# $a_\psi$ in Presence of $c^{\mu\nu}$

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EMMI Workshop, Laboratoire Kastler-Brossel

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# Outline

The Lorentz-Violating Standard Model Extension (SME)

$a_\psi$ in Presence of $c^{\mu\nu}$

Conclusions

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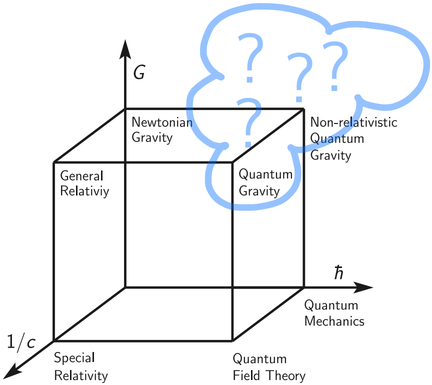
# Lorentz Violation and the SME*




*Don Colladay, Alan Kostelecký, arXiv:9703464,9809521

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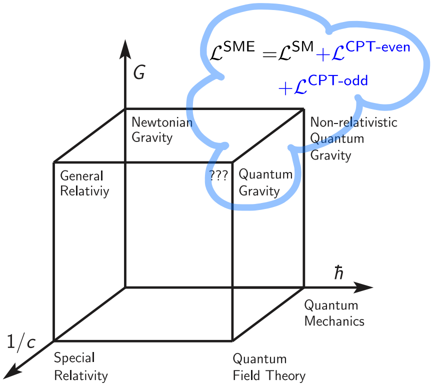


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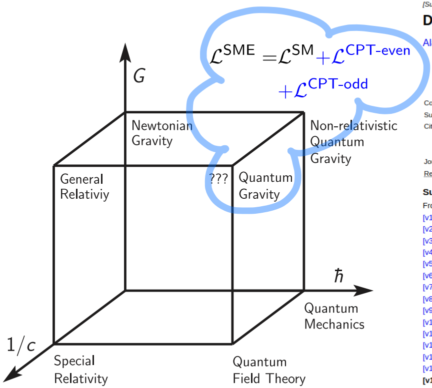


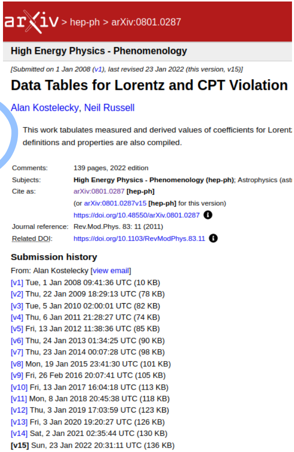
arXiv > hep-ph > arXiv:0801.0287

**High Energy Physics - Phenomenology**

*[Submitted on 1 Jan 2008 (v1), last revised 23 Jan 2022 (this version, v15)]*

**Data Tables for Lorentz and CPT Violation**

Alan Kostelecky, Neil Russell

This work tabulates measured and derived values of coefficients for Lorent
definitions and properties are also compiled.

| | |
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| Subjects: | **High Energy Physics - Phenomenology (hep-ph)**; Astrophysics (ast |
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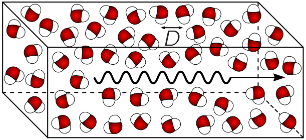
**[v15]** Sun, 23 Jan 2022 20:31:11 UTC (136 KB)

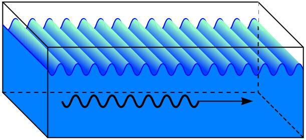
*Don Colladay, Alan Kostelecký, arXiv:9703464,9809521

# SME: An Effective Description of Lorentz Violation




Particle/wave with $\lambda \sim D$ "sees" complicated molecular physics

Particle/wave with $\lambda \sim l_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}$ m "sees" strings? loops? spacetime foam?



Particle/wave with $\lambda \gg D$ "sees" an isotropic medium

Particle/wave with $\lambda \gg l_P$ "sees" a background field

## SME: An Effective Field Theory

- Symmetries: SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$

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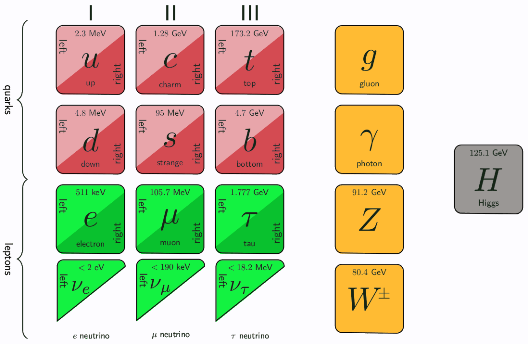

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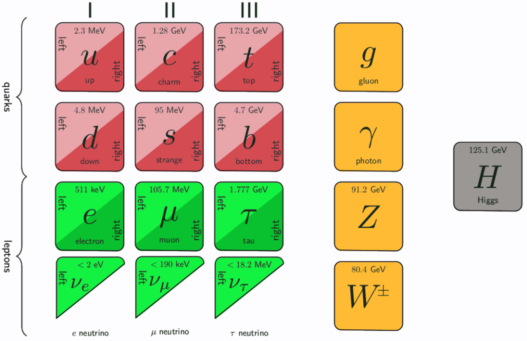


- Expansion coefficient: $\frac{v}{M_P}$, where $v \sim 246$ GeV Higgs vacuum expectation value, $M_\mathrm{P} = \sqrt{\frac{\hbar c}{G}} \sim 10^{19}$ GeV Planck mass

# Standard-Model Extension (SME): The Lepton Sector*

- Strength of Lorentz violation ("Wilson coefficient")
  $\Leftrightarrow$ vevs of tensor-valued background fields: "controlling coefficients"

*Don Colladay, Alan Kostelecký, arXiv:9809521

# Standard-Model Extension (SME): The Lepton Sector*

- Strength of Lorentz violation ("Wilson coefficient")
  $\Leftrightarrow$ vevs of tensor-valued background fields: "controlling coefficients"
- Background fields contracted with SM-fields, e.g.

$$\mathcal{L}^{\text{SM}}_{\text{lepton}} = \frac{1}{2}\overline{L}_A i\partial\!\!\!/ L_A + \frac{1}{2}\overline{R}_A i\partial\!\!\!/ R_A + \text{h.c.}$$

$$\mathcal{L}^{\text{CPT-odd}}_{\text{lepton}} = -\frac{1}{2}\Big(\overline{L}_A(\hat{a}_L)^{\mu}_{AB}\gamma_\mu L_B + \overline{R}_A(\hat{a}_R)^{\mu}_{AB}\gamma_\mu R_B\Big) + \text{h.c.}$$

$$\mathcal{L}^{\text{CPT-even}}_{\text{lepton}} = \frac{1}{2}\Big(\overline{L}_A(\hat{c}_L)^{\mu\nu}_{AB} i\partial_\nu\gamma_\mu L_B + \overline{R}_A(\hat{c}_R)^{\mu\nu}_{AB} i\partial_\nu\gamma_\mu R_B\Big) + \text{h.c.}$$

$L_A$, $L_B$: SM lepton doublets of flavour $A, B$

$(\hat{a})^{\mu}_{AB}$, $(\hat{c})^{\mu\nu}_{AB}$: vectorial, tensorial controlling coefficients connecting the flavours $A$ and $B$

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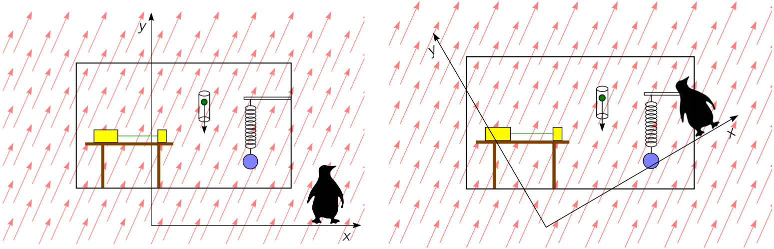
# Observer Lorentz transformation




The observer (the penguin) redefines his x- and y-axes.

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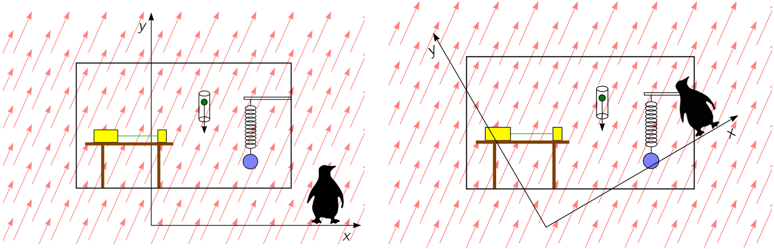
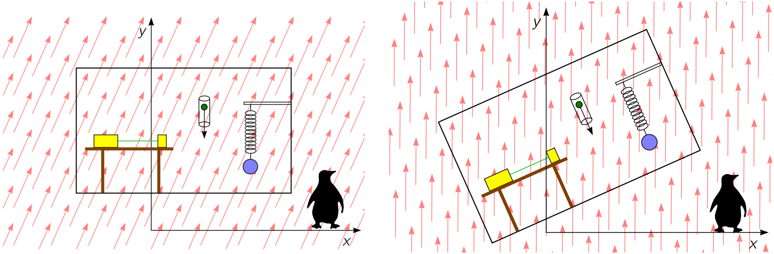



The observer (the penguin) redefines his x- and y-axes.

This does not change the physics.

# Active Lorentz transformation




All coordinates are redefined, but the x- and y-axes of the observer (the penguin) are unchanged.

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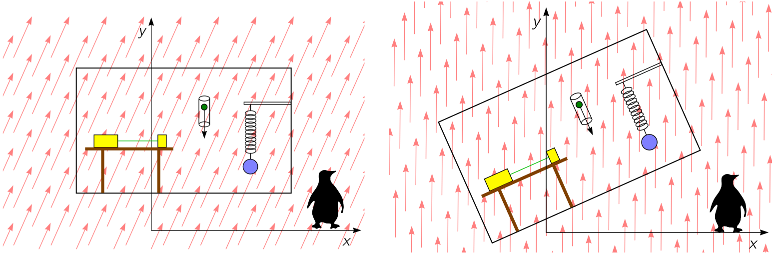


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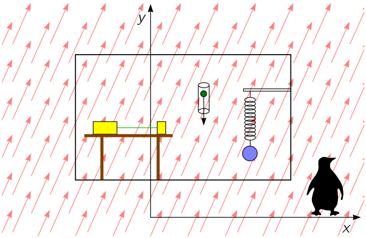
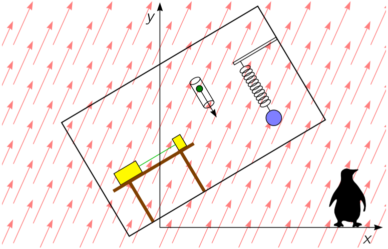
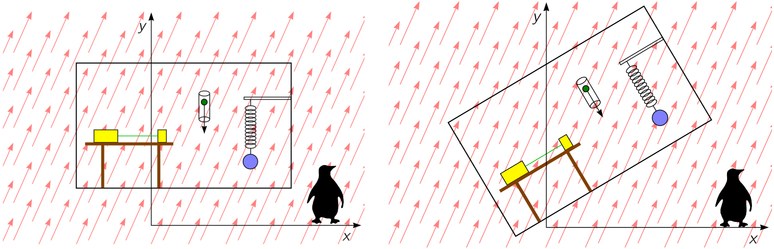
# Particle Lorentz Transformation




Orientation of the laboratory changes wrt. the background field.

# Particle Lorentz Transformation

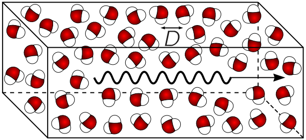



Orientation of the laboratory changes wrt. the background field.

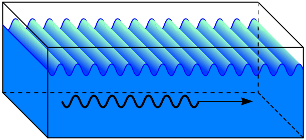
The penguin measures the effect of the background field on the lab.

# Isotropic Lorentz Violation

A medium defines a Lorentz-violating background:




Particle "sees" crazy stuff.



Particle "sees" an isotropic medium, background field.

⇒ modified dispersion relation*

$$E^2 = m^2 + \mathbf{p}^2$$

$$\Rightarrow (1 + \mathring{c})^2 E^2 = m^2 + \left(1 - \frac{\mathring{c}}{3}\right)^2 \mathbf{p}^2$$

*arXiv:1702.03171

# Outline

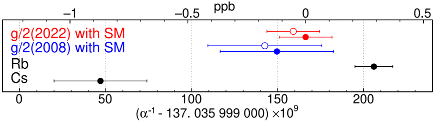
# Electron/Muon Anomalous Magnetic Moments

Electron Anomalous Magnetic Moment*

$$\Delta a_{\mathrm{e}}[\mathrm{Cs}] \equiv a_{\mathrm{e}}^{\mathrm{exp}} - a_{\mathrm{e}}^{\mathrm{SM}}[\mathrm{Cs}] = -0.88(28)(23)[36] \times 10^{-12}\,,$$

$$\Delta a_{\mathrm{e}}[\mathrm{Rb}] \equiv a_{\mathrm{e}}^{\mathrm{exp}} - a_{\mathrm{e}}^{\mathrm{SM}}[\mathrm{Rb}] = +0.48(28)(9)[30] \times 10^{-12}\,,$$

$\alpha$ from Cs/Rb interferometry**, uncertainties on $a_{\mathrm{e}}^{\mathrm{exp}}$, $\alpha$, total uncertainty
update: arXiv:2209.13084




*Hanneke et al., ** Parker et al. / Morel et al. *** Exp. av., Th. Init. WP

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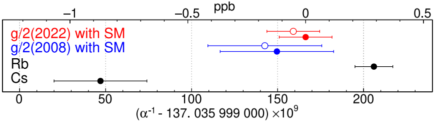


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$$\Delta a_{\mu} \equiv a_{\mu}^{\mathrm{exp}} - a_{\mu}^{\mathrm{SM}} = 2.51(59) \times 10^{-9}\,,$$

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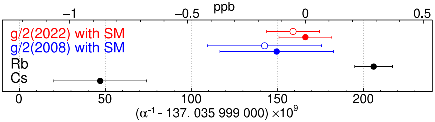


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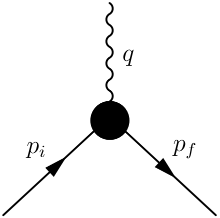
# QED Vertex Function




$$\Gamma^\mu(p_1, p_2, q) = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m}\sigma^{\mu\nu}q_\nu$$

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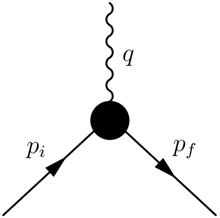


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At tree-level, in the SM,

$$F_1(0) = 1, \qquad F_2(0) = 0$$

$$\Rightarrow a_{\psi} = \frac{g_{\psi} - 2}{2} = F_2(0) = 0$$

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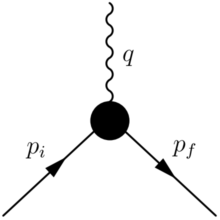


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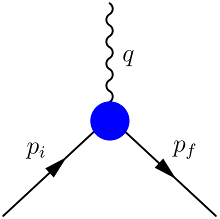


At tree-level, in presence of Lorentz-violating coefficient $c^{\mu\nu}$,

$$F_1(0) = 1, \qquad F_2(0) = ???$$

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## $a_\psi$ in Presence of Lorentz-Violation

**Previous studies:**

Spin-nondegenerate coefficients $b, d, H, g$

$\Rightarrow$ Modified spin precession

$\Rightarrow$ Time-dependent effects

Bluhm et al., Chen & Kunstatter, Muon g-2 collaboration, Kostelecký & Mewes, Gomes et al., Lin et al.

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**Our article:** Spin-degenerate coefficients $c, \tilde{k}$

$\Rightarrow$ Modified QED vertex

$\Rightarrow$ Modified propagation

arXiv: 2208.11420 [hep-ph]

## Modified QED

$$\mathcal{L} = \frac{1}{2}\overline{\psi}\left[\mathrm{i}(\gamma^\mu + c^{\nu\mu}\gamma_\nu)D_\mu - m\right]\psi + \text{h.c.}$$
$$- \frac{1}{4}(\eta_{\mu\varrho}\eta_{\nu\sigma} + (k_F)_{\mu\nu\varrho\sigma})F^{\mu\nu}F^{\varrho\sigma} \,.$$

$D_\mu = \partial_\mu + \mathrm{i}qA_\mu$, $q = -e$, $e > 0$ for electrons.

*A. Kostelecký, hep-th/0312310

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$c^{\mu\nu}$ and $(k_F)_{\mu\nu\varrho\sigma}$ are real. The antisymmetric part of $c^{\mu\nu}$ can, to linear order in $c$, be removed by a field redefinition*

$$\psi(x) \mapsto \psi'(x) = \left(1 + \frac{\mathrm{i}}{4}c^{\mu\nu}\sigma_{\mu\nu}\right)\psi(x)\,, \qquad \sigma^{\mu\nu} \equiv \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]\,.$$

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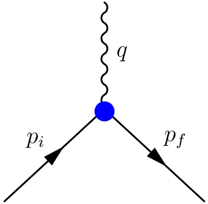
$k_F$ satisfies $(k_F)_{\mu\nu\kappa\lambda} = (k_F)_{\kappa\lambda\mu\nu} = -(k_F)_{\nu\mu\kappa\lambda} = -(k_F)_{\mu\nu\lambda\kappa}$ .

Define the symmetric traceless $4 \times 4$ matrix $\tilde{k}^{\alpha\beta} \equiv (k_F)_\gamma^{\ \alpha\gamma\beta}$ .

*A. Kostelecký, hep-th/0312310

## $a_\psi$ in Presence of $c_{\mu\nu}$




**Modified QED Vertex:**

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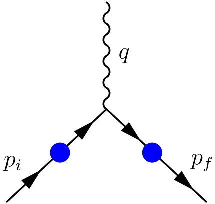


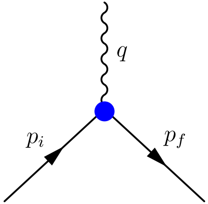

**Modified Dirac Equation:**

$$(\not{p} + c^{\mu\nu}\gamma_\mu p_\nu - m)u(p) = 0$$

**Modified Gordon identity:**

$$\overline{u}(p_f)\gamma^\mu u(p_i) = \frac{1}{2m}\overline{u}(p_f)\Big[(\eta^{\mu\lambda} + c^{\mu\lambda})P_\lambda - \mathrm{i}\sigma^\mu{}_\kappa(\eta^{\kappa\lambda} + c^{\kappa\lambda})q_\lambda\Big]u(p_i)$$

## $a_\psi$ in Presence of $c_{\mu\nu}$ $\Rightarrow$ effective metric

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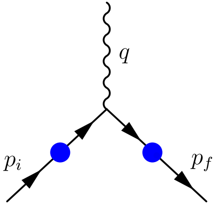


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## $a_\psi$ in Presence of $c_{\mu\nu}$

SM contribution:

$$
\begin{aligned}
\mathrm{i}\mathcal{M}^{(0)}_{ss'} &= -\overline{u}^{(s)}\mathrm{i}e\gamma^\mu u^{(s')}A_\mu(q) \\
&= -\frac{\mathrm{i}e}{2m}\overline{u}^{(s)}\left(P^\mu A_\mu(q) - \frac{1}{2}\sigma_{\mu\nu}F^{\mu\nu}(q)\right)u^{(s')}
\end{aligned}
$$

$$
\Rightarrow \boxed{\mathcal{M}^{(0)}_{ss} \supset \frac{e}{4m}\overline{u}^{(s)}\sigma_{\mu\nu}u^{(s)}F^{\mu\nu}(q) = -\frac{e}{m}BS^z}
$$

Lorentz-violating contribution:

$$
\delta\mathcal{M}^{(0)}_{\psi,ss'} = -\frac{e}{2m}\overline{u}^{(s)}\left[-\frac{1}{2}(c^{\mu\lambda}\sigma_\lambda{}^\nu - c^{\nu\lambda}\sigma_\lambda{}^\mu)F_{\mu\nu}(q) + 2c^{\mu\nu}P_\nu A_\mu(q)\right]u^{(s')}
$$

$$
\Rightarrow \boxed{\delta\mathcal{M}^{(0)}_{\psi,ss} \supset \tfrac{e}{4m}\overline{u}^{(s)}(c^{\mu\lambda}\sigma_\lambda{}^\nu - c^{\nu\lambda}\sigma_\lambda{}^\mu)u^{(s)}F_{\mu\nu} = \tfrac{e}{m}(c^{11}+c^{22})BS^z}
$$

## $a_{\psi,\gamma}$ in Presence of $c^{\mu\nu}$ and $\tilde{k}^{\mu\nu}$

At tree-level,

$$a_{\psi,\gamma} = \frac{g_{\psi,\gamma} - 2}{2} = \left(\frac{\tilde{k}}{2} - c\right)^{11} + \left(\frac{\tilde{k}}{2} - c\right)^{22} .$$

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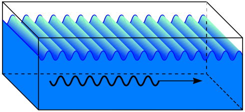
**Isotropic Lorentz violation:** $c^{\mu\nu} = \mathring{c}\ \mathrm{diag}\left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^{\mu\nu}$

$$\Rightarrow\ a_\psi = -\frac{2}{3}\mathring{c}$$

*arXiv:hep-th/0609030

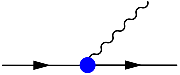
# Isotropic Lorentz Violation: Modified Kinematics*



$$c^{\mu\nu} = \mathring{c}\ \mathrm{diag}\left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^{\mu\nu}$$

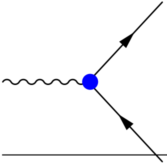
**Modified dispersion relation**

$$(1+\mathring{c})^2 E_\psi^2 = m^2 + \left(1 - \frac{\mathring{c}}{3}\right)^2 \mathbf{p}^2$$



**Vacuum Čerenkov radiation** threshold energy:

$$E_\psi^{\mathrm{th}} = \frac{1}{2}\sqrt{\frac{3}{2}}\frac{m}{\sqrt{-\mathring{c}}}\,.$$

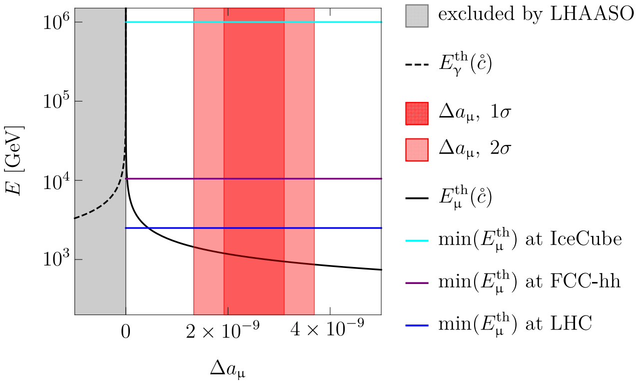


**Photon decay** threshold energy:

$$E_\gamma^{\mathrm{th}} = \sqrt{\frac{3}{2}}\frac{m}{\sqrt{\mathring{c}}}\,.$$

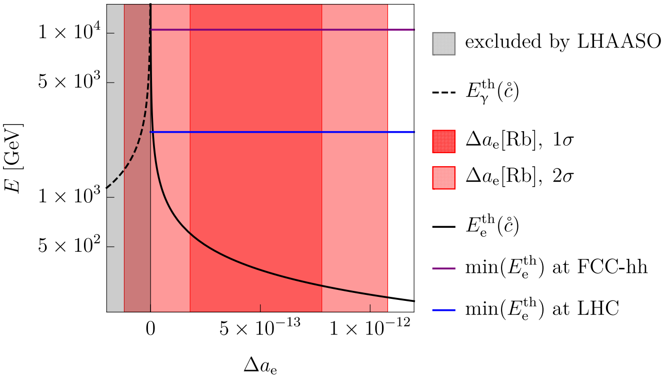
*arXiv:1702.03171

# Can $c_{\mu\nu}$ Explain the Tension in $a_\mu$?*




*A. Crivellin et al., arXiv:2208.11420

# Can $c_{\mu\nu}$ Explain the Tension in $a_\mathrm{e}$?*

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  $\Rightarrow$ Nope, but it was worth checking.

## $a_\gamma$ in Presence of $\tilde{k}^{\mu\nu}$

Field equations for photons modified by $k_F$ *:

$$M^{\alpha\delta}(p)A_\delta(p) = 0\,, \quad M^{\alpha\delta}(p) \quad = \eta^{\alpha\delta}p^2 - p^\alpha p^\delta - 2(k_F)^{\alpha\beta\gamma\delta}p_\beta p_\gamma\,.$$

Insert $\tilde{k}^{\mu\nu}$, linearise, def. $\tilde{q}^\mu \equiv \left(\eta^{\mu\nu} + \frac{1}{2}\tilde{k}^{\mu\nu}\right) q_\nu\,,\ \tilde{A}^\mu \equiv \left(\eta^{\mu\nu} + \frac{1}{2}\tilde{k}^{\mu\nu}\right) A_\nu,$

$$0 = \tilde{A}^\alpha \tilde{q}_\beta \tilde{q}^\beta - \tilde{q}^\alpha \tilde{q}_\gamma \tilde{A}^\gamma + \dots\,.$$

Define $\tilde{F}_{\mu\nu} = \tilde{F}_{\mu\nu}(\tilde{q}) \equiv -\mathrm{i}(\tilde{q}_\mu \tilde{A}_\nu - \tilde{q}_\nu \tilde{A}_\mu)$.

$$\begin{aligned}\mathrm{i}\mathcal{M}^{(0)}_{\gamma,ss'} = -\frac{\mathrm{i}e}{2m}\overline{u}^{(s)}\Big[&\tilde{P}^\mu(\eta_{\mu\nu} - \tilde{k}_{\mu\nu})\tilde{A}^\nu(q)\\ &- \frac{1}{2}\left(\eta_{\mu\varrho} - \frac{1}{2}\tilde{k}_{\mu\varrho}\right)\left(\eta_{\nu\sigma} - \frac{1}{2}\tilde{k}_{\nu\sigma}\right)\sigma^{\mu\nu}\tilde{F}^{\varrho\sigma}(q)\Big]u^{(s')}\end{aligned}$$

$$\Rightarrow \boxed{\delta\mathcal{M}^{(0)}_{\gamma,ss} \supset -\frac{e}{8m}\overline{u}^{(s)}(\tilde{k}^{\varrho\kappa}\sigma_\kappa{}^\nu - \tilde{k}^{\nu\kappa}\sigma_\kappa{}^\varrho)u^{(s)}\tilde{F}_{\varrho\nu} = -\frac{e}{2m}(\tilde{k}^{11} + \tilde{k}^{22})BS^z}$$

*D. Colladay, A. Kostelecký, arXiv:hep-ph/9809521