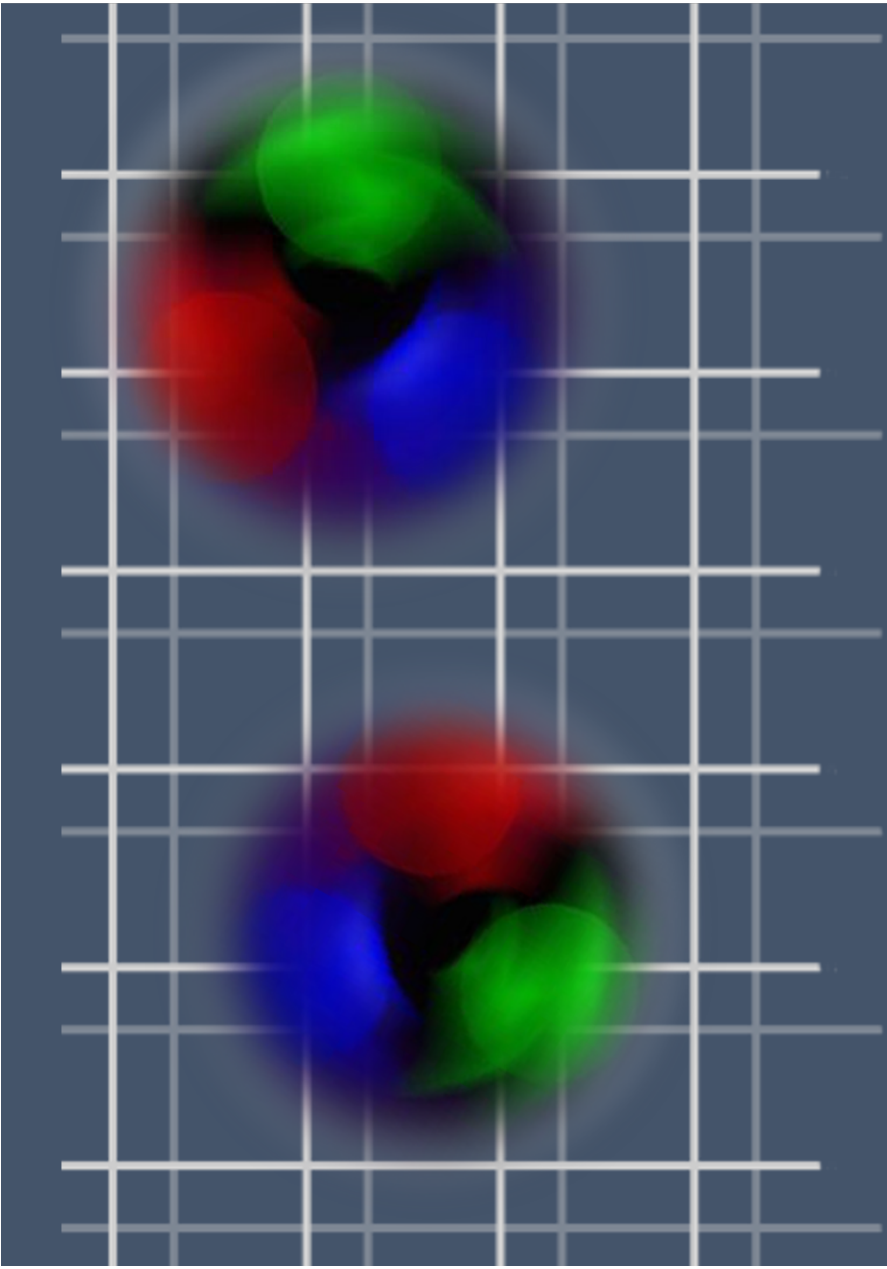


Latest results from the lattice description of bound states and interactions

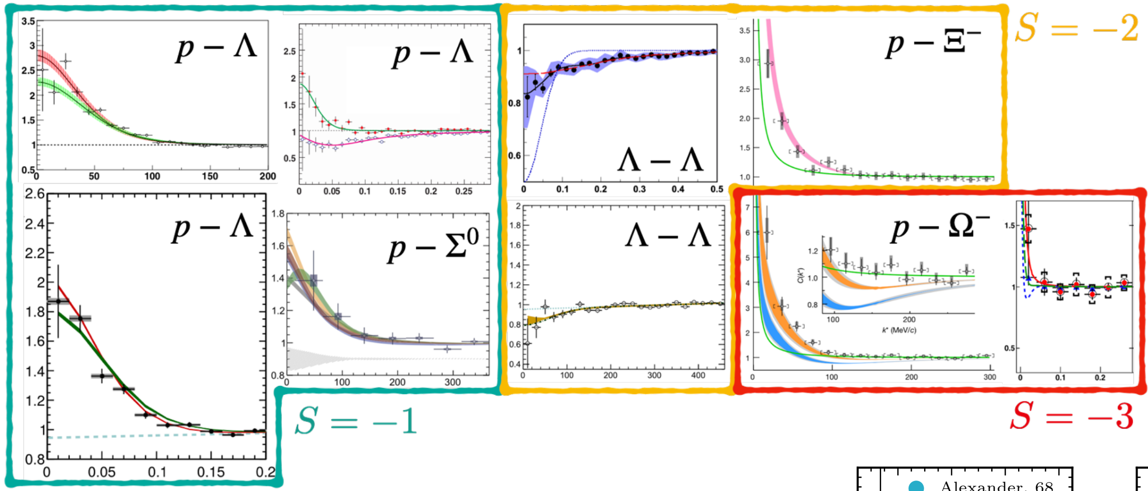
Assumpta Parreño
Universitat de Barcelona

NPLQCD Collaboration
www.ub.edu/nplqcd

Bologna 2023
4th EMMI workshop on (anti)matter, hyper-matter and exotica production at the LHC
February 13-17, 2023

UNIVERSITAT DE BARCELONA
ICCUB
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA
EXCELENCIA MARÍA DE MAEZTU

Femtoscopy: correlation function $C(k)$ as a function of relative momentum k

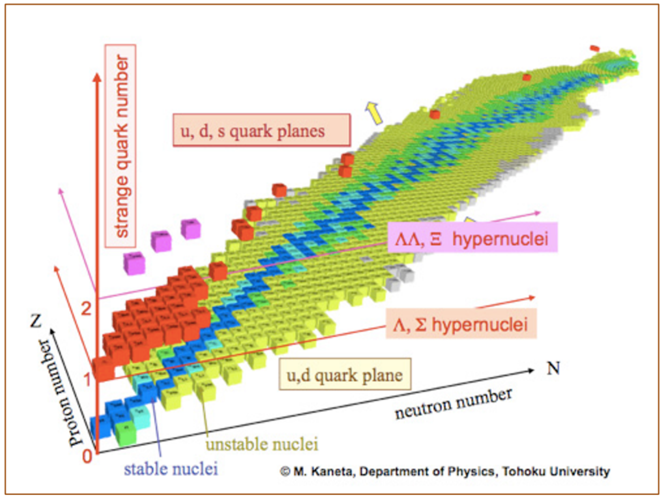


STAR, HADES and ALICE Collaborations

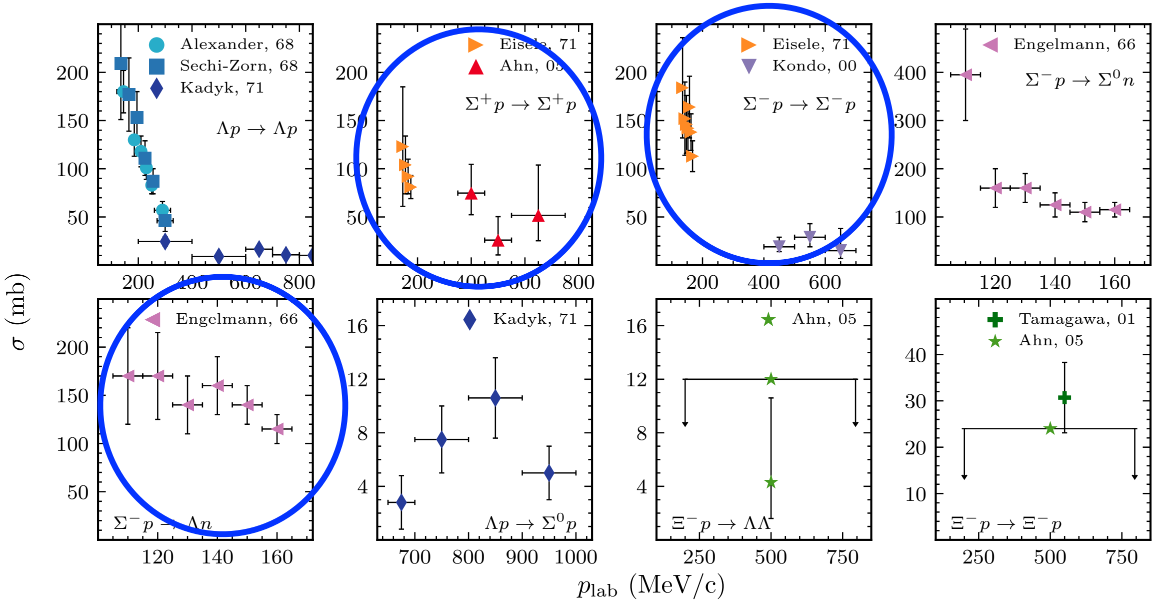
Connection to QCD

Systematically improve the calculation

Control the uncertainties



E40 @ JPARC



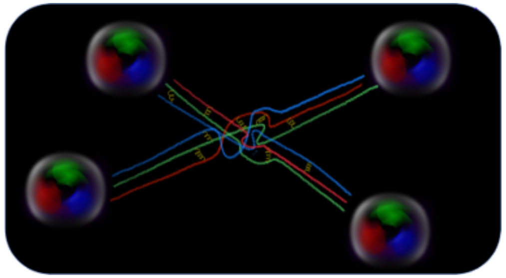
First collected in
Dover and Feschback,
Ann. Phys. 198 (1990)

Updated by Marc Illa, UB

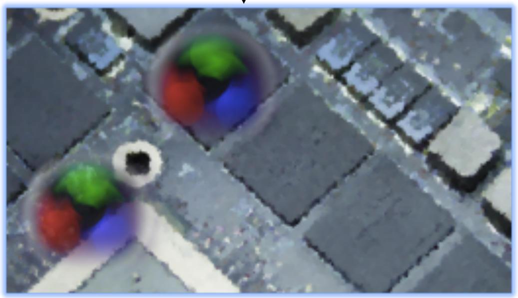
Nuclear physics, the non-perturbative regime of QCD

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F^{\ a}_{uv}F^{\ uv}_{a}$$

$$i = r,g,b \quad j = u,d,c,s,t,b$$



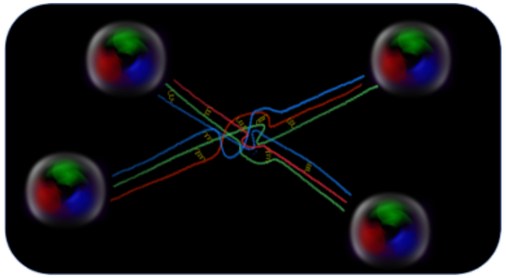
Lattice QCD



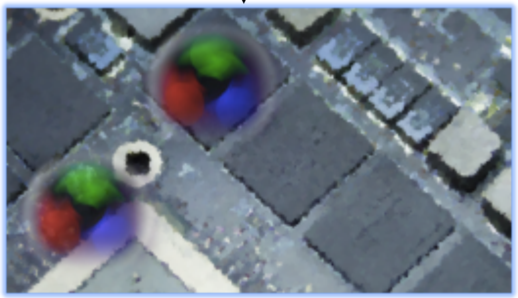
Nonperturbative (numerical) solution

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F^{a}_{uv}F^{uv}_{a}$$

$$i = r,g,b \quad j = u,d,c,s,t,b$$



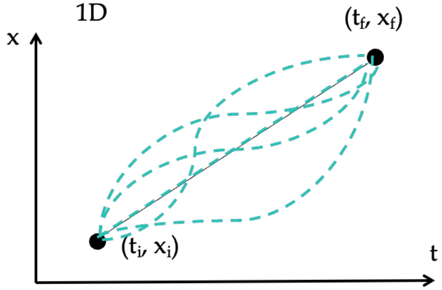
Lattice QCD



Nonperturbative (numerical) solution

PATH INTEGRAL
Feynman, 1948

The quantum propagation
is expressed as a
weighted sum over paths

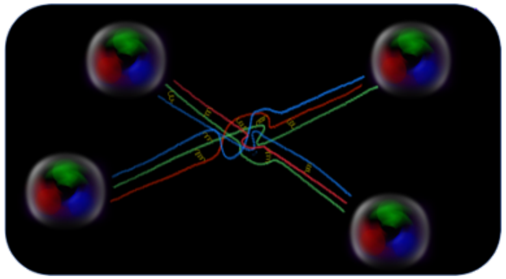


expectation values

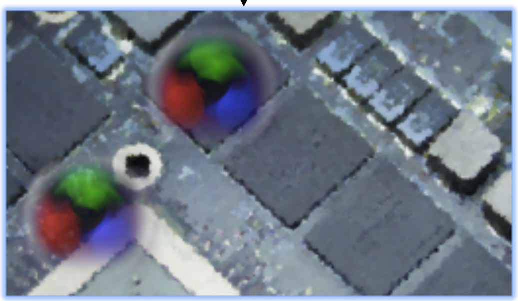
$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}A_{\mu}\,\hat{\mathcal{O}}[q,\bar{q},A]\,e^{iS_{QCD}}$$

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F^{\,a}_{uv}F^{\,uv}_{a}$$

$$i = r,g,b \quad j = u,d,c,s,t,b$$



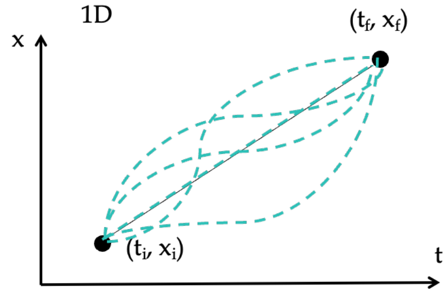
Lattice QCD



Nonperturbative (numerical) solution

PATH INTEGRAL
Feynman, 1948

The quantum propagation is expressed as a weighted sum over paths



expectation values

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}A_{\mu}\,\hat{\mathcal{O}}[q,\bar{q},A]\,e^{iS_{QCD}}$$

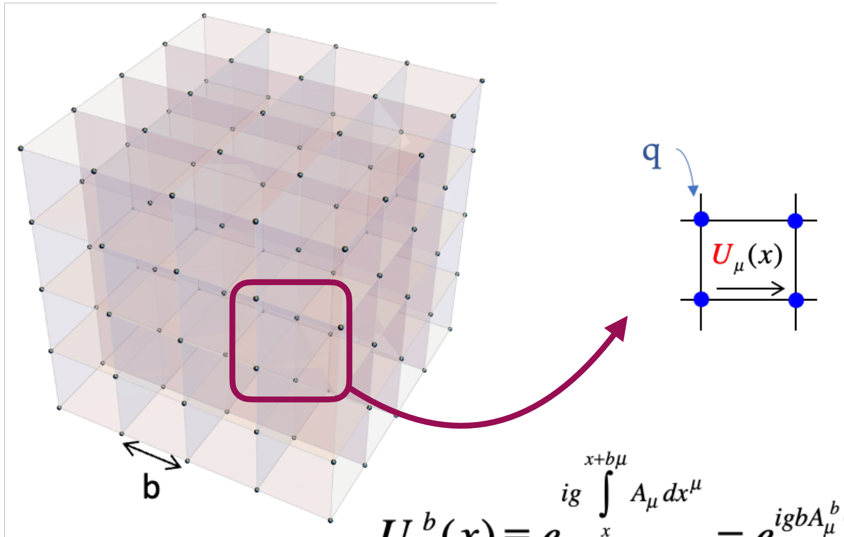
$t \to i\tau$

go to **Euclidean space**
numerical methods/important sampling

$$\langle\hat{O}\rangle = \frac{1}{Z}\int DU\;D\bar{\psi}\;D\psi\,\hat{O}[\psi,\bar{\psi},U]\,e^{-\bar{\psi}Q(U)\psi - S_g[U]}$$

$$\langle\hat{O}\rangle = \frac{1}{Z}\int DU\;\hat{O}[Q(U)^{-1}]\,\det(Q(U))\,e^{-S_g[U]}$$

propagators — configurations (~ P (U))



$$U_\mu^b(x) \equiv e^{ig\int_x^{x+b\mu} A_\mu \, dx^\mu} = e^{igbA_\mu^b(x)}$$

$L_x \times L_y \times L_z \times T$

($SU(3)_c$ matrices, “links”)

$L >> m_\pi^{-1}$ Finite volume

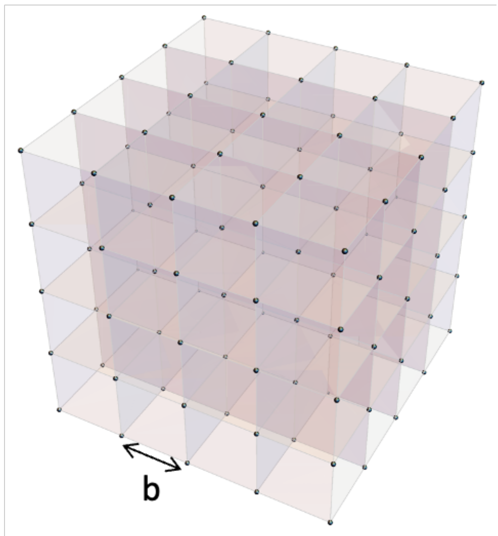
$b << \Lambda_{QCD}^{-1}$ Discretize spacetime

$$\langle \hat{\mathcal{O}} \rangle \approx \frac{1}{N_{\mathrm{cfg}}} \sum_{n=1}^{N_{\mathrm{cfg}}} \hat{\mathcal{O}}(\{U\}_n)$$

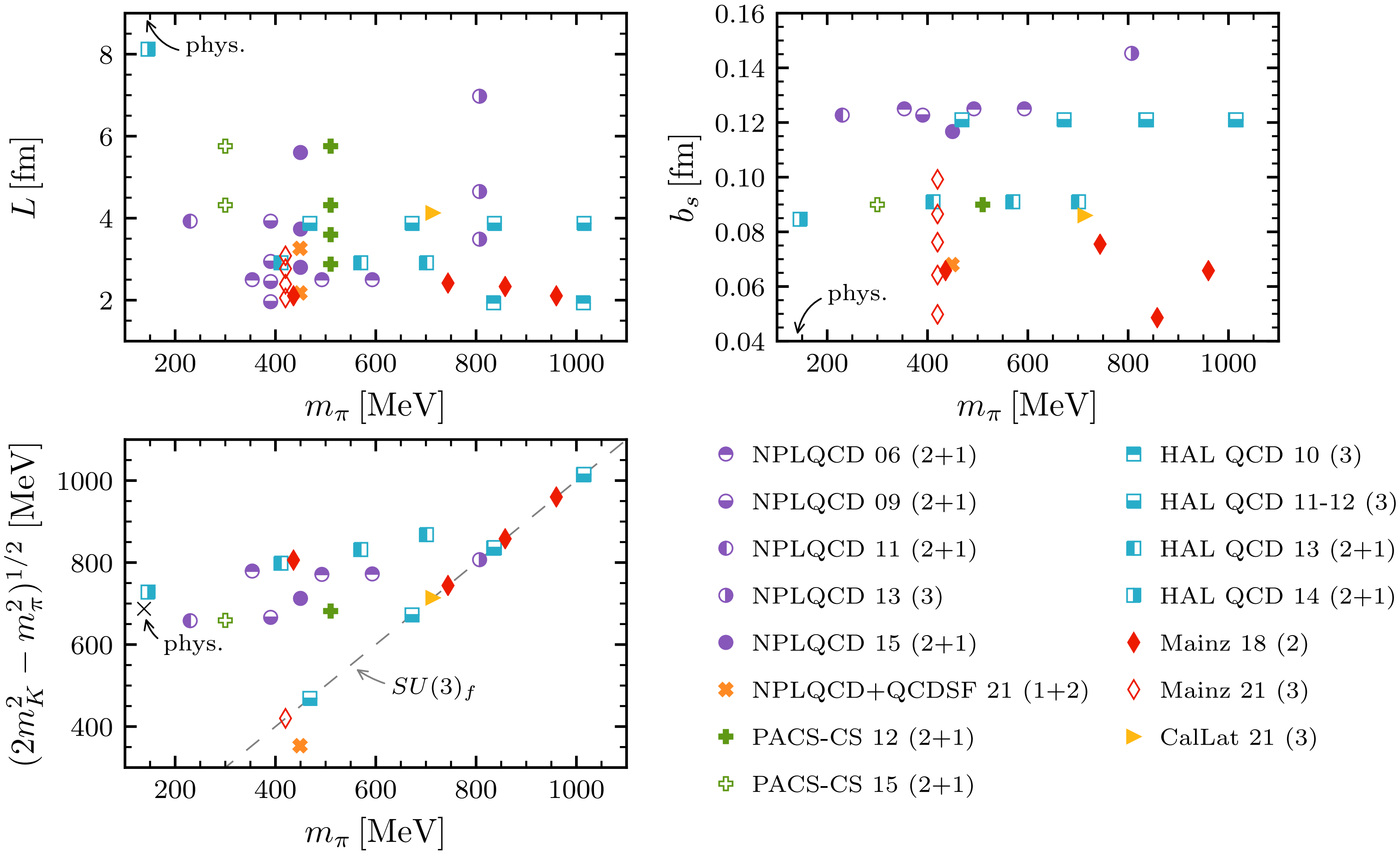
$$\langle \hat{O} \rangle = \frac{1}{Z} \int DU \; \hat{O}\,[Q(U)^{-1}] \, \det(Q(U)) \, e^{-S_g[U]}$$

propagators configurations $(\sim P(U))$

Baryon-Baryon LQCD calculations landscape



$L_x \times L_y \times L_z \times T$

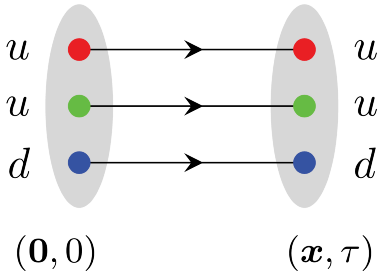


extrapolations to infinite volume,
continous ($b \to 0$)
and physical quark mass values
must be done to connect with Nature

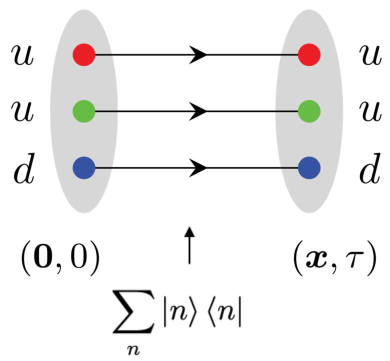
M. Illa, e-Print: 2109.10068 [hep-lat]

Energy levels

$$C_{2pt}(\tau, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\, \Gamma_{\beta\alpha} \langle \mathcal{X}_\alpha(\boldsymbol{x}, \tau) \bar{\mathcal{X}}_\beta(\mathbf{0}, 0) \rangle$$



Energy levels



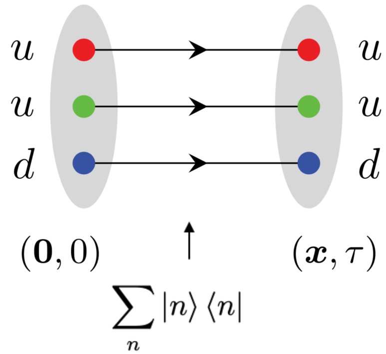
$$C_{2pt}(\tau, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\, \Gamma_{\beta\alpha} \langle \mathcal{X}_\alpha(\boldsymbol{x}, \tau) \bar{\mathcal{X}}_\beta(\mathbf{0}, 0) \rangle$$

$$= Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)} t} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)} t} + \ldots$$

Tower of energy eigenstates
of the system
in the finite volume

LQCD DIRECT METHOD:
FV Energy levels from two-point correlation functions

Energy levels



$$C_{2pt}(\tau, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\, \Gamma_{\beta\alpha} \langle \mathcal{X}_\alpha(\boldsymbol{x}, \tau) \bar{\mathcal{X}}_\beta(\mathbf{0}, 0) \rangle$$

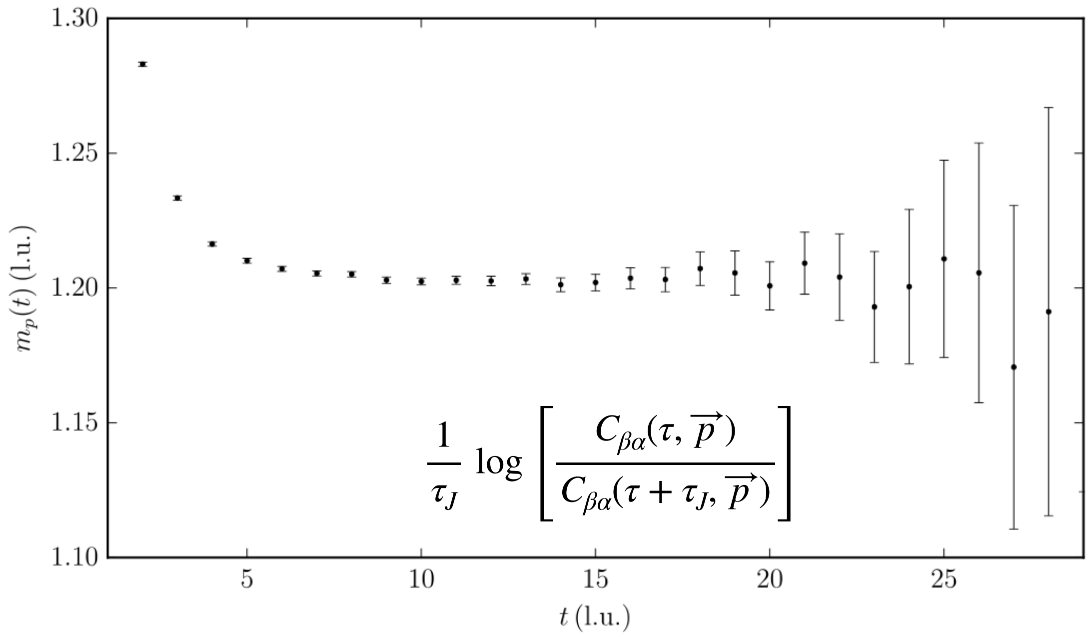
$$= Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)}t} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \ldots$$

dominates at large t

Tower of energy eigenstates of the system in the finite volume

$$E_{\mathfrak{n}}$$

$$p_\alpha(\mathbf{x}, t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x}, t) \left(u^{j\mathsf{T}}(\mathbf{x}, t) C\gamma_5 d^k(\mathbf{x}, t)\right)$$

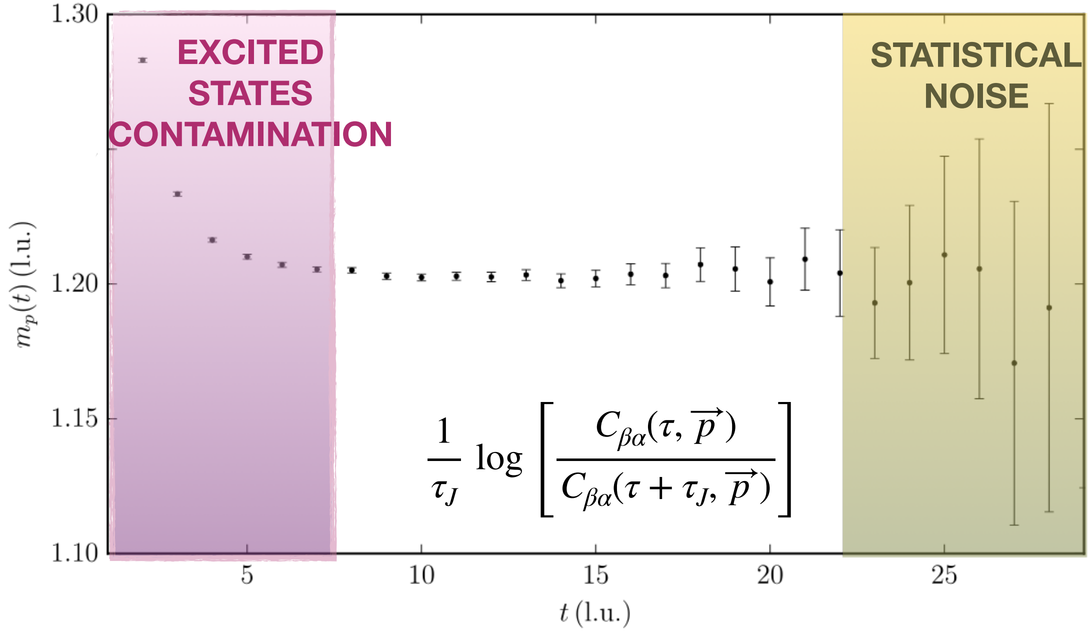


$$C_{2pt}(\tau, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{x}\cdot\boldsymbol{p}}\,\Gamma_{\beta\alpha}\langle \mathcal{X}_\alpha(\boldsymbol{x},\tau)\bar{\mathcal{X}}_\beta(\boldsymbol{0},0)\rangle$$

$$= \underbrace{Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)}t}}_{\text{dominates at large t}} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \dots$$

signal-to-noise degradation

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u_\alpha^i(\mathbf{x},t)\left(u^{j\mathsf{T}}(\mathbf{x},t) C\gamma_5 d^k(\mathbf{x},t)\right)$$



$$C_{2pt}(\tau,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\,\Gamma_{\beta\alpha}\langle \mathcal{X}_\alpha(\boldsymbol{x},\tau)\bar{\mathcal{X}}_\beta(\mathbf{0},0)\rangle$$

$$= \underbrace{Z_0^{snk}\,Z_0^{\dagger src}\,e^{-E^{(0)}t}}_{\text{dominates at large t}} + Z_1^{snk}\,Z_1^{\dagger src}\,e^{-E^{(1)}t} + \ldots$$

more severe degradation for A nucleons

Expectation is that for A nucleons:

$$\frac{\sigma}{\langle C\rangle} \sim \frac{\exp\left[A\left(M_N - \frac{3m_\pi}{2}\right)t\right]}{\sqrt{N}}$$

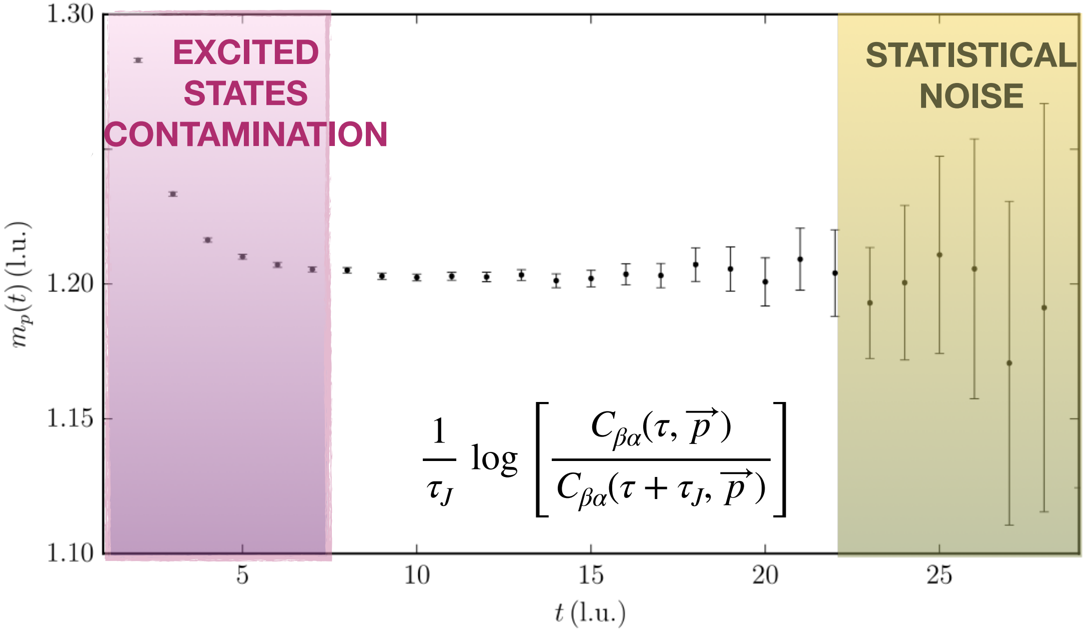
G. Parisi, Phys.Rept. 103 (1984)
G.P. Lepage, Boulder TASI (1989)
M.L. Wagman, M.J. Savage, Phys.Rev.D 96 (2017)

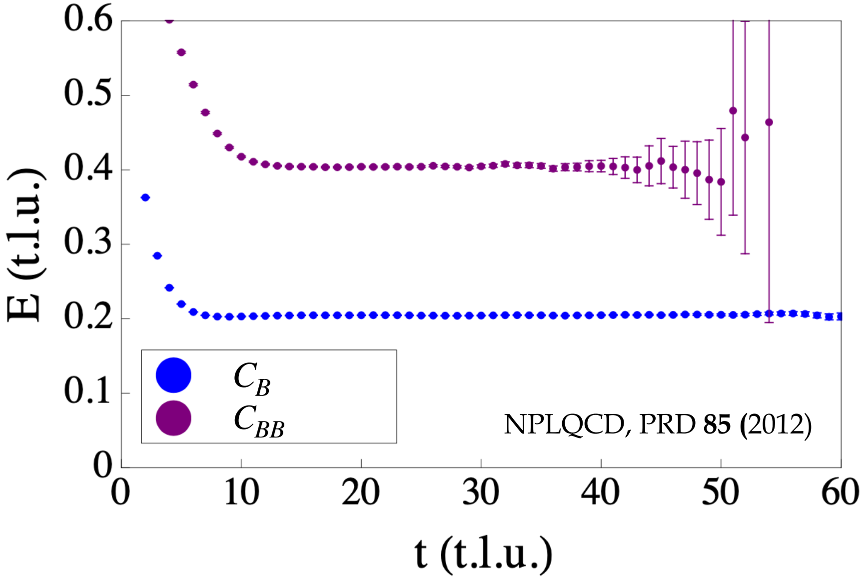
Increase the statistics / Increase the pion mass

Construct operators with a better overlap with the ground state

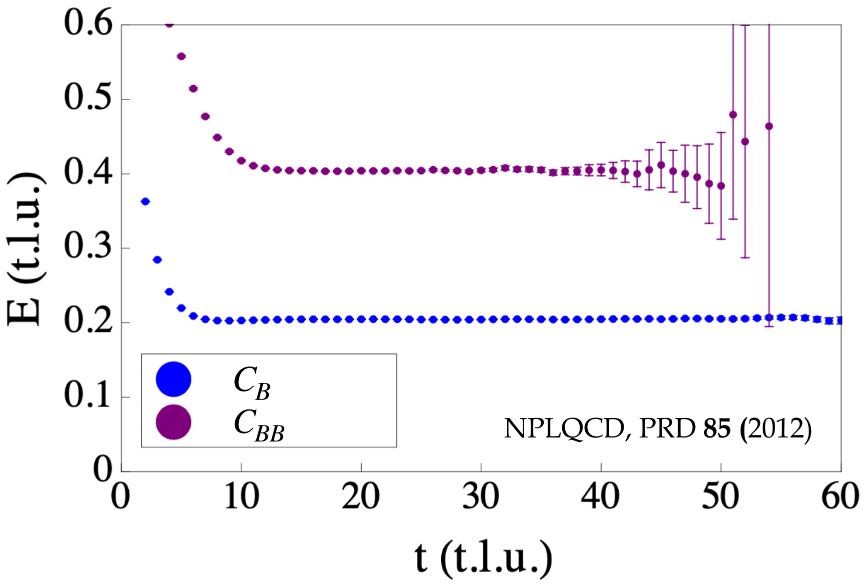
signal-to-noise degradation

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk}u^i_\alpha(\mathbf{x},t)\left(u^{j\mathsf{T}}(\mathbf{x},t)C\gamma_5 d^k(\mathbf{x},t)\right)$$





$$R(\tau, \boldsymbol{p}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{p})}{C_{B_1}(\tau, \boldsymbol{p}) C_{B_2}(\tau, \boldsymbol{p})}$$



$$R(\tau, \boldsymbol{p}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{p})}{C_{B_1}(\tau, \boldsymbol{p}) C_{B_2}(\tau, \boldsymbol{p})}$$

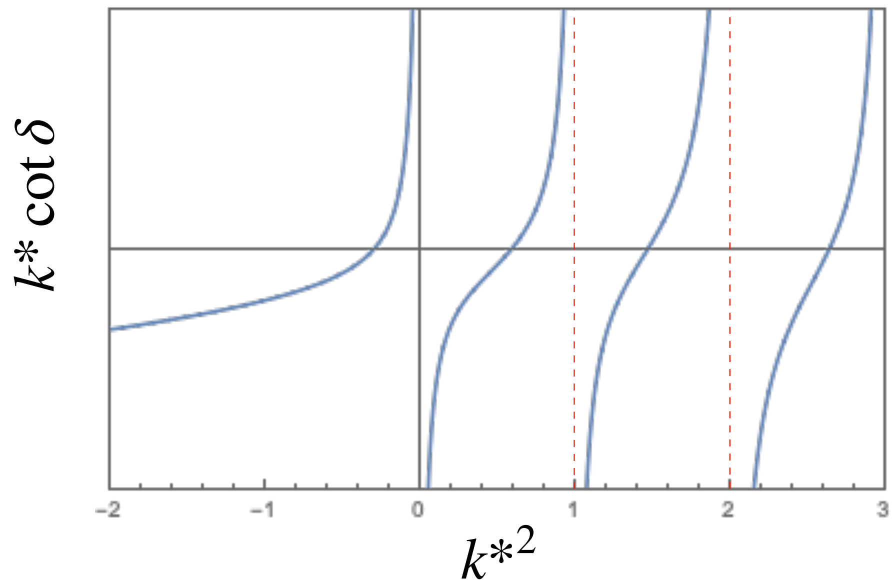
$$\downarrow$$

$$\Delta E_n, \; k^*$$

Lüscher's method $\downarrow$ $\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^V\right] = 0$

$$k^* \cot\delta = \frac{2}{\sqrt{\pi} L} \mathcal{Z}_{00}(1; (\tfrac{k^* L}{2\pi})^2)$$

$$\frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;(\tfrac{k^*L}{2\pi})^2)$$



$$R(\tau,\boldsymbol{p}) = \frac{C_{B_1B_2}(\tau,\boldsymbol{p})}{C_{B_1}(\tau,\boldsymbol{p})C_{B_2}(\tau,\boldsymbol{p})}$$

↓

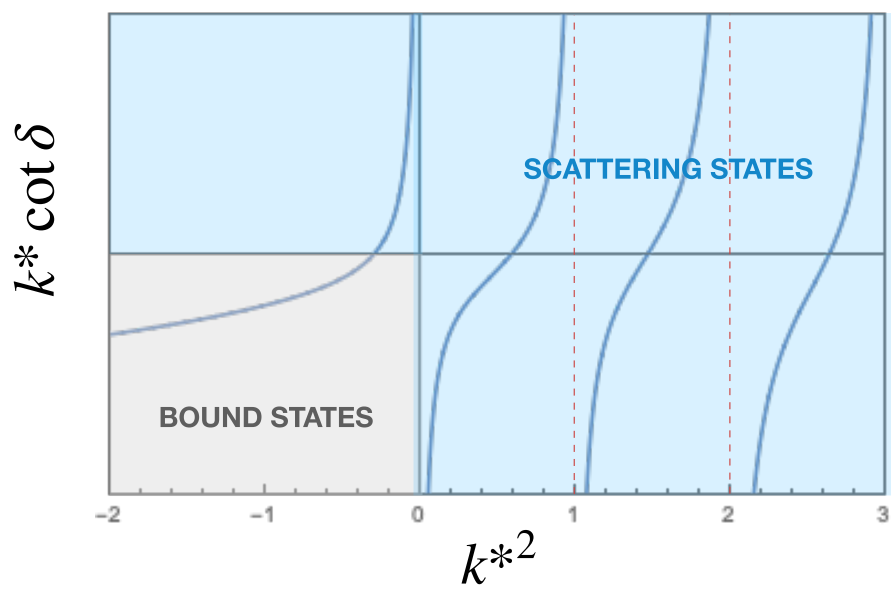
$$\Delta E_n,\ k^*$$

Lüscher's method ↓ $\det\left[(\mathcal{M}^\infty)^{-1} + \delta\mathcal{G}^V\right] = 0$

$$k^*\cot\delta = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;(\tfrac{k^*L}{2\pi})^2)$$

Baryon-Baryon FV Energy levels. Bound states

$$\frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;(\tfrac{k^* L}{2\pi})^2)$$



scattering theory

$$T \sim \frac{1}{k \cot \delta - ik}$$

$\lim_{k\to 0} k \cot \delta < 0$ bound state

$\lim_{k\to 0} k \cot \delta > 0$ unbound state

$$R(\tau, \boldsymbol{p}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{p})}{C_{B_1}(\tau, \boldsymbol{p}) C_{B_2}(\tau, \boldsymbol{p})}$$

↓

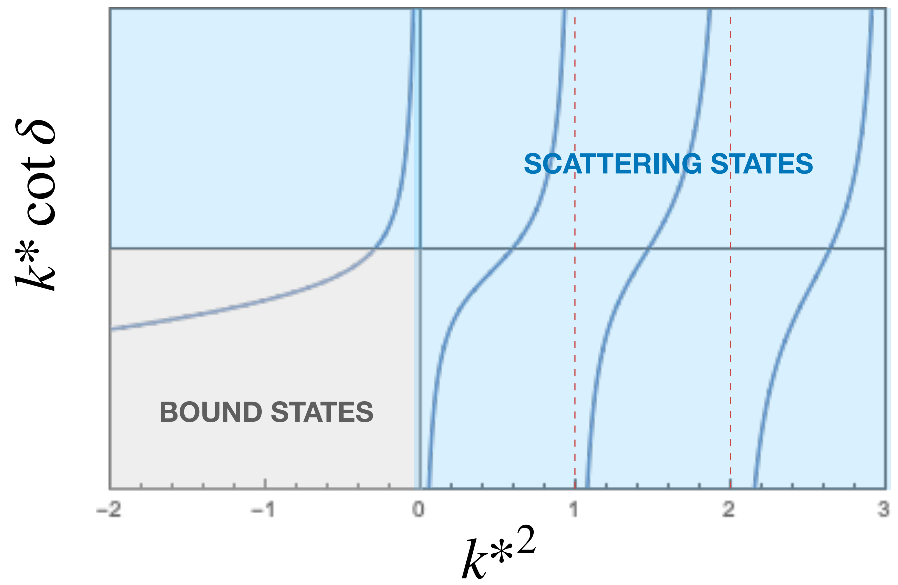
$$\Delta E_n, \ k^*$$

Lüscher's method ↓ $\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^V\right] = 0$

$$k^* \cot \delta = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;(\tfrac{k^* L}{2\pi})^2)$$

Baryon-Baryon FV Energy levels. Bound states

$$\frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;(\tfrac{k^*L}{2\pi})^2)$$



scattering theory

$$T \sim \frac{1}{k\cot\delta - ik}$$

$\lim_{k\to 0} k\cot\delta < 0$ bound state

$\lim_{k\to 0} k\cot\delta > 0$ unbound state

$$R(\tau,\boldsymbol{p}) = \frac{C_{B_1B_2}(\tau,\boldsymbol{p})}{C_{B_1}(\tau,\boldsymbol{p})C_{B_2}(\tau,\boldsymbol{p})}$$

↓

$\Delta E_n,\ k^*$

Lüscher's method ↓ $\det\left[(\mathcal{M}^\infty)^{-1} + \delta\mathcal{G}^V\right] = 0$

$$k^*\cot\delta = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;(\tfrac{k^*L}{2\pi})^2)$$

↓ $k^{*2} < 0$

$$|k^*| = \kappa^{(\infty)} + \frac{Z^2}{L}\left[6e^{-\kappa^{(\infty)}L} + \cdots\right]$$

B

Beane, Bedaque, Parreño, Savage, PLB585 (2004)
Davoudi, Savage, PRD84 (2011)

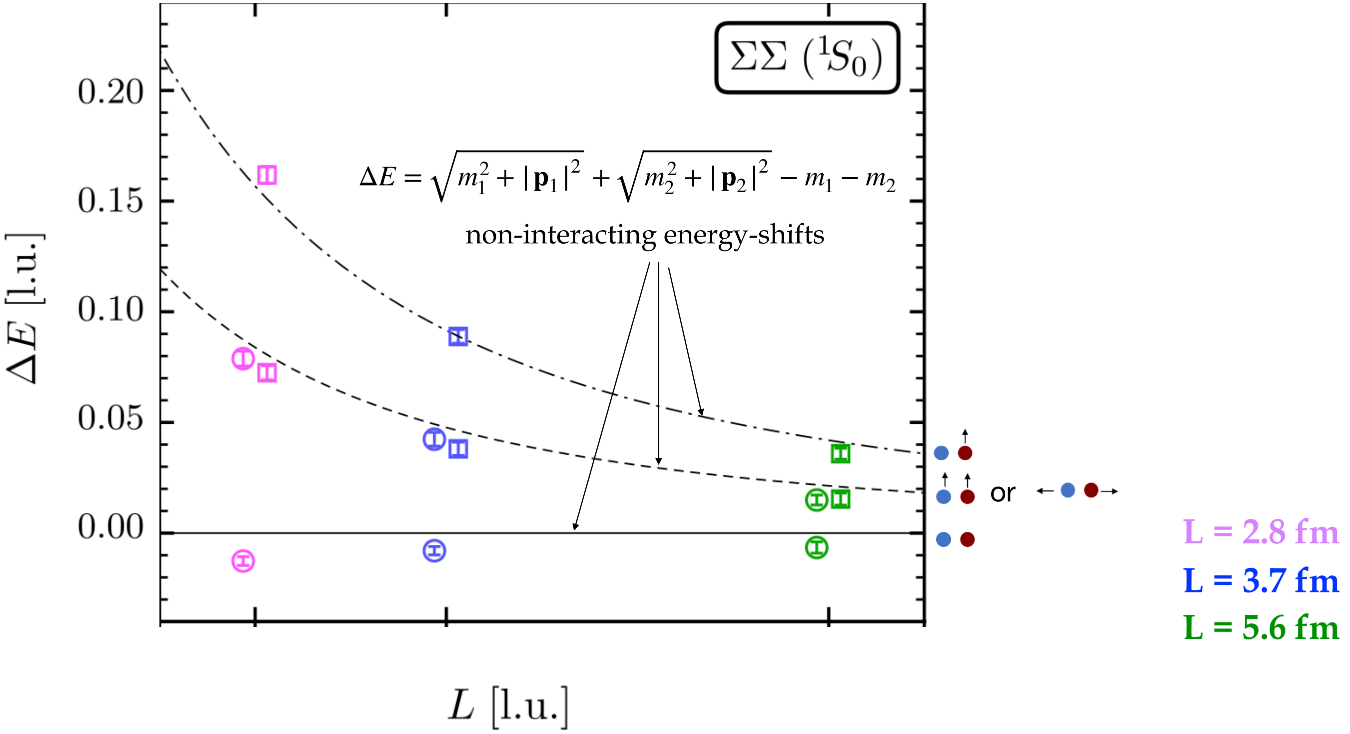
$m_\pi \sim 450$ MeV

Calculation with systems that are boosted and with back-to-back momenta

$|\mathbf{p}_1|^2 = |\mathbf{p}_2|^2 = 0$

$|\mathbf{p}_1|^2 = |\mathbf{p}_2|^2 = \left(\frac{2\pi}{L}\right)^2$

$|\mathbf{p}_1|^2 = 0,\ |\mathbf{p}_2|^2 = \left(\frac{4\pi}{L}\right)^2$



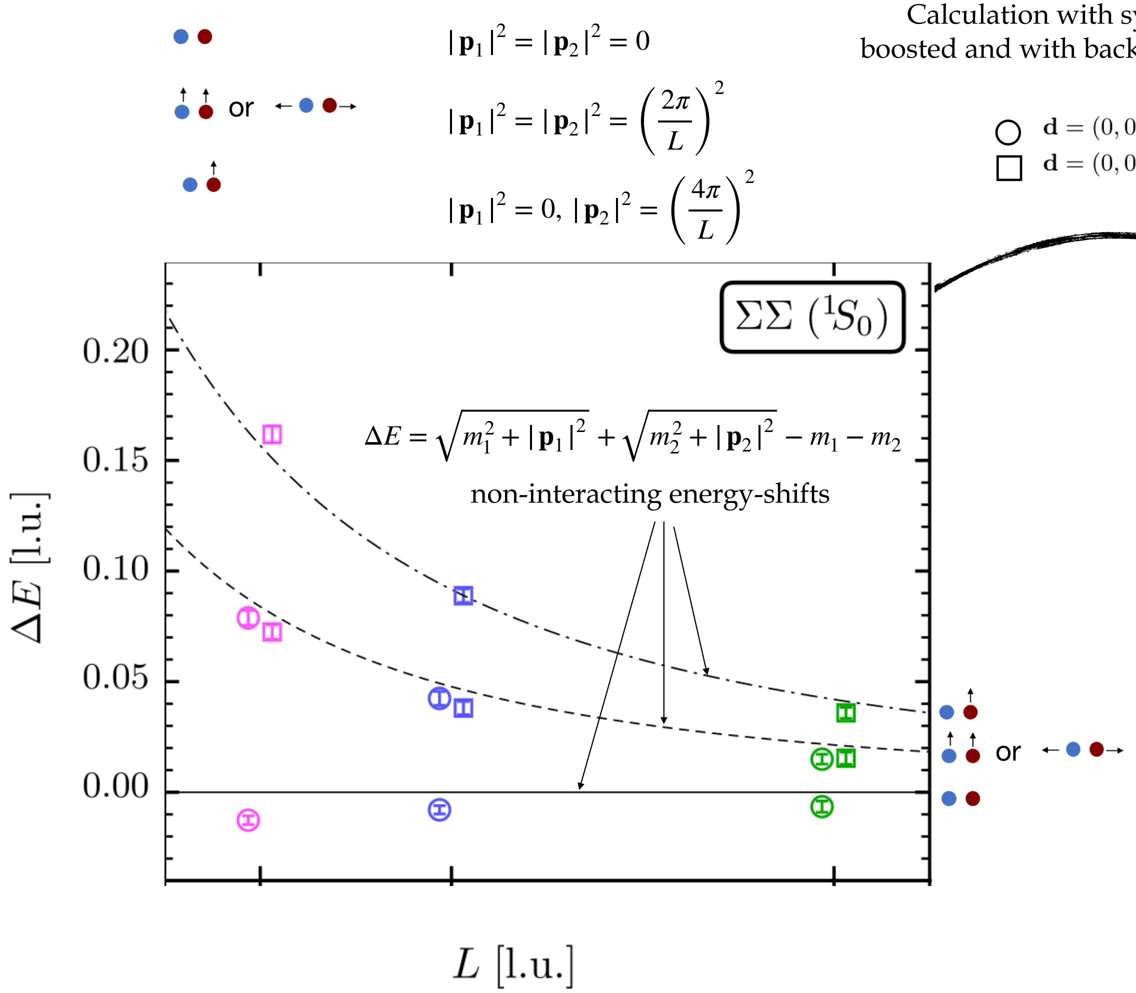
$m_\pi \sim 450$ MeV

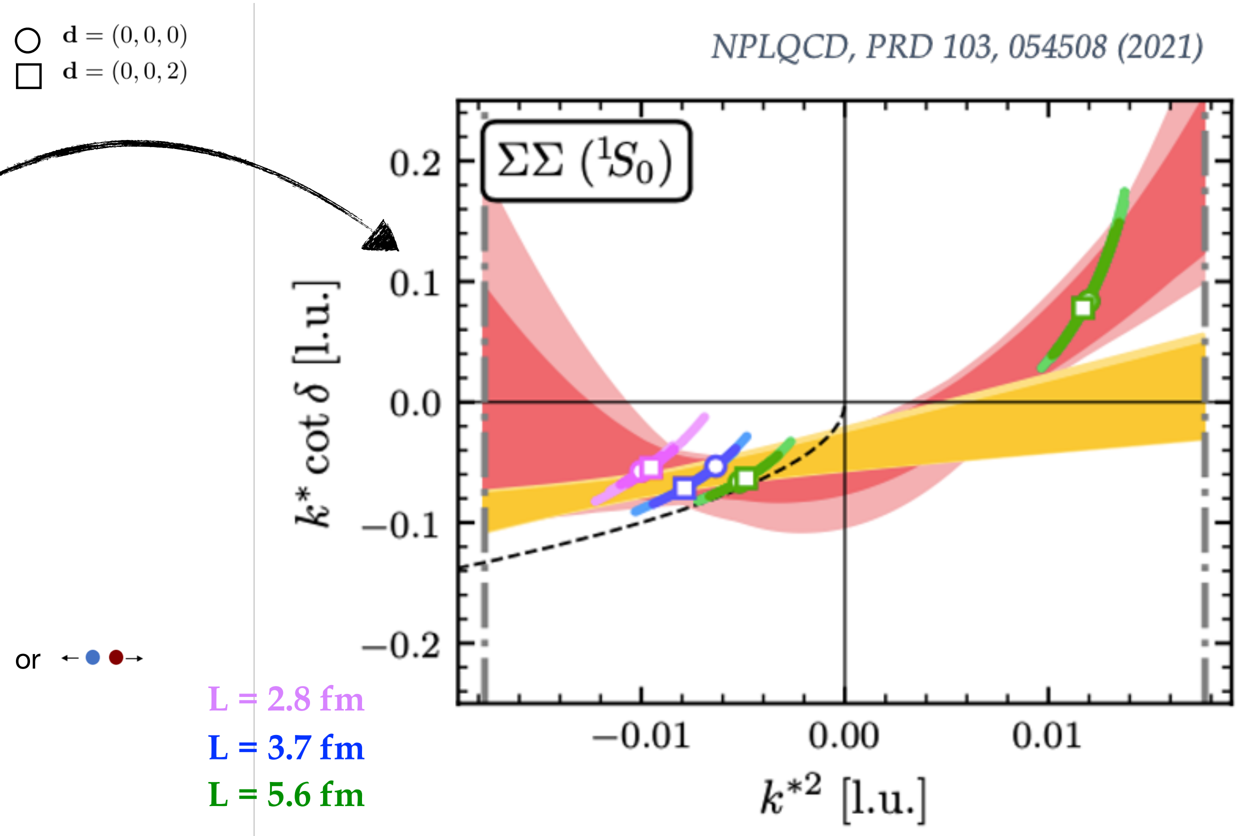
Calculation with systems that are boosted and with back-to-back momenta

$|\mathbf{p}_1|^2 = |\mathbf{p}_2|^2 = 0$

$|\mathbf{p}_1|^2 = |\mathbf{p}_2|^2 = \left(\frac{2\pi}{L}\right)^2$

$|\mathbf{p}_1|^2 = 0,\ |\mathbf{p}_2|^2 = \left(\frac{4\pi}{L}\right)^2$





$$k^* \cot\delta = -\frac{1}{a} + \frac{1}{2} r k^{*2} + P k^{*4} + \mathcal{O}(k^{*6})$$

L [fm]	T [fm]
3.4	6.7
4.5	6.7
6.7	9

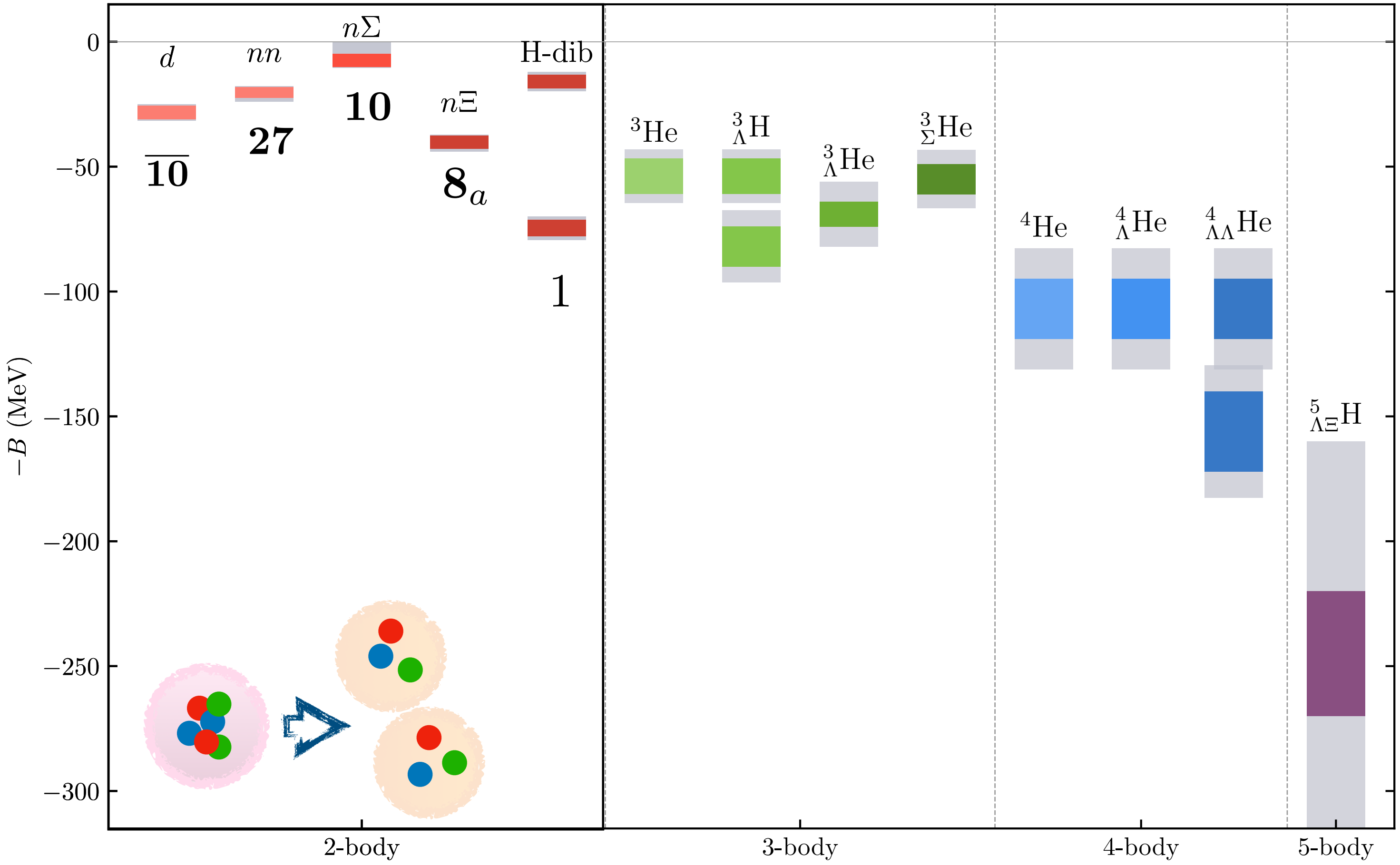
$b[fm] = 0.1453(16)$

$SU(3)_f$

$m_\pi \sim 800$ MeV

no e.m. interactions

NPLQCD, Phys.Rev. D87 (2013) no.3, 034506;



Updated in PRD96 (2017) 114510

L [fm]	T [fm]
3.4	6.7
4.5	6.7
6.7	9

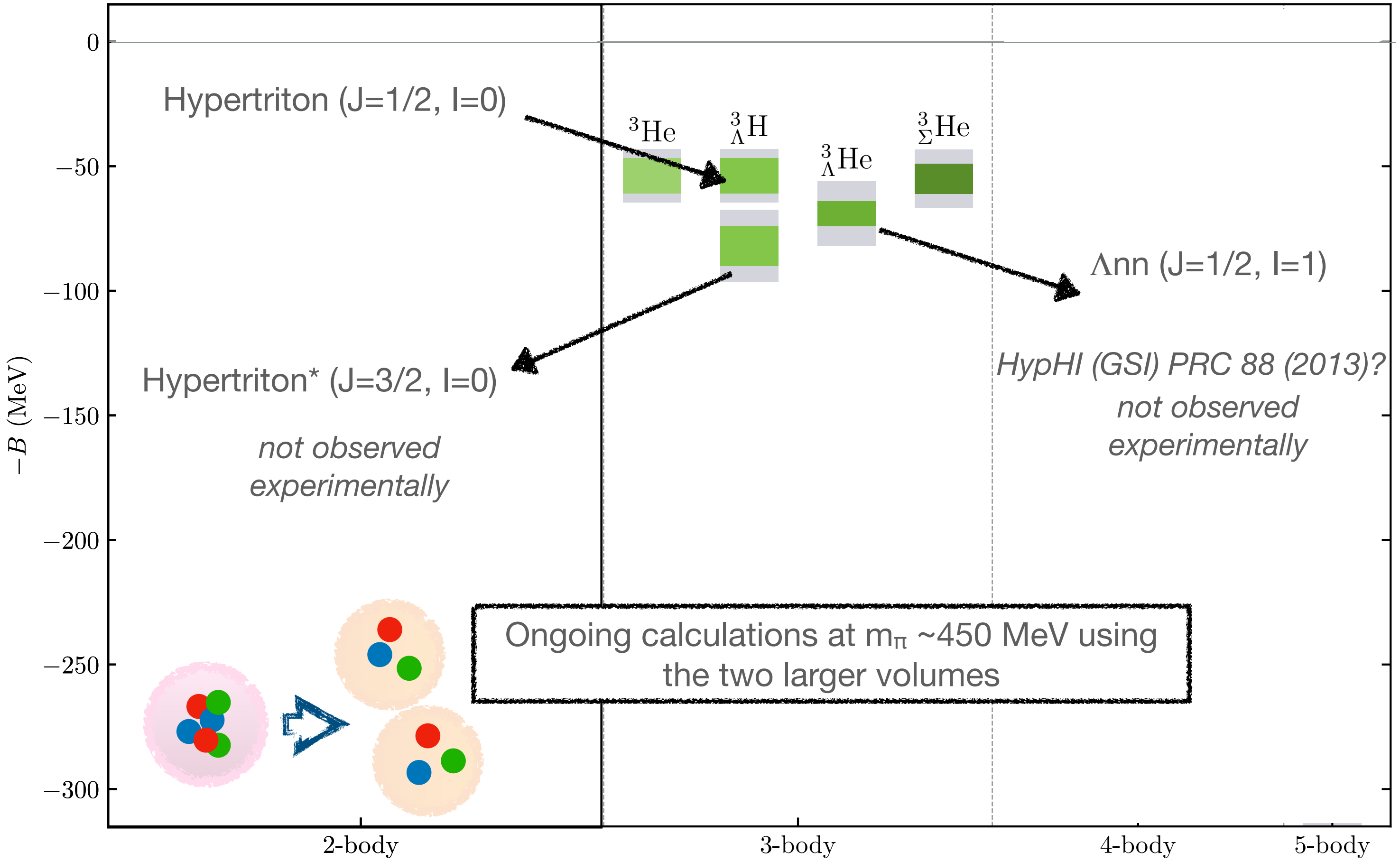
$b[fm] = 0.1453(16)$

$SU(3)_f$

$m_\pi \sim 800$ MeV

no e.m. interactions

NPLQCD, Phys.Rev. D87 (2013) no.3, 034506;



away from the $SU(3)_f$ limit

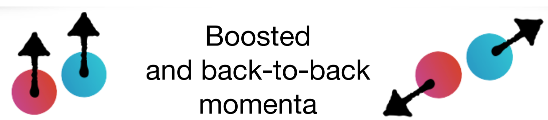
$n_f = 2 + 1$

$m_\pi = 450(5)\text{MeV}$

$b = 0.117(2)$ fm

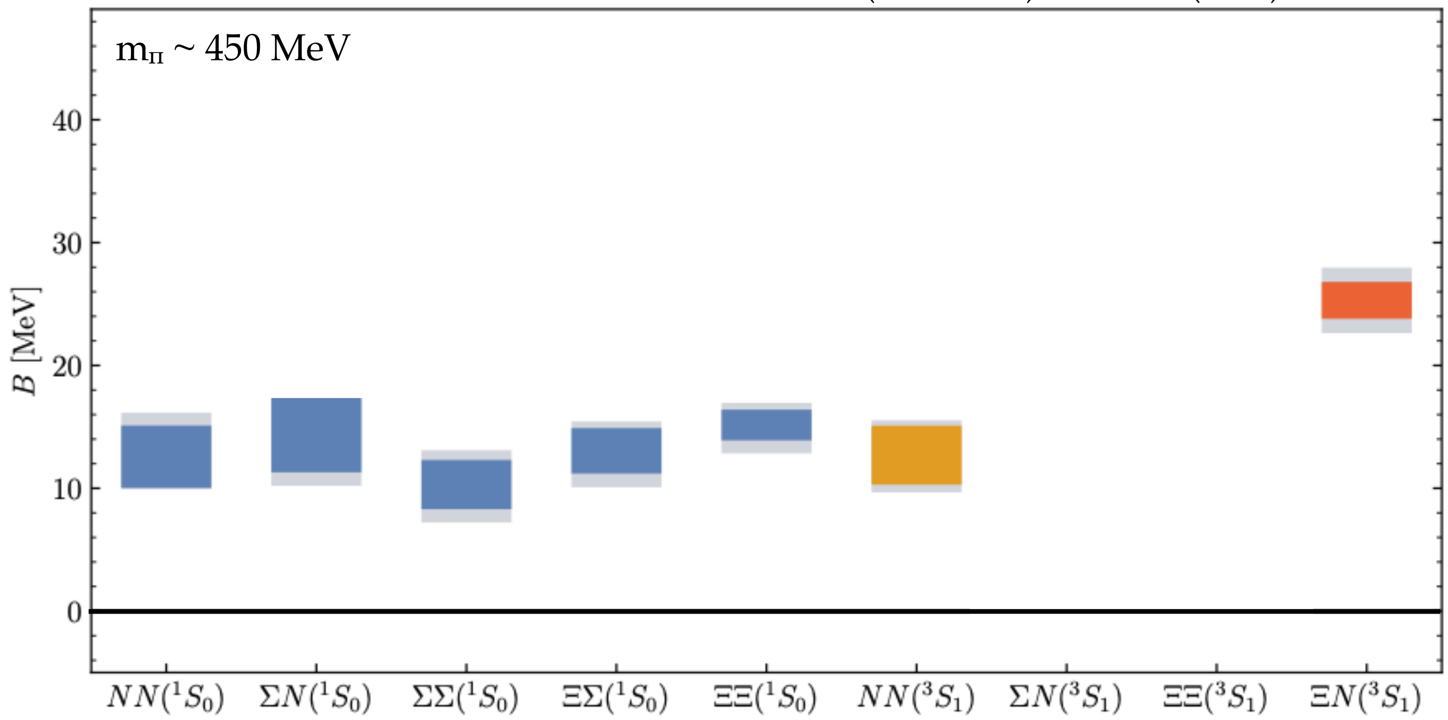
$L = 2.8, 3.7, 5.6$ fm

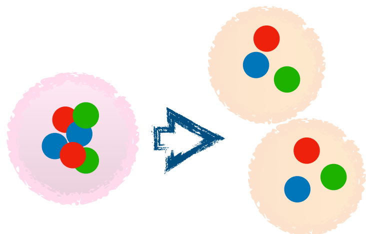
$T = 7.5, 11.2, 11.2$ fm



no e.m. interactions

Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508





away from the SU(3)$_f$ limit

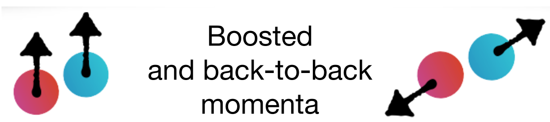
$n_f = 2 + 1$

$m_\pi = 450(5)\text{MeV}$

$b = 0.117(2)$ fm

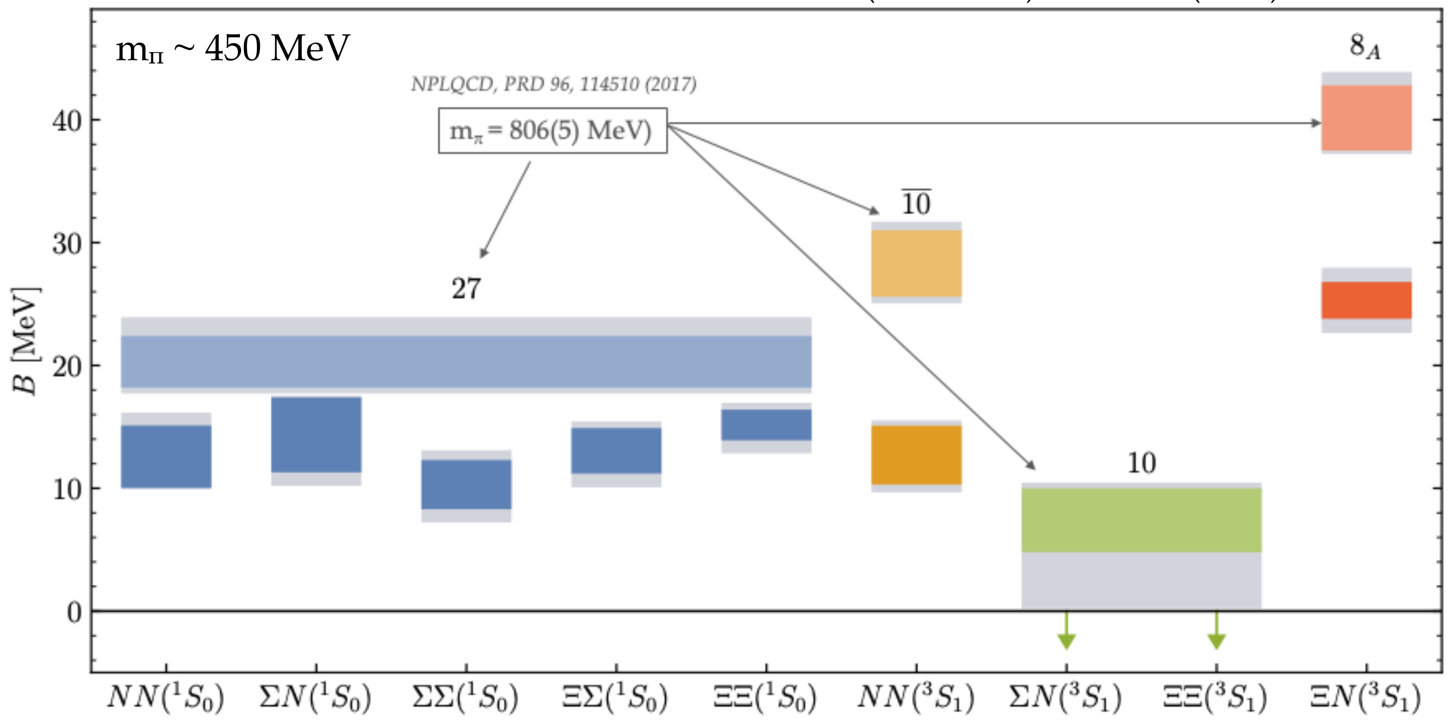
$L = 2.8, 3.7, 5.6$ fm

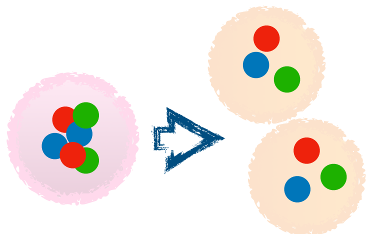
$T = 7.5, 11.2, 11.2$ fm



no e.m. interactions

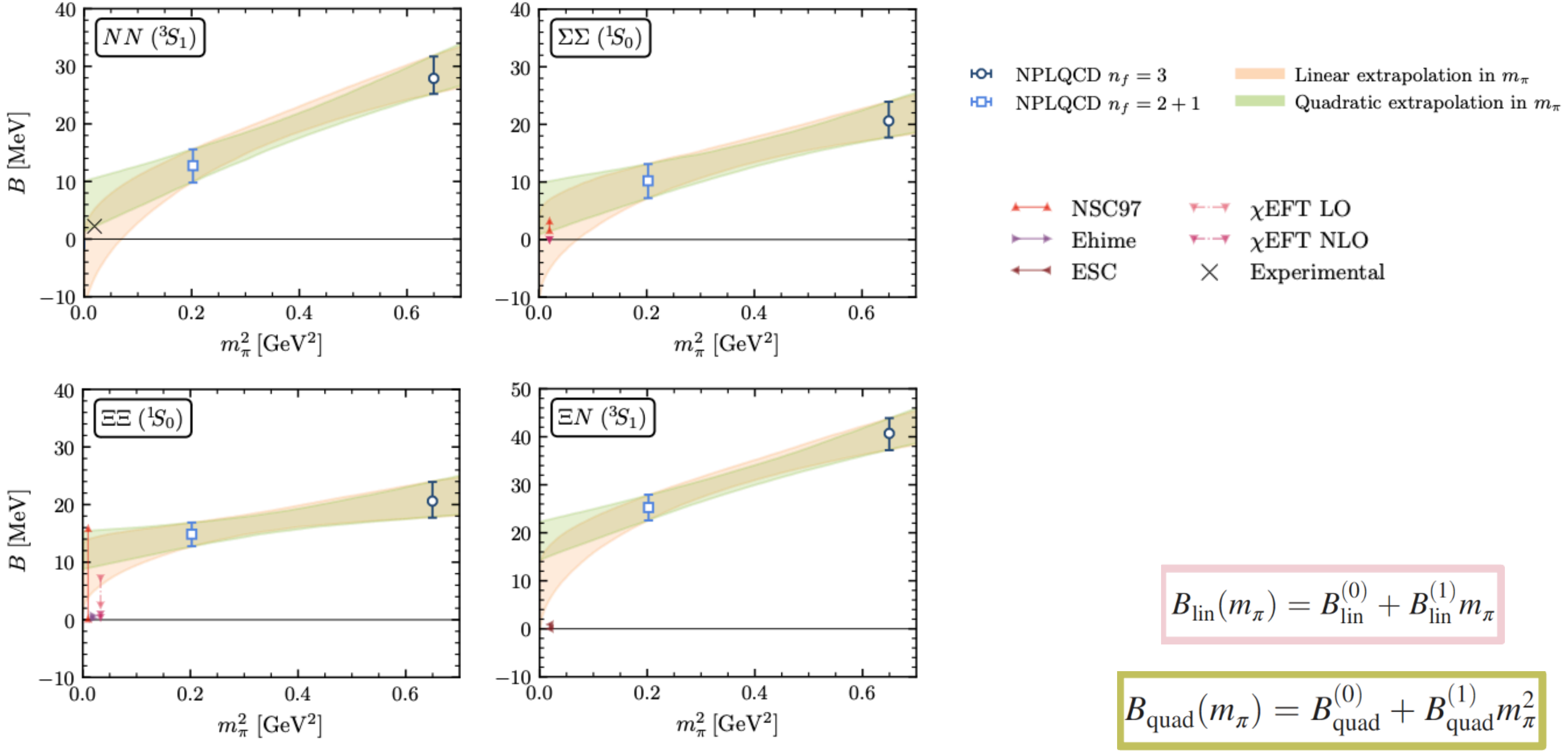
Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508





BB systems, quark mass extrapolations

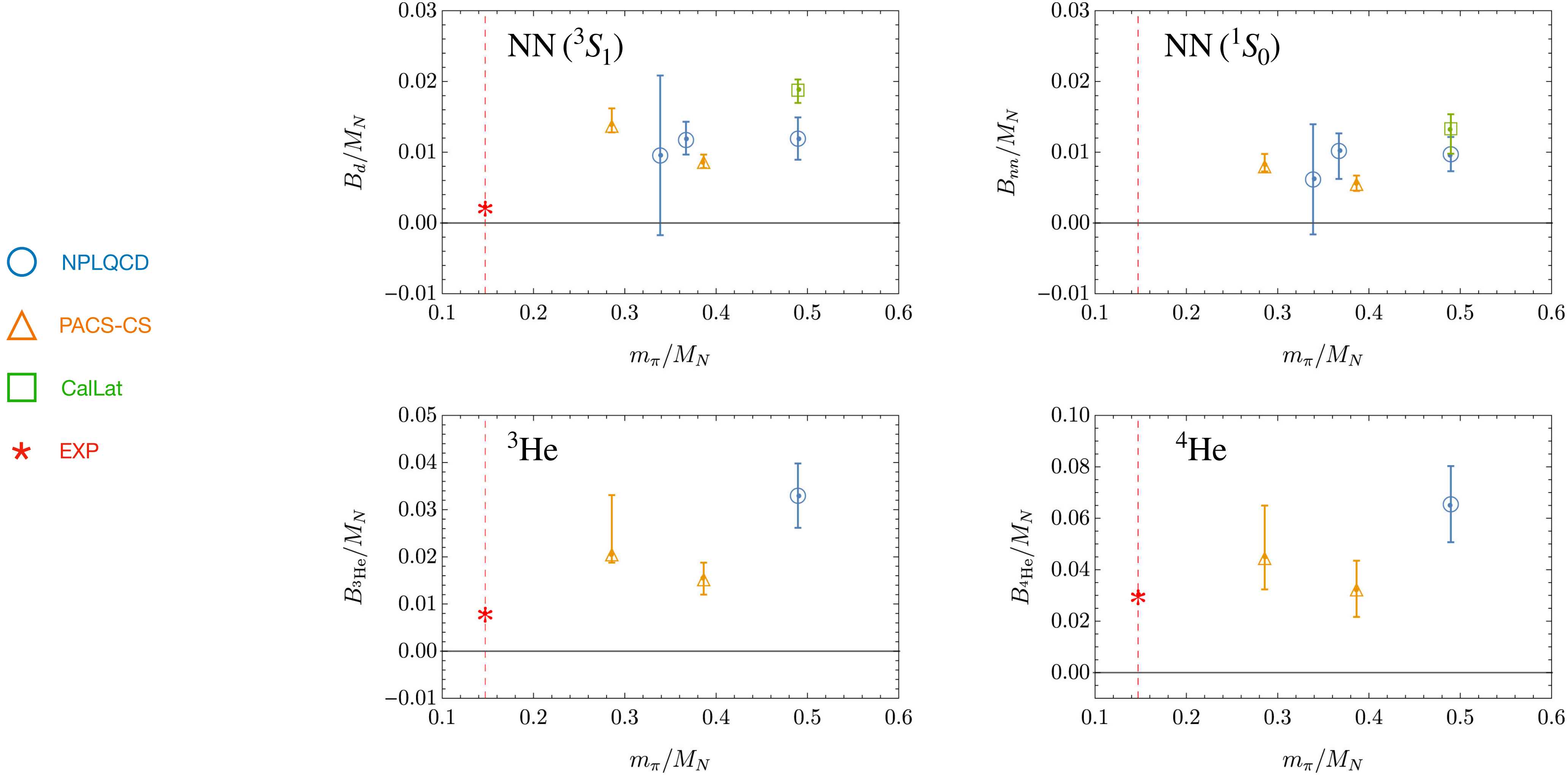
Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508



$$B_{\text{lin}}(m_\pi) = B_{\text{lin}}^{(0)} + B_{\text{lin}}^{(1)} m_\pi$$

$$B_{\text{quad}}(m_\pi) = B_{\text{quad}}^{(0)} + B_{\text{quad}}^{(1)} m_\pi^2$$

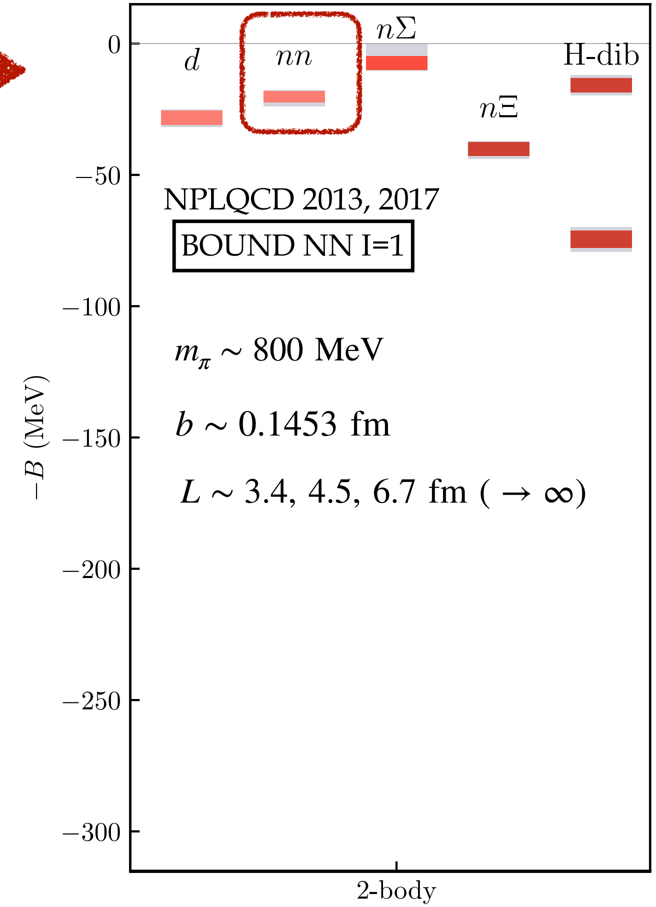
Davoudi, Detmold, Shanahan, Orginos, Parreño, Savage, Wagman, Physics Reports 900 (2021) 1–74



Misidentification of the plateau?

Deep bound states

E. Berkowitz et al. [CalLat], Phys.Lett.B 765 (2017)
S.R. Beane et al. [NPLQCD], arXiv:1705.09239 [hep-lat]
T. Yamazaki et al. [PACS], EPJ Web Conf. 175 (2018)



Small excited-state gaps may lead to incorrect identification of the ground-state energy

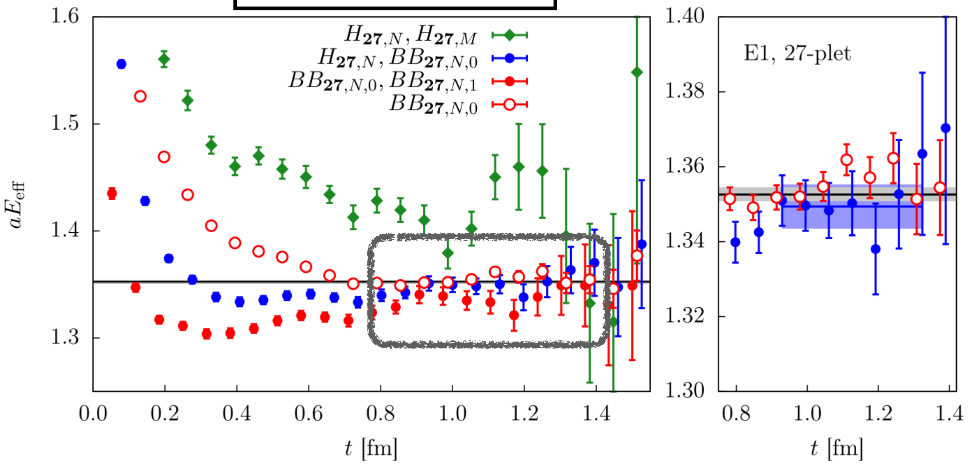
- Is the fitting interval correctly identified?
- Are we missing excited state contributions?
- Is there an operator dependence on the energy levels extracted?

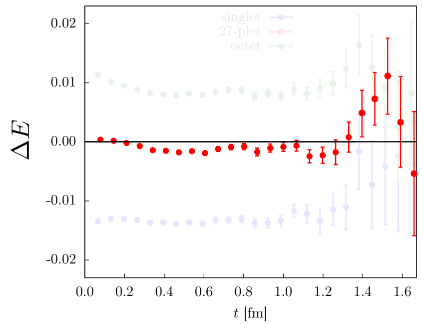
Reduce uncertainty at small time: GPoF, matrix Prony, variational

First variational calculation A. Francis et al., Phys.Rev.D 99 (2019)

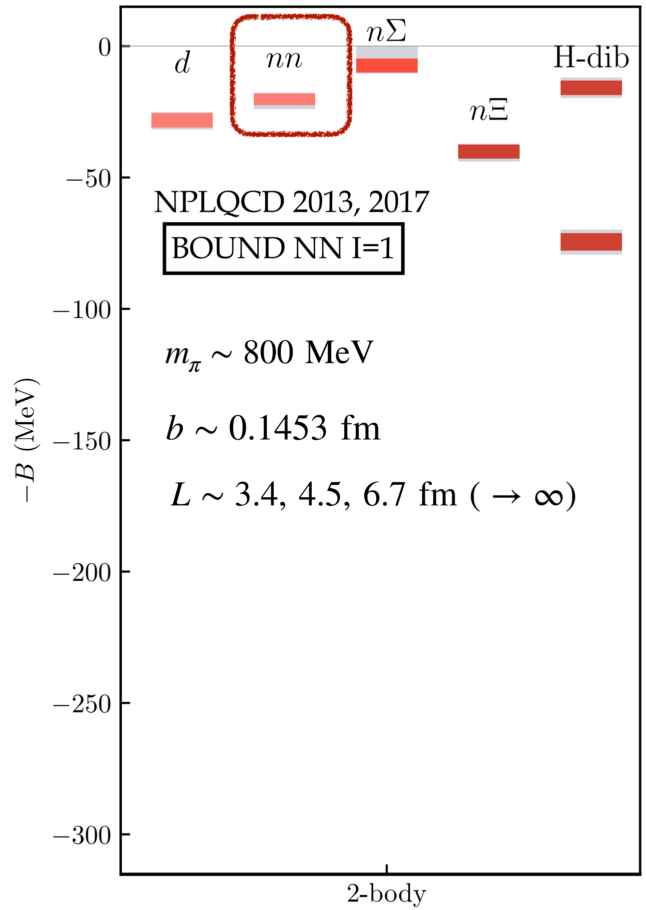
$m_\pi \sim 960$ MeV

UNBOUND NN I=1

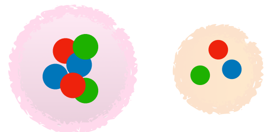




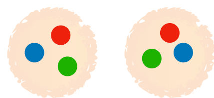
Ground-state energy close to threshold



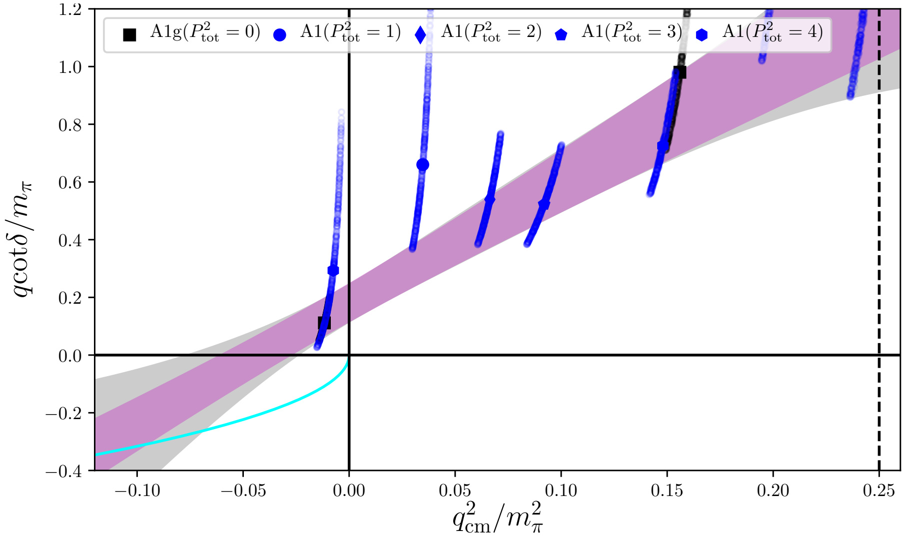
Hermitian 2x2 matrix with hexaquark and dibaryon-like operators



Non-hermitian 2x2 matrix with dibaryon-like operators



CalLat B. Hörz et al., Phys.Rev.C 103 (2021)



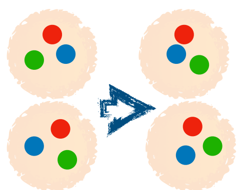
$m_\pi \sim 714$ MeV

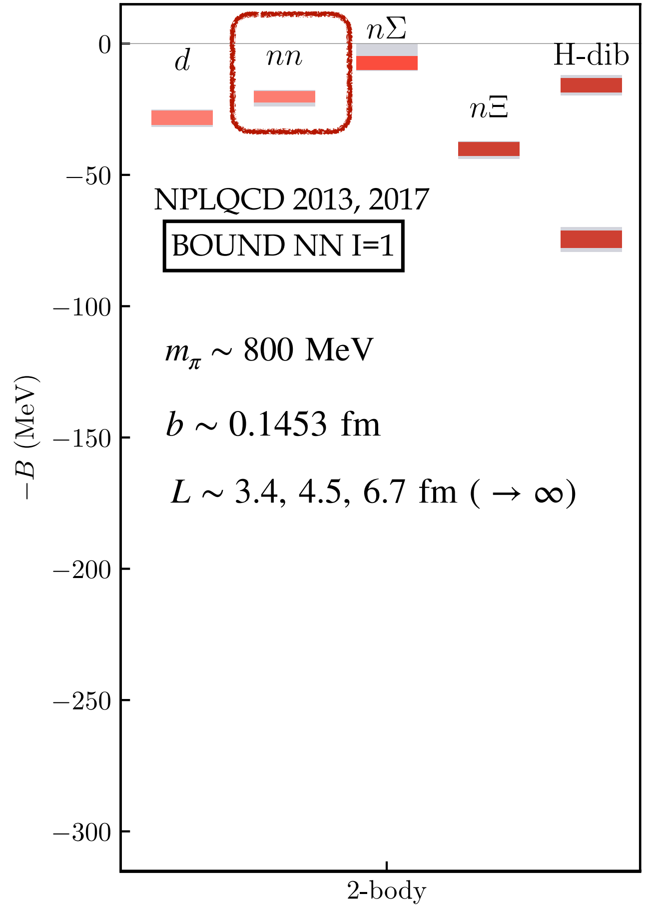
UNBOUND NN I=1

$b \sim 0.086$ fm

$L = 48\,b \sim 4.1$ fm

Hermitian 2x2 matrix with dibaryon-like operators





NPLQCD 2013, 2017

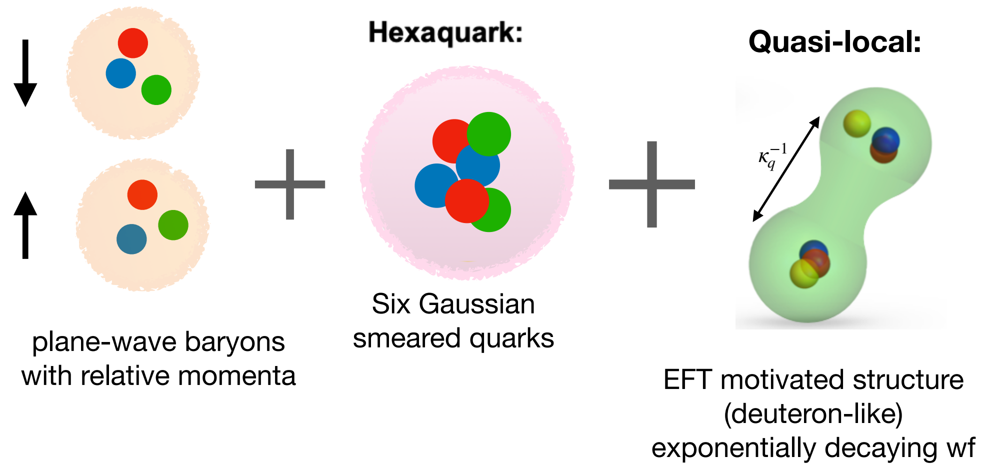
BOUND NN I=1

$m_\pi \sim 800$ MeV

$b \sim 0.1453$ fm

$L \sim 3.4,\ 4.5,\ 6.7$ fm ($\to \infty$)

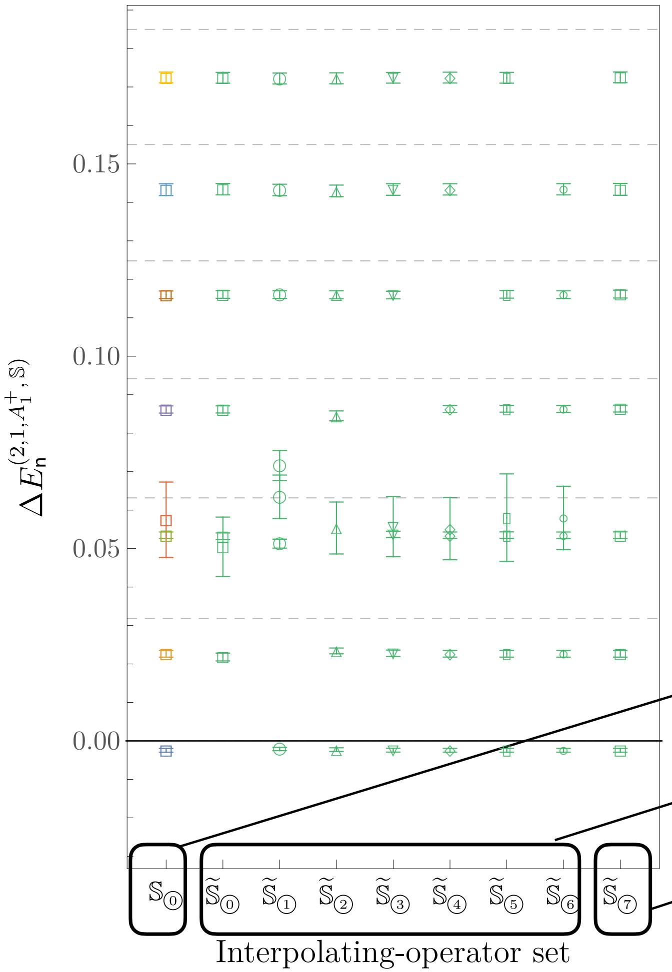
S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



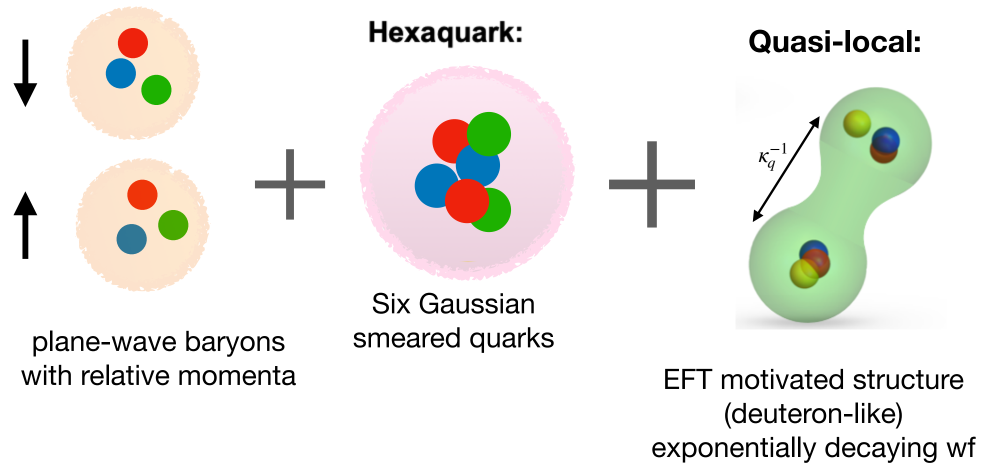
Largest set of operators to date
(ongoing work)

b=0.145 fm , L/b=32 (4.7 fm aprox)

NN (I=1)



S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



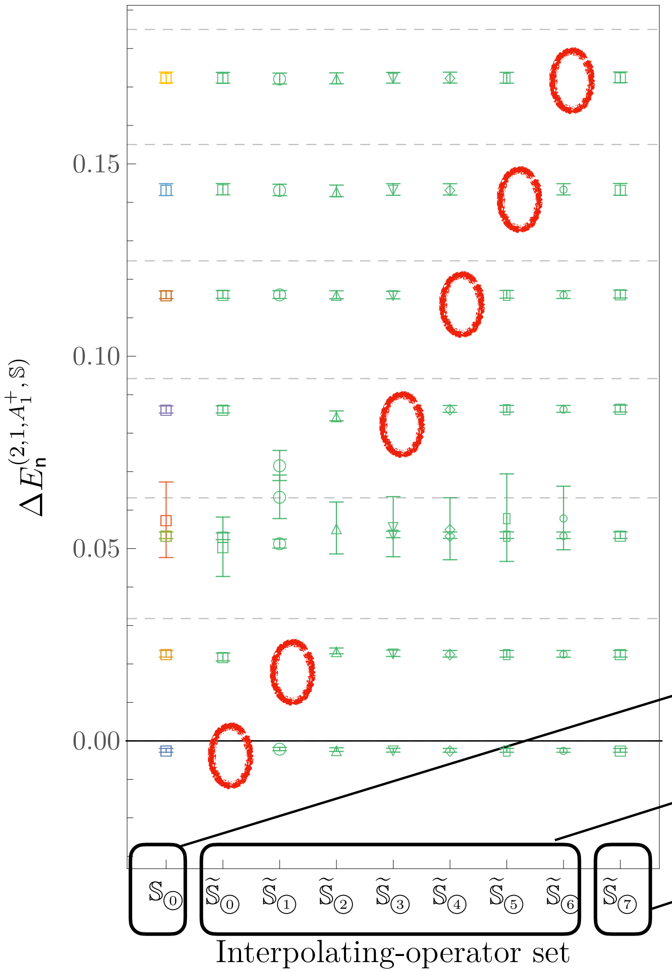
S_0 contains all operators except the quasi-locals (hexaquark and dibaryons ops with different relative momentum)

Set without a particular dibaryon operator (taking out a dibaryon op with a given value of the relative momentum)

Set with only the whole set of dibaryon operators (NO hexaquark)

S. Amarasinghe et al (NPLQCD), arXiv:2108.10835

NN (I=1)



Similarly with what happens in the meson sector, removing the operator structure with maximum overlap on to a given energy level leads to missing energy levels

Importance of using an interpolating-operator set with significant overlap onto all energy levels of interest.

Having a large interpolating-operator set is not sufficient to guarantee that a set will have good overlap onto the ground state or a particular excited state

NEEDS MORE WORK

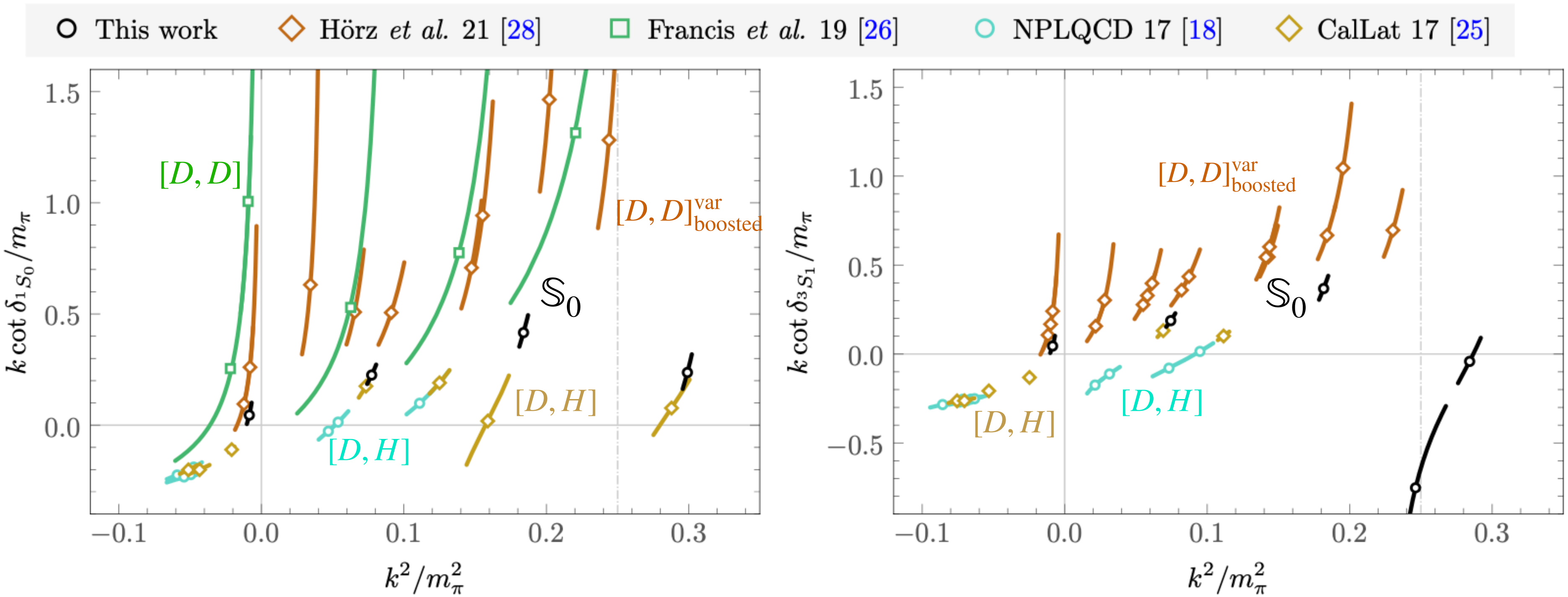
S_0 contains all operators except the quasi-locals
(hexaquark and dibaryons ops with different relative momentum)

Set without a particular dibaryon operator
(taking out a dibaryon op with a given value of the relative momentum)

Set with only the whole set of dibaryon operators

S. Amarasinghe et al (NPLQCD), arXiv:2108.10835

I = 1 (left) and I = 0 (right) 2N S-wave phase shifts

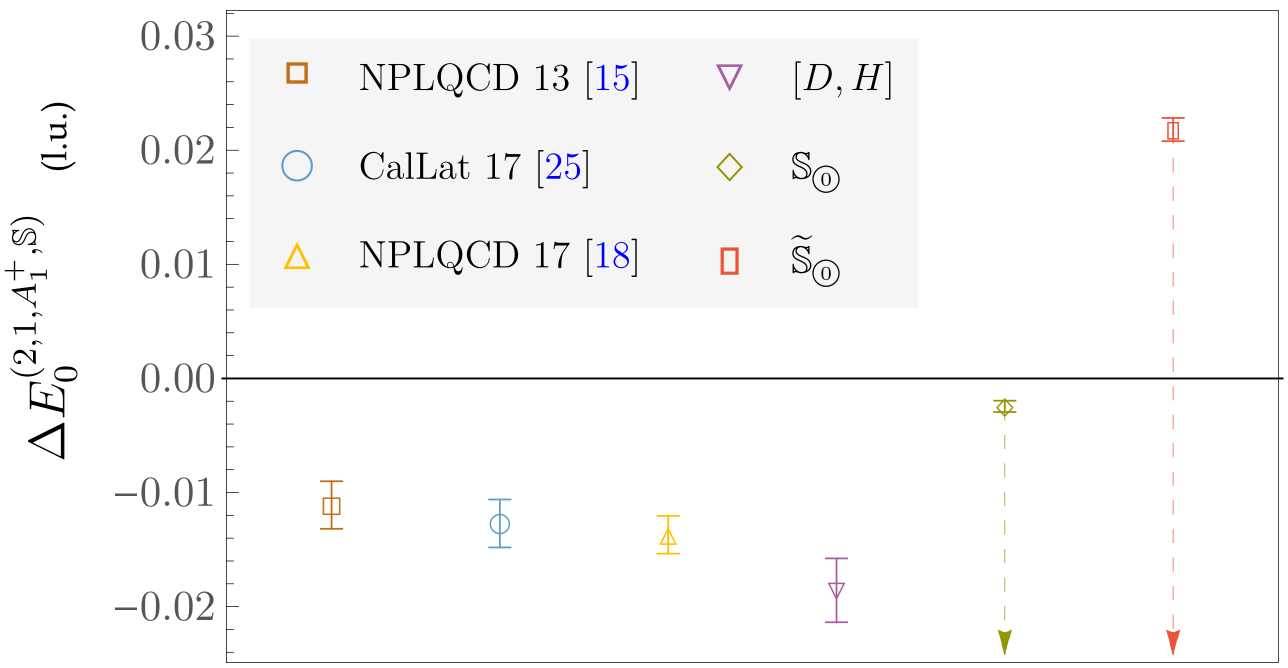


Consistent results when using similar interpolating operators

Discrepancies between asymmetric correlation functions (local hexaquark source, plane-wave dibaryon sink) and recent variational studies.

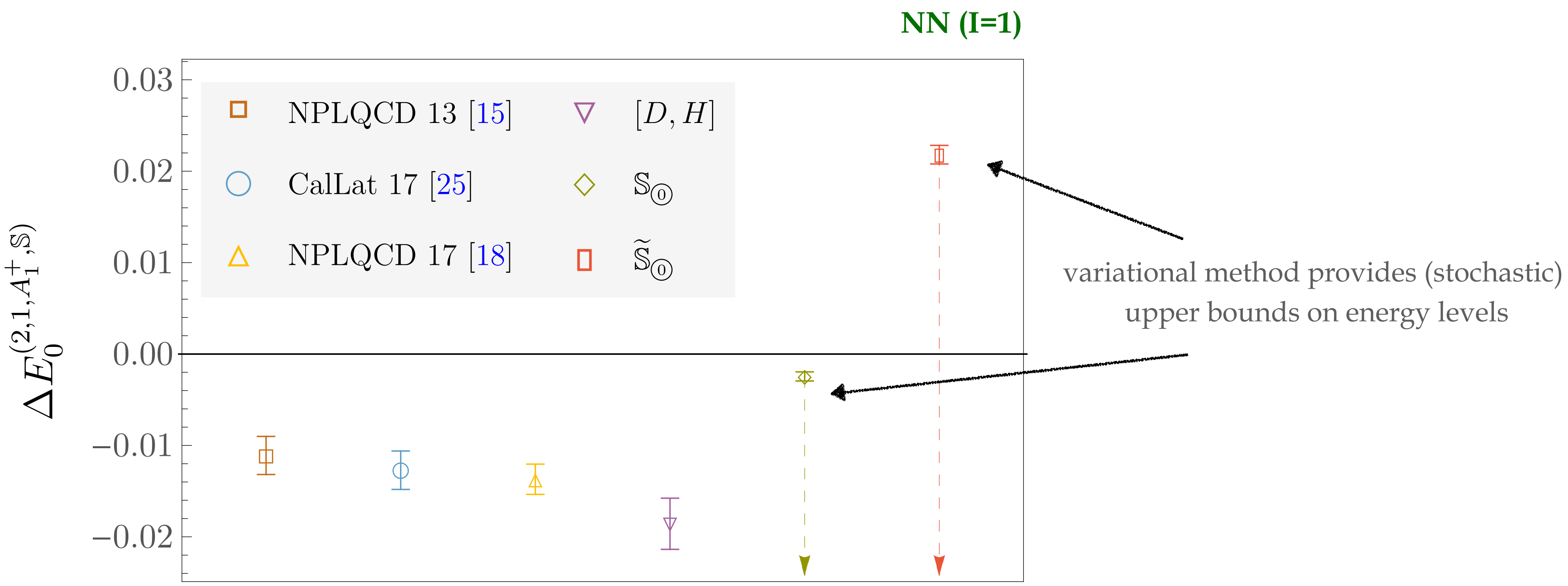
S. Amarasinghe et al (NPLQCD), arXiv:2108.10835

NN (I=1)



calculations made on the same set of configurations

S. Amarasinghe et al (NPLQCD), arXiv:2108.10835

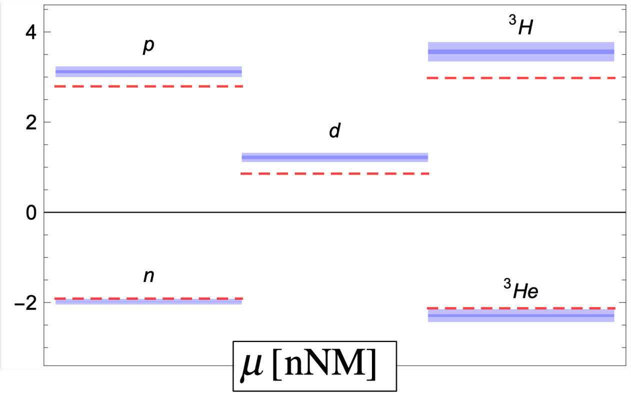


calculations made on the same set of configurations

Successful reproduction of properties of light nuclei direcly from QCD

Nuclear magnetic moments

NPLQCD, Phys. Rev .Lett. 113 (2014)



$$\text{nNM} = \frac{e}{2M_N^{\text{latt}}} = \frac{e}{2M_N(m_\pi^{\text{latt}})}$$

LQCD @ $m_\pi \sim 800$ MeV

experiment

Shell-model predictions

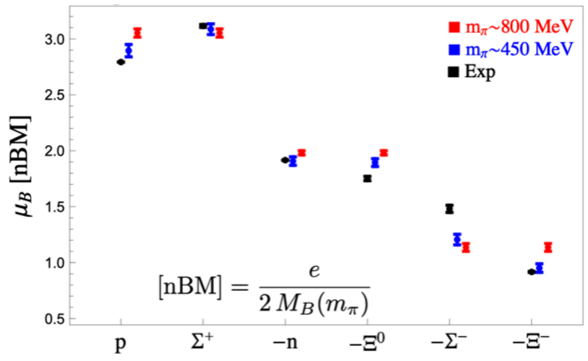
$\mu(^3H) = \mu_p$

$\mu(^3He) = \mu_n$

$\mu_d = \mu_n + \mu_p$

Octet baryon magnetic moments

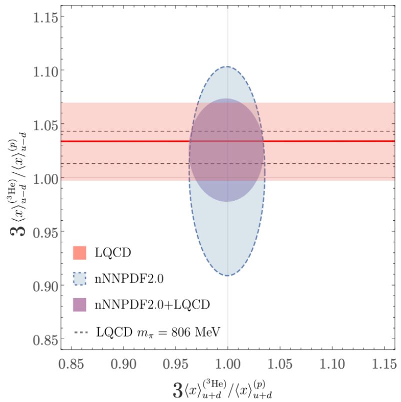
NPLQCD, PRD95, 114513 (2017)



$(\mu_{-B} \equiv -\mu_B)$

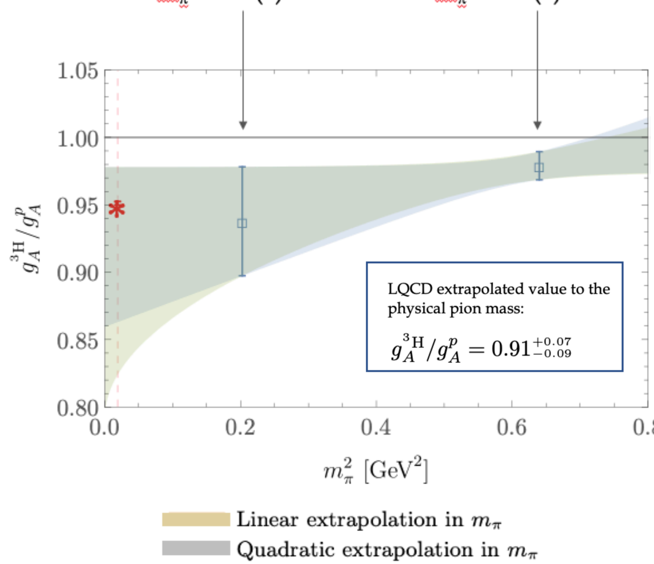
Momentum fraction of ^{3}*He* , $\langle x \rangle_q = \langle h | \bar{q} \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} q | h \rangle$

NPLQCD, PRL 126 (2021)



Triton axial charge, $g_A = \langle h | \bar{q} \gamma_3 \gamma_5 q | h \rangle$

NPLQCD, PRD 103 (2021)



In agreement with the phenomenological result:

$$g_A^{^3H}/g_A^p = 0.9511(13)$$

Baroni, Girlanda, Kievsky, Marcucci, Schiavilla, Viviani, Tritium β-decay in chiral effective field theory, Phys. Rev. C 94, 024003 (2016); Erratum, Phys. Rev. C 95, 059902 (2017).

- During the last 2 decades we have witnessed great advances in the calculation of nuclear interactions with Lattice QCD, partly thanks to the technological development but also thanks to the development of algorithms and communication between different communities (computing science, theoretical physics, nuclear).

- Calculations near the physical pion mass using the direct method are under way (coarse extrapolations at the moment)

- Investigation on lattice artefacts, excited state contamination, …

- Operator dependence: Variational studies have revealed significant interpolating-operator dependence in LQCD calculations of NN energy spectrum with unphysical quark masses

  - Variational bounds don’t provide conclusive evidence for (or against) bound states.

  - Ongoing and future work: include a larger operator set (complete basis of local 6 quark operators), additional volumes, multi-exponential fits vs GEVP

  - The analysis of analogous calculations in the strange sector are underway ($\Lambda\Lambda$)

Acknowledgments

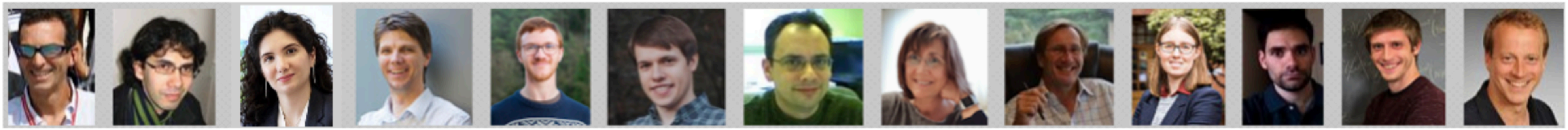
Thank you for your attention

UNIVERSITAT DE BARCELONA

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

NPLQCD
Nuclear Physics with Lattice QCD

USQCD
SDSC
Teragrid
RES
U of Washington
William & Mary
NERSC
LLNL
PRACE
ALCC
TACC
INCITE



Beane *Chang* *Davoudi* *Detmold* *Illa* *Murphy* *Orginos* *Parreño* *Savage* *Shanahan* *Tiburzi* *Wagman* *Winter*

Thank you for your attention

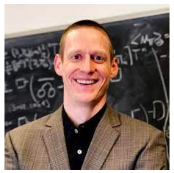
UNIVERSITAT DE BARCELONA

GOBIERNO DE ESPAÑA | MINISTERIO DE CIENCIA E INNOVACIÓN

NPLQCD
Nuclear Physics with Lattice QCD

- USQCD
- SDSC
- Teragrid
- RES
- U of Washington
- William & Mary
- NERSC
- LLNL
- PRACE
- ALCC
- TACC
- INCITE

Davoudi *Detmold* *Illa*



W. Jay

R. Perry

Shanahan

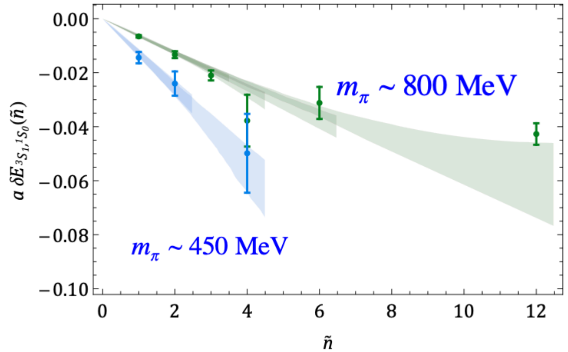
Wagman

The presence of a magnetic field induces mixing of the I_z=I_z=0 components of the $^3S_1$ and $^1S_0$ np systems

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C^{^3S_1,^3S_1}(t;\mathbf{B}) & C^{^3S_1,^1S_0}(t;\mathbf{B}) \\ C^{^1S_0,^3S_1}(t;\mathbf{B}) & C^{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

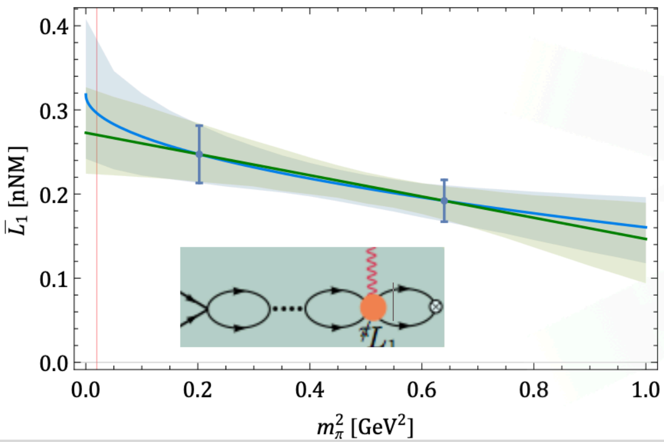
$$\Delta E_{^3S_1,^1S_0}(\mathbf{B}) = 2\left(\kappa_1 + \underbrace{\gamma_0 Z_d^2 \tilde{l}_1}_{\overline{L}_1}\right)\frac{e}{M}|\mathbf{B}| + \mathcal{O}(|\mathbf{B}|^2)$$

isovector nucleon magnetic moment



$$\delta E_{^3S_1,^1S_0} \equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}]$$
$$\rightarrow 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)$$

$$\overline{L}_1^{\rm lqcd} = 0.285(\,^{+63}_{-60}\,)\ \rm nNM$$



$$\sigma(np \rightarrow d\gamma) = \frac{e^2\left(\gamma_0^2 + |\vec{p}^2|\right)^3}{M^4\gamma_0^3|\vec{p}|}\left|\tilde{X}_{M1}\right|^2 + \ldots$$

$\sigma^{lqcd} = 332.4^{+5.4}_{-4.7}$ mb

($\sigma^{\rm exp} = 334.2 \pm 0.5$ mb)

Direct method

Misidentification of the plateau

E. Berkowitz et al. [CalLat], Phys.Lett.B 765 (2017)
S.R. Beane et al. [NPLQCD], arXiv:1705.09239 [hep-lat]
T. Yamazaki et al. [PACS], EPJ Web Conf. 175 (2018)

$$R(\tau, \boldsymbol{p}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{p})}{C_{B_1}(\tau, \boldsymbol{p}) C_{B_2}(\tau, \boldsymbol{p})}$$

↓

$$\Delta E_{\mathfrak{n}}$$

↓ Lüscher's method

$$k^* \cot \delta$$
$$B$$

pre-variational → bound
variational → not clear

Potential method

Only applicable at the energy of the calculation/system

Test convergence expansion

Ground-state saturation requirement

Short-distance operator dependence

T. Iritani et al. [HAL QCD], Phys.Rev.D 99 (2019)

$$R(\tau, \boldsymbol{r}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{r})}{C_{B_1}(\tau, \boldsymbol{r}) C_{B_2}(\tau, \boldsymbol{r})}$$

↓

$$\left(\frac{\partial_\tau^2}{4m_B} - \partial_\tau - H_0\right) R(\tau, \boldsymbol{r}) = \int d^3\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R(\tau, \boldsymbol{r}')$$

↓

$$U(\boldsymbol{r}, \boldsymbol{r}') = V(\boldsymbol{r}) \delta(\boldsymbol{r} - \boldsymbol{r}') + \mathcal{O}(\nabla_{\boldsymbol{r}}^2 / \Lambda^2)$$

↓

$$k^* \cot \delta$$
$$B$$

not bound

NN systems at unphysical m_π

Direct method

Misidentification of the plateau

E. Berkowitz et al. [CalLat], Phys.Lett.B 765 (2017)

S.R. Beane et al. [NPLQCD], arXiv:1705.09239 [hep-lat]

T. Yamazaki et al. [PACS], EPJ Web Conf. 175 (2018)

Potential method

Prof. Aoki's talk this morning

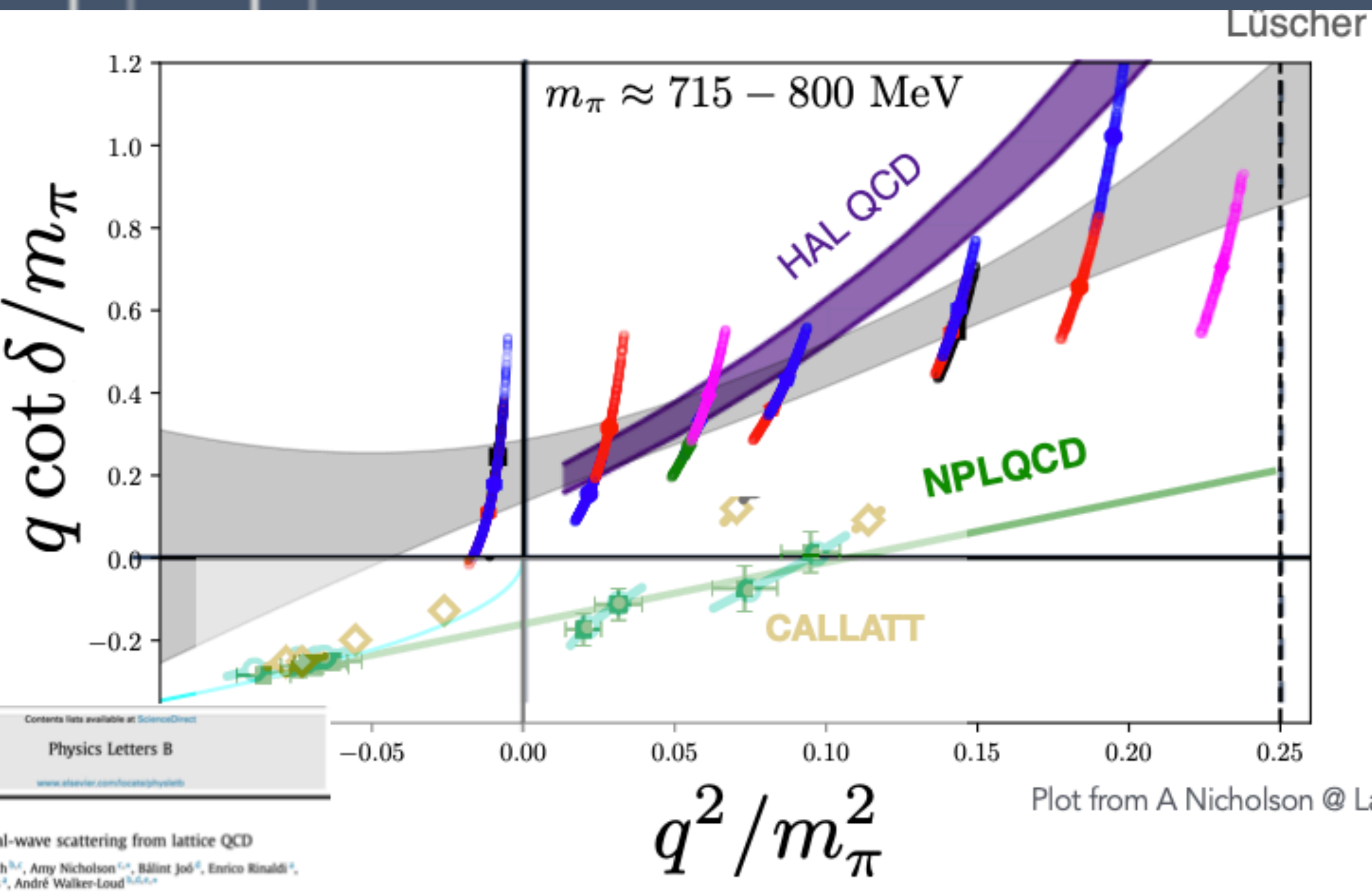
Only applicable at the energy of the system

Ground-state saturation requirement

Test of the convesgence expansion

Short-distance operator dependence

T. Iritani et al. [HAL QCD], Phys.Rev.D 99 (2019)



Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Two-nucleon higher partial-wave scattering from lattice QCD

Evan Berkowitz, Thorsten Kurth, Amy Nicholson, Bálint Joó, Enrico Rinaldi, Mark Strother, Pavlos M. Vranas, André Walker-Loud

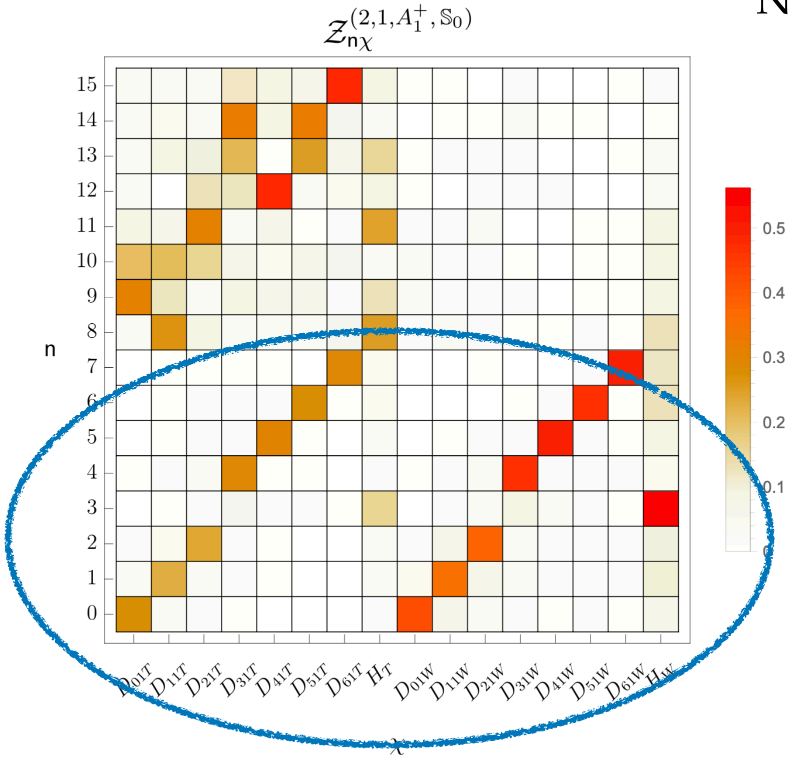
NN

Plot from A Nicholson @ Lattice 2022

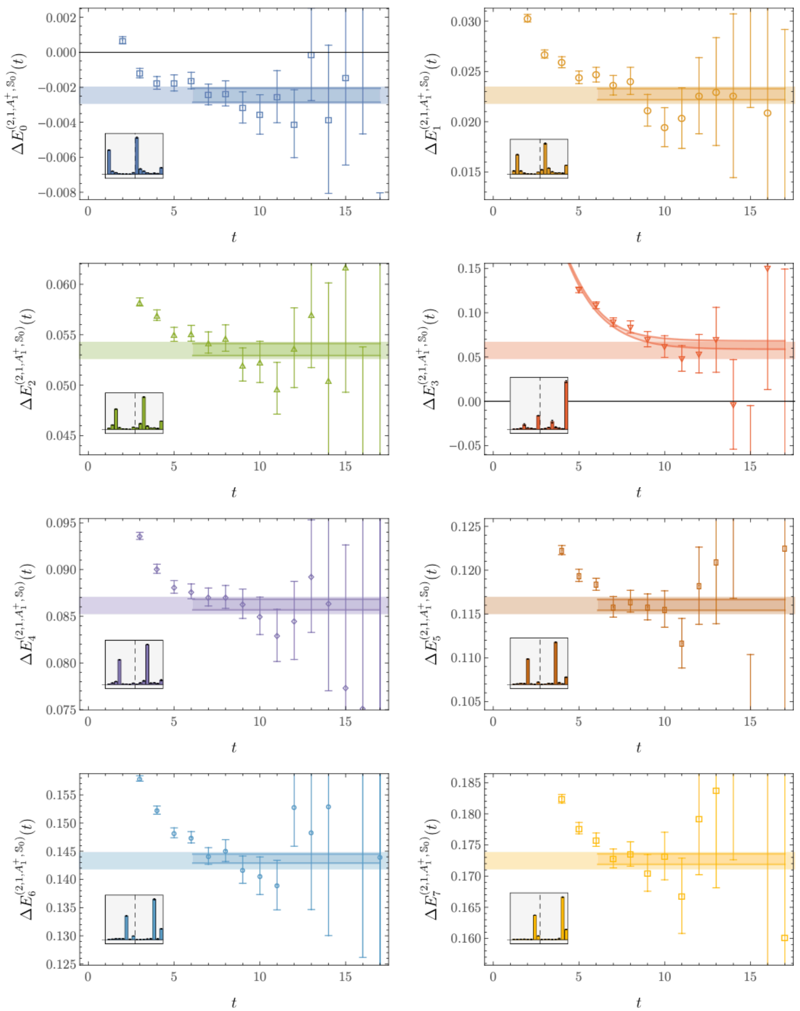
from UNRESOLVING THE NN CONTROVERSY, W. Detmold. Bethe Forum: Multihadron Dynamics in a Box, MIT

NN, I=1

S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



strong overlap with a particular operator



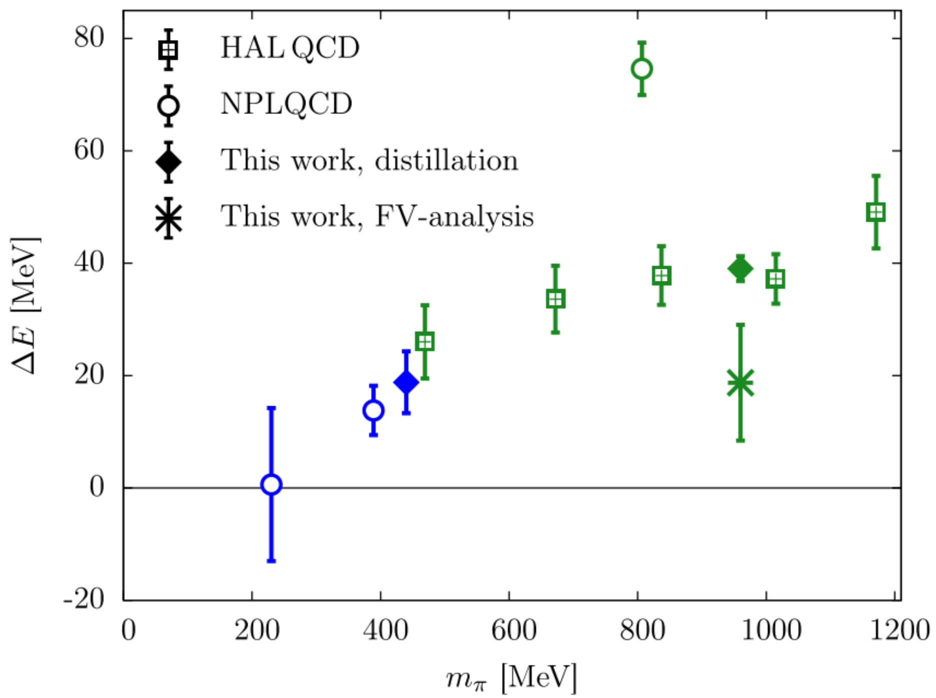
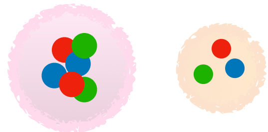


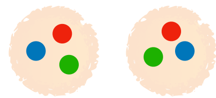
FIG. 11. Comparison of our results in Eqs. (23)–(25) to the estimates quoted by NPLQCD [14,18,20] and HAL QCD [22,23]. Green and blue symbols refer to the SU(3)-symmetric and broken cases, respectively. The data point marked by a star denotes the result in infinite volume.

A. Francis, PRD 99, 074505 (2019)

Hermitian 2x2 matrix with hexaquark and dibaryon-like operators



Non-hermitian 2x2 matrix with dibaryon-like operators



Note:
We should use adimensional variables to compare results obtained using different configurations to minimize the effect of different scale settings and other systematics

away from the $SU(3)_f$ limit

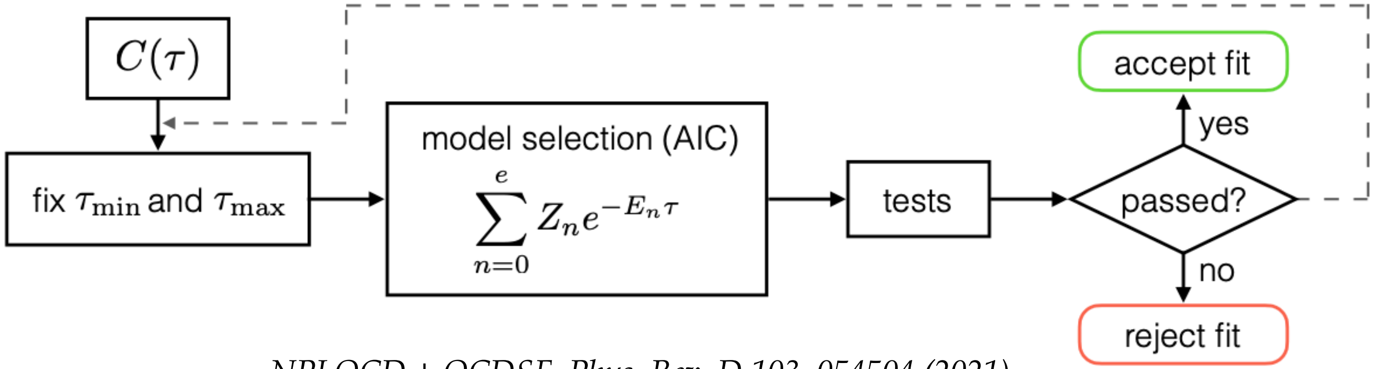
$n_f = 2 + 1$

$m_\pi = 450(5)\text{MeV}$

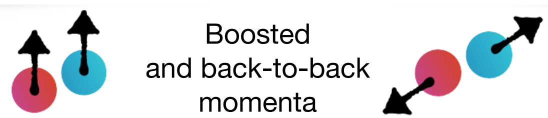
$b = 0.117(2)\ \text{fm}$

$L = 2.8, 3.7, 5.6\ \text{fm}$

$T = 7.5, 11.2, 11.2\ \text{fm}$



NPLQCD + QCDSF, Phys. Rev. D 103, 054504 (2021)

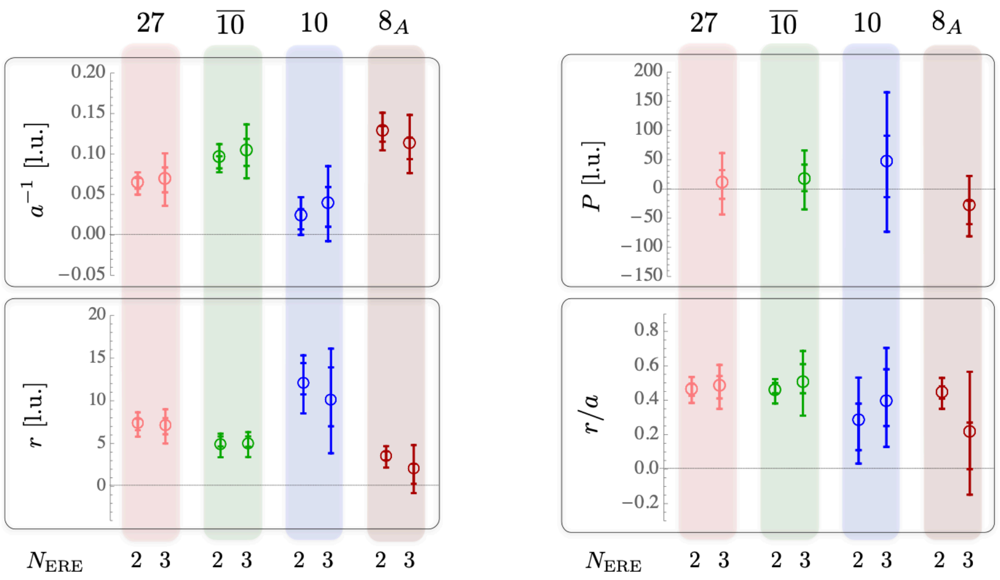


no e.m. interactions

NPLQCD, PRD 96, 114510 (2017)

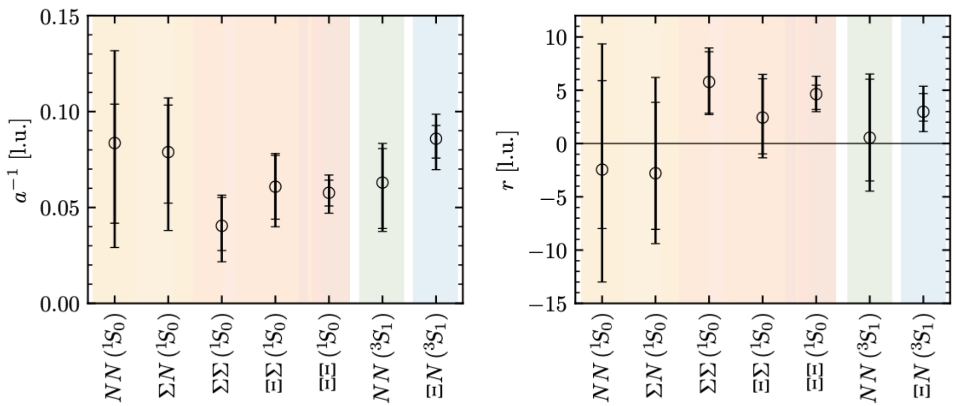
$SU(3)_f$

$m_\pi \sim 800$ MeV

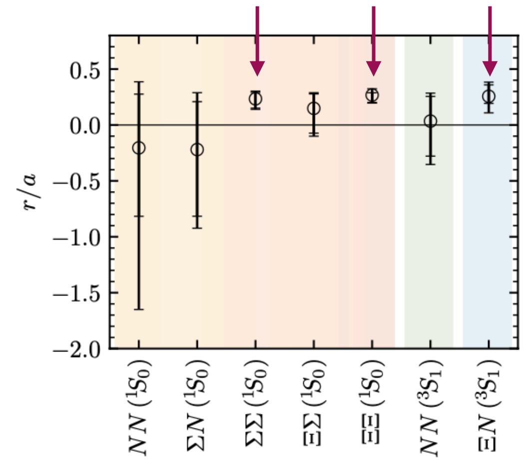


NPLQCD, PRD 103, 054508 (2021)

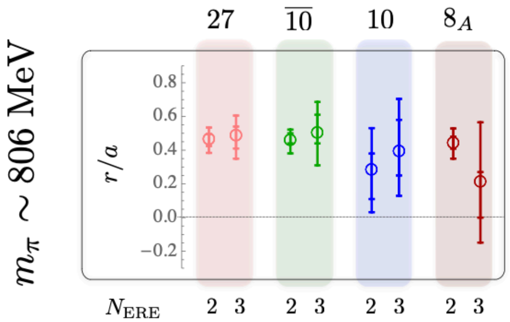
$m_\pi = 450$ MeV

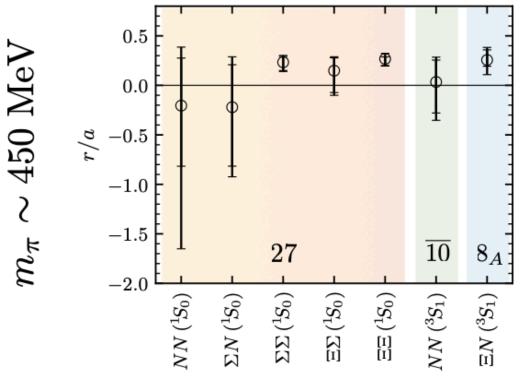


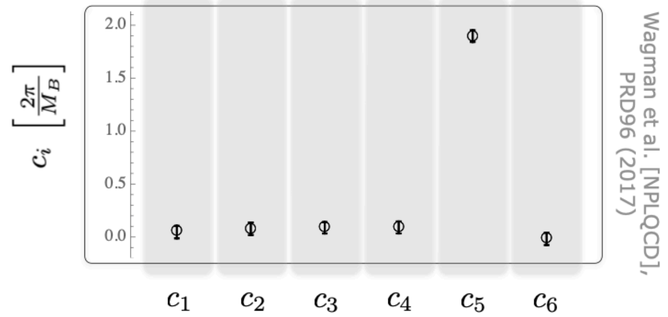
most constrained values show unnaturalness (r/a ~0.2 - 0.3)

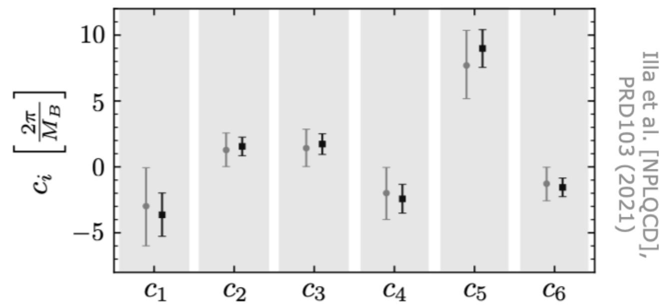


Scattering parameters → EFT matching









Agreement with the large-Nc prediction of an SU(6) symmetry

D. Kaplan, M.J. Savage, Phys.Lett.B 365 (1996)

Assuming $SU(3)_f$, at leading order we have

M.J. Savage, M. Wise, Phys.Rev.D 53 (1996)

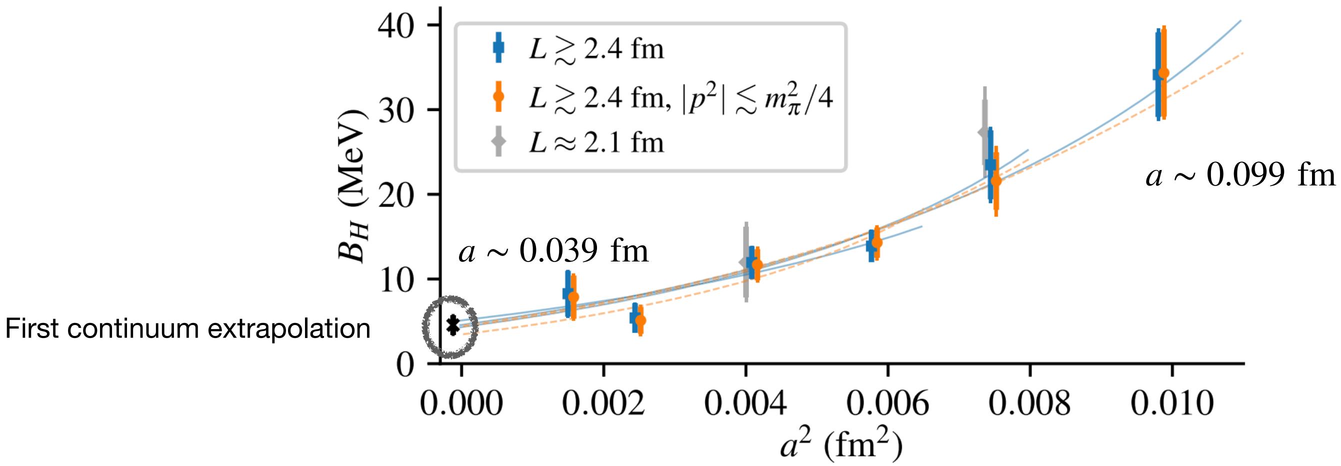
$$\mathcal{L}_{BB}^{(0),\,SU(3)} = -\,c_1\mathrm{Tr}(B_i^\dagger B_i B_j^\dagger B_j) - c_2\mathrm{Tr}(B_i^\dagger B_j B_j^\dagger B_i) - c_3\mathrm{Tr}(B_i^\dagger B_j^\dagger B_i B_j)$$
$$-\,c_4\mathrm{Tr}(B_i^\dagger B_j^\dagger B_j B_i) - c_5\mathrm{Tr}(B_i^\dagger B_i)\mathrm{Tr}(B_j^\dagger B_j) - c_6\mathrm{Tr}(B_i^\dagger B_j)\mathrm{Tr}(B_j^\dagger B_i)$$

$$c_1, \ldots, c_6 \longrightarrow c^{(27)}, \ldots, c^{(8_a)}$$

- Expect artefacts: $\Delta E = \Delta E_0(1 + (a\Lambda)\delta E_1 + \ldots)$ with $\delta E_1 \ll 1$

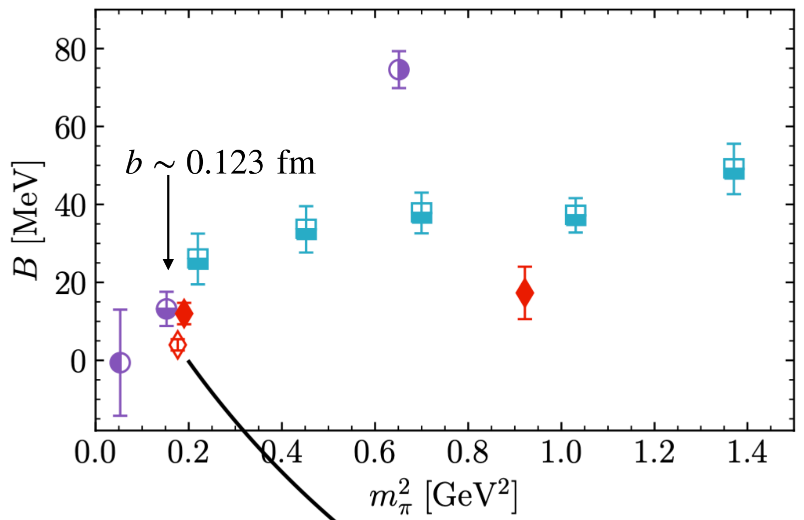
Green, Hanlon, Junnarkar, Wittig, PRL 127 (2021)

$m_\pi \sim m_K \sim 420$ MeV



Lattice artifacts?

Potentially important



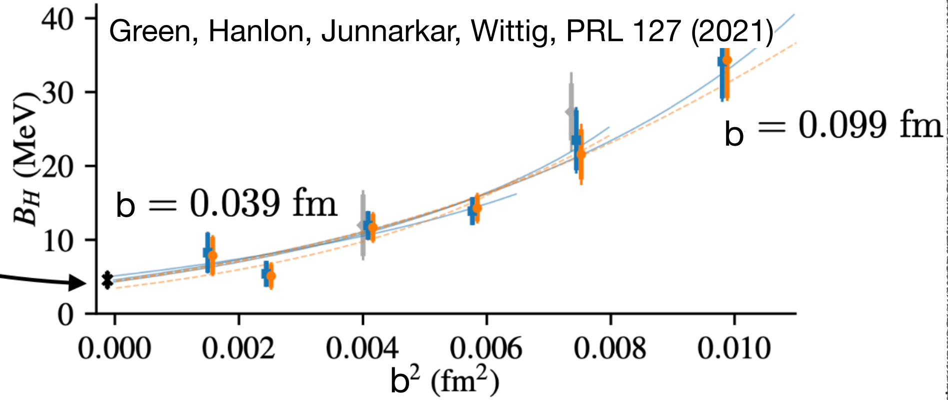
NPLQCD

HAL (Hadrons to Atomic nuclei from Lattice QCD)

JG|U → first application of variational approach to BB (with only dibaryon-like operators)

$m_\pi \sim m_K \sim 420$ MeV

first extrapolation to the continuum for BB systems



Potentially important

Green et al.,
"Weakly bound H dibaryon
from SU(3)-flavor-symmetric QCD"
Phys.Rev.Lett. **127** (2021) 24, 242003