From Exotic Charmonia to \mathcal{T}_{cc} and \mathcal{T}_{bb} Luciano Maiani CERN, Geneve, CH

> *GSI, Darmstadt October 13, 2022*

Introducing quarks to describe the known mesons and baryons, Murray Gell-Mann, in 1964, suggested the existence of further $qq\bar{q}\bar{q}$ mesons (tetraquarks) and $qq\bar{q}\bar{q}$ baryons (pentaquarks).

The firsty *unexpected* hadron, the X(3872), was discovered by BELLE in 2003, confirmed by BABAR and seen in many other High Energy experiments. Since then, a wealth of Exotic Hadrons have been observed, mesons and baryons that cannot be described by the classical Gell-Mann, $q\bar{q}$ and qqq, configurations.

I shall illustrate this new chapter of Particle Spectroscopy with a selection of results on the structure and symmetry pattern of the Exotic Hadrons, recent experimental discoveries and future perspectives.

1. The Standard Theory of Particle Interactions

E. M. INTERACTIONS= QED



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QCD is the answer to (almost) any question



- QCD is asymptotically free
- quarks carry color symmetry, SU(3)_{col}, and are confined inside color singlet hadrons,
- $\Delta^{(++)} = \epsilon^{\alpha\beta\gamma} u^{\dagger}_{\alpha} u^{\dagger}_{\beta} u^{\dagger}_{\gamma}$: Fermi statistics is obeyed

Constituent Quarks

OCD Partons

- increasing q², quarks radiate gluons (the Altarelli-Parisi picture of scaling violations)
- at large q^2 , we see quarks and neutral gluons as almost free partons.

Heavy quarks ($m_Q > > \Lambda_{QCD}$):

- inclusive decays are calculable like deep inelastic processes;
- $c\bar{c}$ or $b\bar{b}$ bound states involve short distance forces: a calculable spectrum of charmonia/bottomonia;
- inside hadrons, $c\bar{c}$ or $b\bar{b}$ pairs are not easily created or destroyed: a hadron decaying into J/Ψ or $\Upsilon + \ldots$ indicates a valence $c\bar{c}$ or $b\bar{b}$ pair: *heavy-quark counting is possible*.

2. Unanticipated charmonia X, Y, Z.. and more

- Hidden charm/beauty resonances not fitting in preedicted charmonium/bottomonium spectrum because of mass/decay properties or because charged.
 - X, e.g. X(3872): neutral, typically seen in Ψ + pions, positive parity, $J^{PC} = 0^{++}$, 1⁺, 2⁺⁺



Figure 1: From Belle [31], the mass recoiling against $\pi^+\pi^-$ pairs, $M_{\rm miss}$, in e^+e^- collision

- Y, e.g. Y(4260): neutral, seen in e⁺e⁻ annihilation with *Initial State Radiation* (ISR) $(e^+e^- \rightarrow e^+e^- + \gamma_{ISR} \rightarrow Y + \gamma_{ISR})$, therefore $J^{PC} = 1^{--}$,
- Z, e.g. Z(4430): charged/neutral, typically positive parity, 4 valence quarks manifest, mostly seen to decay in Ψ + π and some in h_c(1P) + π (valence quarks: $c\bar{c}u\bar{d}$);
- Z_b observed $(b\bar{b}u\bar{d})$.

A new wave of Exotic Hadrons started in 2016:

- Hidden charm and Hidden strangeness seen, e.g. $X(4140) \rightarrow \Psi + \phi$, J^{PC=1++}
- 4 charm tetraquarks seen as di- Ψ resonances by LHCb, e.g. $X(6900) \rightarrow \Psi + \Psi \rightarrow (\mu^+ \mu^-)^2$
- Hidden charm- Open strangeness $(c\bar{c}u\bar{s})$, seen (last year!) by BES III: $Z_{cs}^+(3985) \rightarrow \Psi + K^+$ and by LHCb: $Z_{cs}^+(4003) \rightarrow \Psi + K^+$.
- Double charm tetraquark seen by LHCb: $\mathcal{T}_{cc}^+(3875) \to D^0 D^0 \pi^+$ (valence quarks: $cc\bar{u}\bar{d}$)

The saga of $Z^{\pm}(4430)$

(2008)

42



PRL100, 142001 Belle observed Z(4430)[±] \rightarrow ψ (2S) π^{\pm}

- Found in $\psi(2S)\pi^+$ from $B \rightarrow \psi(2S)\pi^+K$. Z parameters from fit to $M(\psi(2S)\pi^+)$
- Confirmed through Dalitz-plot analysis of $B \rightarrow \psi(2S)\pi^*K$
- B→ψ(2S)π⁺K amplitude: coherent sum of Breit-Wigner contributions
- Models: all known K^{*}→Kπ^{*} resonances only

all known K* \rightarrow K π^+ and Z⁺ $\rightarrow \psi$ (2S) $\pi^+ \Rightarrow$ favored by data



- [cu][cd] tetraguark? neutral partner in ψ'π⁰ expected
- D*D₁(2420) molecule? should decay to D*D*π

LHCB:

confirms BELLE's observation of a bump

- CANNOT be built as a molecule of standard states: $D^*D_1 = in S$ -wave may have J=1 but has negative parity

Argand Plot shows 90⁰ phase: Z is a genuine resonance

BaBar doesn't see a significant Z(4430)+



"For the fit ... equivalent to the Belle analysis...we obtain mass & width values that are consistent with theirs,... but only ~1.9 σ from zero; fixing mass and width increases this to only ~3.10."

 $BF(B^0 \rightarrow Z^+K) \times BF(Z^+ \rightarrow \psi(2S)\pi^+) \leq 3.1 \times 10^{-5}$

0.2

0.4

-0.0

4477 Me\

0.4

0.2

- Belle PRL: (4.1±1.0±1.4)x10-5
- Babar inserts in the fit all K* resonances - is Belle effect due to K* reflections ???

[PRL 112 (2014) 222002]





0.2 Do AZ

LHCb

Z(M.F)

605 Me¹

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The unexpected $J^{PC}=1$ -resonance, Y(4260), and its unexpected $J^{PG}=1$ ++ descendants (2013)

Observed by BES III, BELLE and CLEO: $Y(4260) \rightarrow \pi^{\pm} Z_c^{\mp}(3900) \rightarrow \pi^{+} \pi^{-} \Psi$

tetraquark de-excitation?



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Expected and Unexpected Charmonia



Compact tetraquarks $[cq][\bar{c}\bar{q}']$: the first multiplets

- The spectrum of 1S ground states is characterised by two quantities:
 - the diquark mass, m_[cq]
 - the spin-spin interaction inside the diquark or the antidiquark, κ_{cq} .
- The first radially excited, 2S, states are shifted up by a common quantity, the radial excitation energy, ΔE_r expected to be mildly dependent on the diquark mass: $E_r(cq) \sim E_r(cs)$



J/ Ψ - φ structures and S-wave tetraquarks (2016)



No consensus, yet



In most cases Exotic Hadrons are produced in the decay of Beauty Mesons or Baryons but, for X(3872), production from unelastic reactions at large p_T has been observed (by CMS). This gives an important clue about its structure



Rescaling from Pb-Pb ALICE cross sections to p-p CMS cross section is done with: Glauber model (**left panel**) and blast-wave function (**right panel**}) (R_{AA} or $R_{CP} = 1$)

- There is a vast difference in the probability of producing X(3872) and that of producing light nuclei, true "hadronic molecules", in high energy collisions
- high energy production of suspected exotic hadrons from quark-gluon plasma in HI collisions at colliders can be very effective tool to discriminate different models
- a long list of suspects: $f_0(980)$, X(3872), $Z^{\pm}(3900)$, $Z^{\pm}(4020)$, $Z^{\pm}(4430)$, X(4140)....

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can molecule be saved by mixing with charmonium?

3. Generating multiquark hadrons in QCD



Lowest lying Spectroscopic series (color singlets) in 3 colours QCD

• Spectroscopic series depend upon the number of colors: with 2 colours we have only $q\bar{q}$ mesons and qq "baryons" : all extra states must be molecules.

• Multiquark states can be formulated in arbitrary number of colours, N,

• *With 3 colours*, from baryons *qqq* we generate *three spectroscopic series*: tetraquarks, pentaquarks and dibaryons, all in different flavours.

• BELLE, BESIII, LHCb have produced well established examples of (valence) tetraand pentaquarks, no dibaryon has bee seen thus far.

• A recent lattice QCD calculation shows a deeply bound 6 b-quarks dibaryon: $\Omega_{bbb}\Omega_{bbb}$, $E_B = -.89^{+16}_{-12}$ MeV

N. Mathur et al. ArXiv:2205.02862

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4. Exotics: the New Wave



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new wave (cont'd)

- Starting from 2016, new kinds of exotic hadrons have been discovered:
 - $J/\Psi \phi$ resonances, $di J/\Psi$ resonances,
 - open strangeness Exotics: $Z_{cs}(3082)$ and $Z_{cs}(4003)$
- No pion exchange forces could bind them as hadron molecules made by color singlet mesons
- molecular models applied to the new hadrons have to stand on the existence of "phenomenological forces" with undetermined parameters
- The New Exotics arise very naturally as $([cq]^{\bar{3}}[\bar{c}\bar{q}']^3)_1$ bound in color singlet.

The compact tetraquark model makes a firm prediction: hidden charm tetraquarks must form *complete multiplets of flavor SU(3)* with mass differences determined by the quark mass difference

 $m_s - m_{u,d}$

with $Z_{cs}(3082)$ and $Z_{cs}(4003)$ we can almost fill two tetraquark nonets with the expected scale of mass differences

Two nonets: Solution 1 (preferred)



•There is a *third nonet* associated to $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, Mass=4150 - 4170 •*LHCb sees a Z_{cs}(4220)*, $J^P = 1^+$ or 1^- : *is it too heavy*? A bold proposal:



A well defined shopping list towards completion:

- $X_{s\bar{s}}, M = 4076$ (Sol. 2 : 4121), decays: $\eta \psi, \eta_c \phi, D_s^* \bar{D}_s$ (if phase space allows)
- the I=1 partner of X(3872), decays: $X^+ \to J/\psi \ \rho^{\pm} \to J/\psi \ \pi^+ \pi^0$
- the I=0 partners of $Z_c(3900)$ and $Z_c(4020)$, possibly decaying into: $J/\psi + f_0(500)$ (aka $\sigma(500)$) GSI, 13/10/2022 L. Maiani. From Exotic Charmonia to \mathcal{T}_{cc} and \mathcal{T}_{bb} 16

5. Molecule or compact ? the QCD framework (following Weinberg's argument about deuteron, PRL 1965)

- We know that QCD produces hidden charm, confined hadron states: charmonia, $D^*\overline{D} + \overline{D}^*D$. ??? Do confined tetraquarks exist??
- Suppose we switch off the interactions between confined hadrons. The space of possible hidden charm st
 is made by two components
 - discrete energy states: charmonia and possibly tetraquarks: $|C > \langle C| + |T \rangle \langle T|$
 - continuum charmed meson pairs: $|D^*\overline{D}(\alpha) > < D^*\overline{D}(\alpha)|$

Completeness relation: $\langle X | X \rangle = 1 = Z + \int d\alpha | \langle X | D^* \overline{D}(\alpha) \rangle |^2$, $Z = |\langle X | C \rangle |^2 + |\langle X | T \rangle |^2$

There are *two regimes*

- Z=0: corresponds to a pure molecular state: X results from $D^* \overline{D}$ interactions only (like the deuteron = pn)
- $Z \neq 0$: some compact, discrete state *must* exist: could it be charmonium only?
- unlike charmonium states, X decays violate isospin: $\Gamma(\Psi \rho) \sim \Gamma(\Psi \omega)$ so that:
- $Z \neq 0 \rightarrow$ Tetraquark with X quantum numbers exists.

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How can we know?

S. Weinberg, Phys.Rev. 137, (1965) B672

- The key is the $D^*\bar{D}$ scattering amplitude, f, that near threshold (k=center of mass momentum~0) can be parametrised as $f^{-1} = k \cot \delta(k) ik = -\kappa_0 + \frac{1}{2}r_0k^2 ik + \dots$
- in presence of a shallow, below-threshold, resonance Weinberg finds:

$$\kappa_0^{-1} = 2 \frac{1-Z}{2-Z} \kappa^{-1} + O(1/m_\pi); \ r_0 = -\frac{Z}{1-Z} \kappa^{-1} + O(1/m_\pi)$$

 $\kappa^{-1} = \sqrt{2\mu B}, B = M(D^*) + M(D) - M(X)$ (the "binding energy")

- in the molecular case (Z=0) one has $r_0 = O(1/m_{\pi})$
-and, for attractive potentials, one can show that *the unspecified part* $O(1/m_{\pi})$ *is positive*: $r_0 > 0$

Rev. D 105 (2022), L031503

X and \mathcal{T}_{cc} lineshape: from Breit-Wigner to scattering lengths

R. Aaij et al. (LHCb), PRD 102, (2020) 092005

• Consider $D^{*0}\overline{D}^0$ scattering above threshold. If there is a resonance slight below, the amplitude takes the Breit-Wigner form

$$f = -\frac{\frac{1}{2}g_{\rm BW}^2}{E - m_{\rm BW} + \frac{i}{2}g_{\rm BW}^2k}$$
(A)

• for $E = \frac{k^2}{2\mu}$ and small k, the BW reduces to the scattering amplitude

$$f = \frac{1}{\cot \delta(k) - ik} = \frac{1}{-\kappa_0 + \frac{1}{2}r_0k^2 - ik + \dots}$$
(B)

- from the parameters of the line-shape (A) we can determine κ_0 and r_0 in (B)
- for X(3872), taking into account the experimental errors, the effective radius is found to be in the range
 A. Esposito *et al.* Phys. Rev. D 105 (2022).

$$X(3872): -1.6 \text{ fm} > r_0 > -5.3 \text{ fm}$$

V. Baru et al., arXiv:2110.07484

• a similar analysis can be done for \mathcal{T}_{cc} . M Mikhasenko (Arxiv:2203.04622v1) defines: X = 1 - Z and finds



X=1-Z0.65 > Z > 0.360.62 > Z > 0.09

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The value of Z

• From previous Weinberg formulae, we derive

$$Z = \frac{-r_0\kappa}{1 - r_0\kappa}$$

- Z is often identified with the admixture of the molecule with the compact (tetraquark) state and for small Z one would say that X is "essentially" a molecule
- However, the interpretation of Z as mixing coefficient, *holds true in the free theory only*. With interaction, the state vector corresponding to the compact state may be renormalized and the strenght of Z looses its meaning.
- What counts is that Z is non vanishing, indicating that *there are*, in the Hilbert space, discrete states, different from the D D* continuum, to which X has a non-vanishing projection. This is stated clearly in Weinberg's paper:
 - the true token that the deuteron is composite is an effective range r_0 small and positive rather than large and negative
 - *an elementary deuteron would have* 0<Z<1.
- 0 < Z < 1 does not say anything about the existence of bound states in the inter-hadron $D^*\bar{D}$ potential, i.e. molecules: the interaction could be driven by the compact state only and be consistent with no bound molecule at all.

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My Summary about r_0



No consensus yet, but we are on a very promising road. Stay tuned!!

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6. The new sensation: doubly heavy baryons and doubly heavy tetraquarks

Single heavy-doubly heavy quark symmetry: M. Savage, M. B. Wise, PLB 248,1990; N. Brambilla, A. Vairo nd T. Rosch, PRD 72, 2005; T. Mehen, arXiv:1708.05020v3

• Doubly heavy baryons are related to single quark heavy mesons:



- QCD forces are mainly spin independent, so there is an approximate symmetry relating masses of DH baryons to SH mesons: e.g. $M(\Xi_{cc}^*) M(\Xi) = \frac{3}{4}(M(D^*) M(D))$
- Doubly heavy tetraquarks have been anticipated long ago.
 Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Phys. Rev. D88, 054029 (2013)
- The possibility has been raised that the I = 0, $J^P = 1^+$, $\mathcal{T}_{cc}^+ = bb\bar{u}\bar{d}$ be stable under strong and e.m. decays M. Karliner and I. L. Rosner, PRL **119** (2017) 202001, E. I. Eichten and

M. Karliner and J. L. Rosner, PRL **119** (2017) 202001. E. J. Eichten and C. Quigg, PRL **119** (2017) 202002.; S. Q. Luo et al. Eur. Phys. J. C **77** (2017) 709.

- extended calculations of \mathcal{T}_{cc} and \mathcal{T}_{bb} mass have been presented, in the Born-Oppenheimer approximation, analytical L. Maiani, A. D.Polosa and V. Riquer, Phys. Rev. **D100** (2019) 074002,
- and in Lattice QCD see later for Reffs.

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Radiography of the double beauty tetraquark

0.2

0.3

Born-Oppenheimer approximation in 3 steps:

- take the b quarks as fixed sources at distance R;
- find V_{BO} = energy of the bound state of light $\frac{2}{5}$ quarks in presence of the b quarks;
- solve Schroed. equation of b quarks with potential $V_{BO}(R)+V_{bb}(R)$.

For $R \to +\infty V_{BO}(R)$ vanishes in both cases: why not confined?

- at ∞, each b quark is screened by a q
 : the state is a superposition of
- $B_{color 8} B_{color 8}$ and $B_{color 1} B_{color 1}$
- but in $B_{color 8} B_{color 8}$ color can be screened by soft gluons from vacuum
- the asymptotic state is $B_{color 1} B_{color 1}$ with vanishing interaction at $R \to \infty$
- Projection of the lattice $\bar{q}\bar{q}$ result over Dd (yellow) or BB (blue) gives a picture of the space arrangement of light quarks at a given bb distance, R
- Mainly $[bb]_{\bar{3}}[\bar{q}\bar{q}]_{3}$ at the peak of the bb wave function ~0.2 fm.



0.7

0.8

υυ

0.9

0.6

CC

0.5

r (fm]

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The mass of the lightest double heavy tetraquarks can be computed!

- There are recent estimates of the mass of of the lightest, double heavy tetrquarks, that may indicate that the I=0, $bb\bar{u}\bar{d}$ tetraquark may be stable. We give in the table below a comparison of different theoretical results.
- Q-value is taken with respect to PS-PS threshold (not V-PS!) $M(D^*) M(D) = 140 \text{ MeV}$
- *BO revised*: L. Maiani, A. Pilloni, A. D. Polosa and V.~Riquer, [arXiv:2208.02730 [hep-ph]]

$QQ\bar{u}\bar{d}$	BO revised [1]	K. and R.[2]	E. and Q.[3]	L.[4]	Lattice QCD
$cc\bar{u}\bar{d}$	+136(+119)	+140	+102	+39	-23 ± 11 Junn. <i>et al.</i> [5]
$bbar{u}ar{d}$	-1.8(-9.7)	-170	-121	-75	$\begin{array}{cccc} -143 \pm 34 & \text{Junn.[5] et al.} \\ -143(1)(3) & \text{Francis et al.[6]} \\ -82 \pm 24 \pm 10 & \text{Leskovec et al.[7]} \\ -13^{+38}_{-30} & \text{P.Bicudo et al.[8]} \end{array}$

[1] L. Maiani, et al. (2022) [2]M. Karliner and J. L. Rosner (2017). [3] E. J. Eichten and C. Quigg, (2017); [4] S. Q. Luo et al. (2017)

Lattice QCD:

[5] Junnarkar *et al.* Phys. Rev. **D 99** (2019) 034507; [6] Francis *et al.* Phys. Rev. Lett. **118** (2017), Phys. Rev. **D 99** (2019); [7] L. Leskovec *et al.* Phys. Rev. **D 100** (2019) 014503; [8] P. Bicudo *et al.* (BO in lattice QCD) Phys. Rev. **D 103** (2021) 114506.

Born-Oppenheimer approximation:

- Estimate of \mathcal{T}_{cc} mass close to the observed mass: $M(\mathcal{T}_{cc}^{BO}) = 3871(3854) \leftrightarrow \text{LHCb} : \mathcal{T}_{cc}^+(3875)$
- Stable Double Beauty tetraquarks ? still possible but not so clear
- Searching for \mathcal{T}_{cc} , I = 1 around DD/D^*D thresholds *is essential!!*

7. Pentaquarks



Three non strange pentaquark lines: P^{N} (4312): M = 4311.9 ± 0.7+6.8, Γ = 9.8 ± 2.7+3.7 P^{N} (4440): M = 4440.3 ± 1.3+4.1, Γ = 20.6 ± 4.9+8.7 P^{N} (4457): M = 4457.3 ± 0.6+4.1, Γ = 6.4 ± 2.0+5.7

> $\Lambda_b \rightarrow J/\Psi + p + K^-$, R.~Aaij et al. [LHCb], Phys. Rev. Lett. **122** (2019) 222001, arXiv:1904.03947 [hep-ex]

In addition two (three ?) strange pentaquark lines seen

 $B^- \rightarrow J/\Psi + \Lambda + \bar{p}$, LHCb Paper 2022-031,

R.Aaij *et al.* [LHCb], Sci. Bull. **66** (2021), 1278; arXiv:2012.10380

PΛ (4338) : M = 4338.2 ± 0.7 MeV, Γ=7.0±1.2MeV PΛ (4455) : M = 4454.9 ± 2.7 MeV, Γ = 7.5 ± 9.7 MeV PΛ (4468) : M = 4467.8 ± 3.7 MeV, Γ = 5.2 ± 5.3 MeV



Theory

1. Molecules?

- Non strange Pentaquarks : $\Sigma_c \bar{D}^0$ (spin 1/2), $\Sigma_c \bar{D}^{*0}$ (spin 1/2, 3/2) Karliner & Rosner
- Strange Pentaquarks : $\Xi_c \overline{D}^0$ (spin 1/2), $\Xi_c \overline{D}^{*0}$ (spin 1/2, 3/2)

2. Compact Pentaquarks ?

Maiani, Polosa, Riquer, in preparation

arXiv:2207.07581

- We consider non strange Pentaquark in the Born-Oppenheimer approximation, with $\bar{c} [cu] [ud] \rightarrow [(\bar{c}c)_8 \times (uud)_8]_1$,
- To extend to a full flavour SU(3) multiplet one must consider the restrictions due to Fermi Statistics to the configurations of *three light quarks in colour octet*
- We consider the exchange of colour, coordinates and flavour \times spin (summarised in the $SU(6) \supset SU(3)_f \times SU(2)_{spin}$ symmetry)
- We find that full quark antisymmetry is reached for *two* flavour-spin configurations: $\mathbf{8}_{1/2} + \mathbf{10}_{3/2} = \mathbf{56}$ or $\mathbf{1}_{3/2} + \mathbf{8}_{1/2} = \mathbf{20}$.
- In both case, the ground state is in $\mathbf{8}_{1/2}$. Combining with with $c\bar{c}$ spin= 0,1, this geves rise to three Pentaquark octets: $2 \times \mathbf{8}_{1/2} + \mathbf{8}_{3/2}$
- We *predict three lines*, corresponding to pentaquark decays: $\mathscr{P}^{(S=0)} \to J/\Psi + p$ or to: $\mathscr{P}^{(S=-1)} \to J/\Psi + \Lambda$, as observed
- decays $\mathscr{P}^{(S=-1)} \to J/\Psi + \Sigma$ and $\mathscr{P}^{(S=-2)} \to J/\Psi + \Xi$ are also predicted, e.g.:

 $\Omega_b^-(bss) \to J/\Psi + \Xi^- + K_S; \ B^-(b\bar{d}) \to J/\Psi + \Xi^0 + \bar{\Sigma}^-$

• The two alternatives (56 or 20) would be distinguished by presence or absence of Pentaquarks decaying into spin 3/2 resonances, e.g. : $\mathscr{P}^* \to J/\Psi + \Delta^{++} \to J/\Psi + p + \pi^+$

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Born-Oppenheimer approximation for QCD, in a nutshell

- $c(x_A)$, $\bar{c}(x_B)$ are treated as sources at fixed position (distance R) and fixed color ($\mathbf{8}_{SU(3)}$ colour)
- find the energy eigestate of the light particles $\epsilon(x_A, x_B)$)
- in molecular physics one assumes light particles in orbitals around the sources
- we do the same in QCD, as in the figure



- Solve the Schroedinger equation for $c(0) \bar{c}(x)$ with potential $V(x) = V_{c\bar{c}}(x) + \epsilon(0, x)$
- the eigenvalue, E, is the energy of the whole system

Preparing for Panda

- Panda could provide an additional source of high resolution results on exotic states produced by direct formation
- In particular, a high resolution study of X(3872) and related decay channel could shed light to the greatest challenges of multiquark theory of exotics:
 - 1. is the X(3872) line split into two almost degerate tetraquarks $X_u = [cu][\bar{c}\bar{u}], X_d = [cd][\bar{c}\bar{d}], \text{ with } \Delta M \leq 1 \text{ MeV}?$
 - 2. does the charged partner $X^+ \to J/\Psi + \rho^+ \to J/\Psi + \pi^+ + \pi^0$ exists?

(1.) is compatible with the present LHCb resolution,

for (2.) the present information is limited to:

$$R = \frac{\Gamma(B^0 \to K^+ + X^- \to K^+ + J/\psi + \pi^- + \pi^0)}{\Gamma(B^0 \to K^0 X(3872) \to K^+ + J/\psi + \pi^- + \pi^+)} < 1 \text{ (PdG)},$$
we estimate: $R = 0.6 - 0.3$

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In conclusion....

- First class results on Exotic Hadrons have been obtained in the last decade by BELLE, BES III and LHCb
- The nex decade will see substantial progress by the same collaborations, with upgraded detectores, and with the start of Panda
- The existence of exotic SU(3) flavour multiplets, with a characteristic scale of symmetry breaking is a distinctive prediction of compact tetraquarks.
- The newly found strange exotics are close in mass, like X(3872) and $Z_c(3900)$, and fit into their nonets: a clear score in favour.
- Lineshape analyses of both X(3872) and $\mathcal{T}_{cc}^+(3875)$ seem to indicate negative values of r_0 .
- Much remains to be done, to produce more precise data and to search for still missing particles, to complete the flavour multiples required by QCD bound, multiquark Exotics.
- Among the missing particles:
 - $X(3872)^+$, isX(3872) split into two lines: $X(3872) \rightarrow X_u + X_d$?

-
$$\mathcal{T}_{cc}^{++}(?), \, \mathcal{T}_{bb}^{-}(?)$$

- many other states are still missing, with well defined mass and decay modes as discussed before.
- Quantum numbers of Pentaquarks and of $di J/\Psi$
- Exotic hadrons produced in hadron collisions at large p_T : are there other, besides X(3872)?
- *Tough orders*: more luminosity, better energy definition, detectors with exceptional qualities... a lot of work...
- Close exchange between theory and experiments is essential and it has to continue.

so much accomplished, and so much more left to do (Winston Churchill)

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