Improved coalescence model based on the Wigner function formalism

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CosmicAntiNuclei

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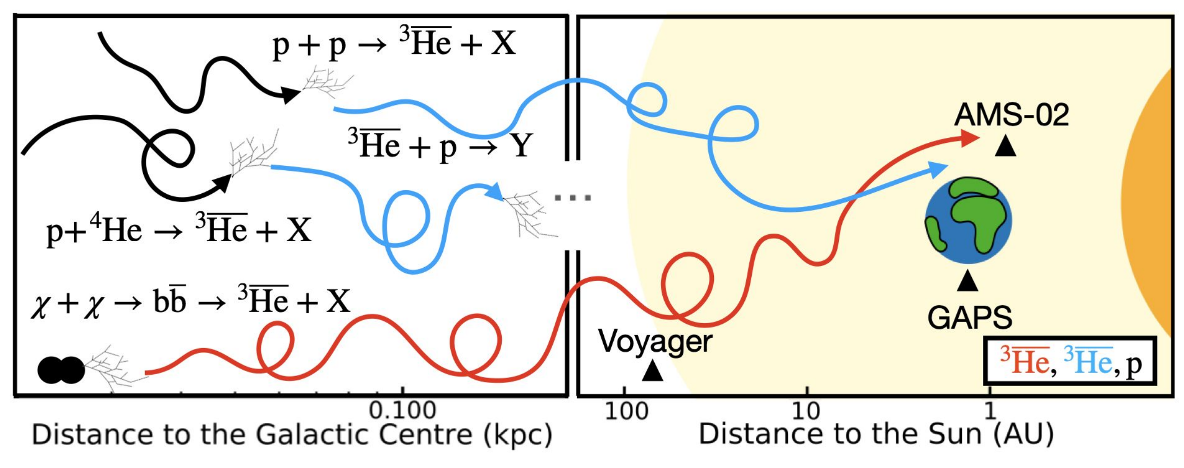
Cosmic Rays

Antinuclei in cosmic rays



Antinuclei production:

- pp, p–A and (few) A–A reactions between primary **cosmic rays** and the interstellar medium
- **dark-matter** annihilation processes

ALICE Collab., Nature Phys. (2022)

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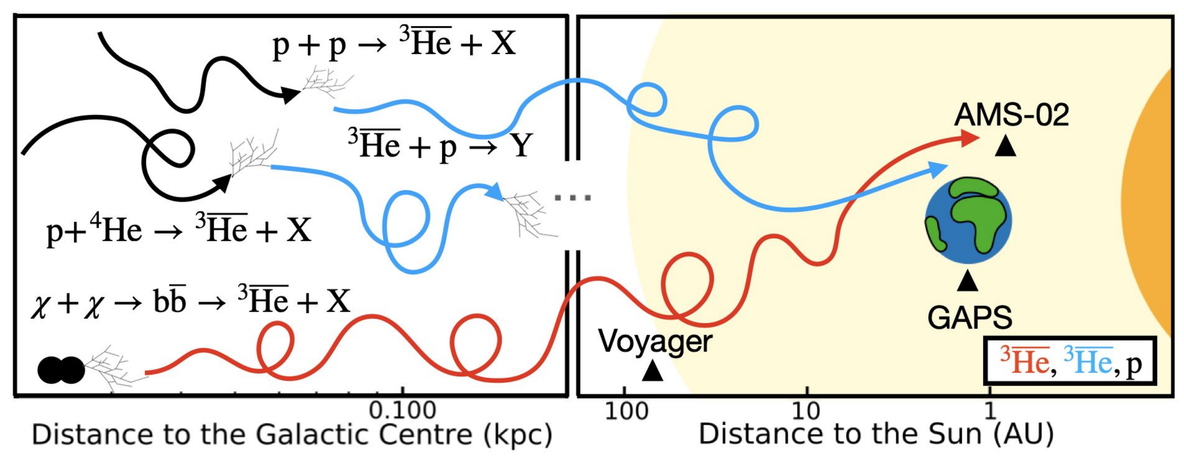

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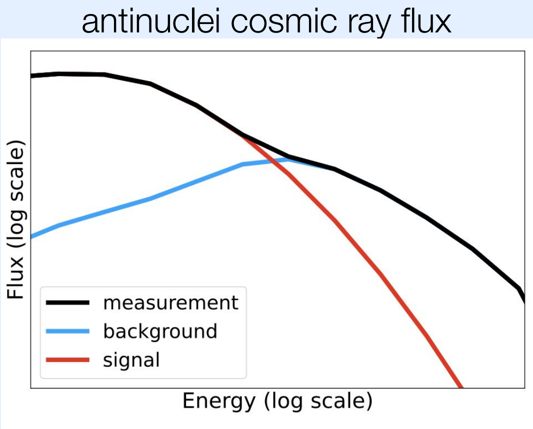
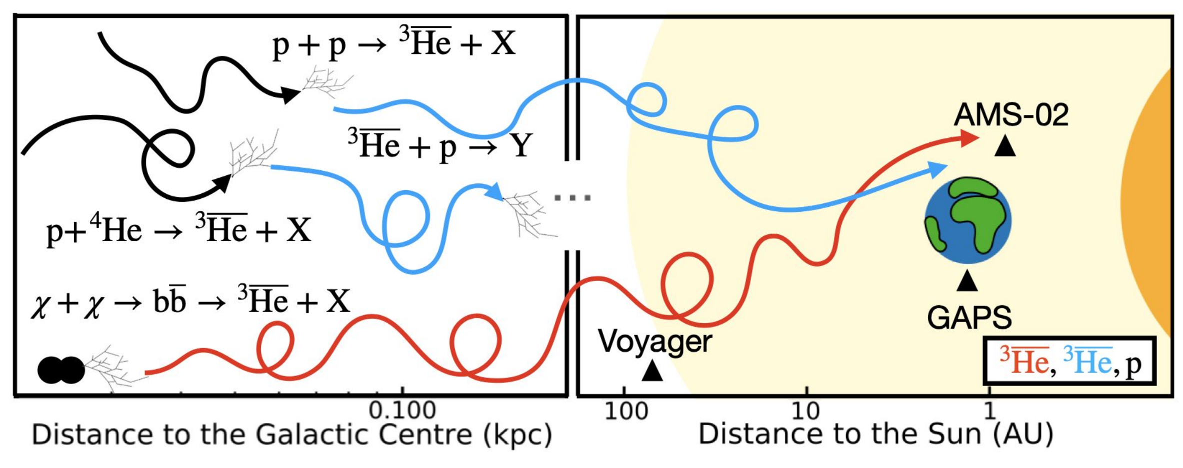


ALICE Collab., Nature Phys. (2022)

- To correctly interpret any future measurement of antinuclear fluxes (only antip measured so far)
- Need to determine exact primary and secondary fluxes → precise knowledge of antinuclei production, propagation and annihilation is needed
- High Signal/Noise ratio (~10^2-10^4) at low E_{kin} expected by models

Cosmic Rays

Antinuclei in cosmic rays



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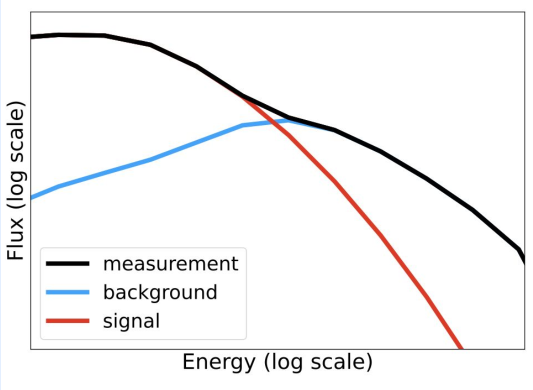
antinuclei cosmic ray flux



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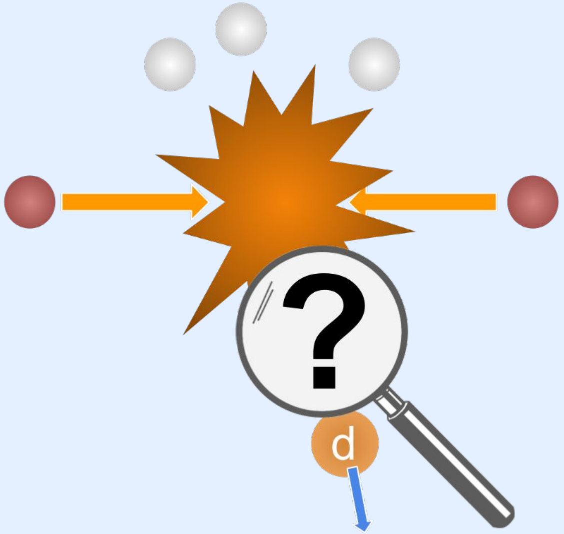
See talk by L. Serksnyte today 15:30!

Modelling (anti)nuclei production

Overview of production models

(Anti)nuclear production described by two models:

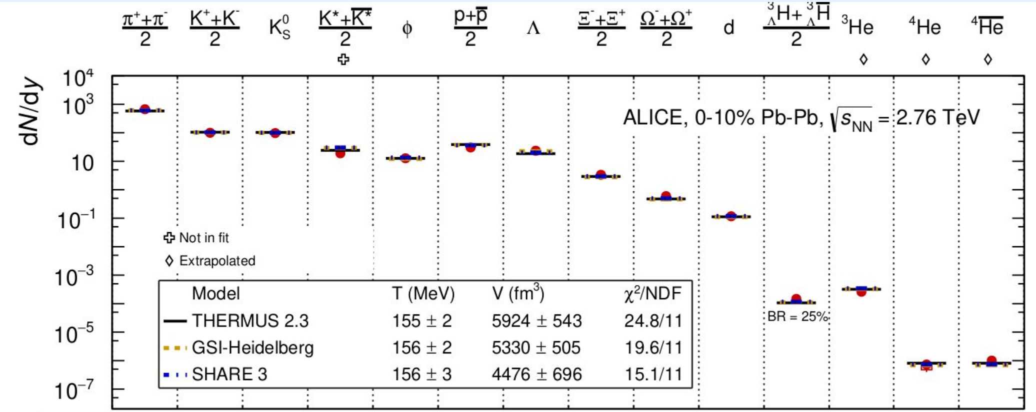


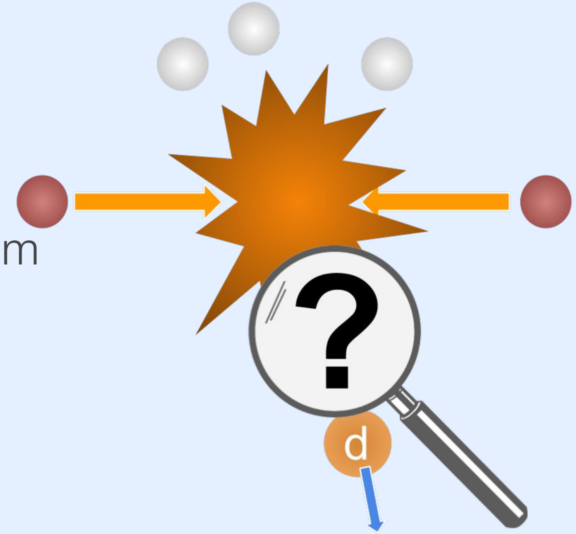
Overview of production models

(Anti)nuclear production described by two models:

Statistical hadronization (SHM)

- ➤ Particle yields (including nuclei) described by filling the available phase-space after the collision
- ➤ Works very well with a common temperature of the medium (T~155 MeV)
- ➤ No dynamical description of nuclei formation





Andronic et al., Nature 561 (2018) 321–330

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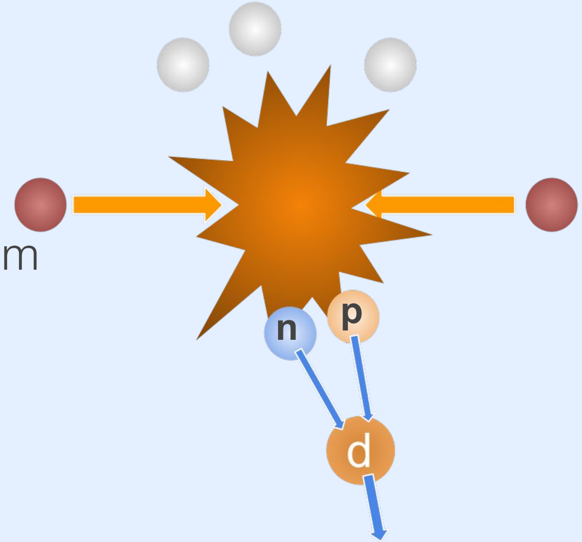

Coalescence model

- Nucleons bind after chemical freeze-out if they are close in phase-space

Butler et al., Phys. Rev. 129 (1963) 836

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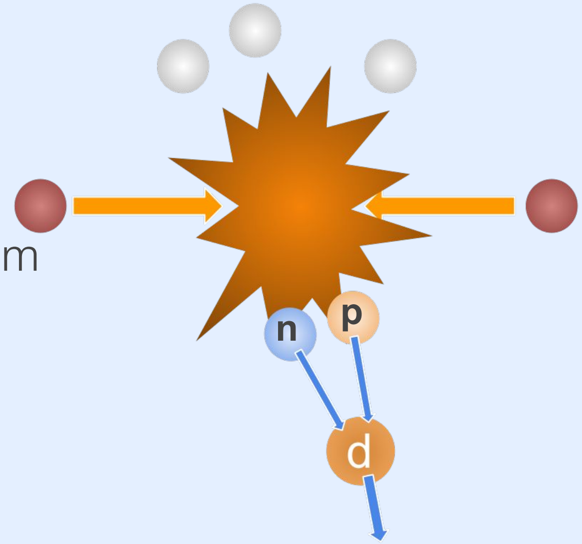
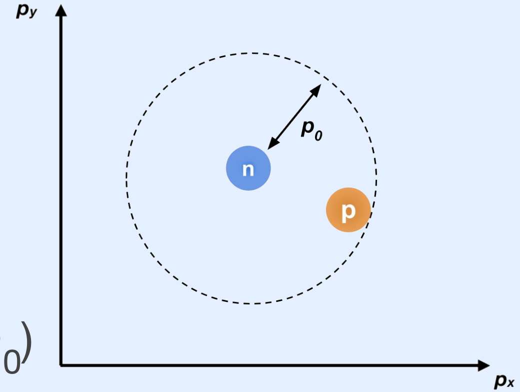


Coalescence model

- Nucleons bind after chemical freeze-out if they are close in phase-space
- Common implementation: Spherical Approximation ($\Delta p < p_0$)



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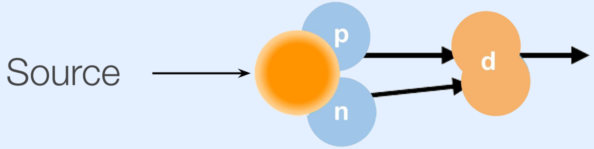
The coalescence model

Wigner function formalism

What do we need for coalescence?



p,n-pair deuteron

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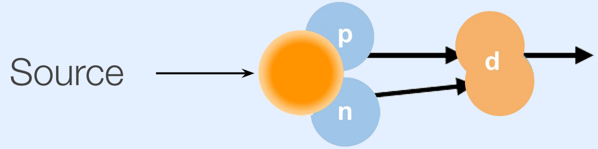

p,n-pair deuteron

Quantum mechanics: $d^3N/dP^3 = Tr(\rho_d \rho_{Nucl})$

Kachelriess et al. EPJA 57 (5) 167, 2021

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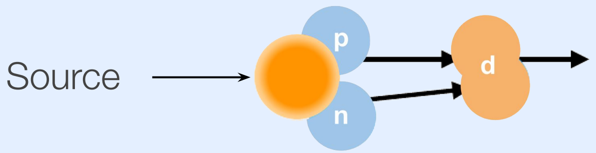

$q = (p_p - p_n)/2$

$r = r_p - r_n$

p,n-pair   deuteron

Quantum mechanics:

$$d^3N/dP^3 = \mathrm{Tr}(\rho_d \rho_{Nucl})$$

$$d^3N/dP^3 = \int d^3q \int d^3r_p \int d^3r_n \, \textit{Deuteron Density} \;\; \textit{Nucleon Density}$$

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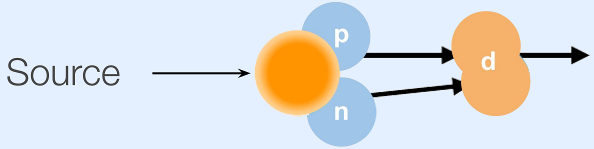

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$$d^3N/dP^3 = S \int d^3q \int d^3r_p \int d^3r_n \, W(q,r) \;\; W_{pn}(p_p,p_n,r_p,r_n)/(2\pi)^6$$

Spin-Isospin statistics factor (=⅜ for deuterons)

Wigner function of deuteron

Wigner function of p-n state

Kachelriess et al. EPJA 57 (5) 167, 2021

The coalescence model

Wigner function formalism

Two-nucleon Wigner function

$$W_{np}(\vec{P}/2+\vec{q},\vec{P}/2-\vec{q},r_n,r_p) = H_{np}(\vec{r}_n,\vec{r}_p)G_{np}(\vec{P}/2+\vec{q},\vec{P}/2-\vec{q})$$

- G_{np} is the momentum distribution of nucleons
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$$H_{np}(\vec{r}_n,\vec{r}_p) = h(\vec{r}_n)h(\vec{r}_p) = \frac{1}{(2\pi\sigma^2)^3}\exp\left(-\frac{\vec{r}_n^{\,2}+\vec{r}_p^{\,2}}{2\sigma^2}\right)$$

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Some simple calculation later

$$\frac{d^3N_d}{dP_d^3} = \frac{3\zeta}{(2\pi)^6}\int d^3q\ \mathrm{e}^{-q^2d^2}G_{np}(\vec{P}_d/2+\vec{q},\vec{P}_d/2-\vec{q})$$

Nucleon momentum phase-space

with

deuteron size (3.2 fm)

$$\zeta \equiv \left(\frac{d^2}{d^2+4\sigma^2}\right)^{3/2}$$

Two-particle emitting source size

Kachelriess et al. EPJA 57 (5) 167, 2021
Kachelriess et al. EPJA 56 (1) 4, 2020

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Constrained from data!

Kachelriess et al. EPJA 57 (5) 167, 2021
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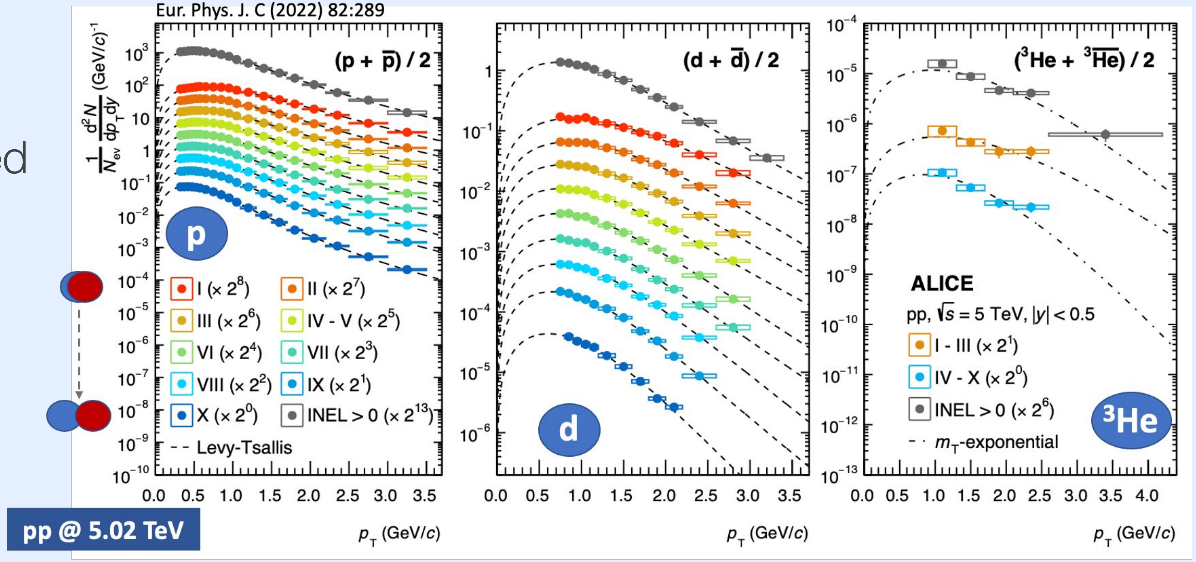
Light (anti)nuclei measured in ALICE

Transverse momentum spectra

- Comprehensive measurements of light (anti)nuclei have been carried out in ALICE, from pp…

- From (anti)deuterons to (anti)^{3}He



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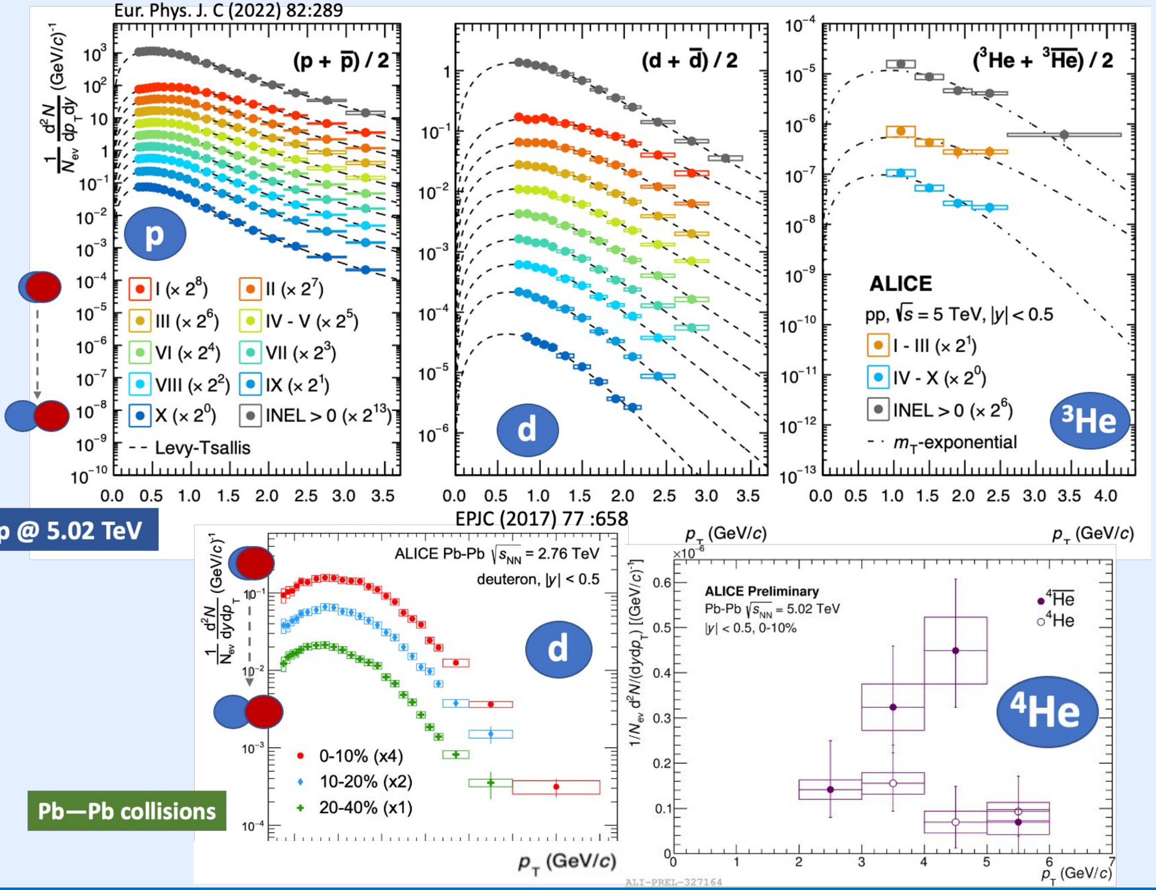

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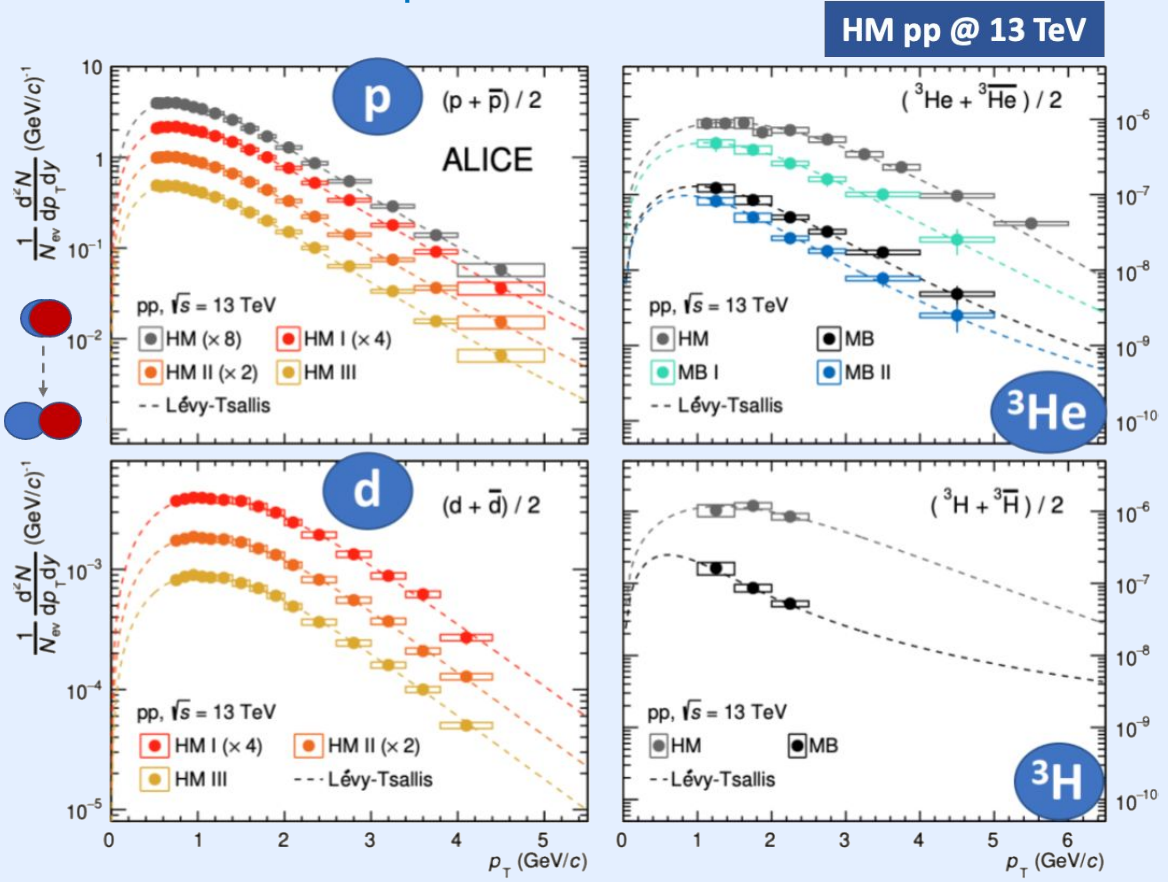
- From (anti)deuterons to (anti)^{3}He and (anti)^{4}He
- **High multiplicity** (HM) class in **pp collisions at 13 TeV** (→ 0-0.17% centrality class)
- In HM class both production spectra and emitting source size measurements available

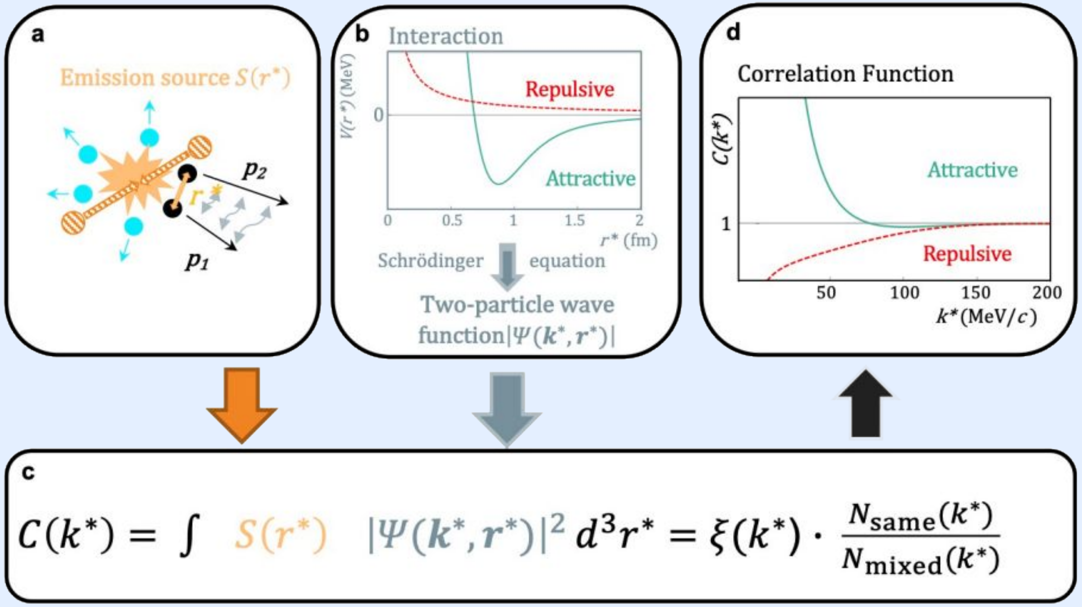


ALICE Coll., JHEP 01 (2022) 106

Emission source size measured in ALICE

Femtoscopy

- ALICE is pioneering the study of the strong interaction using femtoscopic correlations
- Momentum correlations can be employed to explore two-particle dynamics
- The **correlation function** depends on two ingredients:
  - Emission source function
  - Two-particle wave function (quantum statistics + Coulomb + strong interaction)



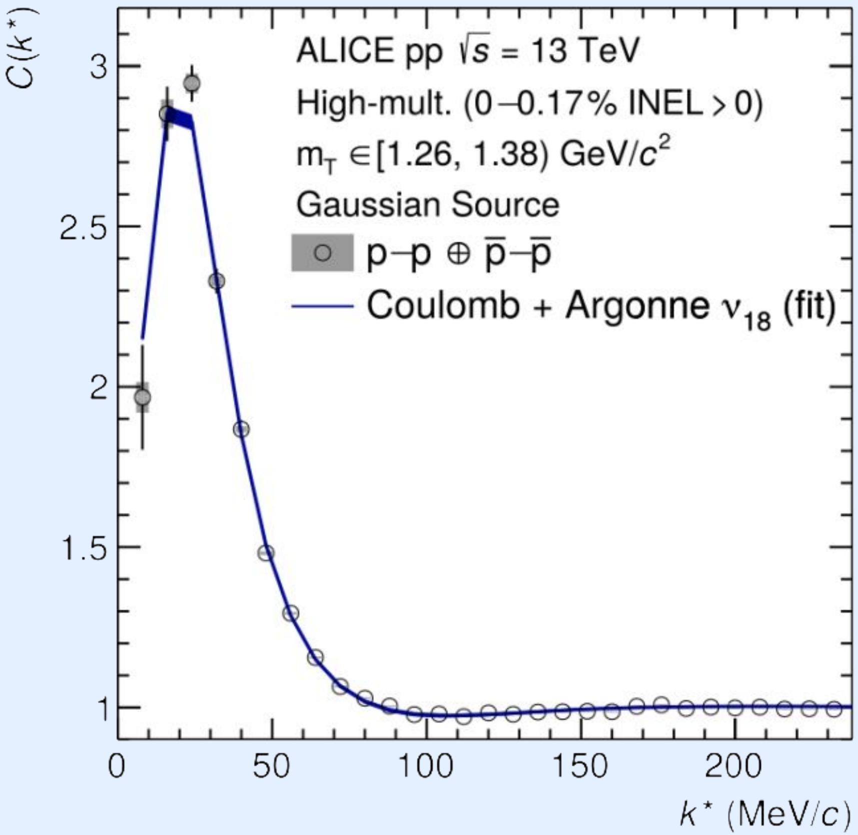
If we measure C(k*) and use a known interaction (e.g. nucleon-nucleon) we can study the emission source

ALICE Coll., Nature 588 (2020) 232-238

Emission source size measured in ALICE

Femtoscopy

- Good description of the interaction with Fermi-Dirac statistics, Coulomb and strong interaction (using v18)
- Only free parameter: the source size



ALICE Coll., PLB 811 (2020) 135849

Emission source size measured in ALICE

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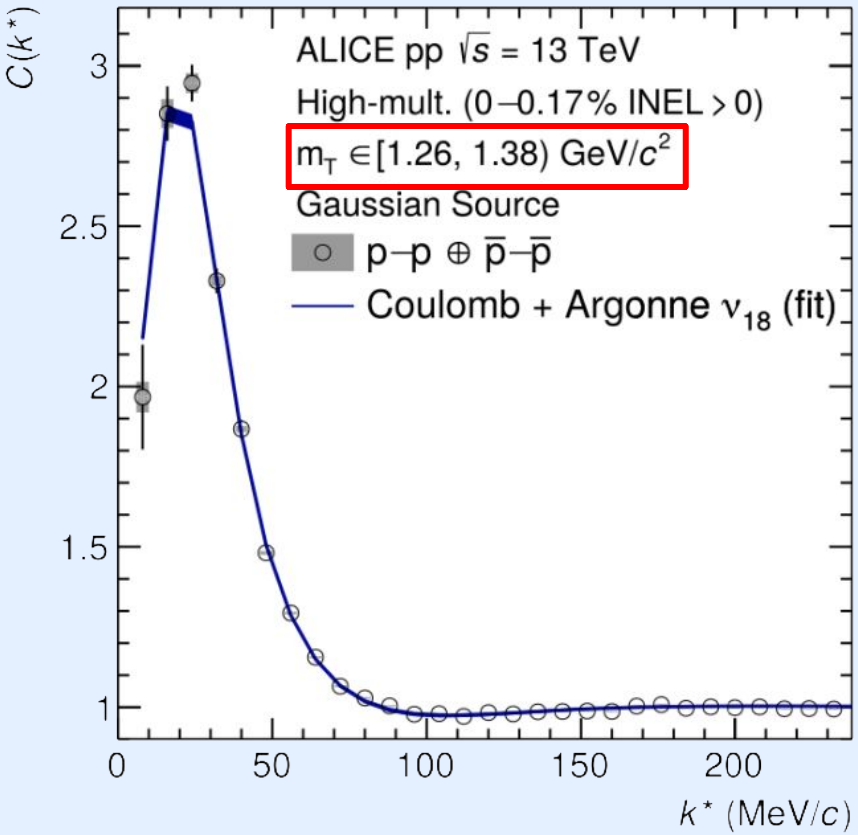

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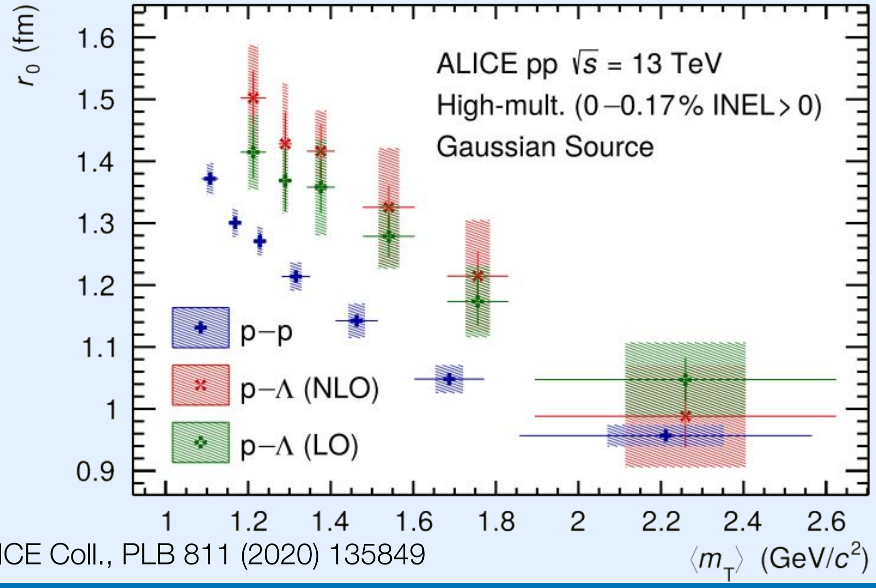

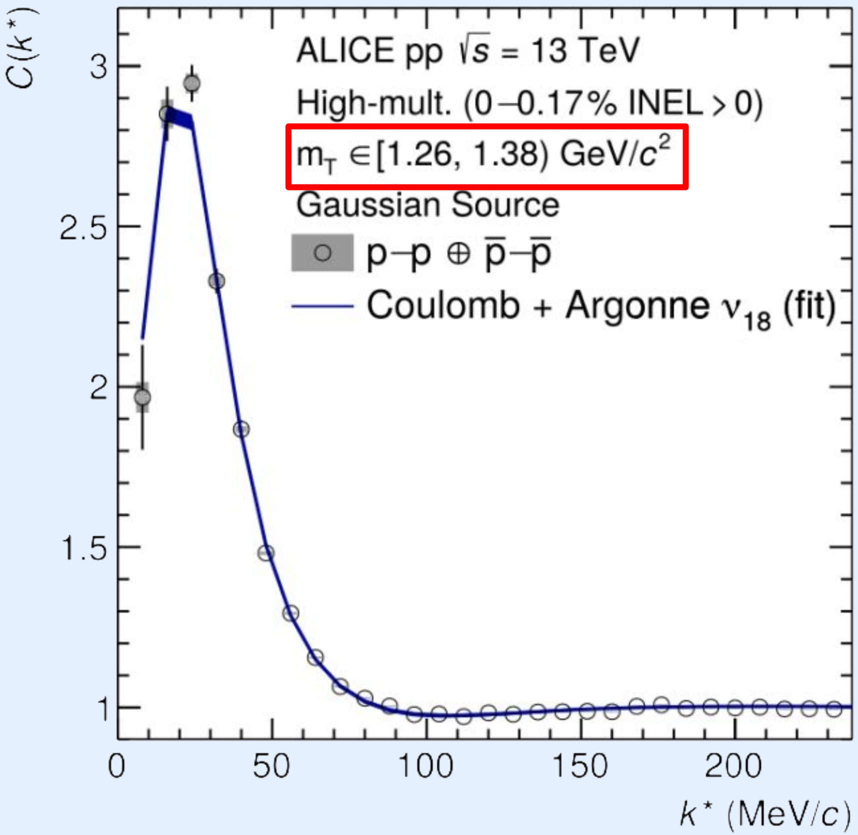

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The coalescence model

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Constrained from data!

Kachelriess et al. EPJA 57 (5) 167, 2021

Kachelriess et al. EPJA 56 (1) 4, 2020

The coalescence model

Wigner function formalism, tuned to ALICE measurements

Let's remember:

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Constrained from data!

- The term $3\zeta\mathrm{e}^{-q^2d^2}$ can be interpreted as a coalescence probability depending on the relative momentum *q* and the source size σ
- More in general:

$$p(\sigma, q) = \int d^3r_p d^3r_n h(r_n)h(r_p)W(q,r)$$

- This allows us to calculate the coalescence probability for arbitrary Wigner functions

⇨ Probe different hypotheses for the deuteron wave function

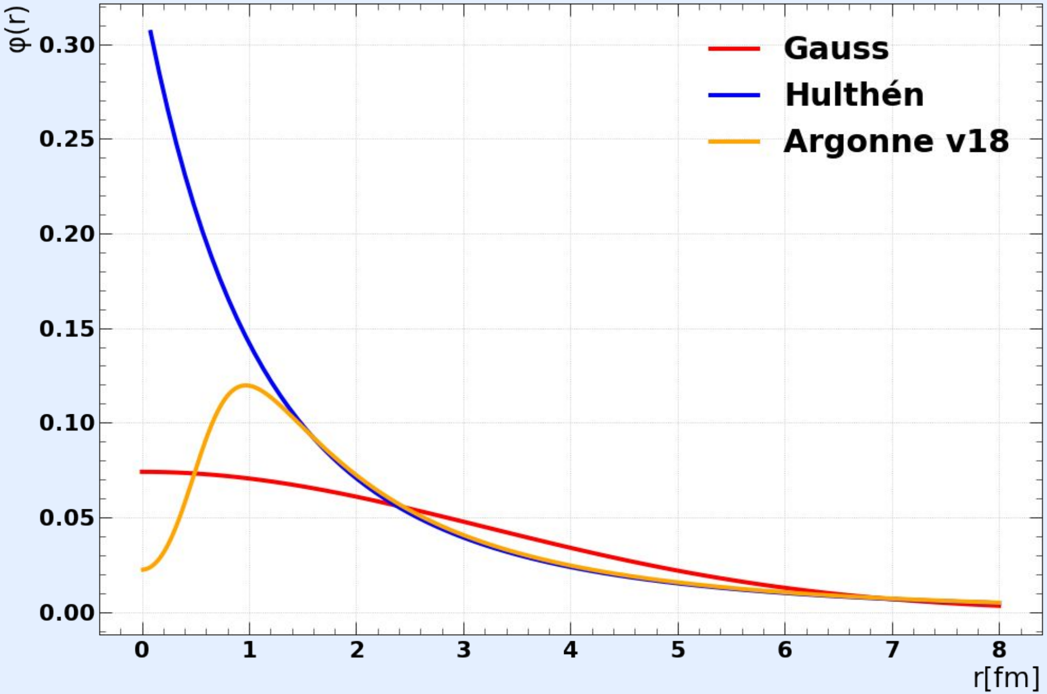
$$W(\vec{q},\vec{r}) = \int d^3\zeta\;\Psi(\vec{r}+\vec{\zeta}/2)\Psi^*(\vec{r}-\vec{\zeta}/2)e^{i\vec{q}\vec{\zeta}}$$

State of the art coalescence predictions

Wigner function formalism → wave functions

There are multiple models for the deuteron wave function

- ➤ Simplistic:
  Single Gaussian
- ➤ From *pion field theory* (Yukawa-like potential) ('50s)*:
  Hulthén
- ➤ From pn scattering measurements**:
  Argonne v_{18}



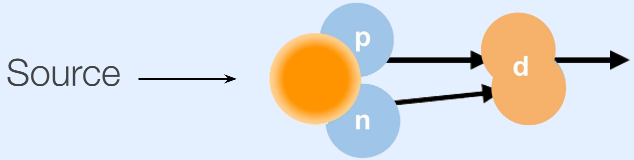
* Scheibl et al., PRC 59 (1999) 1585-1602

** Wiringa et al., PRC 51 (1995) 38-51

State of the art coalescence predictions

Wigner function formalism tuned to ALICE measurements

- Event-by-event coalescence afterburner with Wigner function formalism
- EPOS 3/Pythia 8.3 as event generator



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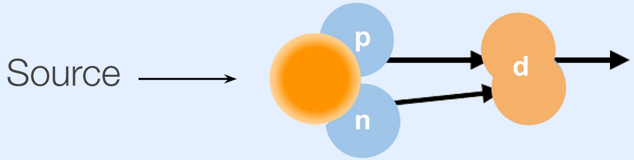
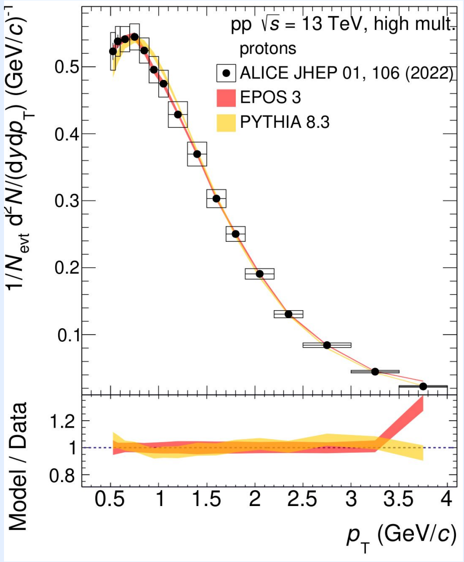


Ingredients

- Protons (and neutrons) are tuned to p measurements from ALICE



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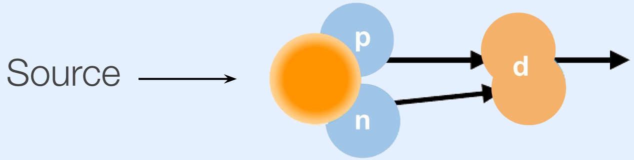
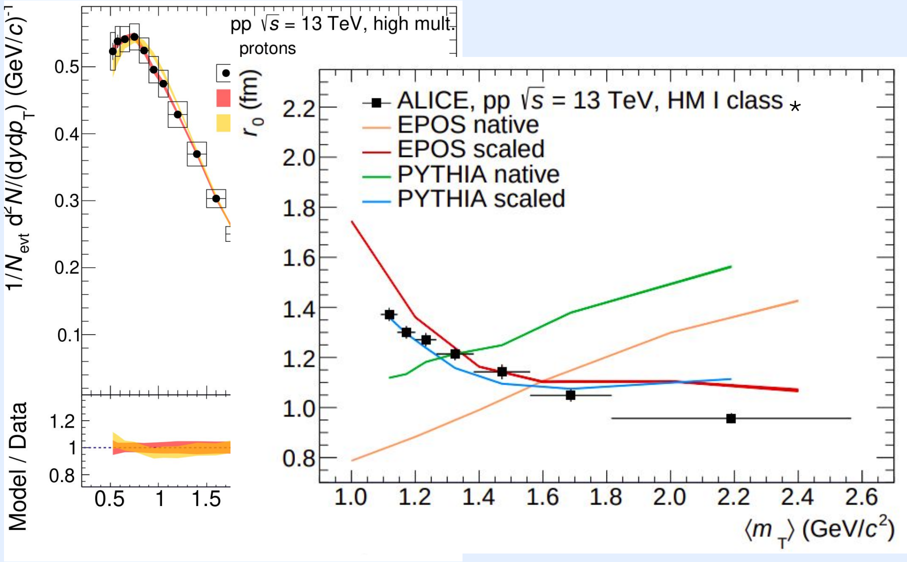
- Protons (and neutrons) are tuned to p measurements from ALICE
- Improved source model
  - source size
  - resonance cocktail
- charged-particle multiplicity (35.8±0.5)





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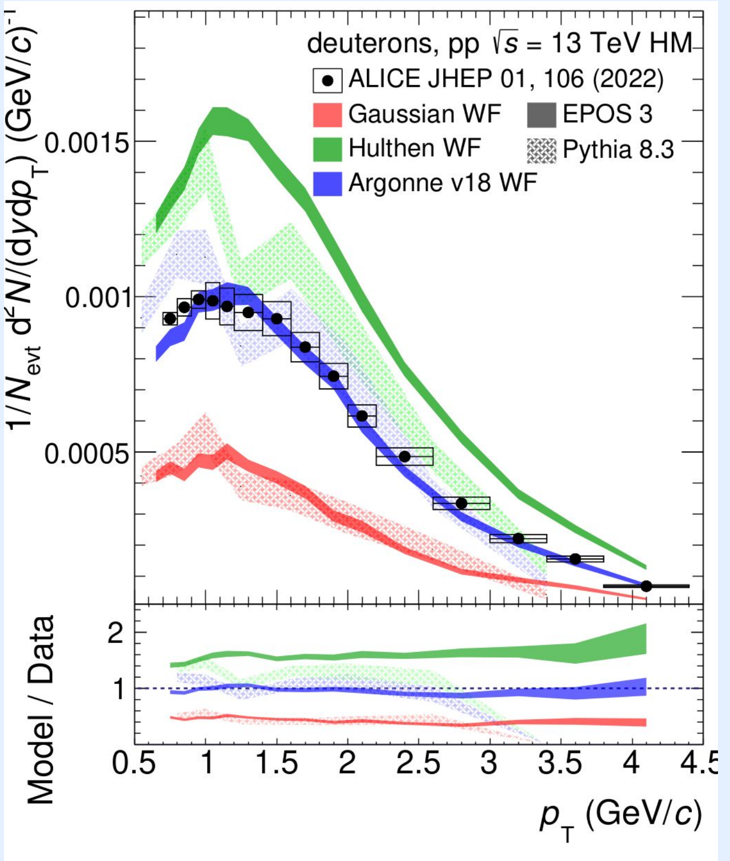
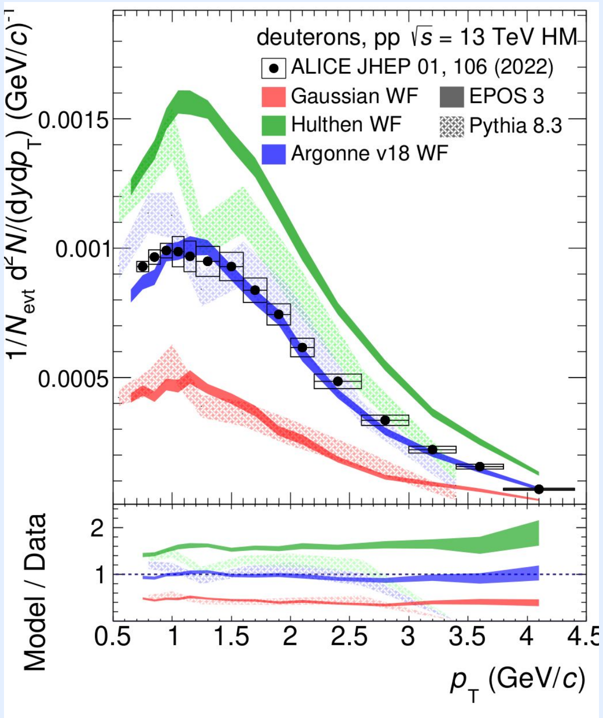
  - resonance cocktail
- charged-particle multiplicity (35.8±0.5)
- Argonne WF shows the best agreement with measurements



Summary

- Novel approach for coalescence based on Wigner function formalism is developed
- Deuteron production in high multiplicity pp collisions $\sqrt{s}$ = 13 TeV
- *If* we have control of the underlying physics
  - emission source size
  - (anti)nucleon momentum distributions
  - resonance cocktail
  - charged-particle multiplicity
  - realistic nucleus wavefunction

- Model successfully reproduces data with no free-parameters!



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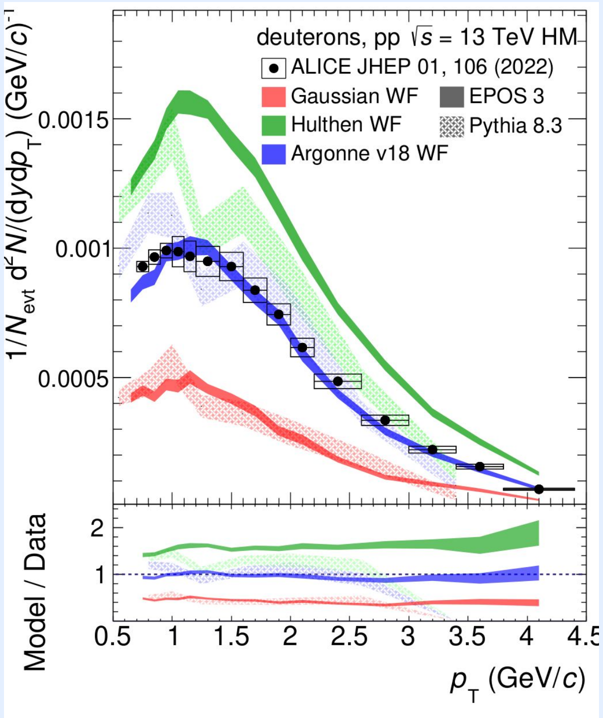
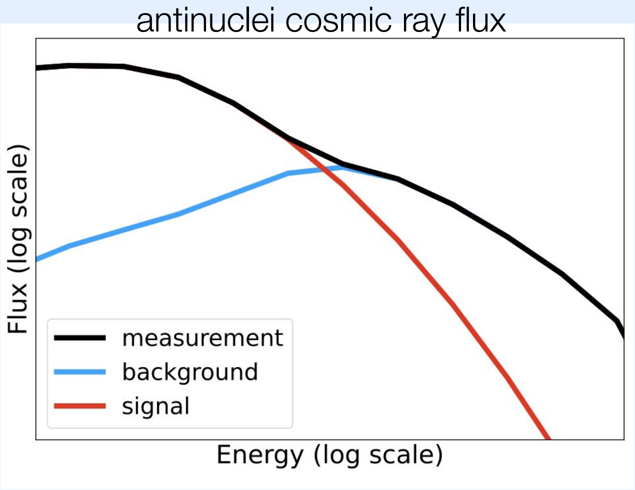
- ➤ Mod[...] no free-parameters!

Thank you for your attention!
Let's have a great workshop!



Backup slides

Conclusions



- Modelling antinuclei production is an essential backbone to interpret any future measurement of cosmic ray antinuclear fluxes

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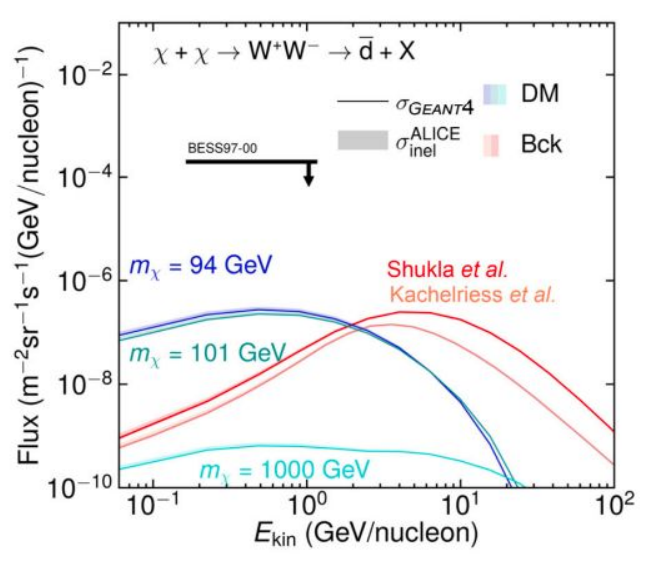

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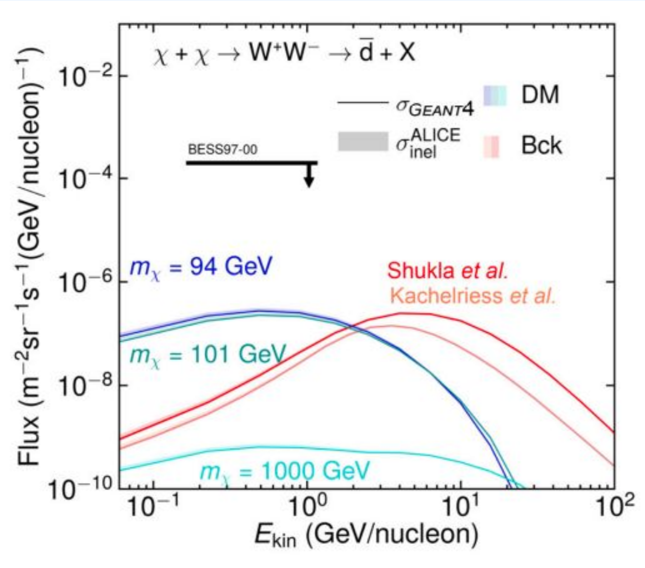
PRD 105 (2022) 083021

Conclusions



- ➤ Modelling antinuclei production is an essential backbone to interpret any future measurement of cosmic ray antinuclear fluxes
- ➤ Antideuteron production predominantly from collisions of protons of E_{kin}~200-500 GeV ($\sqrt{s}$ ~ 19-30 GeV for p-H)

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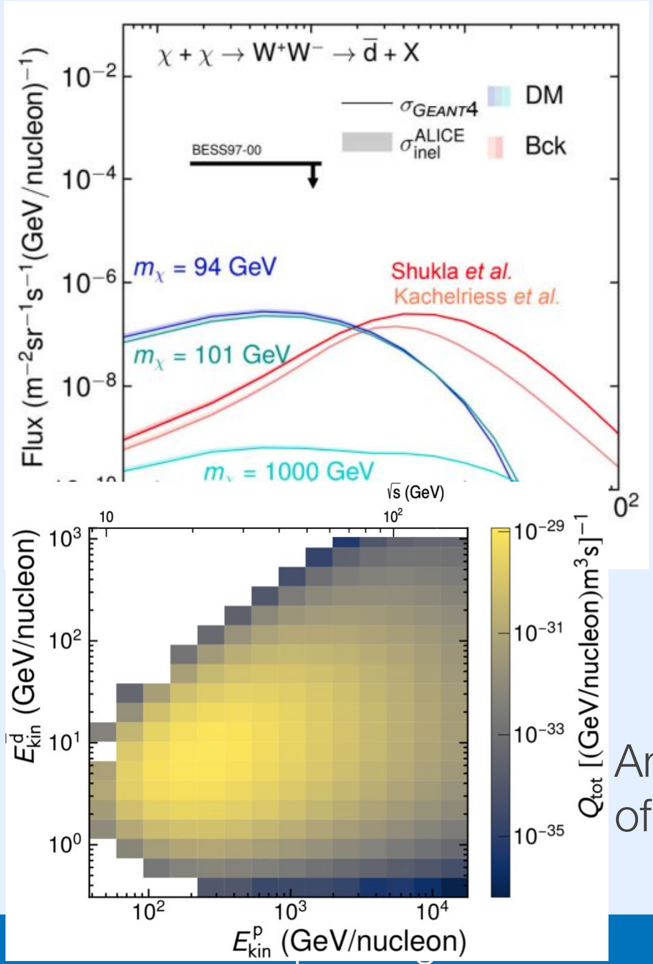

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Antideuteron source function as a function of kinetic energy of the incoming proton and produced antideuteron

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Conclusions

- Modelling antinuclei production is an essential backbone to interpret any future measurement of cosmic ray antinuclear fluxes
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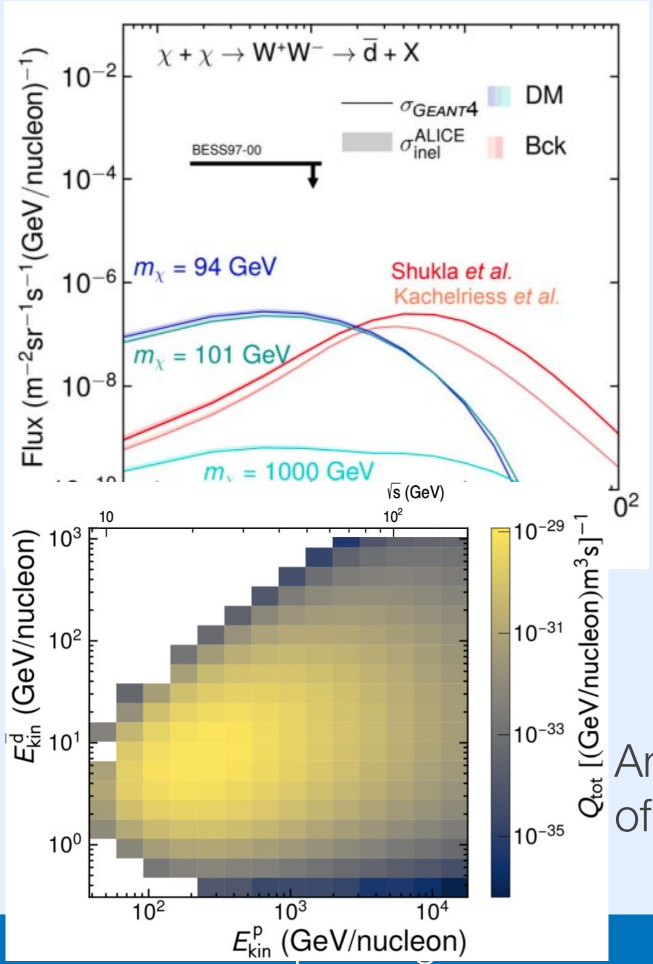

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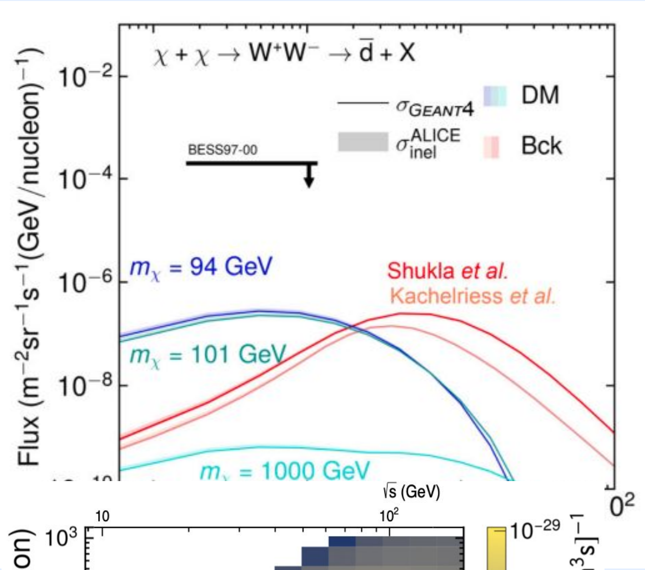

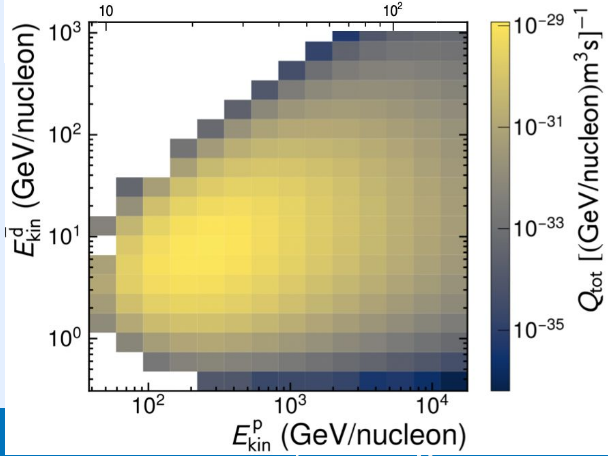

- Modelling antinuclei production is an essential backbone to interpret any future measurement of cosmic ray antinuclear fluxes
- Antideuteron production predominantly from collisions of protons of E_{kin}~200-500 GeV (√s ~ 19-30 GeV for p-H)
- Modelling production of antideuterons for HM pp collisions at 13 TeV is only the first piece of a much more complicated puzzle
- Extrapolation in the energy range of interest
- More experimental data at lower energies needed!

Antideuteron source function as a function of kinetic energy of the incoming proton and produced antideuteron

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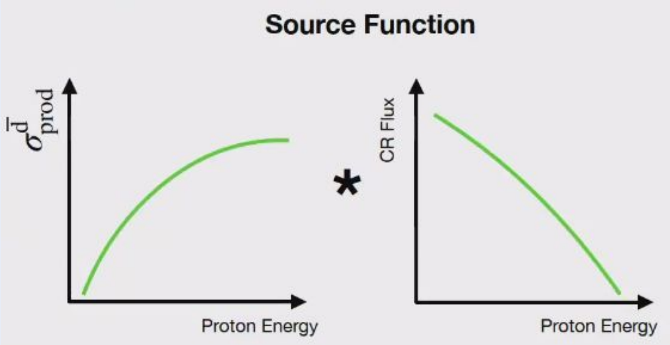
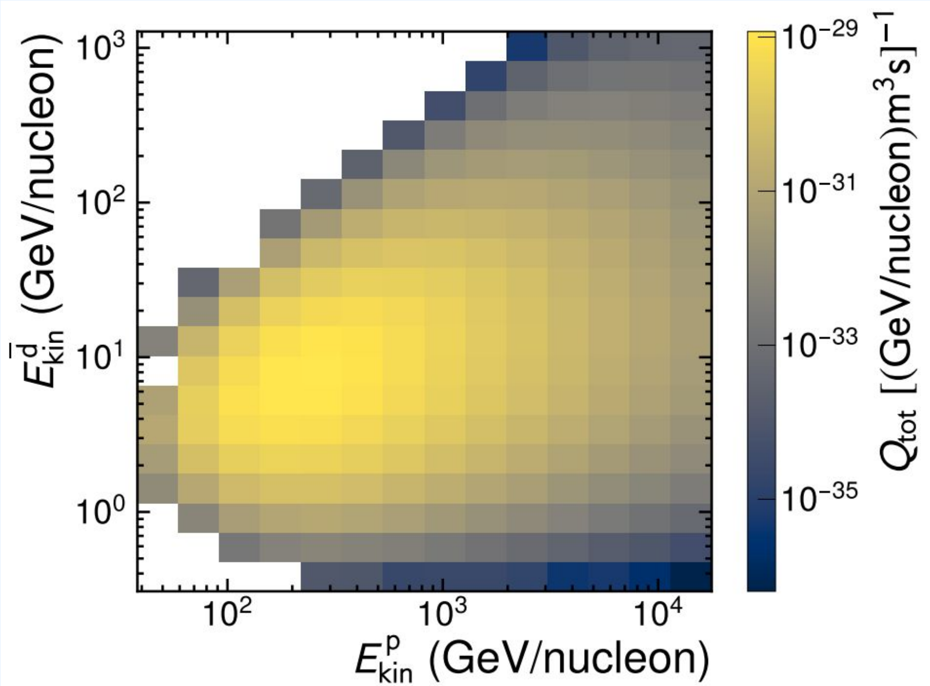
Cosmic Rays
Production energy of antinuclei

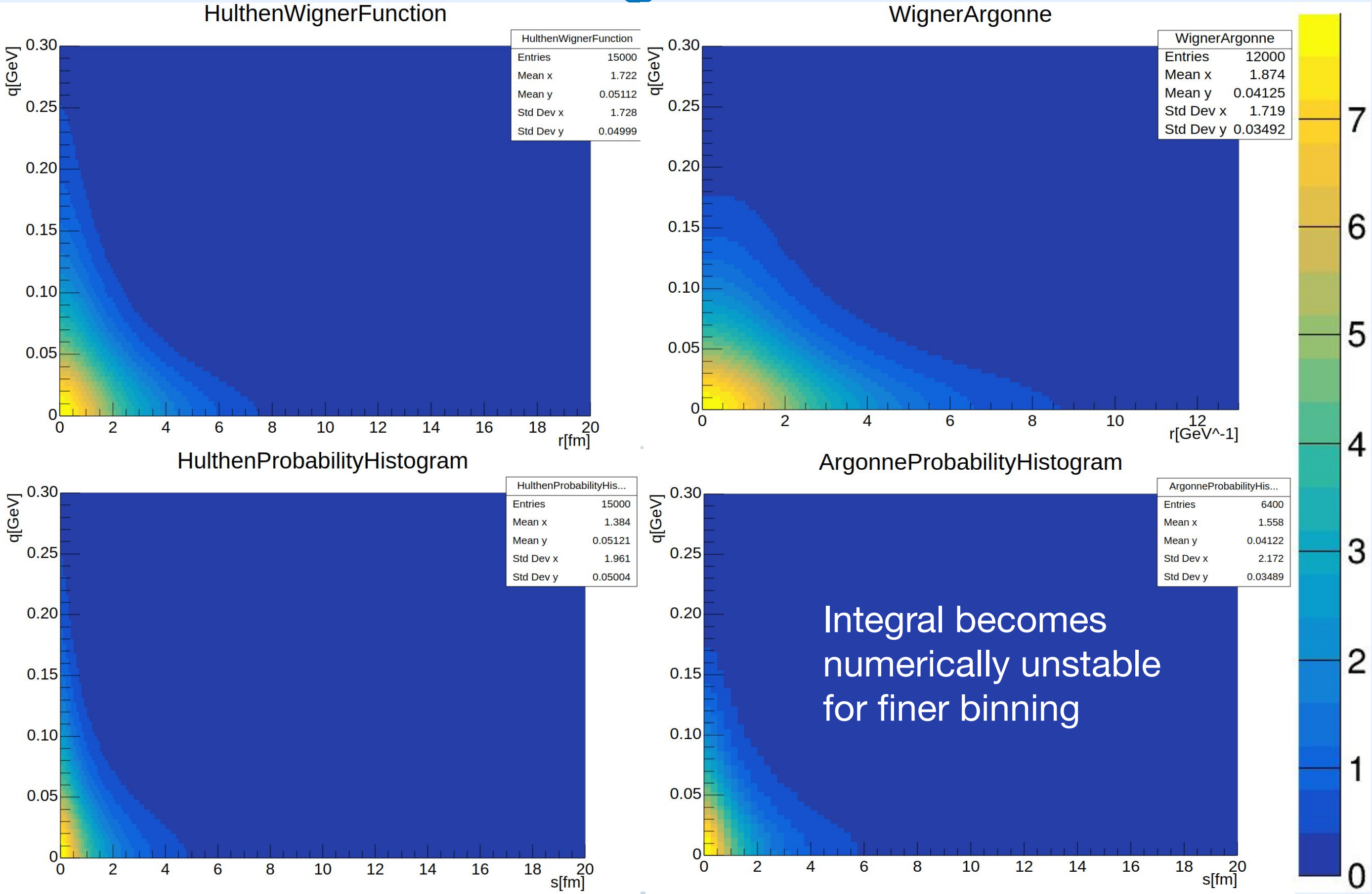
- ➤ Antideuteron source function as a function of kinetic energy of the incoming proton and produced antideuteron
- ➤ Antideuteron production predominantly for protons of E_{kin}~200-500 GeV ($\sqrt{s}$ ~ 19-30 GeV for p-H)





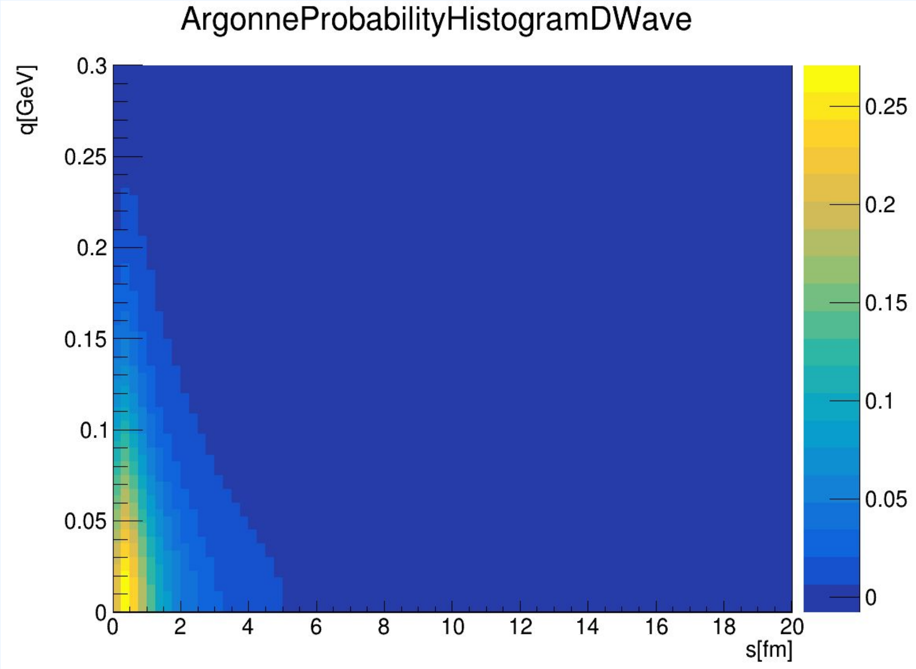
Šerkšnytė, et al. PHYSICAL REVIEW D 105, 083021 (2022)

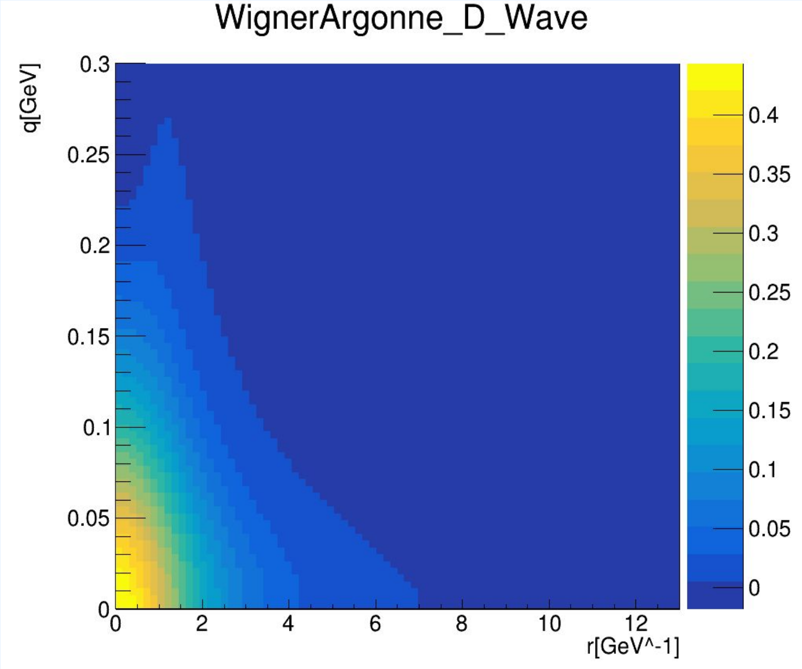
New Wiger functions/Probabilities



Integral becomes numerically unstable for finer binning

Argonne D-State probability





D-State probability is 6%

Overview of (anti)nuclei data

(anti)nuclei measurements

- No measurement of antideuterons in the energy region (~19-30 GeV) relevant for astrophysics
- Most measurements are very old (~60s and 70s)
- NA61's energy (17.3 GeV) would be a perfect candidate to study antinuclei for astrophysics

We need precise measurements at the energies of interest to constrain (anti)nuclei production!

Experiment or Laboratory	Collision	p_{lab} (GeV/c)	$\sqrt{s}$ (GeV)
CERN	p + p	19	6.15
CERN	p + p	24	6.8
Serpukhov	p + p p + Be	70	11.5
CERN-SPS	p + Be p + Al	200	19.4
Fermilab	p + Be	300	23.8
CERN-ISR	p + p	1497.8	53
CERN-ALICE	p + p	4.3×10^5	900
CERN-ALICE	p + p	2.6×10^7	7000

No antideuteron data!

Modelling (anti)nuclei production

B_A predictions

- Important observable in accelerator measurements: $\mathbf{B_A}$

$$B_A(p_T^p) = E_A \frac{d^3N_A}{dp_A^3} \Big/ \left(E_p \frac{d^3N_p}{dp_p^3} \right)^A$$

- Theoretical prediction [1]

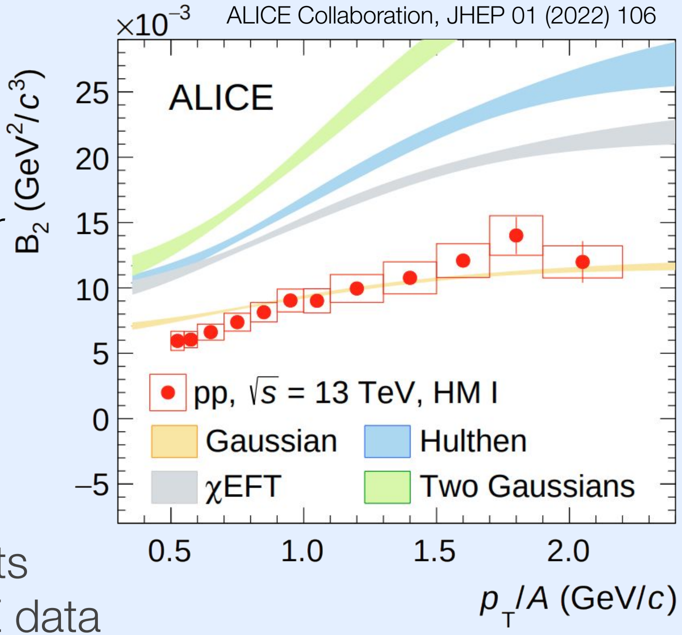
$$B_2(\vec{p}) \approx \frac{3}{2m} \int d^3q D(\vec{q}) e^{-R^2(p_T)\ q^2}$$

$$D(\vec{q}) = \int d^3r |\phi_d(\vec{r})|^2 e^{-i\vec{q}\cdot\vec{r}}$$

Emission source size

* *keep it in mind for later!*

Deuteron wave function



Testing different wave functions:

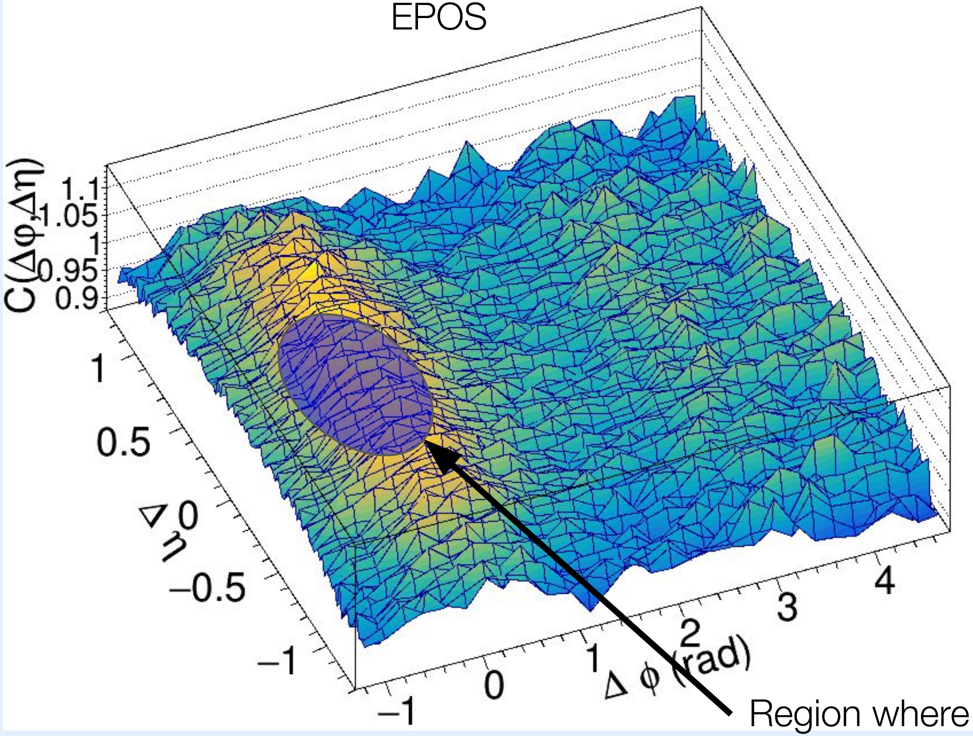
- **Hulthén:** Favoured by low energy scattering experiments
- **Gaussian:** Best description of currently available ALICE data
- **Two Gaussians:** Approximates Hulthén, easy to use in calculations
- **χEFT:** Favoured by modern nuclear interaction experiments (e.g. Femtoscopy)

[1] Blum, Takimoto, PRC 99 (2019) 044913

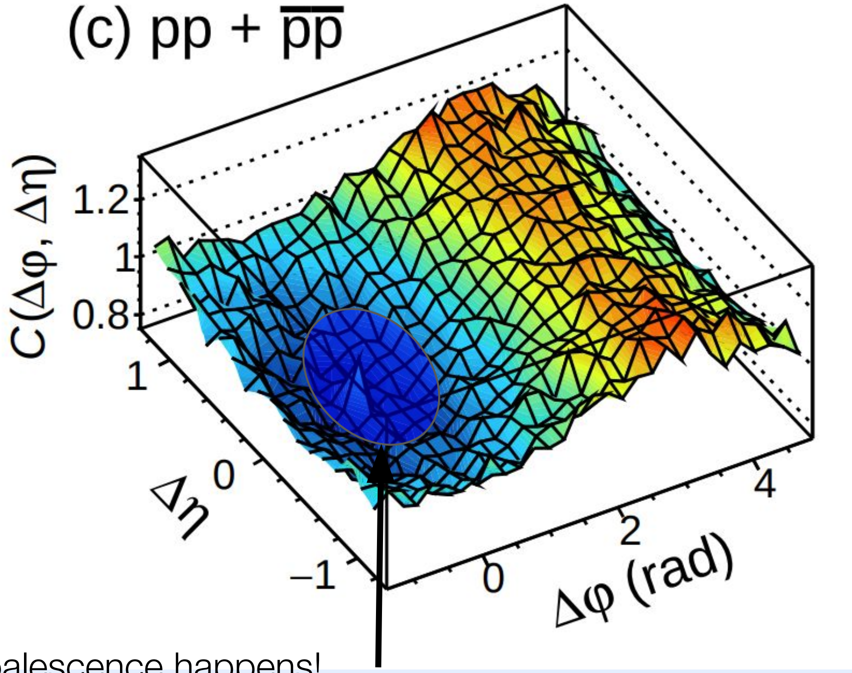
Correlations comparison

Δη-Δφ Correlation function

EPOS



ALICE (*Eur.Phys.J.C* 77 (2017) 8, 569)



Region where coalescence happens!

The advanced source model in EPOS

Scheme

Propagation scheme:

- We obtain a scaling factor as a function of m_T from the source size measurement
- We move the primordials out radially until we reach the scaled distance
- This distance ($\tilde{x}$) is the same for both primordials of the pair