Many-body effects of collective neutrino oscillations

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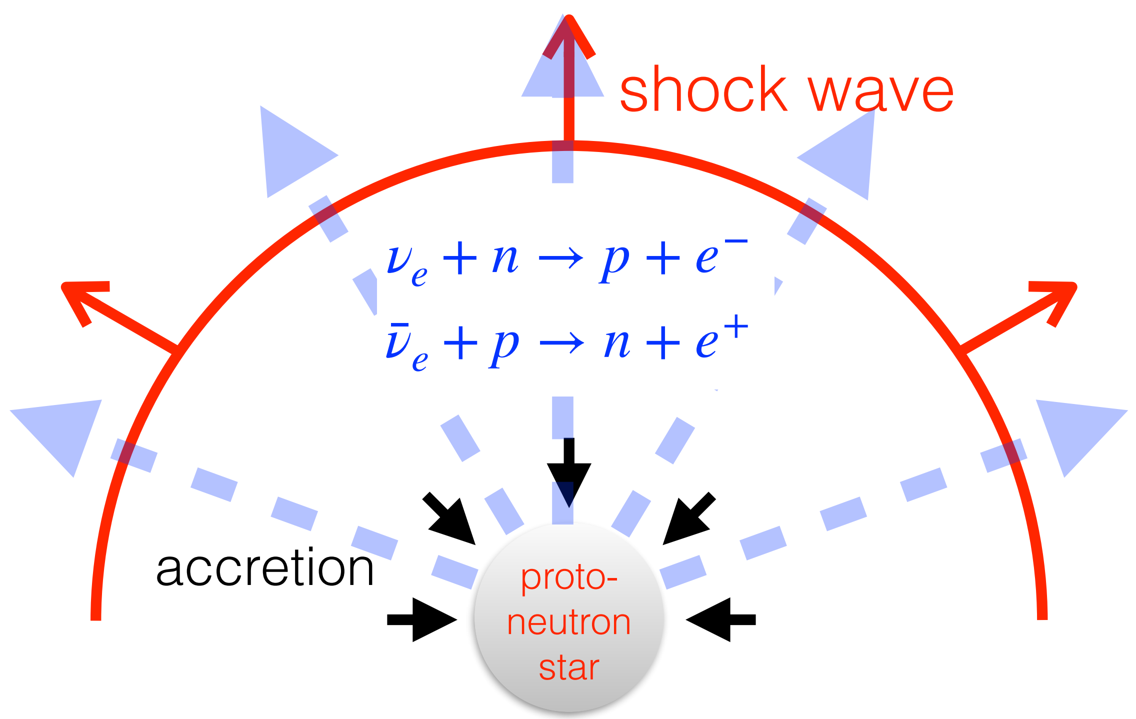
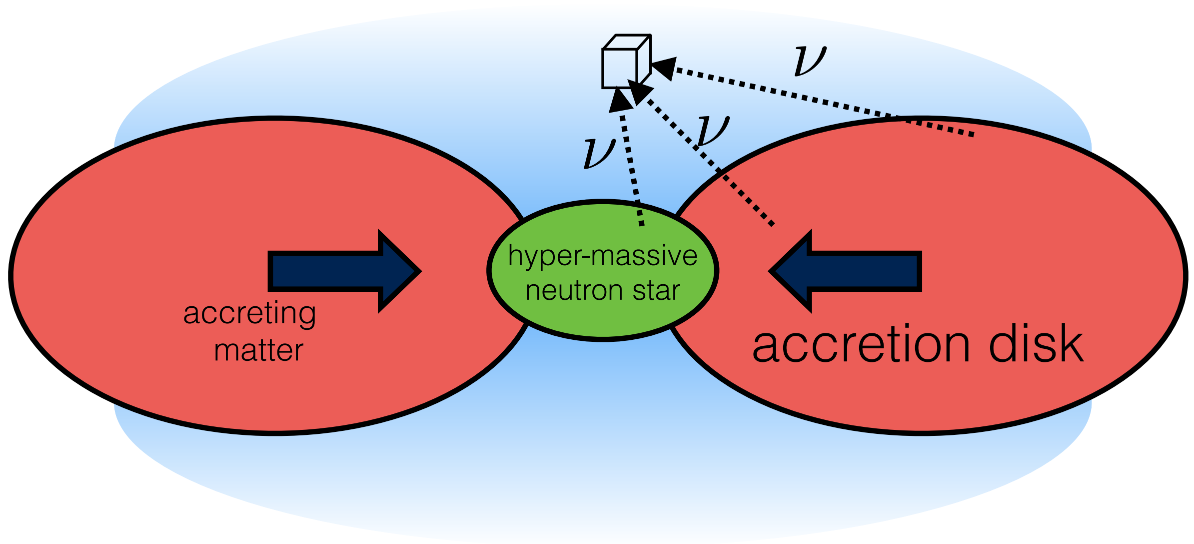
Neutrinos in compact objects

Supernova



Binary merger remnant



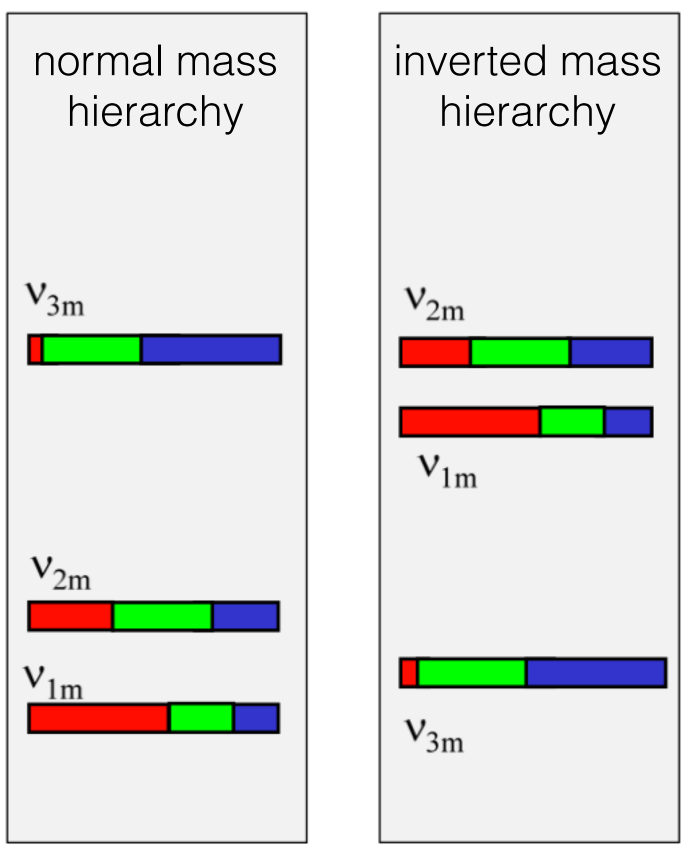
- intense sources of neutrinos in all flavors
- play important roles in dynamics and nucleosynthesis

Neutrino oscillations

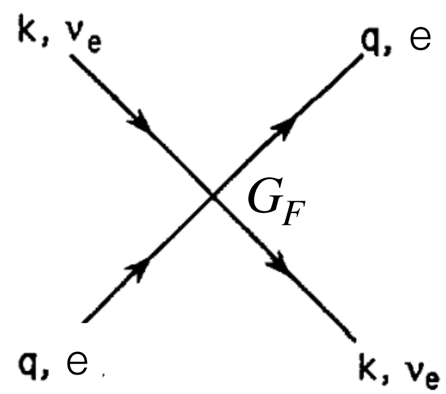
Flavor mixing in vacuum

flavor eigenstate $\nu_\alpha = U_{\alpha l} \nu_l$ mass eigenstate

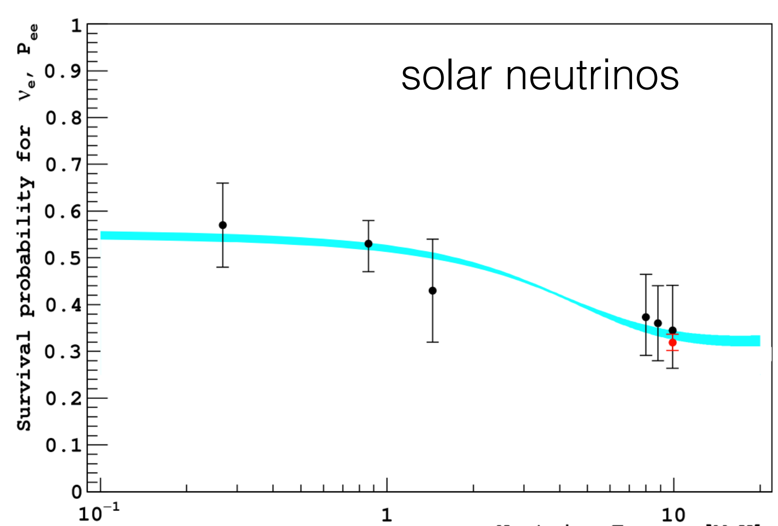
with Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary matrix



Mikheyev–Smirnov-Wolfenstein (MSW) matter effect

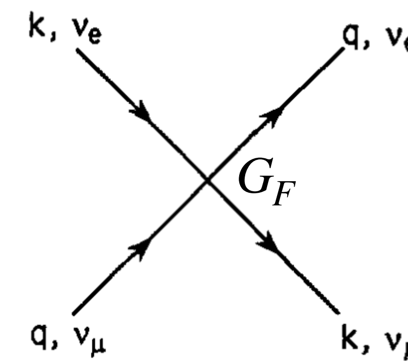


redefine the effective mixing angle



(particle data group)

Neutrino-neutrino coherent forward scattering



collective phenomena in neutrino flavors

$$|\nu_e(\mathbf{k})\nu_\mu(\mathbf{q})\rangle$$

$$\Updownarrow$$

$$|\nu_\mu(\mathbf{k})\nu_e(\mathbf{q})\rangle$$

possible entanglement?

Collective neutrino oscillation as a many-body problem

$$\hat{H} = \hat{H}_{\text{vac}} + \frac{G_F}{\sqrt{2}} \int d^3x \sum_{\alpha,\beta} [\hat{\bar{\nu}}_\alpha(\mathbf{x})\gamma_\mu \frac{1-\gamma_5}{2}\hat{\nu}_\alpha(\mathbf{x})][\hat{\bar{\nu}}_\beta(\mathbf{x})\gamma^\mu \frac{1-\gamma_5}{2}\hat{\nu}_\beta(\mathbf{x})] + \hat{H}^{\text{NC}}_{e\nu} + \hat{H}^{\text{CC}}_{e\nu} + \hat{H}_{\nu N} + \cdots$$

- Simplifications and assumptions:
  - coherent "forward" scattering only: interested in those which can be coherent and so restrict the state space to the states where each of the neutrino's energy and momentum remain unchanged (Pantaleone 1992)
  - $t_{\text{coherent}} \sim (G_F n_\nu)^{-1} \ll t_{\text{collision}} \sim (G_F^2 |\mathbf{p}|^2 n_{\text{baryon}})^{-1}$
  - no "incorrect" helicity
  - energy of free neutrinos: $\sqrt{|\mathbf{p}|^2 + m_l^2} \approx |\mathbf{p}| + m_l^2/(2|\mathbf{p}|)$
  - only neutrinos, excluding antineutrinos
  - two flavors in SU(2) symmetry: ν_e and ν_x
  - within a quantization volume V

$$\hat{H} \approx \sum_p \sum_{\alpha,\beta,l} \underbrace{\left(U_{\alpha l} \frac{m_l^2}{2|\mathbf{p}|} U^*_{l\beta} + \sqrt{2} G_F n_e \delta_{\alpha e} \right)}_{\mathfrak{B}_{\alpha\beta}} \hat{a}^\dagger_{\alpha p} \hat{a}_{\beta p} + \frac{\sqrt{2} G_F}{V} \sum_{p,q} \underbrace{\left(1 - \frac{\mathbf{p}\cdot\mathbf{q}}{|\mathbf{p}||\mathbf{q}|}\right)}_{J_{pq}} \sum_{\alpha,\beta} \hat{a}^\dagger_{\alpha p} \hat{a}_{\beta p} \hat{a}^\dagger_{\beta q} \hat{a}_{\alpha q}$$

Collective neutrino oscillation as a many-body problem

- By constructing the flavor isospin operators $\hat{\tau}^+ = \hat{a}_x^\dagger \hat{a}_e$, $\hat{\tau}^- = \hat{a}_e^\dagger \hat{a}_x$, and $\hat{\tau}^z = (\hat{a}_e^\dagger \hat{a}_e - \hat{a}_x^\dagger \hat{a}_x)/2$ as well as $\mu \equiv \sqrt{2} G_F N/V$,

$$\hat{H} = \sum_p \vec{\mathfrak{B}}_p \cdot \hat{\vec{\tau}}_p + \frac{\mu}{N} \sum_{p \neq q} J_{pq} \hat{\vec{\tau}}_p \cdot \hat{\vec{\tau}}_q$$

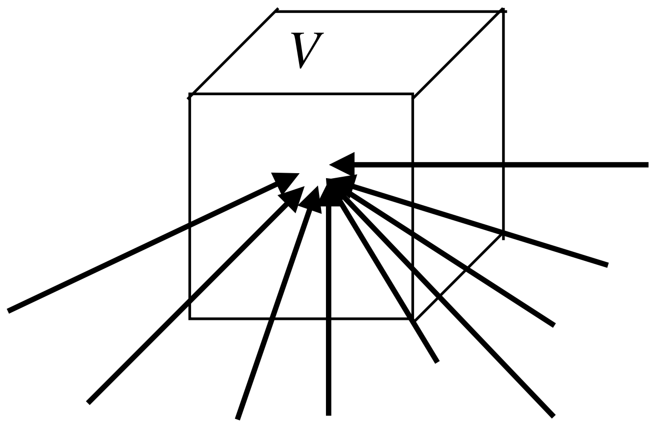
- comparison to Heisenberg model

flavor isospin	spin
vacuum mixing & MSW matter term	magnetic field
all-to-all coupling	nearest-neighbor coupling
dynamical evolution of flavor given an initial state	ground state

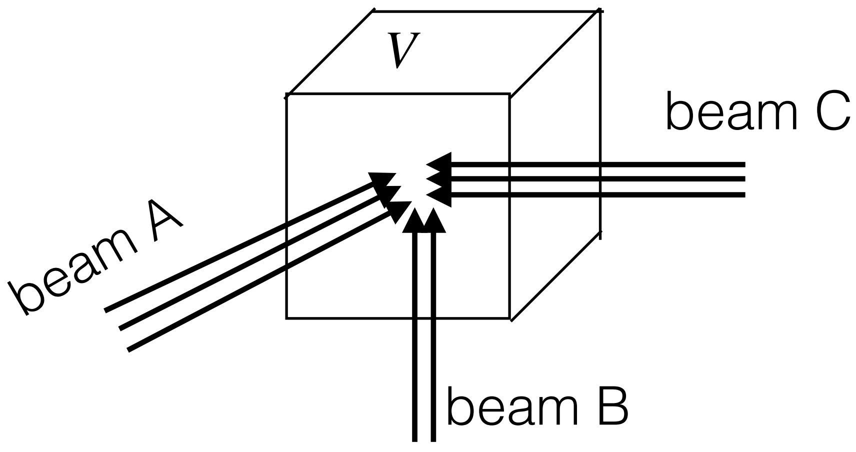
Beam geometry

- state space
  - increases exponentially $\sim 2^N$ for a general momentum configuration (up to $N \sim 20$) (Rrapaj 2020; Patwardhan, Cervia, Balantekin 2022; Martin, Roggero, Duan, Carlson 2023; ...)
  - $\sim (N/N_{\rm beam} + 1)^{N_{\rm beam}}$: **momenta are grouped** into $N_{\rm beam}$ beams where neutrinos in each beam have similar values for $\vec{\mathfrak{B}}_p$ and J_{pq}

$$\hat{H} \approx \sum_{A}^{N_{\rm beam}} \vec{\mathfrak{B}}_A \cdot \hat{\vec{\tau}}_A + \frac{\mu}{N} \sum_{A \neq B} J_{AB} \hat{\vec{\tau}}_A \cdot \hat{\vec{\tau}}_B \quad \text{with} \quad \hat{\vec{\tau}}_A = \sum_{p \in A} \hat{\vec{\tau}}_p$$



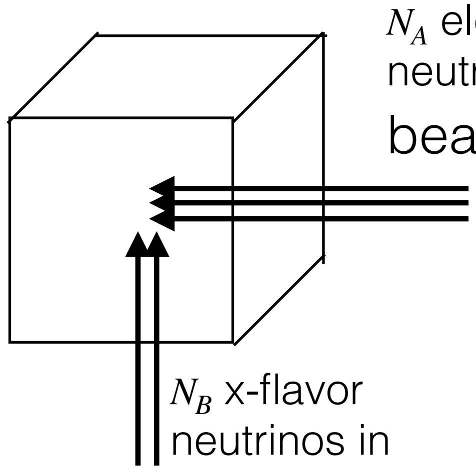
$|\nu_{e,p_1}\rangle \otimes \cdots \otimes |\nu_{e,p_{N_e}}\rangle \otimes |\nu_{x,q_1}\rangle \otimes \cdots \otimes |\nu_{x,q_{N_x}}\rangle$



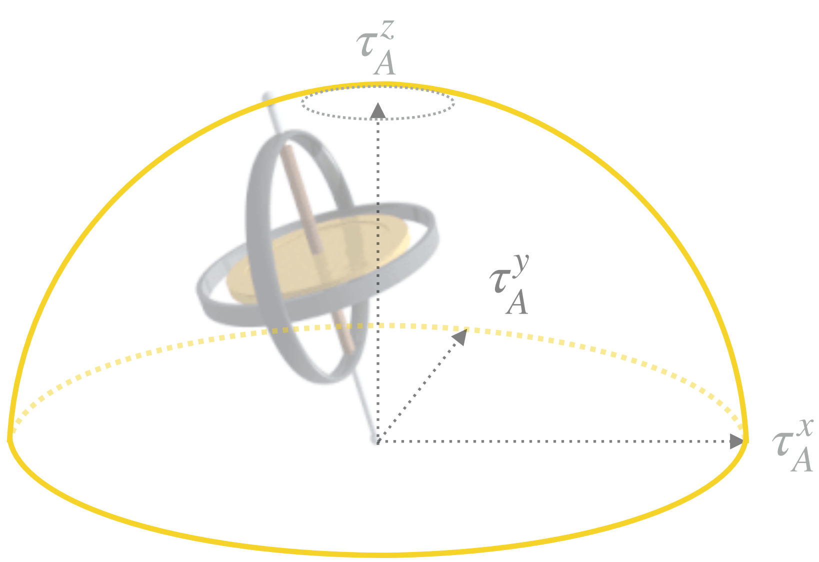
$|\tau_A, \tau_A^z\rangle \otimes |\tau_B, \tau_B^z\rangle \otimes |\tau_C, \tau_C^z\rangle$

Mean-field solution in two-beam geometry

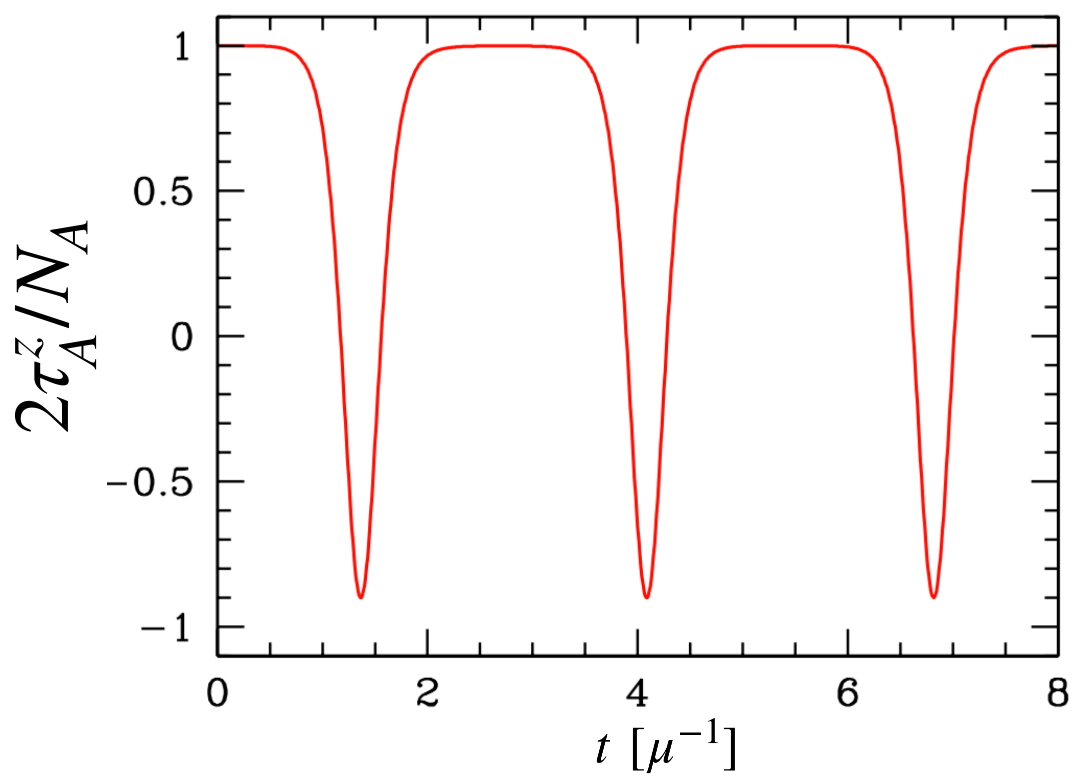
$$H = \mathfrak{B}_A^z \tau_A^z + \mathfrak{B}_B^z \tau_B^z + \frac{2\mu}{N} \vec{\tau}_A \cdot \vec{\tau}_B$$



- neglect effective mixing angle, $\mathfrak{B}^x \approx \mathfrak{B}^y \approx 0$
- Precession;
  Bipolar nutation: flavor conversion



Bloch sphere for flavor isospin

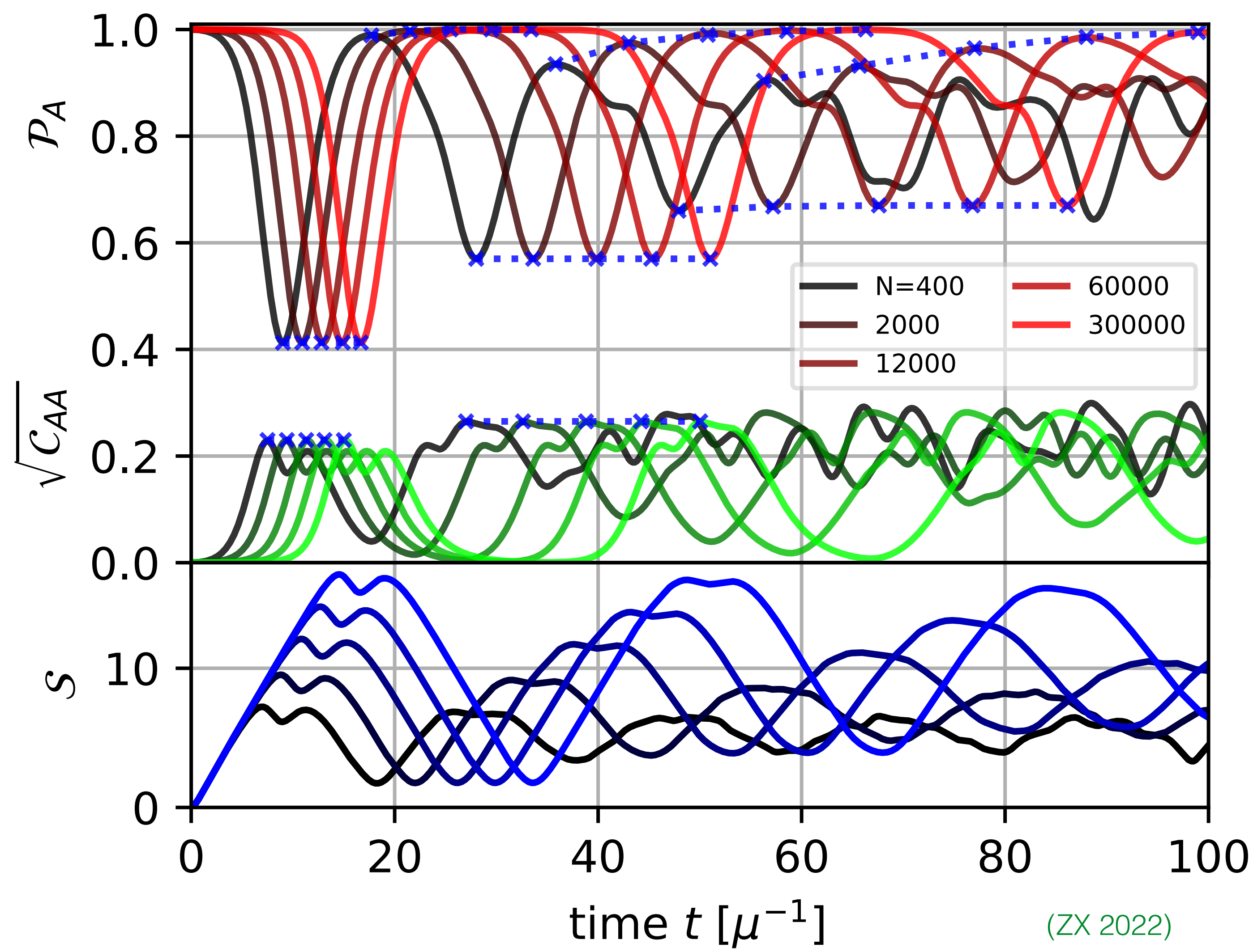


(Hannestad, Raffelt, Sigl, Wong 2006; Duan, Fuller, Yong 2007)

Many-body flavor evolution

$$\hat{H} = \mathfrak{B}_A^z \hat{\tau}_A^z + \mathfrak{B}_B^z \hat{\tau}_B^z + \frac{2\mu}{N} \hat{\vec{\tau}}_A \cdot \hat{\vec{\tau}}_B$$

- $\mu = 4(\mathfrak{B}_B^z - \mathfrak{B}_A^z)$, $3N_A = N_B$
- Evolve $|\Psi\rangle = \sum c_{\tau_A^z} |\tau_A, \tau_A^z\rangle \otimes |\tau_B, \tau_B^z\rangle$ from $|N_A/2, N_A/2\rangle \otimes |N_B/2, -N_B/2\rangle$ with constant τ_A, τ_B and $\tau_A^z + \tau_B^z$
- Survival probability: $P_A = \langle \Psi | \hat{\tau}_A^z / N_A | \Psi \rangle + 1/2$
- Pair correlation: $C_{AA} = \langle \Psi | (\hat{\tau}_A^z / N_A + 1/2)^2 | \Psi \rangle - P_A^2$
- Von Neumann entanglement entropy: $S = -\mathrm{Tr}[\rho_A \log_2 \rho_A]$ with $\rho_A = \mathrm{Tr}_B[|\Psi\rangle\langle\Psi|]$

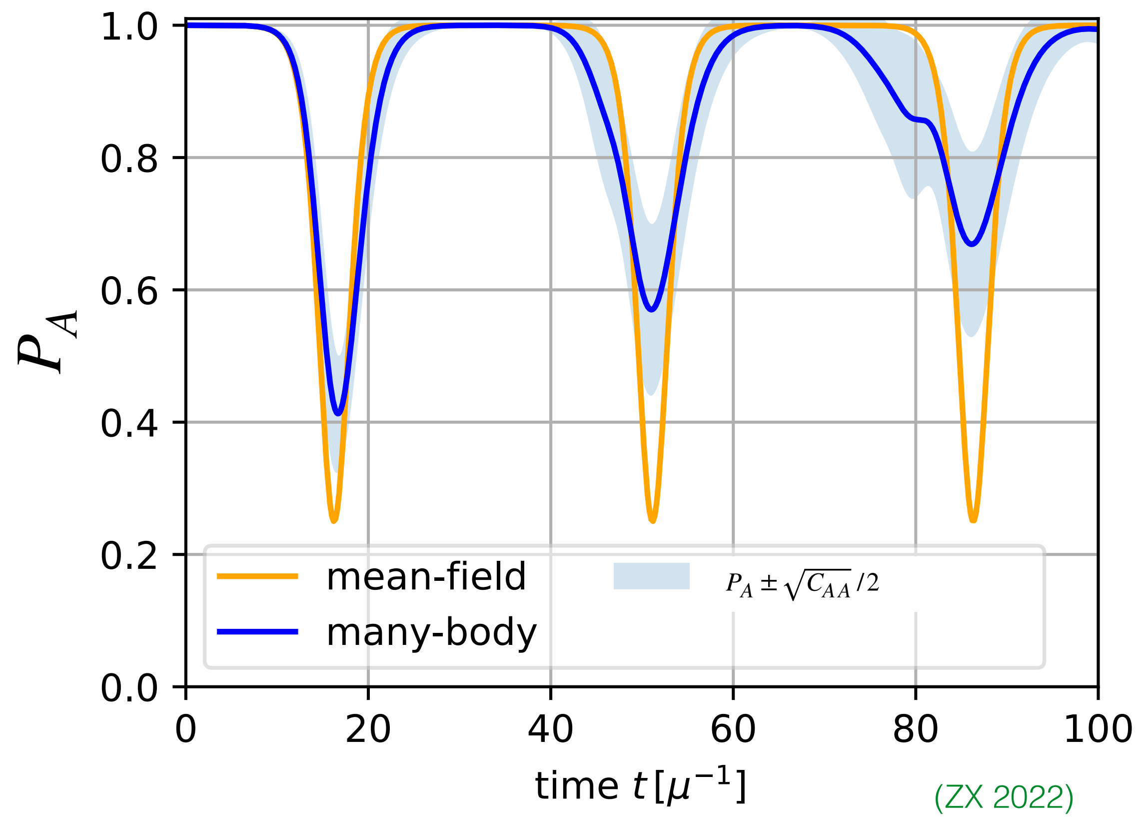


(ZX 2022)

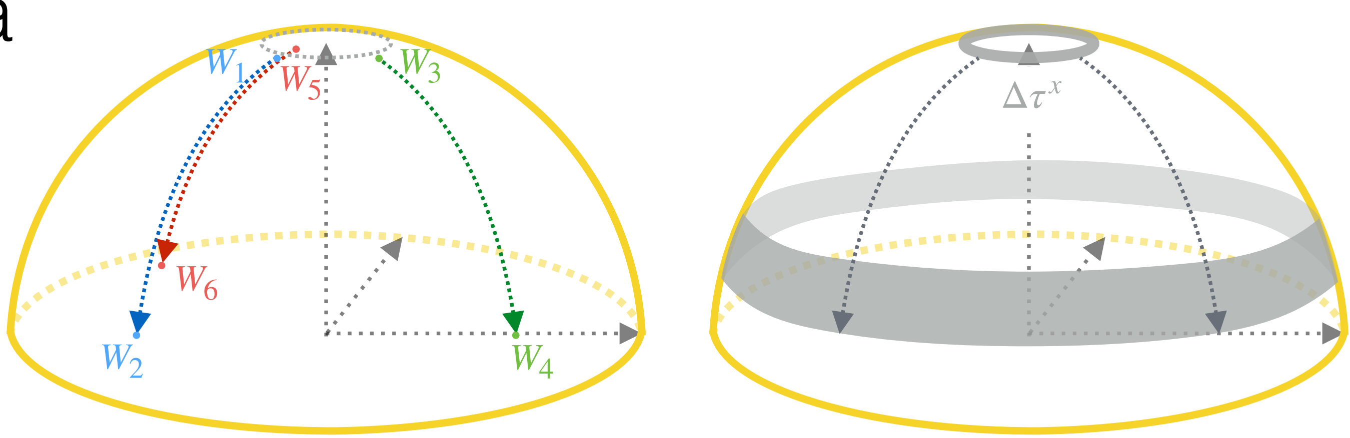
Comparison

- Both many-body and mean-field calculations show a similar exponential growth at the beginning and a bipolar nutation later

- Decoherence effects in the many-body calculation lead to a decrease in the transition probability and a growth in both entanglement and correlation



(ZX 2022)



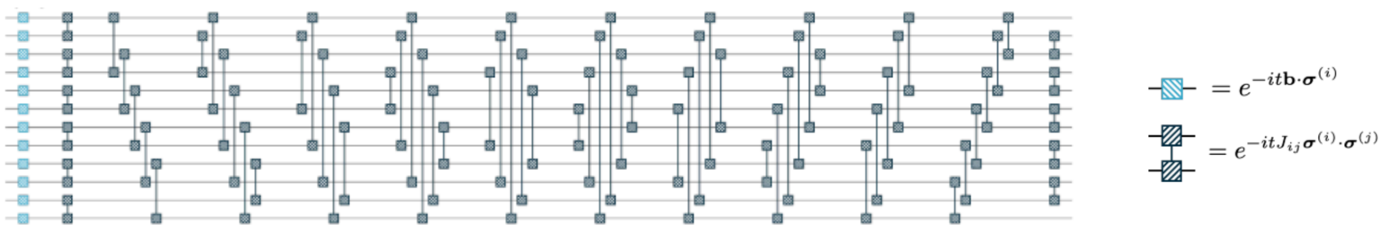
Summary and outlook

- Summary
  - We study the possible many-body effects of collective neutrino oscillations in simplified configurations with zero vacuum mixing
  - We find decoherence in the flavor-isospin space as the many-body solution deviates from that in the mean-field approximation
  - The entanglement and correlation develop in the similar time scale as flavor converts

- Outlook:
  - BBGKY (Bogoliubov–Born–Green –Kirkwood–Yvon) hierarchy?
    (Volpe, Väänänen, Espinoza 2013; ...)

$$\begin{cases} i\dot{\rho}_1 &= [H_0(1), \rho_1] + tr_2[V(1,2), \rho_{12}] \\ i\dot{\rho}_{12} &= [H_0(1) + H_0(2) + V(1,2), \rho_{12}] \\ & \quad + tr_3[V(1,3) + V(2,3), \rho_{123}] \\ \vdots & \\ i\dot{\rho}_{1\ldots s} &= [\sum_{k=1}^{s} H_0(k) + \sum_{k'>k=1}^{s} V(k,k'), \rho_{1\ldots s}] \\ & \quad + \sum_{k=1}^{s} tr_{s+1}[V(k, s+1), \rho_{1\ldots s+1}] \end{cases}$$

  - quantum computing?
    (Hall, Roggero, Baroni, Carlson 2021; Illa, Savage 2022)



  - seeking for more tools from nuclear theory and many-body physics