Theory of heavy muonic atoms

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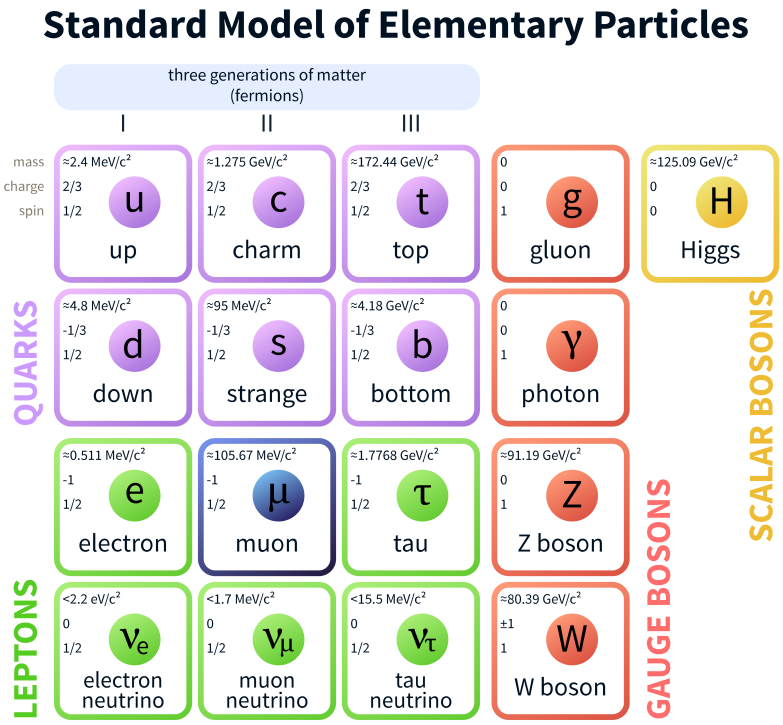
Max Planck Institute for Nuclear Physics (Heidelberg)

EMMI Workshop on Super-heavy elements
Paris, 25 October 2022

MAX-PLANCK-GESELLSCHAFT

Muons and Exotic Atoms



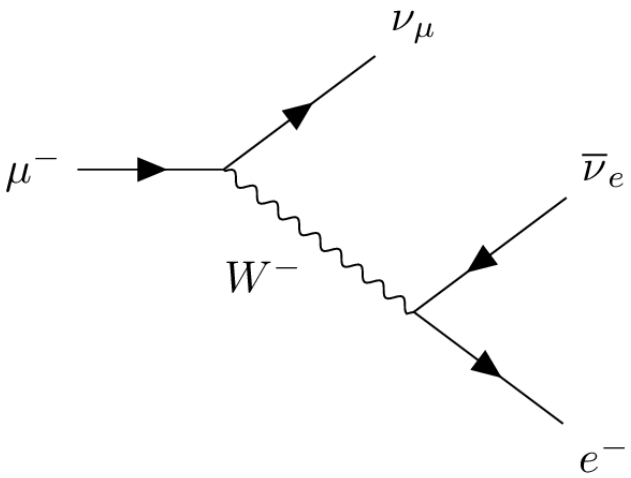
muon:

similar properties as electron
$q_e = q_\mu$

- much bigger mass
  $m_\mu \approx 207\, m_e$
- "heavy electron"
- lifetime free muon $\approx 2\mu$s

$\rightarrow$ β-decay:

- lives long enough to form bound states with other particles



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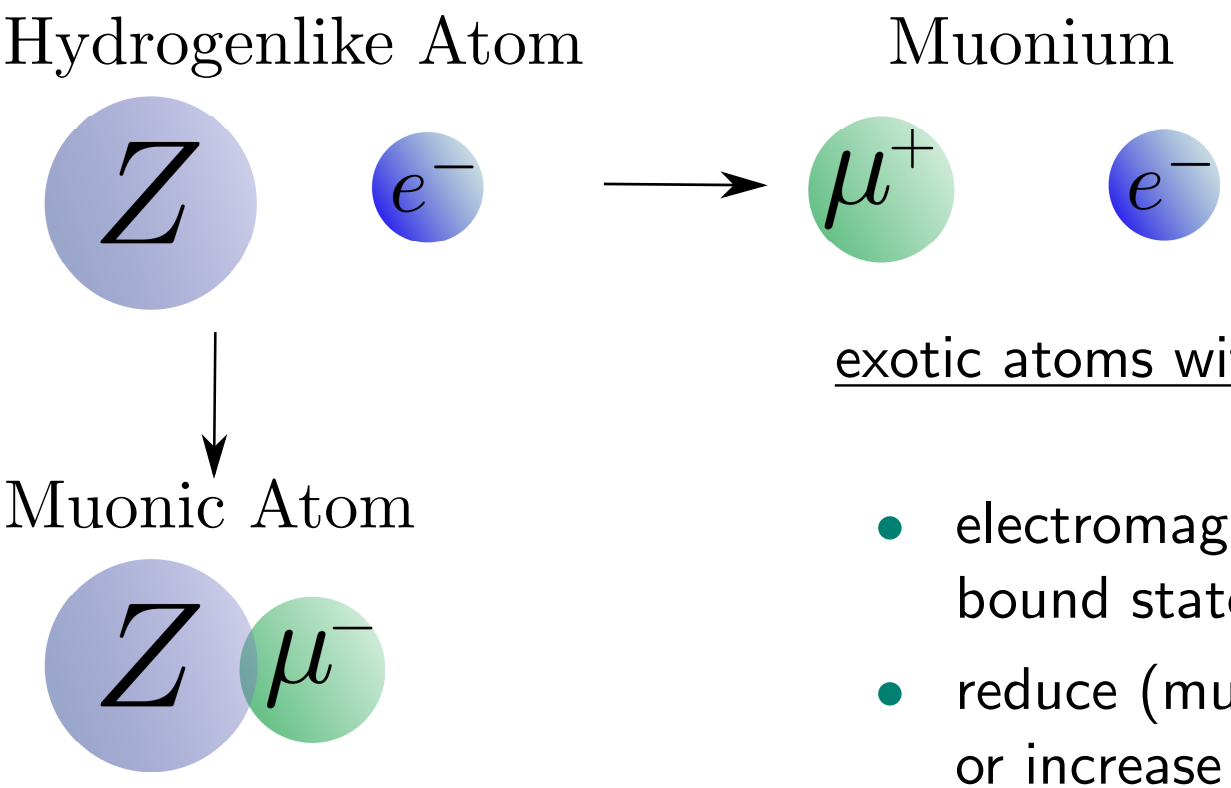


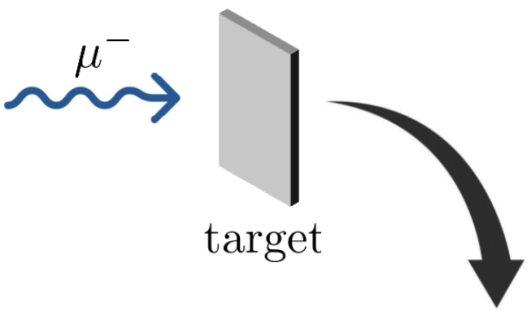
exotic atoms with muons

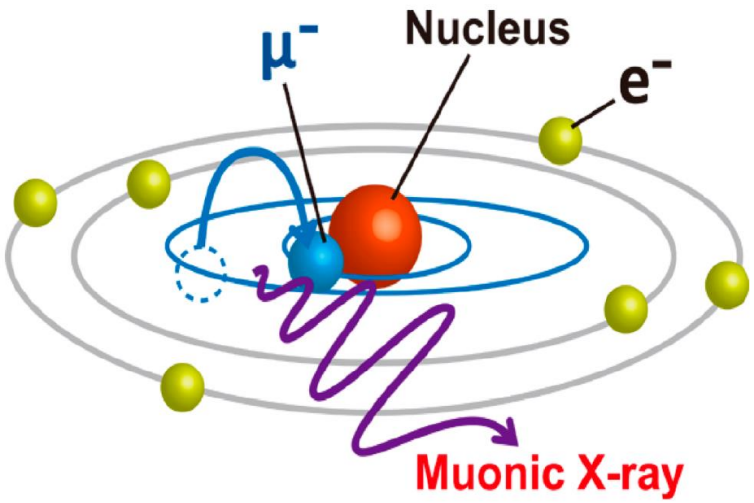
- electromagnetically bound states
- reduce (muonium) or increase (muonic atom) nuclear structure effects
- theory: QED

Literature eg.:
J. M. Bailey et al., Phys. Rev. A 3, 871 (1971)
C. J. Oram et al., Phys. Rev. Lett. 52, 910 (1984)
K. S. Khaw et al., Phys. Rev. A 94, 022716 (2016)

Access to muonic atoms

- capture and cascade: $10^{-12} - 10^{-9}$ s





http://www.mdpi.com/2412-382-X/1/1/11/htm

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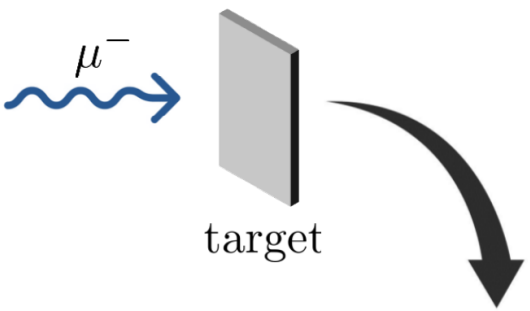

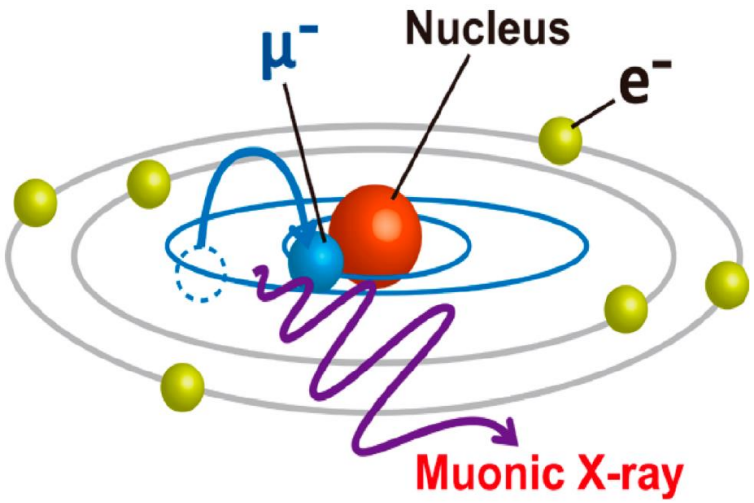

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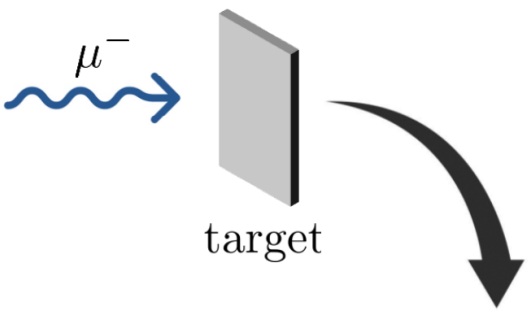

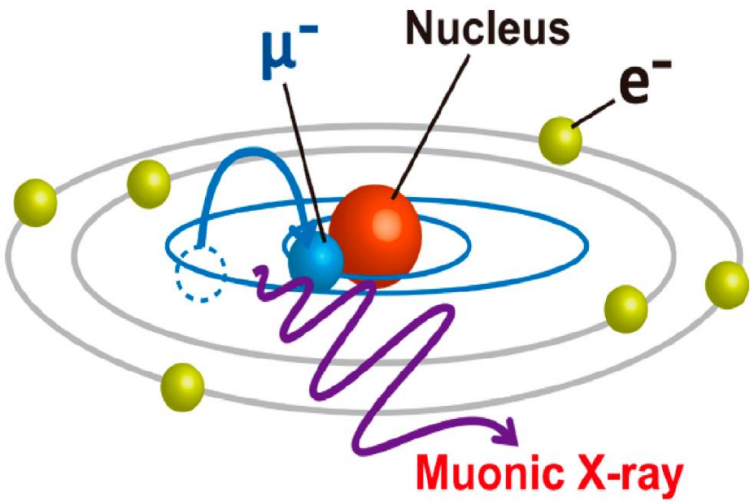

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$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$

$$\mu^- + p \to n + \nu_\mu$$

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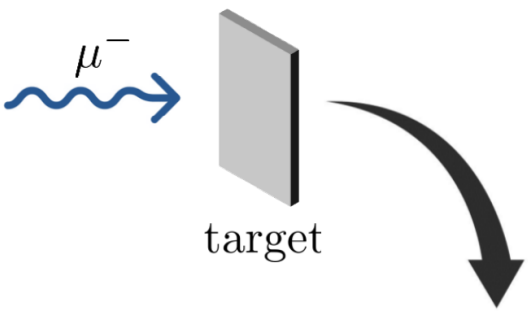

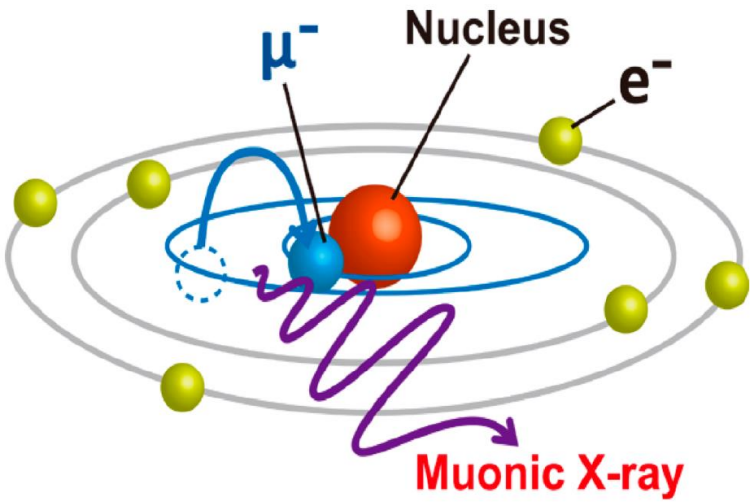

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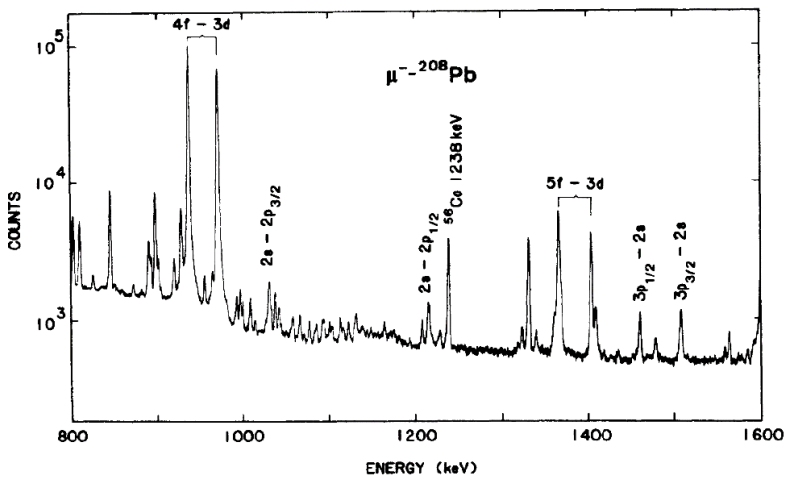


Motivation for muon physics: Theory

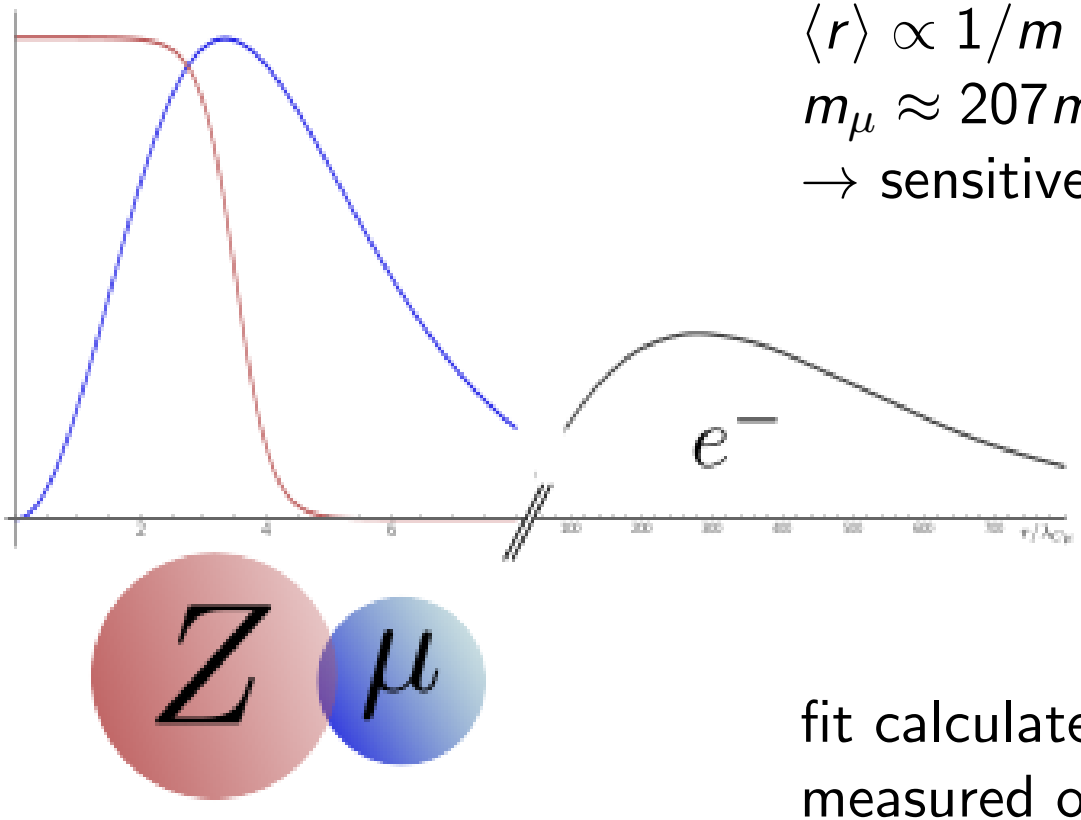


Fig: Niklas Michel

$\langle r \rangle \propto 1/m$
$m_\mu \approx 207 m_e$
$\rightarrow$ sensitive to nuclear structure

fit calculated spectra to measured ones
$\rightarrow$ determine nuclear parameters

Motivation for muon physics 2: Experiments

Proton size nopuzzle

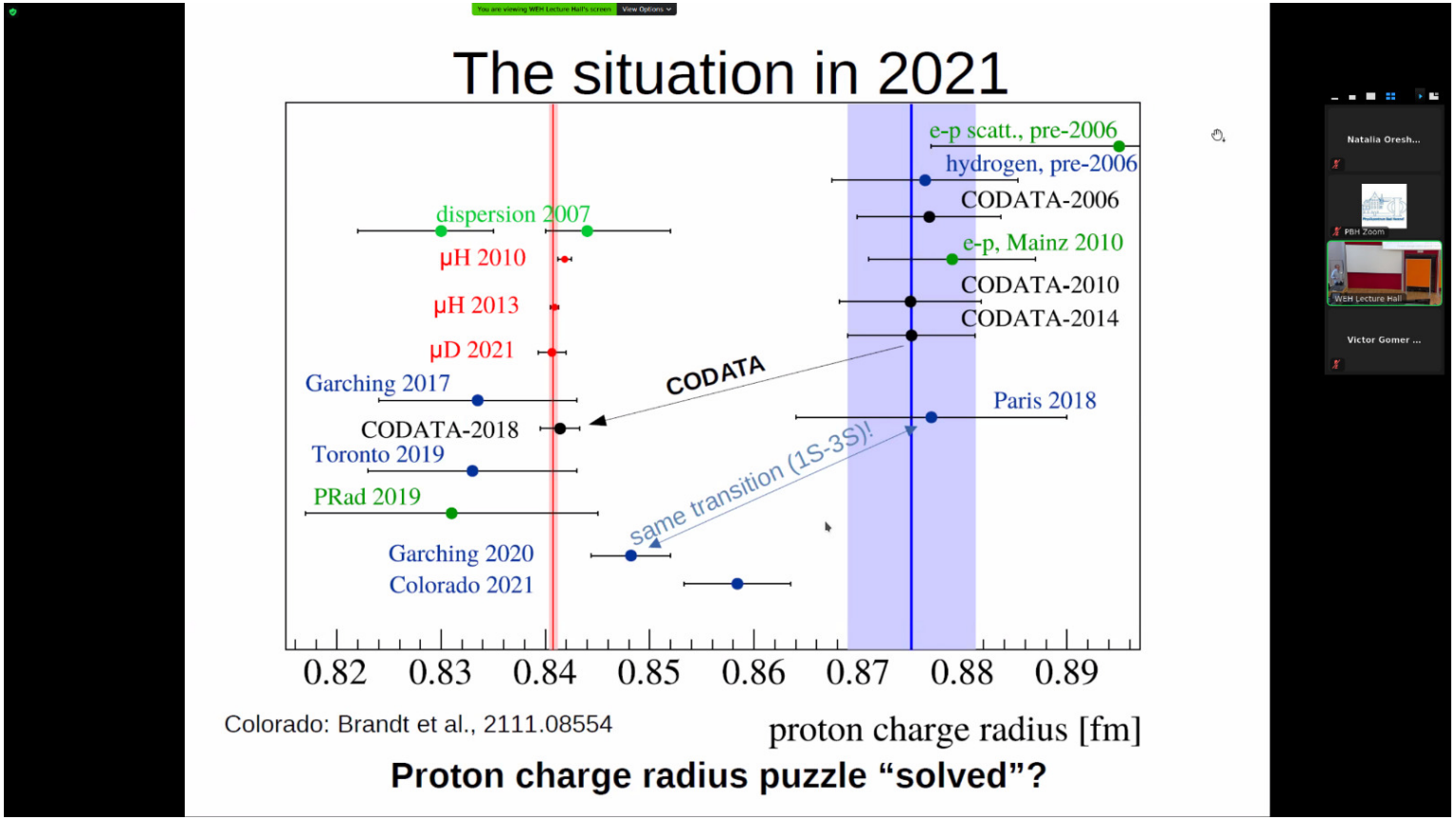
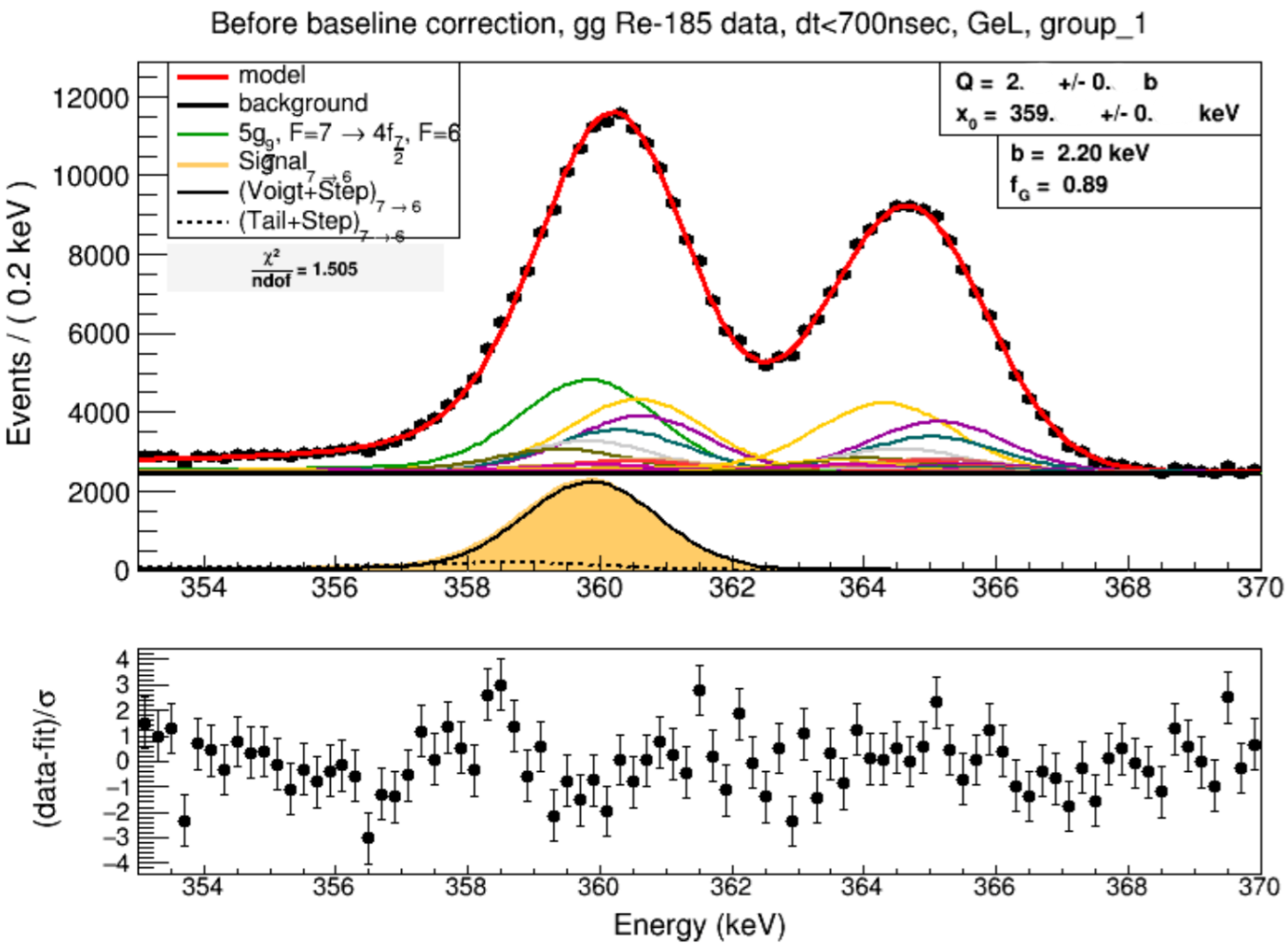


Figure: A presentation of R. Pohl on Heraeus Seminar, 2022

Motivation for muon physics 2: Experiments

Experiment on Re: extraction of quadrupole moment Q



A. Antognini *et al.*, Phys. Rev. C **101**, 054313 (2020)

Motivation for muon physics 2: Experiments

Further muonic experiments with heavy muonic atoms in PSI

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Further muonic experiments with heavy muonic atoms in PSI

and other institutes and facilities

Motivation for muon physics 3: A fine-structure anomaly

muonic ^{90}Zr, $^{112-124}$Sn, ^{208}Pb: very poor fit, $\chi^2/\mathrm{DF} = 187$

P. Bergem *et al.*, Phys. Rev. C **37** 2821 (1988)

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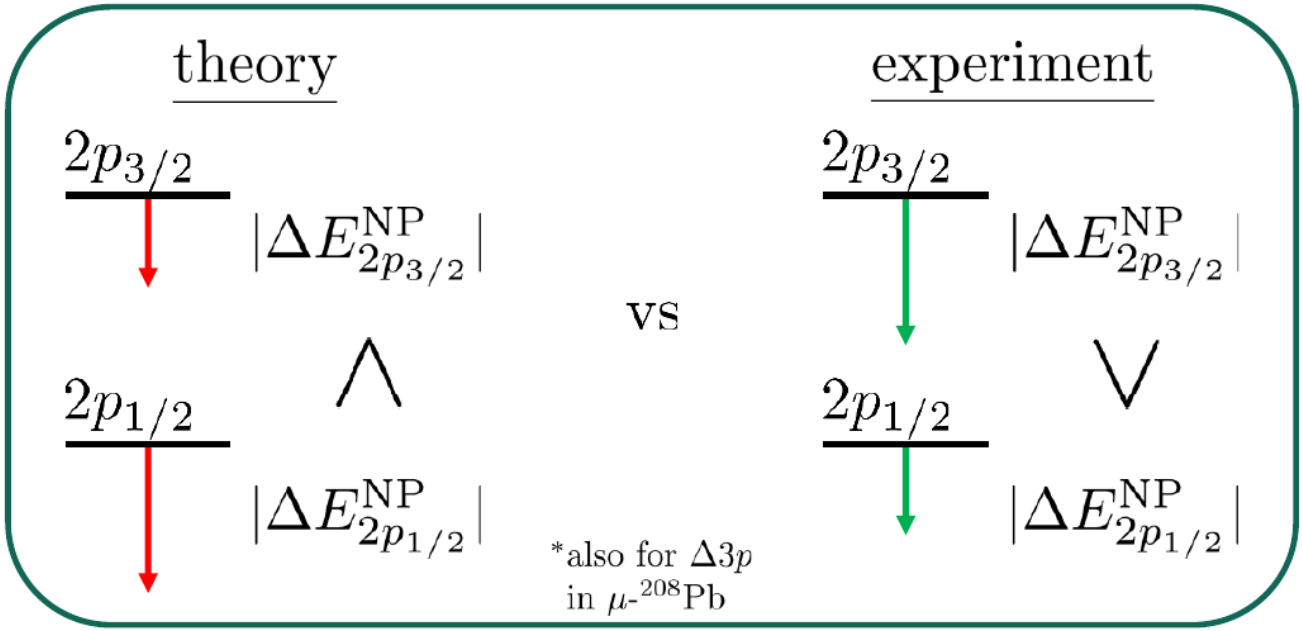

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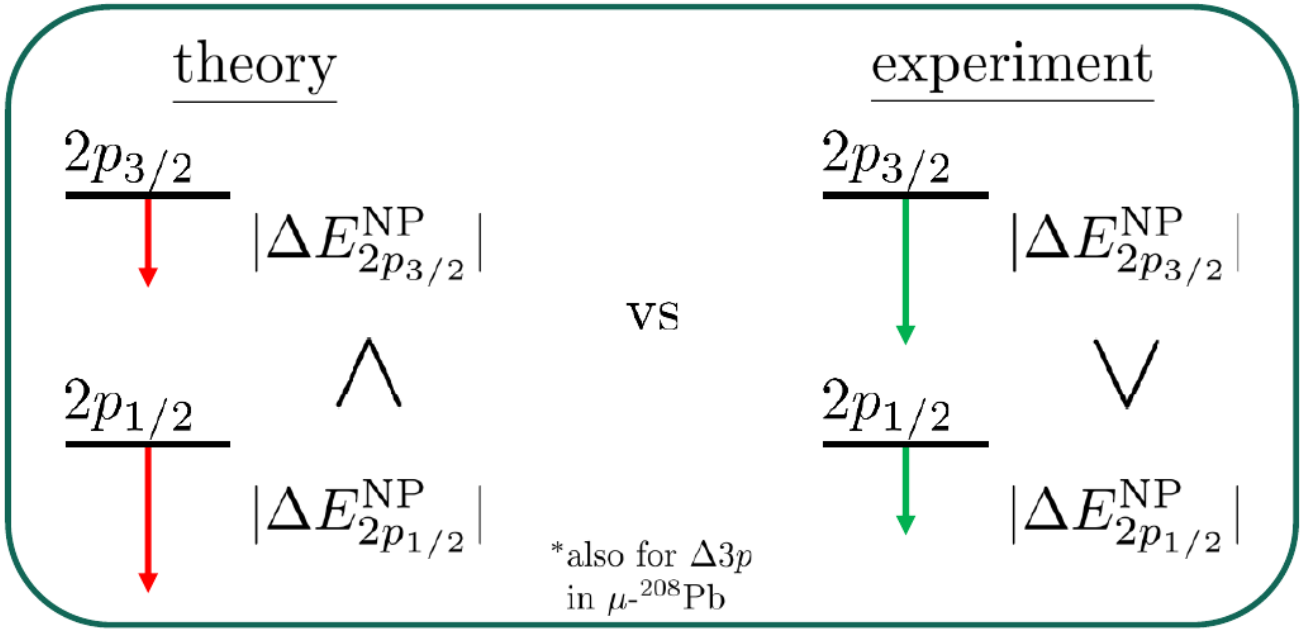
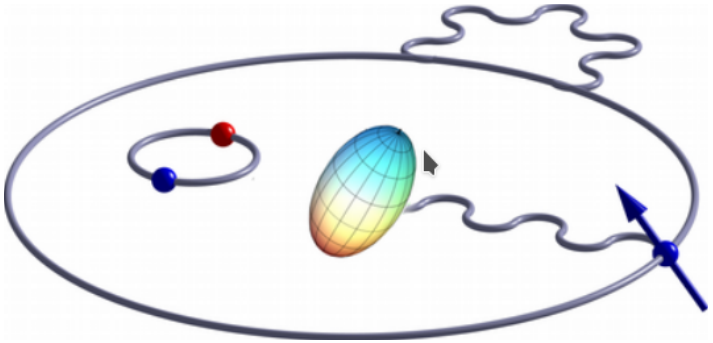
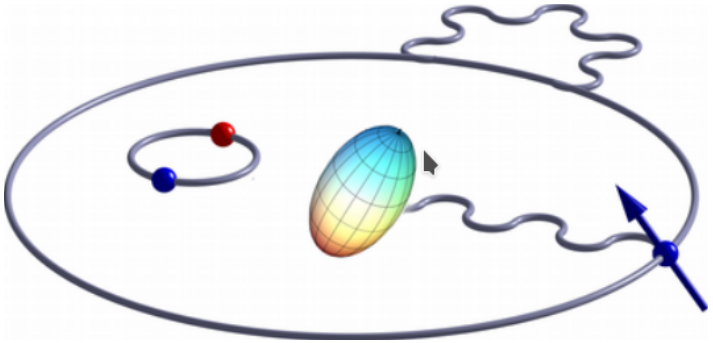


$2p_{1/2}$ is closer to a nucleus and should be affected more strongly

P. Bergem *et al.*, Phys. Rev. C **37** 2821 (1988)

Outline





Basic and brief theory

- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation

A. S. M. Patoary and NSO, EPJD **72**, 54 (2018)
N. Michel, NSO, and C. H. Keitel, PRA **96**, 032510 (2017)
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- Extended nucleus: sphere,

$$V_{\mathrm{Sph}}(r) = \begin{cases} a + br^2; & r \leq R \\ -\frac{Z\alpha}{r}; & r \geq R \end{cases}$$

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$$\rho^F_{a,c}(r_\mu) = \frac{N}{1 + \mathrm{e}^{(r-c)/a}}$$

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$$\rho_{a,c,\beta}(r_\mu, \vartheta_\mu) = \frac{N}{1 + \mathrm{e}^{[r - c(1+\beta \mathrm{Y}_{20}(\vartheta_\mu))]/a}}$$

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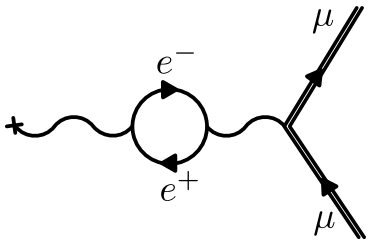

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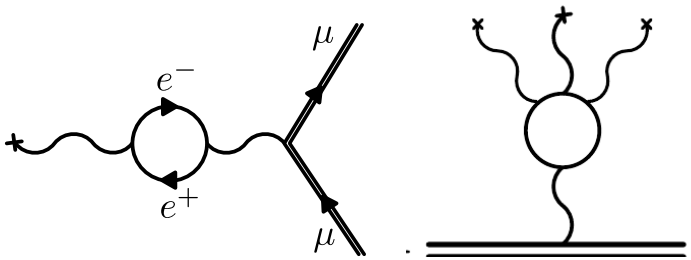

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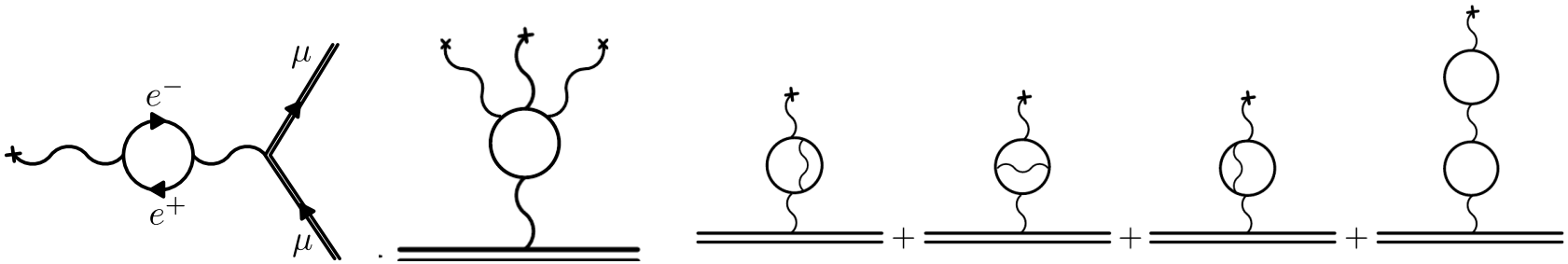

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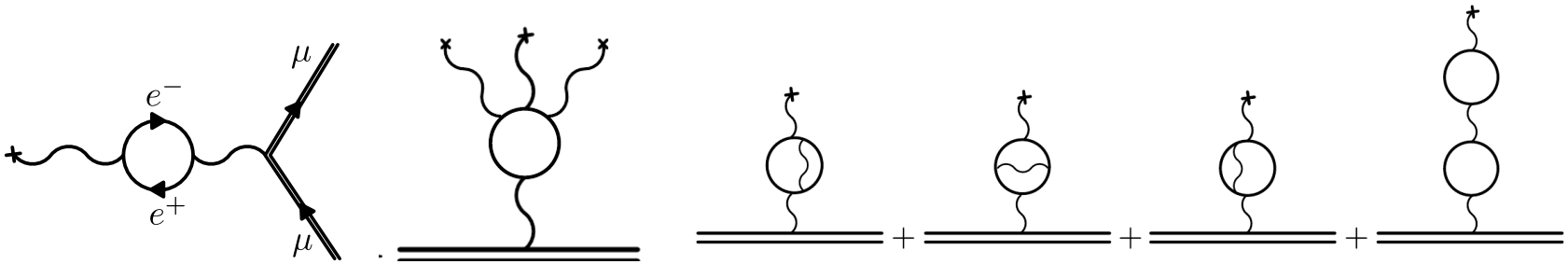

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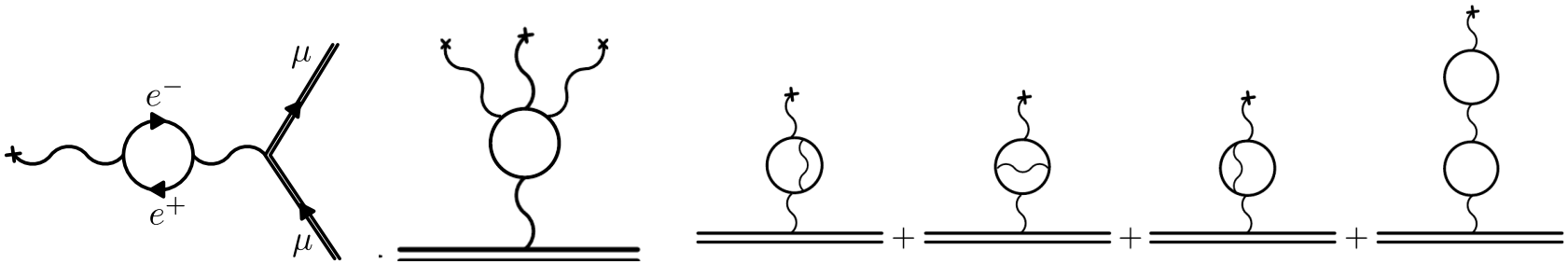

- Electron screening effect

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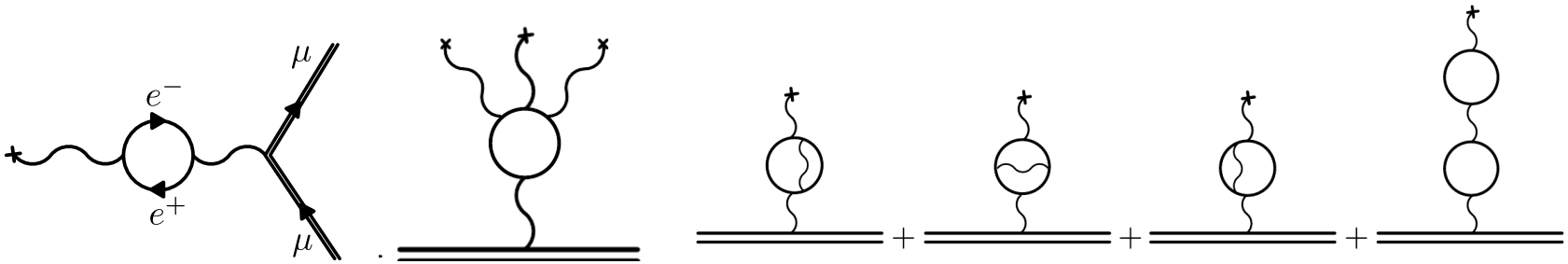

- Electron screening effect
- HFS: electric quadrupole and magnetic dipole

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Dynamic hyperfine structure in muonic atoms

Muonic FS ond/or HFS $\approx$ nuclear rotational states energies
$\rightarrow$ Muonic transitions can excite nucleus!

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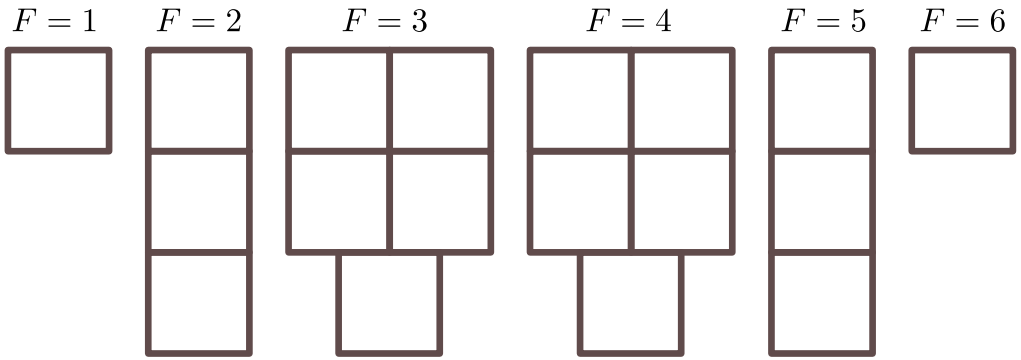
for given F, M_F: 6 unperturbed states: $|FM_F(n\kappa)(IK)\rangle$



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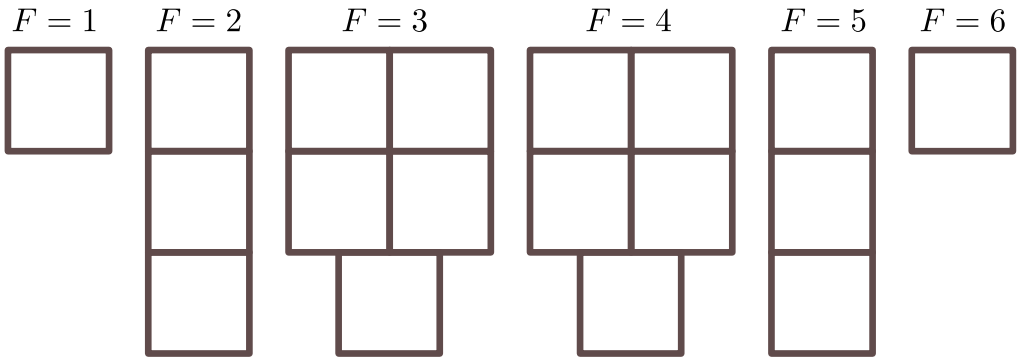

diagonalise quadrupole interaction in every F-block

Dynamic hyperfine structure in muonic atoms for Re

$$E_{5/2} = 0\,\text{keV}$$
$$E_{7/2} = 125\,\text{keV}$$
$$E_{9/2} = 284\,\text{keV}$$

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Figure courtesy: N. Michel

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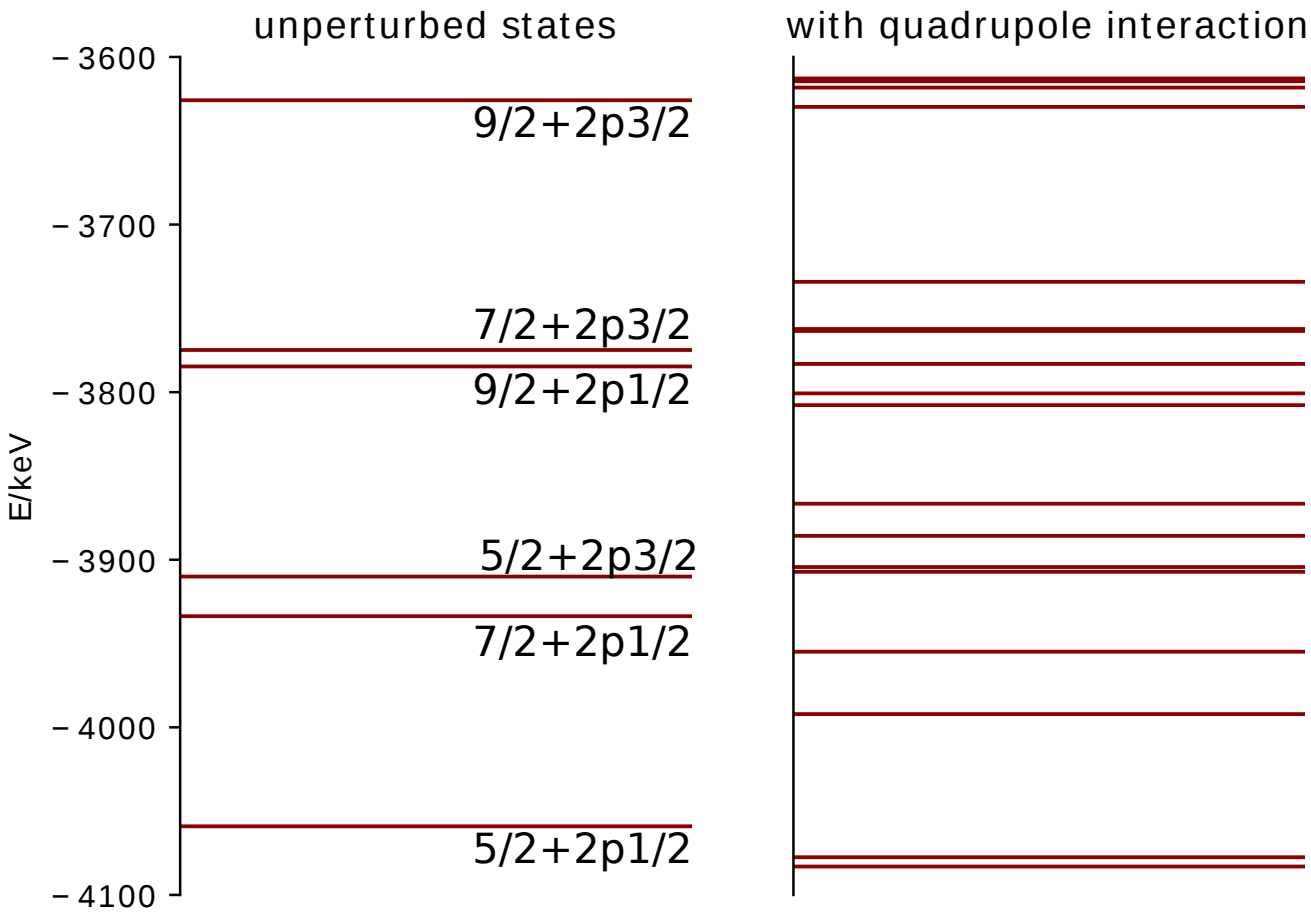

Figure courtesy: N. Michel

PHYSICAL REVIEW LETTERS **128,** 203001 (2022)

Evidence Against Nuclear Polarization as Source of Fine-Structure Anomalies in Muonic Atoms

Igor A. Valuev,[1,*] Gianluca Colò,[2,3] Xavier Roca-Maza,[2,3] Christoph H. Keitel,[1] and Natalia S. Oreshkina[1,†]
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[2]*Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy*
[3]*INFN, Sezione di Milano, via Celoria 16, I-20133 Milano, Italy*

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Self-energy correction to the energy levels of heavy muonic atoms

Natalia S. Oreshkina[1,*]
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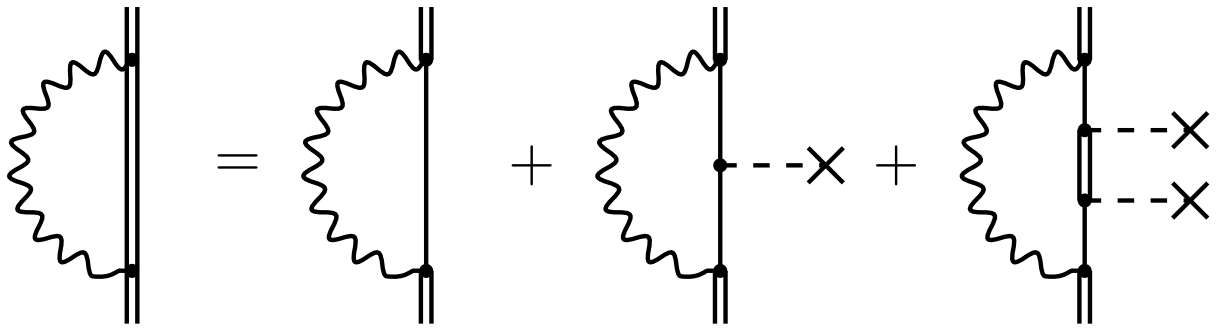
(Dated: June 14, 2022)

Self-energy correction

$$\langle a|\Sigma(E)|b\rangle = \frac{i}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \sum_n \frac{\langle an|I(\omega)|nb\rangle}{E-\omega-\varepsilon_n(1-i0)},$$

$$I(\omega, \mathrm{x}_1, \mathrm{x}_2) = \frac{(1-\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)\exp\left(i\sqrt{\omega^2+i0}x_{12}\right)}{4\pi x_{12}}.$$



Self-energy correction

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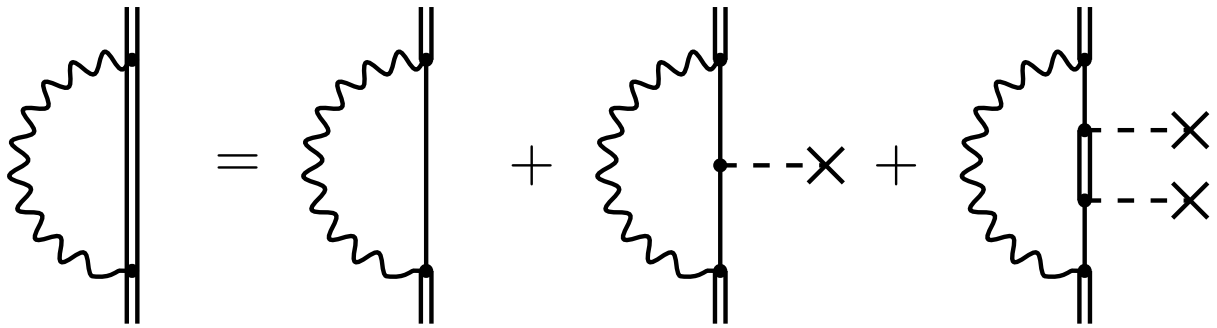
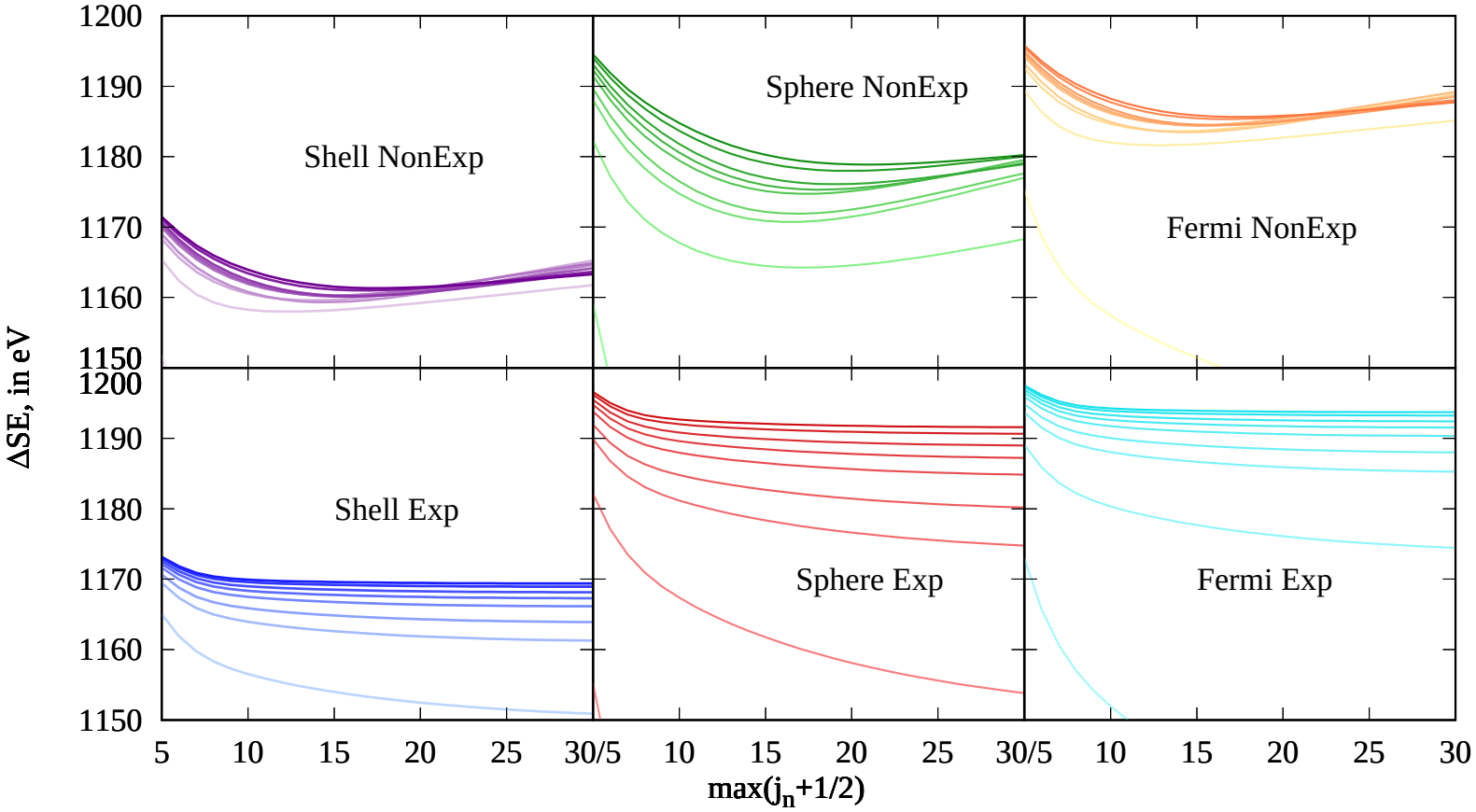
$$I(\omega, \mathrm{x}_1, \mathrm{x}_2) = \frac{(1-\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)\exp\left(i\sqrt{\omega^2+i0}x_{12}\right)}{4\pi x_{12}}.$$



- Order-of-magnitude improvement for $1s_{1/2}$ state
- Rigorous for $2p_{1/2}$, $2p_{3/2}$
- Three nuclear models: shell, sphere, Fermi
- Two integration grids
- Contains infinite summation over intermediate κ

Models and grid compared: $\mu - {}^{90}_{40}\mathrm{Zr}$



ΔE_{SE} contribution to the $1s_{1/2}$ state of the muonic zirconium in units of eV as a function of maximal intermediate angular momentum j_n for different nuclear models and numerical grids. The colors of the lines on every panel change depending on the number of used DKB basis functions from light for $n_{\mathrm{DKB}} = 50$ to dark for $n_{\mathrm{DKB}} = 150$

Nuclear polarization effect

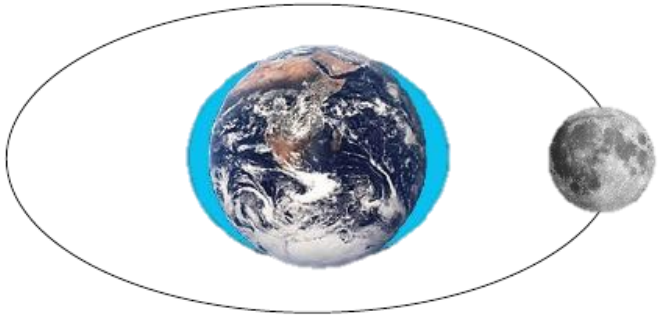


Image source: www.universetoday.com

$$V_{\mathrm{Coul}}(r) = -\frac{\alpha Z}{r}$$

$$V_{\mathrm{ext}}(r) = -\alpha \int \frac{\rho(\mathrm{r}')}{|\mathrm{r} - \mathrm{r}'|} d\mathrm{r}'$$

$$V_{\mathrm{NP}}(r) = -\alpha \sum_{Z} \frac{1}{|\mathrm{r} - \mathrm{r}_{\mathrm{N_i}}|}$$

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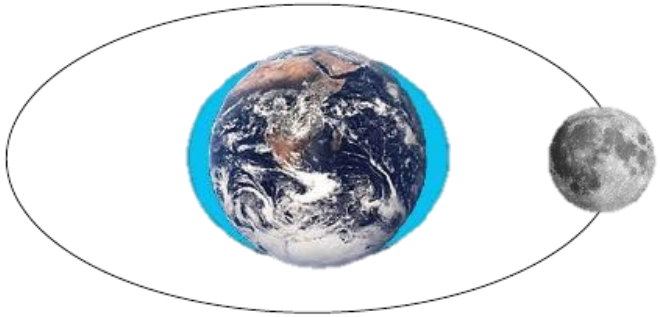

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$$\Delta E_I = \sum_N{}' \frac{\langle I|\Delta V|N\rangle\langle N|\Delta V|I\rangle}{E_I - E_N}$$

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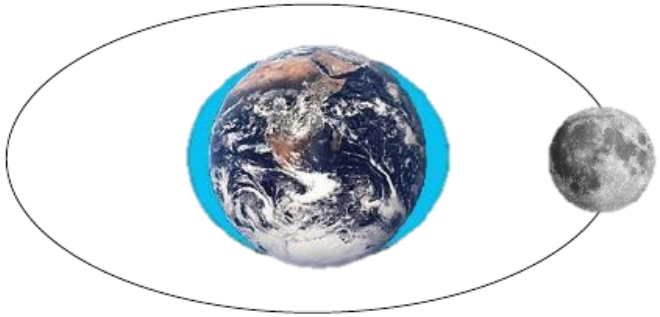

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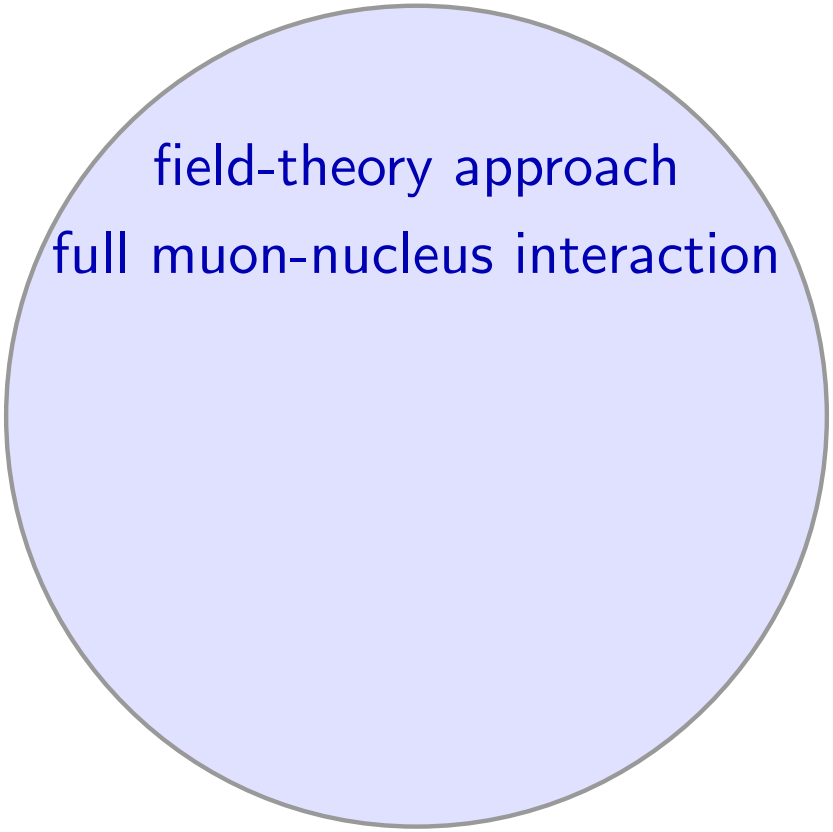
Only longitudinal (Coulomb) part

Also: transverse part, only via field-theory approach

Our goal



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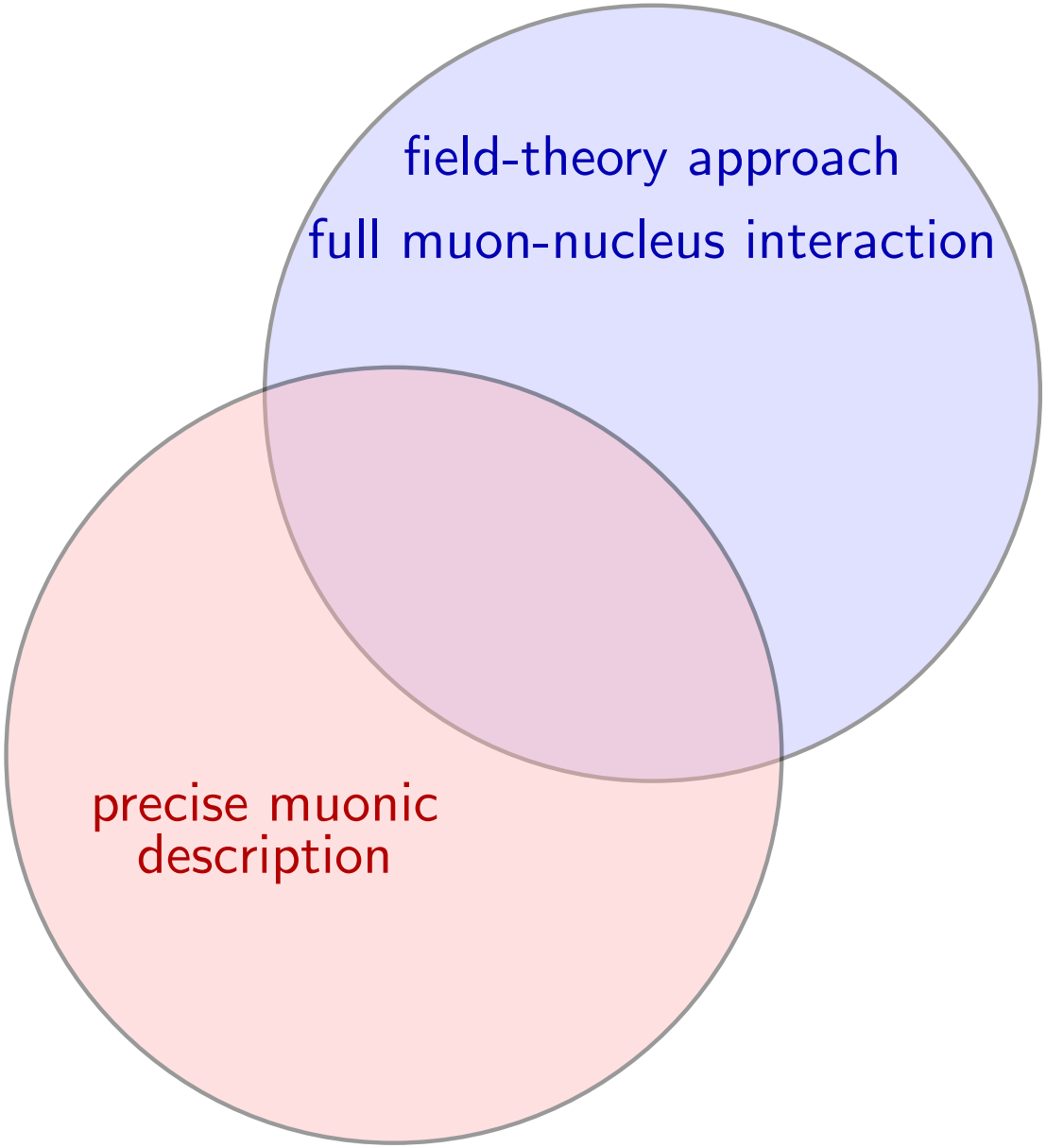
field-theory approach
full muon-nucleus interaction
precise muonic description

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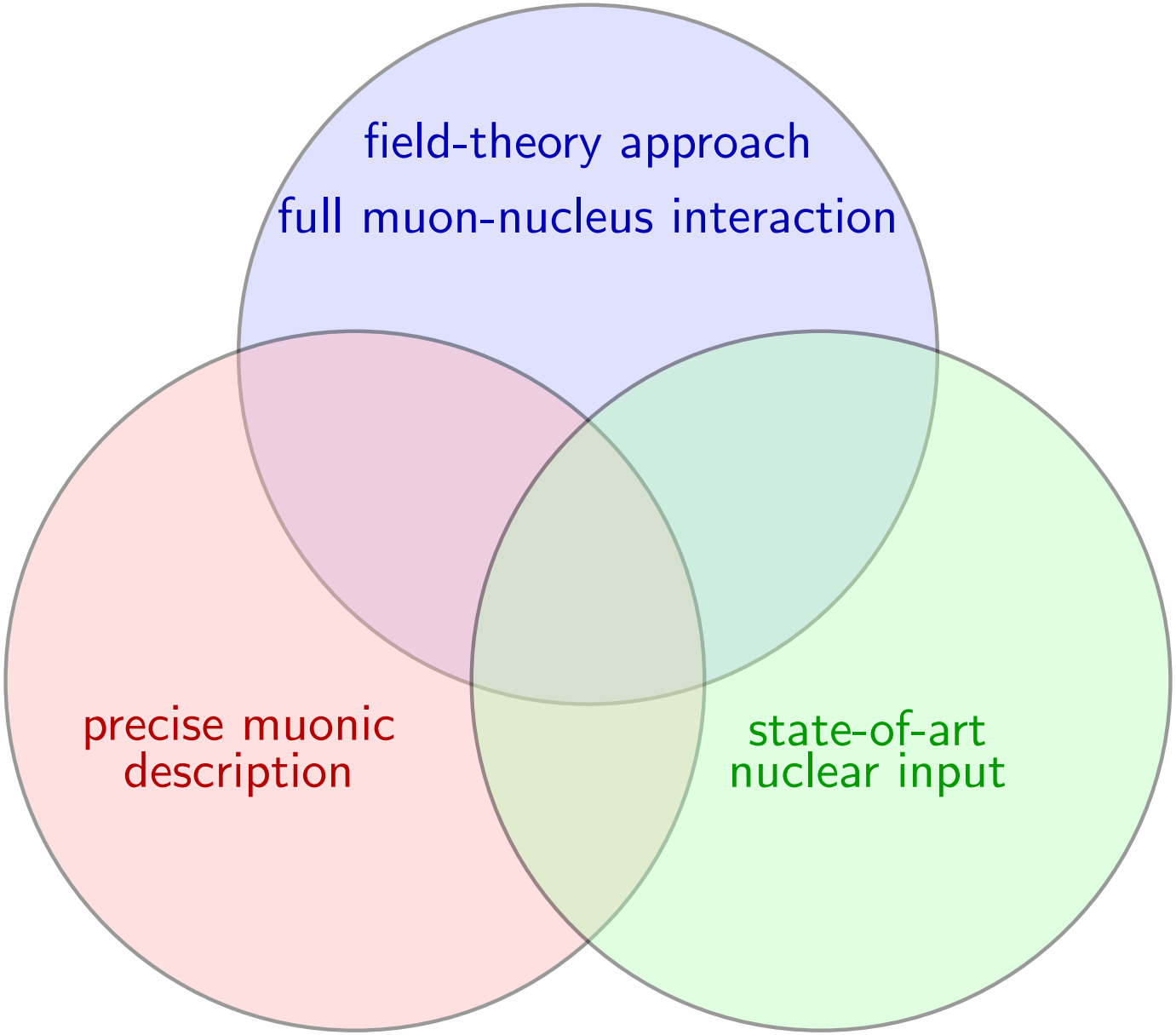
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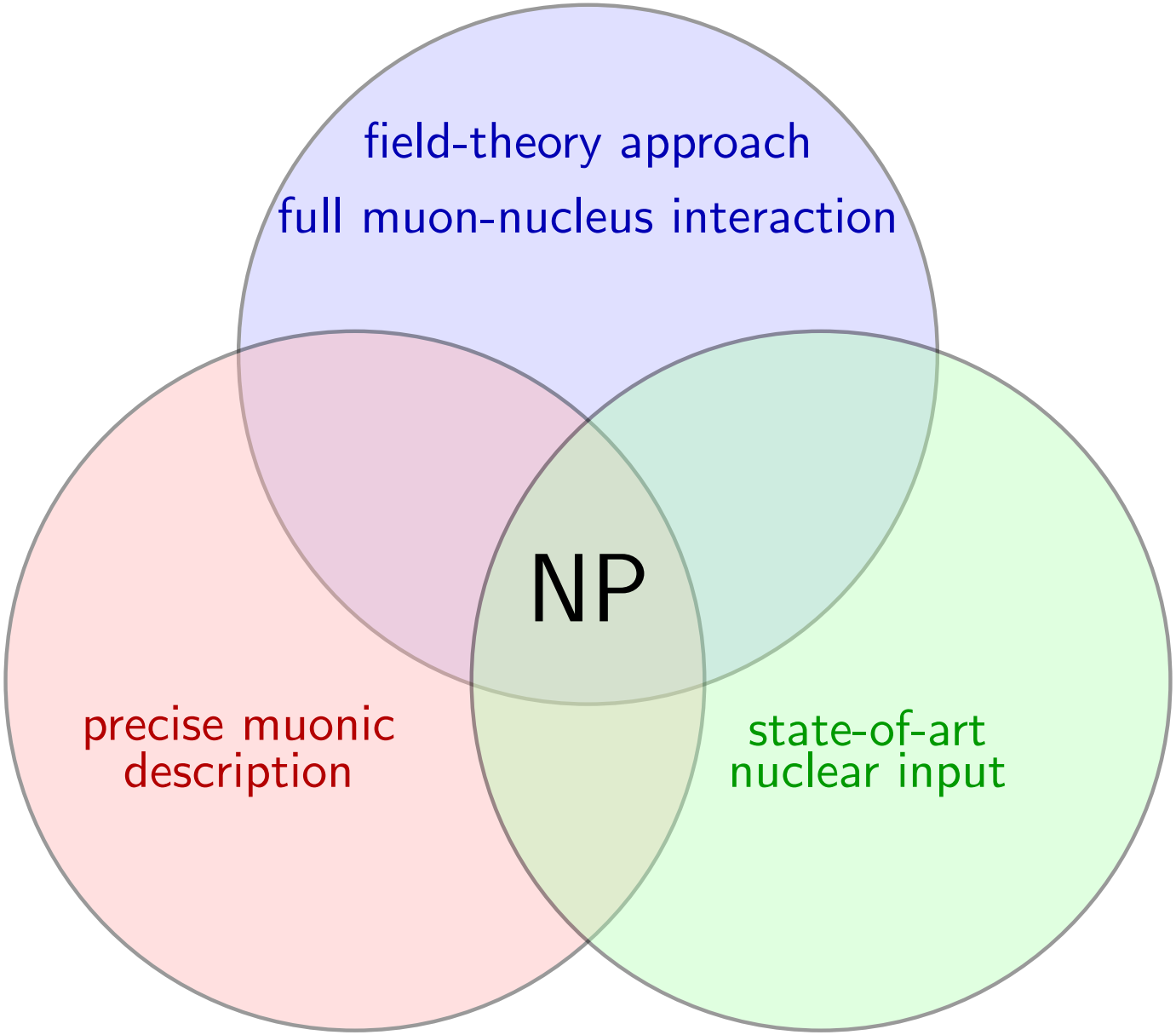
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Transverse part of muon-nucleus interaction

$$H = H_N + \boldsymbol{\alpha}\mathrm{p} + \beta m_\mu + V(\mathrm{r}, \mathrm{r}_{N_i})$$

$$\Downarrow$$

$$H = H_N + \boldsymbol{\alpha}(\mathrm{p} - e\mathrm{A}(\mathrm{r}, \mathrm{r}_{N_i})) + \beta m_\mu + V(\mathrm{r}, \mathrm{r}_{N_i})$$

- Longitudinal (or Coulomb) interaction $V(\mathrm{r}, \mathrm{r}_{N_i})$ always $|\Delta E^{\mathrm{NP}}_{2p_1/2}| > |\Delta E^{\mathrm{NP}}_{2p_3/2}|$

Tanaka and Horikawa, Nucl. Phys. **A580**, 291 (1994)

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- Transverse interaction $\mathrm{A}(\mathrm{r}, \mathrm{r}_{N_i})$ contributes with the opposite muon-spin dependence
- However, the anomalies still persisted (for more than 40 years)

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Calculations details

- complete muonic Dirac spectrum

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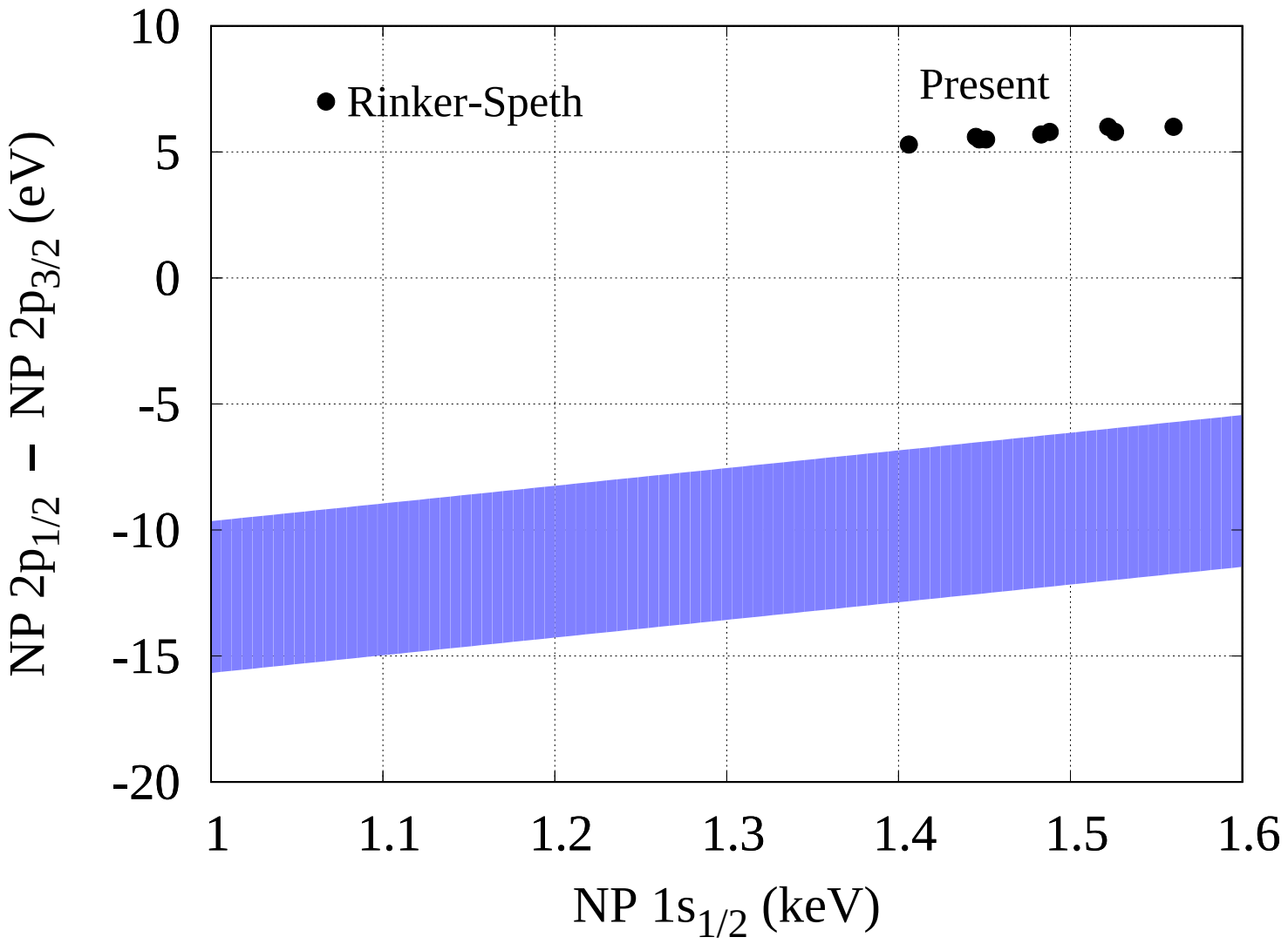
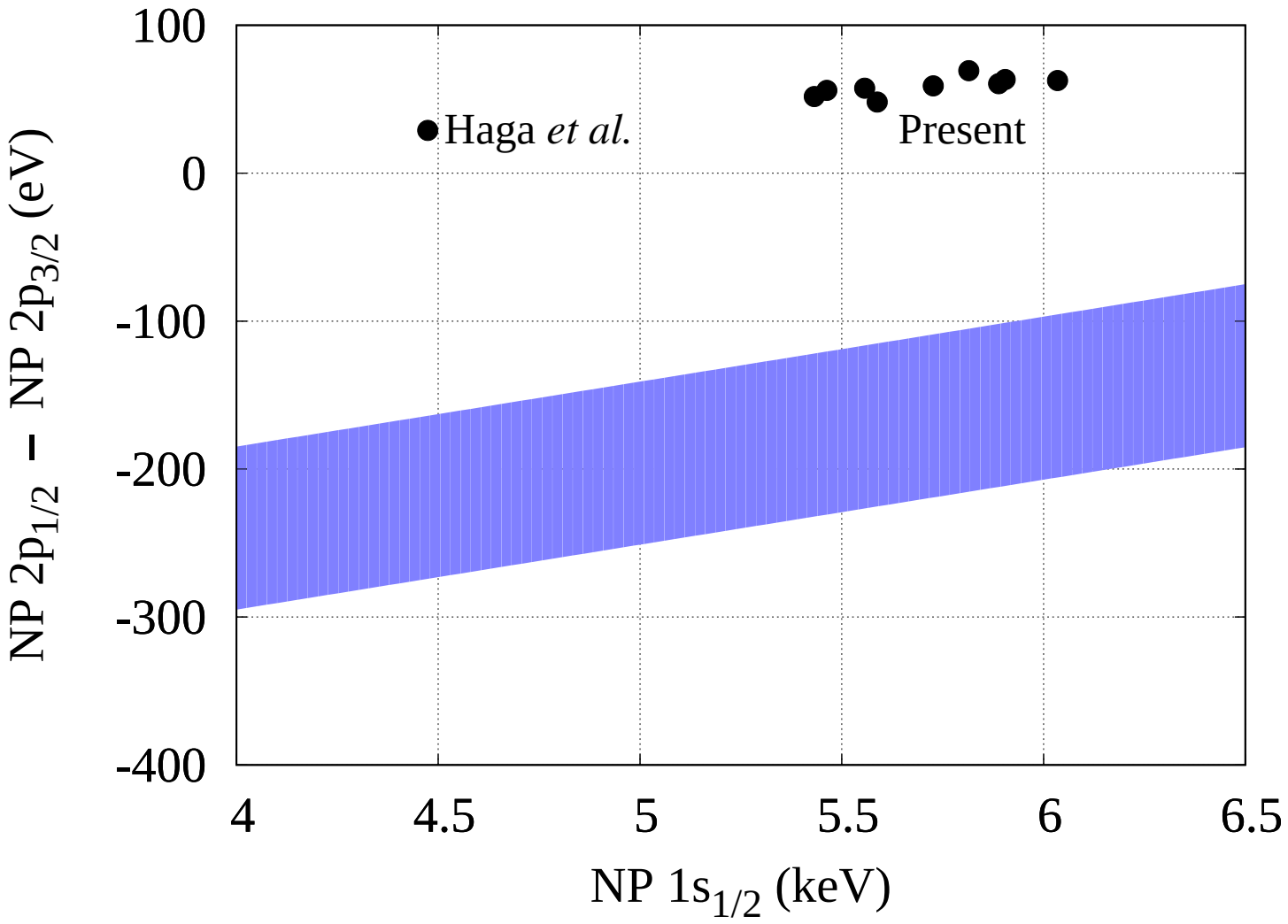
- RMS value changes the NP predictions
- Comparison between theory and free-parameter fit of the experimental data

Nuclear polarization correction ^{90}Zr



around 15 eV (5 σ) gap remains practically constant

Nuclear polarization correction ^{208}Pb



around 150 eV, or 4 σ standard deviations gap

RMS values: why should we care?

Image: Homer Simpson

- Muonic spectroscopy provides RMS for majority of stable nuclei
- High importance for QED, PNC, VFC, ...
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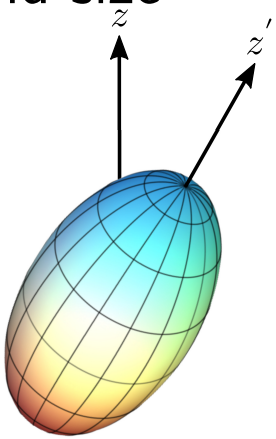
However!

- Fine-structure anomaly
- Poor fit $\chi^2/\mathrm{DF} = 187$
- Estimation for theory
- How much can we trust it?

Summary and Outlook

Nuclear shape and size



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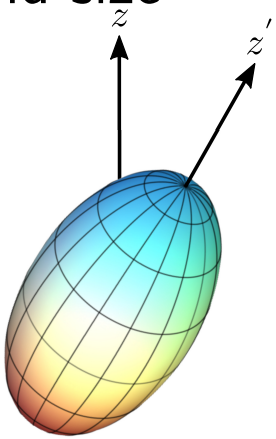
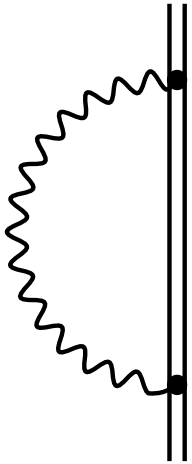


QED effects



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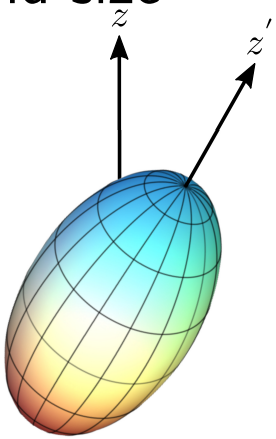

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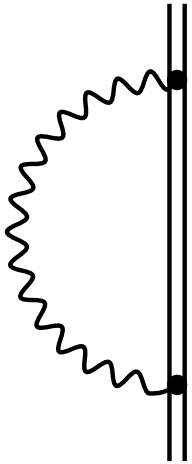
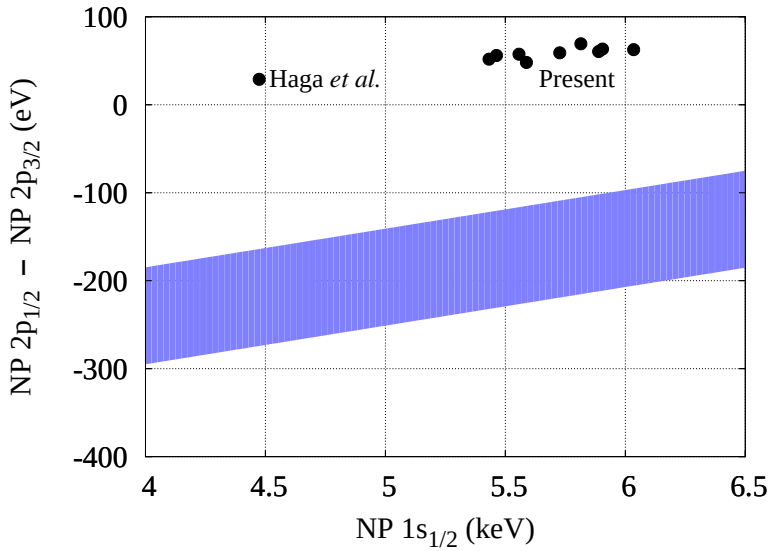


Microscopic nuclear properties



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