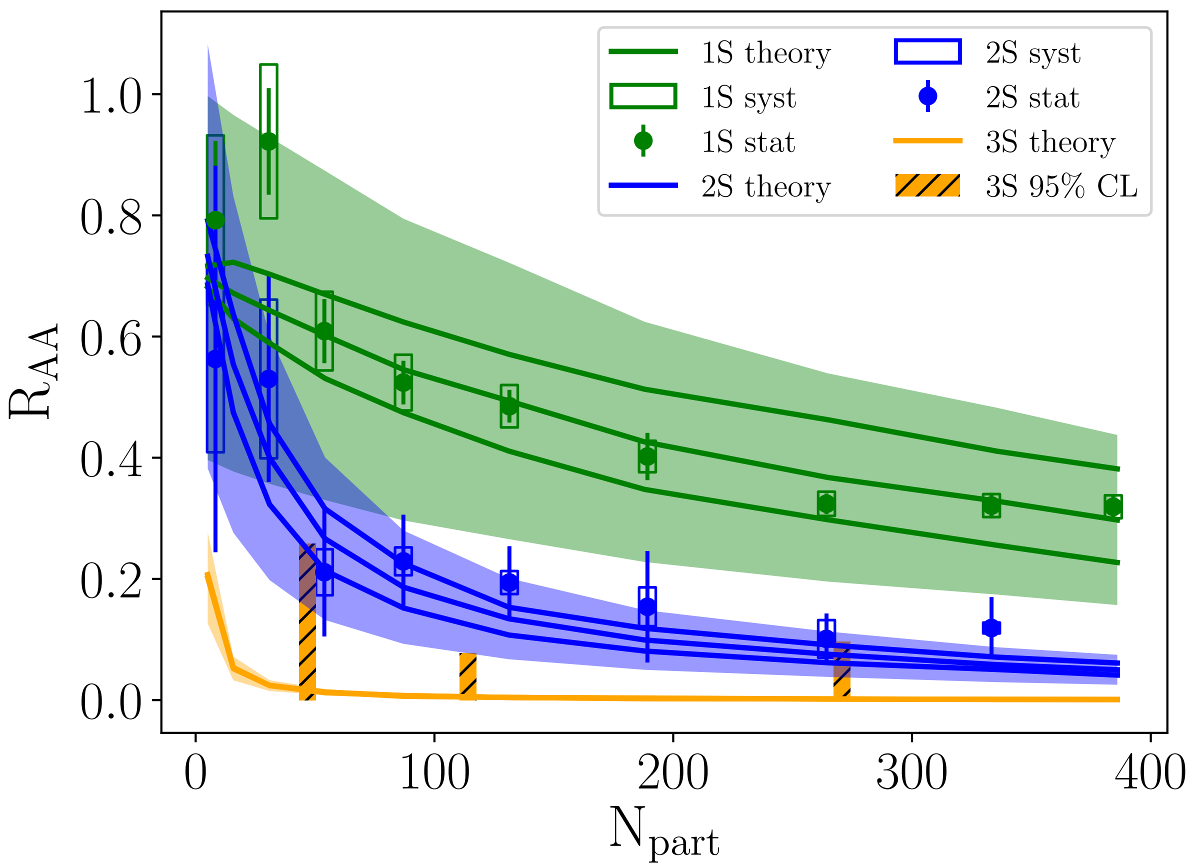
- **Initial condition**
- Momentum distribution and initial yields: Pythia w/ EPPS 16
- Spatial distribution: binary collision density from Trento model—> corona effect included
- Hydro evolution starts at 0.6 fm/c, from t=0 to 0.6 fm/c free streaming
- At low p_T, Υ(1S) formed before 0.6 fm/c. Others (2S, 1P, etc) may have not fully formed —> fine, because if they go inside high temperature region, dissociate in one time step

In other words, Pythia gives initial p_T and y distributions for Υ(1S) and correlated $Q\bar{Q}$ pairs

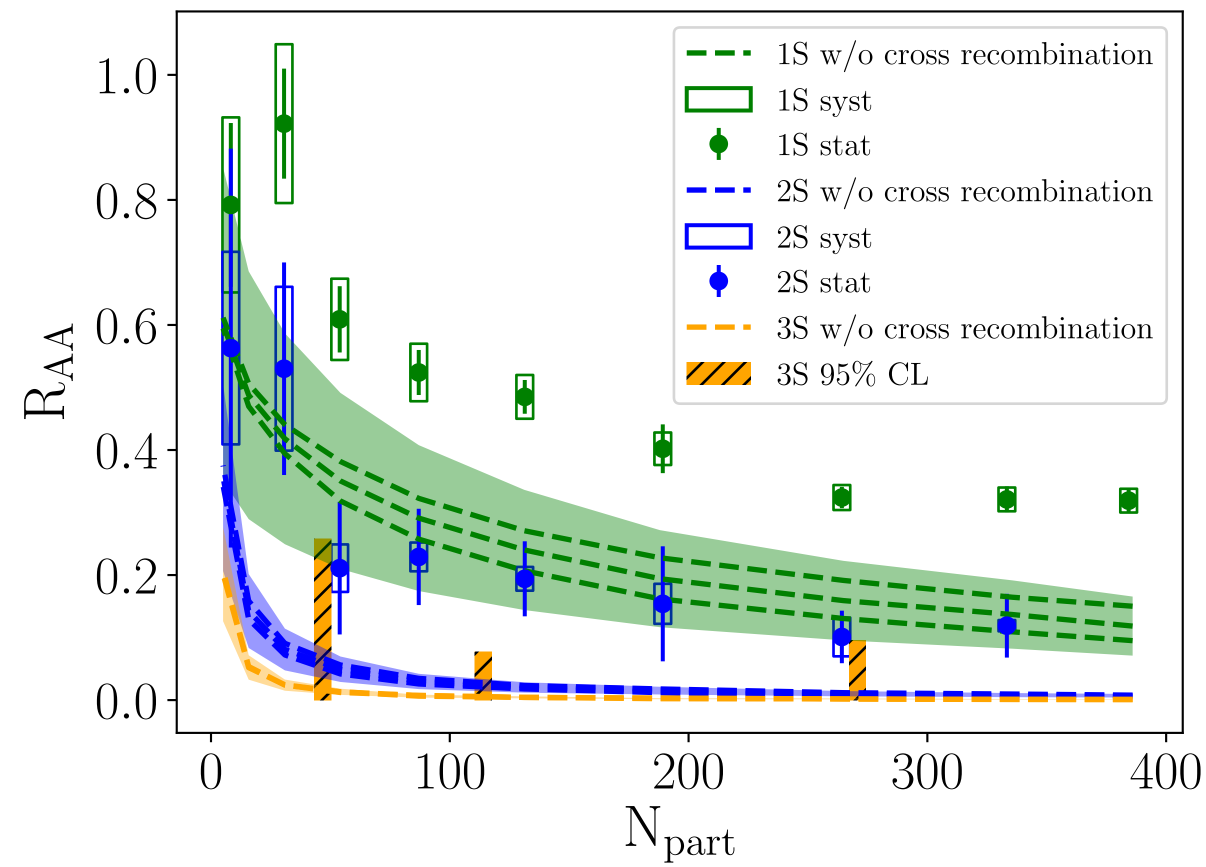
- **In-medium evolution**
- 2+1D viscous hydro: VISHNew, calibrated w/ Trento by bayesian analysis
- Test particle Monte Carlo: at each time step, consider HQ elastic scattering + radiation, dissociation of each quarkonium state, recombination of each pair w/ $x_{\rm rel}$ and $p_{\rm rel}$
- Calculate rates in rest frame of $Q\bar{Q}$ pair, medium boosted—> momentum dependent rates
- If dissociation/recombination occurs, sample final-state momentum according to $C^{\pm}_{nl}$
- Stop coupled Boltzmann equations at $T_c = 154$ MeV

- **Hadronic feed-down**

Results on Bottomonium: Regeneration



W/ correlated recombination

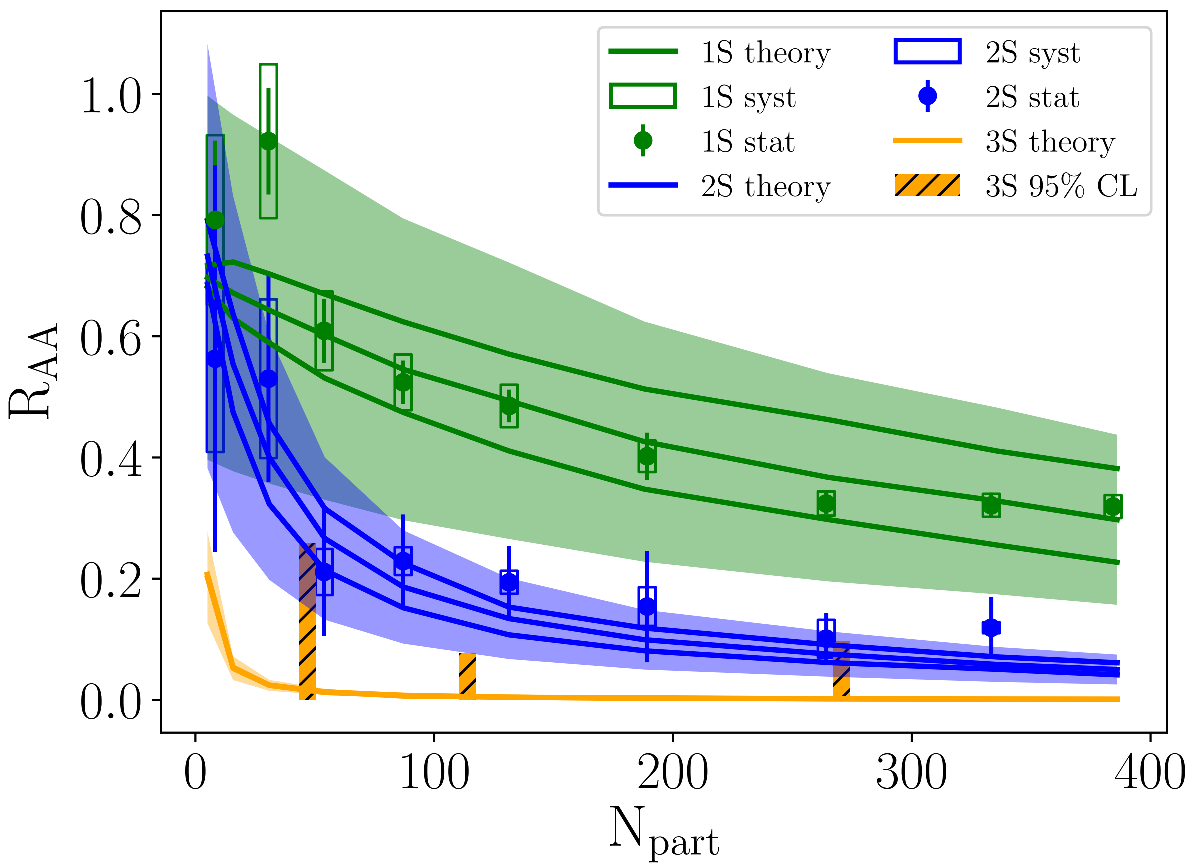


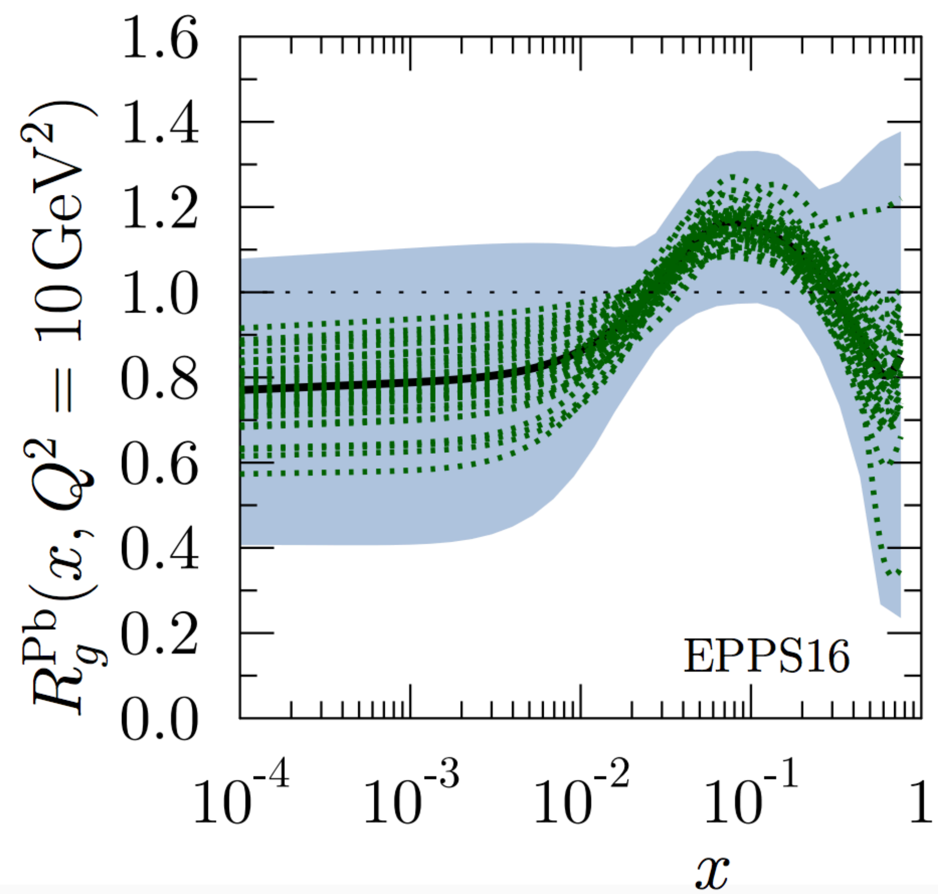
Roughly w/o correlated recombination

In our approach, almost all 2S, 1P states are regenerated at later stage in central heavy ion collisions, at the moment no regeneration of 3S and 2P implemented, 1S—>unbound—>2S is important

At high temperature, dissociation/recombination rates are large for excited states, effectively, they are at chemical equilibrium and thus have tiny densities

Results on Bottomonium: nPDF Uncertainty





Uncertainties from nPDF large

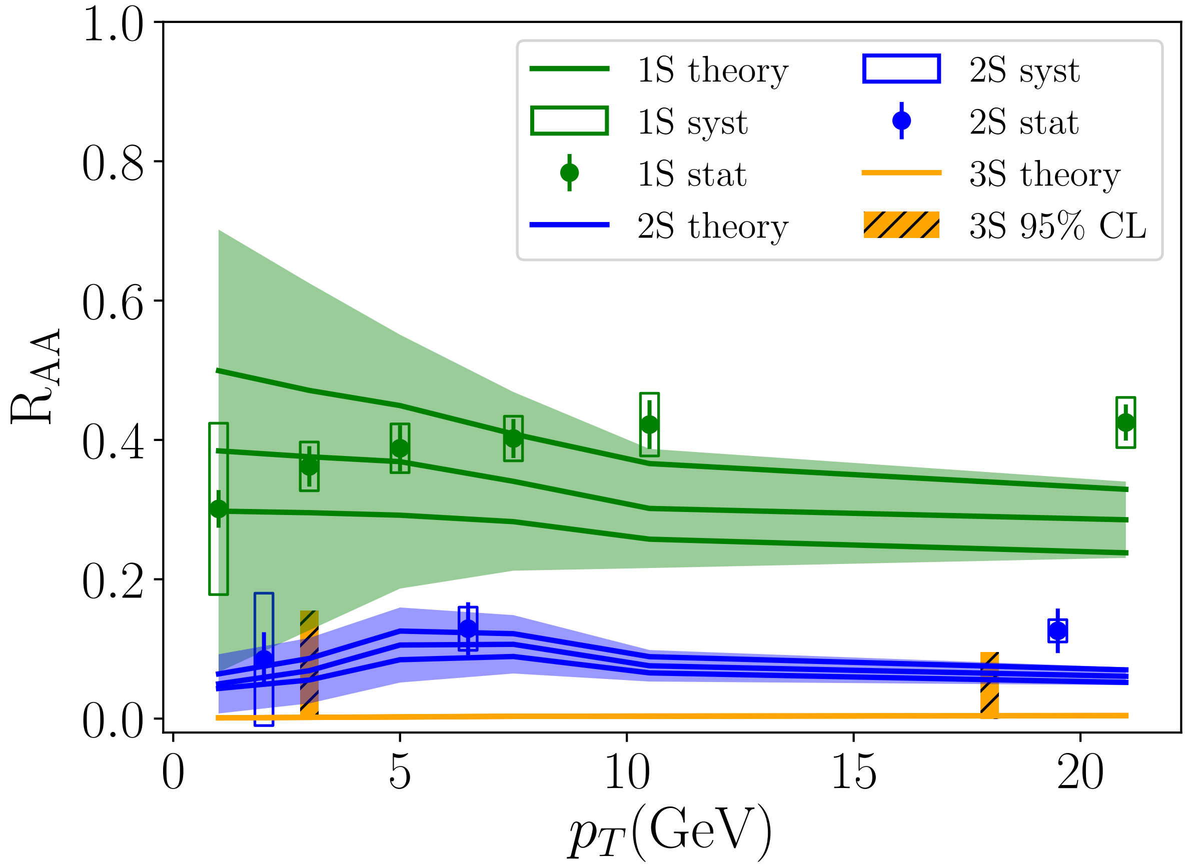
At mid rapidity

$$x \sim \frac{2m_T}{\sqrt{s}}$$

CNM

$$\sim [R_g^{Pb}(x)]^2$$

Results on Bottomonium: p_T Dependence



Start to see deviation at $p_T \gtrsim 10$ GeV ~ $M_{b\bar{b}}$

Relativistic correction becomes important

In vacuum, one can always go to rest frame, where ultrasoft gluons have energy ~ binding energy $\ll M_Q$ —> nonrelativistic expansion works —> NRQCD works for high p_T production

In medium, one can always go to rest frame, but gluons from medium get boosted to have larger energies —> nonrelativistic expansion fails —> at high enough p_T, we start to talk about quarkonium production from jet

Future Prospects

- Potential effect for unbound pair in transport equations: for color singlet, potential effect is on for some time scale related to imaginary part

- Include regeneration of 3S and 2P states

- Beyond Coulomb potential, need an efficient way to sample final-state momentum

- EPPS21 for nPDF

- High p_T quarkonium

- Nonperturbative determination of $[g_E^{\pm\pm}]^>(q)$, which describes on-shell and off-shell gluons in QGP that are relevant for quarkonium screening, dissociation and recombination—> joint effort of theory, computation and experiment

$$[g_E^{++}]^>(t) = \left\langle E_i^a(t) W^{ac}(t,+\infty) W^{cb}(+\infty,0) E_i^b(0) \right\rangle$$

XY, T.Mehen 2009.02408 T.Binder, K.Mukaida, B.Scheihing-Hitschfeld, XY, 2107.03945

P.Petreczky, B.Scheihing-Hitschfeld, XY, in preparation See also Munich+KSU approach