Latest developments on hadron interaction study in connection with femtoscopy

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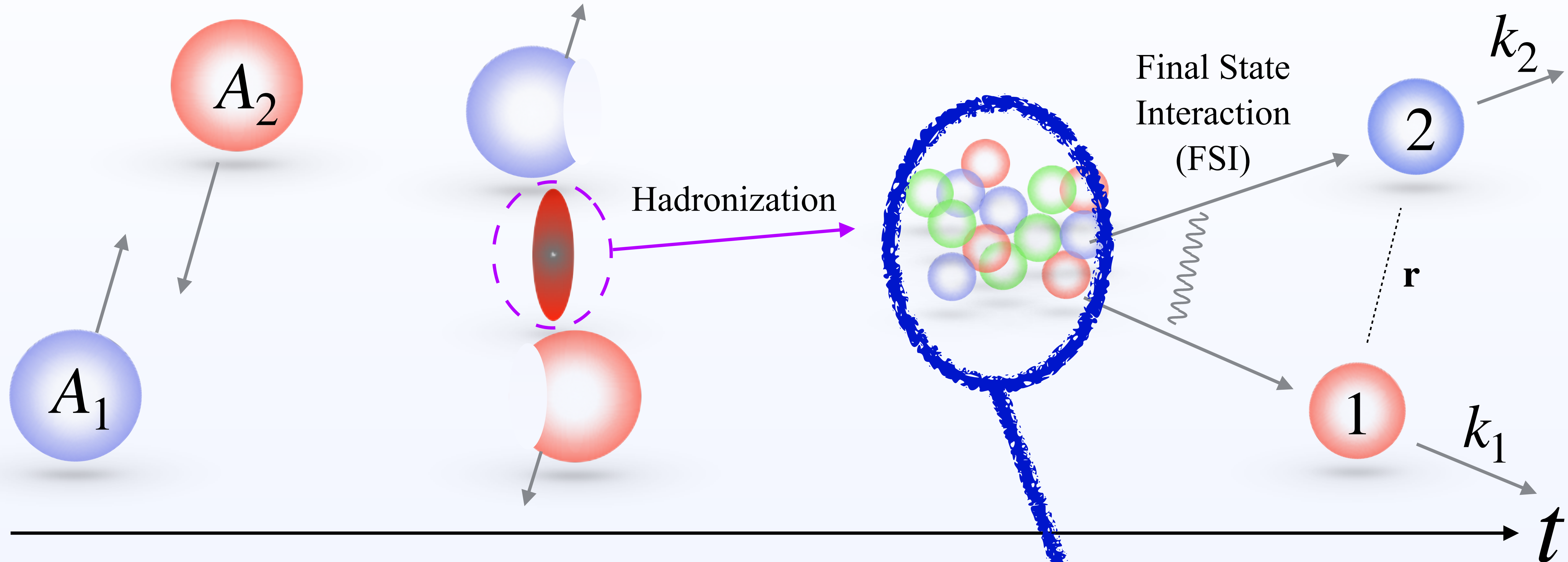
Hadron correlation in high energy nuclear collision

- High energy nuclear collision and FSI



- Hadron-hadron correlation
  - Koonin-Pratt formula : S.E. Koonin, PLB 70 (1977) S. Pratt et. al. PRC 42 (1990)

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r}\ S(\mathbf{r})\,|\varphi^{(-)}(\mathbf{q},\mathbf{r})|^2$$

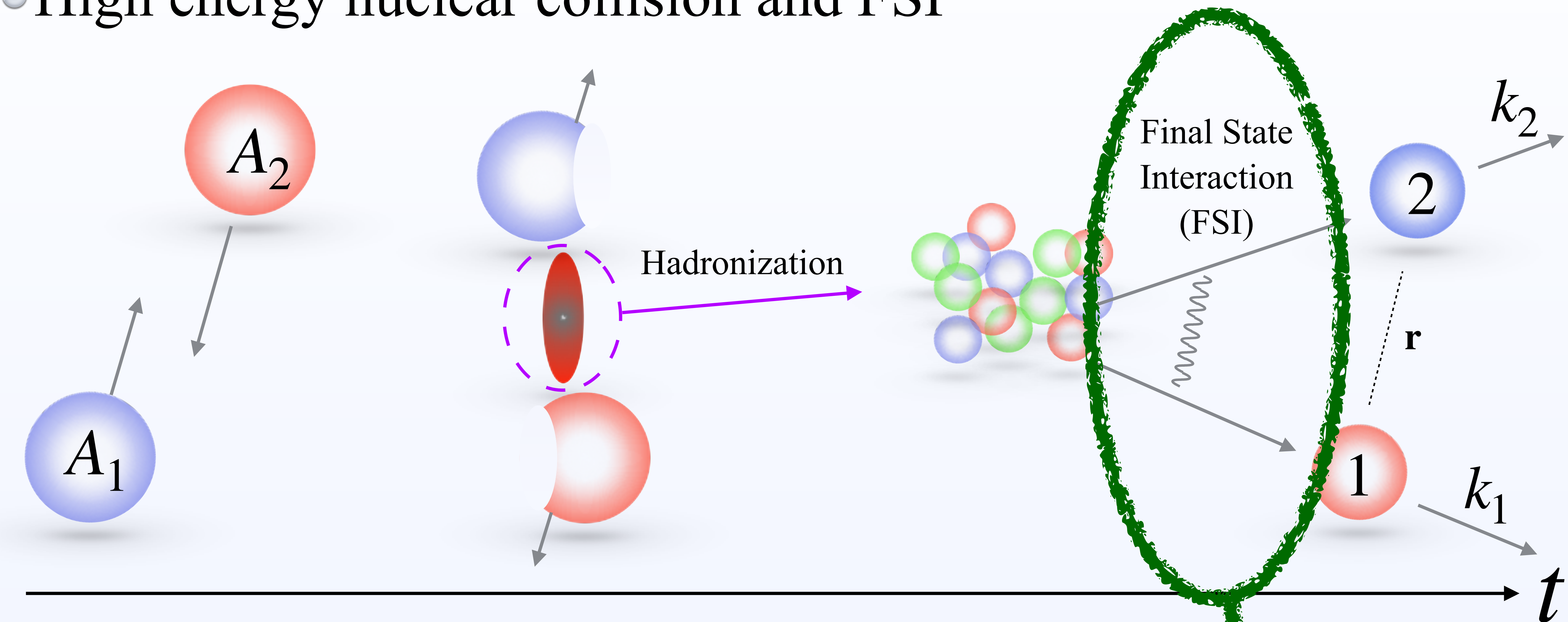
$$\mathbf{q} = (m_2\mathbf{k}_1 - m_1\mathbf{k}_2)/(m_1+m_2)$$

$S(\mathbf{r})$: Source function

$\varphi^{(-)}(\mathbf{q},\mathbf{r})$: Relative wave function

- Depends on …
  Collision detail (A_i, energy, centrality)
- Including information of…
  size of hadron source,
  momentum dependence, weight…

- High energy nuclear collision and FSI



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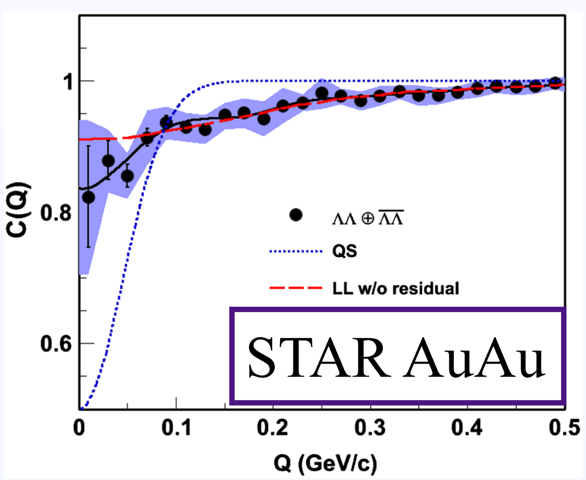
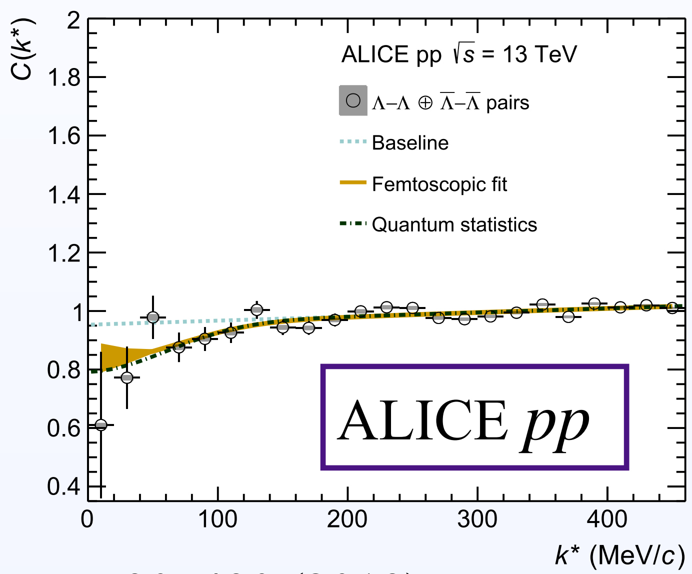
- Depends on …
  Interaction (strong and Coulomb)
  quantum statistics (Fermion, boson)

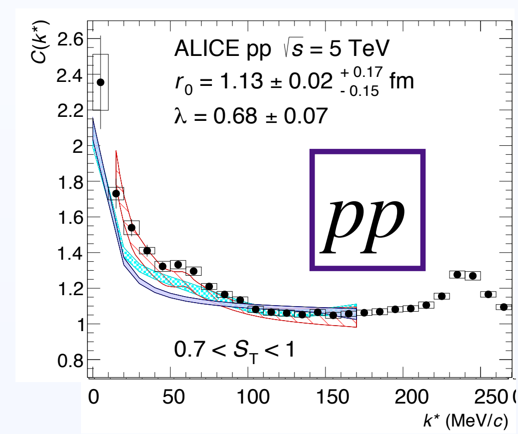
Experimental data in various sectors

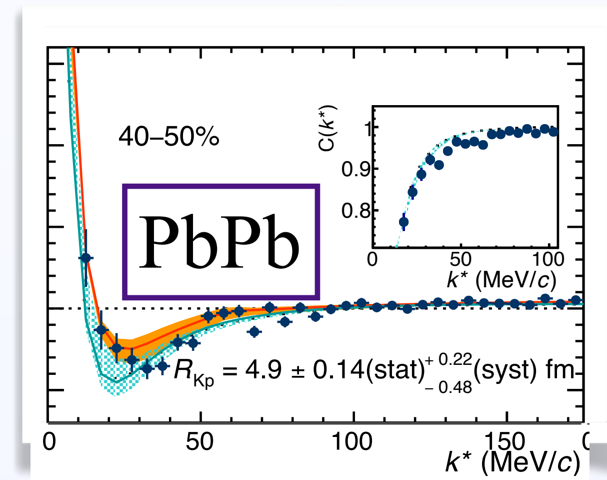
- $\Lambda\Lambda$ — STAR AuAu: PRL 114,022301(2015)
  ALICE *pp*: PLB 797 (2019) 134822
  PbPb: PRC99, 024001 (2019)



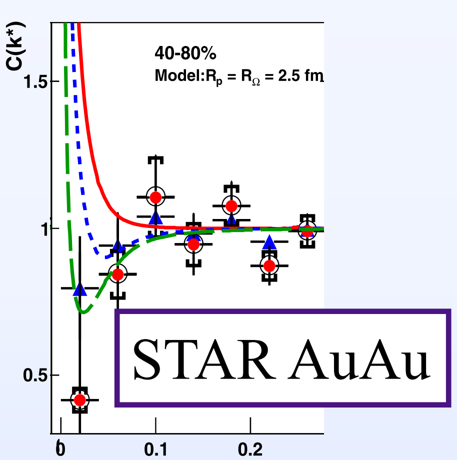


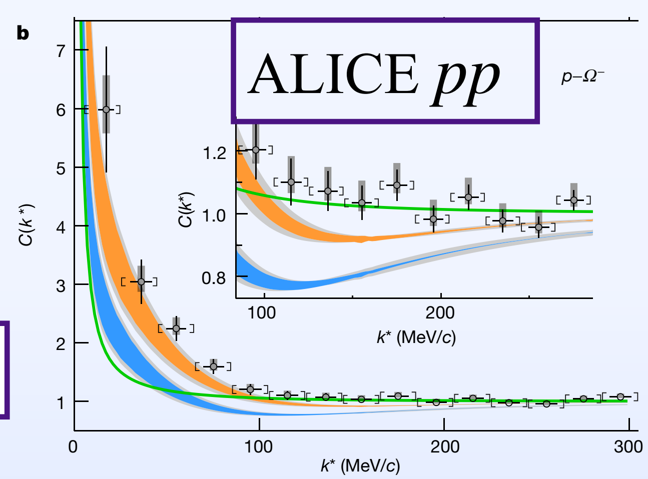
- $K^{\pm}p$ — ALICE *pp*: PRL 124 (2020) 9, 092301
  PbPb: PLB 822 (2021) 136708
  STAR AuAu: NPA 982 (2019) 359



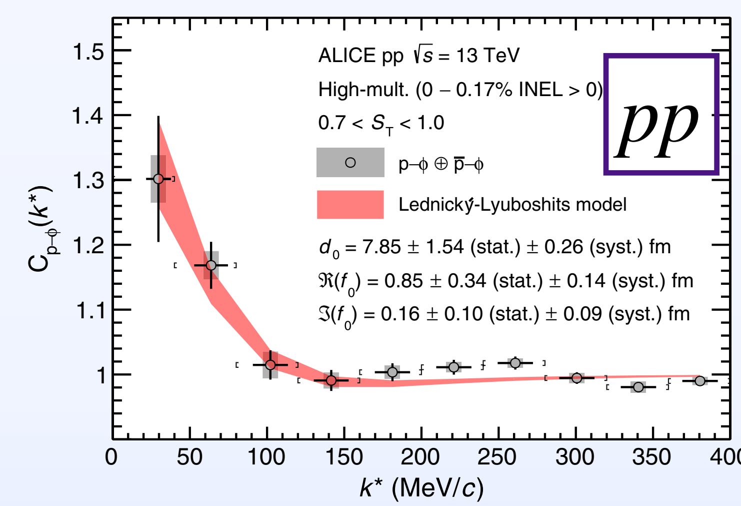


- $p\Omega$ — STAR AuAu: PLB 790, 490 (2019)
  ALICE *pp*: Nature 588 (2020) 232





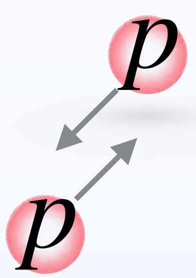
- $p\phi$ — ALICE *pp*: PRL 127 (2021) 17, 172301

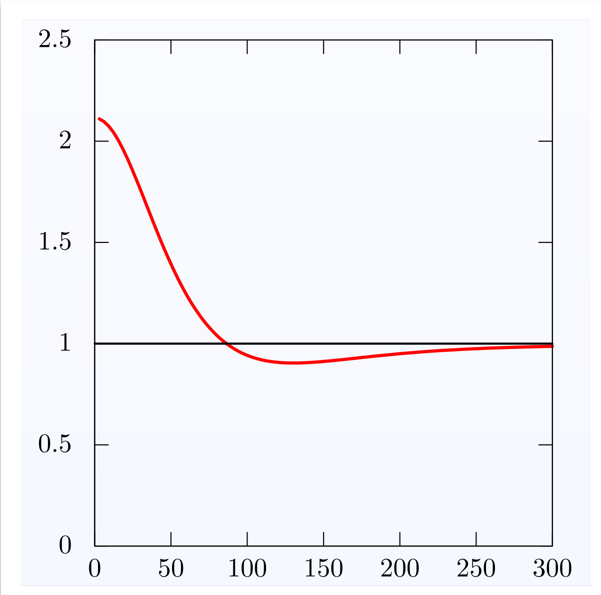


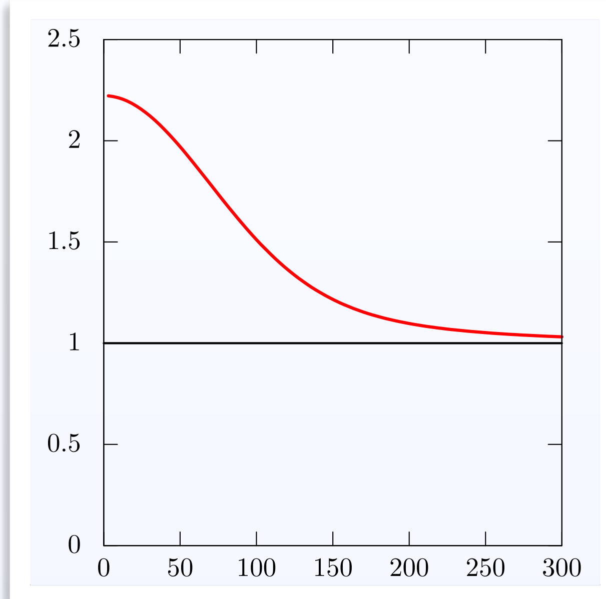
Line shapes of $C(q)$: relation to interaction

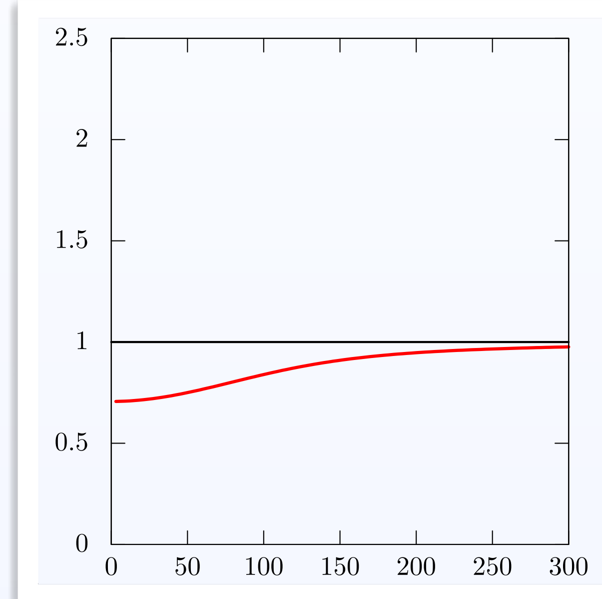
- Attractive interaction
  - w/ bound state
  - w/o bound state
- Repulsive interaction

- Small source

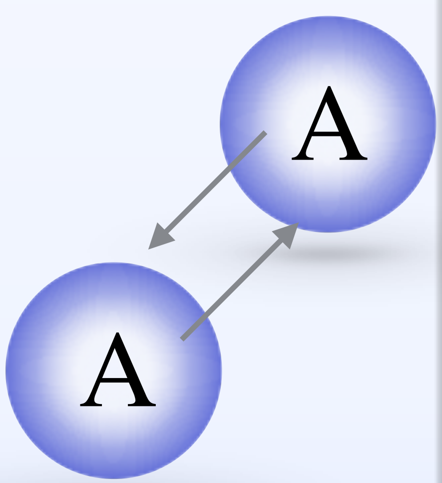


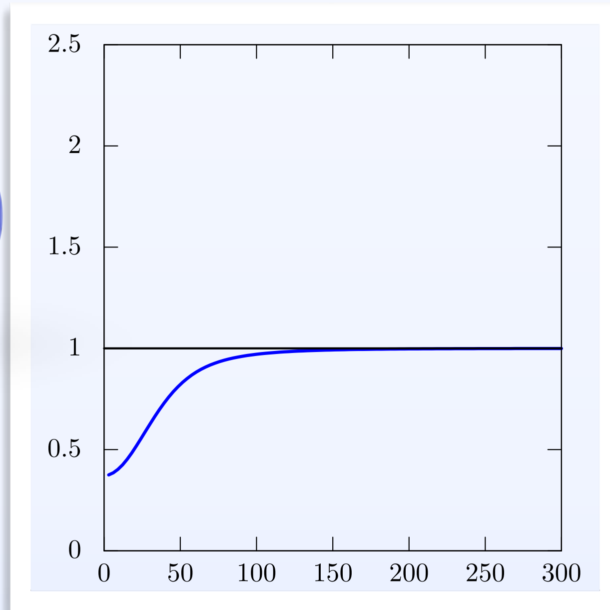


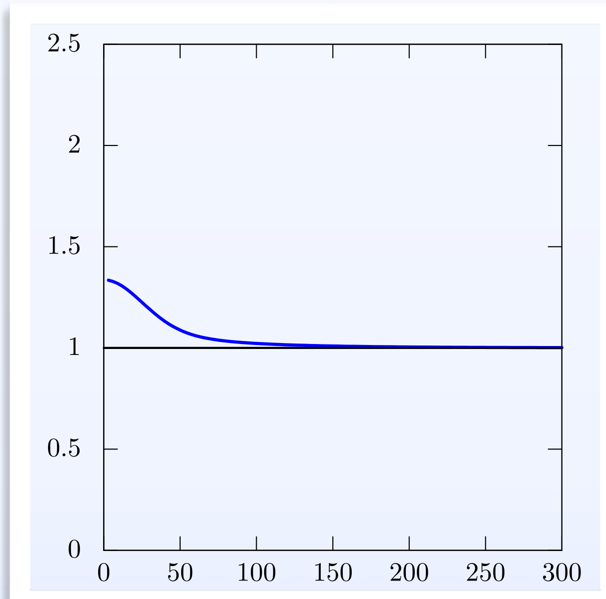


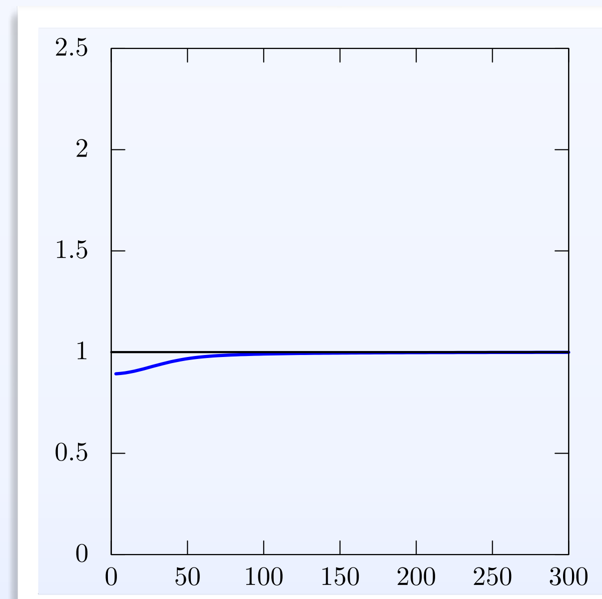


- Large source









How to construct correlation model from theory; $\mathscr{F}(q) \rightarrow C(q)$

- Using effective potential
  - Construct the eff. potential by reproducing the amplitude $\mathscr{F}$ (or threshold parameters (a_0, r_e))
  - Solving the Schrödinger eq. $\longrightarrow \varphi$
- Using half offshell T-matrix $T_l(q, k; E)$ Haidenbauer, Nuclear Physics A 981 (2019) 1–16
  - $T_l(q, k; E) \longrightarrow \varphi$

$$\tilde{\psi}(k, r) = j_l(kr) + \frac{1}{\pi} \int j_l(qr)\, dq q^2 \frac{1}{E - E_1(q) - E_2(q) + i\epsilon} T_l(q, k; E)$$

- Using Lednicky-Lyuboshitz formula
  - Approximation for the simple interaction
  - Direct relation between $C(q)$ and $\mathscr{F}(q)$

→ Comparison of model predictions and correlation data

How to extract interaction from Correlation data; $C(q) \to \mathscr{F}(q)$

- Potential method

$$C(q) \to V(r) \to \mathscr{F}(q)$$

- Parametrize the potential

e.g. $V(r) = V_0 \exp(-(mr)^2) \xrightarrow{H\varphi = E\varphi} \varphi \xrightarrow{C(\mathbf{q}) = \int d^3\mathbf{r}\, S(\mathbf{r}) |\varphi^{(-)}(\mathbf{q},\mathbf{r})|^2} C(q)$

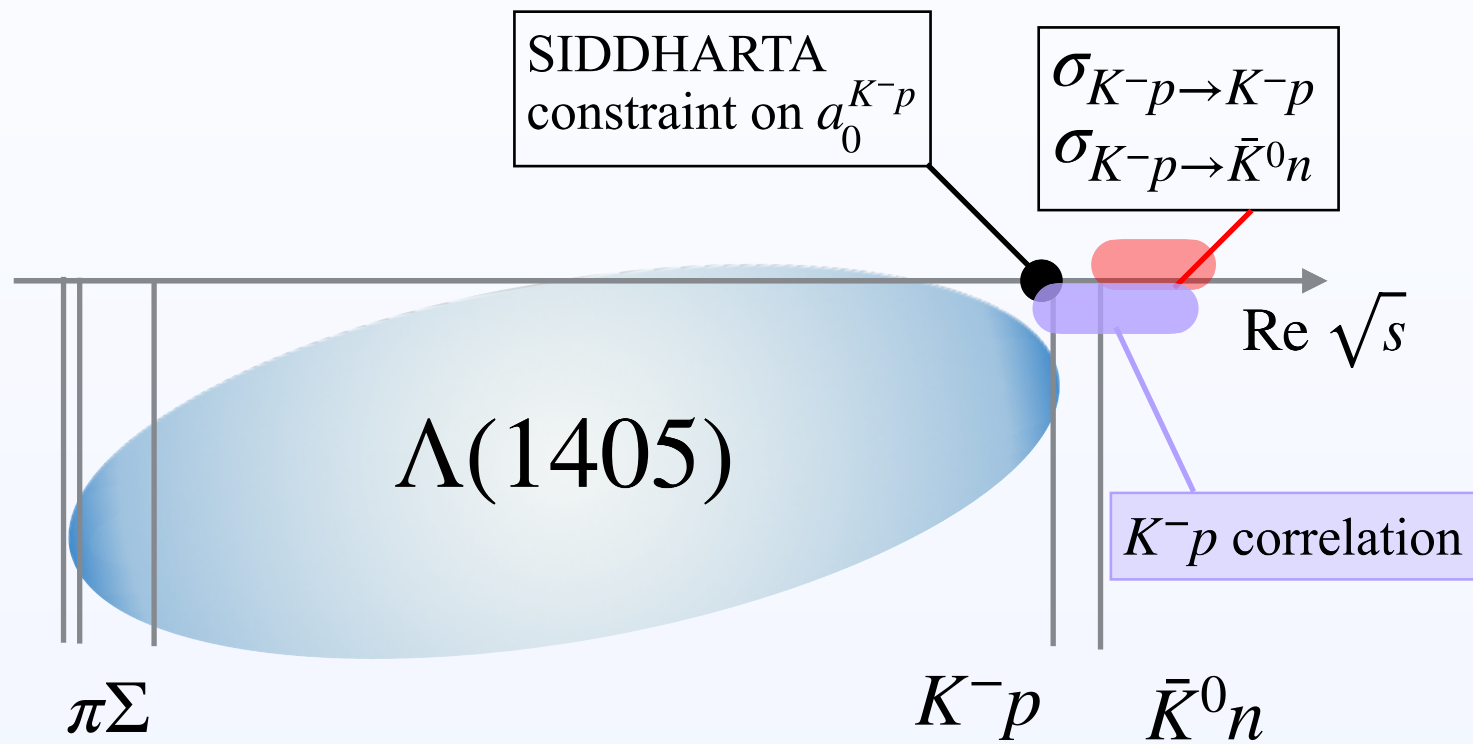
- Determine the parameters by fitting the data
- Calculate the amplitude or threshold parameters (a_0, r_e) from $V(r)$
  - More fitting costs (needs to solve Schrödinger eq. for every change of parameters.)
  - Easy to introduce coupled-channel effect
  - Coulomb effect can be precisely calculated by adding Coulomb pot. in H.

⇒ Amplitude can be directly determined from correlation data

$\bar{K}N$ interaction and K^-p correlation

- $\bar{K}(s\bar{l})N$ interaction and $\Lambda(1405)$

  - Coupled-channel system of $\pi\Sigma$-$\pi\Lambda$-$\bar{K}N$
  - Strong attraction reproducing quasi-bound state $\Lambda(1405)$
  - Strong constraint on $a_0^{K^-p}$ by SIDDHARTA experiment of Kaonic hydrogen
    M. Bazzi, et al.. PLB 704 (2011)



- Chiral SU(3) based $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential Miyahara, Hyodo, Weise, PRC 98 (2018)

  - Constructed based on the amplitude with NLO chiral SU(3) dynamics <— $a_0^{K^-p}$, σ fitted
    Ikeda, Hyodo, Weise, NPA881 (2012)
  - Constructed to reproduce the chiral SU(3) amplitude around the $\bar{K}N$ sub-threshold region

Coupled-channel effect

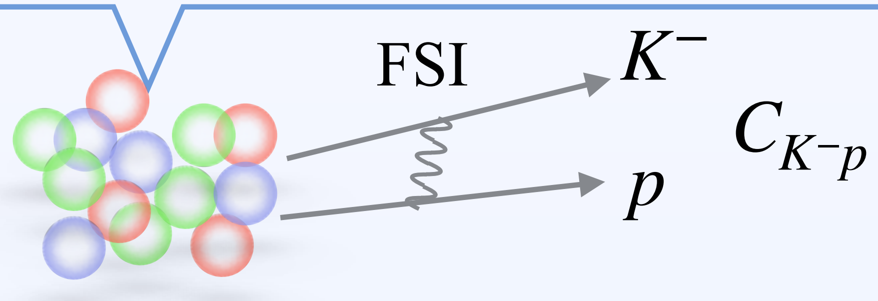
- Koonin-Pratt-Lednicky-Lyuboshits-Lyuboshits (KPLLL) formula

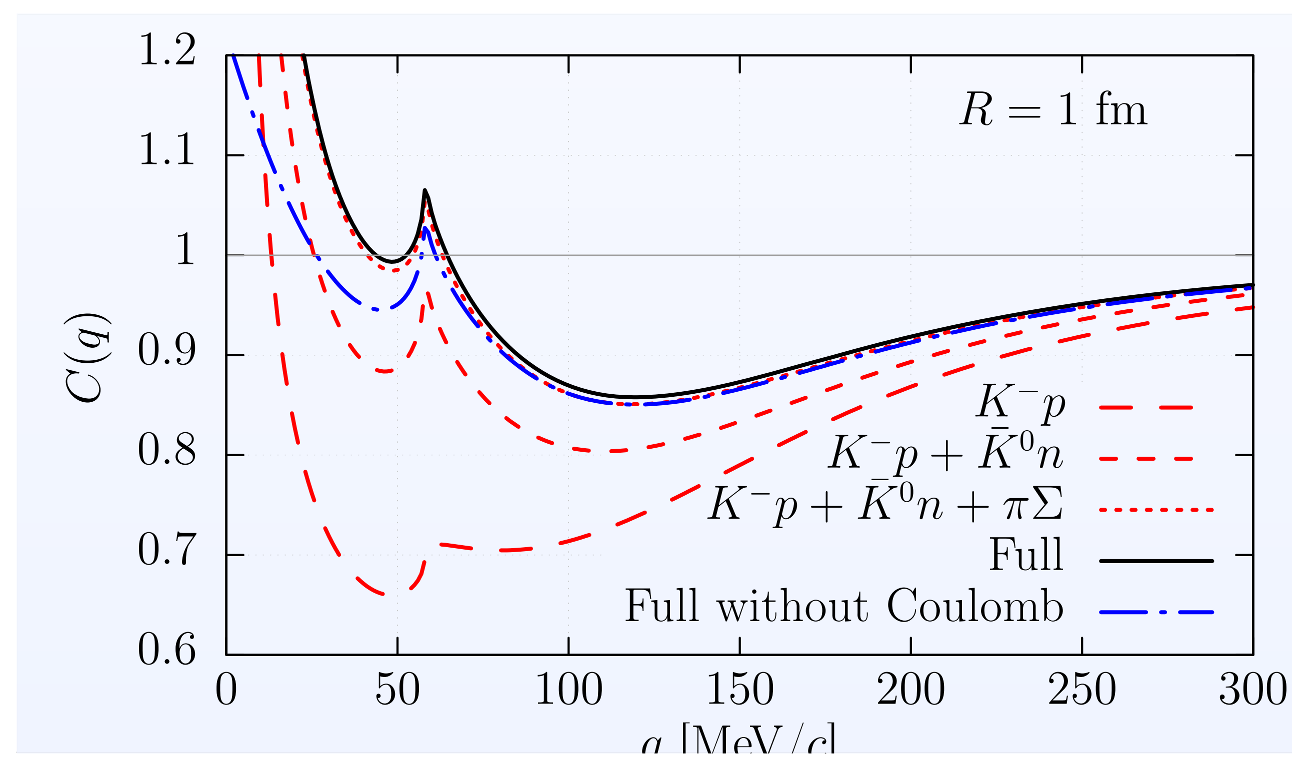
$$C(\mathbf{q}) = \int d^3\mathbf{r}\ S(\mathbf{r})\,|\psi^{(-)}(q;r)|^2 + \sum_{j\neq i} \omega_j \int d^3\mathbf{r}\ S_j(\mathbf{r})\,|\psi_j^{(-)}(q;r)|^2$$

S.E. Koonin, PLB 70 (1977)
S. Pratt et. al. PRC 42 (1990)
R. Lednicky, et.al. Phys. At. Nucl. 61(1998)

- Contribution from coupled-channel source

K^-p, $\bar{K}^0 n$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\pi^0\Lambda$





- Enhance $C(q)$
- Enhance cusp structure
- ω_i : production rate
  (compared to measured channel)

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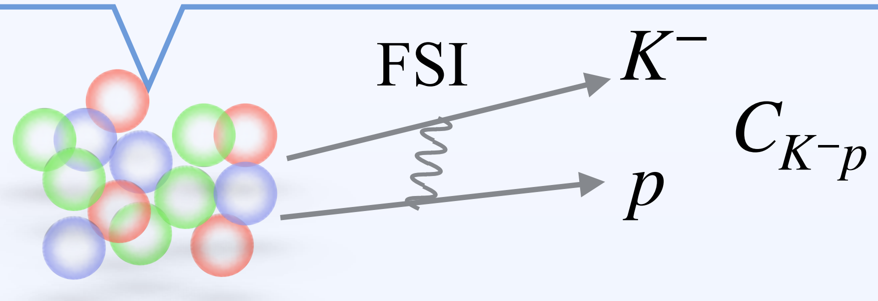
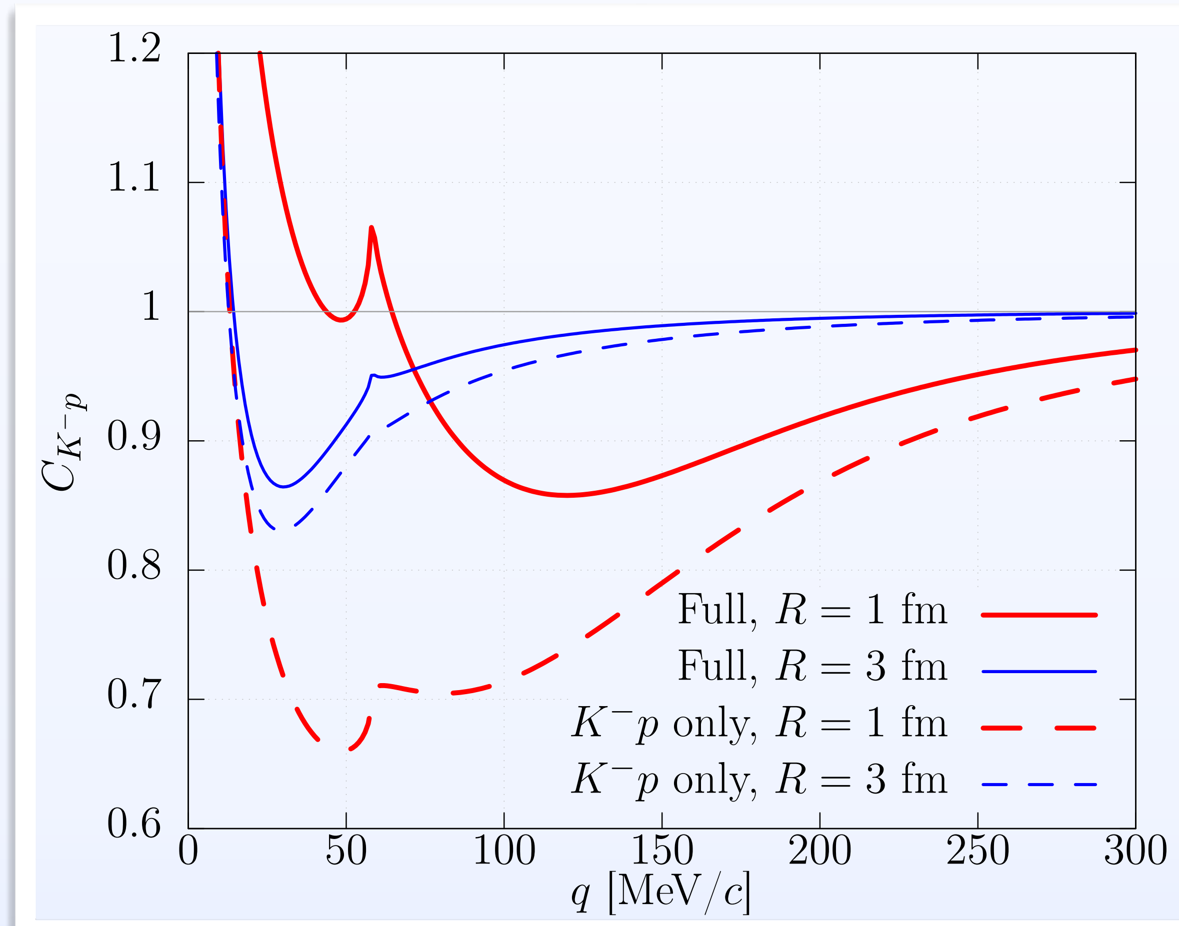




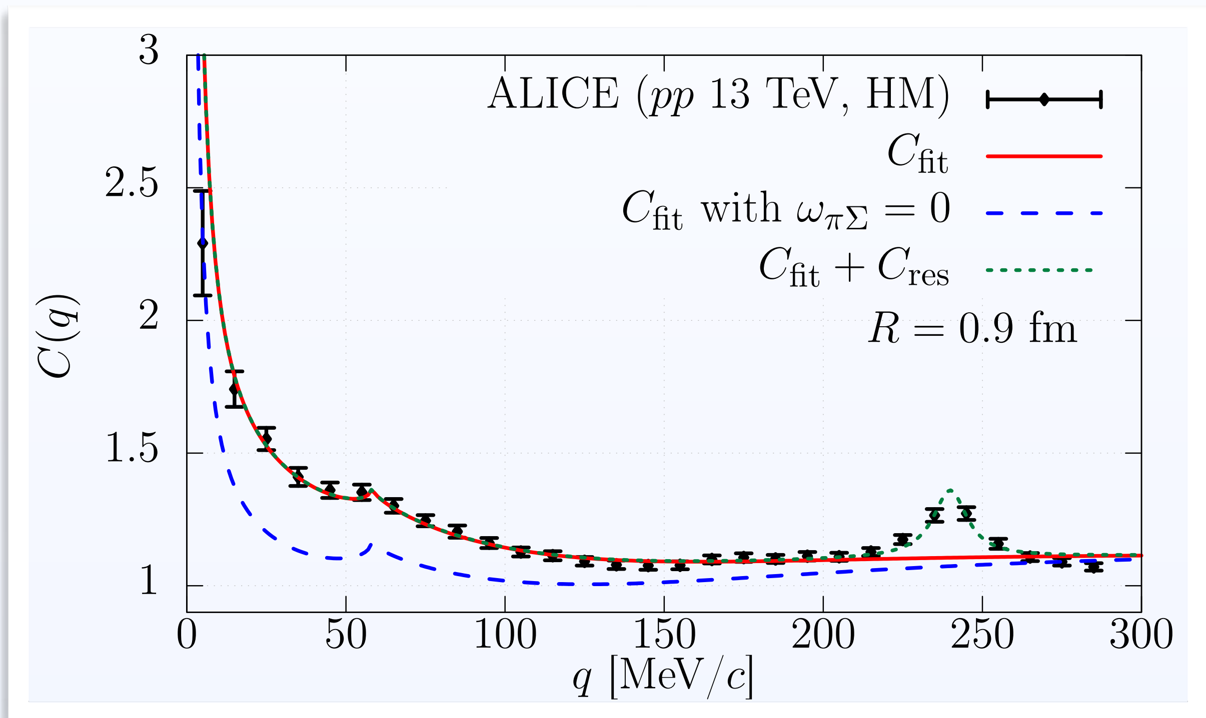
- Enhance $C(q)$
- Enhance cusp structure
- ω_i : production rate (compared to measured channel)

$\bar{K}N$ interaction and $K^{-}p$ correlation

Source size dependence with $K^{-}p$ data

- ALICE *pp* collision data

ALICE PRL 124, 092301 (2020)

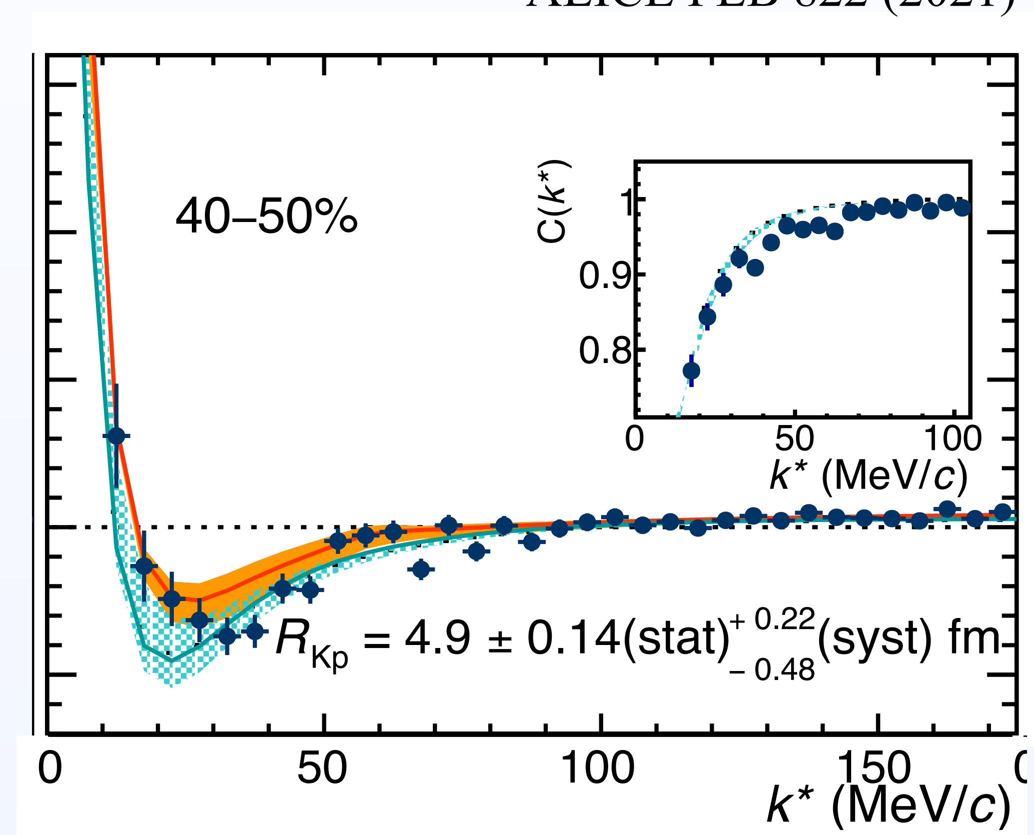


Kamiya, Hyodo, Morita, Ohnishi, Weise, PRL 124 (2020) 13, 132501

- Small source
  - Clear $\bar{K}^0 n$ cusp structure
  - Sizable contribution from coupled-channel source required to reproduce data

- ALICE PbPb collision data

ALICE PLB 822 (2021) 136708

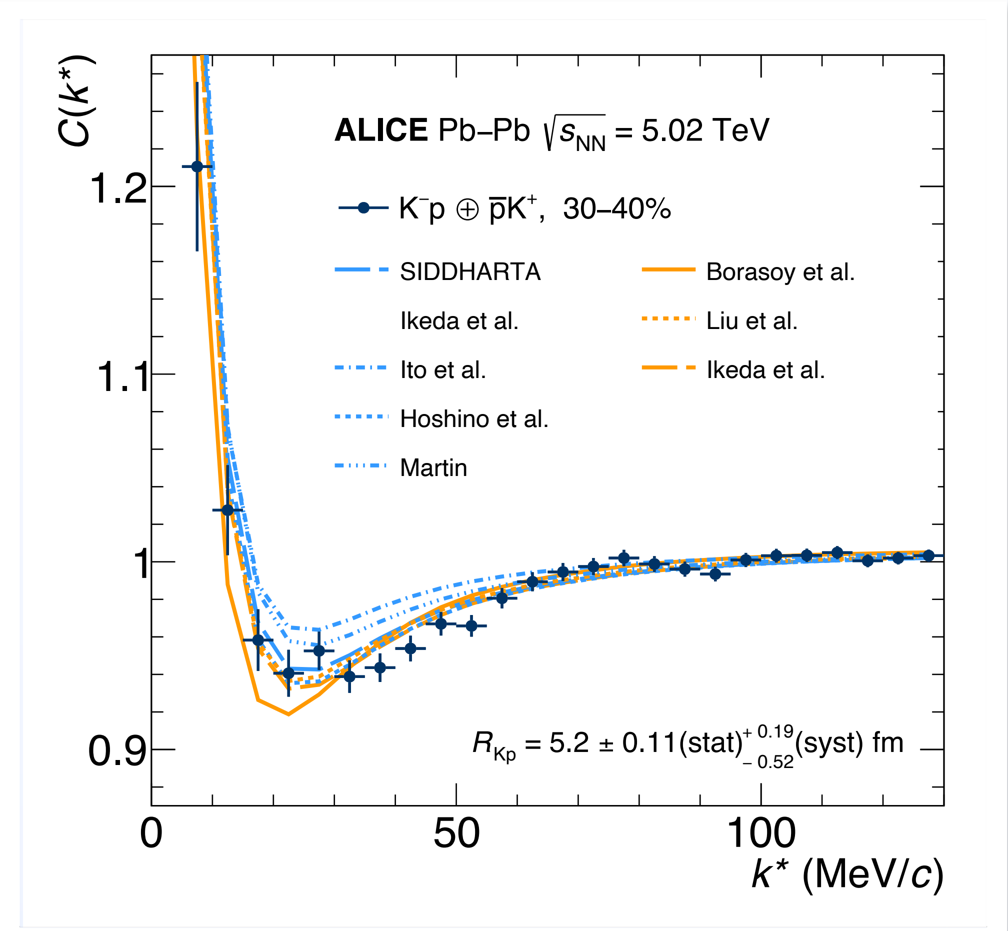


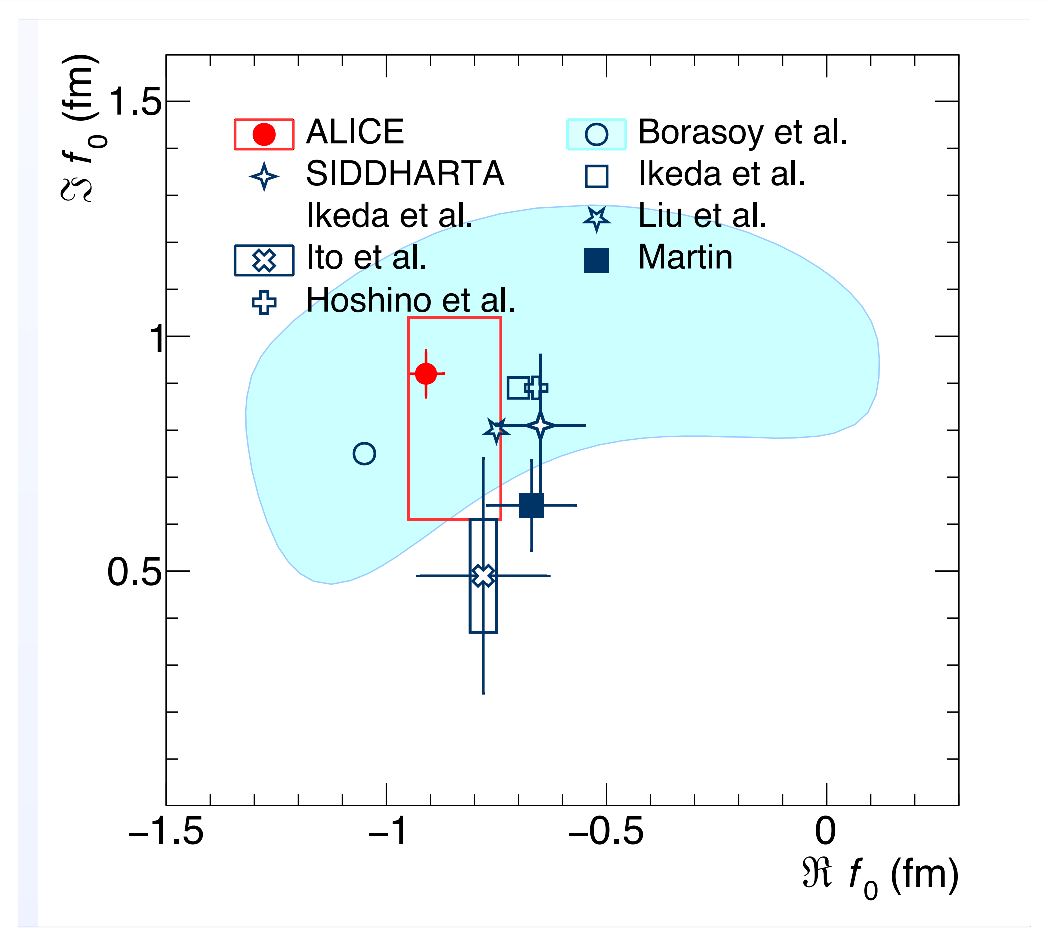
- Large source
  - Weaker cusp
  - Consistent with analysis only with $K^{-}p$ source

→ Chiral SU(3) dynamics describes the both correlation data well.

$\bar{K}N$ interaction and $K^{-}p$ correlation

- Source size dependence of $K^{-}p$ data
  - ALICE data PbPb collisions data   ALICE PLB 822 (2021) 136708
    - Large source —> weaker coupled-channel effect
      —> more direct approach to interaction of the measured channel
    - Extraction of the $K^{-}p$ scattering length from correlation function
      * Fitting with 1 channel LL model with Gaussian source





$\bar{K}N$ interaction and $K^{-}p$ correlation

- Latest $K^{-}p$ correlation results

  ALICE [2205.10258]

  - pPb : 0-20%, 20-40% 40-100%
  - PbPb : 60-70%, 70-80% 80-90%

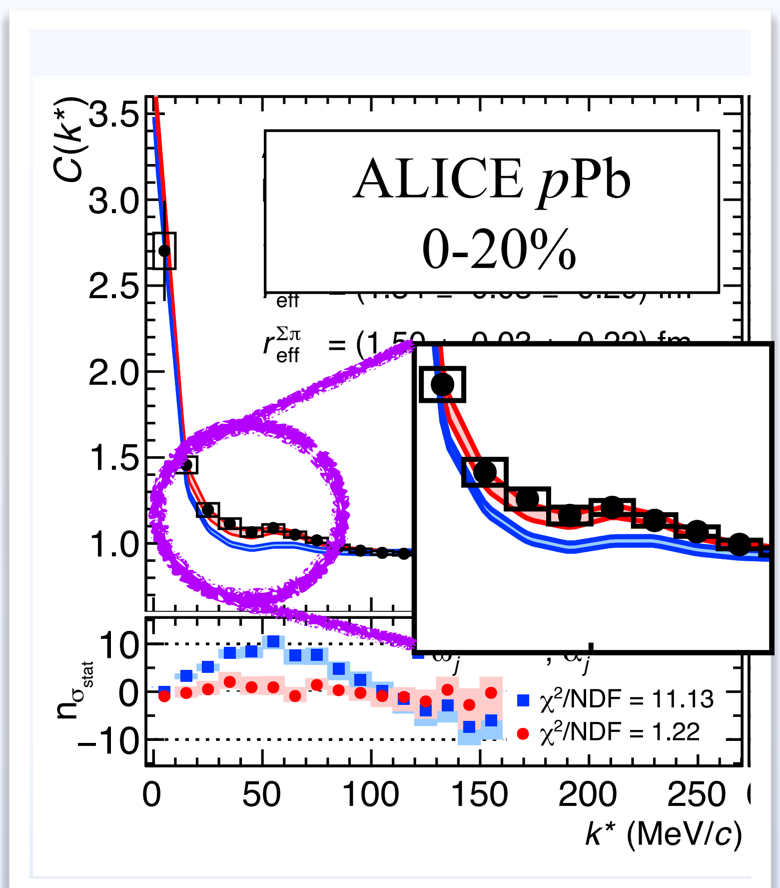
- Discrepancy around $\bar{K}^0 n$ threshold between chiral SU(3) model and exp. data for small source data

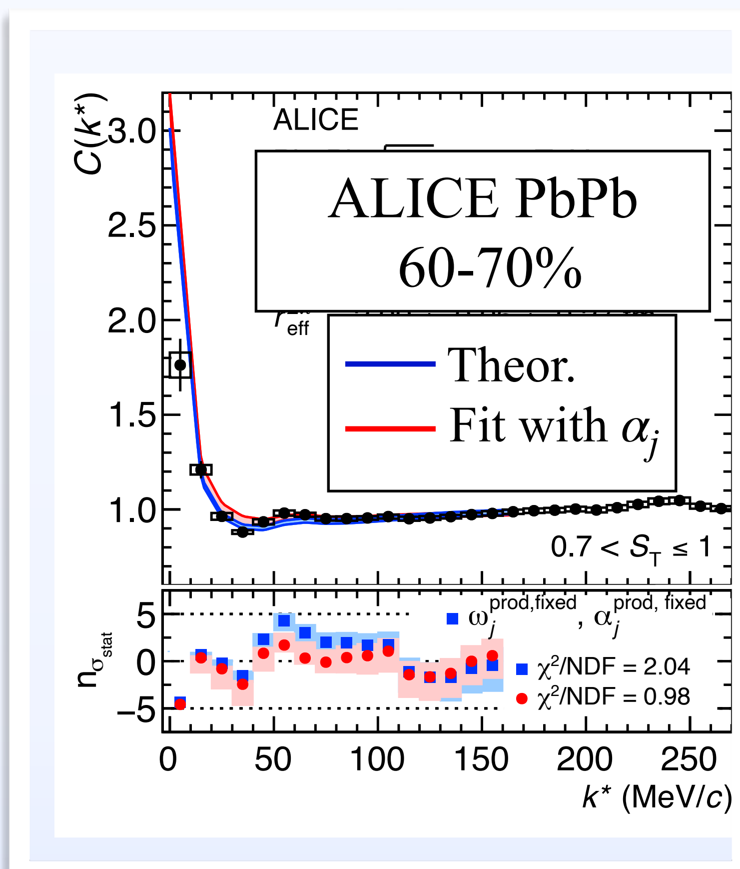
- Analysis with scale factor α_j
  - Scale the coupled-channel source contribution by scaling factor

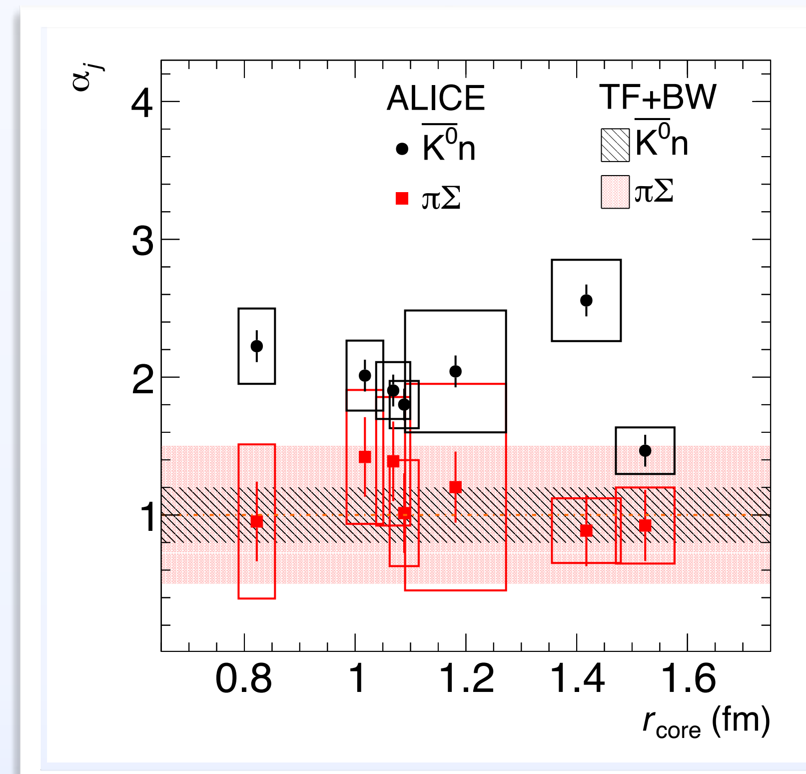
$$C_{K^-p} = C^{\mathrm{el}}_{K^-p} + \sum_j \alpha_j C^{\mathrm{inel}}_j$$

  - $\alpha_{\bar{K}^0 n} \sim 2$ gives better agreement

  → implying the stronger coupling

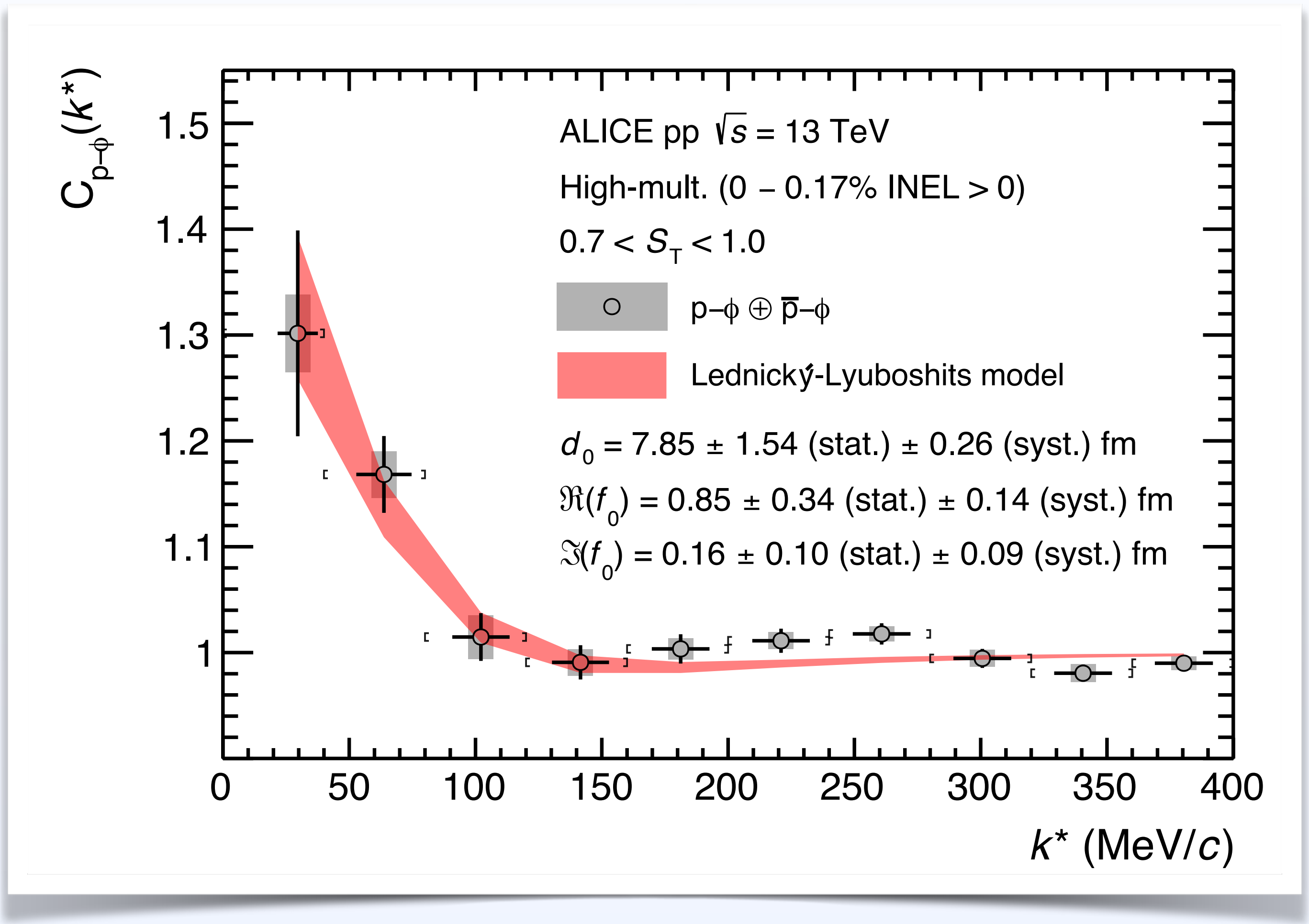






$N\phi$ interaction

- $p\phi$ correlation data from pp collisions



ALICE, PRL 127 (2021) 17, 172301

- Enhancement in the low momentum region
  - attractive $p\phi$ interaction
- Analysis with Lednický–Lyuboshits formula

$$\text{Re } a_0 = 0.85 \pm 0.34(\text{stat.}) \pm 0.14(\text{syst.}) \text{ fm}$$
$$\text{Im } a_0 = 0.16 \pm 0.10(\text{stat.}) \pm 0.09(\text{syst.}) \text{ fm}$$

- Decomposition for spin channels?

$$C_{p\phi}(k^*) = \frac{2}{3}C_{3/2}(k^*) + \frac{1}{3}C_{1/2}(k^*)$$

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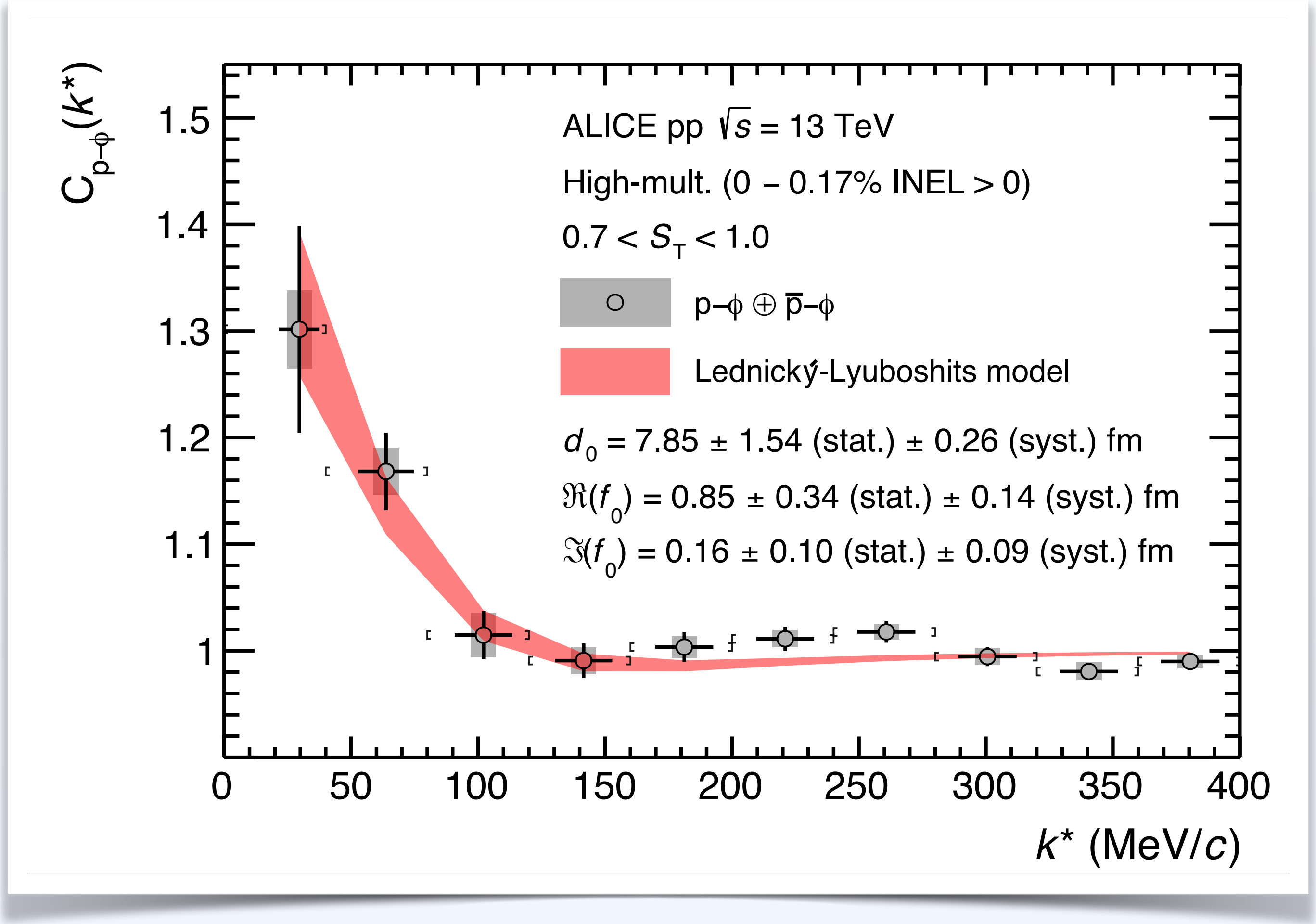

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use the latest lattice potential — determine from data

Reanalyze data to extract spin 1/2 int.

$N\phi$ interaction

HAL QCD potential for spin 3/2

Y. Lyu *et al*, PRD 106, 074507 (2022).

- Fitting function

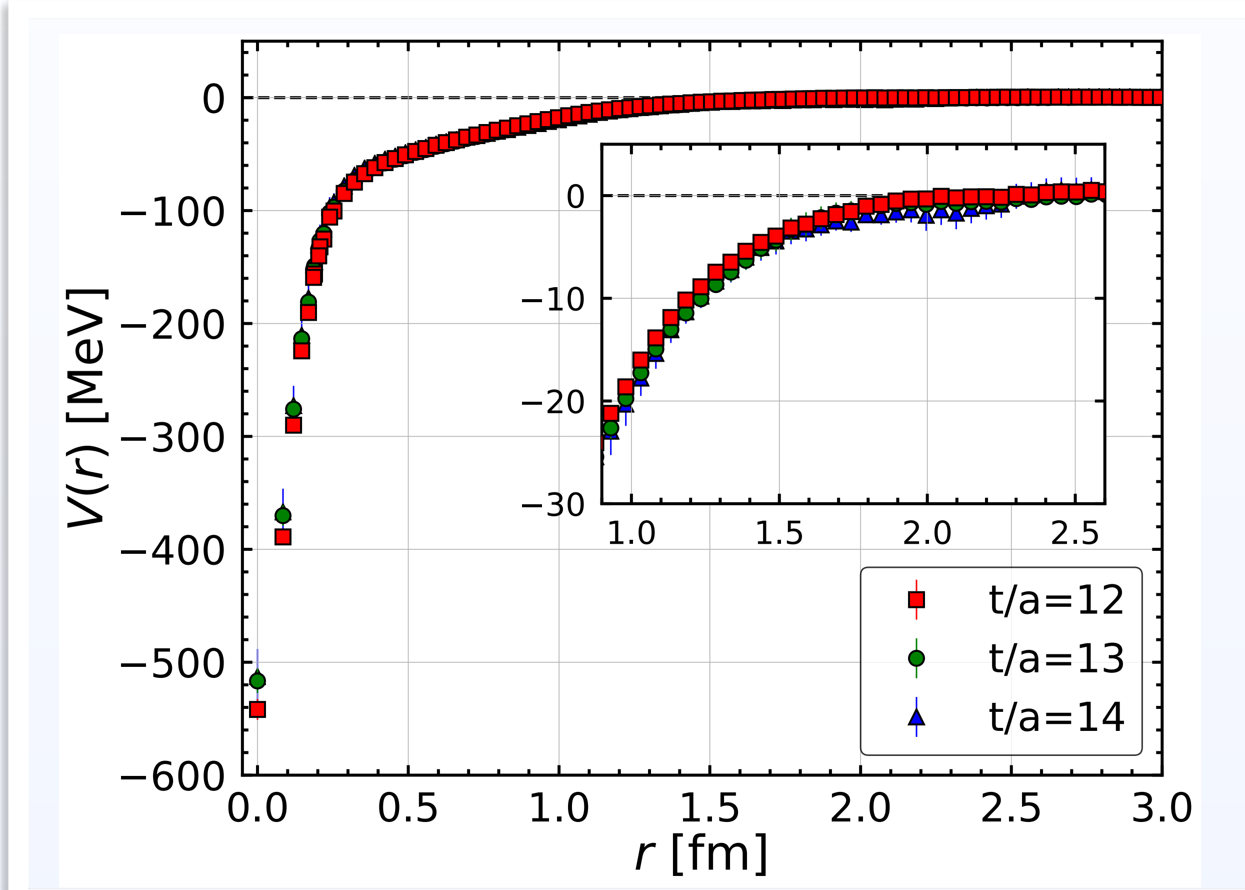
$$V_{\mathrm{fit}}(r) = \underbrace{\sum_{i=1,2} a_i e^{-(r/b_i)^2}}_{\text{short range part}} + \underbrace{a_3 m_\pi^4 f(r; b_3) \frac{e^{-2m_\pi r}}{r^2}}_{2\pi \text{ exchange int.}}$$

short range part

2π exchange int.

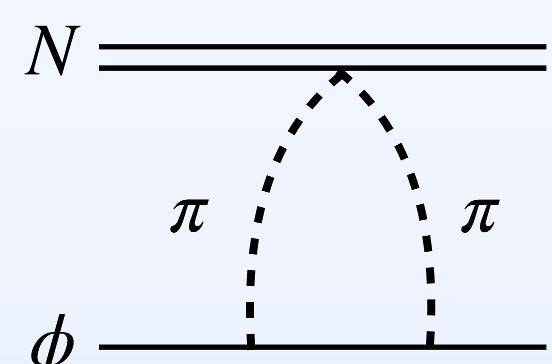
- Decay effect to $\Lambda K/\Sigma K$:
  strongly suppresses by *d*-wave coupling
  → well described with real potential
- Long range tail
  → 2 π exchange int.

J. Tarrús Castella` and G. a. Krein, PRD 98, 014029 (2018).



- Threshold parameters from fitted potential

m_π [MeV]	$a_0^{(3/2)}$ [fm]	$r_{\mathrm{eff}}^{(3/2)}$ [fm]
146.4	$-1.43(23)_{\mathrm{stat.}} \left({}^{+36}_{-06}\right)_{\mathrm{syst.}}$	$2.36(10)_{\mathrm{stat.}} \left({}^{+02}_{-48}\right)_{\mathrm{syst.}}$
138.0	$\simeq -1.25$	$\simeq 2.49$



- Strongly attractive but no bound state
  (nuclear physics convention for a_0)

$N\phi$ interaction

- Spin 1/2 $N\phi$ int. from femtoscopic data and HAL QCD potential

E.~Chizzali, et. al. [arXiv:2212.12690 [nucl-ex]].

$$C^{(\beta,\gamma)}_{\rm model}(k^*) = \frac{2}{3}C_{3/2}(k^*) + \frac{1}{3}C^{(\beta,\gamma)}_{1/2}(k^*)$$

HAL QCD potential

Fit with effective potential

- Fitting function for spin 1/2 potential

$$V_{1/2} = \beta \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 f(r;b_3)\frac{e^{-2m_\pi r}}{r^2} + i\gamma\sqrt{f(r;b_3)}\frac{e^{-m_K r}}{r}$$

  - Inspired by HAL QCD potential for spin 3/2: $V_{3/2} = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 f(r;b_3)\frac{e^{-2m_\pi r}}{r^2}$
  - Two fitting parameters

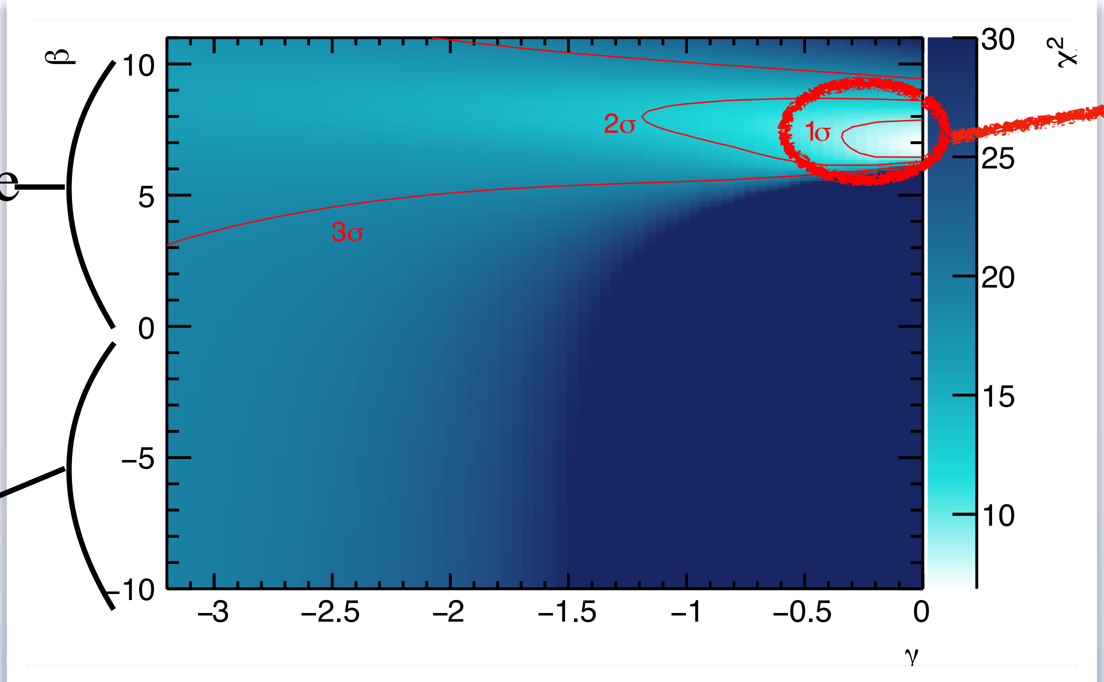
    β: relative strength of short range int.

    γ: strength of imaginary part

- Fitting result

Attractive ($\beta > 0$)

Repulsive ($\beta < 0$)



- Well fitted range

$$\beta = 7.0^{+0.8}_{-0.2}({\rm stat.})^{+0.2}_{-0.2}({\rm syst.})$$

$$\gamma = 0.0^{+0.0}_{-0.2}({\rm stat.})^{+0.0}_{-0.2}({\rm syst.})$$

- No good parameter sets for repulsive interactions

→ Strongly attractive interaction with small decay effect

$N\phi$ interaction

- Spin 1/2 $N\phi$ int. from femtoscopic data and HAL QCD potential

$$C^{(\beta,\gamma)}_{\mathrm{model}}(k^*) = \frac{2}{3}C_{3/2}(k^*) + \frac{1}{3}C^{(\beta,\gamma)}_{1/2}(k^*)$$

E.~Chizzali, et. al. [arXiv:2212.12690 [nucl-ex]].

HAL QCD potential

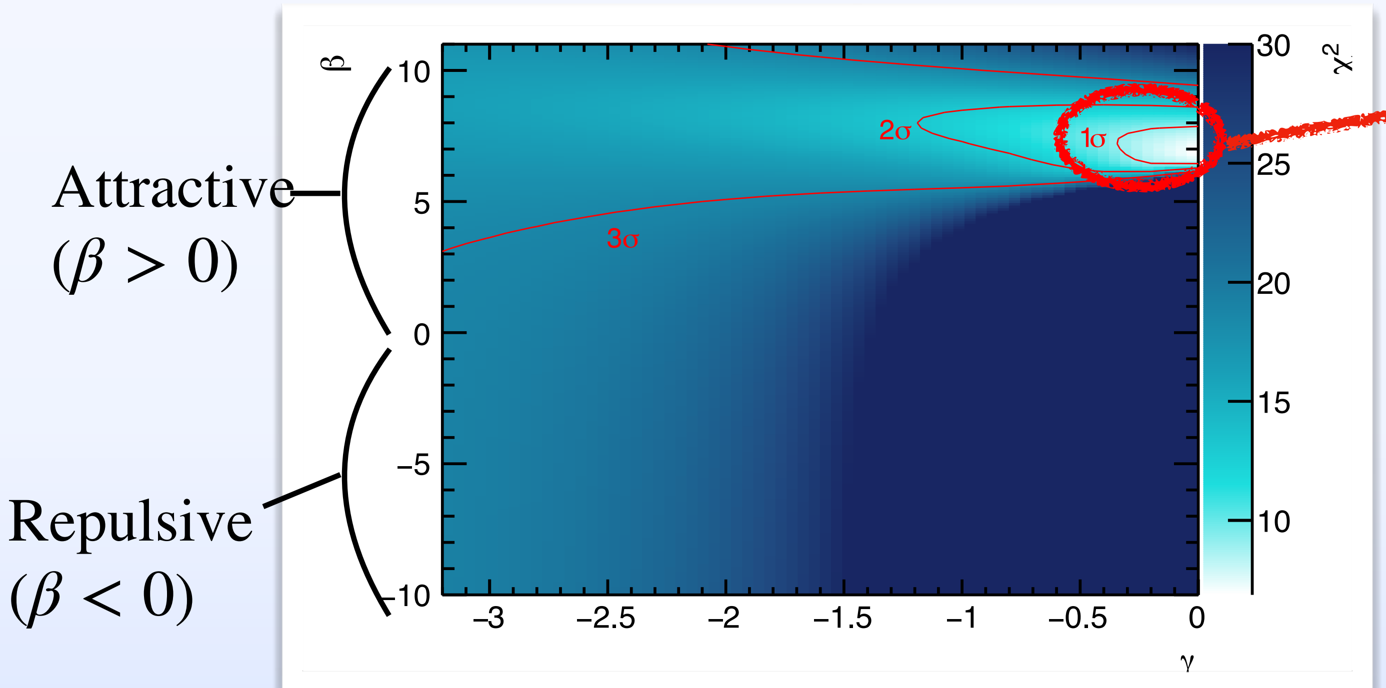
Fit with effective potential

- Fitting function for spin 1/2 potential

$$V_{N\phi} = -\beta\,(\text{short range part})$$
$$+(2m_\pi \text{ exchange})$$
$$+i\gamma(\text{imag. part})$$

- Gaussian having same range b_i with spin 3/2 $-\exp(-r^2/b_i^2)$
- Same strength with spin 3/2 (2 pion exchange does not depends on spin)
- Given with K exchange $\propto \exp(-m_K r)/r$

- Fitting result



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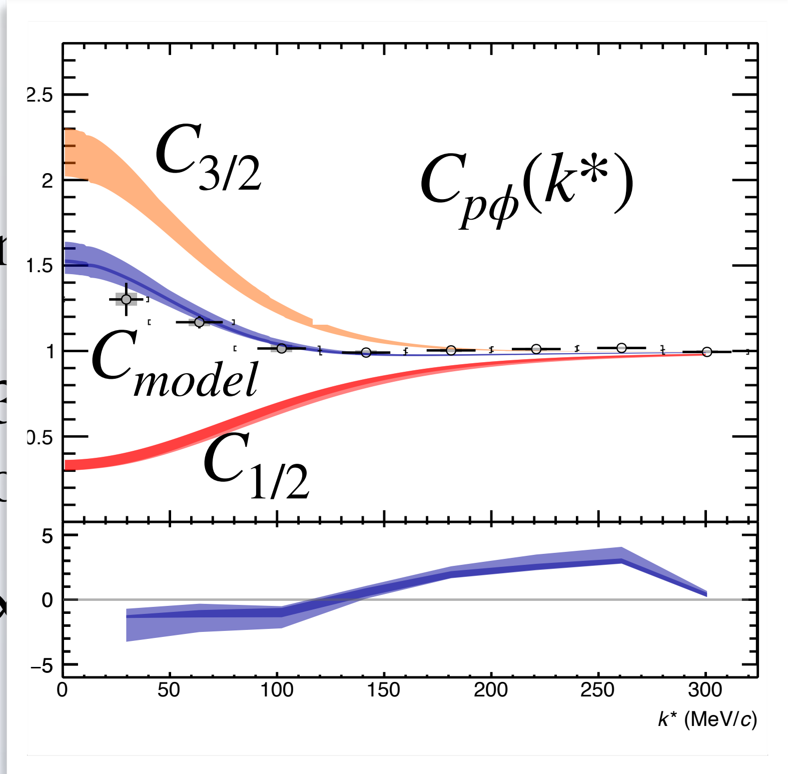

- Fitting function for spin 1/2 potential

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  (2 pion exchange does no
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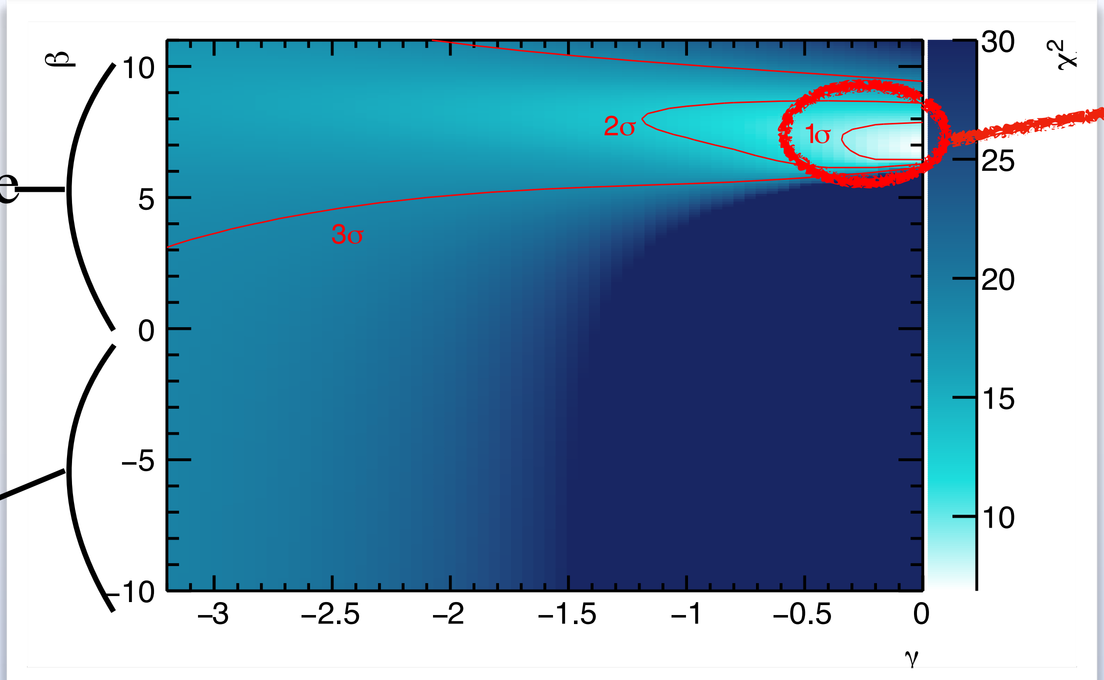
Attractive ($\beta > 0$)

Repulsive ($\beta < 0$)



- Well fitted range

$$\beta = 7.0^{+0.8}_{-0.2}(\text{stat.})^{+0.2}_{-0.2}(\text{syst.})$$
$$\gamma = 0.0^{+0.0}_{-0.2}(\text{stat.})^{+0.0}_{-0.2}(\text{syst.})$$

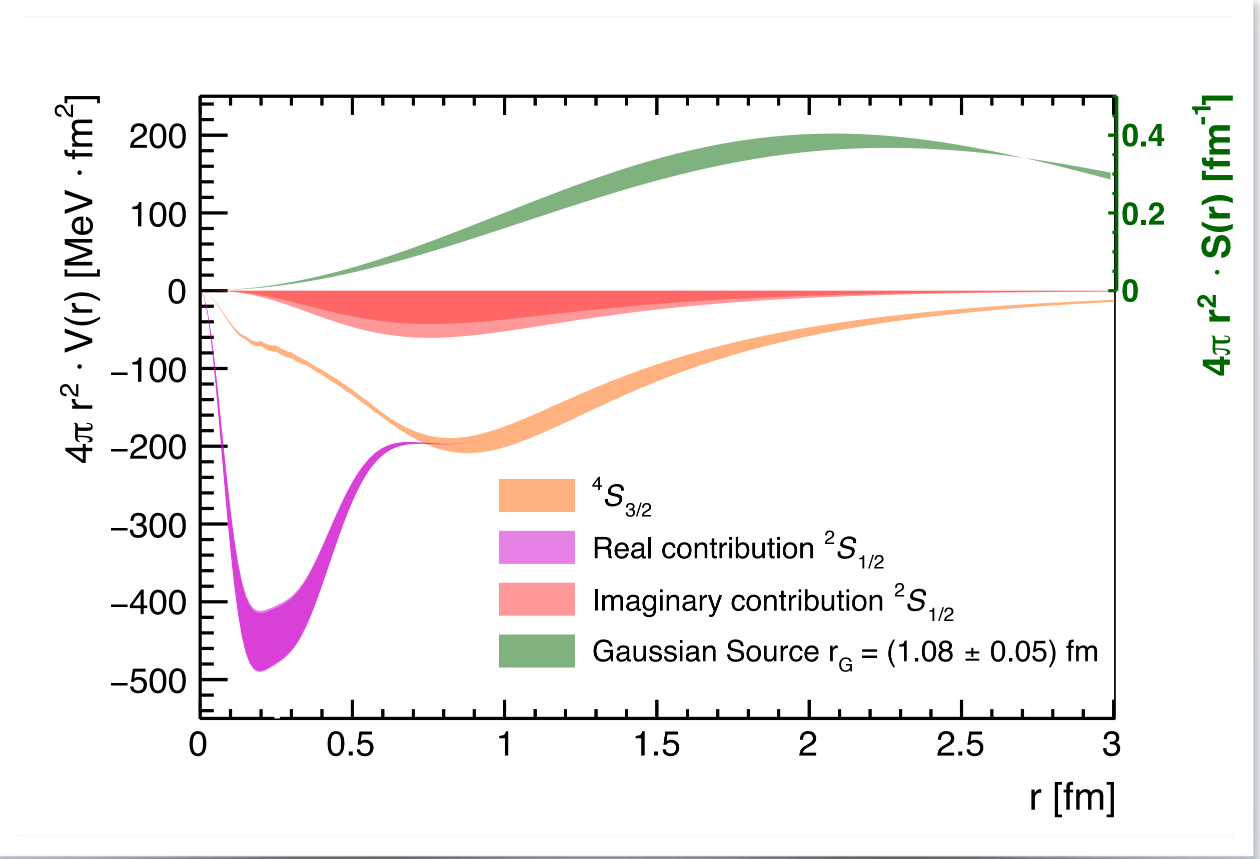
- No good parameter sets for repulsive interactions

→ Strongly attractive interaction with small decay effect

$N\phi$ interaction

Analysis with fitted potential

E.~Chizzali, et. al. [arXiv:2212.12690 [nucl-ex]].



- Threshold parameters (high energy phys. convention)
  - Scattering length

$$\mathrm{Re}\ f_0^{(1/2)} = -1.47^{+0.44}_{-0.37}(\mathrm{stat.})^{+0.14}_{-0.17}(\mathrm{syst.})\ \mathrm{fm},$$
$$\mathrm{Im}\ f_0^{(1/2)} = \quad 0.00^{+0.26}_{-0.00}(\mathrm{stat.})^{+0.15}_{-0.00}(\mathrm{syst.})\ \mathrm{fm},$$

  - Effective range

$$\mathrm{Re}\ d_0^{(1/2)} = +0.37^{+0.07}_{-0.08}(\mathrm{stat.})^{+0.03}_{-0.03}(\mathrm{syst.})\ \mathrm{fm},$$
$$\mathrm{Im}\ d_0^{(1/2)} = \quad 0.00^{+0.00}_{-0.02}(\mathrm{stat.})^{+0.00}_{-0.01}(\mathrm{syst.})\ \mathrm{fm}.$$

- Eigenenergy of quasibound state

$$E = -\,26.6^{+10.5}_{-29.4}(\mathrm{stat.})^{+5.5}_{-6.2}(\mathrm{syst.})$$
$$-i0.0^{+0.0}_{-7.8}(\mathrm{stat.})^{+0.0}_{-6.6}(\mathrm{syst.})\ [\mathrm{MeV}]$$

  - Comparable or larger binding energy compared to model calculations

QCD van der Waals attractive potential (Yukawa-type) $\longrightarrow E_B = 1.8\ \mathrm{MeV}$

H. Gao, T.-S. H. Lee, and V. Marinov, PRC 63, 022201 (2001).

SU(3) chiral quark model $\longrightarrow E_B \sim 3\ \mathrm{MeV}$

F. Huang, Z. Y. Zhang, and Y. W. Yu, PRC 73, 025207 (2006).

DD^* and $D\bar{D}^*$ int. from femtoscopy

- DD^* and $D\bar{D}^*$ sector

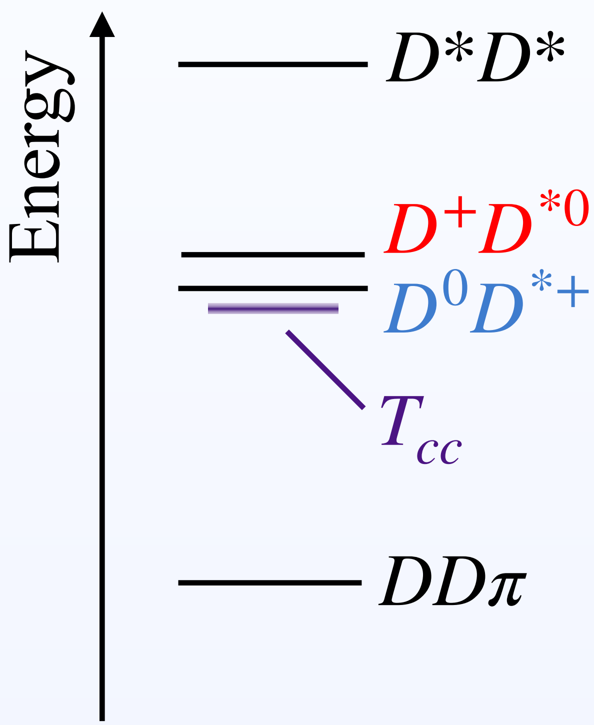
$a_0 \equiv \mathcal{F}(E = E_{\mathrm{th}})$

+ : attractive w/o bound

- : repulsive
  or attractive w/ bound

$C = 2$

LHCb, Nature Com. 13 (2022) 1



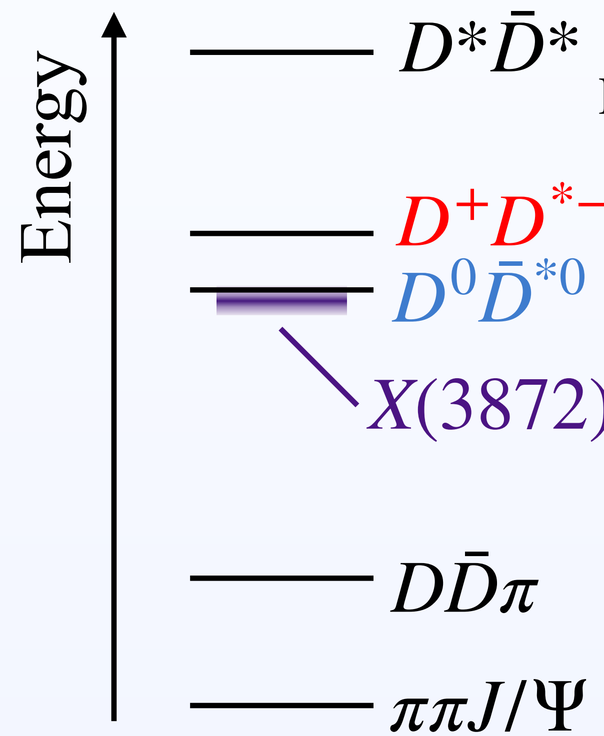
$$E_{T_{cc}} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.36 \text{ MeV}$$

$$\Gamma = 0.048 \text{ MeV}$$

$$a_0 = -7.16 + i1.85 \text{ fm}$$

$C = 0$



PDG, PTEP 2020, 083C01 (2020).

$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

$$\Gamma = 1.19 \text{ MeV}$$

↓

$$a_0^{D^0\bar{D}^{*0}} = -4.23 + i3.95\text{fm}$$

- T_{cc}/ $X(3872)$ lies nearby $DD^*/D\bar{D}^*$

  ==> meson-meson molecule?

  ==>Strong attractive interaction

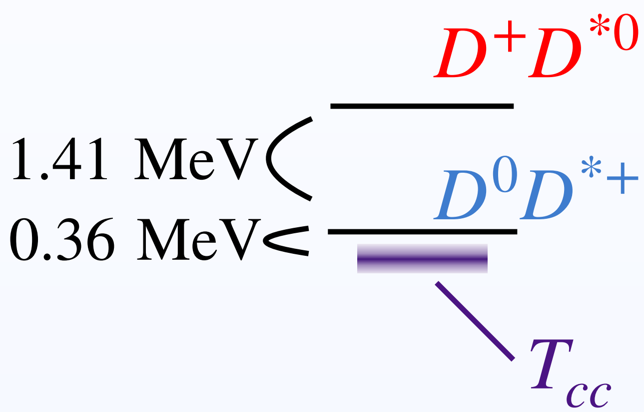
- Gaussian potential

$$V(r) = V_0 \exp(-m^2 r^2)$$

- m <— π exchange ($m = m_\pi$)
- V_0<— scattering lengths
- Assume dominant contribution from exotic channel ($I = 0$)
- Coupled-channel of two isospin channels

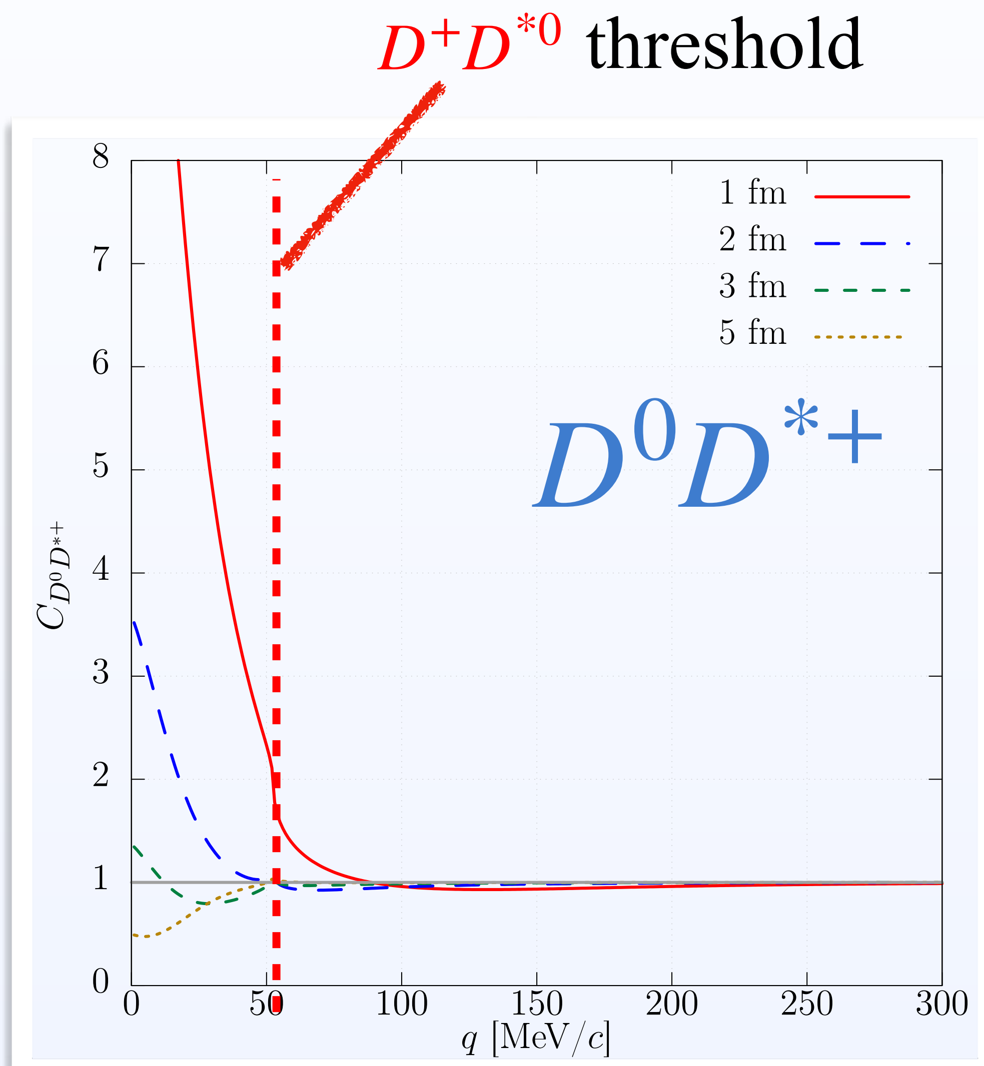
DD^* and $D\bar{D}^*$ int. from femtoscopy

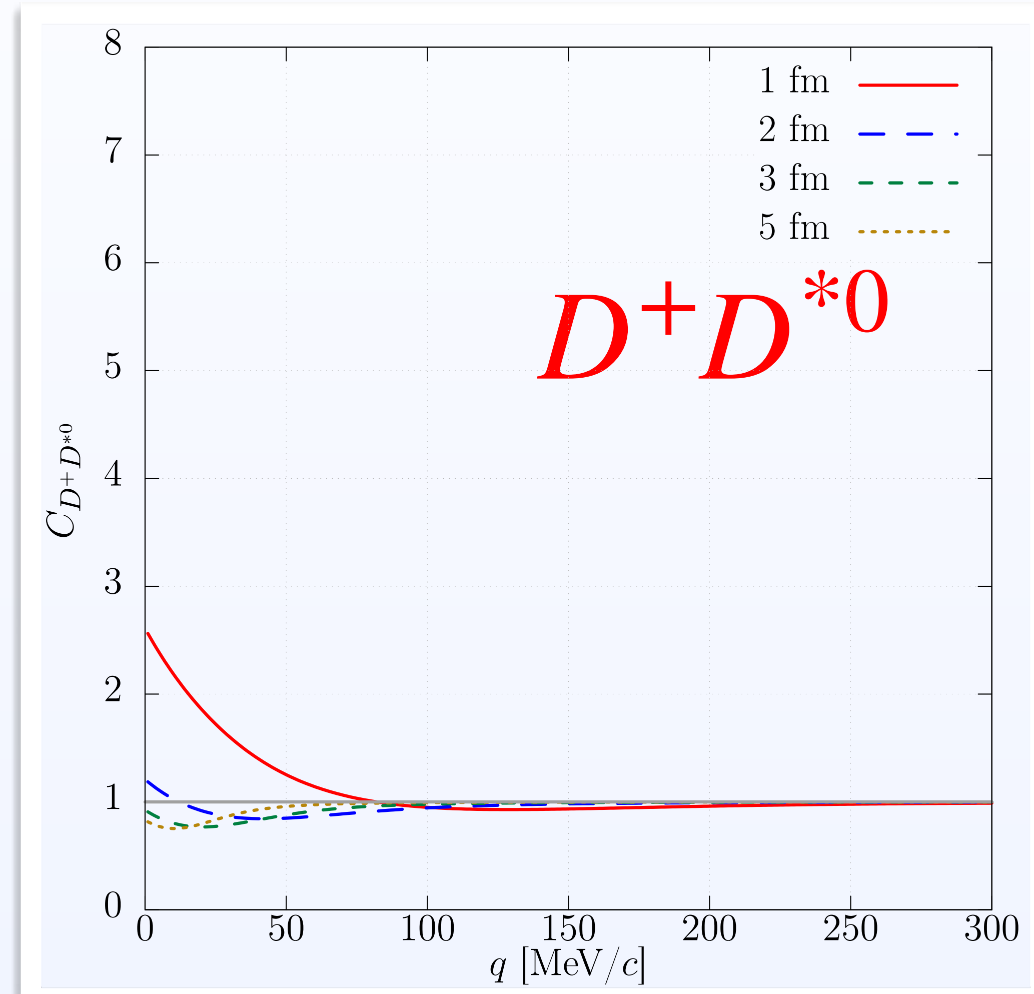
- DD^* correlation and T_{cc} state



$$E_{T_{cc}} = \delta m - \frac{i}{2}\Gamma,$$
$$\delta m = -360 \text{ keV},$$
$$\Gamma = 48 \text{ keV}$$
$$a_0 = -7.16 + i1.85 \text{ fm}$$

LHCb, Nature Com. 13 (2022) 1

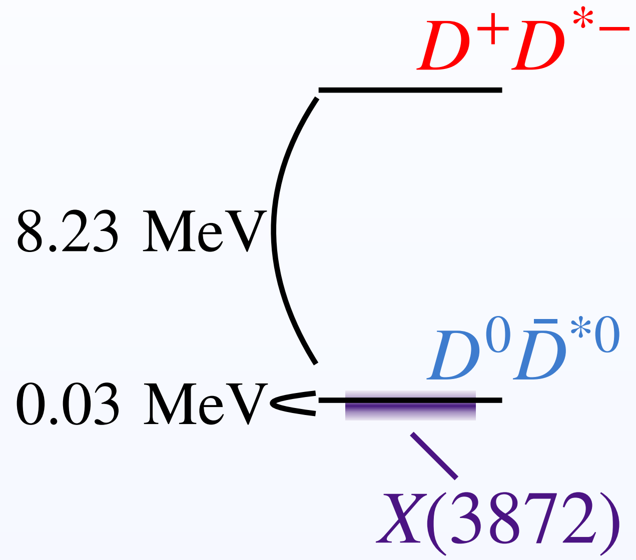




- Bound state like behavior for both pairs
- Stronger source size dep. for D^0D^{*+}
- D^+D^{*0} cusp is not prominent

$D D^*$ and $D\bar{D}^*$ int. from femtoscopy

- $D\bar{D}^*$ correlation and $X(3872)$ state



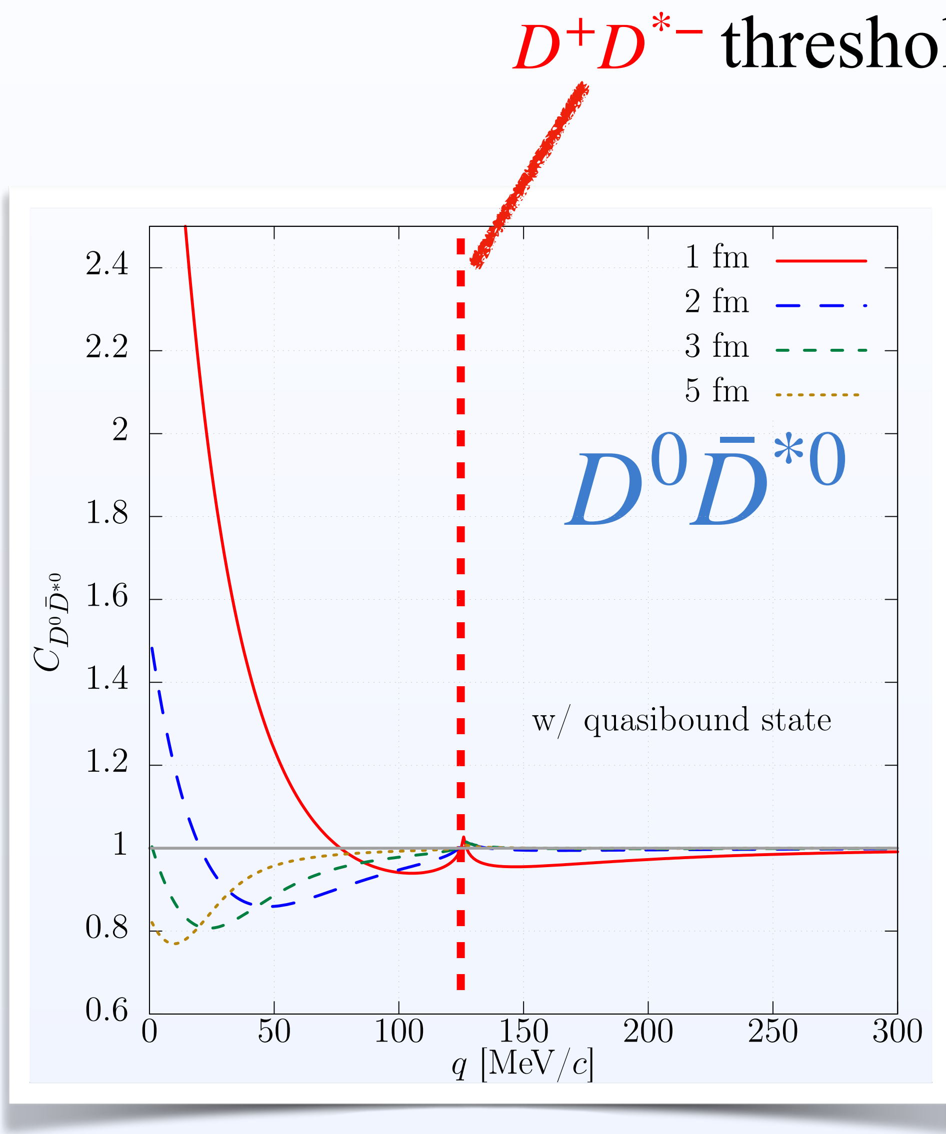
PDG, PTEP 2020, 083C01 (2020)

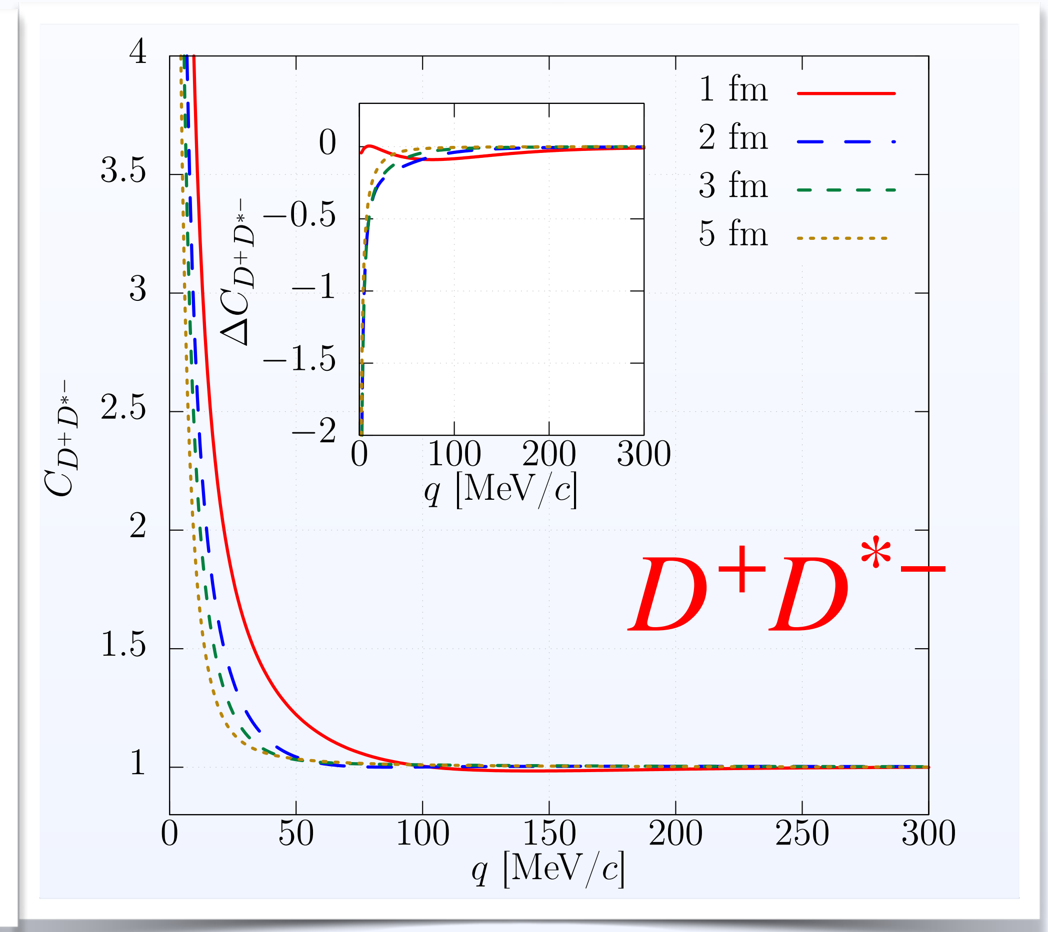
$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

$$\Gamma = 1.19 \text{ MeV}$$

$a_0^{D^0\bar{D}^{*0}} = -4.23 + i3.95\text{fm}$

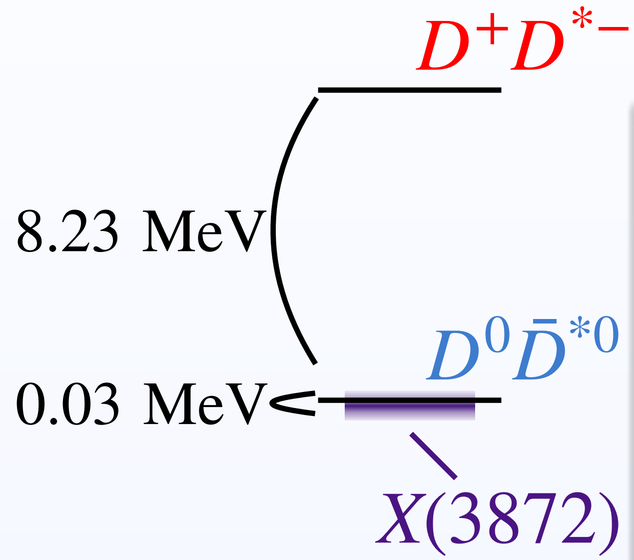




- D^0D^{*+} : Strong source size dep.
- D^+D^{*-} : Small effect of the strong int. (Coulomb int dominance)
- Moderate D^+D^{*+} cusp

DD^* and $D\bar{D}^*$ int. from femtoscopy

- $D\bar{D}^*$ correlation and $X(3872)$ state



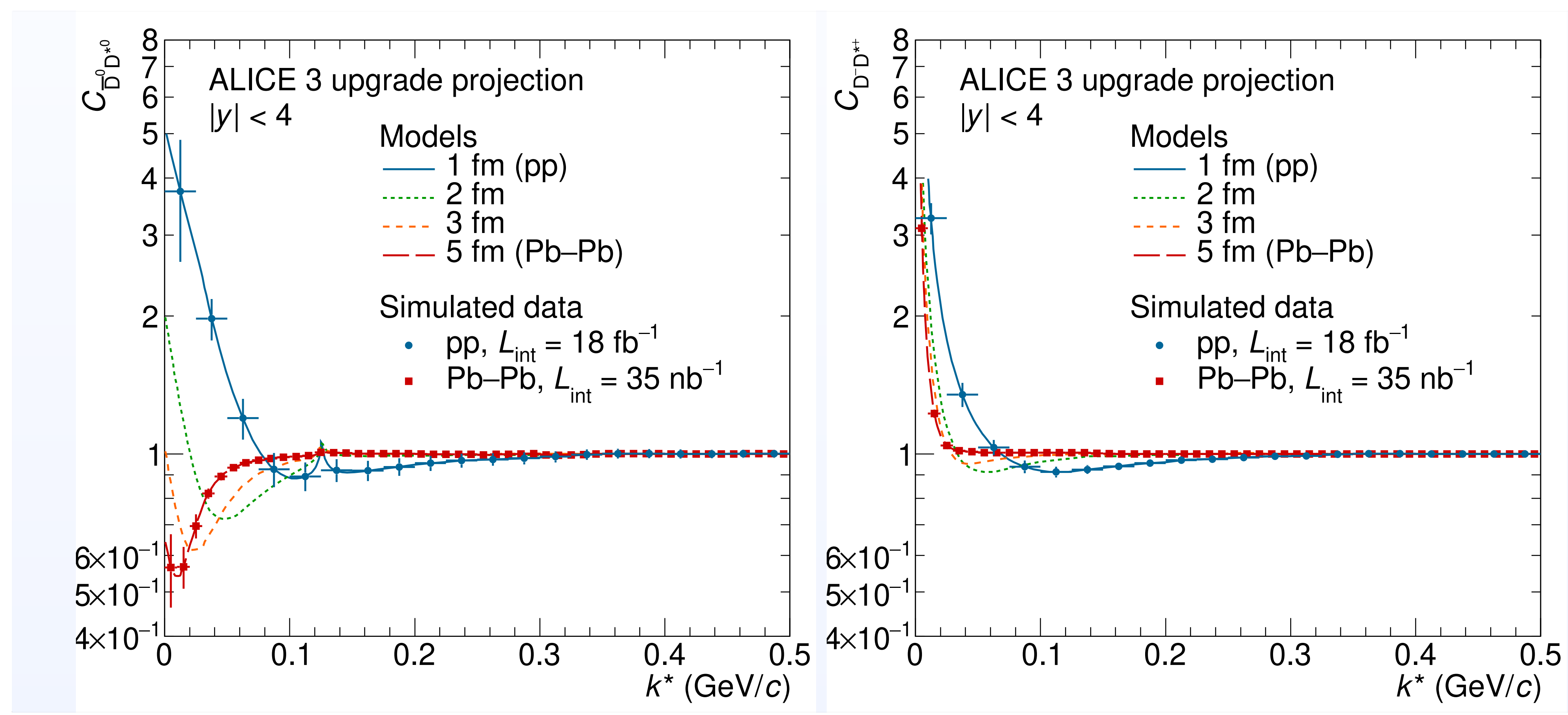
PDG, PTEP 2020, 083C01 (2020)

$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

$$\Gamma = 1.19 \text{ MeV}$$

$$a_0^{D^0\bar{D}^{*0}} = -4.23 + i3.95\text{fm}$$



ALICE collab., CERN-LHCC-2022-009 (2022).

- D^0D^{*+} : Strong source size dep.
- D^+D^{*-} : Small effect of the strong int. (Coulomb int dominance)
- Moderate D^+D^{*+} cusp

Summary

- Femtoscopic correlation function in high energy nuclear collisions is a powerful tool to investigate the nature of bound state.
  - Comparison to model prediction
  - Direct extraction from $C(q)$ data

- $K^{-}p$
  The coupled-channel effect performs significant role. Comparison with chiral SU3 based model $C(q)$ leads;
  - Large R data: well described
  - Small R data: finite deviation implying the stronger coupling to $\bar{K}^0 n$.
- $p\phi$
  Spin 1/2 interaction is extracted by the reanalysis with the correlation data and spin 3/2 Lattice HAL QCD potential.The potential is found to be attractive enough to support a $N\phi$ bound state.

- $DD^*/D\bar{D}^*$
  The lower isospin partner channels are expected to show the strong source size dependence due to the near threshold $T_{cc}/X(3872)$ states.

Thank you for your attention!