Latest developments of the PHQMD model

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& Gabriele Coci, Viktar Kireyeu, Elena Bratkovskaya, Joerg Aichelin, Vadym Voronyuk, Christoph Blume, Vadim Kolesnikov

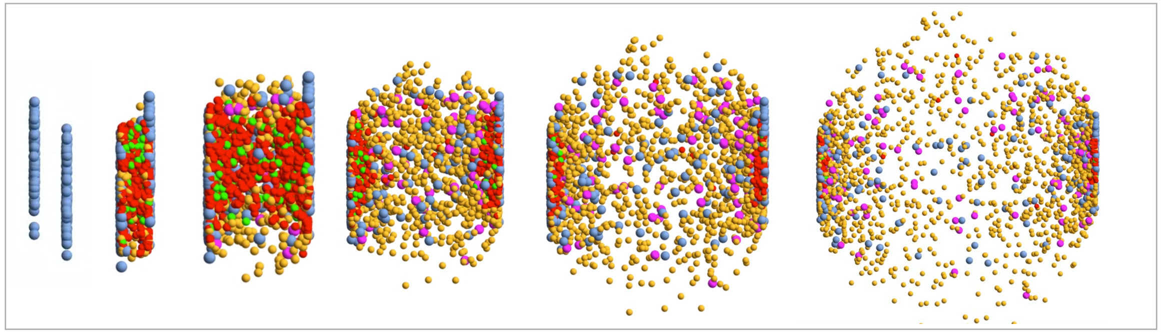
4th EMMI workshop on anti-matter, hyper-matter and exotica production at the LHC
Bologna, Italy, February 13-17, 2023

Agenda

PHQMD model & dynamic cluster formation

PHQMD results for AGS, SIS & RHIC energies

New development: Stabilisation of clusters



Cluster and hyper-cluster production in HICs with PHQMD

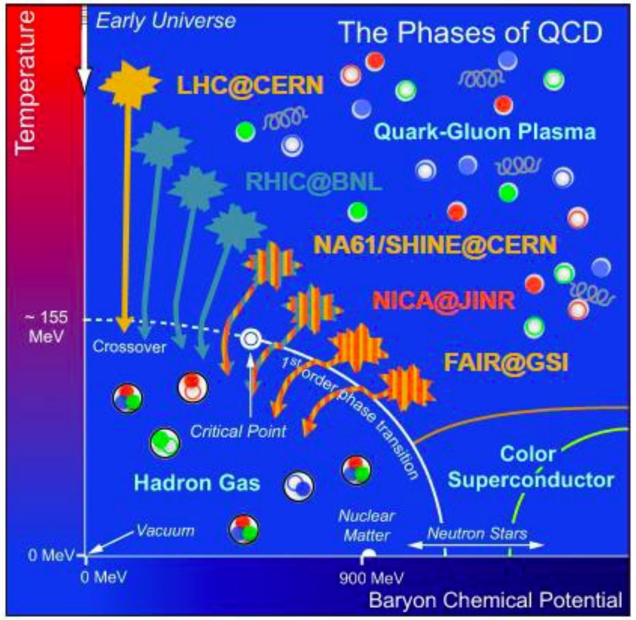
Motivation

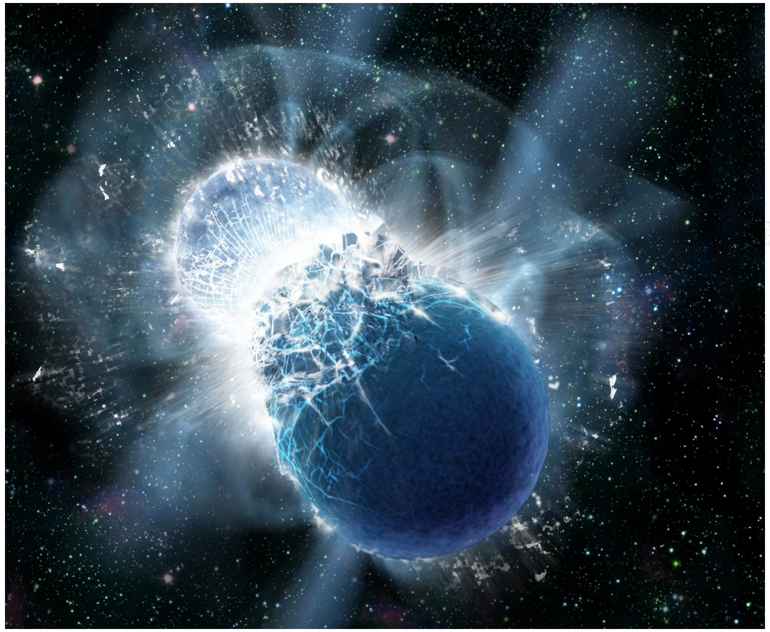
Exploring the **QCD-phase-diagram** with clusters as **experimental observables.**

Understanding the **production of clusters** in relativistic heavy-ion collisions:

How can **weakly bound** clusters survive in the **hot and dense** environment of a HICs?

’Ice in fire puzzle‘





Challenge

Modeling the **time evolution of cluster formation** and the origin of their production.

Modelling of cluster formation in HIC

Statistical models

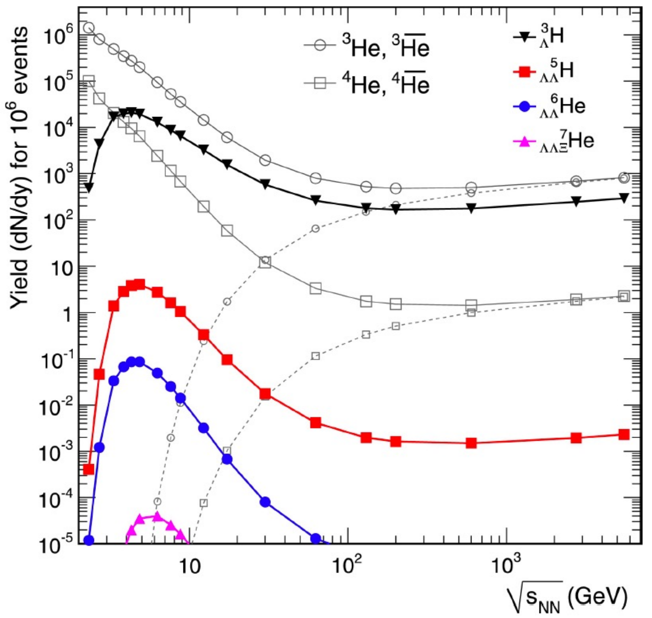
- Production of nuclei depending on T and μ_B at chemical freeze-out & particle mass

Coalescence models

- Formation of nuclei by nucleons & hyperons that are close in coordinate and momentum spaces at freeze-out time

Hybrid models for cluster production

- sudden transition from a dynamical model to clusterisation e.g . UrQMD + afterburner



A. Andronic et al., Phys. Lett. B697 (2011) 203-207.

=> no dynamical cluster formation during time evolution

=> no information on the dynamics of clusters formation & microscopic origin

Modeling of dynamic cluster formation in HIC

Dynamical modeling of cluster formation based on interactions.

Interaction mechanism in transport models:

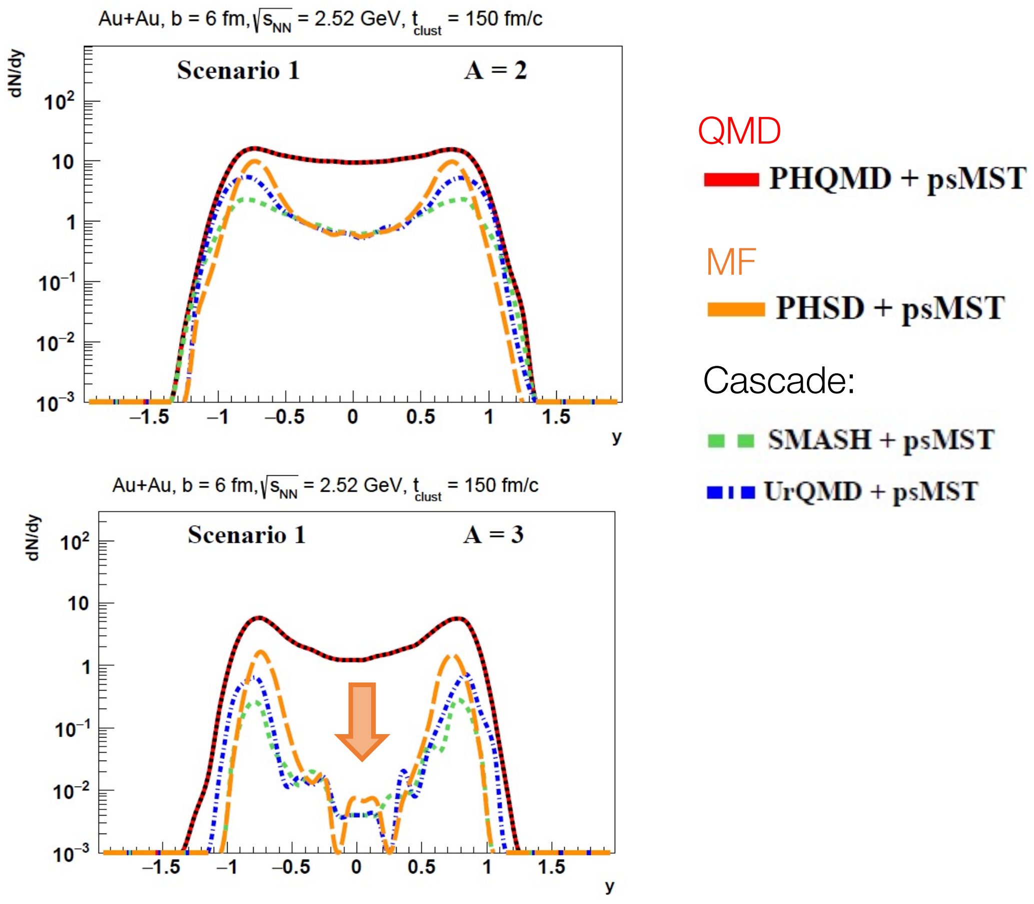
- via potential interaction - potential mechanism this talk
- by scattering - kinetic mechanism

=> Cluster formation is sensitive to nucleon dynamics.

Modeling of nucleon-nucleon potential interactions:

- QMD (quantum-molecular dynamics) – allows to keep correlations
- MF (mean-field based models) – correlations are smeared out
- Cascade – correlations by potential interactions missing

Cluster at final time



V. Kireyeu, Phys.Rev.C 103 (2021) 5

Parton-Hadron-Quantum-Molecular Dynamics

PHQMD

= n-body microscopic transport approach for the description of heavy-ion dynamics with dynamical cluster formation from low to ultra-relativistic energies

Relativistic considerations + Correlations between nucleons + Cluster recognition

Initial A+A collisions | Formation of QGP | Partonic phase | Hadronization | Hadronic phase

QMD Initialization nuclei | propagation of baryons

local $\varepsilon > \varepsilon_c$ dissolution of pre-hadrons

propagation of partons

interactions of partons | collision integral
interactions of hadrons

PHSD Primary collisions pre-hadronic states

propagation of mesons

Cluster recognition: MST or SACA

J. Aichelinet al., PRC 101 (2020) 044905

PHSD: W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168(2009)

QMD propagation

= n-body transport approach

Generalized Ritz variational principle $\delta \int_{t_1}^{t_2} dt < \psi(t)|i\frac{d}{dt} - H|\psi(t) >= 0.$

Ansatz: trial wave function for one particle "i" : Gaussian with width L centered at r_{i0}, p_{i0} Aichelin Phys. Rept. 202 (1991)

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

$L = 4.33\ \mathrm{fm}^2$

Assume that $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$ for N particles (neglecting antisymmetrization!)

Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:

$$\dot{r_{i0}} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p_{i0}} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

J. Aichelin et al., PRC 101 (2020) 044905

QMD potentials and EoS

expectation value of Hamiltonian

$$\langle H\rangle = \sum_i \langle H_i\rangle = \sum_i (\langle T_i\rangle + \sum_{j\neq i}\langle V_{i,j}\rangle) \;=\; \sum_i (\sqrt{p_{i0}^2+m^2}-m) \;+\; \sum_i \langle V_{Skyrme}(\mathbf{r}_{i0},t) + V_{coul}(\mathbf{r}_{i0},t)\rangle$$

Skyrme interaction ('static'): Effective density dependent nucleon-nucleon interactions

$$\langle V_{Skyrme}(\mathbf{r_{i0}},t)\rangle \;=\; \alpha\left(\frac{\rho_{int}(\mathbf{r_{i0}},t)}{\rho_0}\right) + \beta\left(\frac{\rho_{int}(\mathbf{r_{i0}},t)}{\rho_0}\right)^{\gamma}$$

Interaction density (with relativistic extension):

$$\rho_{int}(\mathbf{r_{i0}},t) \;\to\; C\sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r^T_{i0}}(t)-\mathbf{r^T_{j0}}(t))^2}\;\; \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r^L_{i0}}(t)-\mathbf{r^L_{j0}}(t))^2}$$

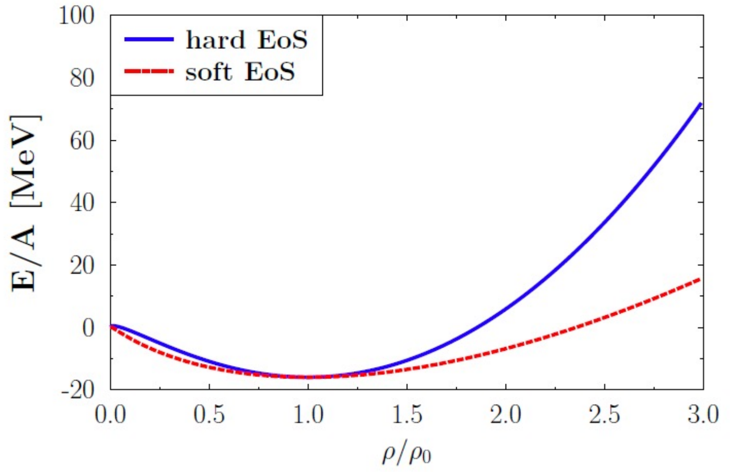
Parameter of the nuclear equation of state in PHQMD

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

compression modulus K

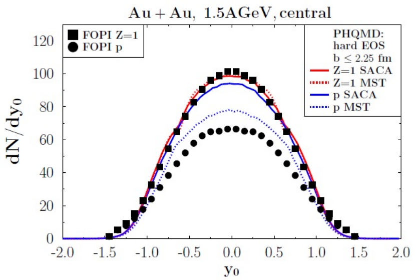
$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}\Big|_{\rho=\rho_0}.$$

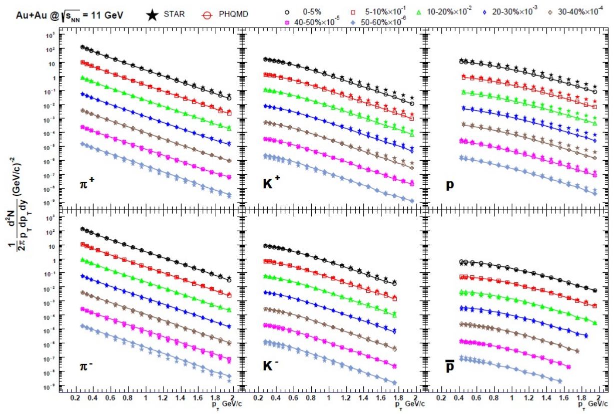
EoS for infinite matter at rest

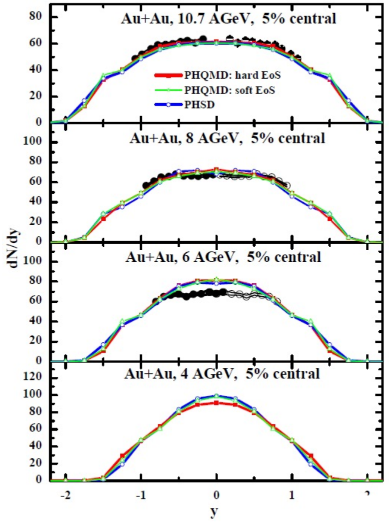


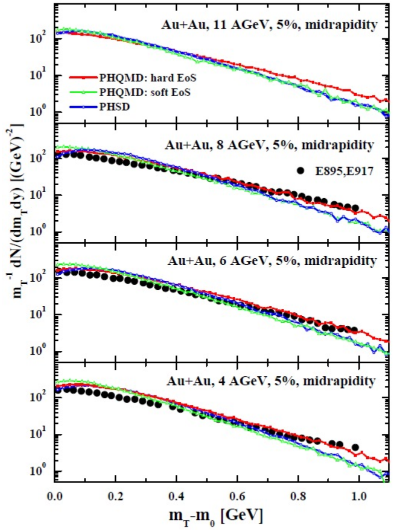
*Work in progress: implementation of momentum-dependent potential (M. Winn)

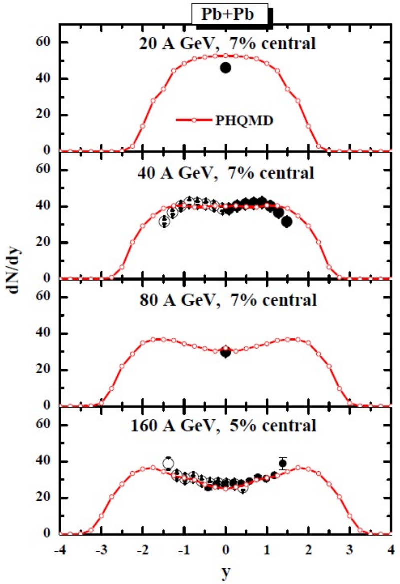
Highlights: PHQMD ‚bulk‘ dynamics from SIS to RHIC

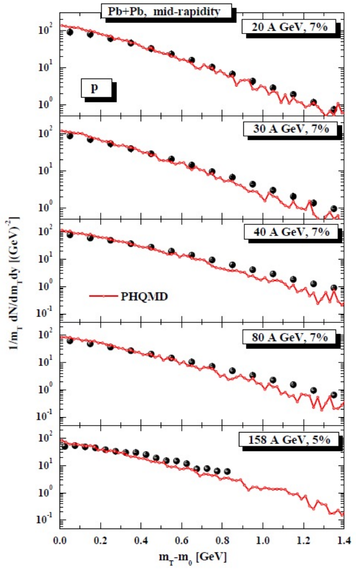


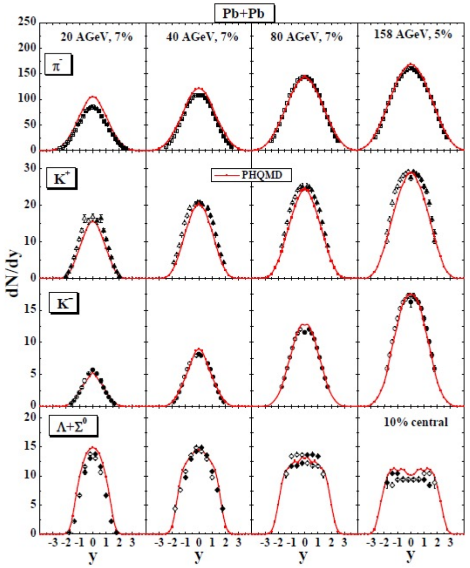


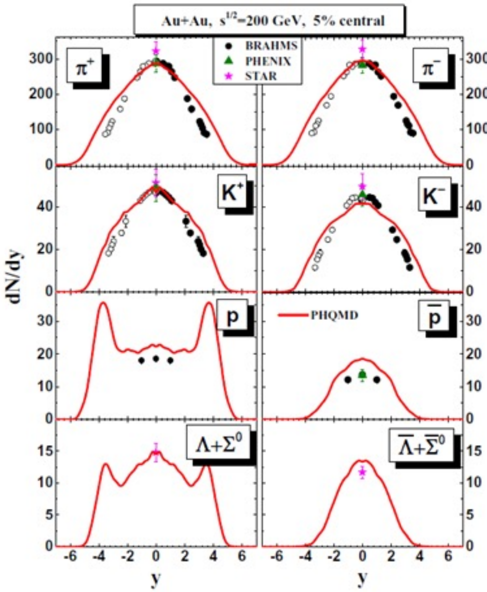












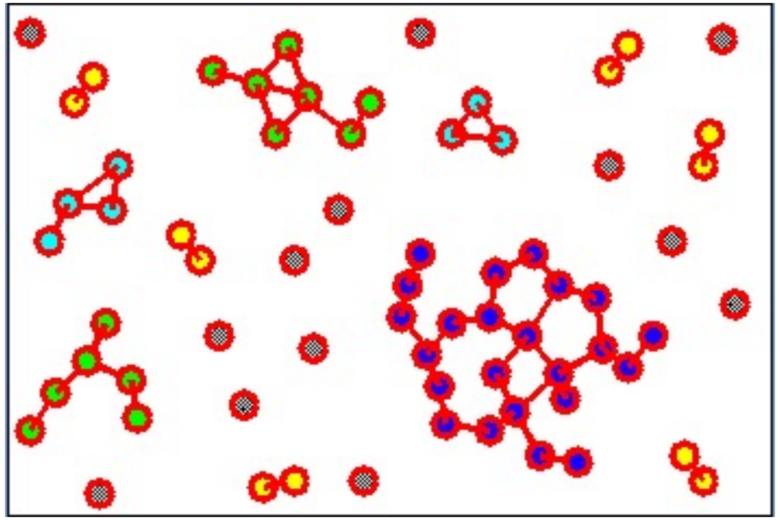
PHQMD: J. Aichelin et al., PRC 101 (2020) 044905

=> PHQMD provides a good description of hadronic ‘bulk’ observables from SIS to RHIC energies.

Minimum Spanning Tree (MST)

Cluster criterion: distance of nuclei

Algorithm: search for accumulations of particles in coordinate space



1. Two particles i & j are bound if:

$$|r_i - r_j| < 4.0 \text{ fm}$$

2. Particle is bound to cluster if bound with at least one particle of cluster

Remark: additional momentum cuts lead to a small changes: particles with large relative momentum are mostly not at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)

Cluster stability over time

Semiclassical model QMD:

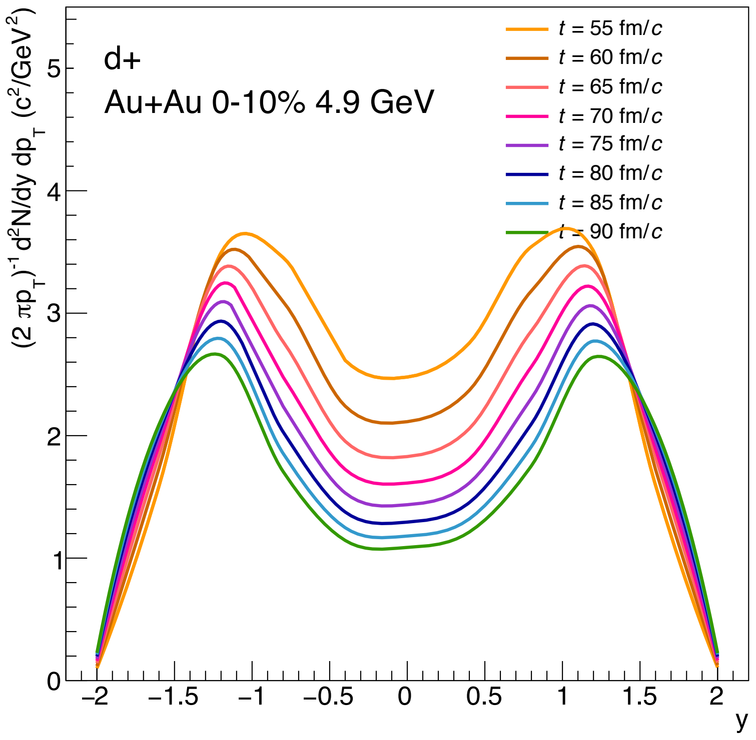
Clusters not described as ‘quantum objects’
with minimal average kinetic energy for nucleons
=> cluster-nucleon can have enough kinetic energy to escape cluster
=> “bound” cluster at time t can spontaneously dissolve at t + Δt

Skyrme potential is not relativistic:
calculation of cluster binding energy in cluster-system after Lorentz-boost
=> baryons have different times in cluster-frame
=> sign of binding energy can change

=> cluster multiplicities decrease over time

=> time selection needed for every nuclei specie for correct multiplicities
=> form of y-, p_T- distribution & ratio of particles don’t change over time

New: Stabilisation procedure aMST



Relativistic fragment formation time for spectra

Consideration of time dilatation for the cluster identification:

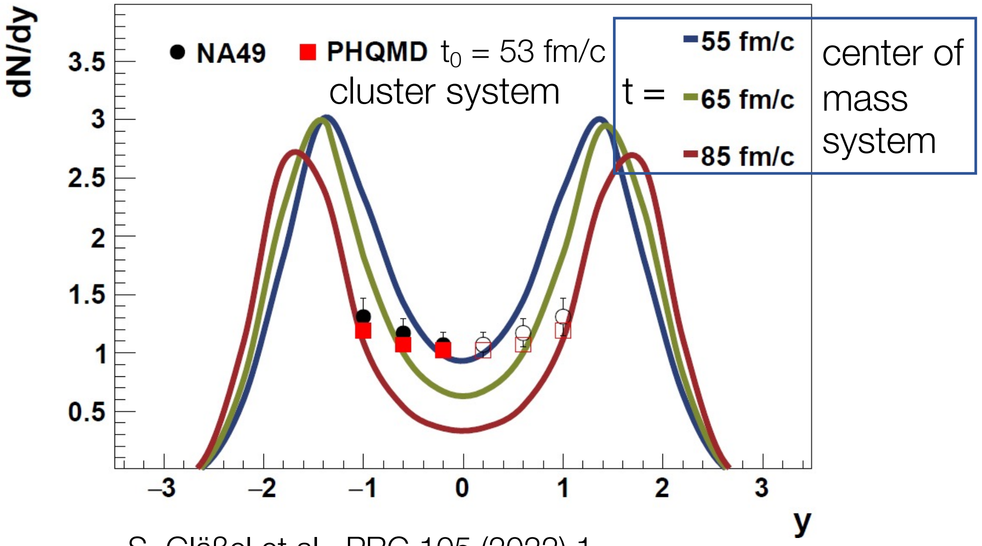
time t_0 in cluster rest-system is delayed vs. time t in the center of mass system depending on y

=> create spectra at the same time in the cluster rest-system t_0

=> transformation: $t = t_0 \cosh(y)$

center of mass system cluster system

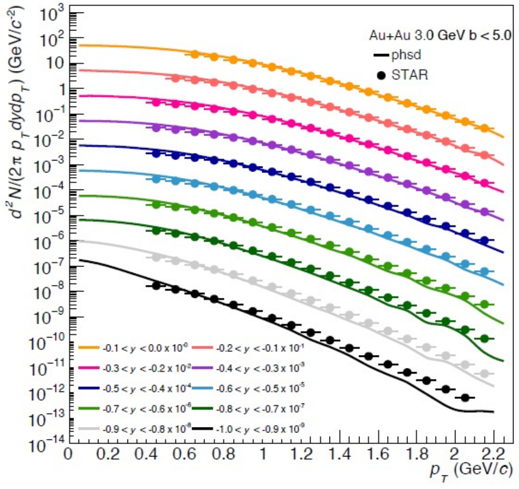
Pb-Pb 8.8 GeV 0 - 10 %



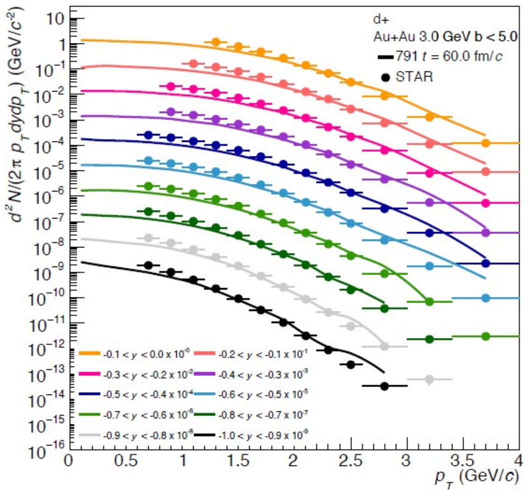
S. Gläßel et al., PRC 105 (2022) 1

Light cluster production at $\sqrt{s_{NN}}$ = 3 GeV

protons



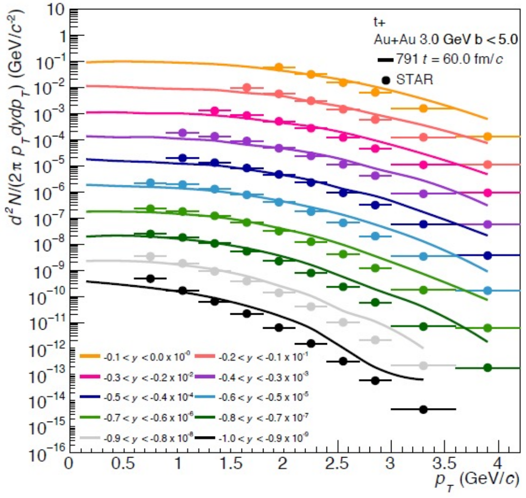
deuterons



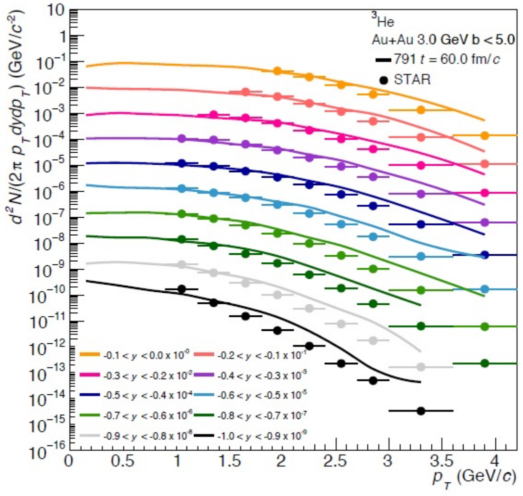
=> PHQMD reproduces STAR* p_T-spectra for protons, deuterons, tritions ^{3}He and ^{4}He.

* (preliminary) STAR data – talk by Hui Liu at QM'2022

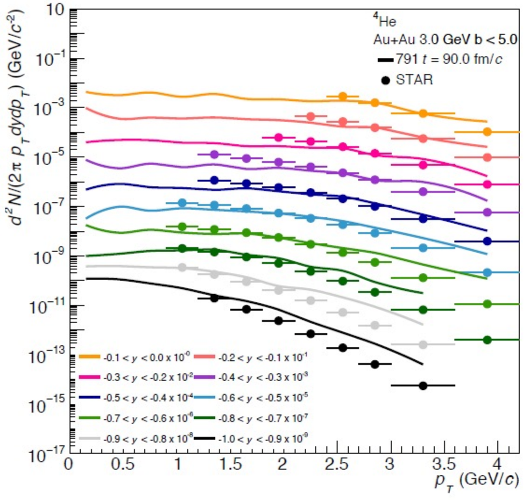
tritons



^{3}He

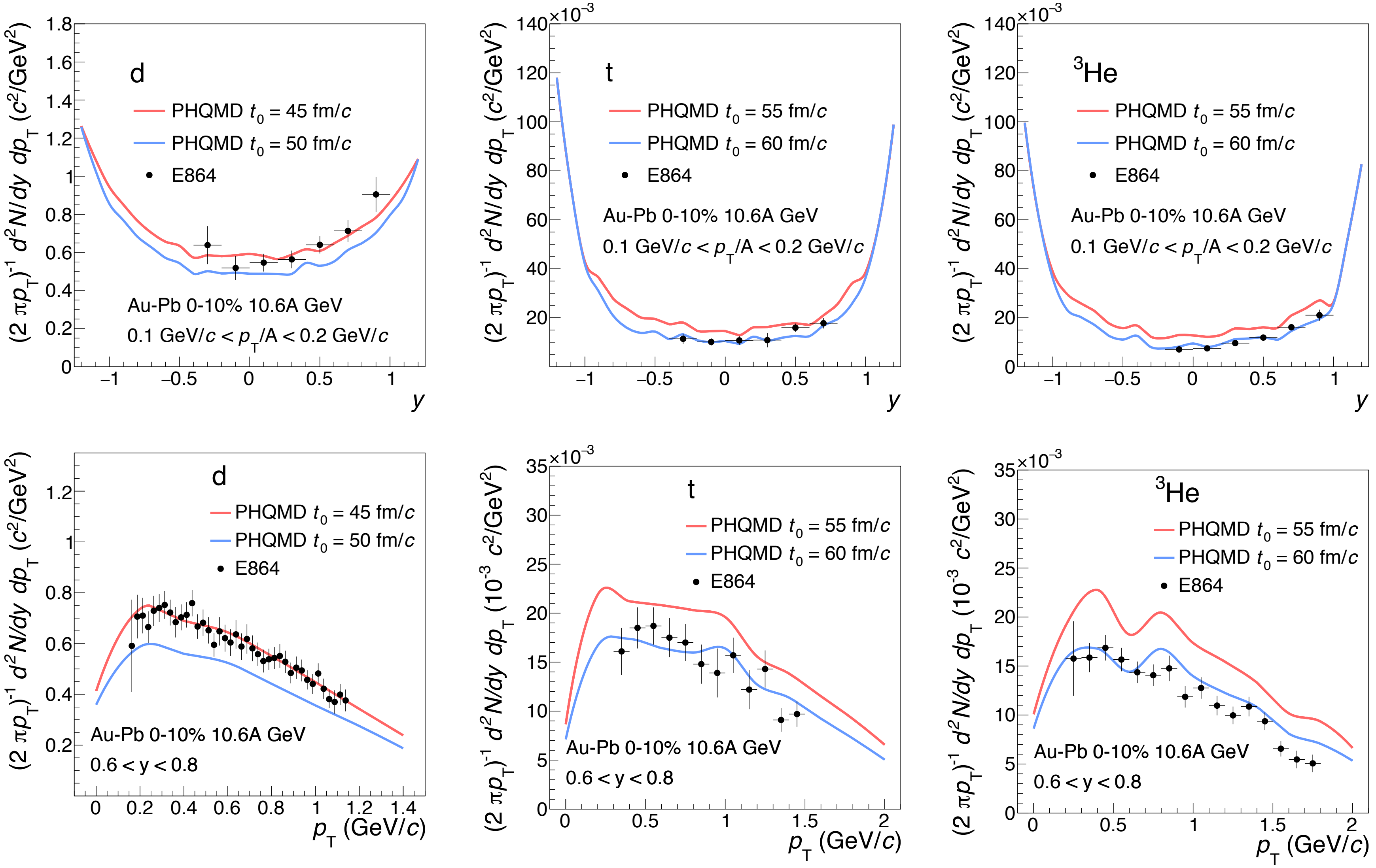


^{4}He



S. Gläßel, PHQMD calculations: arXiv:2208.11802

Light cluster production at AGS energies



$t = t_0 \cosh(y)$

=> y- & p_T-spectra for deuterons, tritions and ^{3}He from E864* at $\sqrt{s_{NN}}$ = 4.9 GeV are nicely described by PHQMD.

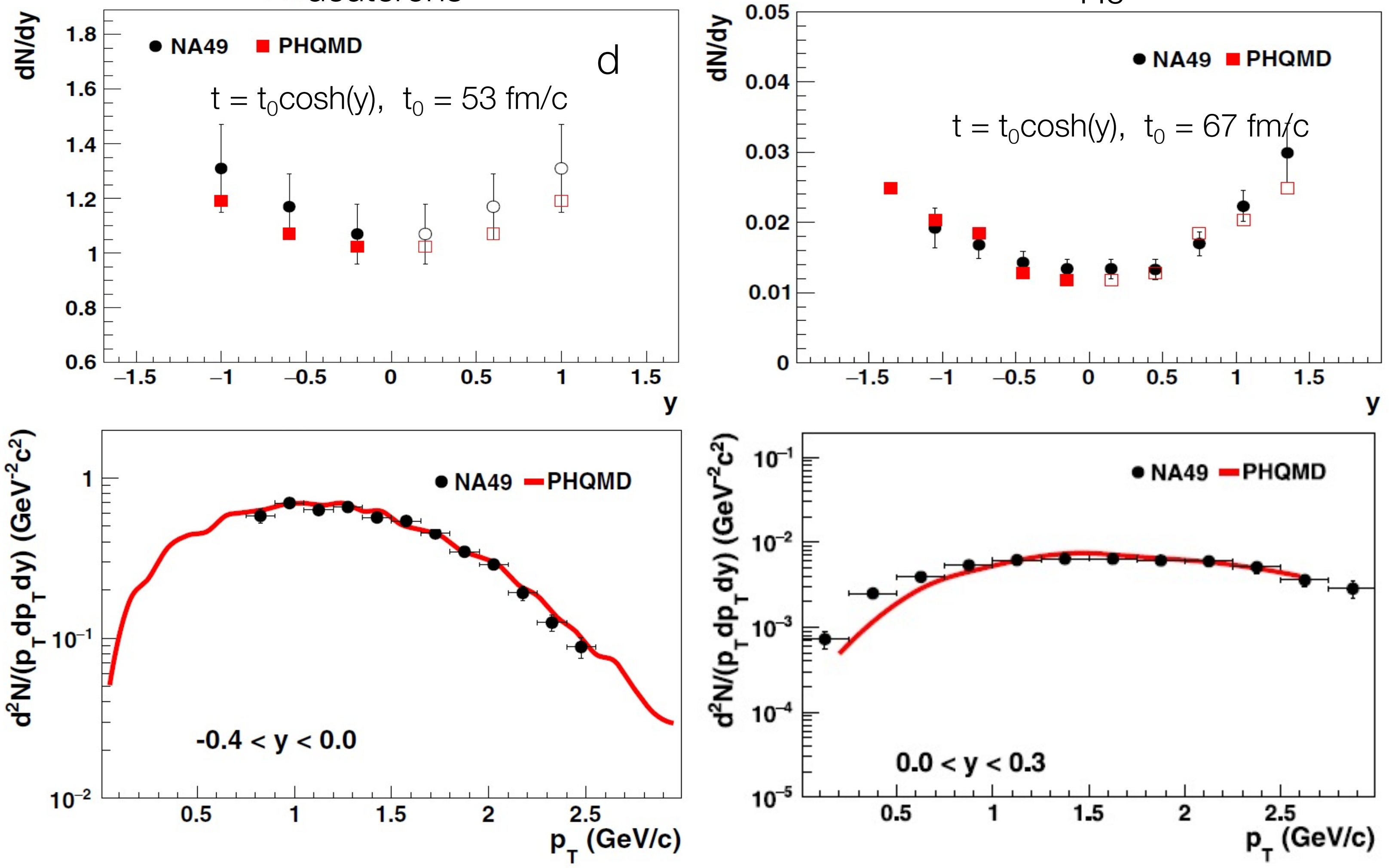
S. G. et al., PRC 105 (2022) 1

*T. A. Armstrong et al: „Measurements of light nuclei production in pbeam=11.5A GeV/c Au+Pb heavy-ion collisions", PHYSICAL REVIEW C, VOLUME 61, 064908 (2002)

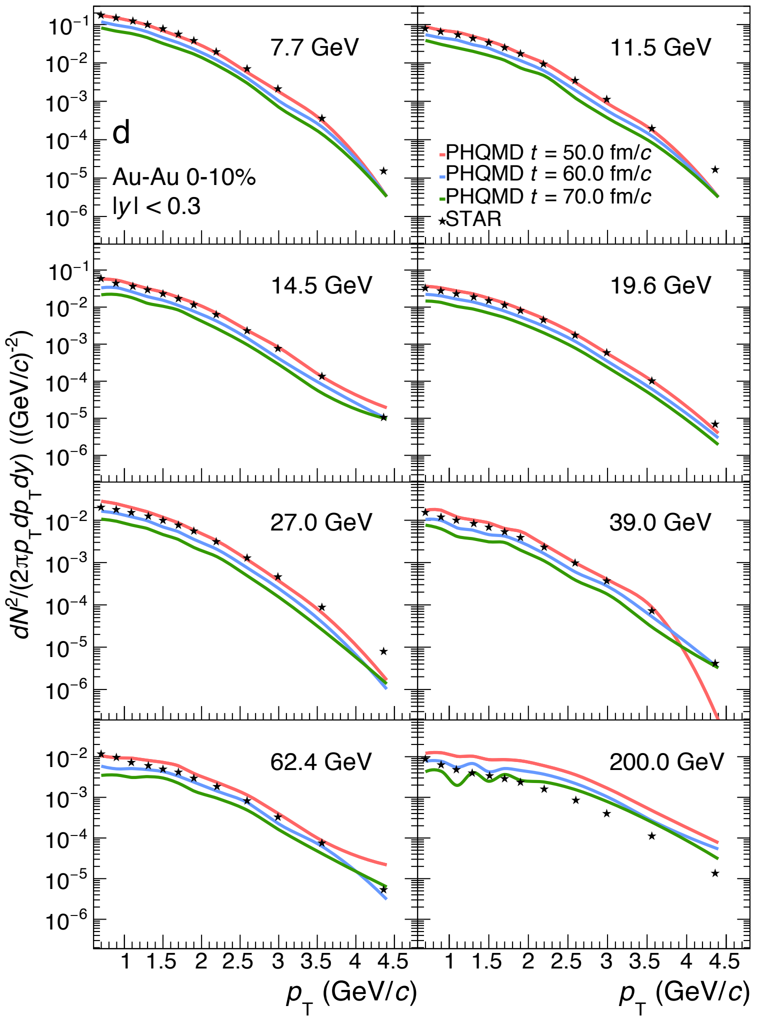
Cluster production at SPS & RHIC energies

NA49 $\sqrt{s_{NN}}$ = 8.8 GeV

deuterons　　　　^{3}He



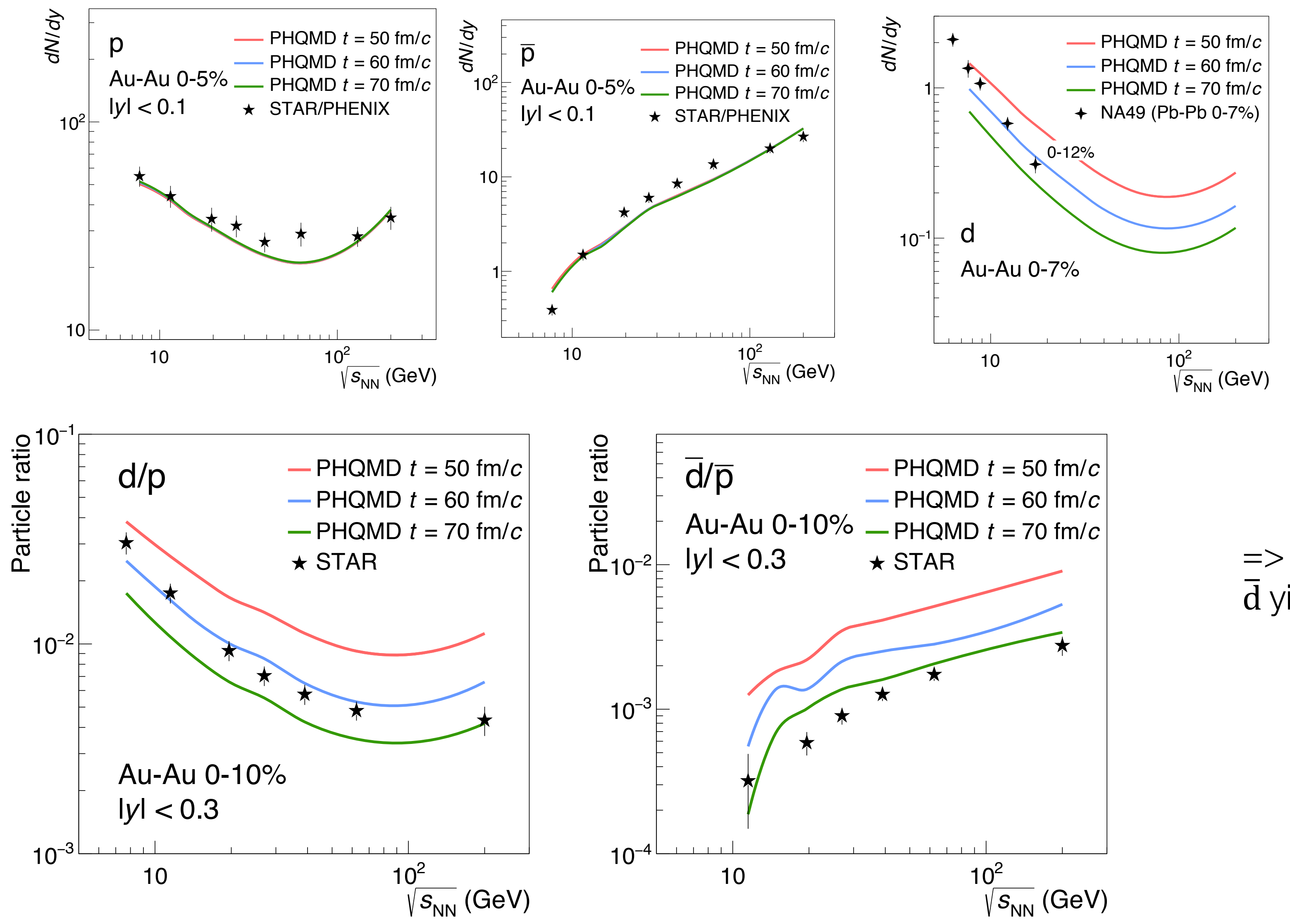
STAR $\sqrt{s_{NN}}$ = 7.7 GeV – 200 GeV



=> The PHQMD results for d and ^{3}He agree with NA49 and STAR data.

T. Anticic et al. (NA49), Phys. Rev. C 94, 044906 (2016)　J. Adam et al. (STAR), Phys. Rev. C 99, 064905 (2019)　S. G. et al., Phys. Rev. C 105 (2022) 1

Excitation function of multiplicity of p, $\overline{p}$, d & p/d-ratio



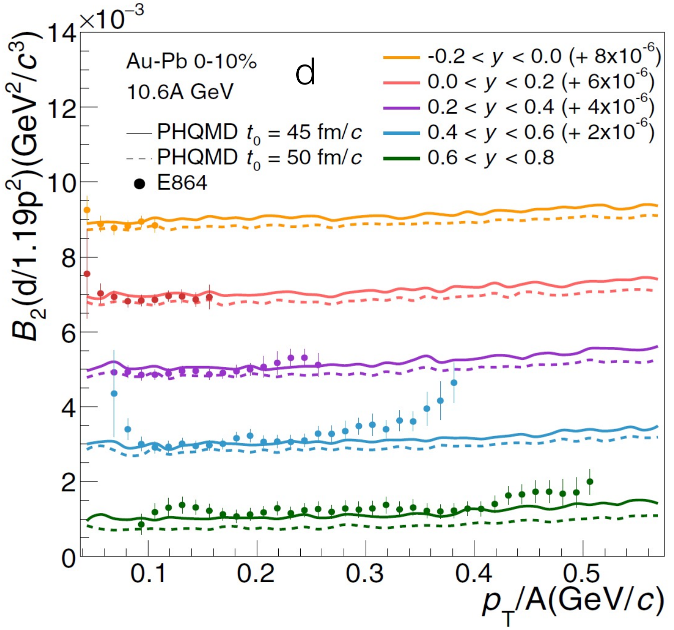
=> The p, $\overline{p}$ yields at y~0 are stable, the d, $\overline{d}$ yields are best described at t= 60-70 fm/c.

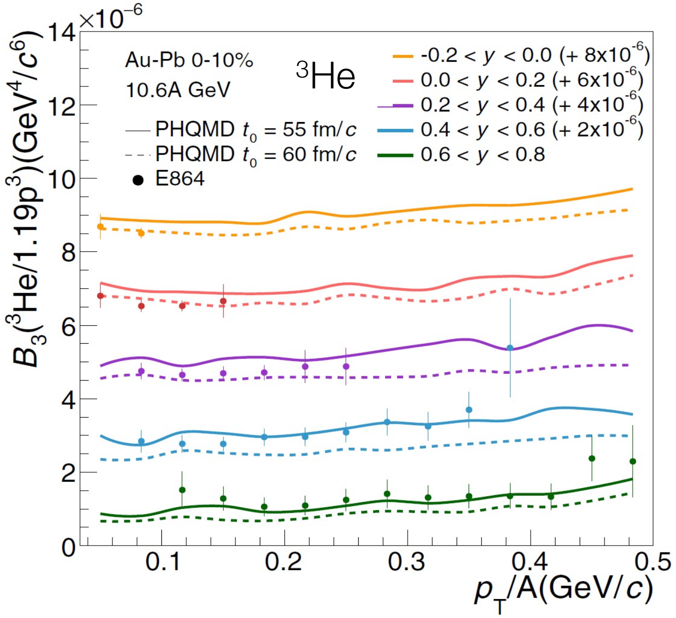
S. G. et al., PRC 105 (2022) 1 J. Adam et al. (STAR), Phys. Rev. C 99, 064905 (2019)

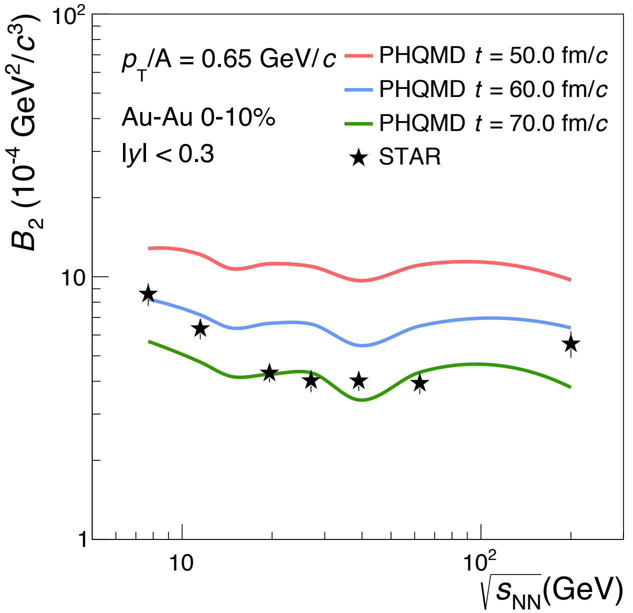
Coalescence parameter for d and ^{3}He

E864 $\sqrt{s_{NN}}$ = 4.9 GeV

STAR $\sqrt{s_{NN}}$ = 7.7 GeV – 200 GeV







Coalescence parameter:

$$B_2 = \frac{E_d \frac{d^3 N_d}{d^3 P_d}}{\left(E_p \frac{d^3 N_p}{d^3 p_p}\Big|_{p_p = P_d/2}\right)^2}$$

=> Good description of B_A for energies between 5 – 200 GeV.

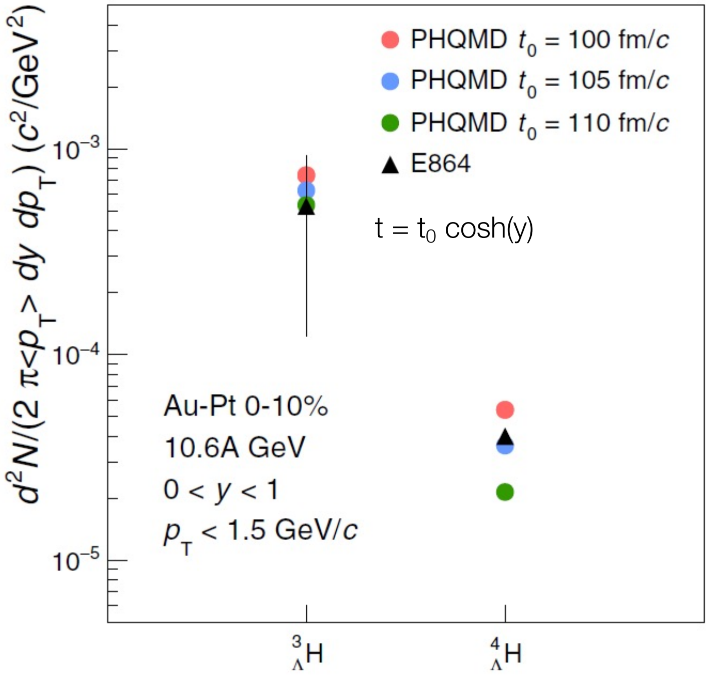
=> No remarkable structure for B_A as a function of p_T, only slight increase.

=> Probability that baryons with p_T/A form a cluster with size A more or less independent of p_T, only increases slightly.

S. G. et al., Phys. Rev. C 105 (2022) 1

Hypernuclei production at $\sqrt{s_{NN}}$ = 3.0 and 4.9 GeV

E864 $\sqrt{s_{NN}}$ = 4.9 GeV

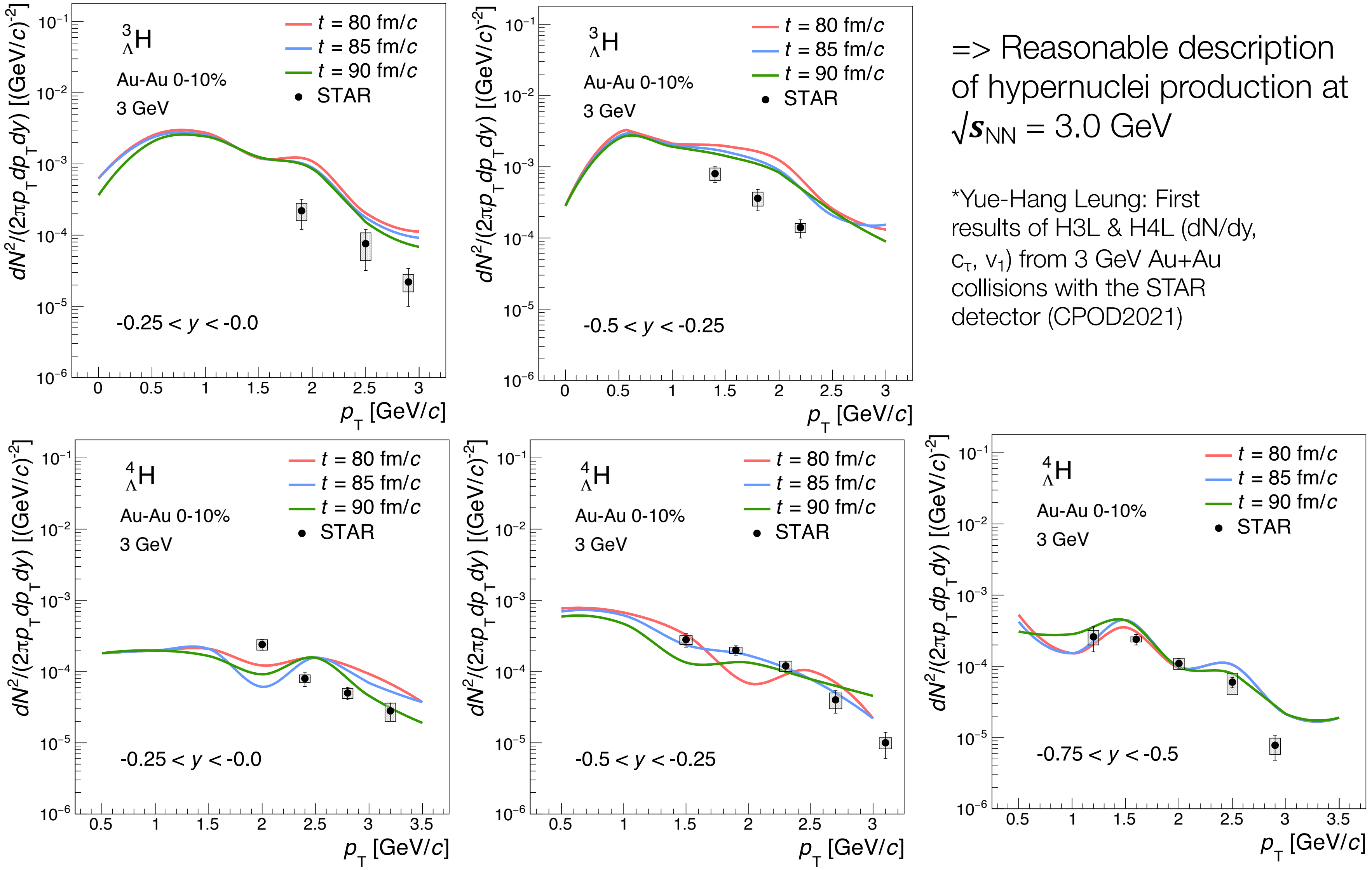


Assumption for nucleon-hyperon potential: $V_{NL} = 2/3\ V_{NN}$

=> trend of the experimental STAR* $^3_\Lambda$H & $^4_\Lambda$H p_T-spectra at $\sqrt{s_{NN}}$=3 is produced well

=> yields are slightly overpredicted

STAR $\sqrt{s_{NN}}$ = 3.0 GeV



S. G. et al., Phys. Rev. C 105 (2022) 1

=> Reasonable description of hypernuclei production at $\sqrt{s_{NN}}$ = 3.0 GeV

*Yue-Hang Leung: First results of H3L & H4L (dN/dy, c_T, v_1) from 3 GeV Au+Au collisions with the STAR detector (CPOD2021)

Stabilisation procedure Advanced MST

Clusters are not stable over time

1) Clusters not described as 'quantum objects' : cluster can dissolve

2) Skyrme potential is not relativistic: binding energy can change

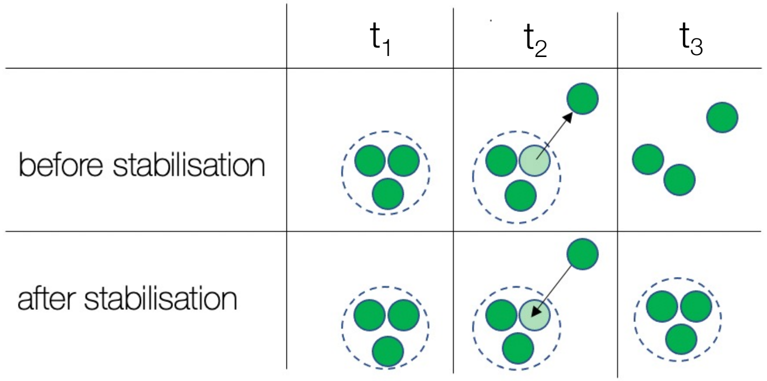
Advanced MST

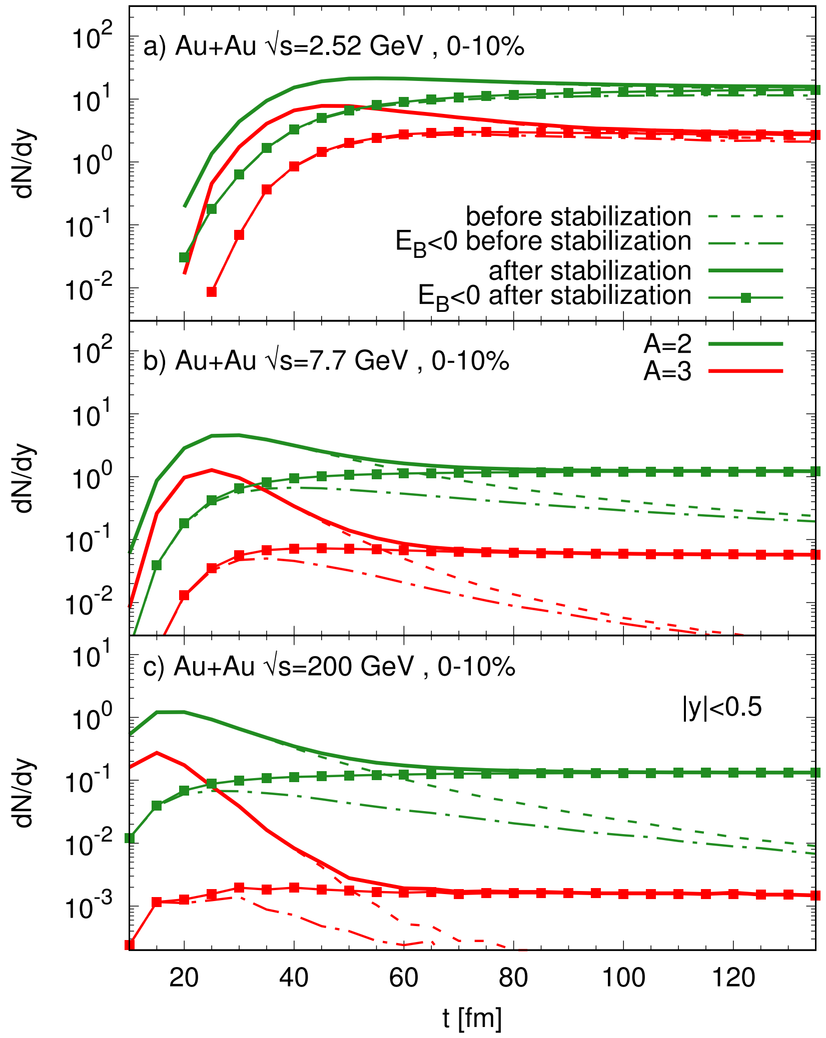
- Consider asymptotic state: clusters and free nucleons
- Track the freezeout-time for each baryon = time of the last collision

a) If time of cluster disintegration > baryon freeze-out time (and if $E_B < 0$) :
   => restore cluster

b) If sign of binding energy changes after baryon freeze-out time:
   => restore E_B

+ extra condition: $E_B < 0$ negative binding energy for identified clusters

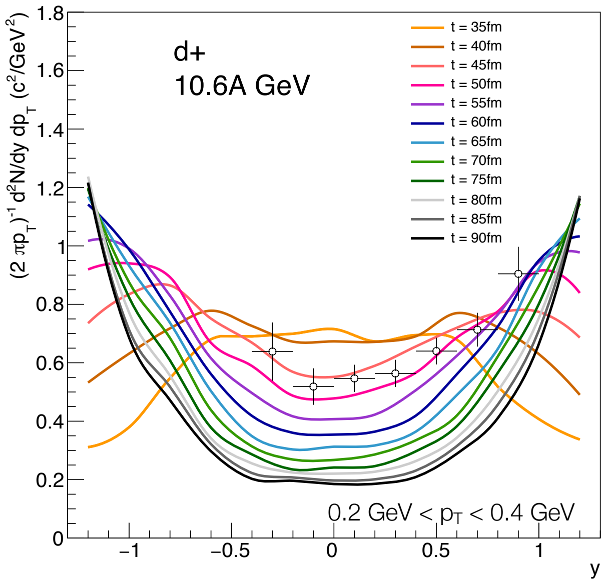


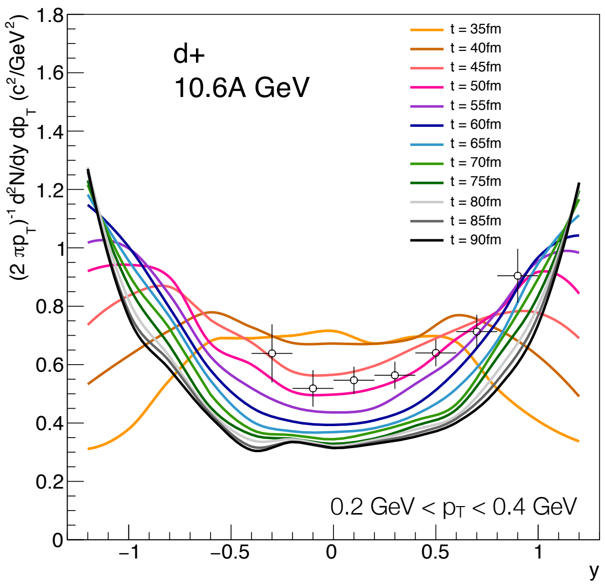


dN/dy time evolution for deuteron at $\sqrt{s_{NN}} = 4.9$ GeV

no stabilisation | with stabilisation

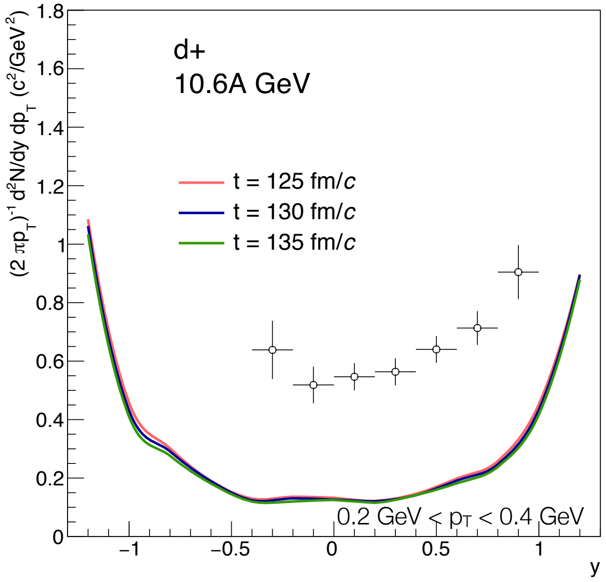
35 fm < t < 90 fm

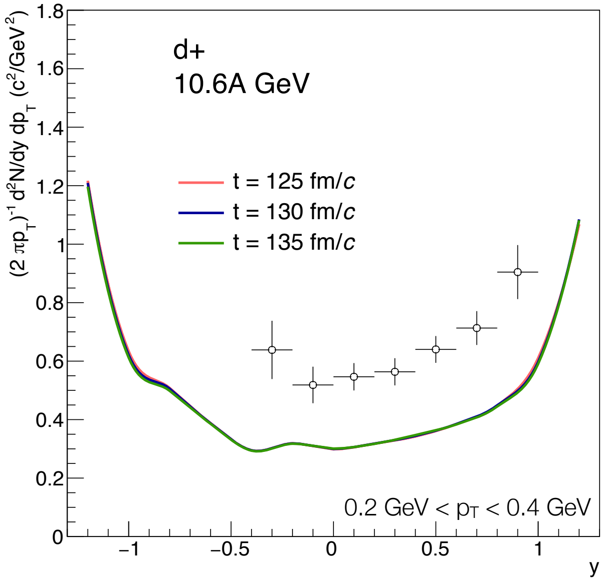




=> With stabilisation, multiplicity starts to drop slower at around 60 fm/c.

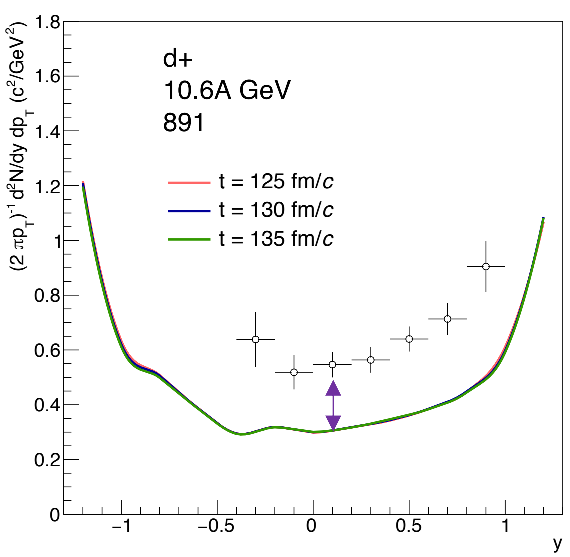
125 fm < t < 135 fm

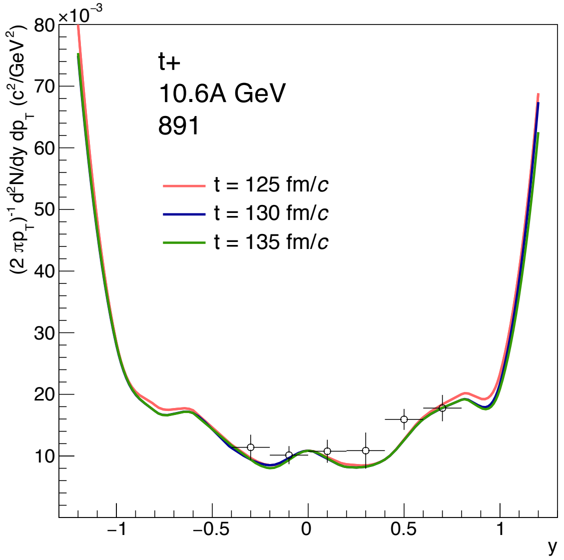


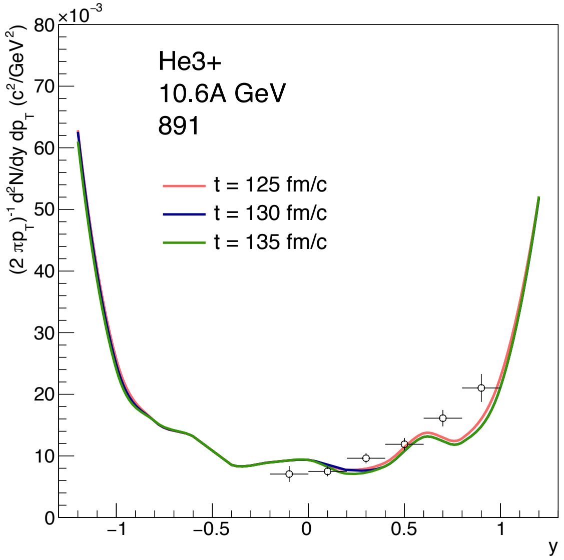


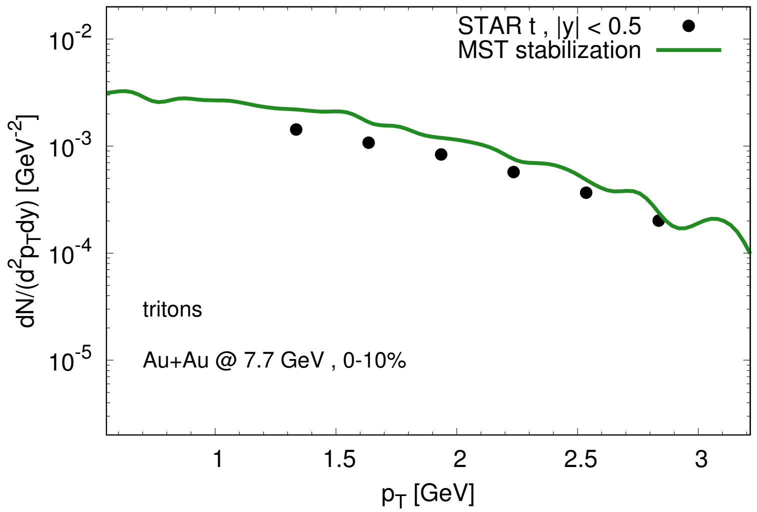
=> At the final time, deuteron multiplicity is ~2 times higher with stabilisation.

Stable light nuclei at $\sqrt{s_{NN}}$ = 4.9 and 7.7 GeV with aMST









PHQMD with stabilisation procedure fits the experimental data at $\sqrt{s_{NN}}$ = 4.9 and 7.7 GeV very well for triton and ^{3}He.

Deuterons are underestimated => contribution of deuterons formed by inelastic scattering.

See talk on Tues, 4.30 pm by Elena Bratkovskaya.

Summary & Conclusion

PHQMD

- is a microscopic n-body transport approach to describe HIC and dynamical cluster formation
- is a combined model: PHSD + QMD + MST | SACA
- transports baryons with QMD to keep potential interactions among baryons and allow cluster formation
- identifies with MST the clusters based on the distance between baryons

=> **reproduces cluster and hypercluster** data on dN/dy, dN/dp_T and ratios from **AGS to top RHIC energies**

- The newly developed stabilisation routine in advanced MST overcomes time dependency of cluster multiplicities.

Outlook

- study of hyper-nuclei with more realistic potentials
- extension to LHC energies

Talk on Tuesday, 4.30 pm by Elena Bratkovskaya:

How can weakly bound clusters survive in the hot and dense environment of a heavy ion collision?

Deuteron production based on kinetic interactions / scattering

THANK YOU FOR YOUR ATTENTION.