

ExtreMe Matter Institute (EMMI)

4th Workshop on Anti-Matter, Hyper-Matter and Exotica Production at the LHC
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Extreme Matter in Neutron Stars and in related astrophysical phenomena



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The **EOS of extreme dense and hot matter** is a basic ingredient to describe a variety of astrophysical phenomena

- **Core-Collapse Supernovae**
- **Neutron star birth (protoneutron stars)**
- **Neutron Stars structure**
- **Merging of Binary Neutron Stars (BNS) and**
- **Gravitational Wave (GW) signals from these mergers**

Using a complementary strategy

Observational data from the above mentioned astrophysical phenomena give us a unique opportunity to test our present models of the EOS of extreme matter.

Summary of the talk

□ EOS of nuclear matter and Neutron Stars (**Nucleon Stars**)

- a. Basic requirements of nuclear matter EOS models (empirical saturation properties, measured NS masses, ... etc.....)
- b. Dependence of the nuclear matter EOS on the **quantum many-body approach**.

□ Hyperons in dense matter and (a possible solution of) the **hyperon puzzle**

□ Quark Matter in Neutron Stars and in BNS mergers.

Neutron Stars: bulk properties

Mass	$M \sim 1.5 M_{\odot}$
Radius	$R \sim 10 \text{ km}$
Centr. Density	$\rho_c = (4 - 8) \rho_0$
Compactness	$R/R_g \sim 2 - 4$
Baryon number	$A \sim 10^{57}$
Binding energy	$B \sim 10^{53} \text{ erg}$
	$B/A \sim 100 \text{ MeV} \quad B/(Mc^2) \sim 10\%$

Stellar structure:
General Relativity

Giant “atomic nucleus”
or “hypernucleus”
bound by gravity

$$M_{\odot} = 1.989 \times 10^{33} \text{ g} \quad R_{\odot} = 6.96 \times 10^5 \text{ km} \quad R_{g \odot} = 2.95 \text{ km}$$

$$\rho_0 = 2.6 \times 10^{14} \text{ g/cm}^3 \quad (n_0 \sim 0.16 \text{ fm}^{-3} \text{ nuclear saturation density})$$

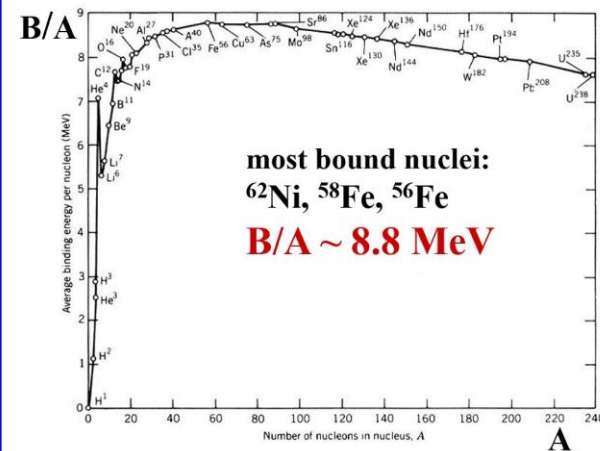
$$R_g \equiv 2GM/c^2 \quad (\text{Schwarzschild radius})$$

Atomic Nuclei: bulk properties

Mass number $A = 1 - 238$ (natural stable isotopes)

Radius $R = r_0 A^{1/3} \sim (2 - 8) \text{ fm}$

Density $\rho \sim \rho_0 = 2.6 \times 10^{14} \text{ g/cm}^3$



$B/(Mc^2)$
 $\sim (0.1-1)\%$

bound by
nuclear
interactions

Relativistic equations for stellar structure

Static and spherically symmetric self-gravitating mass distribution

Tolman – Oppenheimer – Volkov equations (TOV)

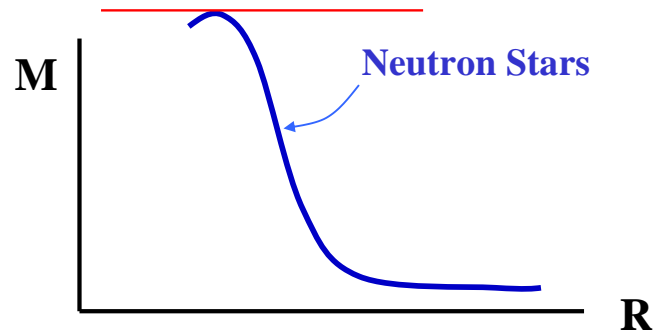
$$\frac{dP}{dr} = -G \frac{m(r)\rho(r)}{r^2} \left(1 + \frac{P(r)}{c^2 \rho(r)} \right) \left(1 + 4\pi \frac{r^3 P(r)}{m(r) c^2} \right) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{d\Phi}{dr} = -\frac{1}{\rho(r)c^2} \frac{dP}{dr} \left(1 + \frac{P(r)}{\rho(r)c^2} \right)^{-1}$$

One needs the **equation of state (EOS)** of dense matter, $P = P(\rho)$, up to **very high densities**

Black Holes



$M_{\max}(\text{EOS}) \geq$ all measured neutron star masses

Measured Neutron Star masses in relativistic binary systems: three “heavy” Neutron Stars

Measuring post-Keplerian parameters: very accurate NS mass measurements;
model independent measurements within GR

PSR J1614–2230

NS – WD binary system (He WD) $M_{\text{NS}} = 1.97 \pm 0.04 M_{\odot}$
 $M_{\text{WD}} = 0.5 M_{\odot}$ (companion mass)
 $P_b = 8.69$ hr (orbital period) $P = 3.15$ ms (PSR spin period) $i = 89.17^\circ \pm 0.02^\circ$ (inclination angle)

[P. Demorest et al., Nature 467 \(2010\) 1081](#)

PSR J0348+0432

NS – WD binary system $M_{\text{NS}} = 2.01 \pm 0.04 M_{\odot}$
 $M_{\text{WD}} = 0.172 \pm 0.003 M_{\odot}$ (companion mass)
 $P_b = 2.46$ hr (orbital period) $P = 39.12$ ms (PSR spin period) $i = 40.2^\circ \pm 0.6^\circ$ (inclination angle)

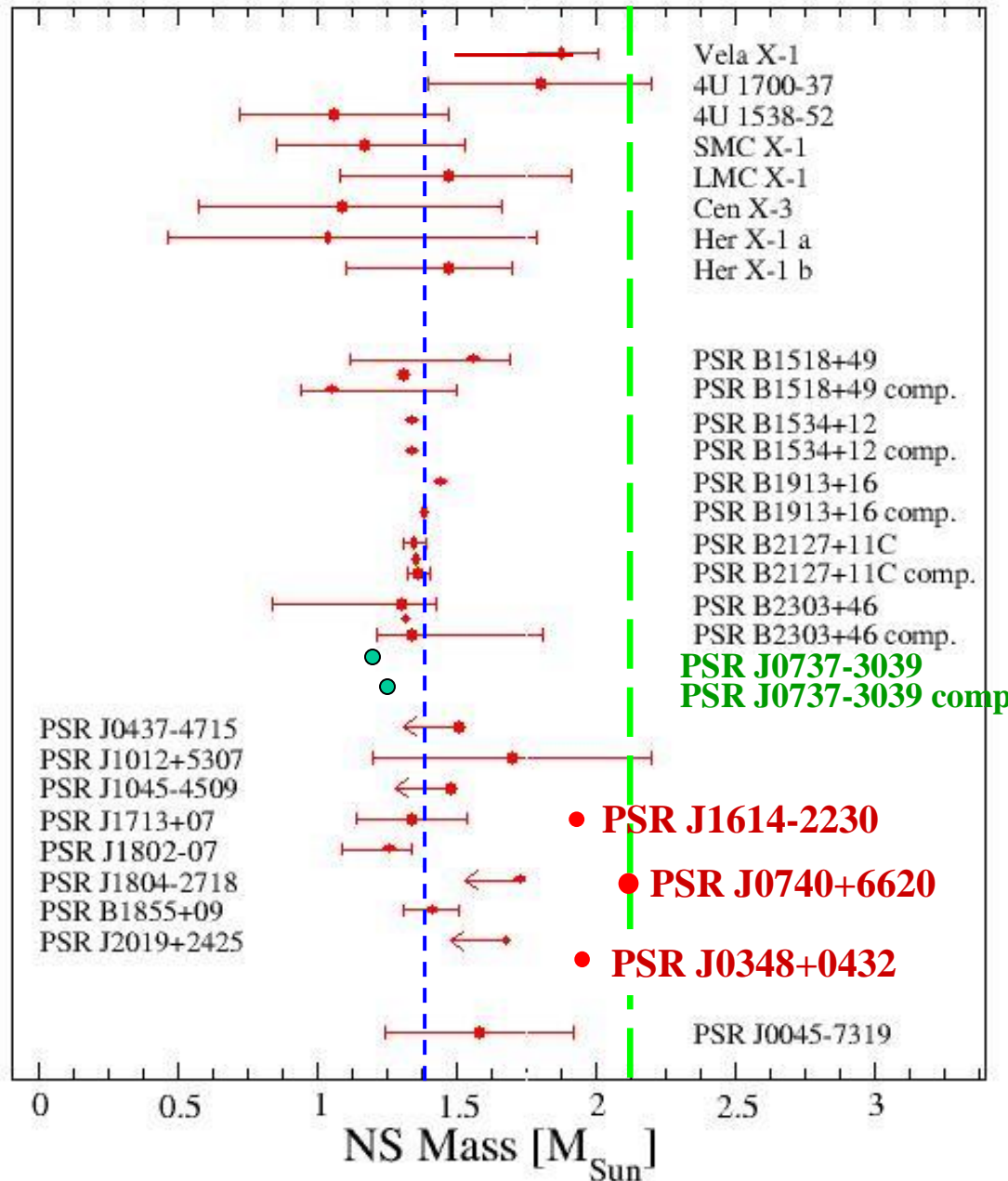
[Antoniadis et al., Science 340 \(2013\) 448](#)

PSR J0740+6620 ($M_{\text{NS}} = 2.14^{+0.10}_{-0.09} M_{\odot}$) $M_{\text{NS}} = 2.08 \pm 0.007 M_{\odot}$

NS – WD binary system $M_{\text{WD}} = 0.258 M_{\odot}$ (companion mass)
 $P_b = 4.7669$ days (orbital period) $P = 2.89$ ms (PSR spin period) $i = 87.38^\circ \pm 0.20^\circ$ (inclination angle)

[\(H. T. Cromartie et al., Nature Astronomy 4 \(2020\) 72\)](#) [E. Fonseca et al, ApJL 915 \(2021\) L12](#)

Measured Neutron Star Masses



$$M_{\text{max}} \geq 2 M_{\odot}$$



Very stringent
constraint on the
EOS

soft EOS
are
ruled out

Neutron star physics in a nutshell

1) **Gravity** compresses matter at very high density

2) **Pauli principle**

Stellar constituents are different species of **identical fermions** (n, p, ..., e⁻, μ⁻)

→ antisymmetric wave function for particle Exchange → Pauli principle

Chemical potentials $\mu_n, \mu_p, \dots, \mu_e$ rapidly increasing functions of the density

3) **Weak interactions** change the **isospin** and **strangeness** content of dense matter to **minimize the energy**

Cold catalyzed matter (Harrison, Wakano, Wheeler, 1958)

The ground state (minimum energy per baryon) of a system of **hadrons** and **leptons** with respect to their mutual **strong** and **weak interactions** at a given total baryon density n and temperature $T = 0$.

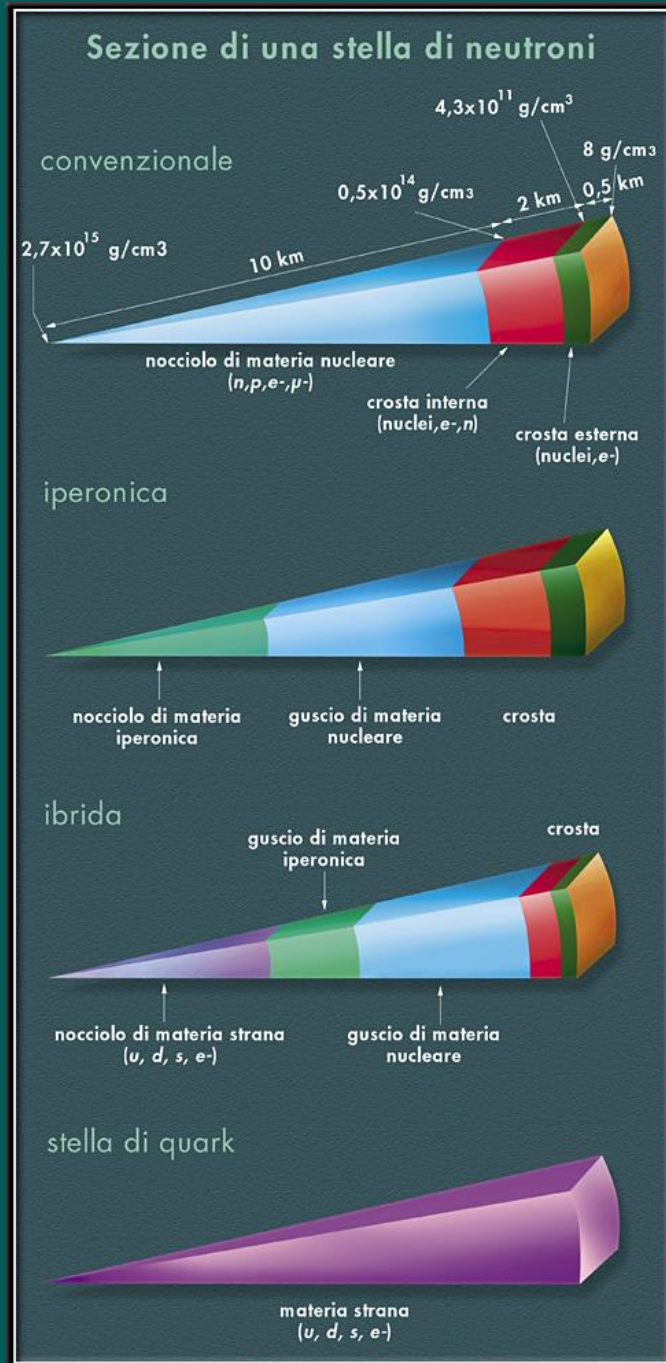
“Neutron Stars”

Nucleon Stars

Hyperon Stars

Hybrid Stars

Strange Stars



EOS of nuclear matter and Neutron Stars (Nucleon Stars)

β -stable nuclear matter

$$p + e^- \leftrightarrow n + \nu_e$$

$$\bar{\nu}_e + p \leftrightarrow n + e^+$$

$$\mu_e \geq m_\mu$$

$$e^- \rightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

Equilibrium with respect to the weak interaction processes

Charge neutrality

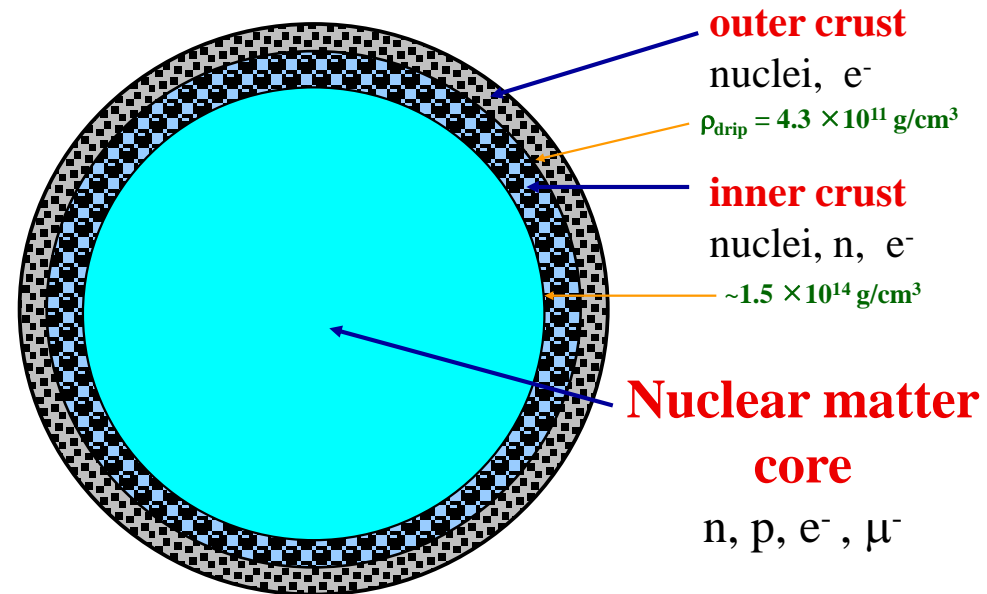
$$\mu_n - \mu_p = \mu_e$$

$$\mu_\mu = \mu_e$$

$$n_p = n_e + n_\mu$$

neutrino-free matter

$$\mu_\nu = \mu_{\bar{\nu}} = 0$$



To be solved for any given value of the total baryon number density n

Microscopic approach to asymmetric nuclear matter EOS

input

Two-body nuclear interactions: V_{NN}

Parameters fitted to
NN scattering data with $\chi^2/\text{datum} \sim 1$

Three-body nuclear interactions: V_{NNN}

Parameters fitted to

- binding energy of $A = 3, 4$ nuclei or
- empirical saturation point of symmetric nuclear matter:
 $n_0 = (0.16 \pm 0.01) \text{ fm}^{-3}$, $E/A = (-16 \pm 1) \text{ MeV}$

	AV18	AV18/UIX	Exp.
B(3H)	7.624	8.479	8.482
B(3He)	6.925	7.750	7.718
B(4He)	24.21	28.46	28.30

Values in MeV

Nuclear Matter at $n = 0.16 \text{ fm}^{-3}$

$E^{\text{pot}}(2\text{BF})/A \sim -40 \text{ MeV}$

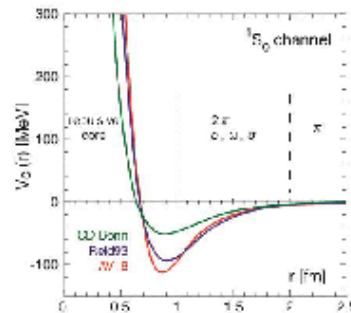
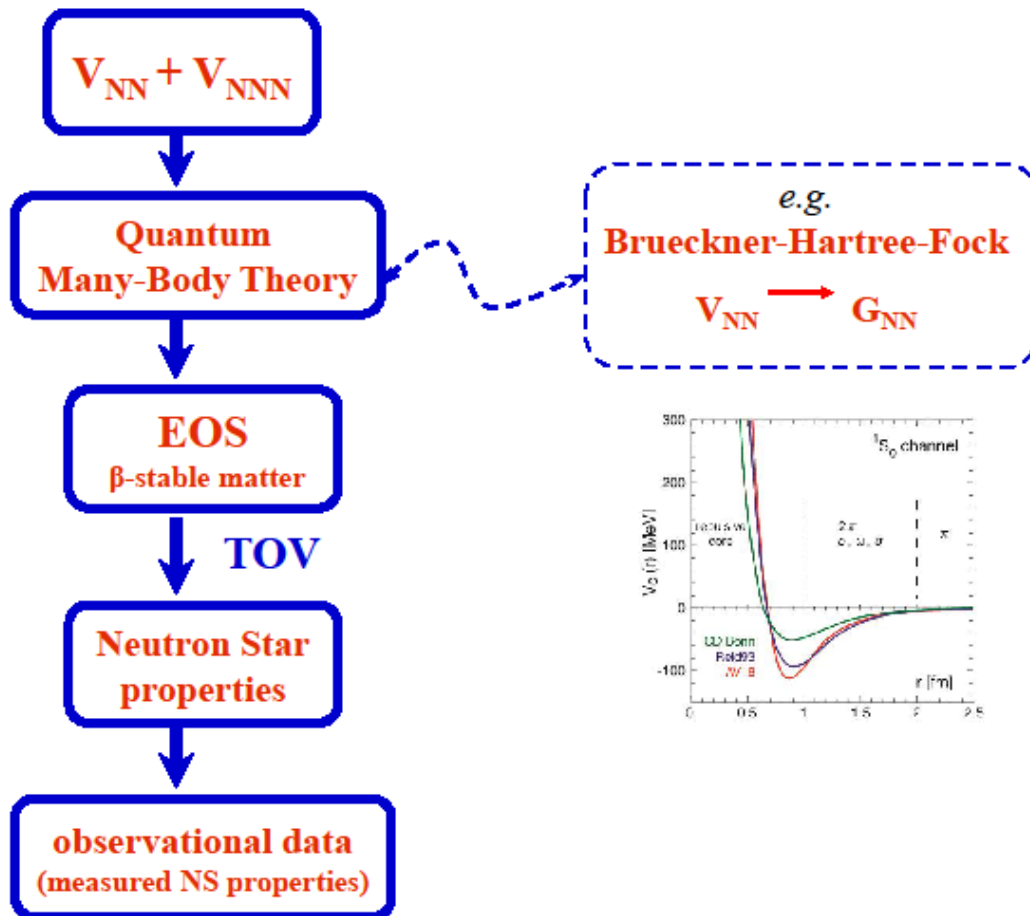
$E^{\text{pot}}(3\text{BF})/A \sim -1 \text{ MeV}$

A new microscopic EOS model for nuclear matter

I. Bombaci, D. Logoteta, *Astron. and Astrophys.* 609 (2018) A128

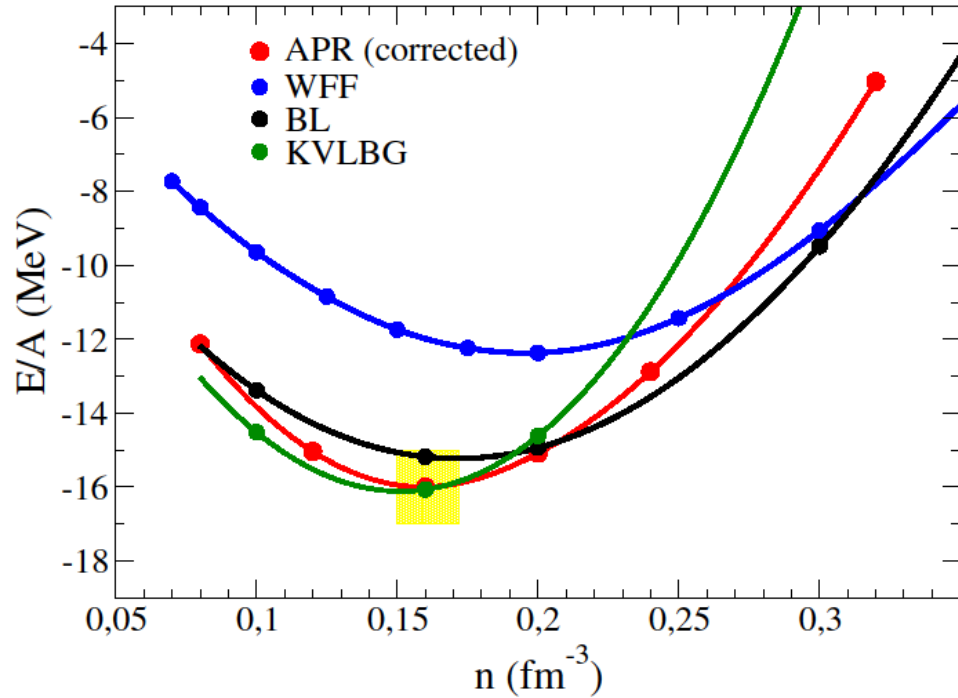
Available in tabular form on **CompOSE** web site

<https://compose.obspm.fr/eos/120/>



Interactions			
NN	ChEFT	N3LO	$\Delta(1232)$
Piarulli et al., <i>Phys. Rev. C</i> 94 (2016) 054007			
NNN	ChEFT	N2LO	$\Delta(1232)$
Epelbaum et al., <i>Phys. Rev. C</i> 66 (2002) 064001			
Navratil, <i>Few-Body Syst.</i> 41 (2007) 117			
average $\rightarrow V_{NN}^{\text{eff}}(\mathbf{n})$			
Logoteta et al., <i>Phys. Rev. C</i> 94 (2016) 064001			

Energy per nucleon (Symmetric NM)



Nuclear matter properties at the saturation density

EOS	n_0 (fm^{-3})	E_0 (MeV)	E_{sym} (MeV)	L (MeV)	K_0 (MeV)
BL	0.17	-15.2	35.4	76.0	190
KVLBG	0.15	-16.1	35.2	70.2	251
WFF	0.19	-12.4	31.0	56.5	209
APR	0.16	-16.0	33.9	59.4	266
APR _{micro}	0.18	-12.4	32.8	69.4	----
empirical	0.16 ± 0.01	-16 ± 1	25 – 37	30 – 90	180 – 260

BL : I. Bombaci, D. Logoteta, Astron. and Astrophys. 609 (2018) A128

WFF : R.B. Wiringa, V. Ficks and A. Fabrocini, Phys. Rev. C **38** (1988) 1010.

APR :A. Akmal, V.R. Pandharipande and D.G. Ravenhall, Phys. Rev. C **58** (1998) 1804.

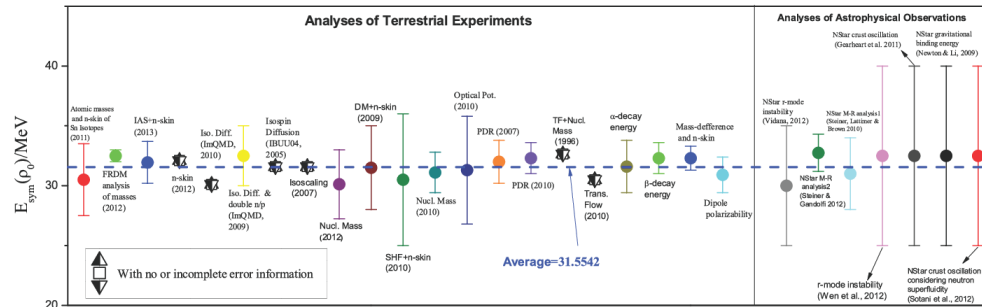
KVLBG :A. Kievsky, M. Viviani, D. Logoteta, I. Bombaci and L. Girlanda, Phys. Rev. Lett. **121** (2018) 072701.

Selecting Nuclear Matter EOS: basic requirements

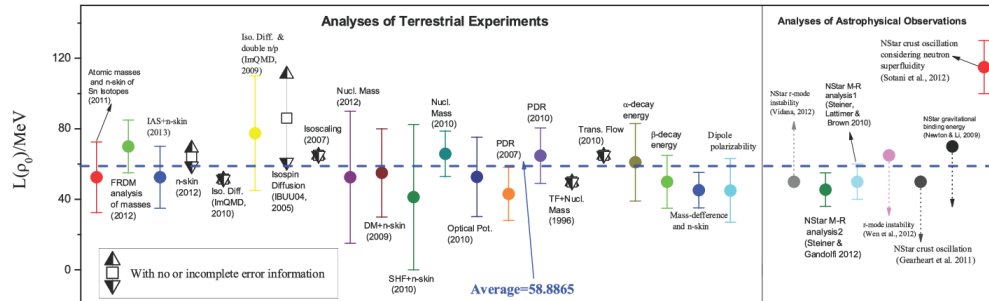
Nuclear Symmetry Energy

$$E_{sym}(n) = E_{sym}(n_0) + L \left(\frac{n - n_0}{3n_0} \right) + \frac{1}{2!} K_{sym} \left(\frac{n - n_0}{3n_0} \right)^2 + \frac{1}{3!} Q_{sym} \left(\frac{n - n_0}{3n_0} \right)^3 + \dots$$

Symmetry energy slope parameter $L = 3n_0 \left. \frac{\partial E_{sym}(n)}{\partial n} \right|_{n_0}$



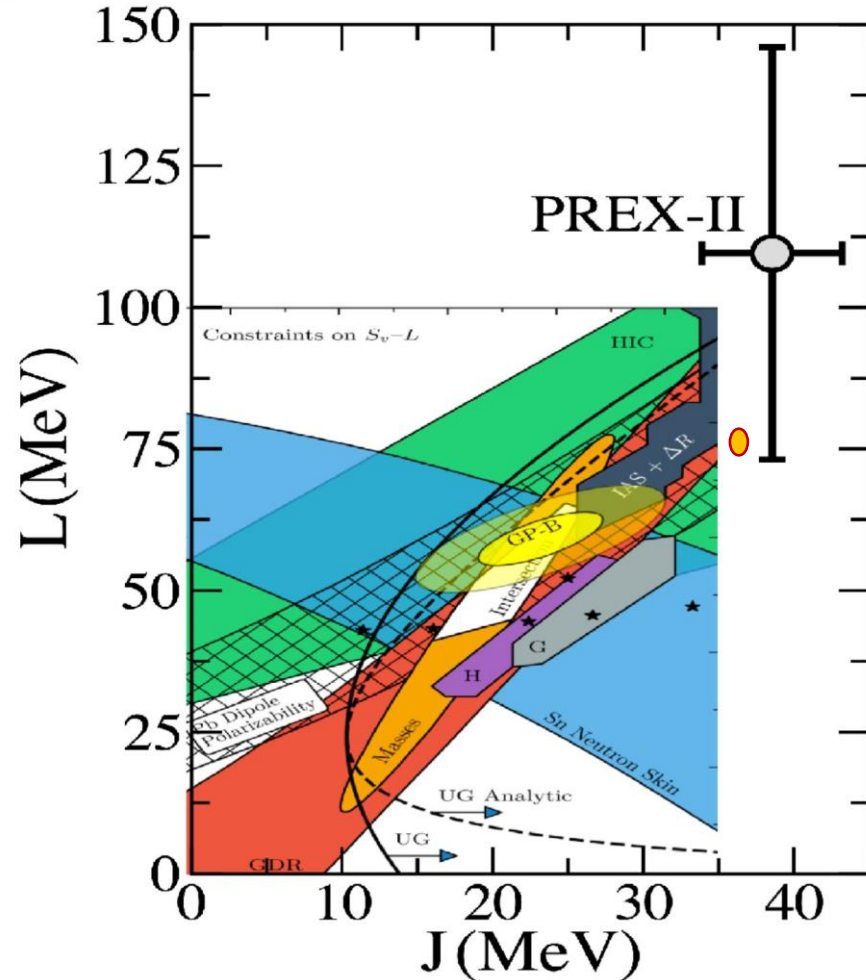
$$E_{sym}(n_0) = (25 - 37) \text{ MeV}$$



$$L = (30 - 90) \text{ MeV}$$

From various nuclear physics experimental data: B.A. Li and X. Han, Phys. Lett. B 727 (2013) 276

Neutron Skin Thickness: the PREX-II experiment



Accurate measurements of the neutron skin thickness Δr_{np} in neutron rich nuclei via **parity-violating electron scattering** at JLab can constrain the **symmetry energy $E_{\text{sym}}(n_0) \equiv J$** at the saturation density and **its slope parameter L** .

D. Adhikari et al. (PREX Collaboration), Phys. Rev. Lett. 126 (2021).

$$\Delta r_{np}(^{208}\text{Pb}) = 0.283 \pm 0.071 \text{ fm}$$

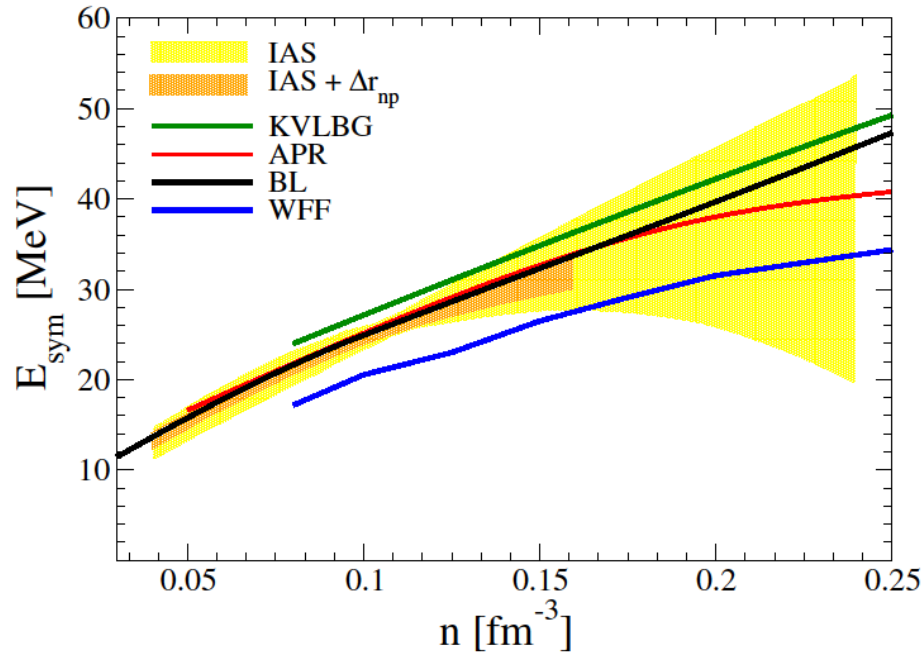
$$E_{\text{sym}}(n_0) \equiv J = 33.4 - 42.8 \text{ MeV}$$

$$L = 69 - 143 \text{ MeV}$$

$$\text{BL EOS: } E_{\text{sym}}(n_0) = 35.4 \text{ MeV, } L = 76.0 \text{ MeV}$$

For asymmetric Nuclear Matter as a mixture of neutrons and protons free Fermi gases $L = 2E_{\text{sym}}(n_0)$

Symmetry energy



IAS = constraint from Isobaric Analog States in nuclei

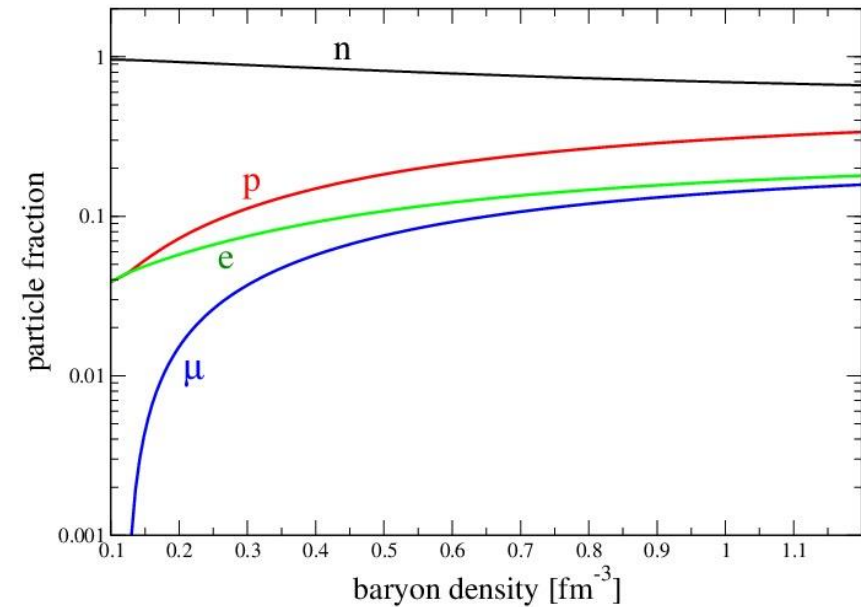
(P. Danielewicz, J. Lee, Nucl. Phys. A922 (2014) 1)

Δr_{np} = neutron skin thickness of heavy nuclei

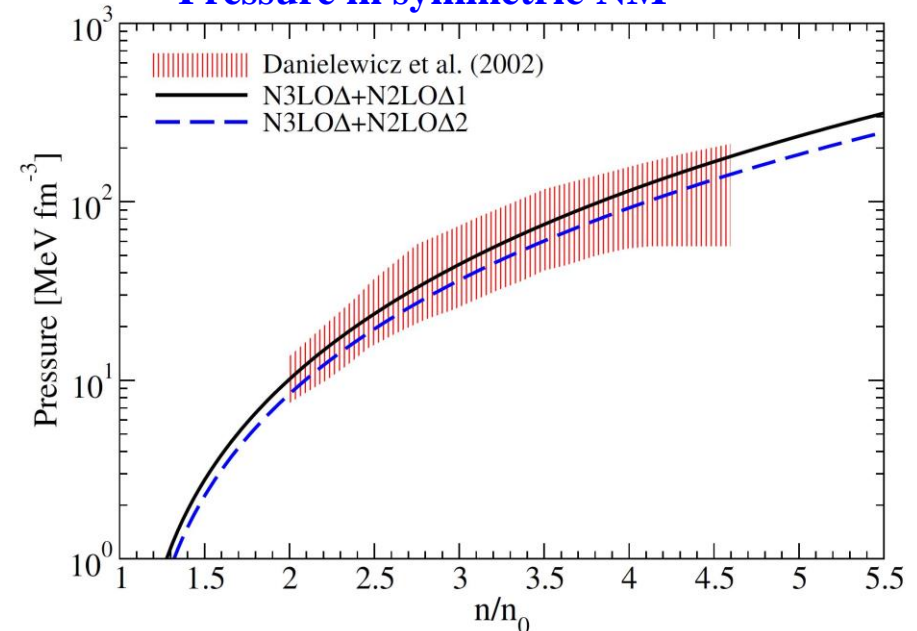
(X. Roca-Maza et al., Phys. Rev. C 87 (2013) 034301)

The **red hatched area** represents the region in the pressure–density plane for **SNM** which is consistent with the measured **elliptic flow of matter in collision experiments between heavy atomic nuclei** (Danielewicz et al. 2002).

Particle fractions



Pressure in symmetric NM



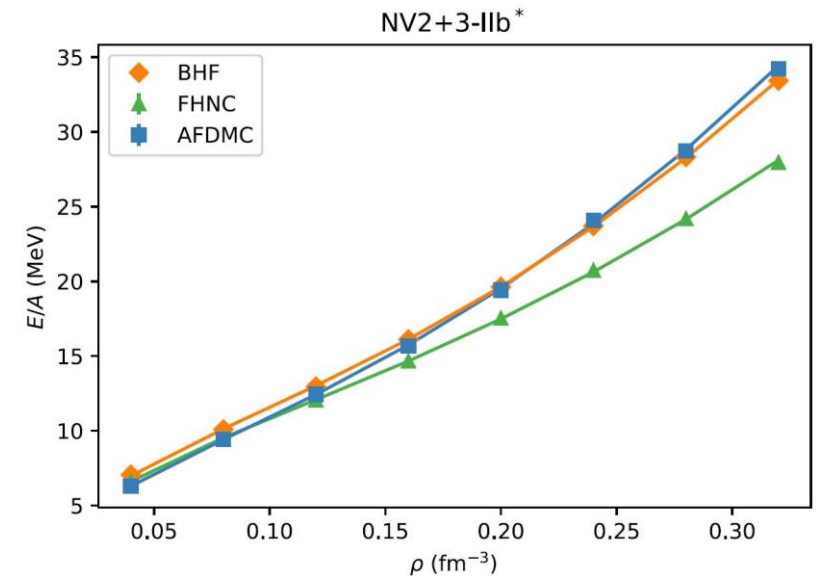
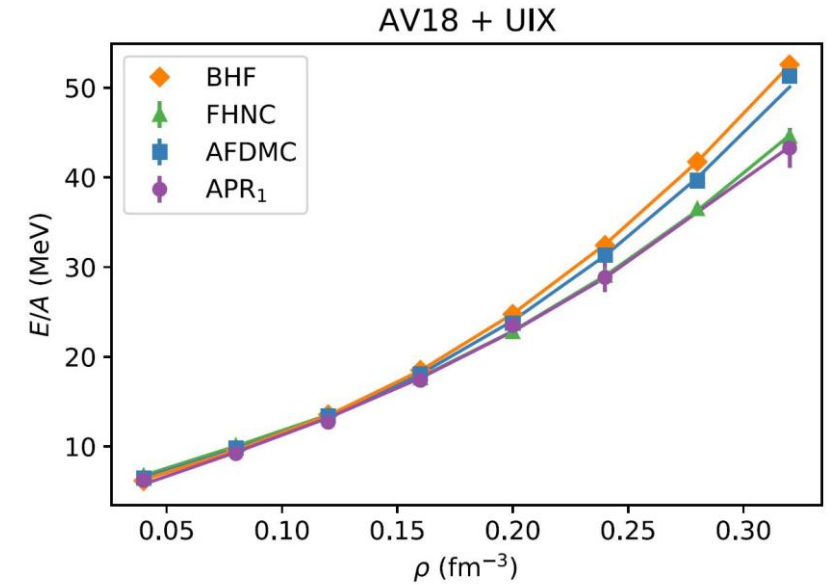
Dependence of the nuclear EOS on the quantum many-body approach

Benchmark calculations of **pure neutron matter** with realistic nuclear interactions

Brueckner-Hartree-Fock (BHF)

Fermi hypernetted chain/ single-operator chain (FHNC)

auxiliary-field diffusion Monte Carlo (AFDMC)

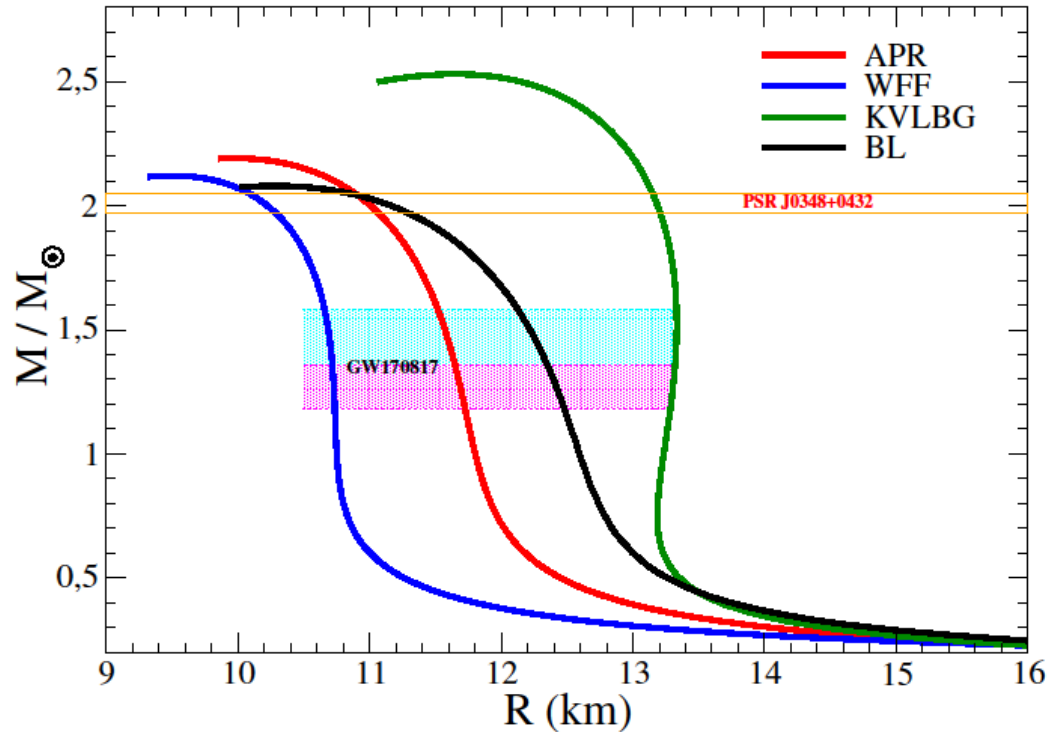


M. Piarulli, I. Bombaci, D. Logoteta, A. Lovato, R.B. Wiringa, Phys. Rev. C 101 (2020)

A. Lovato, I. Bombaci, D. Logoteta, M. Piarulli, R.B. Wiringa, Phys. Rev. C 105 (2022)

Neutron Stars

mass-radius relation



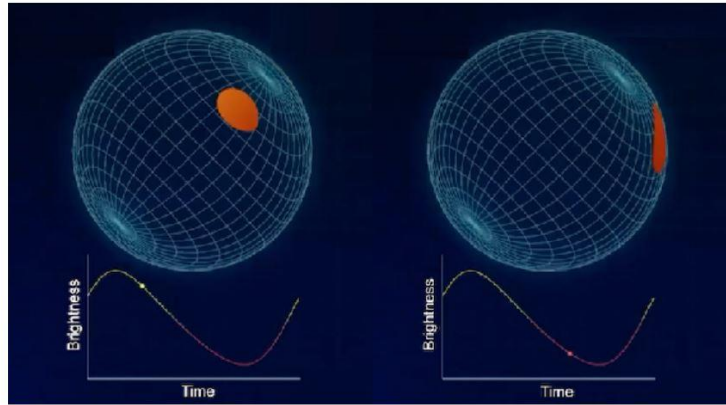
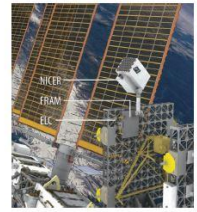
Properties of the Maximum mass configuration

EOS	n_c (fm^{-3})	n_c / n_0	ρ_c (10^{14}g/cm^3)	M (M_{\odot})	R (km)
BL	1.156	7.23	27.37	2.08	10.28
KVLBG	0.845	5.28	20.25	2.53	11.65
WFF	1.247	7.79	30.12	2.12	9.50
APR	1.146	7.16	27.87	2.19	9.97

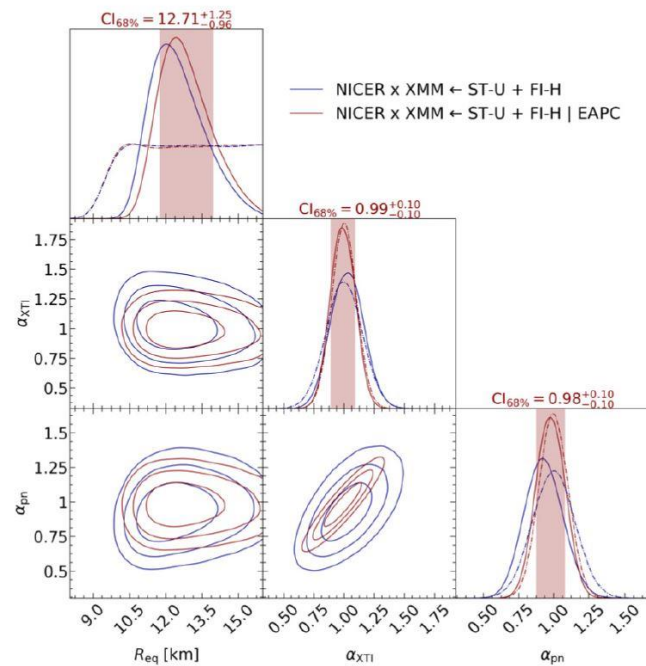
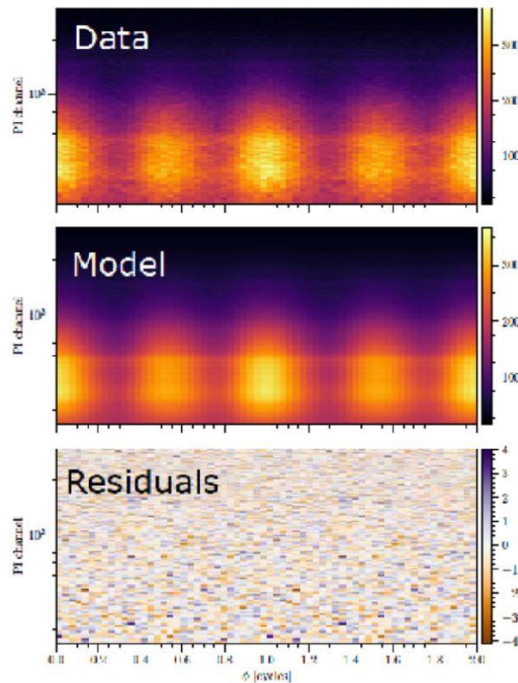
GW170817 data from: **B. P. Abbot et al. (LIGO-Virgo collaboration), Phys. Rev. Lett. 119 (2017) 161101**

$R_1 = (11.9 \pm 1.4) \text{ km}$ and $R_2 = (11.9 \pm 1.4) \text{ km}$ for the radius of the two neutron stars and $1.36 \leq M_1/M_{\odot} \leq 1.58$ and $1.18 \leq M_2/M_{\odot} \leq 1.36$ for their masses at the 90% credible level, for the case of “low-spin” prior.

NICER: Neutron Star Interior Composition Explorer



A new way of measuring NS radius by tracking the X-ray emission from “hot spots” on the star’s surface as the star rotates. M/R is extracted by modeling the Pulse Profile of the hot spots



✧ PSR J0740+6620

$$M = 2.072^{+0.067}_{-0.066} M_{\odot}$$

$$R = 13.7^{+2.6}_{-1.5} \text{ km} \quad \text{Miller et al., arXiv:2105.06979}$$

$$R = 12.39^{+1.30}_{-0.98} \text{ km} \quad \text{Riley et al., arXiv:2105.06980}$$

✧ PSR J0030+0451

$$M/R = 0.156^{+0.008}_{-0.010}$$

$$R = 13.02^{+1.24}_{-1.06} \text{ km} \quad \text{Miller et al., ApJ 887 L24 (2019)}$$

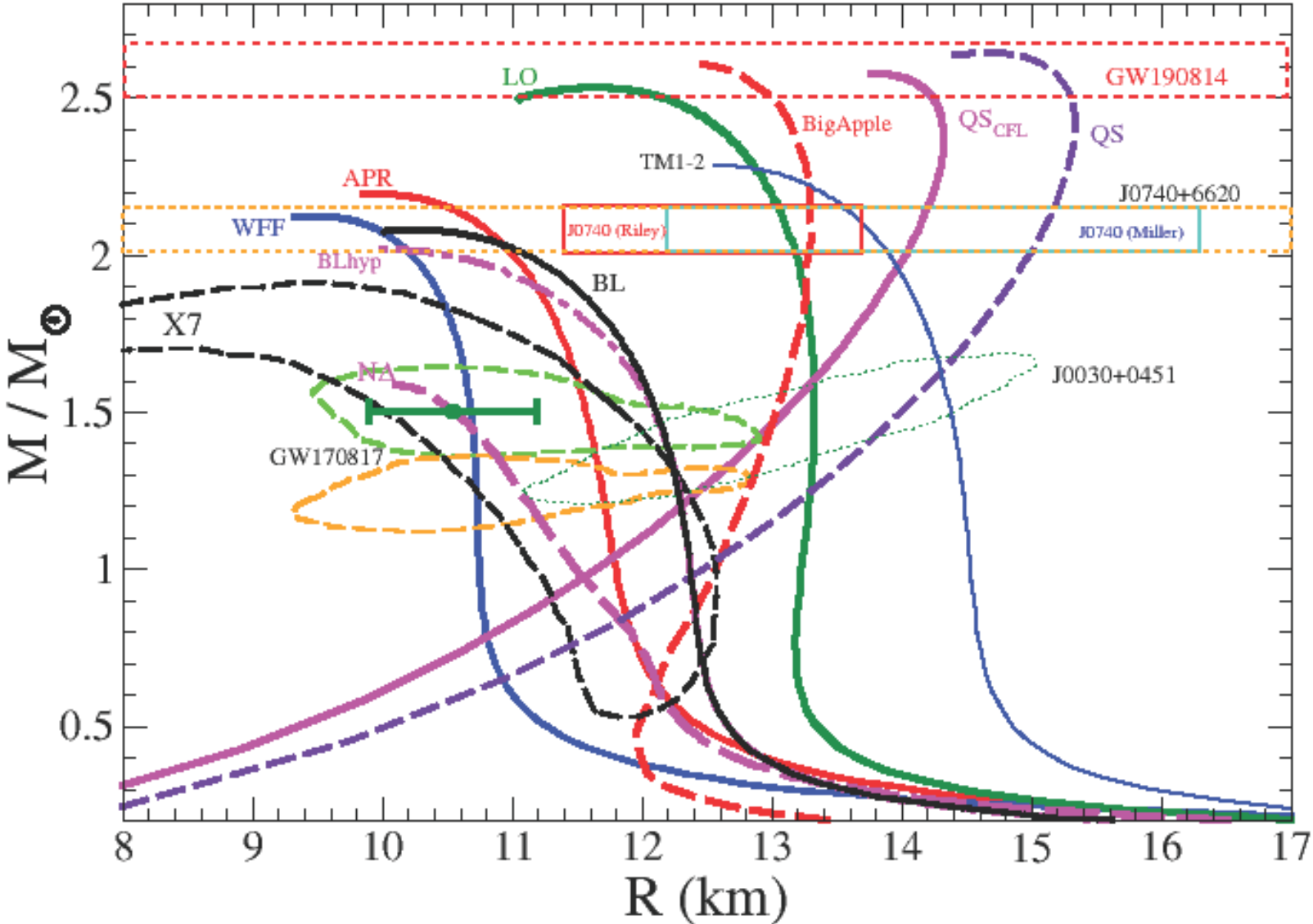
$$R = 12.71^{+1.14}_{-1.19} \text{ km} \quad \text{Riley et al., APJ 887 L21 (2019)}$$

Constraints from astrophysical observations

PSR J0740+6620

NICER data

T.E. Riley et al. (2021) arXiv:2105:06980
M.C. Miller et al. (2021) arXiv:2105:06979



GW190814

In August 2019, the LIGO-Virgo gravitational-wave network observed a GW signal produced by the merger of a **23 M_{\odot} black hole** and a **compact star with mass $M_2 = (2.50 - 2.67) M_{\odot}$** .

GW190814's secondary mass lies in the hypothesized lower **mass gap of 2.5–5 M_{\odot} between known NSs and BHs**.

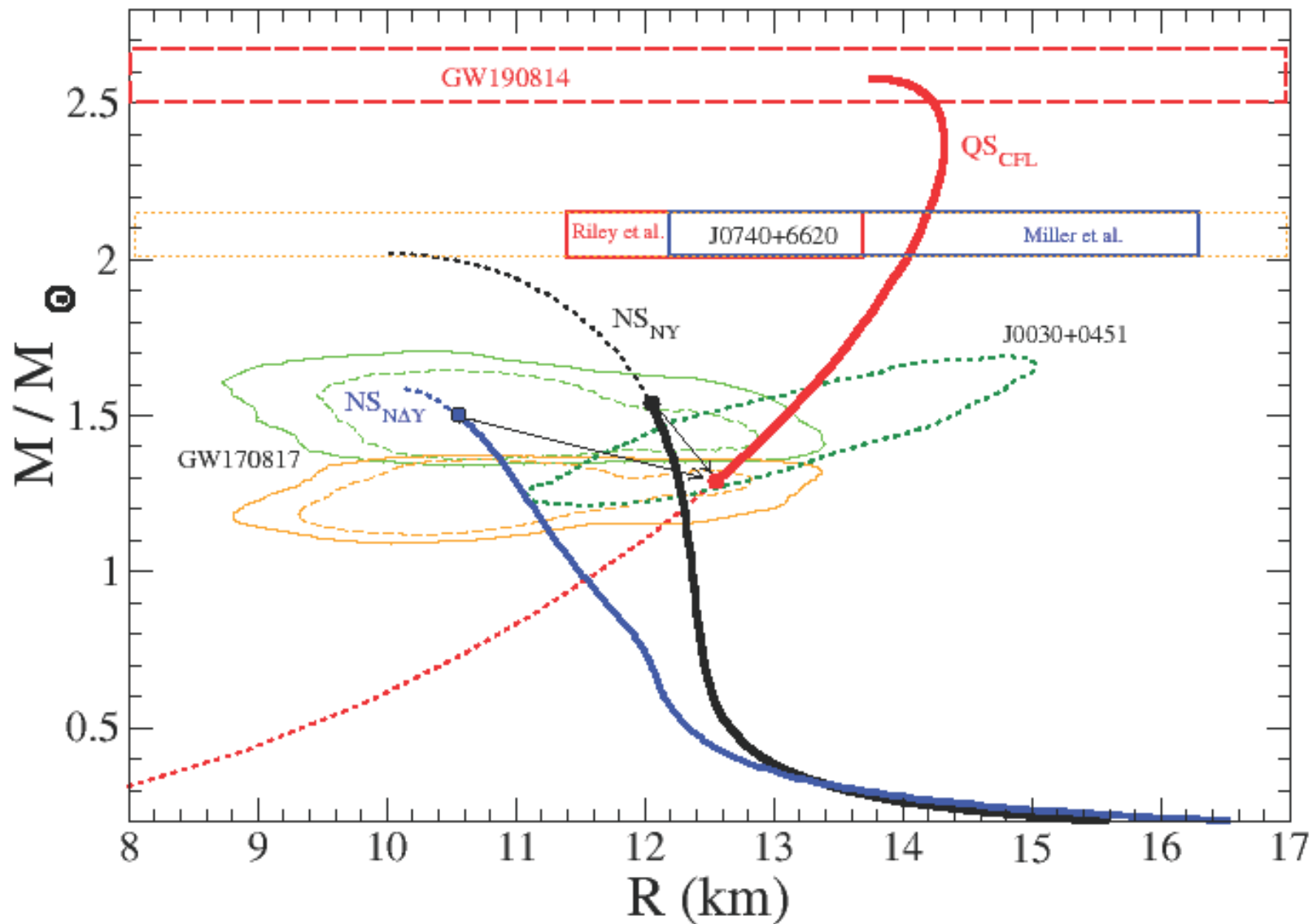
GW190814's secondary could be the **heaviest neutron star** or the **lightest black hole** ever observed.

R. Abbott et al., *Astrophys. J. Lett.* 896 (2020) L44

or could GW190814's secondary be a **Strange Quark Star?**

I. Bombaci, A. Drago, D. Logoteta, G. Pagliara, I. Vidaña, *Phys. Rev. Lett.* 126 (2021) 162702

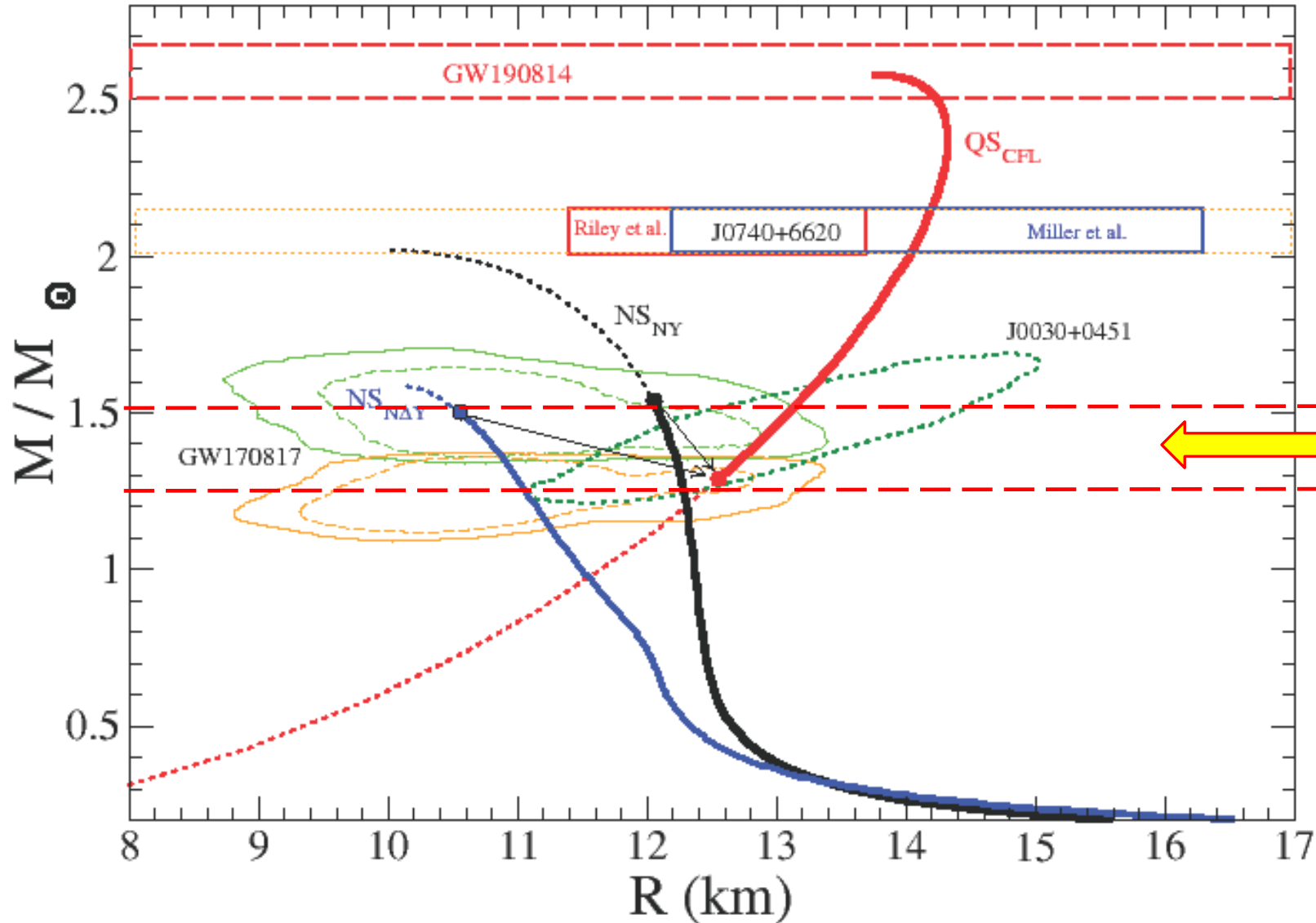
Two coexisting families of Compact Stars



consistent with the interpretation of the secondary component of **GW190814** as a **strange quark star** and relieve the tension between some measurements of **NS radii** (stars with «small» radii are Hadronic Stars and stars with «large» radii are Strange Quark Stars)

I. Bombaci, A. Drago, D. Logoteta,
G. Pagliara, I. Vidaña,
Phys. Rev. Lett. 126 (2021) 162702

Two coexisting families of Compact Stars



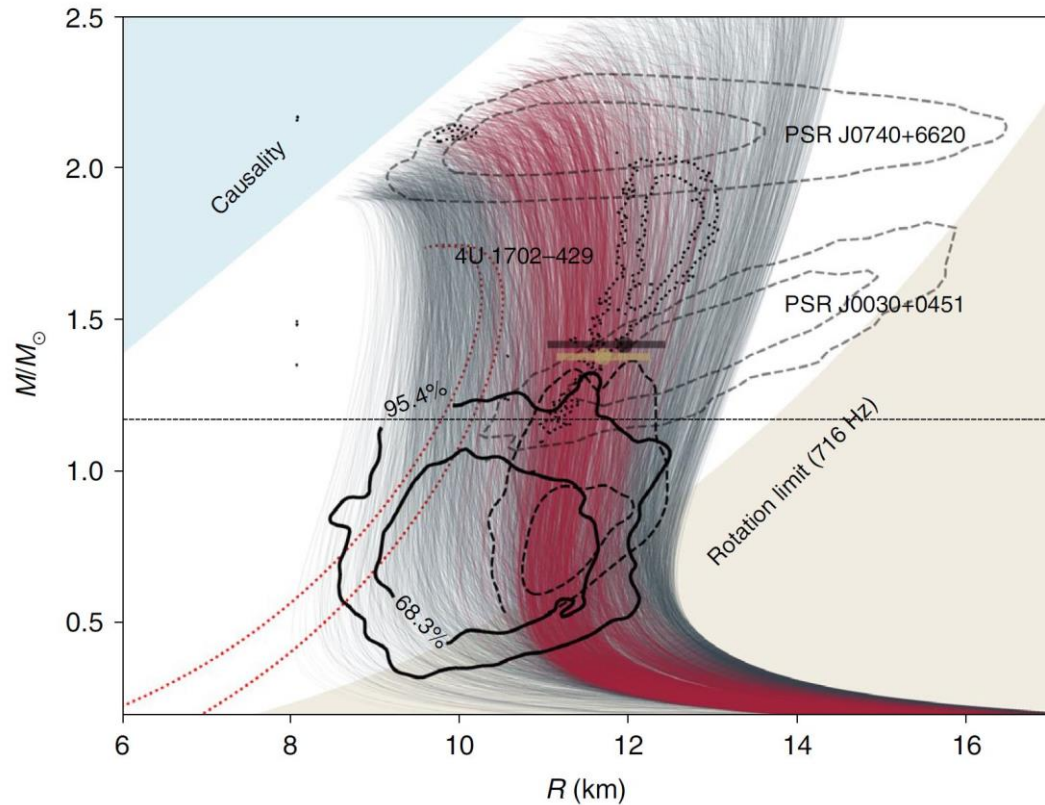
An observationally testable prediction of the two-family scenario:

existence in nature of compact stars having the same mass but different radii.

This observation would falsify the scenario with a single family of compact stars (hadronic stars).

I. Bombaci, A. Drago, D. Logoteta, G. Pagliara, I. Vidaña, Phys. Rev. Lett. 126 (2021) 162702

HESS J1731-347: a neutron star with an unusually small mass



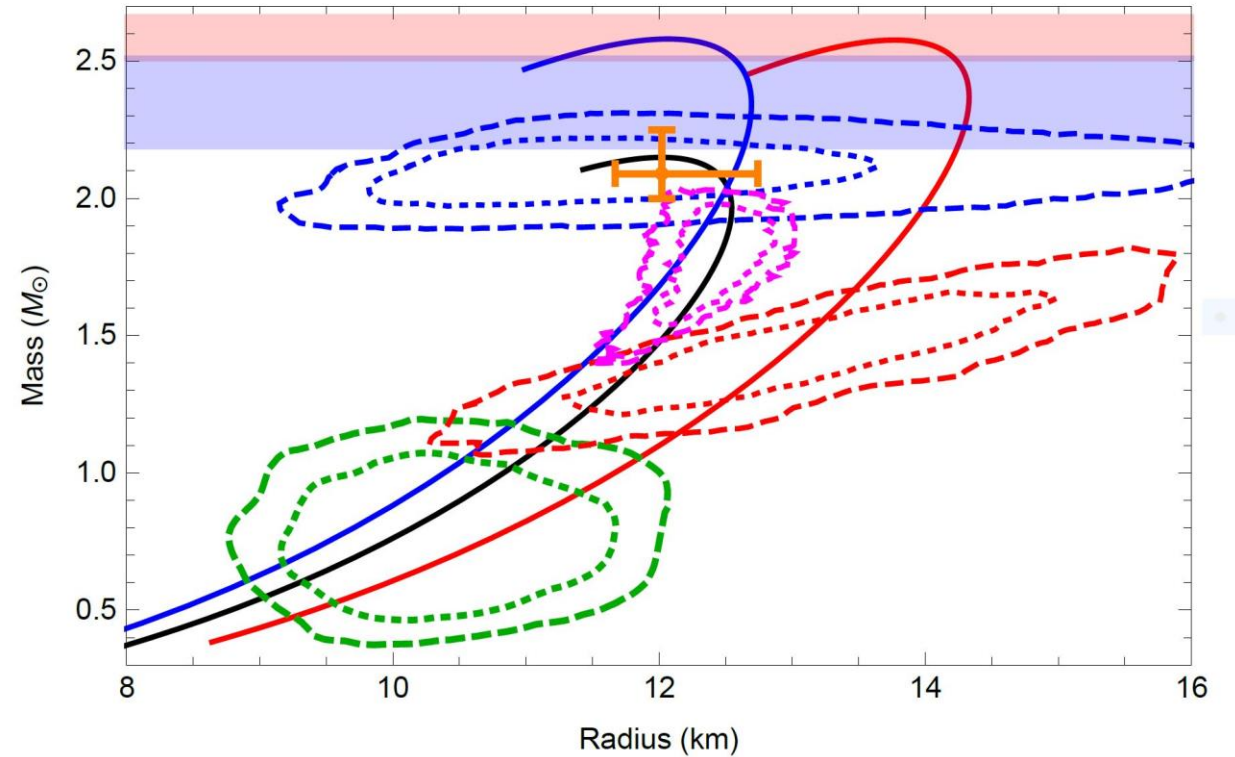
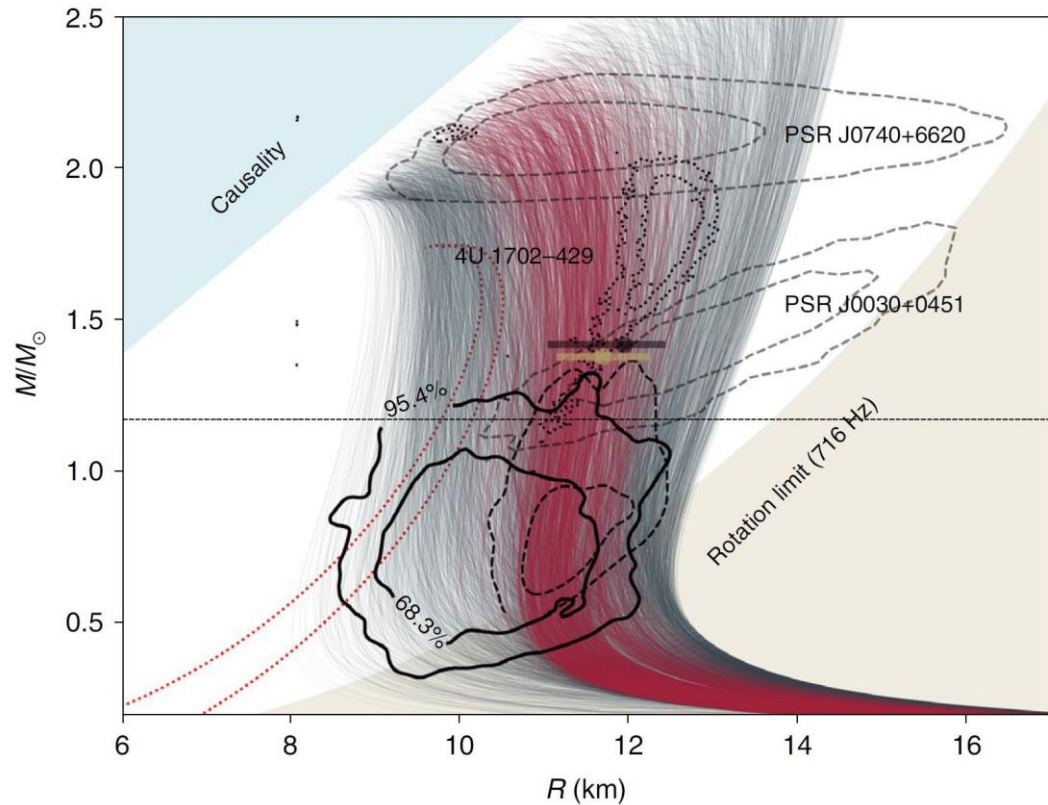
Central Compact Object in
the Supernova Remnant HESSE J1731-347

$$M = 0.77_{-0.17}^{+0.20} M_{\odot}$$

$$R = 10.4_{-0.78}^{+0.86} \text{ km}$$

V. Doroshenko et al., *Nature Astronomy* 6, 1444 (2022)

HESS J1731-347: a neutron star with an unusually small mass



Is the compact object associated with HESS J1731-347 a strange quark star?

F. Di Clemente, A. Drago, G. Pagliara,
arXiv:2211.07485 (Nov. 2022)

Message taken from **Nucleon Stars** (i.e. Neutron Stars with a pure **nuclear matter core**)

NN interactions are essential to have “large” stellar mass

For a free neutron gas $M_{\max} = 0.71 M_{\odot}$ (Oppenheimer and Volkoff, 1939)

NNN interactions are essential

- (i) to reproduce the correct **empirical saturation point of nuclear matter**
- (ii) to reproduce measured neutron star masses, i.e. to have **$M_{\max} > 2 M_{\odot}$**

models of **Nucleon Stars**

(i.e. Neutron Stars with a pure **nuclear matter core**)

are able to explain measured Neutron Star masses
as those of

PSR J1614-2230, PSR J0348+0432 and PSR J0740+6620

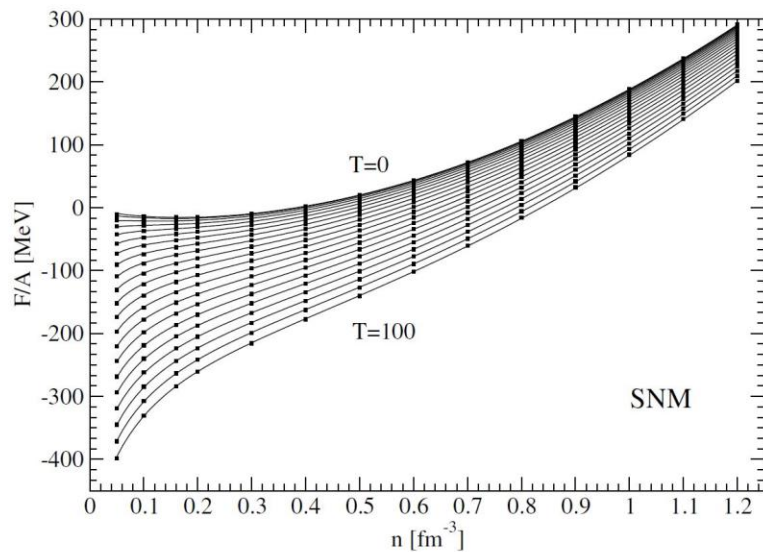
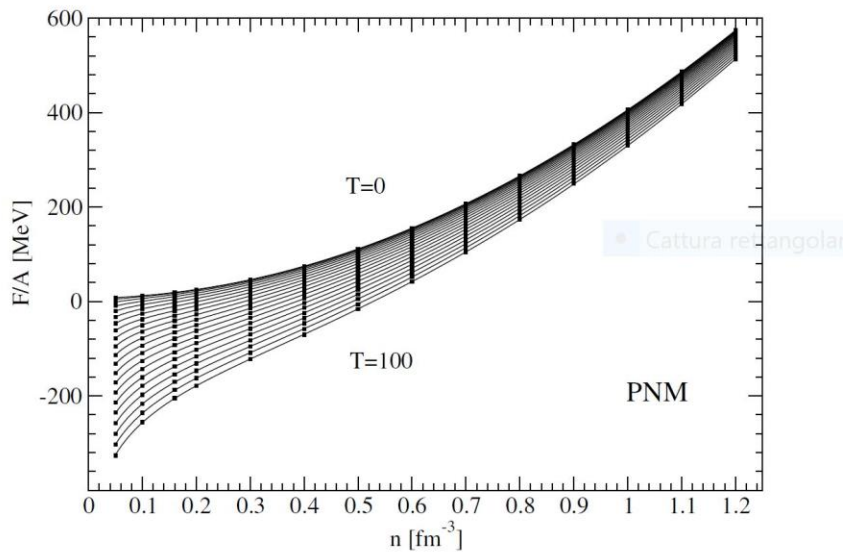
$$M_{\text{NS}} \approx 2 M_{\odot}$$

Happy?

Not the end of the story!

The EOS of Nuclear Matter at finite temperature

Extension of the BHF approximation at $T \neq 0$



Free energy per particle F/A

D. Logoteta, A. Perego, I. Bombaci, *Astron. & Astrophys.* 646 (2021)

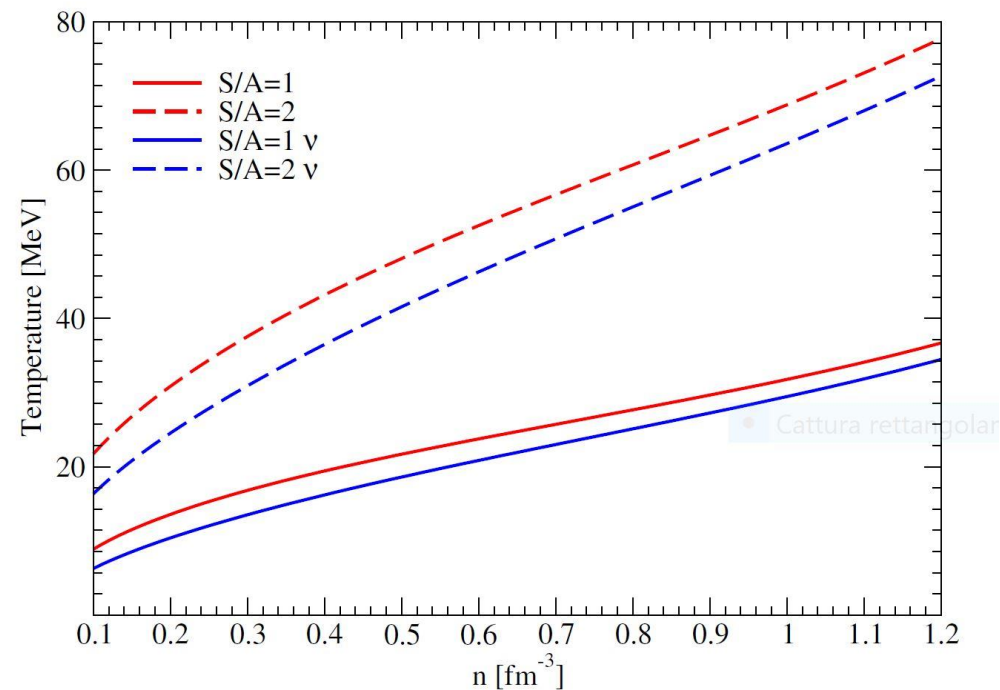


Fig. 5. Temperature profiles as a function of nuclear density n for isentropic ($S/A = \text{const}$) β -stable EOSs in the case of neutrino-free (red lines) and neutrino trapped (blue lines) matter.

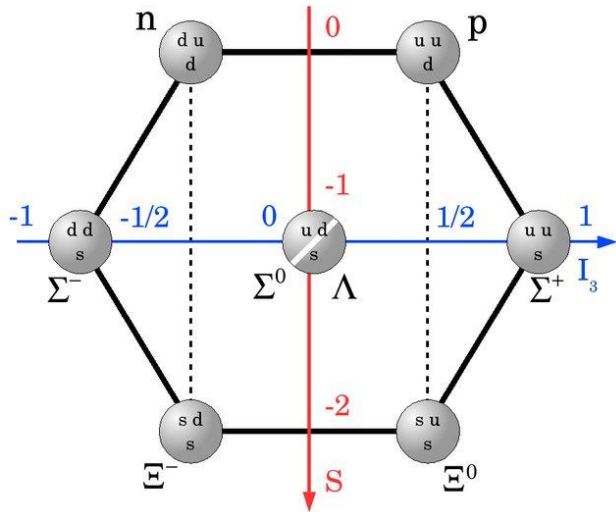
$$T_S = \chi(\beta) \frac{S}{A} n^{2/3},$$

where

$$\chi(\beta) = \frac{\hbar^2}{m} \left(\frac{3}{\pi}\right)^{2/3} \left[\left(\frac{1+\beta}{2}\right)^{1/3} + \left(\frac{1-\beta}{2}\right)^{1/3} \right]^{-1}.$$

Hyperons in dense matter and Hyperon Stars

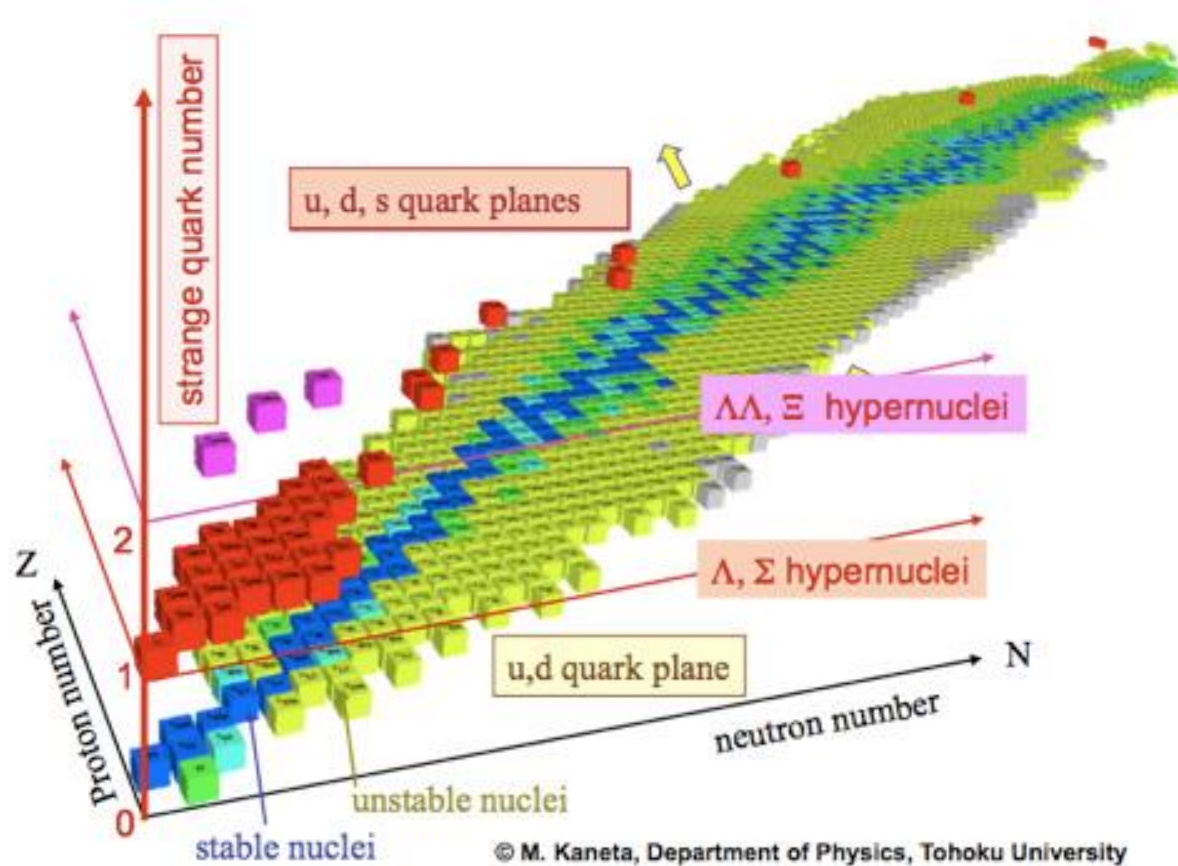
Strange baryons: the **baryon octet** : $\mathbf{J}^\pi = (1/2)^+$



baryon name	Isospin	I_3	S	quarks	mass (MeV/c ²)	mean life
n	1/2	-1/2	0	udd	939.56	880 s
p	1/2	+1/2	0	uud	938.27	stable
Λ	0	0	-1	uds	1115.68	2.63×10^{-10} s
Σ ⁻	1	-1	-1	dds	1197.43	1.48×10^{-10} s
Σ ⁰	1	0	-1	uds	1192.55	7.4×10^{-20} s [†]
Σ ⁺	1	+1	-1	uus	1189.37	0.80×10^{-10} s
Ξ ⁻	1/2	-1/2	-2	dss	1321.32	1.64×10^{-10} s
Ξ ⁰	1/2	+1/2	-2	uss	1314.90	2.90×10^{-10} s

[†] The Σ^0 decays via the electromagnetic process $\Sigma^0 \rightarrow \Lambda + \gamma$ followed by the weak decay of the Λ .

Hypernuclei: (n, p, Y) bound states



valuable informations on the interactions
between nucleons and hyperons

Hyperons formation in dense nuclear matter



etc.

$$\mu_p = \mu_n - \mu_e = \mu_{\Sigma^+}$$

$$\mu_n = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_{\Lambda}$$

$$\mu_n + \mu_e = \mu_{\Sigma^-} = \mu_{\Xi^-}$$

$$\mu_{\mu} = \mu_e$$

$$n_p + n_{\Sigma^+} = n_e + n_{\mu} + n_{\Sigma^-} + n_{\Xi^-}$$

(neutrino-free matter)

At sufficiently high density nuclear matter
turns into hyperonic matter

Hyperons threshold densities

(1) a simple estimate: $\{n, p, e\}$ ideal gas in β -equilibrium (**no strong interaction**) [non-rel. (n, p), ultrarel. e^-]

$$\mu_n = \mu_p + \mu_e = m_\Lambda c^2 \quad \longrightarrow \quad n^{*\Lambda} = 0.86 \text{ fm}^{-3} = 5.4 n_0$$

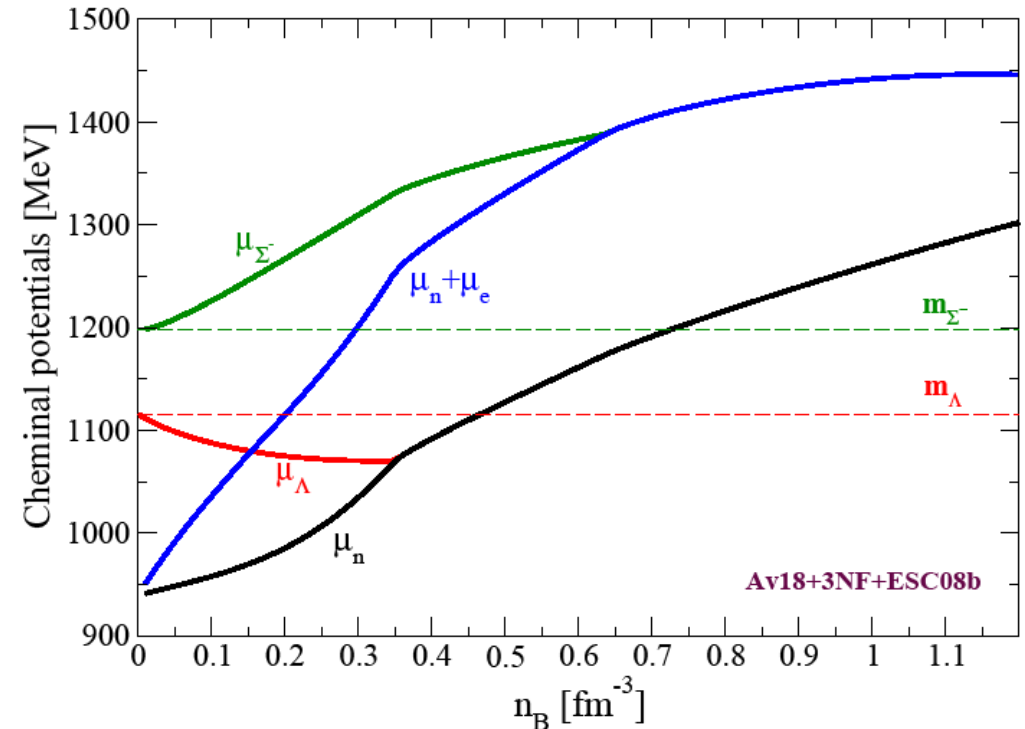
(2) Effect of the **strong interaction**

NN(Av18) + NNN + NY(ESC08b)
(no hyperonic TBF)

$$n^{*\Lambda} = 0.35 \text{ fm}^{-3} = 2.19 n_0$$

$$n^{*\Sigma^-} = 0.64 \text{ fm}^{-3} = 4 n_0$$

$$n^{*\Lambda}, n^{*\Sigma^-} < n_c(M_{max})$$



Neutron stars: the maximum mass configuration

EOS	n_c (fm^{-3})	n_c / n_0	ρ_c (10^{14} g/cm^3)	M (M_\odot)	R (km)
BL	1.156	7.23	27.37	2.08	10.28
KVLBG	0.845	5.28	20.25	2.53	11.65
WFF	1.247	7.79	30.12	2.12	9.50
APR	1.146	7.16	27.87	2.19	9.97

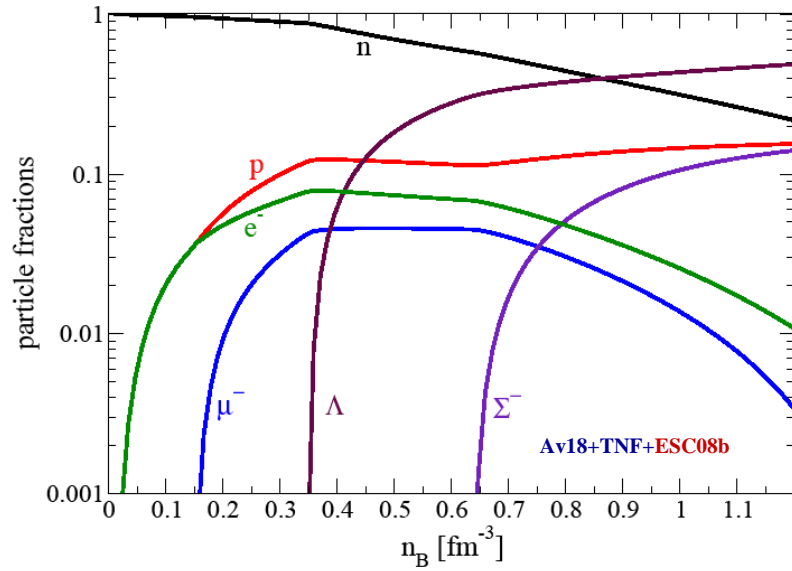
I. Bombaci, D. Logoteta, Astron. and Astrophys. 609 (2018) A128

A. Kievsky, M. Viviani, D. Logoteta, I. Bombaci and L. Girlanda, Phys. Rev. Lett. **121** (2018) 072701

R.B. Wiringa, V. Ficks and A. Fabrocini, Phys. Rev. C **38** (1988) 1010.

A. Akmal, V.R. Pandharipande and D.G. Ravenhall, Phys. Rev. C **58** (1998) 1804.

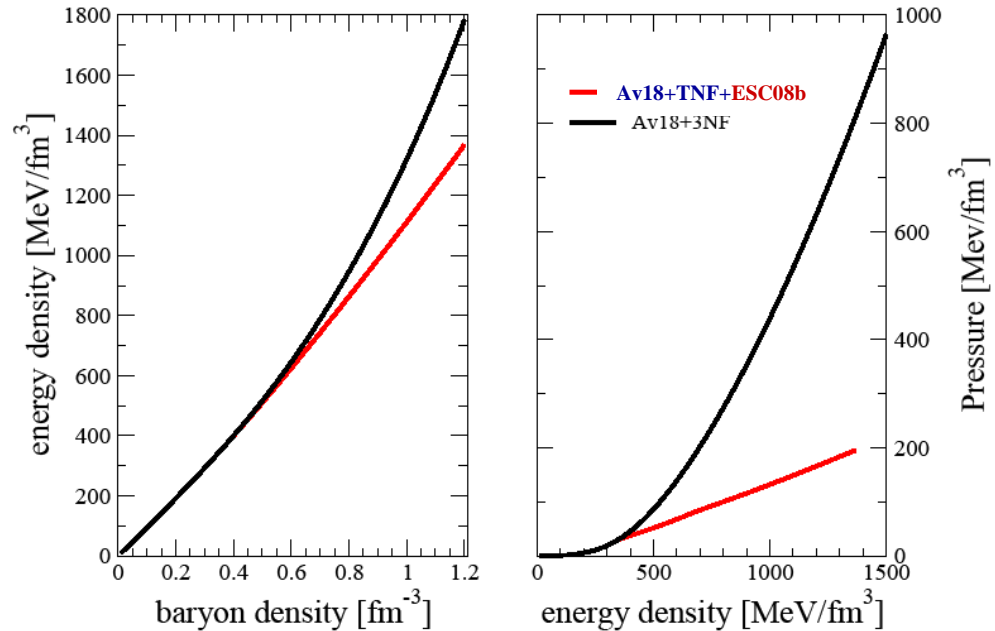
Particle fractions



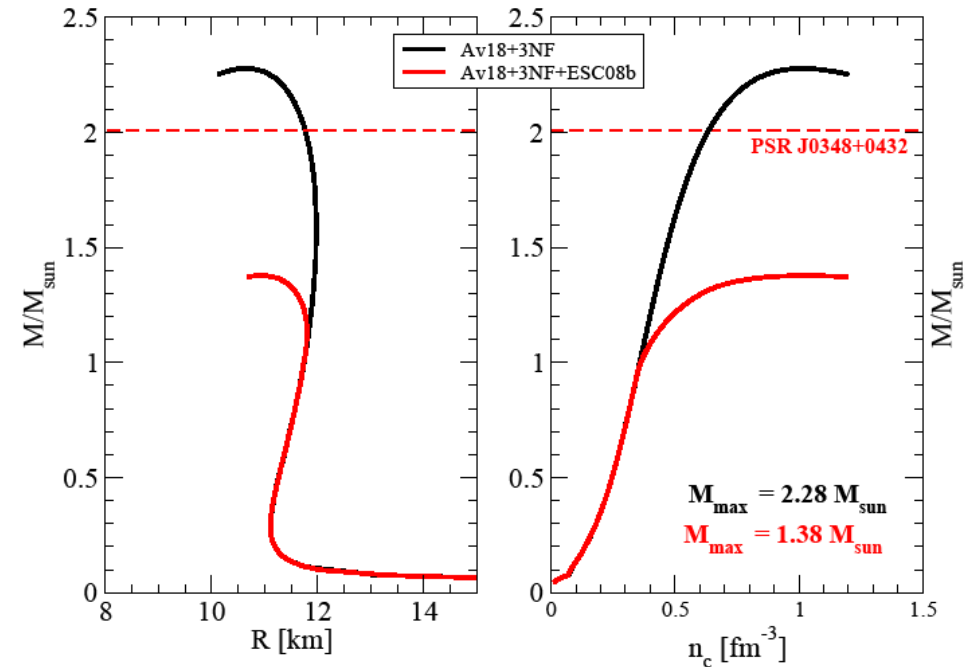
NN(Av18) + NNN + NY(ESC08b)
no hyperonic TBF

hyperons produce a strong softening of the EOS

Equation of State of Hyperonic Matter



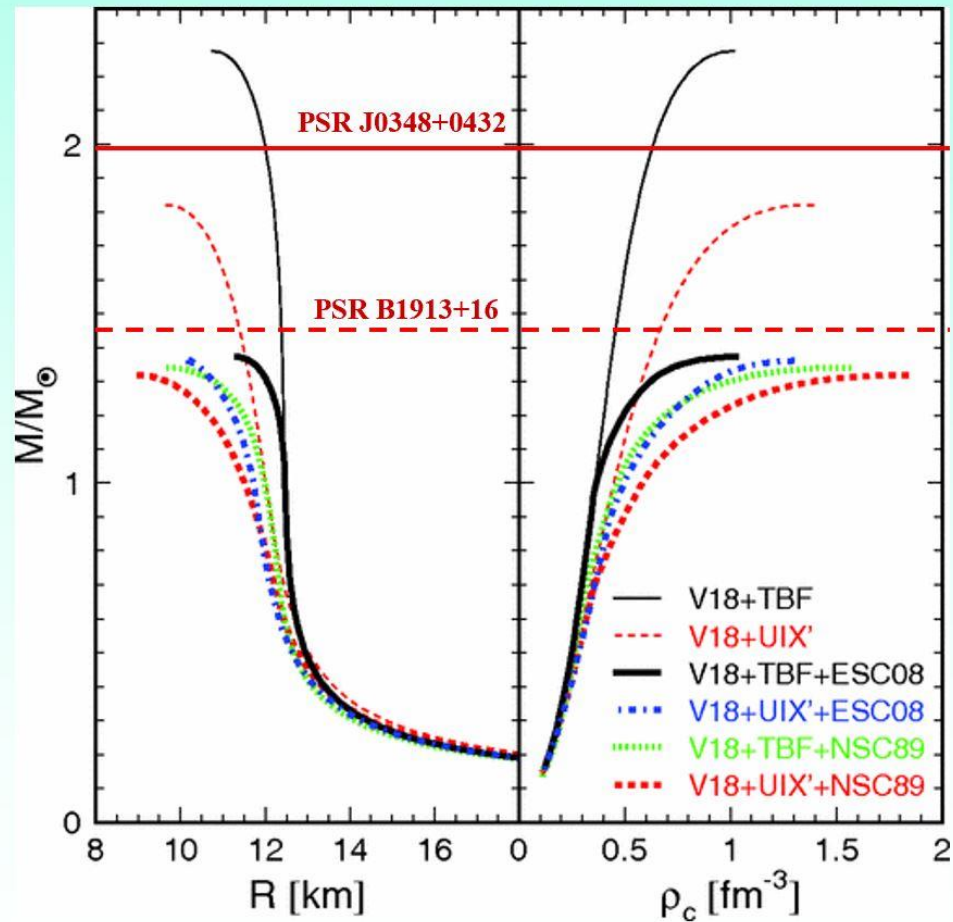
Stellar mass



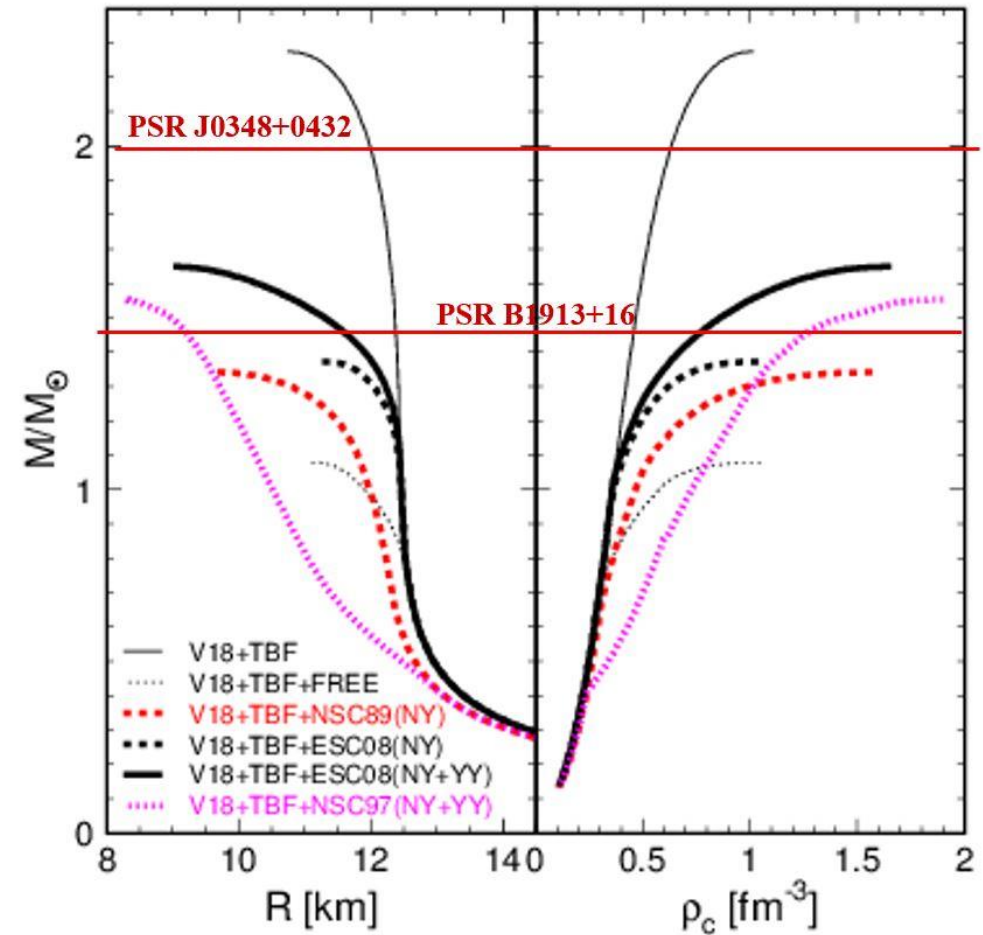
Effect of the YY interaction

$\text{NN(Av18)} + \text{NNN} + \text{NY} + \text{YY (ESC08c)}$
no hyperonic TBF

$\text{NN(Av18)} + \text{NNN} + \text{NY(ESC08 or NSC89)}$
no YY int. no hyperonic TBF



H.-J. Schulze, T. Rijken, Phys. Rev. C 84 (2011) 035801



T. Rijken, H.-J. Schulze, Eur. Phys. J. A 52 (2016) 21

The hyperon puzzle in Neutron Stars

Many hyperonic matter EOS models (mostly microscopic models) predict the presence of hyperons in the NS core, but they give

$$M_{\max} < 2 M_{\odot}$$

not compatible with measured NS masses

A problem which likely originates from our incomplete knowledge of the baryonic interactions.

Hyperonic three-body interactions as a possible solution of the hyperon puzzle

As mentioned before, **three-nucleon interactions** are a necessary ingredient for an accurate description of nuclei and nuclear matter within non-relativistic approaches.

Thus, it is rather obvious to suppose the existence of **hyperonic three-body interactions** (of the type **NNY, NYY, YYY**).

The **NN Λ interaction** was in fact first hypothesized at the end of the 1950s (*), *i.e.* during the early days of hypernuclear physics, as an important ingredient to calculate the **binding energy of hypernuclei**.

It is thus reasonable to expect that **hyperonic three-body interactions** can influence dense-matter EoS and represent a **likely candidate to solve the hyperon puzzle**.

- (*)
- R. Spitzer, Phys. Rev., 110 (1958) 1190.
 - G. G. Bach, Nuovo Cimento XI (1959) 73.
 - R. H. Dalitz, 9th Int. Ann. Conf. on High-Energy Physics, Academy of Sciences, USSR, Vol I (1960) 587.

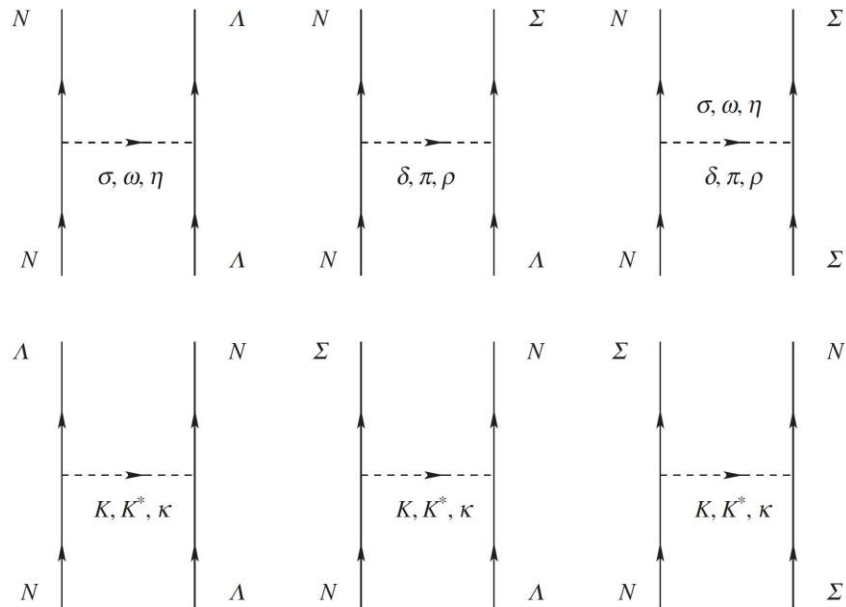
Microscopic approach to hyperonic matter EOS

input

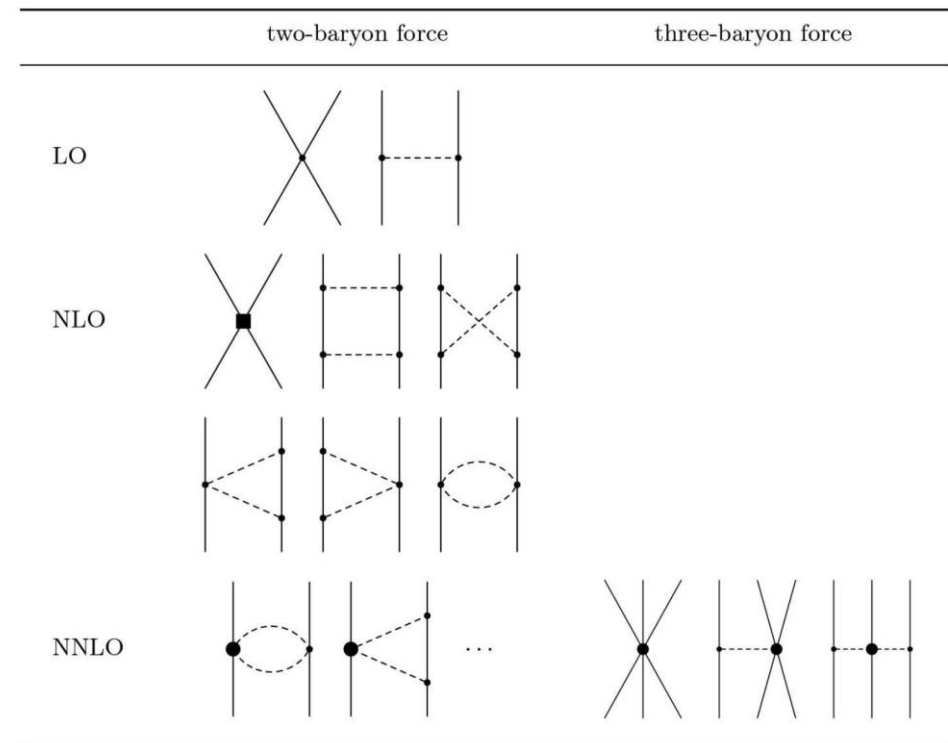
2BF: nucleon-nucleon (NN), nucleon-hyperon (NY), hyperon-hyperon (YY)
3BF: NNN, NNY, NYY, YYY, **4BF:**

Phenomenological meson-exchange models:

(Jülich, Nijmegen)



Chiral Effective Field Theory (ChEFT)



Microscopic approach to hyperonic matter EOS

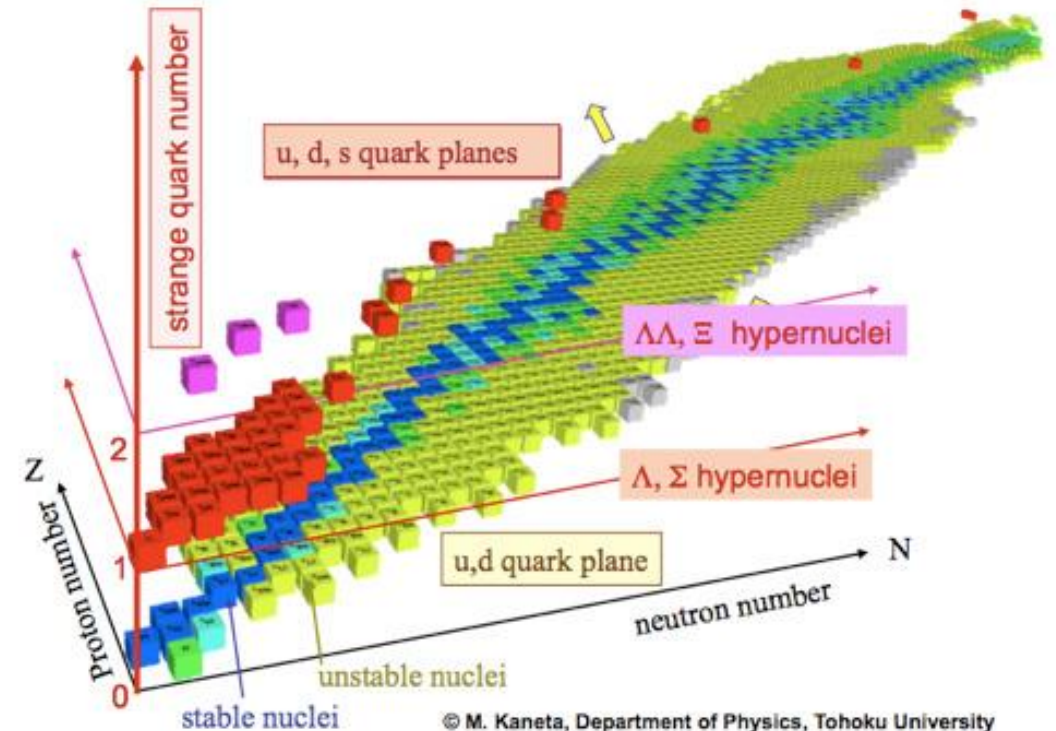
input

2BF: nucleon-nucleon (NN), nucleon-hyperon (NY), hyperon-hyperon (YY)
3BF: NNN, NNY, NYY, YYY, 4BF:

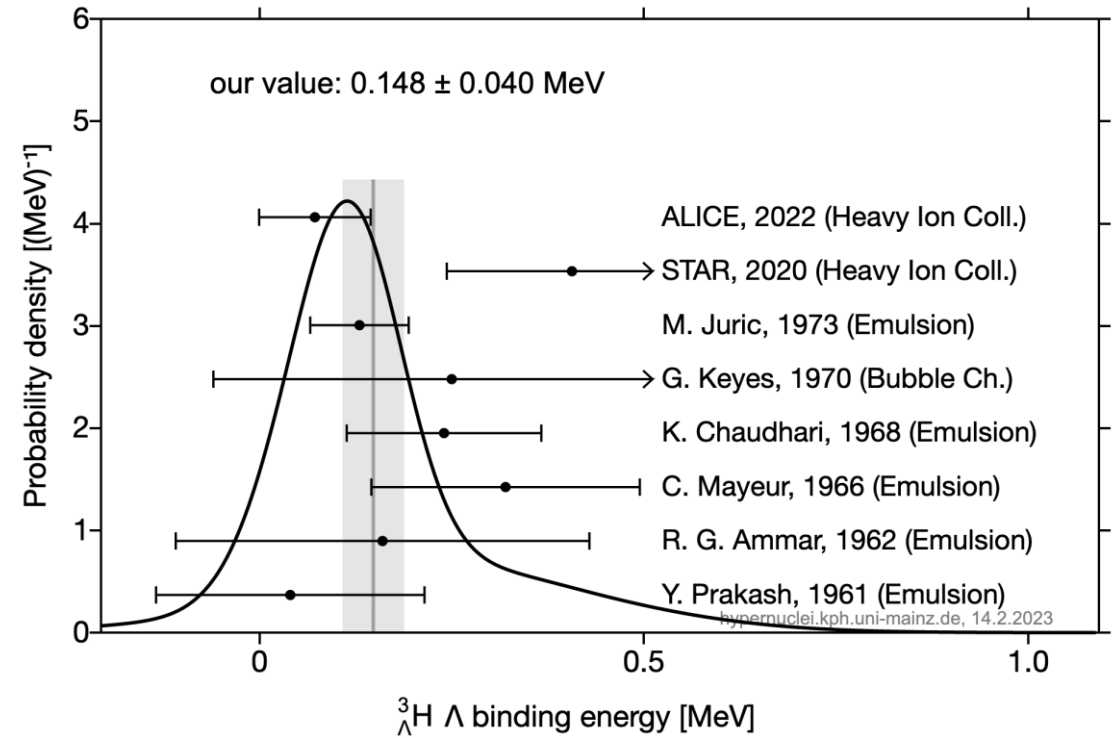
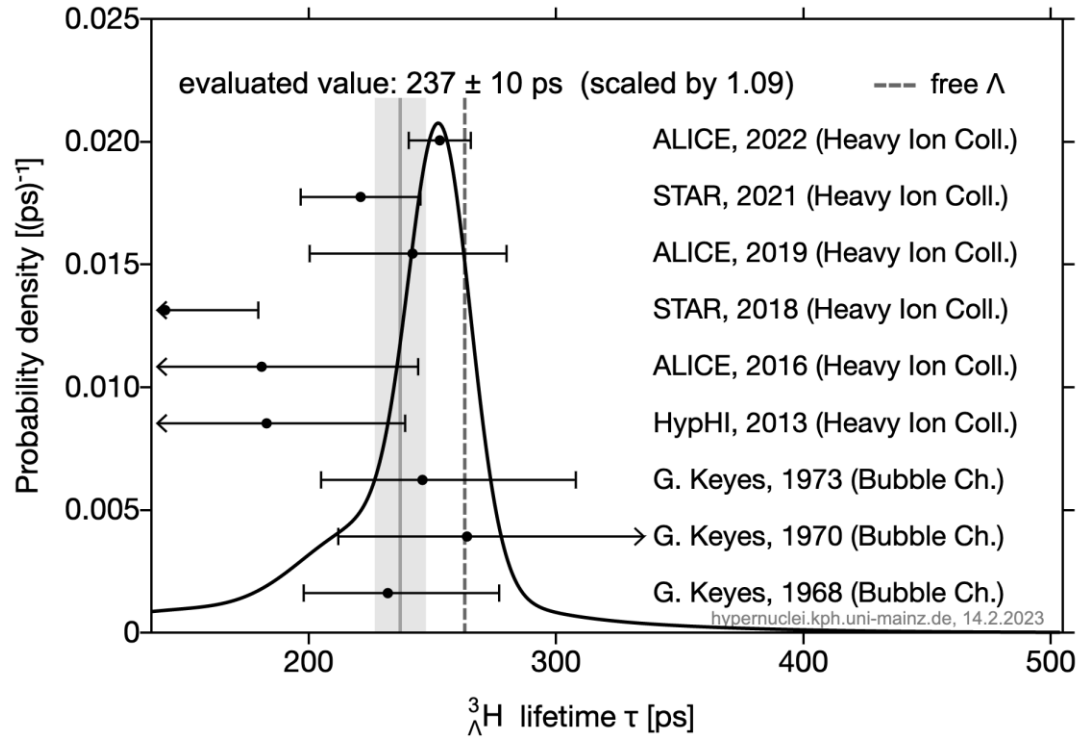
Hyperonic sector: experimental data

- 1. YN scattering data ≈ 50**
(NN data ≈ 4300)
- 2. Hypernuclei : $\approx 40 \Lambda X$, $\approx 5 \Lambda\Lambda X$**
a few Ξ^- -hypernuclei
(nuclei ≈ 3300)

E. Botta, T. Bressani, G. Garbarino, Eur. Phys. J. A 48 (2012) 41
C. Curceanu, *et al.*, Rev. Mod. Phys. 91 (2019) 025006
O. Hashimoto, H. Tamura, Prog. Part. Nucl. Phys. 57, (2006) 564.
L. Tolos, and L. Fabbietti, Prog. Part. Nucl. Phys. 112 (2020) 103770



Hypertriton lifetime and binding energy



M. Puccio, Day 2 Wrap-Up

A breakthrough for determining the NY and YY interactions

Recently, the **ALICE Collaboration** has demonstrated that **NY** and **YY interactions** can be investigated with high precision using **p - p and p -Pb collisions at the LHC** (using the technique called **femtoscropy**).

ALICE Collaboration. p - p , p - A and A - A correlations studied via femtoscopy in pp reactions at $\sqrt{s} = 7$ TeV. *Phys. Rev. C* 99, 024001 (2019).

ALICE Collaboration. Scattering studies with low-energy kaon-proton femtoscopy in proton-proton collisions at the LHC. *Phys. Rev. Lett.* 124, 092301 (2020).

ALICE Collaboration. Study of the A - A interaction with femtoscopy correlations in pp and p -Pb collisions at the LHC. *Phys. Lett. B* 797, 134822 (2019).

ALICE Collaboration. First observation of an attractive interaction between a proton and a cascade baryon. *Phys. Rev. Lett.* 123, 112002 (2019).

ALICE Collaboration. Investigation of the p - Σ^0 interaction via femtoscopy in pp collisions. *Phys. Lett. B* 805, 135419 (2020).

ALICE Collaboration. Unveiling the strong interaction among hadrons at the LHC, *Nature* 588, 232(2020)

Alberto Calivà, Day 1 Wrap-Up

Microscopic EOS for hyperonic matter: **extended Brueckner theory**

$$G(\omega)_{B_1 B_2 B_3 B_4} = V_{B_1 B_2 B_3 B_4} + \sum_{B_5 B_6} V_{B_1 B_2 B_5 B_6} \frac{Q_{B_5 B_6}}{\omega - e_{B_5} - e_{B_6}} G(\omega)_{B_5 B_6 B_3 B_4}$$

$$e_{B_i}(k) = M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + U_{B_i}(k)$$

$$U_{B_i}(k) = \sum_{B_j} \sum_{k' \leq k_{F B_j}} \langle \vec{k} \vec{k}' | \mathbf{G}_{B_i B_j B_i B_j}(\omega = e_{B_i} + e_{B_j}) | \vec{k} \vec{k}' \rangle$$

\mathbf{V} is the **baryon -baryon interaction** for the baryon octet: **n, p, Λ , Σ^- , Σ^0 , Σ^+ , Ξ^- , Ξ^0**

- **Energy per baryon in the **BHF approximation****

$$E/N_B = 2 \sum_{B_i} \int_0^{k_{F[B_i]}} \frac{d^3 k}{(2\pi)^3} \left\{ M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + \frac{1}{2} U_{B_i}^N(k) + \frac{1}{2} U_{B_i}^Y(k) \right\}$$

M. Baldo, G.F. Burgio, H.-J. Schulze, *Phys.Rev. C*61 (2000) 055801

I. Vidaña, A. Polls, A. Ramos, Engvik, M. Hjorth-Jensen, *Phys.Rev. C*62 (2000) 035801

I. Vidana, Ph. D. Thesis, **Barcelona University** (2001)

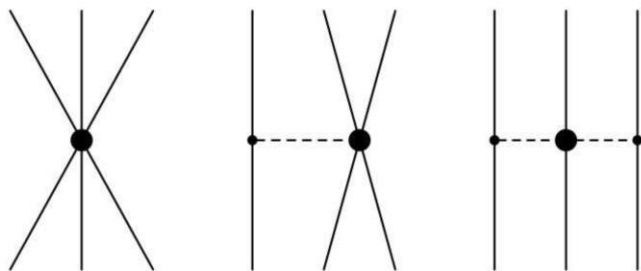
I. Vidaña, I. Bombaci, A. Polls, A. Ramos, *Astron. Astrophys.* 399, (2003) 687.

A simplified version for hyperonic matter: $\{n, p, \Lambda\}$ -matter

(D. Logoteta, I. Vidaña, I. Bombaci, Eur. Phys. J. A 55 (2019) 207)

Interactions:

NN	ChEFT	N3LO	$\Delta(1232)$	Piarulli et al., Phys. Rev. C 94 (2016) 054007
NNN	ChEFT	N2LO	$\Delta(1232)$	Epelbaum et al., Phys. Rev. C 66 (2002) 064001 Navratil, Few-Body Syst. 41 (2007) 117
	average \rightarrow	$V_{NN}^{\text{eff}}(n)$		Logoteta et al., Phys. Rev. C 94 (2016) 064001
NΛ	Nijmegen Soft Core 97 (NSC97a,e)			Rijken et al., Phys. Rev. C 59, (1999) 21; Stocks, Rijken, Phys. Rev. C 59 (1999) 3009
NNA	ChEFT	N2LO		Petschauer et al., Phys. Rev. C 93 (2016) 014001
	average \rightarrow	$V_{N\Lambda}^{\text{eff}}(n)$		Petschauer et al., Nucl. Phys. A 957 (2017) 347



Saturation by decuplet baryons



Only one **LEC (H')** is left

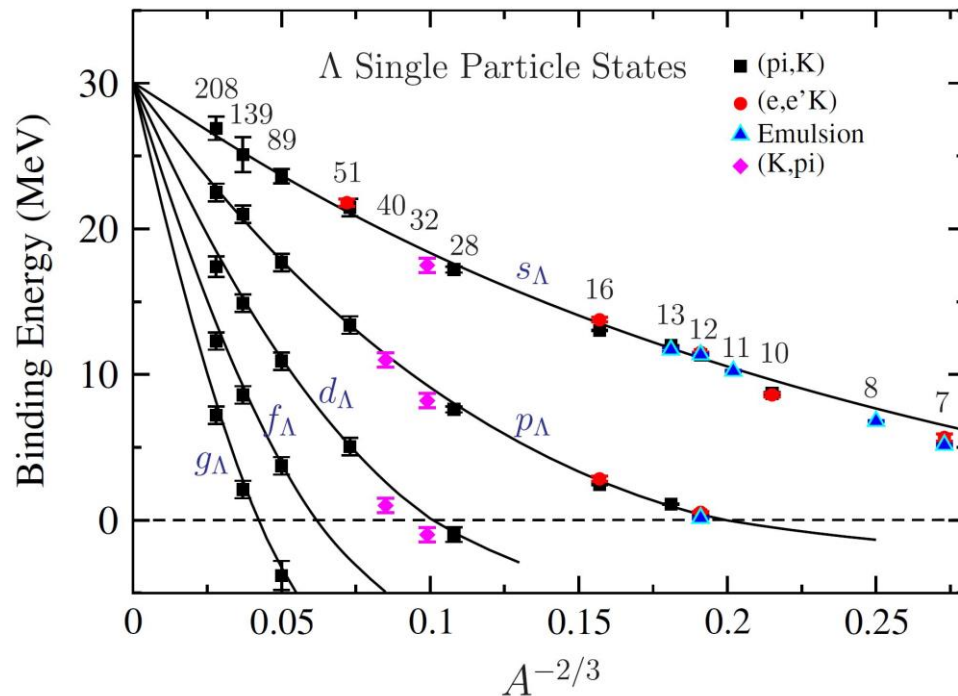
$$H' = \frac{\beta}{f_\pi^2}$$

$f_\pi = 93 \text{ MeV}$ pion decay constant

We fix β to have, in **symmetric nuclear matter** at the empirical saturation density $n_0 = 0.16 \text{ fm}^{-3}$:

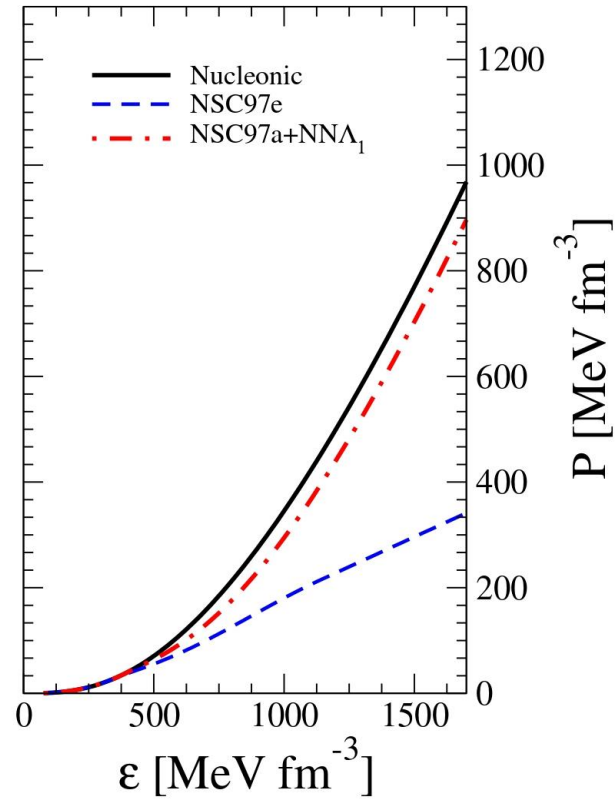
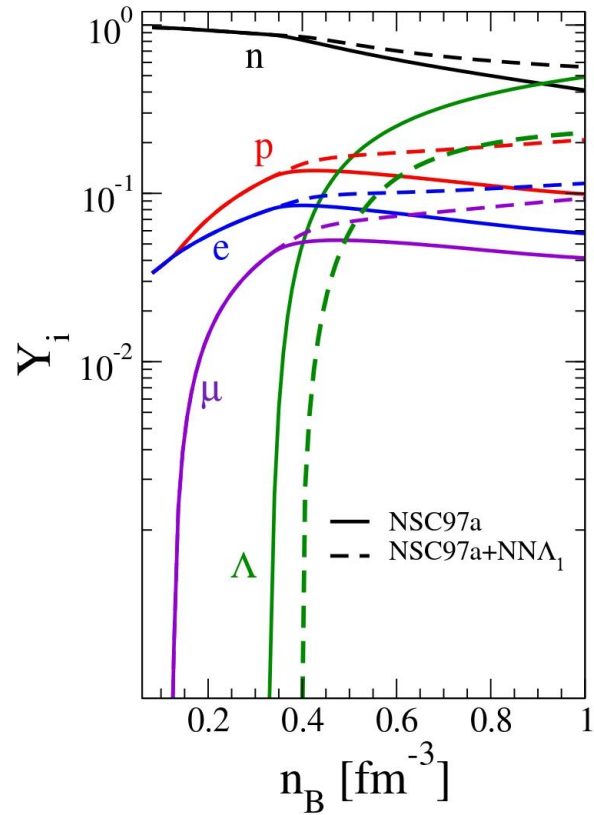
$$U_\Lambda(k = 0, n_0) = B_\Lambda^\infty$$

where B_Λ^∞ is the extrapolation to infinite nuclear matter of the **binding energy B_Λ** of Λ in hypernuclei

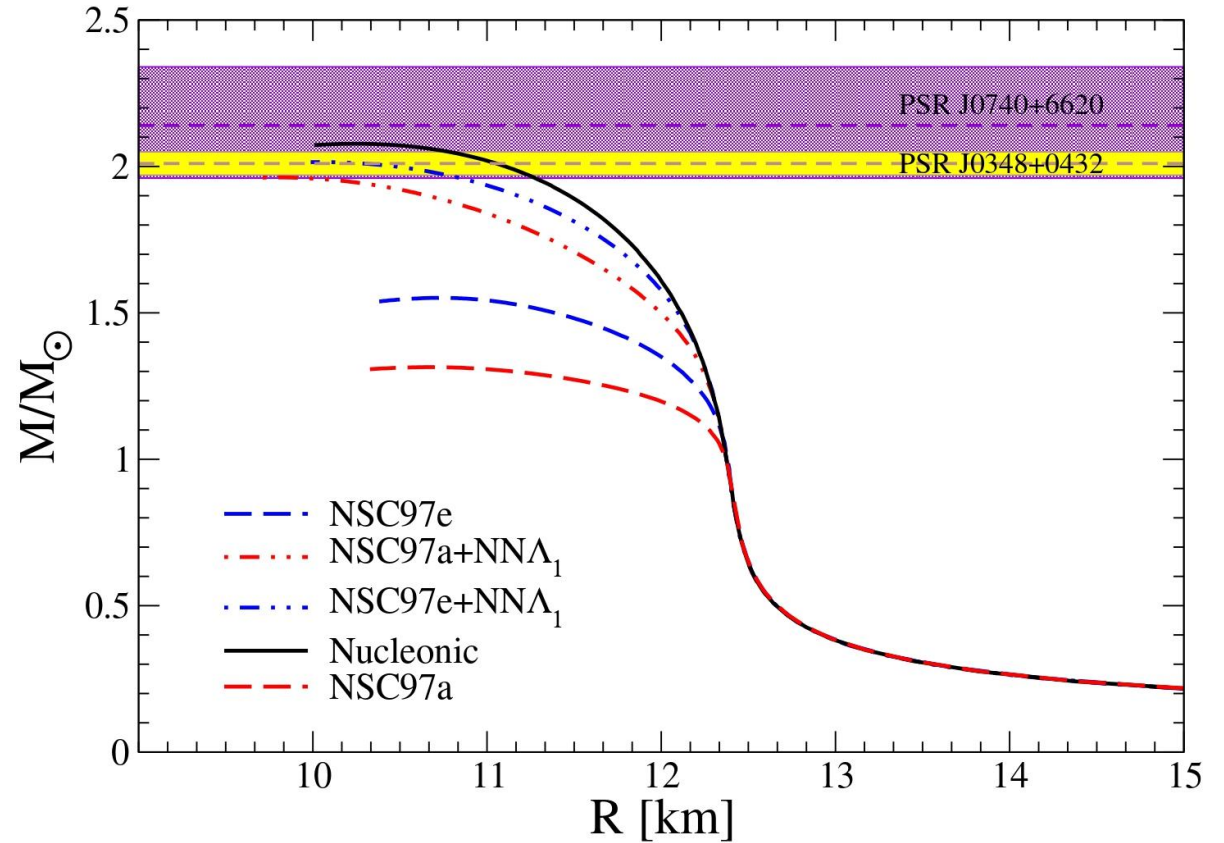


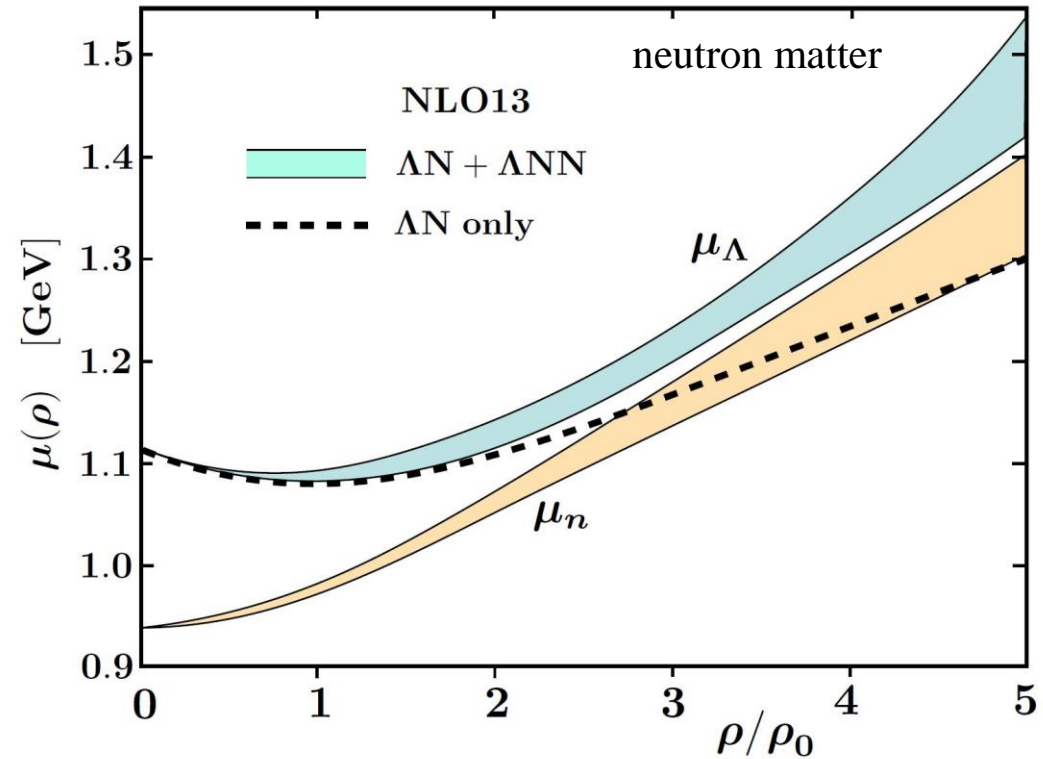
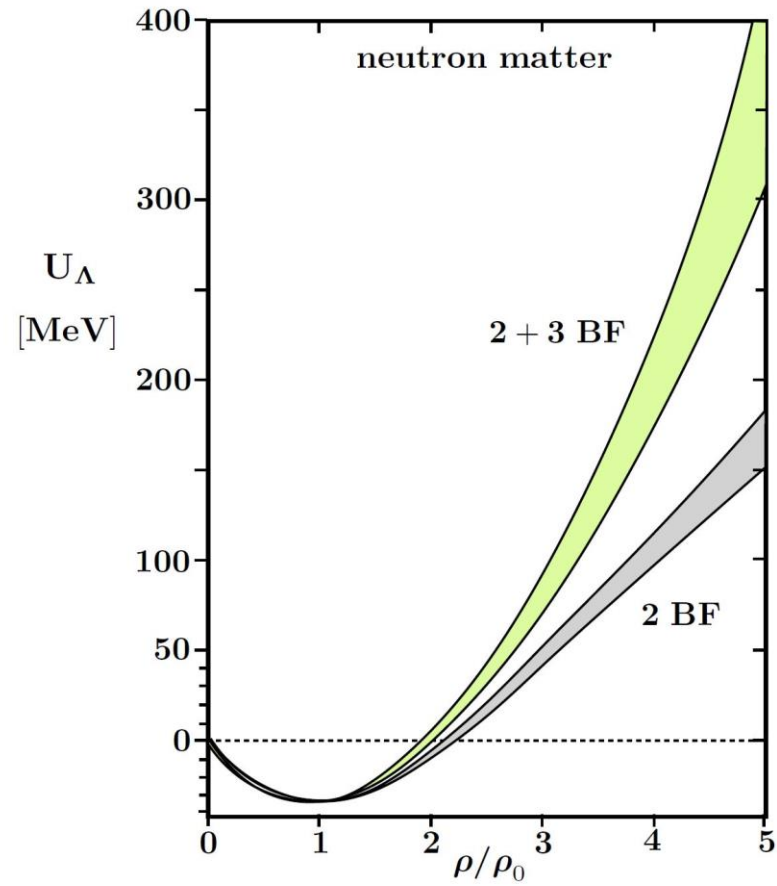
$$U_\Lambda(k = 0, n_0) = -28 \text{ MeV} \quad (\text{NNA}_1)$$

$$U_\Lambda(k = 0, n_0) = -30 \text{ MeV} \quad (\text{NNA}_2)$$



Hyperonic three-body interactions could represent a likely solution of the hyperon puzzle in Neutron Stars





D. Gerstung, N. Kaiser, W. Weise, Eur. Phys. J. A 56 (2020) 175

«The combined repulsion from two- and three-body hyperon-nuclear interactions can indeed be potentially strong enough to avoid the appearance of Λ hyperons in neutron stars»

The hyperon puzzle in Neutron Stars

Hyperonic three-body interactions could represent a likely solution of the hyperon puzzle in Neutron Stars

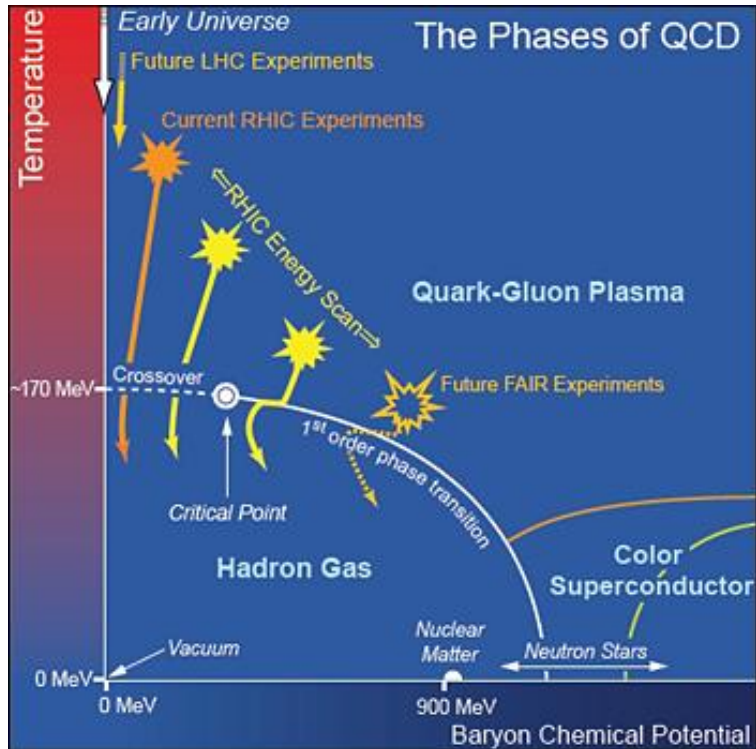
but

if hyperonic three-body interactions are repulsive enough, hyperons will never appear at the densities encountered in neutron star cores.

In this case “hyperon stars” would not contain hyperons at all, that is, we will go back to the case of **nucleonic stars**.

This could be a physically acceptable conclusion if the “true” hyperonic three-body interactions make **hyperon formation non convenient from an energetic point of view, contrary to the simple expectation based on ideal Fermi gas argument.**

Quark Matter in Neutron Stars and in BNS mergers



Can GW observations test the possible occurrence of a **quark-deconfinement phase transition** in NS matter ?

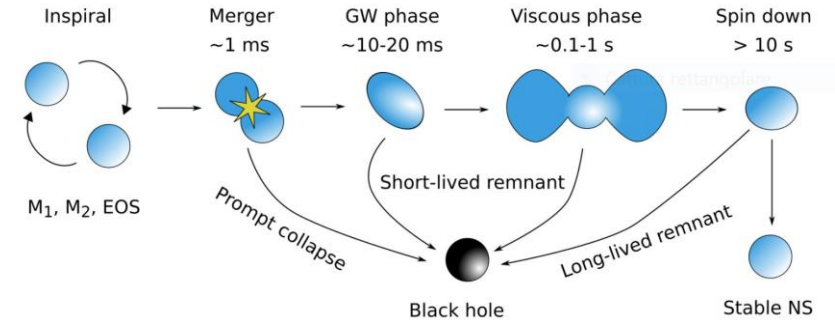
The Dynamics of Binary Neutron Star Mergers

1) Early-inspiral phase

point-like objects,

$$d \gg d_{\text{merg}} \equiv \min \left[(R_1 + R_2), \frac{6G}{c^2} (M_1 + M_2) \right]$$

No EoS dependence (except $M_{\text{max}}(\text{EoS})$)



2) Inspiral phase

$$d \approx (1 \div 10) d_{\text{merg}}$$

tidal deformations of NSs

$$Q_{ij} = \lambda \varepsilon_{ij}$$

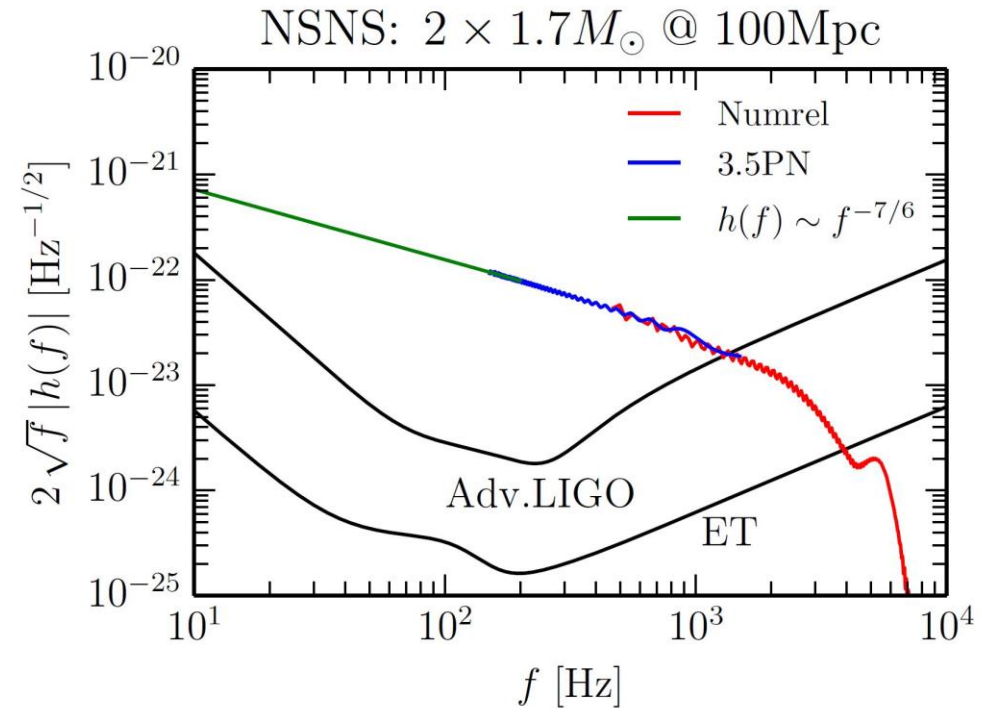
EoS : cold ($T = 0$), neutrino-free matter

3) Merger, post-merger compact object

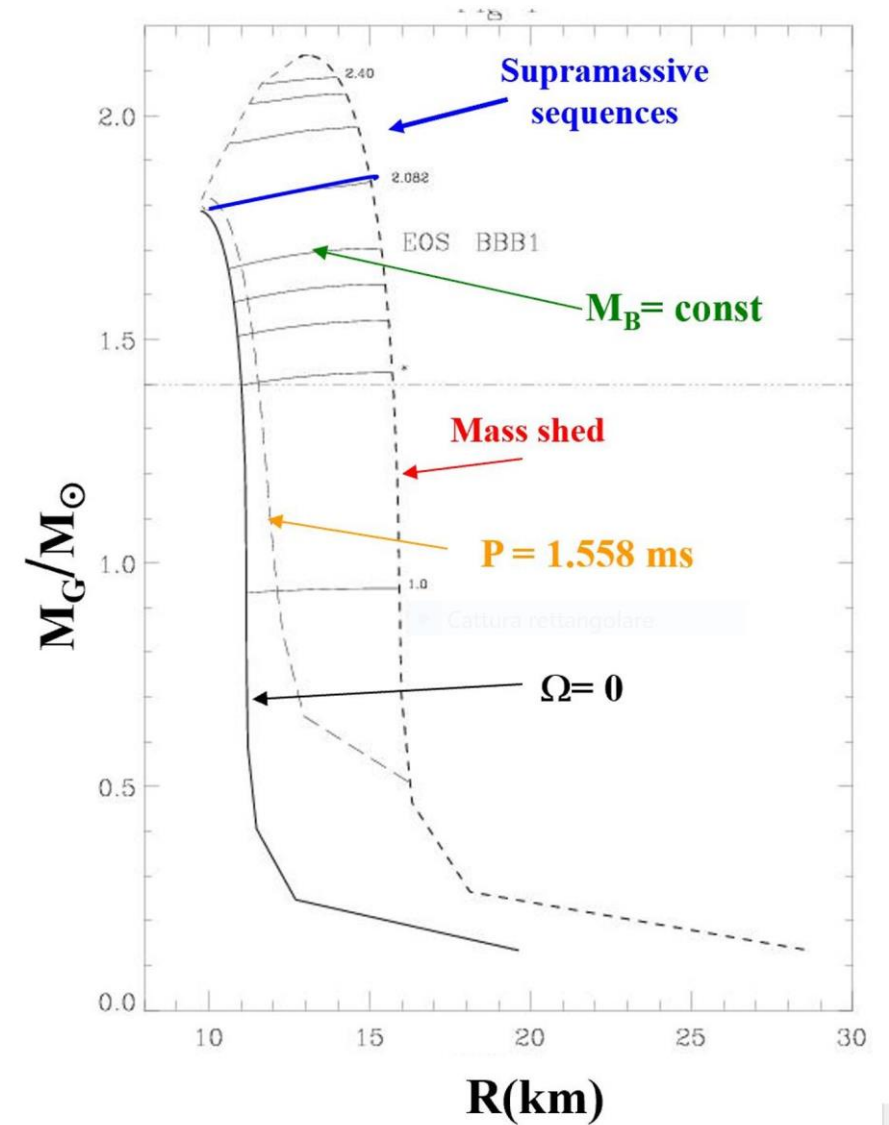
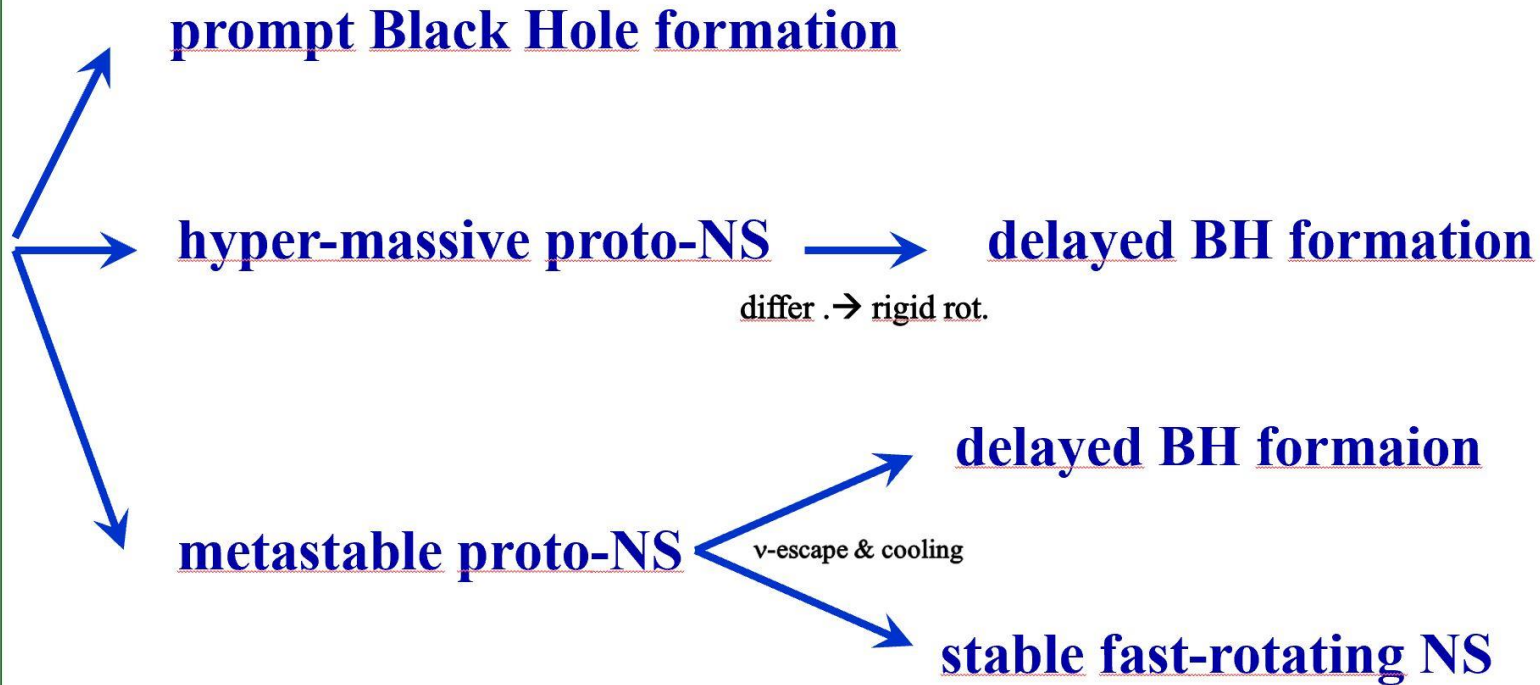
rapidly-differentially-rotating proto-NS

EoS : hot ($T = 10\text{--}100$ MeV), neutrino-trapped matter

$$t_{\text{weak}} \sim 10^{-9} \text{ s}, \quad t_{\text{trapp}} \sim (10 - 30) \text{ s}$$



4) Post-merger evolution: final merger remnant



Modeling extreme matter and GWs with piecewise-polytropic EOS

$$P(\rho) = K_i \rho^{\Gamma_i}; \quad \rho_{i-1} \leq \rho \leq \rho_i$$

Advantages:

very easy and efficient to use in GR numerical simulations;
Easy to include physical requirements as e.g. **causality condition**,
and $M_{\max} \geq 2 M_{\odot}$

Disadvantages:

No information on the **particle composition of matter**.

No connection with the underlying microphysics of the strong interactions.

No fundamental physics of dense matter modeling BNS mergers with polytropic EOS

G. Raaijmakers, T.E. Riel, A.L. Watts,

A pitfall of piecewise-polytropic equation of state inference, MNRAS 478 (2018) 2177

EOS for SQM: extended bag model

(at finite temperature)

$$\Omega = \sum_{i=u,d,s} \Omega_i^0 + \frac{3}{4\pi^2} (1 - a_4) \left(\frac{\mu_b}{3} \right)^4 + B_{eff}$$

$$\varepsilon = \sum_{i=u,d,s} \Omega_i^0 + \frac{3}{4\pi^2} (1 - a_4) \left(\frac{\mu_b}{3} \right)^4 + \sum_{i=u,d,s} \mu_i n_i + B_{eff} \quad \hbar = 1 \quad c = 1$$

2nd term = perturbative QCD correction up to $O(\alpha_s^2)$

$a_4 = 1$ (ideal relativistic gas + bag constant)

$$\mu_b = \mu_u + \mu_d + \mu_s$$

$$\mu_s = \mu_d = \mu_u + \mu_e$$

$$n_b = \frac{1}{3} (n_u + n_d + n_s)$$

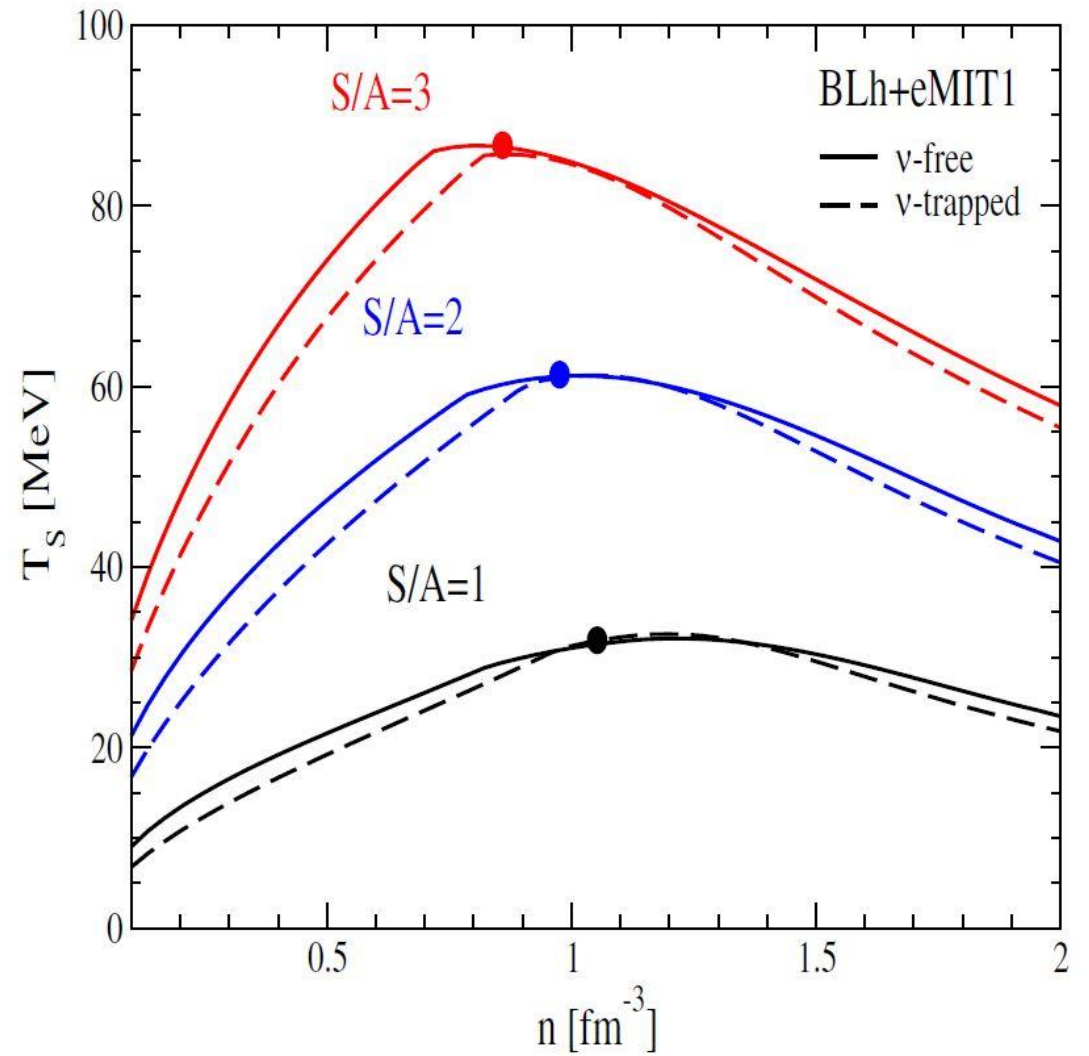
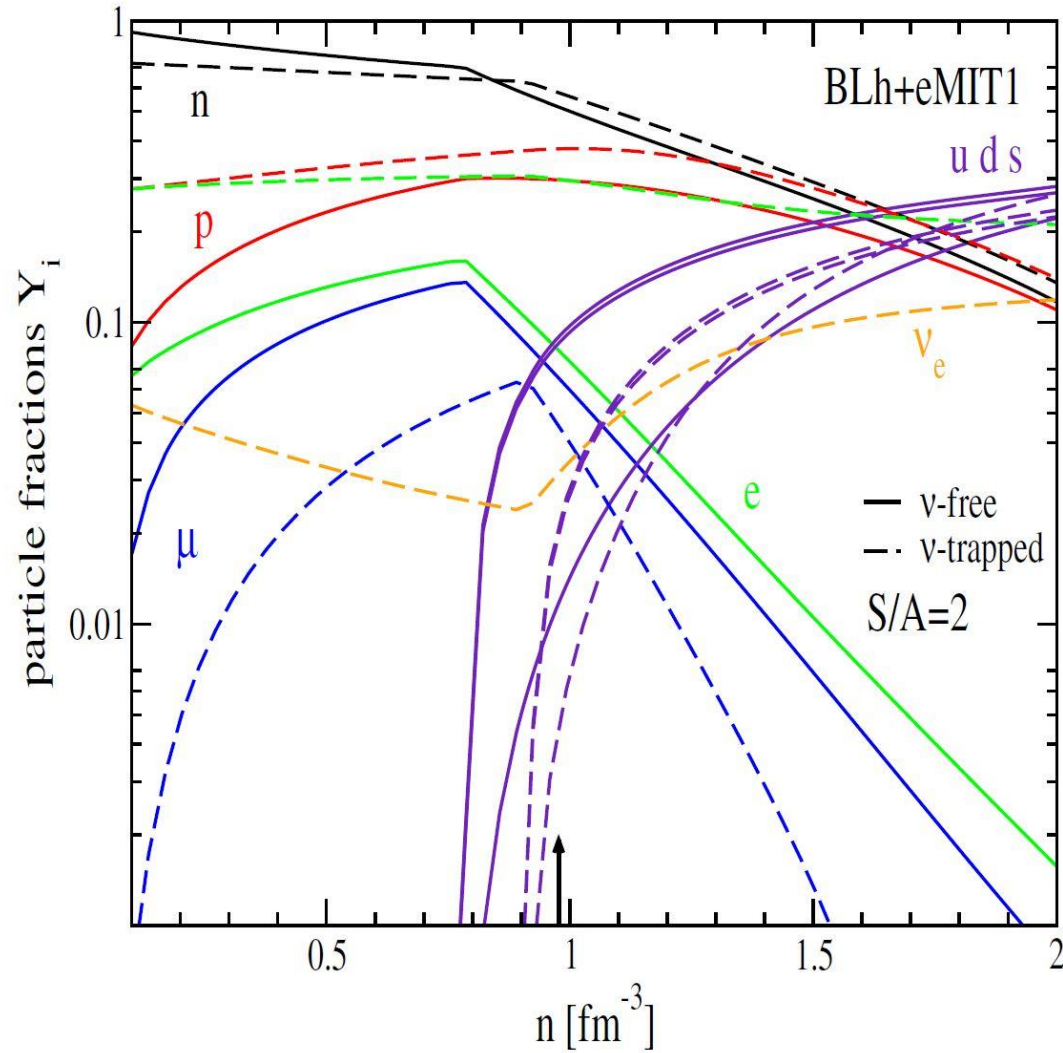
$$\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0$$

E. Fraga, R.D. Pisarki, J. Schaffner-Bielich, Phys. Rev. D 63 (2001) 12172(R)

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ 629 (2005) 969

S. Weissenborn, I. Sagert, G. Pagliara, M. Hempel, J. Schaffner-Bielich, ApJ 740 (2011) L14

Isoentropic β -stable hybrid matter



BNS mergers numerical simulations

Full General Relativistic Hydrodynamics

Code: Wisky THC

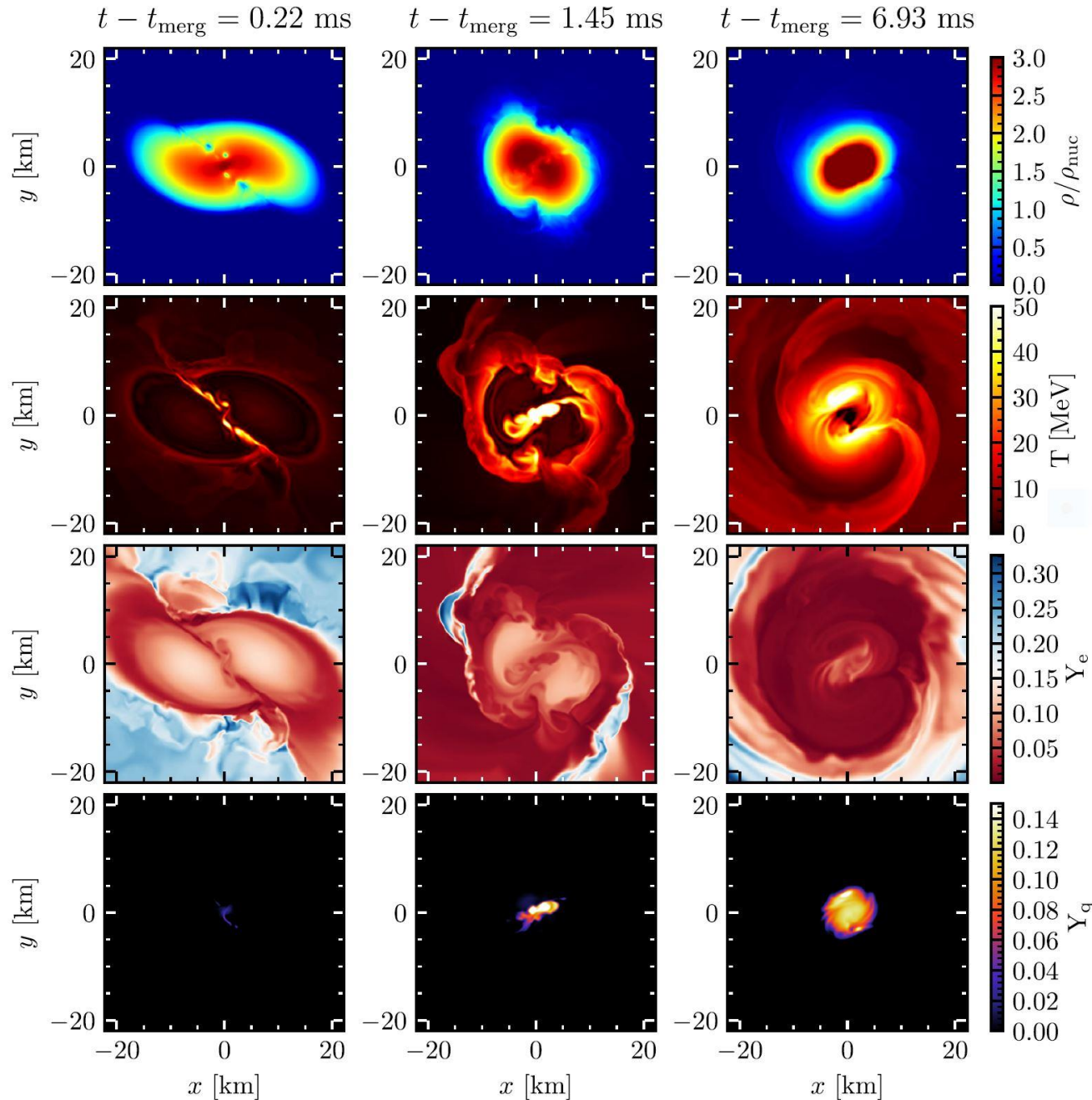
D. Radice, L. Rezzolla, *Astron. & Astrophys.* 547 (2012)

EOS:

BLh finite temperature nuclear matter (BHF)

BLQ finite temperature EOS for
hybrid nuclear-quark matter

A. Prakash, D. Radice, D. Logoteta, A. Perego,
V. Nedora, I. Bombaci, R. Kashyap,
S. Bernuzzi, A. Endrizzi, *Phys. Rev. D*104 (2021)



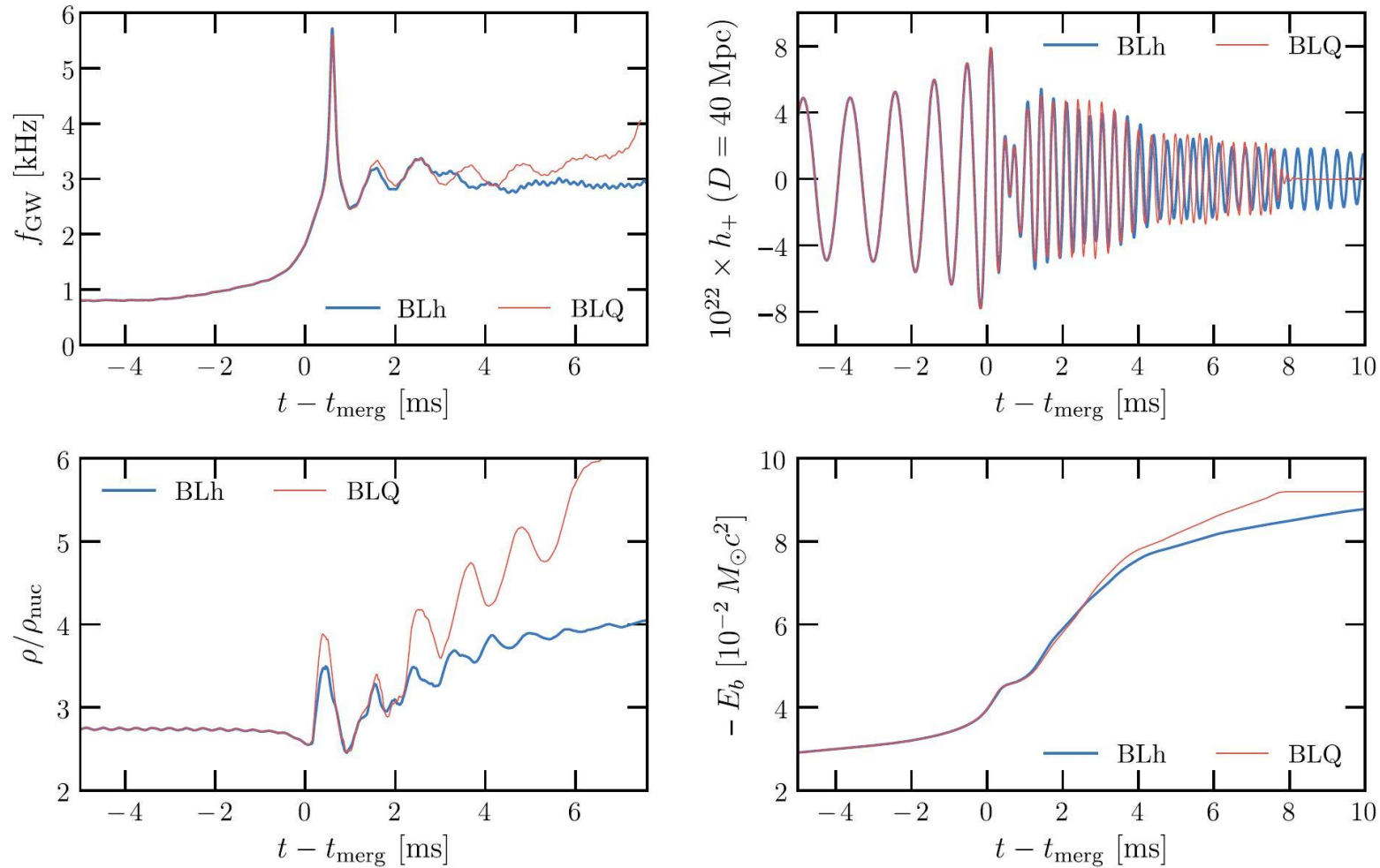


FIG. 7. Evolution of the instantaneous GW frequency f_{GW} , the “+” polarization strain amplitude for the ($l = 2, m = 2$) mode of the GW signal, the central density ρ , and the binding energy E_b of the $1.3325\text{--}1.3325 M_{\odot}$ binary. The inspiral ($t \leq t_{\text{merg}}$) evolution predicted by both the BLh and BLQ EOSs is identical. The appearance of quarks is imprinted on the postmerger dynamics and GW signal.

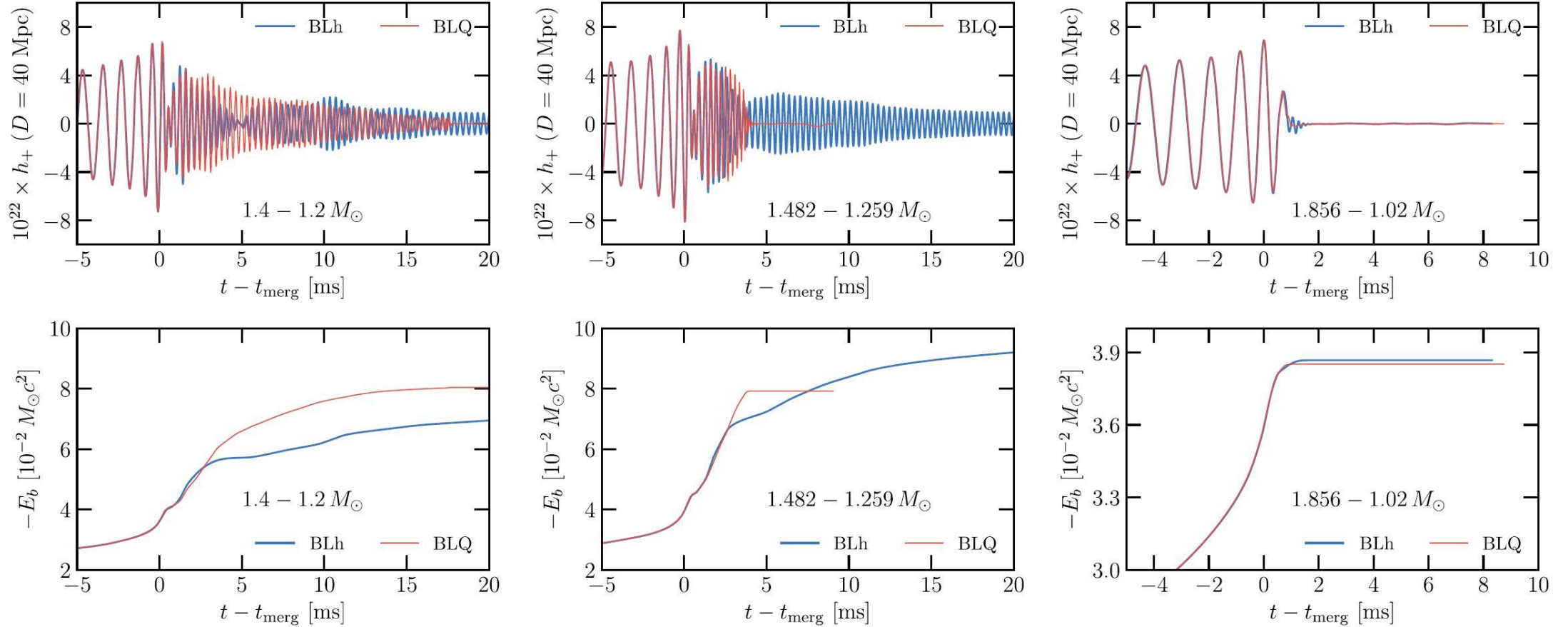
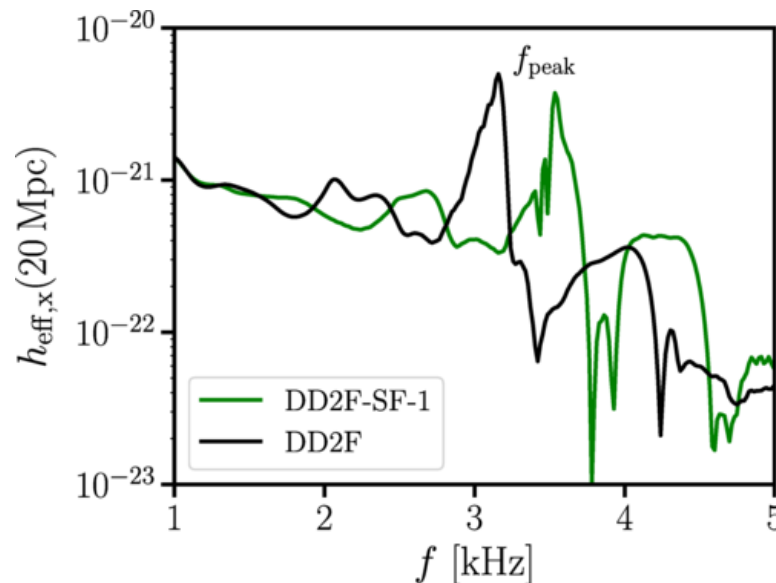
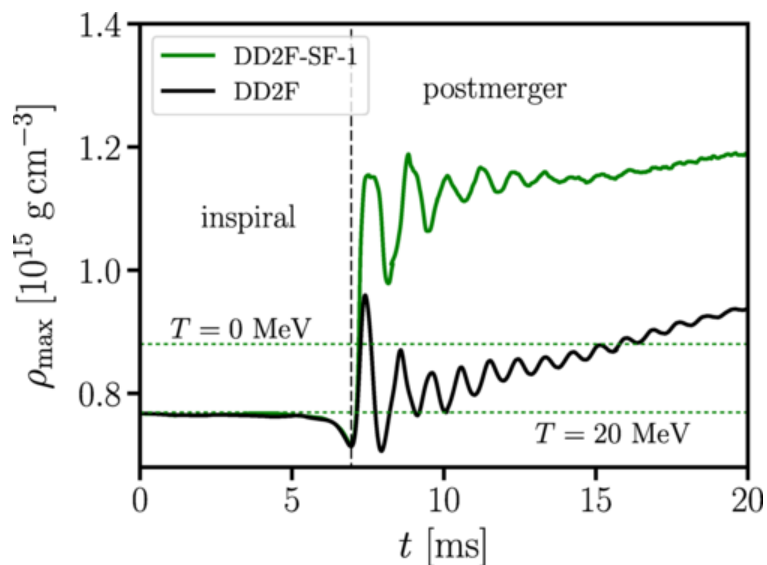


FIG. 8. Amplitude of the $(l = 2, m = 2)$ mode of the GW strain h_+ and binding energies for the $1.4\text{--}1.2 M_{\odot}$, $1.482\text{--}1.259 M_{\odot}$, and $1.856\text{--}1.020 M_{\odot}$ binaries. As the binaries become more massive or more asymmetric, the length of the postmerger signal decreases. The postmerger is further shortened by an onset of deconfinement phase transition.

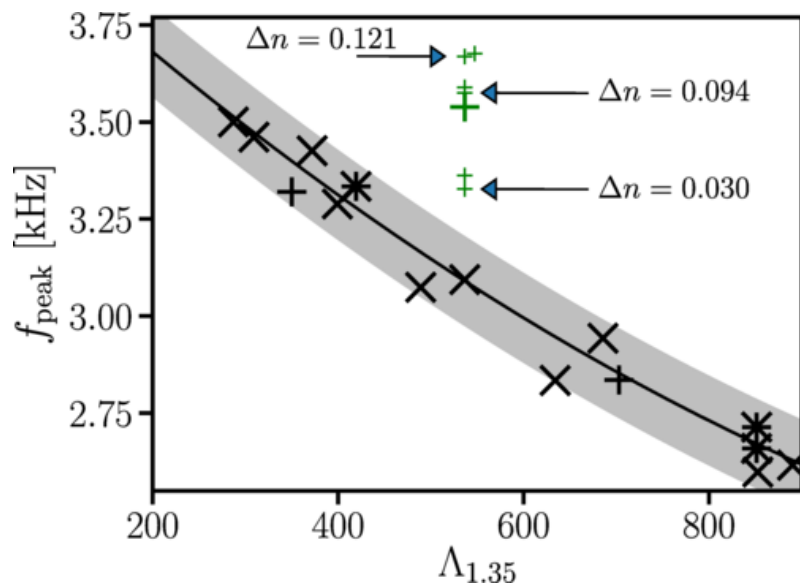
Identifying a first-order phase transition in NS mergers through GWs

A. Bauswein et al. Phys. Rev. Lett. 122 (2019) 061102



f_{peak} = frequency of the dominant oscillation of the **post-merger object**

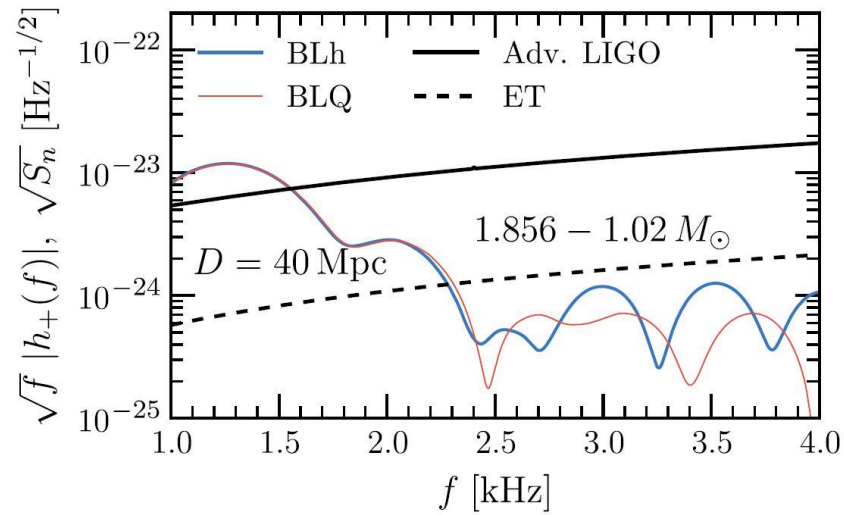
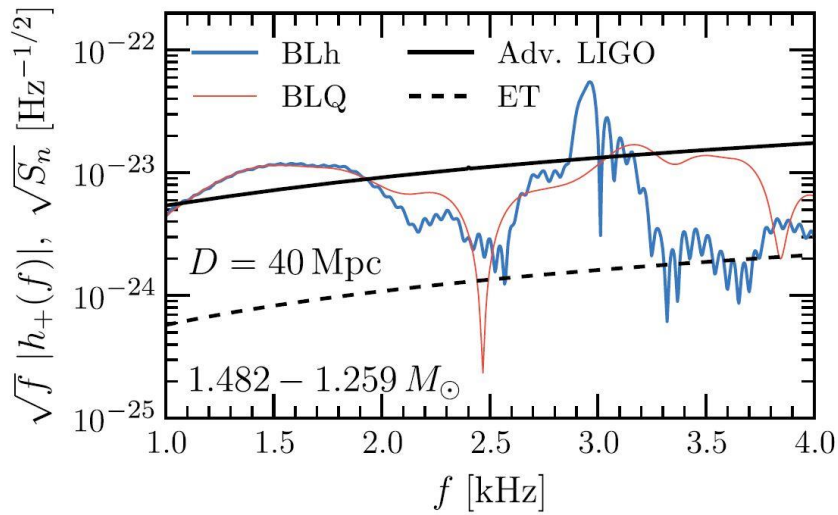
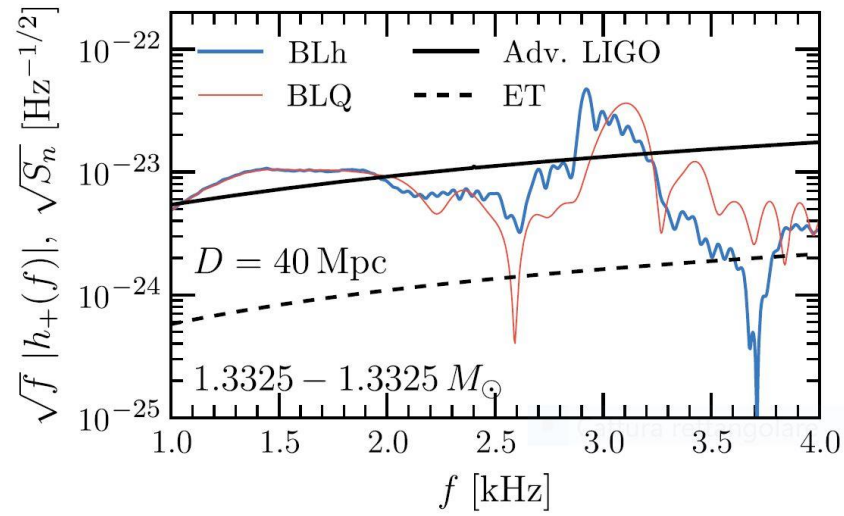
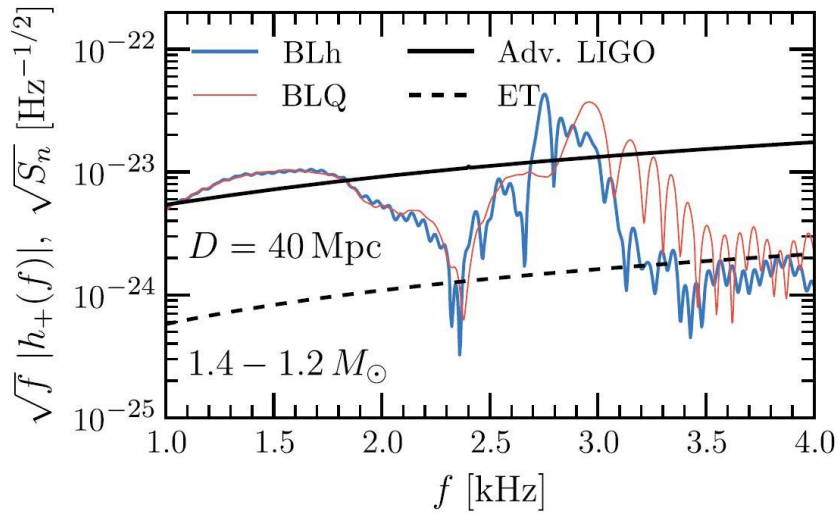
This frequency peak is a robust feature that occurs in all the simulations which do not form directly a black hole
(no prompt BH formation)



$$f_{\text{peak}} = f_{\text{peak}}(\Lambda) \quad \text{empirical relation}$$

A significant deviation of the $f_{\text{peak}}(\Lambda)$ empirical relation would provide evidence for a **strong first-order phase transition to quark matter**.

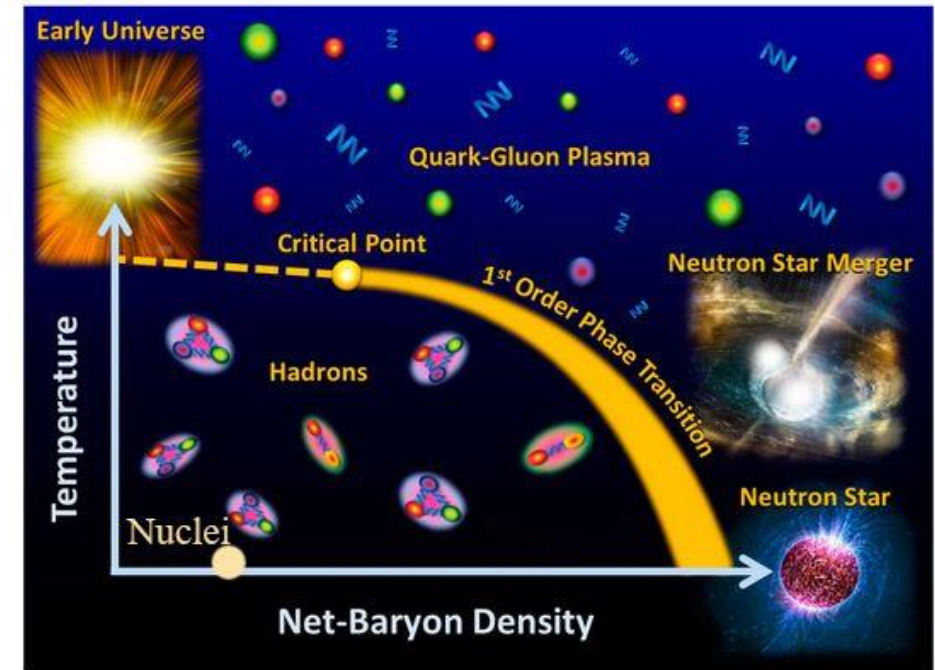
This results depends on the **phase-transition construction**.
The authors used the **Maxwell construction**



Power spectrum of the ($\ell = 2, m = 2$) mode of the GW strain

Probing dense QCD matter in the laboratory

Whether the **quark-deconfinement phase transition** is of the **first-order with a critical endpoint**, or whether it proceeds smoothly by a **crossover** is still an open question.



New dedicated experiments are under construction at future facilities

Compressed Baryonic Matter (CBM) experiment

at the Facility for Antiproton and Ion Research (**FAIR**) in **Darmstadt**,

J-PARC Heavy-Ion-project at the Japan Proton Accelerator
Research Complex (**J-PARC**) in **Tōkai**,

Heavy-Ion-program at the Nuclotron-based Ion Collider fAcility
(**NICA**) at the Joint Institute for Nuclear Research in **Dubna**.



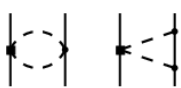

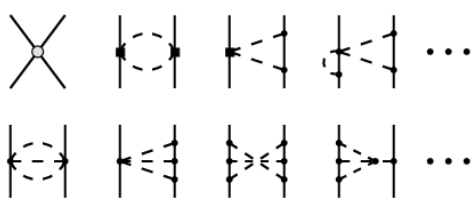
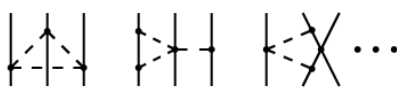
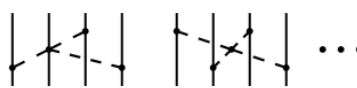
Adapted from P. Senger, and N. Herrmann,
Nuclear Physics News, Vol. 28, No 2 (2018) 23.

Summary

- The **EoS of matter at supranuclear densities** is a key ingredient to describe **Neutron Stars** and related **astrophysical phenomena** (SNEs, NS-NS merging, GRBs)
- **Experimental data of low-energy nuclear physics** (NN scattering, properties of atomic nuclei, ...) can constrain the EoS around the empirical saturation point of nuclear matter.
- At supranuclear density various particle species (e.g. **hyperons**) and phases of matter (e.g. **quark deconfined matter**) are expected.
Needs of new experimental data e.g. from **hypernuclear physics** to constraint models of NY (nucleon-hyperon), YY, NNY, NYY, YYY interactions.
Relativistic Heavy Ions experiments to explore the QCD phase diagram in the region relevant for NSs, SNe, BNS mergers
- **Gravitational wave astronomy and multimessenger astrophysics** opens a new window to explore the properties of extreme matter

Extra slides

Nuclear interactions derived in Chiral Perturbation Theory (ChPT)

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO			
NLO			
N2LO			
N3LO			

Two-nucleon (NN), three-nucleon (NNN) and many-nucleon interactions can be calculated perturbatively (i.e. order by order) according to a well-defined scheme based on a low-energy effective QCD Lagrangian.

[S. Weinberg, Physica A 96 \(1979\) 327; Phys. Lett. B 251 \(1990\) 288; Phys. Lett B 259 \(1992\) 114](#)

[E. Epelbaum, H.-W. Hammer, U. G. Meißner, Rev. Mod. Phys. 81 \(2009\) 1773](#)

[R. Machleidt and D. R. Entem, Phys. Rep. 503 \(2011\) 1](#)

[S. Petschauer, J. Haidenbauer, N. Kaiser, , U. G. Meißner, W. Weise, Front. Phys. 8 \(2020\) 12](#)

Quantum many-body approach

Brueckner-Bethe-Goldstone theory

$$G_{\tau\tau'}(\omega) = V + V \sum_{k_a k_b} \frac{|k_a k_b\rangle Q_{\tau\tau'} \langle k_a k_b|}{\omega - e_\tau(k_a) - e_{\tau'}(k_b) + i\varepsilon} G_{\tau\tau'}(\omega)$$

$$e_\tau(k) = \frac{\hbar^2 k^2}{2m} + U_\tau(k)$$

$$U_\tau(k) = \Re \left\{ \sum_{\tau'} \sum_{k'} \langle k k' | G_{\tau\tau'}(e_\tau(k) + e_{\tau'}(k')) | k k' \rangle_A \right\}$$

Energy per baryon in the **Brueckner-Hartree-Fock (BHF)** approximation

$$\tilde{E}(n_n, n_p) \equiv \frac{E}{A} = \frac{1}{A} \left\{ \sum_{\tau} \sum_k \frac{\hbar^2 k^2}{2M} + \frac{1}{2} \sum_{\tau} \sum_k U_\tau(k) \right\}$$

$$n_n = \frac{1}{2}(1 + \beta)n$$

$$n_p = \frac{1}{2}(1 - \beta)n$$

Saturation by decuplet baryons

three-baryon force

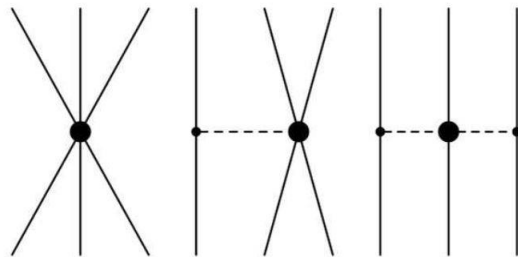
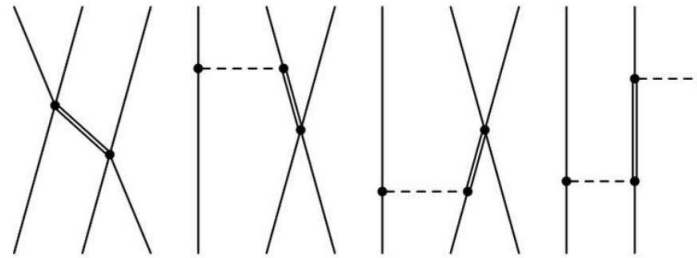
decuplet-less EFT

decuplet-contribution

LO

NLO

NNLO



...

Tidal deformability and Love number

$$Q_{ij} = \lambda \varepsilon_{ij}$$

$$\lambda = \frac{2}{3} \frac{k_2}{G} R^5$$

$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 R}{GM} \right)^5$$

Binary tidal deformability

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2) m_1^4 \Lambda_1 + (m_2 + 12m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12q + 1) \Lambda_1 + (12 + q) q^4 \Lambda_2}{(1 + q)^5}$$

$$M_{chirp} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

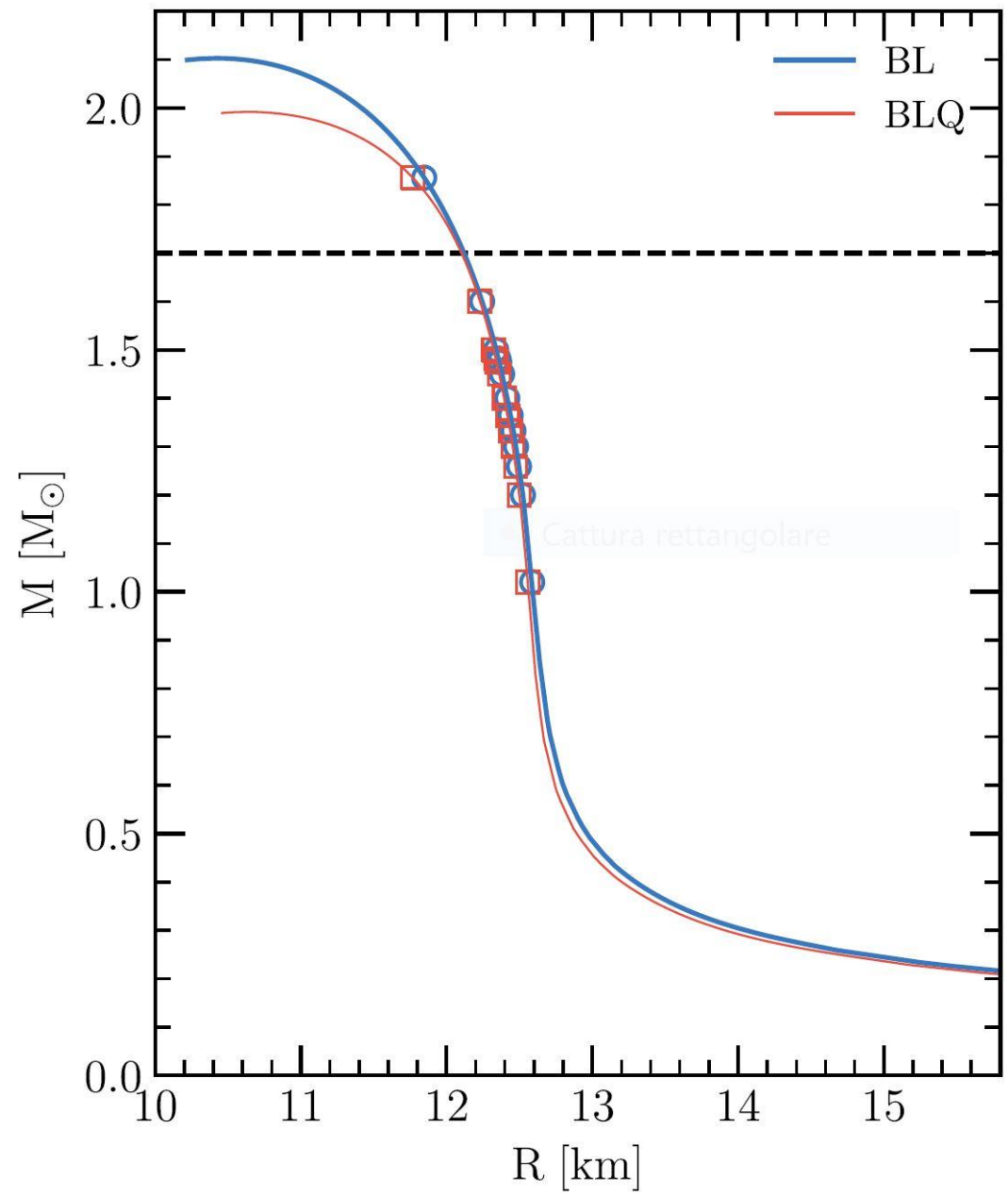
$$M_{chirp} = \left[\frac{q}{(1+q)^2} \right]^{3/5} M_{tot} \quad q = \frac{m_2}{m_1} \leq 1$$

T. Hinderer, ApJ 677 (2008) 1216; erratum arXiv:0711.2420

T. Damour and A. Nagar, Phys. Rev. D 80 (2009) 084035

T. Damour, A. Nagar and L. Villain, Phys. Rev. D 85 (2012) 123007

S. Postikov, M. Prakash and J. M. Lattimer, Phys. Rev. D 82 (2010)

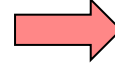


First-order phase transitions: phase equilibrium

The **Gibbs construction**

Neutron star matter is a **multi-component system** with **two conserved "charges"** (electric charge and **baryon number**)

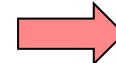
Global charge neutrality:
each of the two phases can have a net and opposite electric charge



The two phases can coexist (**mixed phase**) in a finite range of pressure

The **Maxwell construction**
One-component system (e.g. water)

Local charge neutrality:
each phase in equilibrium is separately charge neutral.



Constant pressure in the mixed phase. Since $P(r)$ must be monotonic in NS, there is a **sharp density discontinuity in the stellar core** at the phase boundary.

N. Glendenning, *Phys. Rev. D* 46 (1992) 1274
N. Glendenning, *Compact Stars*, Springer, 1997

