

# Latest developments on the coalescence model and the kinetic approach

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# Outline

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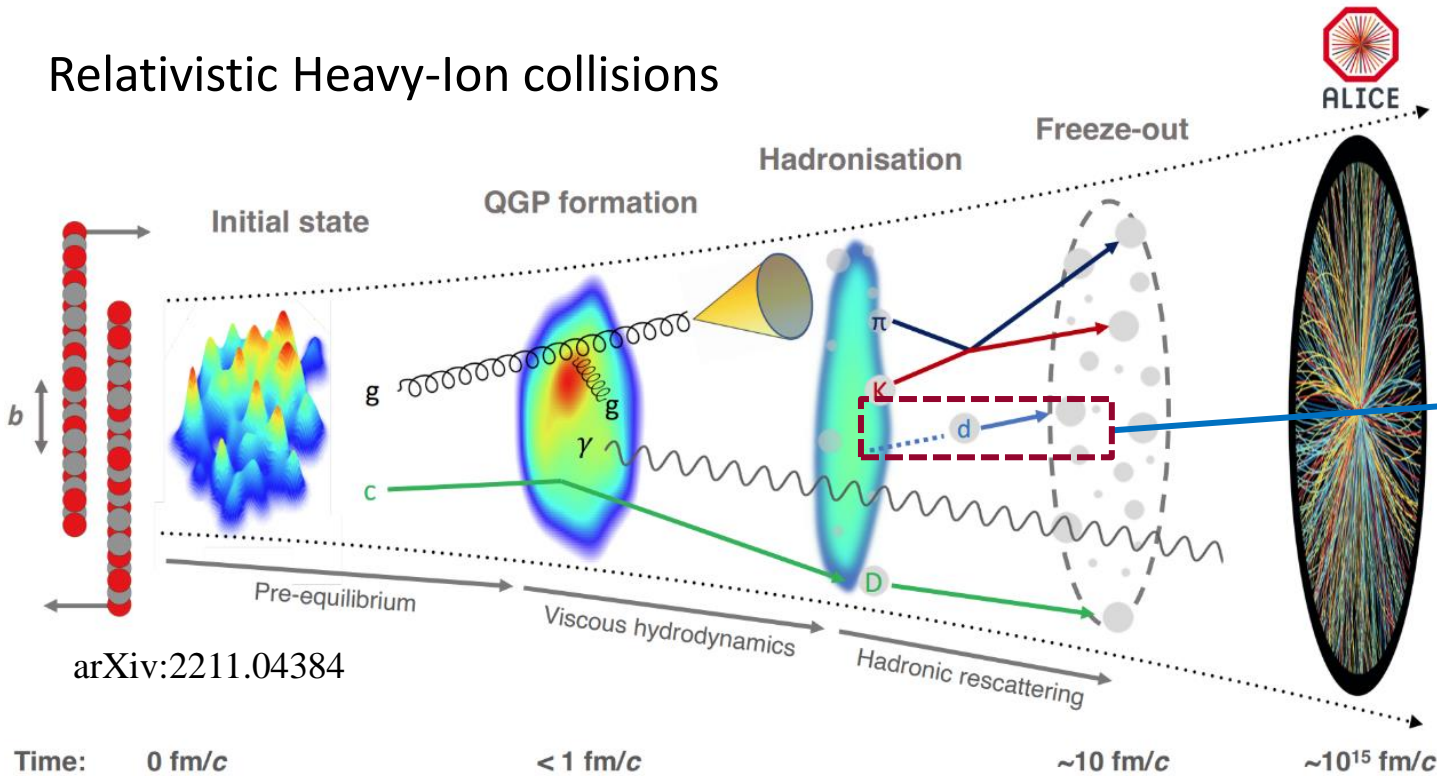
1. Little Bang Nucleosynthesis
2. Quantum correction from coalescence
3. **Hadronic re-scattering effects within a kinetic approach**
4. Summary and Outlook

# Little Bang Nucleosynthesis

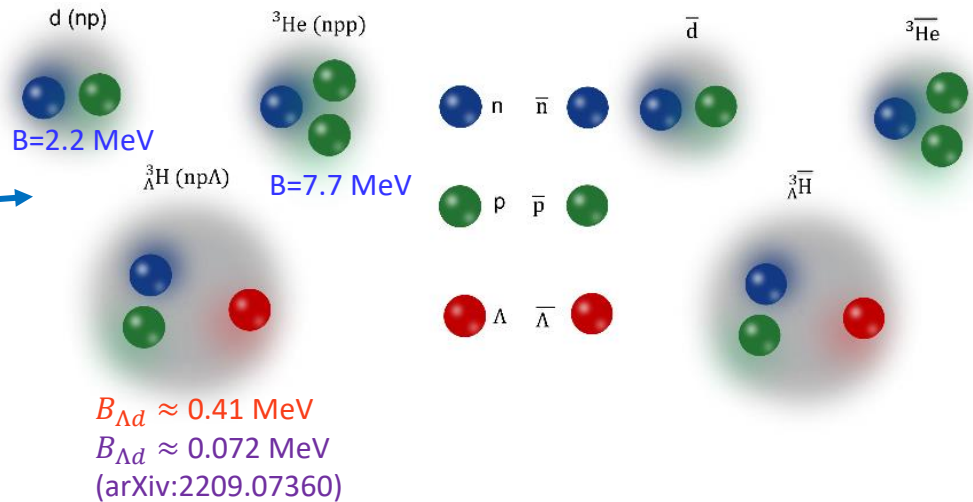
(1)

J. Chen et al., Phys. Rep. 760, 1 (2018); P. Braun-Munzinger and B. Donigus NPA987, 144 (2019)

## Relativistic Heavy-Ion collisions



## Light (anti-)(hyper-)nuclei



STAR (Science 328, 58(2010); Nature 473,353(2011); Nat. Phys. 16 409(2020))

ALICE (Nature Phys. 11,811(2015); Phys. Rev. Lett. 128, 252003 (2022); Nature Phys. 19, 61 (2023);)

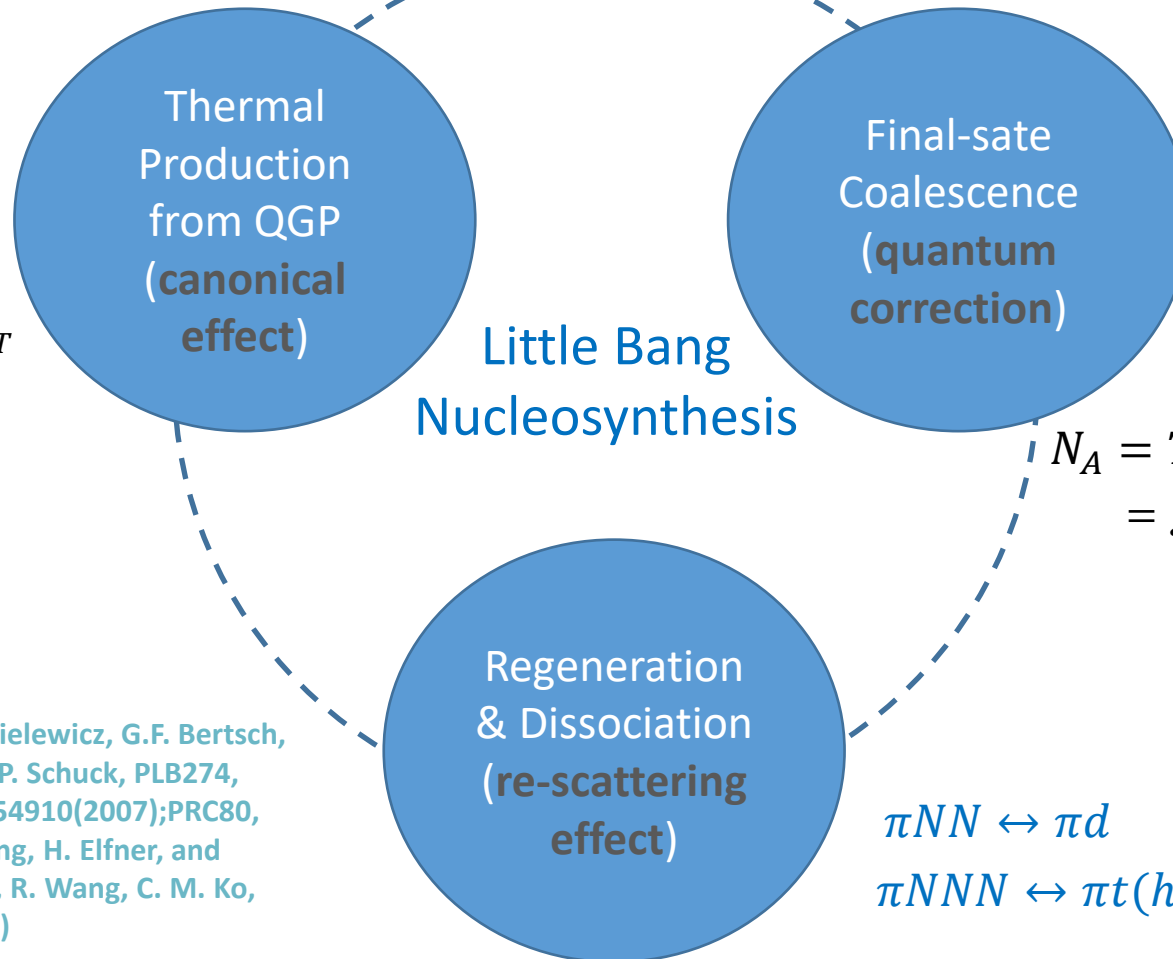
1. Rarely produced, suppressed by  $e^{-m_A/T}$
2. Binding energies ( $E_B$ )  $\ll T_c$  ( $\sim 154$  MeV)  $\ll m_N$  (938 MeV)

The size  $r \sim \frac{1}{\sqrt{4\mu E_B}}$ , ( $r_d \sim 2$  fm,  $r_{3\text{He}} \sim 2$  fm,  $r_{\Lambda H} \sim 5$  fm)

# Main Mechanisms

(2)

A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB 697, 203 (2011)  
 A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018)  
 V. Vovchenko et al., PLB785, 171 (2018); PLB800, 135131 (2020) (Saha Eq.);  
 T. Neidig et al., PLB827,136891(2022)(Rate Eq.);...



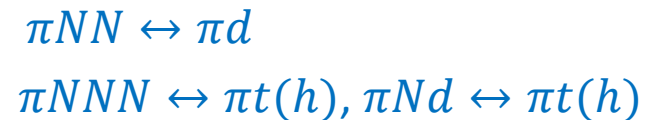
$$N_A \approx g_A V (2\pi m_A T)^{3/2} e^{(A\mu_B - m_A)/T}$$

J. I. Kapusta, Phys. Rev. C 21, 1301 (1980)  
 H. Sato and K. Yazaki, PLB98, 153 (1981);  
 E. Remler, Ann. Phys. 136, 293 (1981);  
 M. Gyulassy, K. Frankel, and E. Remler,  
 NPA402,596 (1983);  
 S. Mrowczynski, J. Phys. G 13, 1089 (1987);  
 S. Leupold and U. Heinz, PRC50, 1110 (1994);  
 R. Scheibl and U. W. Heinz, PRC59. 1585(1999);  
 K. J. Sun, C. M. Ko and B. Donigus,  
 PLB 792, 132 (2019);  
 S. Sombun et al., PRC99, 014901 (2019)  
 F. Bellini et al., PRC99,054905(2019);  
 PRC 103, 014907(2021);  
 W. Zhao et al., PLB 820, 136571(2021);  
 S. Wu et al., arXiv:2205.14302(2022);...

$$N_A = Tr(\hat{\rho}_s \hat{\rho}_A)$$

$$= g_c \int d\Gamma \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

A.Z. Mekjian, PRC17,1051 (1978); P. Danielewicz, G.F. Bertsch,  
 NPA533, 712 (1991); P. Danielewicz and P. Schuck, PLB274,  
 268 (1992); Y. Oh and C. M. Ko PRC76, 054910(2007); PRC80,  
 064902(2009); D. Oliinychenko, L. G. Pang, H. Elfner, and  
 V. Koch, PRC99, 044907 (2019); K. J. Sun, R. Wang, C. M. Ko,  
 Y. G. Ma, and C. Shen, 2207.12532(2022)

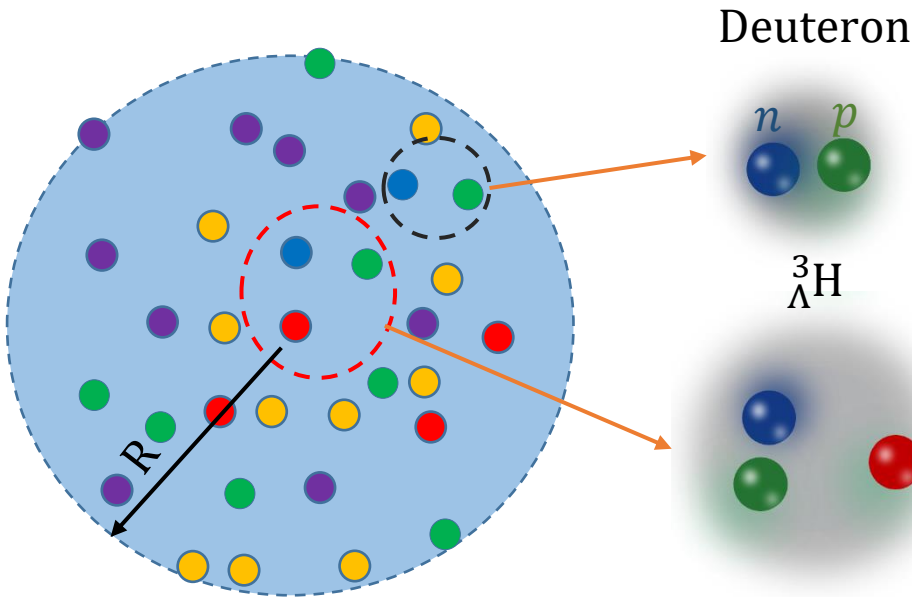


# **Quantum Correction from Coalescence**

# Quantum Correction from Coalescence

(3)

## Coalescence Model



## Quantum Correction (Size Effects)

*K. J. Sun, C. M. Ko, and B. Dögnius,  
Phys. Lett. B792, 132-137(2019)*

$$N_d \propto \frac{1}{\left[1 + \left(\frac{2r_d^2}{3R^2}\right)\right]^{\frac{3}{2}}}$$

Yield

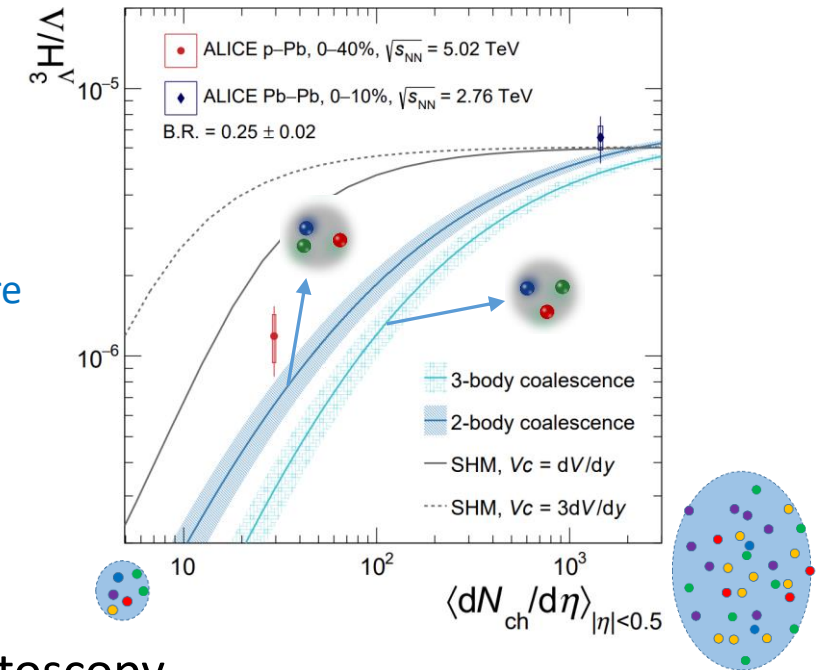
$$N_{\Lambda^3\text{H}} \propto \frac{1}{\left[1 + \left(\frac{r_{\Lambda^3\text{H}}^2}{2R^2}\right)\right]^3}$$

Structure

can be inferred from Femtoscopy

## ALICE Results ( ${}^3\Lambda\text{H}$ )

*Phys. Rev. Lett. 128, 055203(2022)*



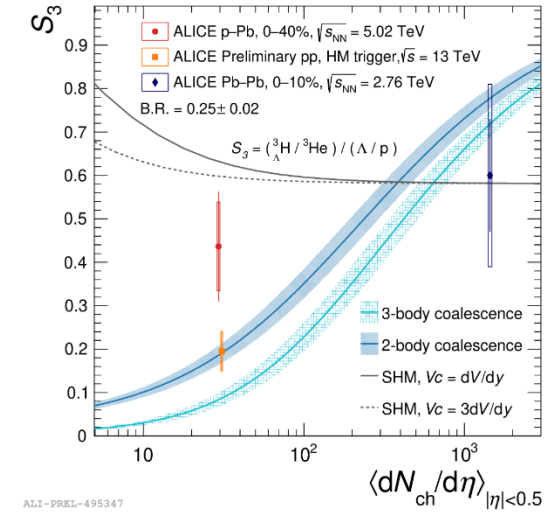
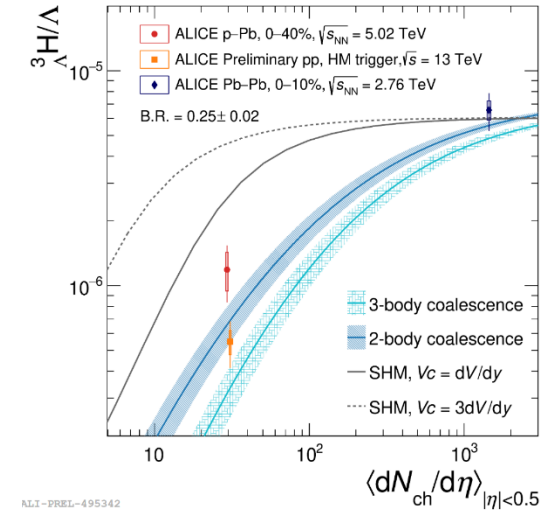
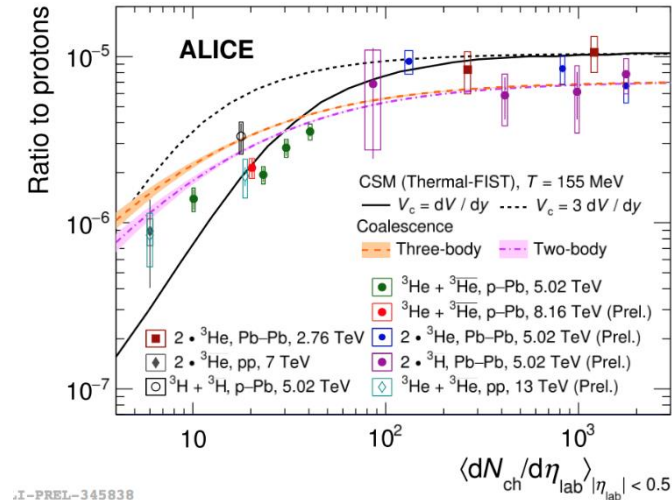
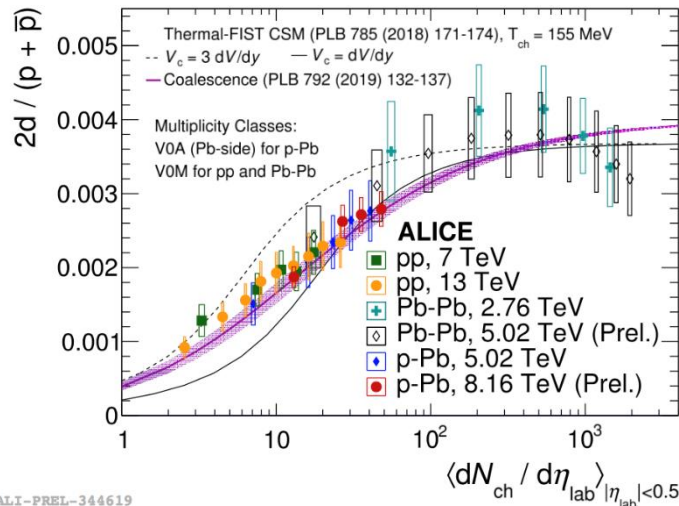
- R. Scheibl and U. W. Heinz, PRC59, 1585(1999);
- F. Bellini et al., PRC99,054905(2019);
- K. J. Sun, C. M. Ko and B. Dögnius, PLB 792, 132 (2019);

# LHC Energies

(4)

L. Barioglio for ALICE Collaboration. PoS LHCP2021 (2021) 056;

See Luca's Talk



**CSM: Baryon number conservation leads to canonical suppression of light nuclei production**

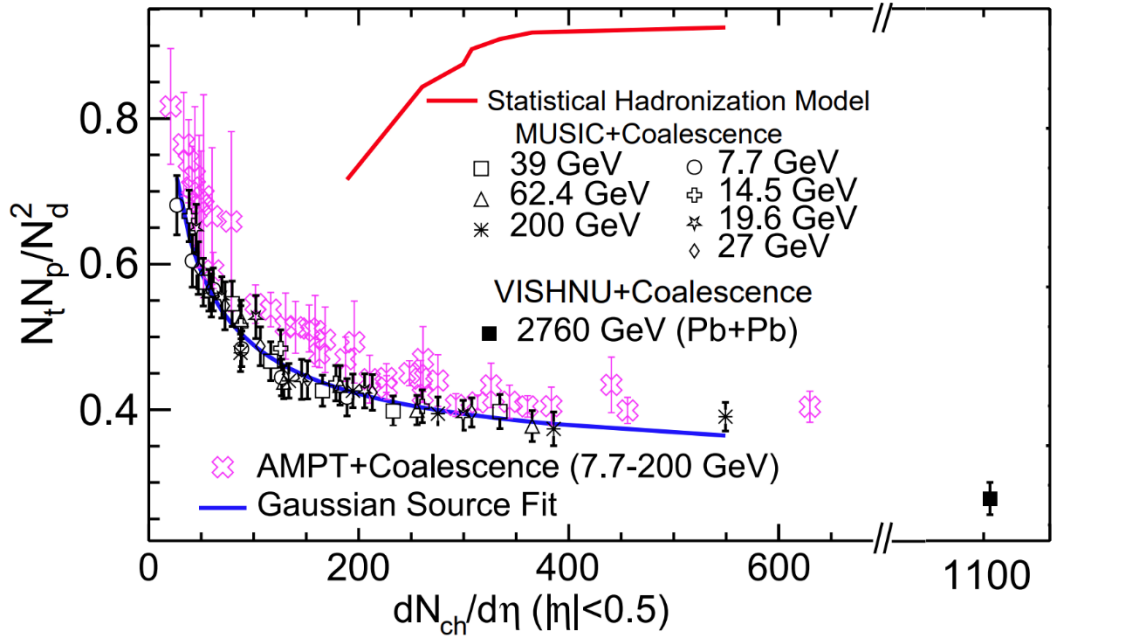
**Coal: Finite nuclei sizes lead to suppression of deuterium and helium-3 yields in collision of small system**

*(better description on hypertriton production in p+p collisions)*

Canonical Statistical Model: V. Vovchenko et al., PLB 785, 171 (2019), PRC 100,054906 (2019)

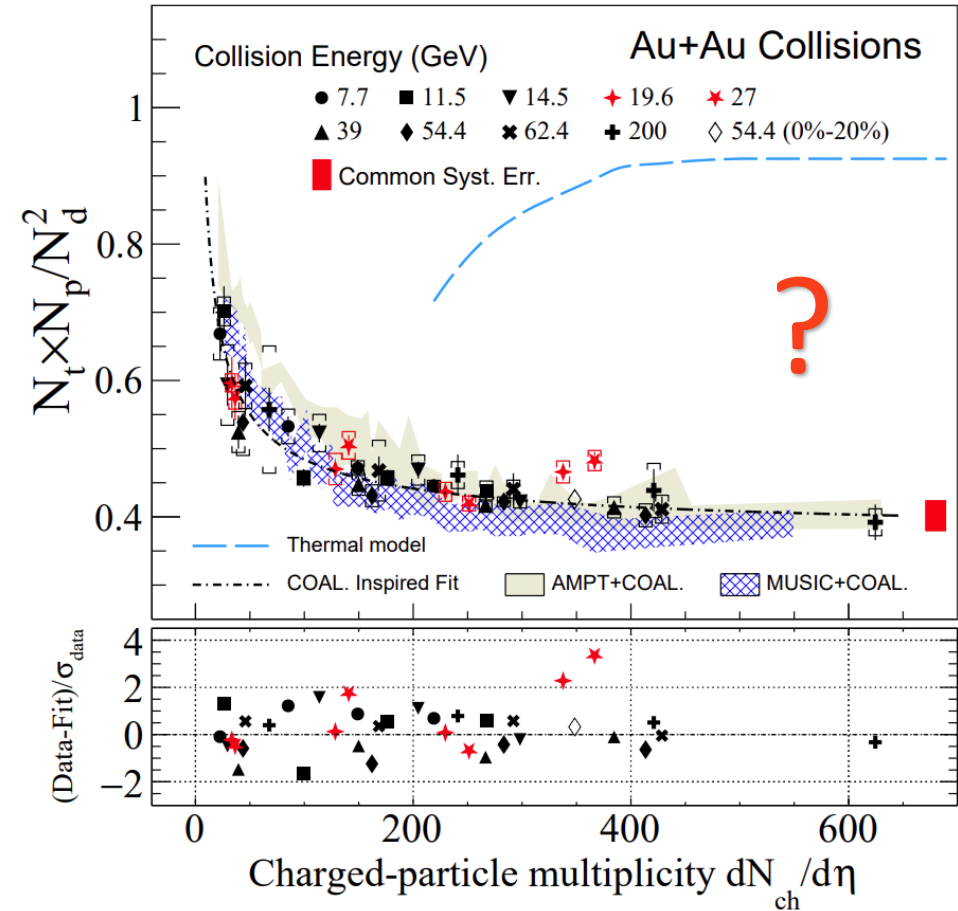
Coalescence: K. J. Sun, C. M. Ko and B. Donigus, Phys. Lett. B 792, 132 (2019)

## Model Prediction



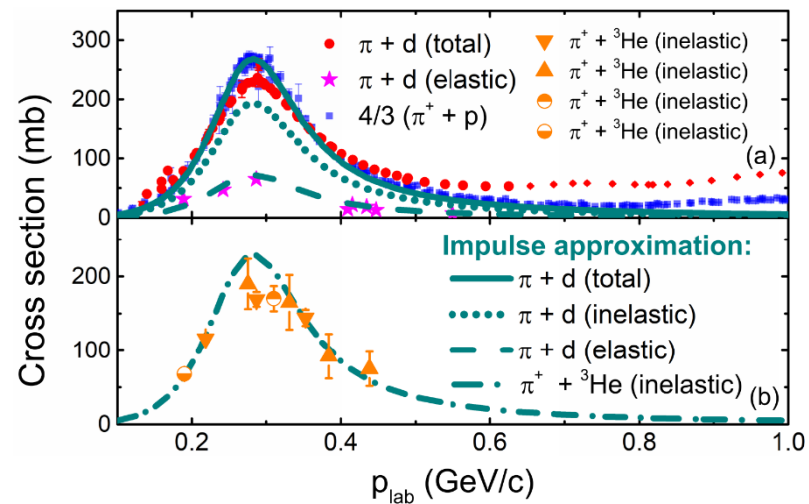
$$\frac{N_t \times N_p}{N_d^2} = p_0 \times \left( \frac{R^2 + \frac{2}{3}r_d^2}{R^2 + \frac{1}{2}r_t^2} \right)^3, r_d > r_t$$

## STAR Measurements





# Hadronic Re-scattering Effects within a Kinetic Approach

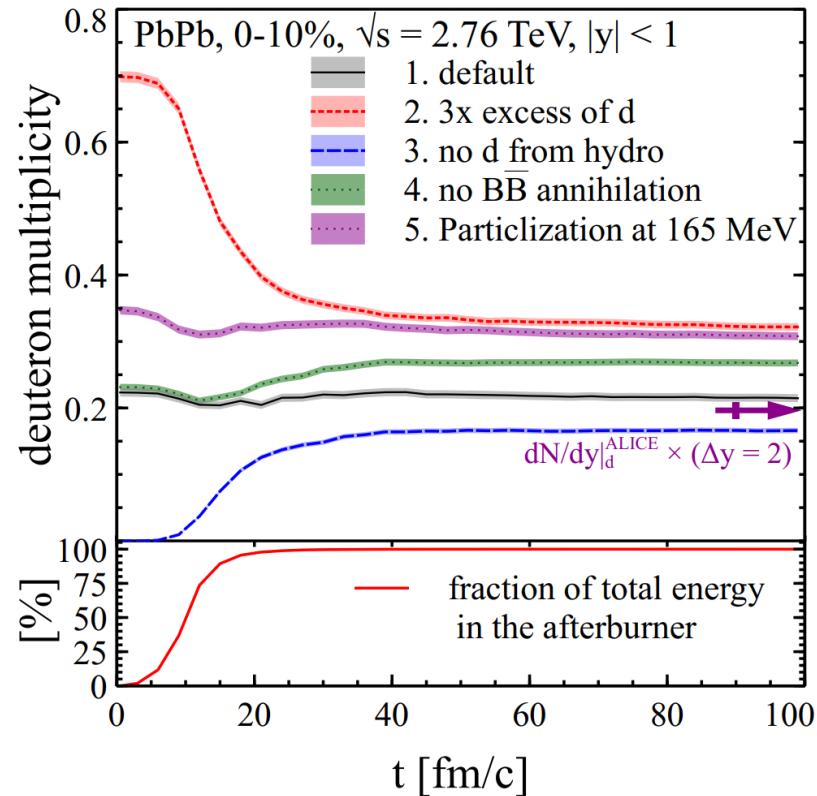


# Hadronic Re-scattering Effects

(6)

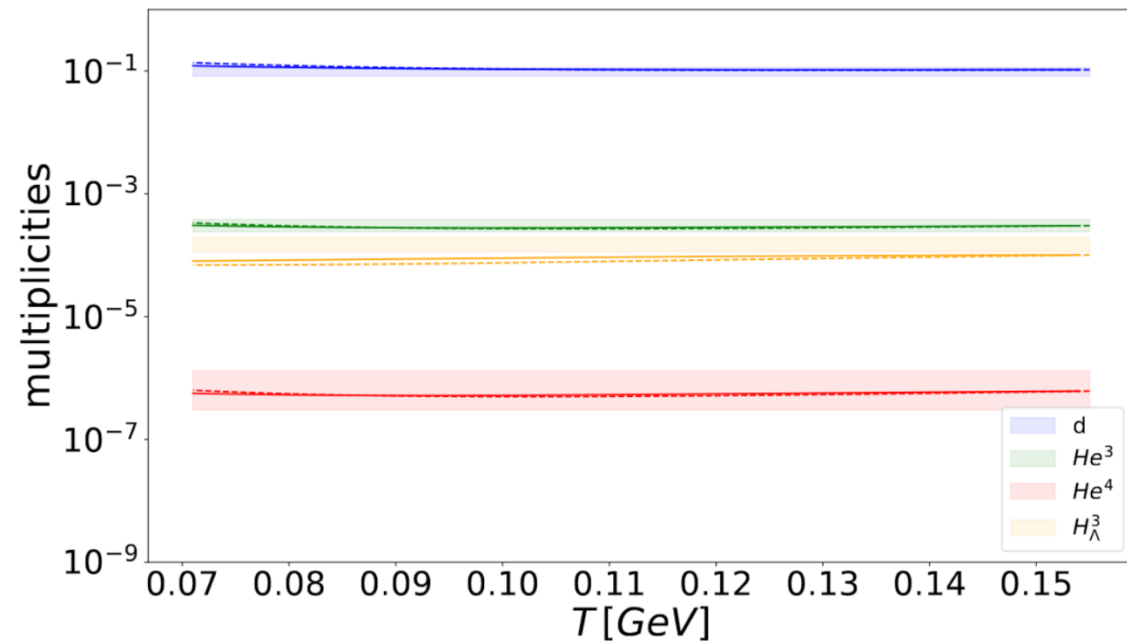
$\pi NN \leftrightarrow \pi d$

D. Oliinychenko, et al., PRC99, 044907 (2019)



V. Vovchenko, et al., PLB800, 135131 (2020)

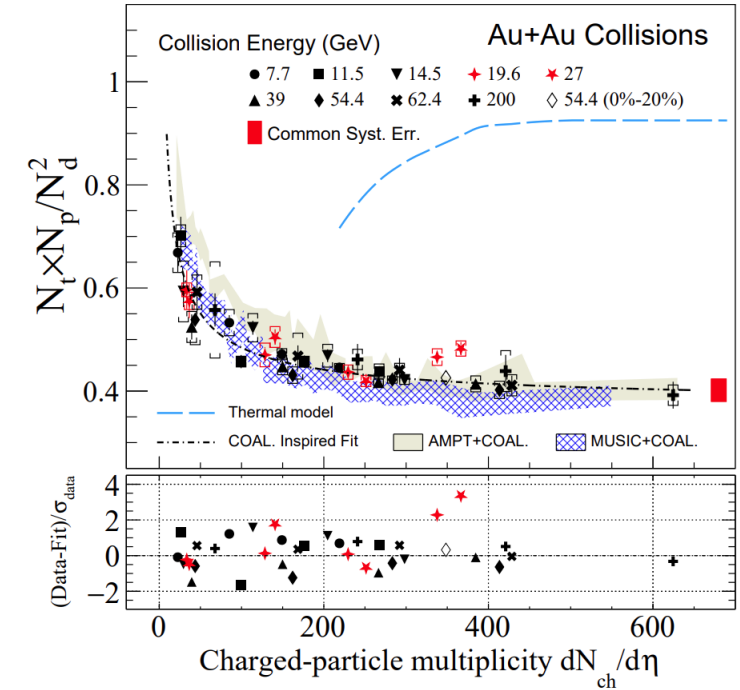
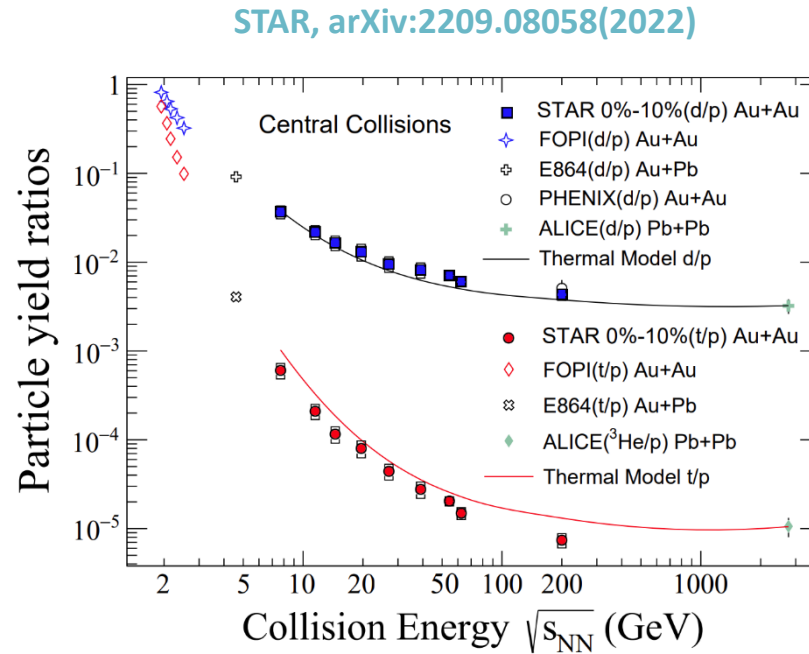
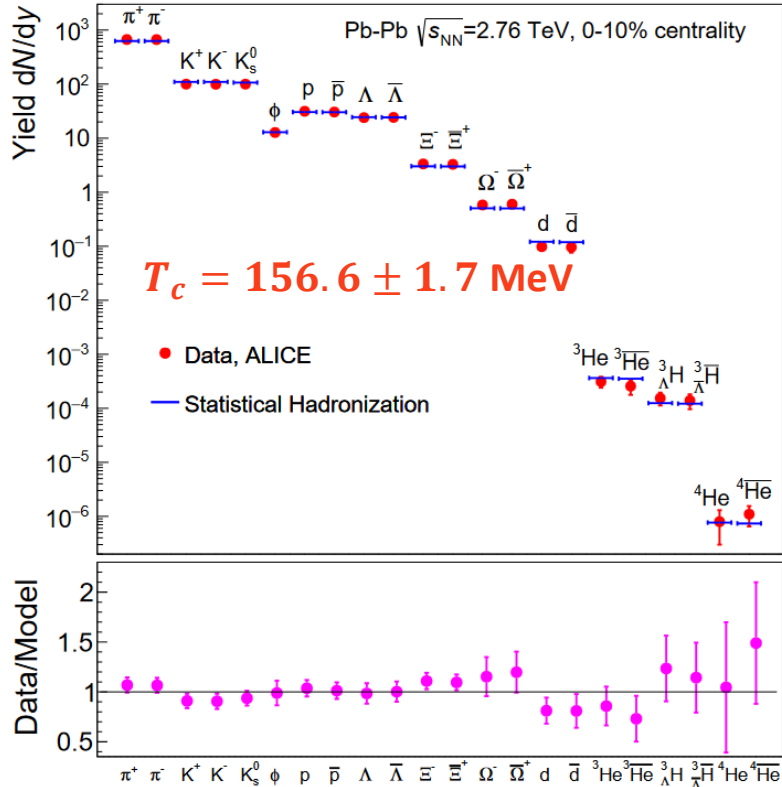
T. Neidig, et al., PLB827, 136891 (2022)



The obtained hadronic effects on light nuclei production are small

# The Triton Puzzle

(7)



**Triton yields at RHIC are overestimated by the statistical hadronization model!  
The effects of hadronic re-scatterings need to be re-examined.**

- $A = 2$   $\pi NN \leftrightarrow \pi d, NNN \leftrightarrow Nd$
- $A = 3$   $\pi NNN \leftrightarrow \pi t(h), \pi Nd \leftrightarrow \pi t(h), NNNN \leftrightarrow Nt(h), NNd \leftrightarrow Nt(h)$

A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB 697, 203 (2011)

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018)

# A novel approach (relativistic kinetic equation)

(8)

K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022)

Relativistic kinetic equation for  $\pi NN \leftrightarrow \pi d$

$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = -\mathcal{K}^> f_d + \mathcal{K}^<(1 + f_d)$$

with collision integral:

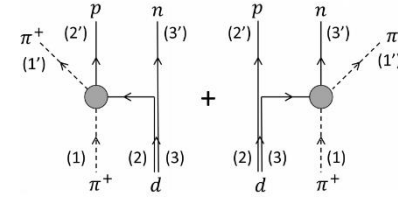
$$\begin{aligned} \text{R.H.S.} = & \frac{1}{2g_d E_d} \int \prod_{i=1'}^{3'} \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d d^3 \mathbf{r}}{m_d} \\ & \times 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}) (|\mathcal{M}_{\pi+n \rightarrow \pi+n}|^2 + n \leftrightarrow p) \\ & \times \left[ - \left( \prod_{i=1'}^{3'} (1 \pm f_i) \right) g_\pi f_\pi g_d f_d + \frac{3}{4} \left( \prod_{i=1'}^{3'} g_i f_i \right) \right. \\ & \left. \times (1 + f_\pi)(1 + f_d) \right] \times (2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}}) \end{aligned}$$

Nonlocal collision integral to take into account the effects of **finite nuclei sizes**.

$W_d$  denotes deuteron Wigner function.

P. Danielewicz et al., NPA533, 712 (1991); PLB274, 268 (1992);  
Annals of Physics 152, 239(1984);

Impulse approximation (IA): Length/energy scale:



$$\lambda_{\text{thermal}} \sim 0.5 \text{ fm} \ll r_{np} \sim 4 \text{ fm}$$

FIG. 1. Diagrams for the reaction  $\pi^+ d \leftrightarrow \pi^+ np$  in the impulse approximation. The filled bubble indicates the intermediate states such as a  $\Delta$  resonance.

**Solving kinetic equations with the stochastic method using test particles**

Probability for reaction  $\pi d \leftrightarrow \pi NN$  to take place in volume  $\Delta V$  and time interval  $\Delta t$  are given by

$$\begin{aligned} \rightarrow P_{23}|_{\text{IA}} & \approx F_d v_{\pi+p} \sigma_{\pi+p \rightarrow \pi+p} \frac{\Delta t}{N_{\text{test}} \Delta V} + (p \leftrightarrow n). \\ P_{32}|_{\text{IA}} & \approx \frac{3}{4} F_d v_{\pi+p} \sigma_{\pi+p \rightarrow \pi+p} \frac{\Delta t W_d}{N_{\text{test}}^2 \Delta V} + (p \leftrightarrow n) \end{aligned}$$

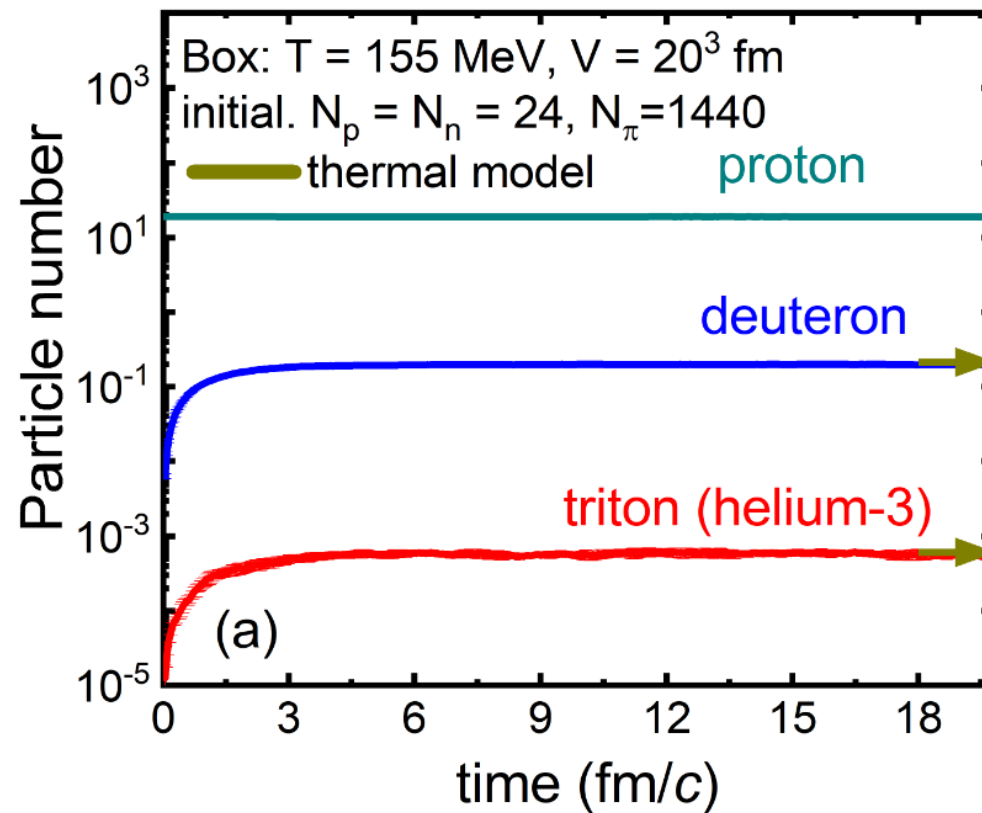
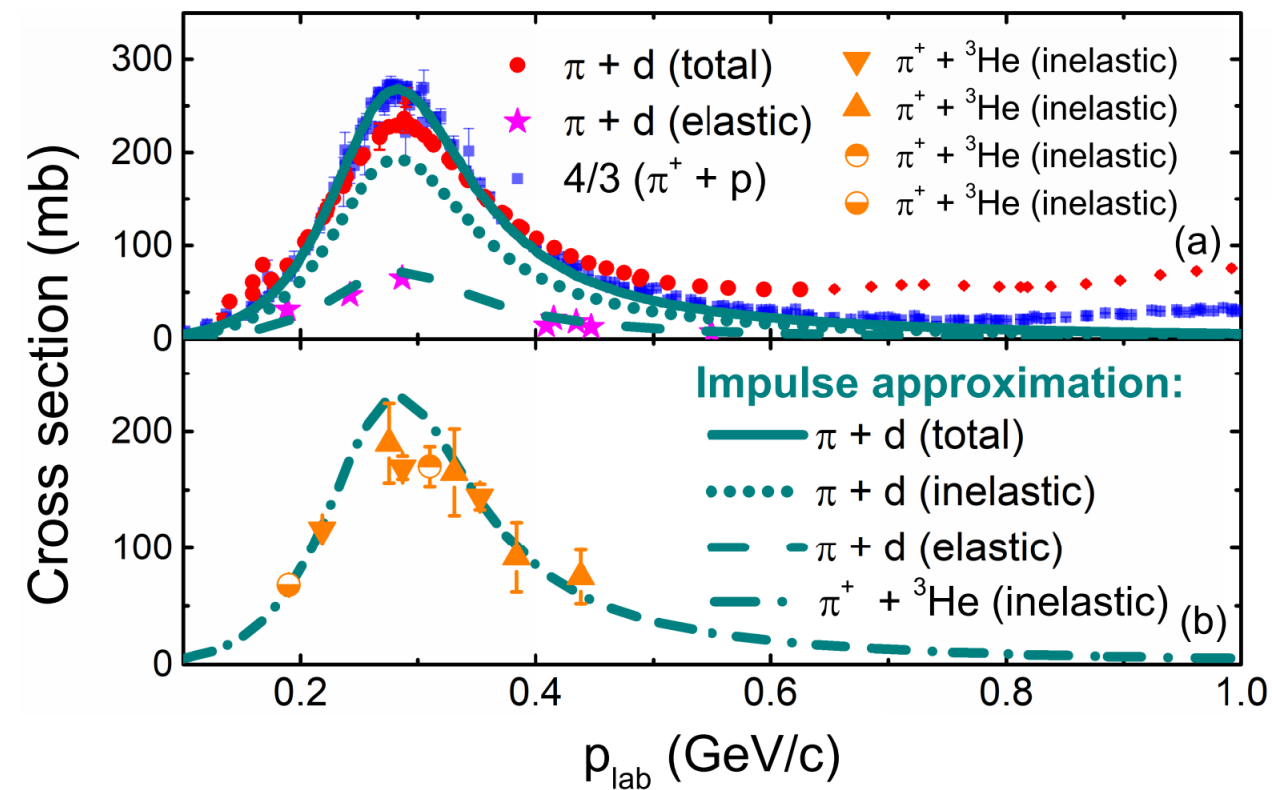
For triton or helium-3:

$$P_{42}|_{\text{IA}} \approx \frac{1}{4} F_t \frac{v_{\pi N} \sigma_{\pi N \rightarrow \pi N} \Delta t}{N_{\text{test}}^3 \Delta V} W_t$$

'renormalization' factor  $F_d, F_t$  which can be fixed by  $\pi d$  and  $\pi t$  cross sections.

# Box calculation

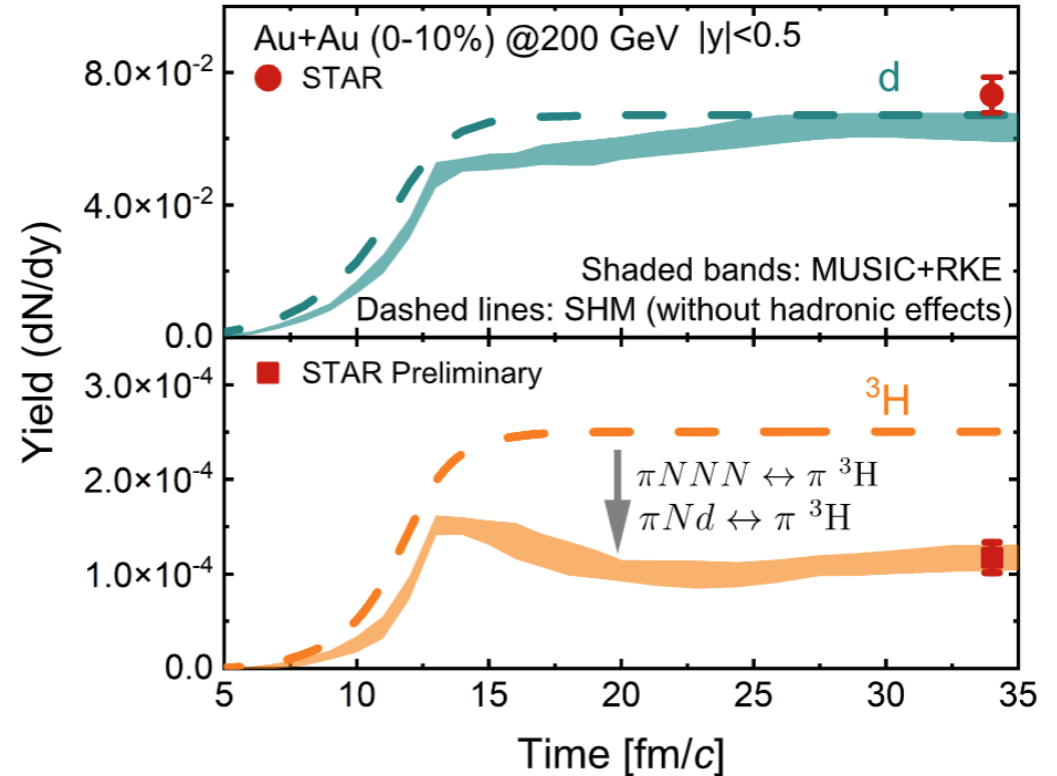
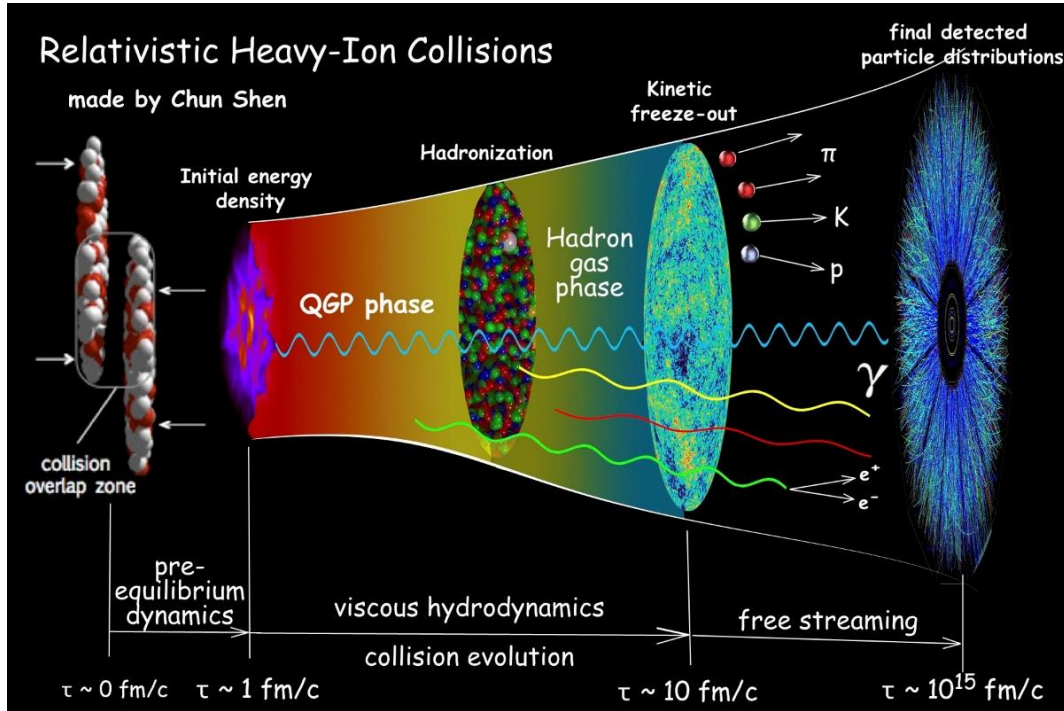
(9)



# Hadronic Re-scattering Effects in Au+Au @200 GeV

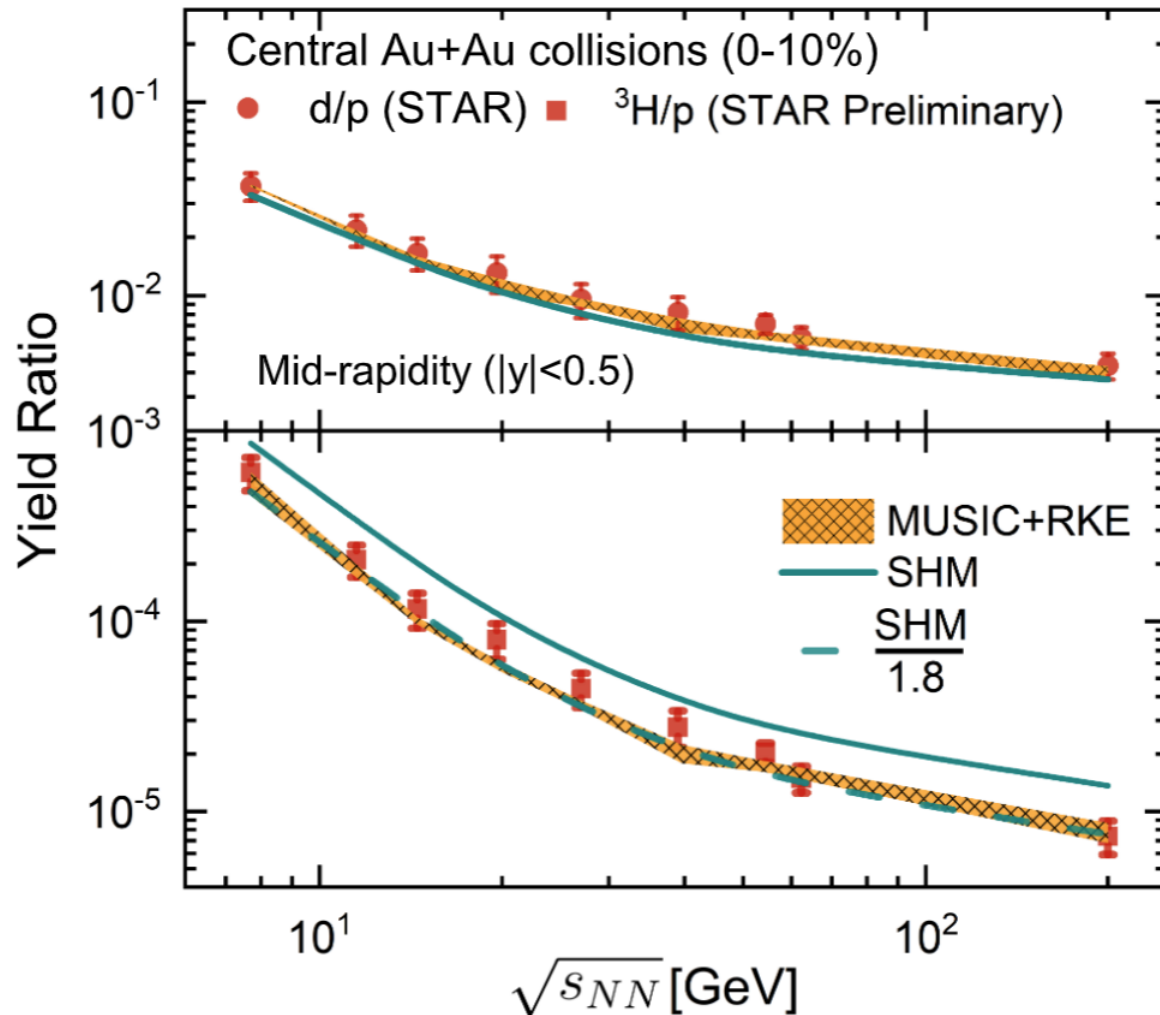
(10)

arXiv:2207.12532(2022)



Hadronic re-scatterings have small effects on the final deuteron yield, but they reduce the triton yield by about a factor of 2

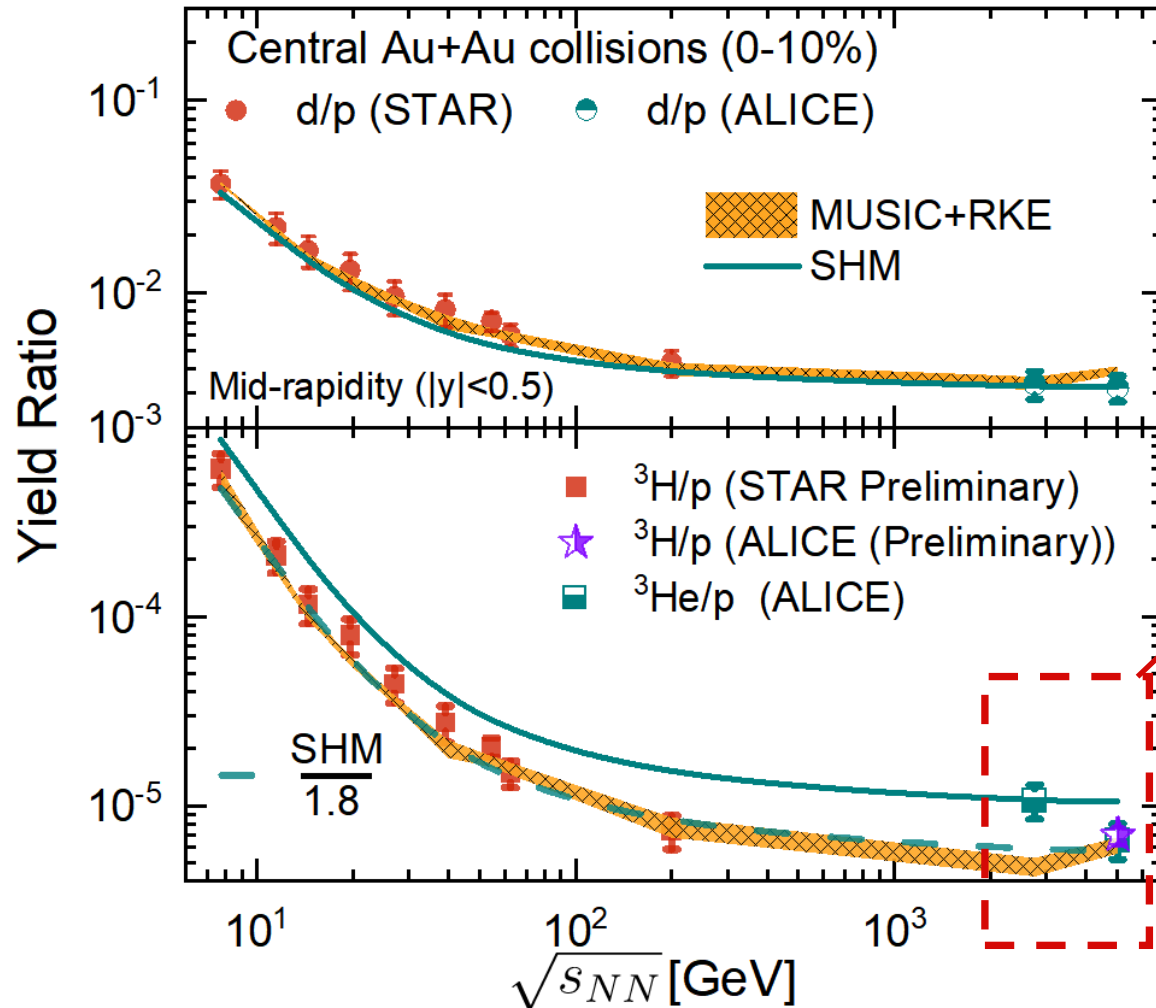
arXiv:2207.12532(2022)



Hadronic re-scatterings reduce the triton yields by about a factor of 1.8

The triton puzzle is resolved.

arXiv:2207.12532(2022)



The hadronic re-scattering effect on triton production is consistent with the measurements in pb+pb collisions at 5.02 TeV, but not 2.76 TeV (uncertainties are still large). More precise measurements help clarify the situation.

ALICE, arXiv:2211.14015(2022)



# Summary and Outlook

1. The quantum correction on light nuclei production due to finite nuclei sizes is consistent with the observation at LHC and RHIC. High precision data on hypernucleus is of particular importance.
2. We have developed a novel kinetic approach to light nuclei production in high-energy nuclear collisions, with the inclusion of many-body scatterings and finite nuclei sizes. **Through this approach, the overestimation on triton production in the thermal model can be resolved after taking into account the effect of hadronic re-scatterings.**
3. The discussed quantum effects and hadronic re-scattering effects may also occur in the production of more exotic and loosely-bound states.

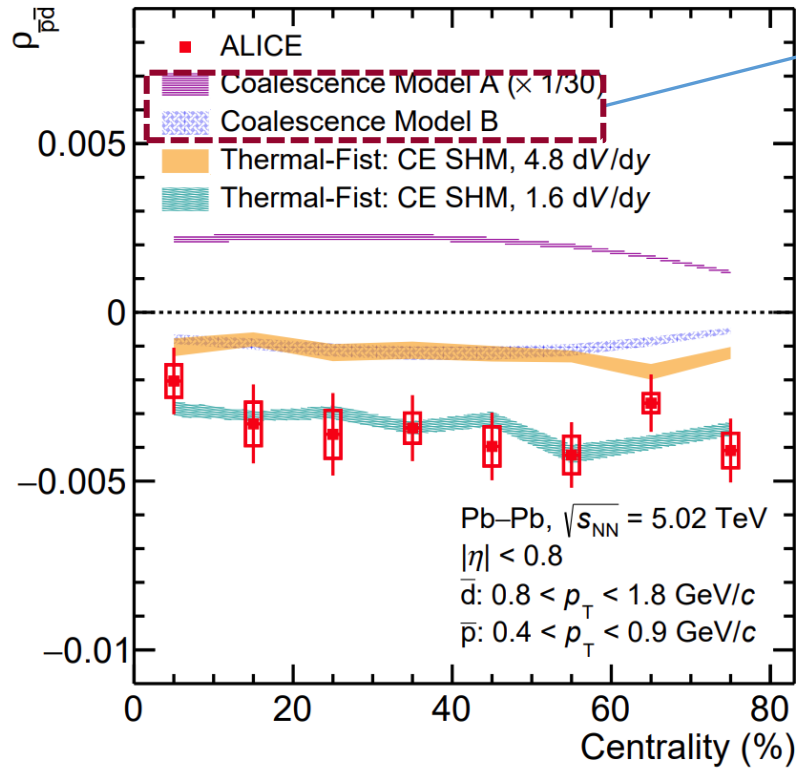
***Future ALICE experiments provide a unique opportunity for studying the phenomenon of little bang nucleosynthesis and related physics!***

**Backup**

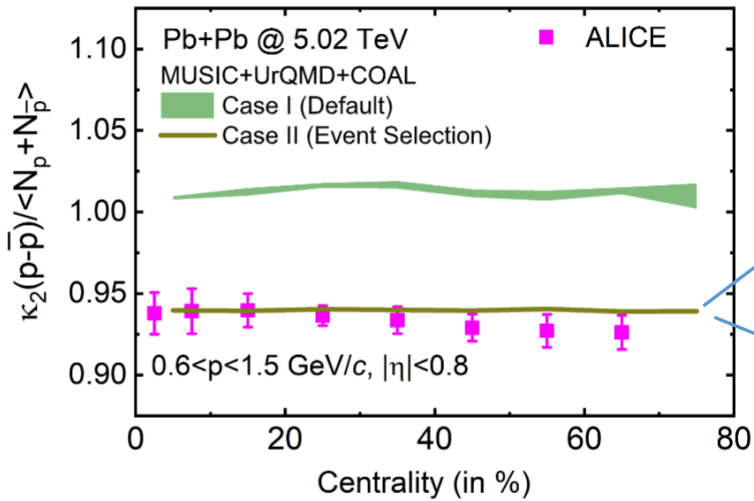
# Event-by-Event Fluctuation

## ALICE: First measurement of antideuteron fluctuation

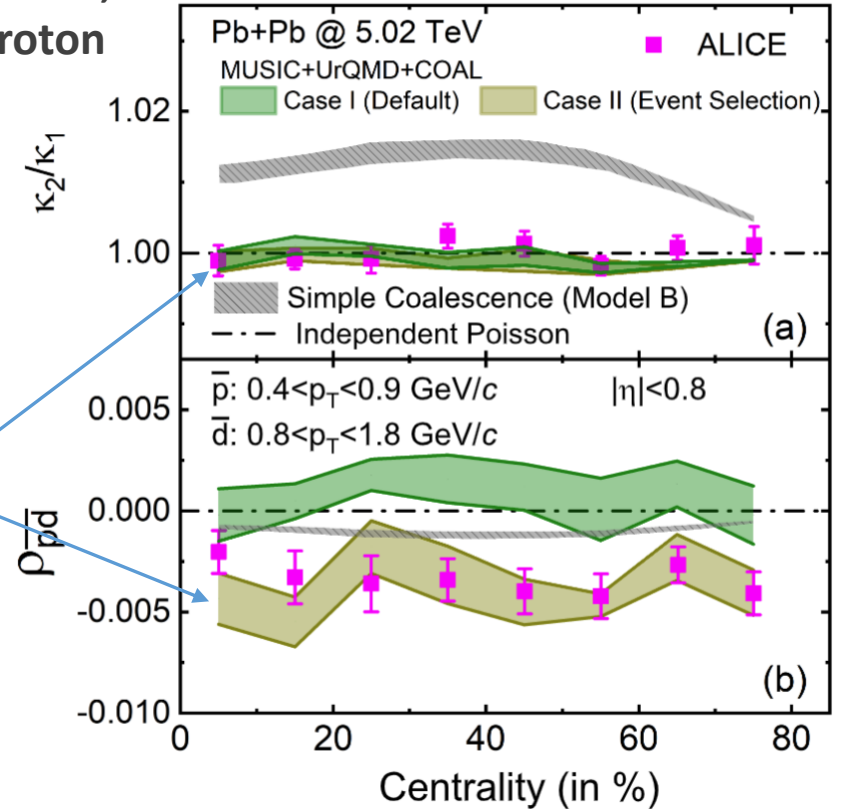
ALICE, arXiv:2204.10166(2022)



Nucleons follow the Poisson distribution, which overestimates the (net)antiproton fluctuation



K. J. Sun and C. M. Ko, arXiv:2204.10879



$$\bar{n} + \bar{p} \rightarrow \bar{d}: \quad \rho_{\bar{p}\bar{d}} \sim \sqrt{\frac{\bar{d}}{\bar{p}}} \left( \frac{\kappa_2(\bar{p})}{\kappa_1(\bar{p})} - 1 \right) < 0 \quad \text{when} \quad \frac{\kappa_2(\bar{p})}{\kappa_1(\bar{p})} < 1$$

To describe the antideuteron fluctuation and the correlation between antiproton and antideuteron, baryon conservations at both the particlization and the nucleon coalescence must be accounted.