Large-N_c Constraints for Nuclear Effective Field Theories

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Implications of Large-N_c QCD for the NN Interaction

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We present a method for ordering two-nucleon interactions based upon their scaling with the number of QCD colors, N_c, in the limit that N_c becomes large. Available data in the two-nucleon sector shows general agreement with this ordering, indicating that the method may be useful in other contexts where data is less readily available. However, several caveats and potential pitfalls can make the large-N_c ordering fragile and/or vulnerable to misinterpretation. We discuss the application of the large-N_c analysis to two- and three-nucleon interactions, including those originating from weak and beyond-the-standard-model interactions, as well as two-nucleon external currents. Finally, we discuss some open questions in the field.

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N=3 is Large?

- Witten's "wisecrack"

Coleman "Aspects of Symmetry," Witten NPB 160

$$\frac{e^2}{4\pi} = \frac{1}{137} \iff e \approx 0.3$$

- Saddle point expansion is often $1/N_c^2$

$$Z[J] = \int D\varphi\, e^{iN_c \hat{S}(\varphi)}$$

- Meson and single-baryon phenomenology is good
- Prototypes of QCD have exact solution in large-N limit

Low Energy Coefficients

$$\mathcal{L}_{\text{eff}} = \sum_n \left(\frac{p}{\Lambda}\right)^n c_{\mathcal{O}} \mathcal{O}_n$$

- LECs must be obtained from:
  - fit to data
    - lacking for many low-energy processes
  - Matching calculations
    - lattice QCD
- Theoretical constraints from large-N_c QCD

Large-N Constraints in Nuclear EFTs

- Chiral EFT and pionless EFT possess symmetries of QCD
  - Map scalings to operators with same spin-flavor structure
- Caveats:
  - Δ degenerate with nucleon
  - chiral limit vs. large-N limit $\qquad \dfrac{m_\pi}{m_\Delta - m_N}$
  - η' is a Goldstone boson
- **Fierz transformations can obscure large-N_c scaling**

Large-N$_c$ QCD

- One-loop beta function

$$\mu \frac{dg}{d\mu} = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right]$$

- Rescale coupling constant

$$g \to \frac{g}{\sqrt{N_c}} \quad \Longrightarrow \quad \mu \frac{dg}{d\mu} = -\left(\frac{11}{3} - \frac{2N_f}{3N_c}\right)\frac{g^3}{(4\pi)^2}$$

- QCD becomes expansion of planar diagrams
  - Planar gluons $\lesssim O(N_c^2)$
  - Single quark along edge with planar gluons $\lesssim O(N_c)$

't Hooft NPB 72

Large-N_c Baryons

- Baryon must be made of N_c quarks
- Baryon mass $m_B \sim O(N_c)$
- Two-baryon interaction $O(N_c)$

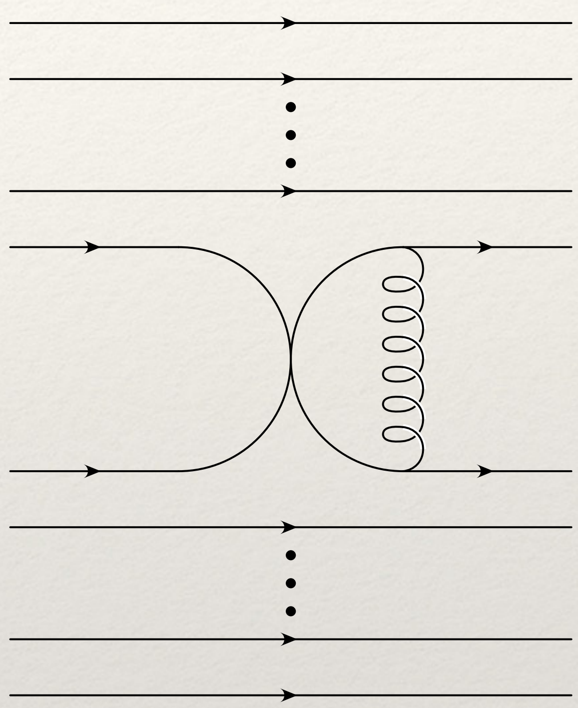


Figure: Baryon-baryon scattering
Manohar, 1998

Witten NPB 160

Baryon-Baryon Interaction

- Quarks with different colors
  - $N_c\,(N_c - 1)$ ways to choose two quarks
  - $1/N_c$ from quark-gluon vertices
- Quarks of same color
  - N_c choices

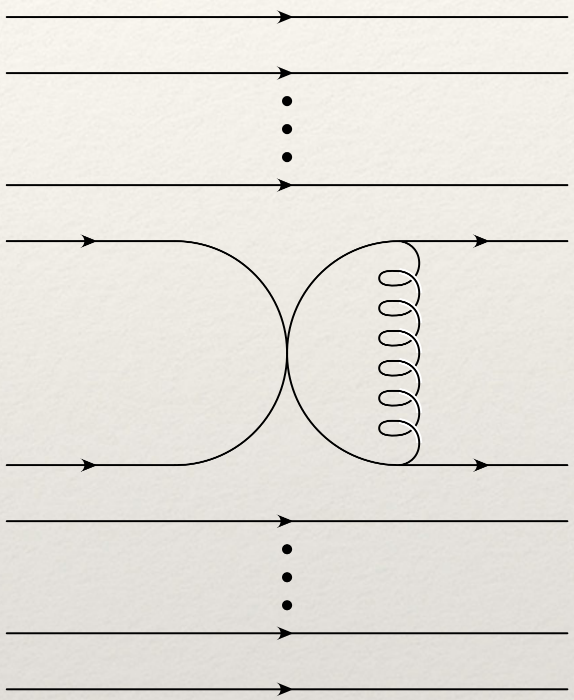


Figure: Baryon-baryon scattering
Manohar, 1998

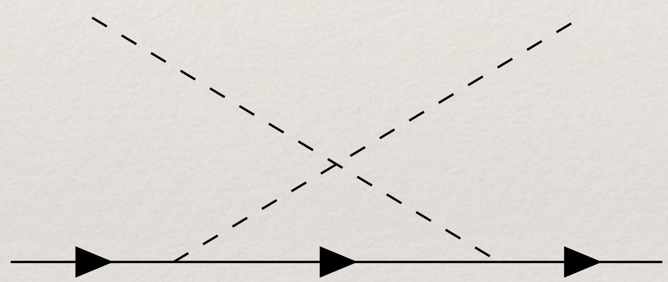
Witten NPB 160

Consistency Conditions

- Axial current matrix elements

$$\langle B'|\,\bar{q}\gamma^i\gamma^5\tau^a q\,|B\rangle = \hat{g}_A N_c\,\langle B'|\,X^{ia}\,|B\rangle$$

- Baryon-meson scattering amplitude *should be* $O(1)$



$$\frac{\hat{g}_A^2}{F^2}N_c^2\left[X^{ia},X^{jb}\right] \Rightarrow \left[X^{ia},X^{jb}\right] \lesssim O(1/N_c^2)$$

- Baryons transform under contracted $\mathrm{SU}(2N_F)$

Gervais and Sakita PRL 52, PRD 30; Dashen and Manohar PLB 315

Spin-Flavor Symmetry

- Large-N$_c$ baryons transform under SU(4)

$$S^i = q^\dagger \frac{\sigma^i}{2} q \qquad I^a = q^\dagger \frac{\tau^a}{2} q \qquad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

$$\langle B' | \frac{\mathcal{O}^{(n)}}{N_c^n} | B \rangle \sim N_c^{-|I-S|} \qquad \langle B' | \hat{1} | B \rangle \sim \mathcal{O}(N_c)$$

- Expand QCD operators in basis of SU(4) generators

$$\mathcal{O}^{(m)}_{\mathrm{QCD}} = N_c^m \sum_{n,s,t} c_n \left(\frac{S^i}{N_c}\right)^s \left(\frac{I^a}{N_c}\right)^t \left(\frac{G^{ia}}{N_c}\right)^{n-s-t}$$

Dashen PRD 49, 51; Carone PLB 322;
Luty and March-Russell NPB 426

Large-N$_c$ NN Force

- Two-nucleon potential

$$V(\vec{p}_-, \vec{p}_+) = \langle N_\alpha N_\beta | \, H \, | N_\gamma N_\delta \rangle$$

- Hamiltonian takes Hartree form

(t-channel)

$$H = N_c \sum_{n,s,t} v_{stn} \left(\frac{S^i}{N_c}\right)^s \left(\frac{I^a}{N_c}\right)^t \left(\frac{G^{ia}}{N_c}\right)^{n-s-t} \qquad p_- \sim 1 \qquad p_+ \sim 1/N_c$$

- Two-nucleon matrix elements factorize

$$\langle N_3 N_4 | \, \mathcal{O}_1 \mathcal{O}_2 \, | N_1 N_2 \rangle \xrightarrow{N_c \to \infty} \langle N_3 | \, \mathcal{O}_1 \, | N_1 \rangle \langle N_4 | \, \mathcal{O}_2 \, | N_2 \rangle + \text{crossed}$$

Kaplan and Savage PLB 365,
Kaplan and Manohar PRC 56

Central Potential

- Large-N analysis shows $1/N_c^2$ expansion

$$V_{\text{central}} = V_0^0 + V_\sigma^0 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_0^1 \vec{\tau}_1 \cdot \vec{\tau}_2 + V_\sigma^1 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2$$

Isospin	V_0	V_σ	V_{LS}	V_T	V_Q
$\mathbf{1} \cdot \mathbf{1}$	N_c	$1/N_c$	$1/N_c$	$1/N_c$	$1/N_c^3$
$\boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2}$	$1/N_c$	N_c	$1/N_c$	N_c	$1/N_c$

- Wigner SU(4) symmetry valid at leading order

Kaplan and Manohar PRC 56

Large-N_c and Nijmegen

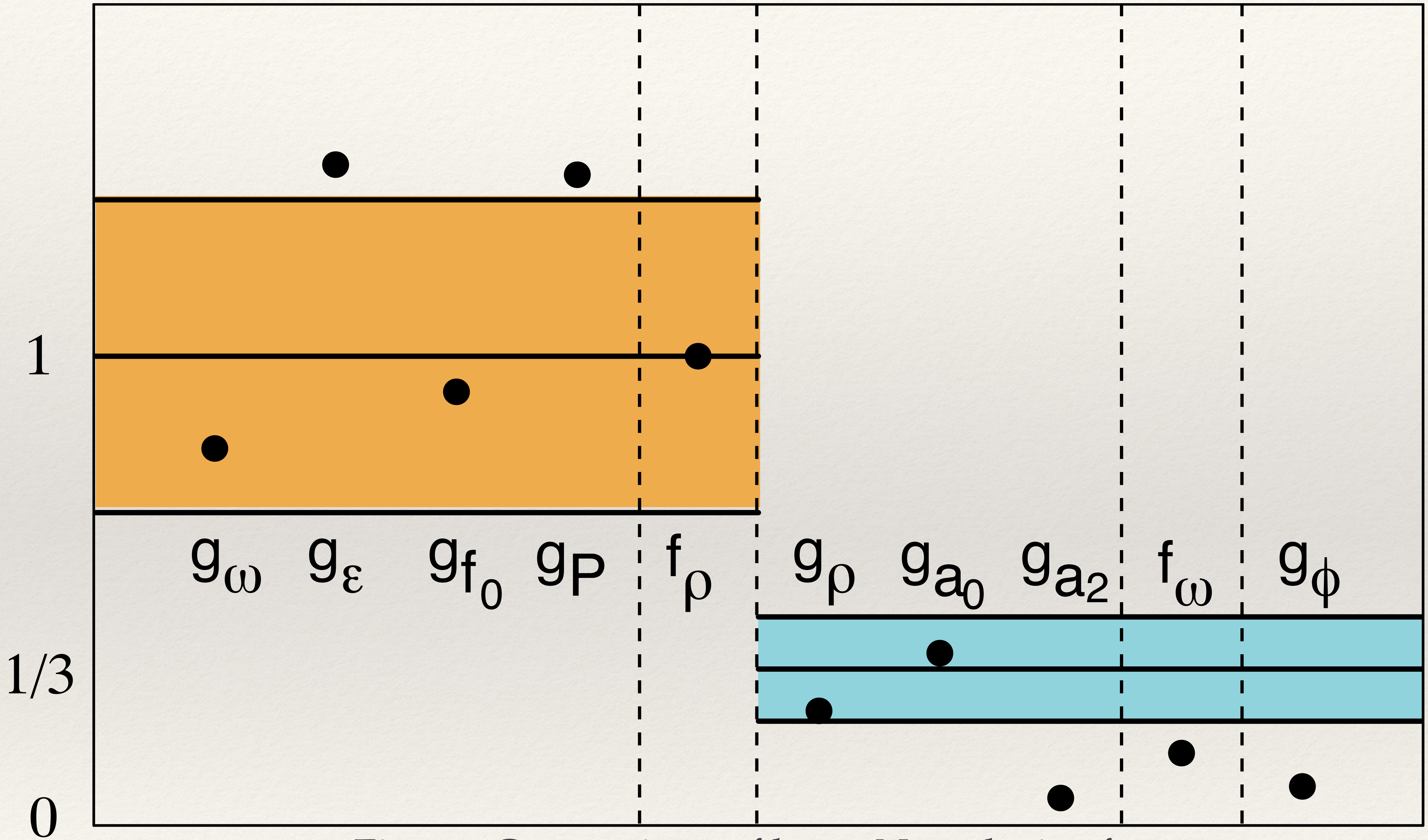
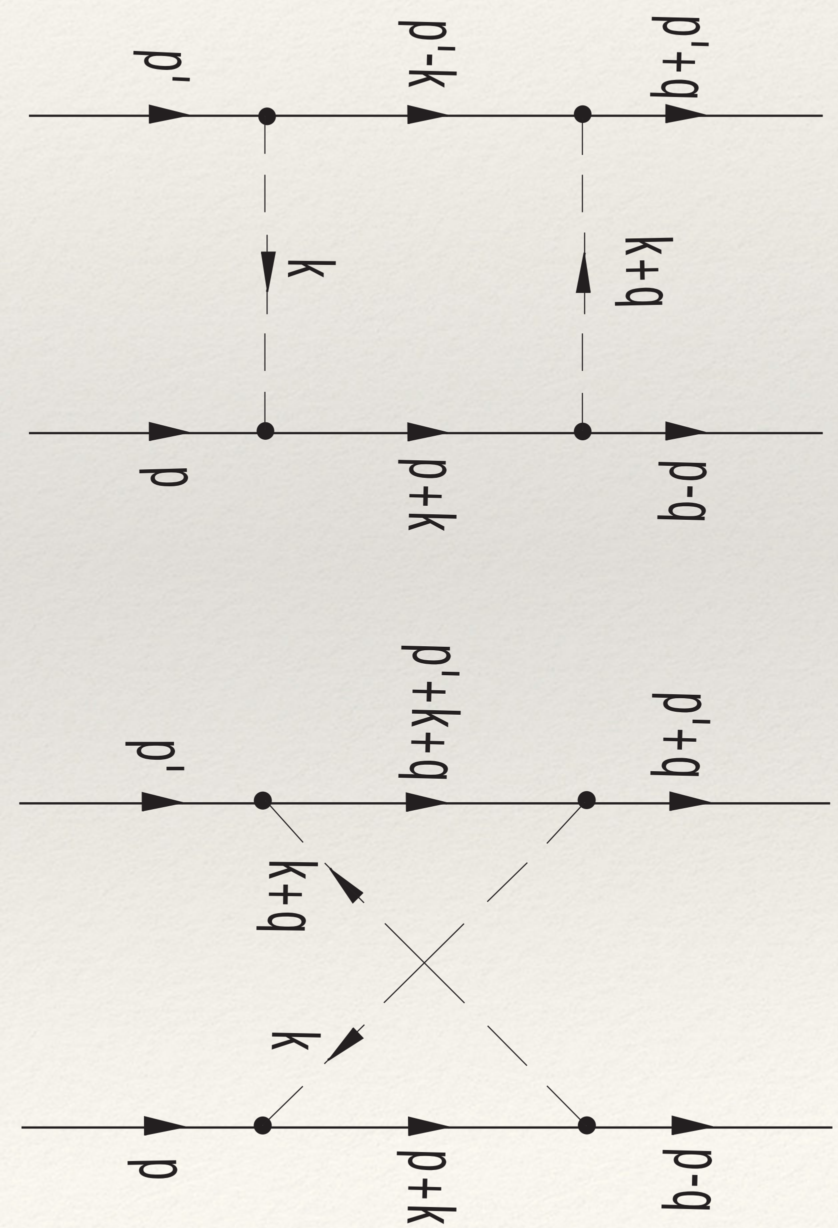


Figure: Comparison of large-N analysis of two-nucleon potential to Nijmegen

Kaplan and Manohar PRC 56

Other Aspects and Open Questions

- Parity and time-reversal-invariance violating forces
- Does nuclear matter bind
- Relations to Skyrme models
- What about the delta resonance
- Alternative large-N scalings
- Renormalization group



Banerjee et al., PRC 65

Two Nucleons in Magnetic Field

- Lagrangian in partial wave basis $P^i = \frac{1}{\sqrt{8}}\sigma^2\sigma^i\tau^2 \qquad \bar{P}^a = \frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^a$

$$\mathcal{L} = eB^i\left[{}^{\not\pi}L_1\left(N^T P^i N\right)^\dagger\left(N^T\bar{P}^3 N\right) - i\epsilon^{ijk}\,{}^{\not\pi}L_2\left(N^T P^j N\right)^\dagger\left(N^T P^k N\right) + \text{h.c.}\right]$$

- Isovector from $np \to d\gamma$ cross section

$$^{\not\pi}L_1(\mu = m_\pi) = 7.24\ \text{fm}^4$$

- Isoscalar determined from fit to deuteron magnetic moment

$$^{\not\pi}L_2(\mu = m_\pi) = -0.149\ \text{fm}^4$$

$$\left|\frac{^{\not\pi}L_2}{^{\not\pi}L_1}\right|_{\text{exp}} = 0.021$$

- Resolution from Large-N_c analysis?

Chen et al., 1999

Large-N$_c$ Basis

- Lagrangian in basis more convenient for large-N$_c$

$$\mathcal{L} = eB^i \left[C_s^{(M)} \left(N^\dagger \sigma^i N\right) \left(N^\dagger N\right) + C_v^{(M)} \epsilon^{ijk} \epsilon^{3ab} \left(N^\dagger \sigma^j \tau^a N\right) \left(N^\dagger \sigma^k \tau^b N\right) \right]$$

$$C_s^{(M)} \sim O(N_c^0) \qquad\qquad C_v^{(M)} \sim O(N_c)$$

- Fierz transformations relate these couplings to the partial wave couplings

$$^{\not{\pi}}L_1 = 8C_v^{(M)} \qquad\qquad ^{\not{\pi}}L_2 = -C_s^{(M)}$$

Richardson and Schindler, 2020

Large-N_c Prediction

- Partial wave basis

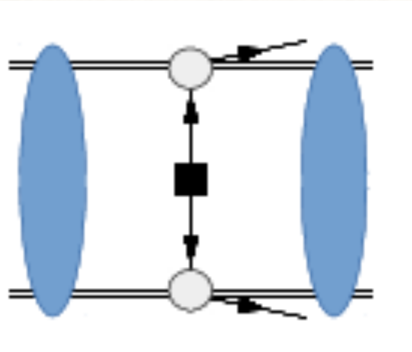
$$\left|\frac{\not{\pi} L_2}{\not{\pi} L_1}\right|_{\text{Large}-N_c} \sim \frac{1}{8N_c} \approx 0.042 \qquad \left|\frac{\not{\pi} L_2}{\not{\pi} L_1}\right|_{\text{exp}} = 0.021$$

- Large-N basis

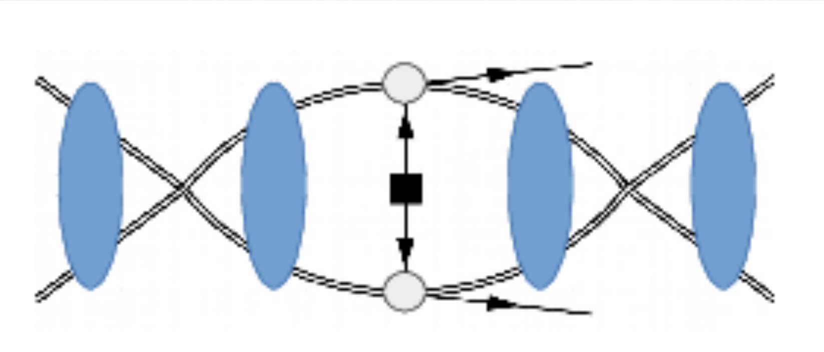
$$\left.\frac{C_s^{(M)}}{C_v^{(M)}}\right|_{\text{large-}N_c} \sim \frac{1}{N_c} \qquad \left.\frac{C_s^{(M)}}{C_v^{(M)}}\right|_{\text{exp}} = 0.165 \approx \frac{1}{2N_c}$$

- Standard power counting + large-N explains suppression
- Use large-N to inform Bayesian priors?

Light Majorana Exchange Mechanism



$$V_{\nu L}^{^1S_0}(q) = \frac{\tau^{(1)+}\tau^{(2)+}}{q^2}\left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(q^2+m_\pi^2)^2}\right]$$



- Contact term renormalizes log-type divergence

$$\mathcal{L}_{\Delta L=2}^{NN} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta}\bar{e}_L C\bar{e}_L^T \frac{g_\nu^{NN}}{4}\left[\left(N^\dagger u\tau^+ u^\dagger N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tau^+\tau^+\right)\left(N^\dagger\tau^a N\right)^2\right] + \mathrm{H.c}$$

$$u = \exp\left(\frac{i}{F_0}\phi_a\tau^a\right)$$

Cirigliano et al. PRC 97, PRL 120, PRC 100

Relation to Charge Independence Breaking

- CIB isotensor Lagrangian

$$\tilde{Q}_\pm = \frac{1}{4}\left[u^\dagger \tau^3 u \pm u \tau^3 u^\dagger\right]$$

$$\mathcal{L}^{NN}_{CIB} = \frac{e^2}{2}\left\{(\mathcal{C}_1 + \mathcal{C}_2)\left[\left(N^\dagger \tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger \tau^a N\right)^2\right]\right.$$
$$\left.(\mathcal{C}_1 - \mathcal{C}_2)\left[\left(N^\dagger \tilde{Q}_- N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger \tau^a N\right)^2\right]\right\}$$

Epelbaum and Meißner 1999, Walzl et al. 2001

- Chiral symmetry dictates

$$g_\nu^{NN} = \mathcal{C}_1$$

- Approximate

$$g_\nu^{NN} = \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$$

Cirigliano et al. 2018a, 2018b, 2019, 2021

Large-N Lagrangian

- LNV contact term

TRR et al. PRC 103

$$\left(N^\dagger\sigma^i\tau^+N\right)\left(N^\dagger\sigma^i\tau^+N\right) = -3\left(N^\dagger\tau^+N\right)\left(N^\dagger\tau^+N\right)$$

$$g_\nu^{NN} \sim O(N_c)$$

- CIB contact terms

$$\mathcal{L}^{\Delta I=2}_{\text{LO-in-}N_c} = \bar{\mathcal{C}}_3\left[\left(N^\dagger\sigma^i\tilde{Q}_+N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger\sigma^i\tau^aN\right)^2\right]$$
$$\mathcal{L}^{\Delta I=2}_{\text{NLO-in-}N_c} = \bar{\mathcal{C}}_6\left[\left(N^\dagger\sigma^i\tilde{Q}_-N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger\sigma^i\tau^aN\right)^2\right]$$

$$\bar{\mathcal{C}}_3 \sim O(N_c) \qquad \bar{\mathcal{C}}_6 \sim O(1)$$

Large-N$_c$ Consistency

- CIB LECs same size and sign

$$\mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$
$$\mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$

- LNV and CIB scale the same way

$$g_\nu^{NN} \sim O(N_c) \qquad\qquad \mathcal{C}_1 + \mathcal{C}_2 \sim O(N_c)$$

$$\Longrightarrow \; g_\nu^{NN} = \frac{1}{2}\left(\mathcal{C}_1 + \mathcal{C}_2\right)$$

Comparison to Cottingham

Cirigliano et al., PRL 126, JHEP 05

- Central Cottingham values fall within large-N_c estimate

$$\tilde{\mathcal{C}}_1\,(\mu = m_\pi) = 1.3(6)$$

$$\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2\,(\mu = m_\pi) = 2.9(1.2)$$

$$\Rightarrow \quad \frac{\tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2} \approx 0.81 \qquad \frac{\tilde{\mathcal{C}}_1 - \tilde{\mathcal{C}}_2}{\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2} \approx 0.1$$

- Large-N_c with experiment

$$\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2\,(\mu = m_\pi) = 5.1 \qquad \tilde{\mathcal{C}}_1\Big|_{\text{Large-}N_c} \approx 2.5$$

$$1.7 \lesssim \tilde{\mathcal{C}}_1 \lesssim 3.3 \qquad 1.8 \lesssim \tilde{\mathcal{C}}_2 \lesssim 3.4$$

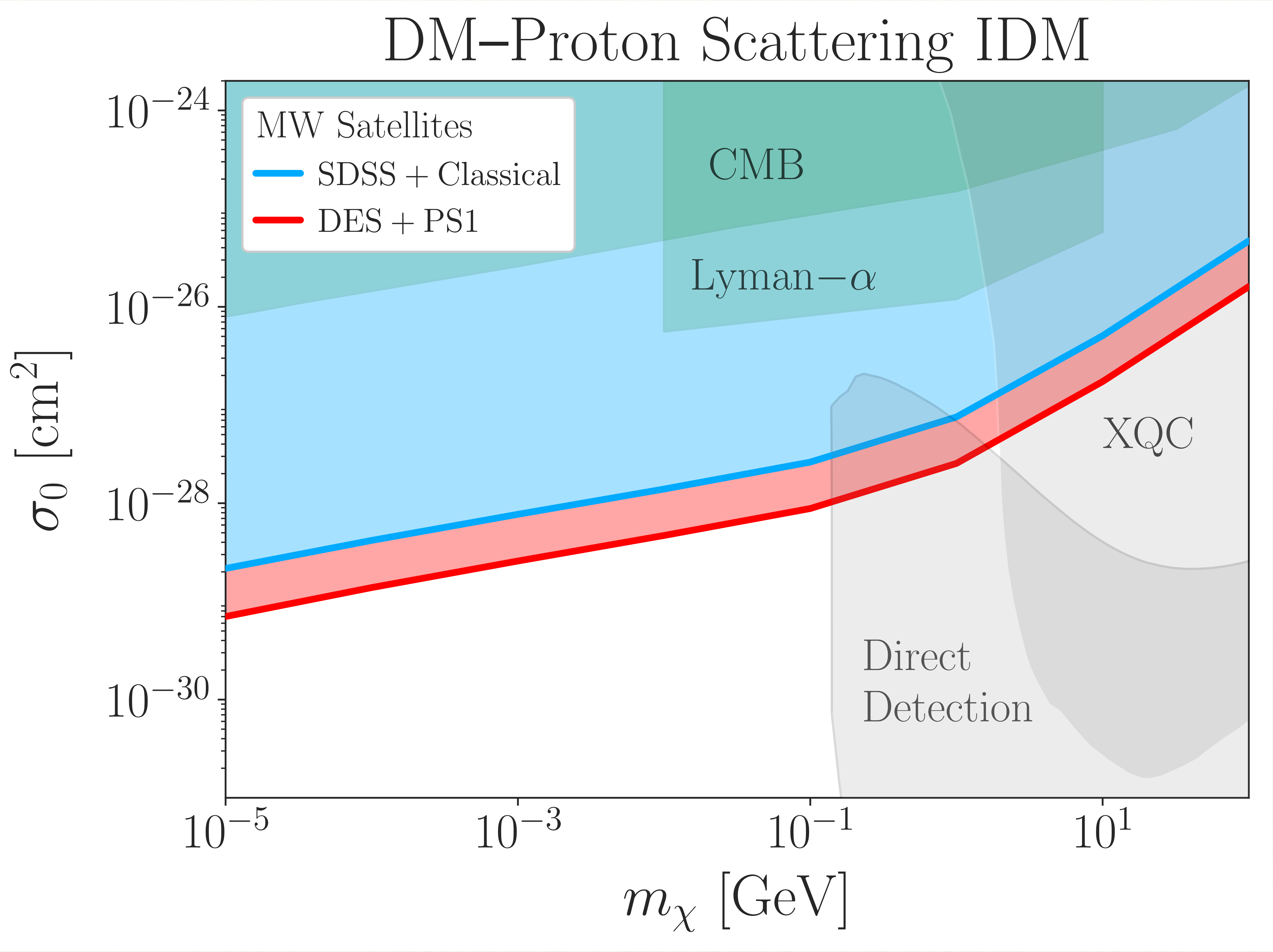
Dark Matter Direct Detection

- Recent interest in light nuclear targets

- Momentum bounded from above by a few MeV

$$\frac{d\sigma}{dE_R} = \frac{1}{32\pi m_\chi^2 m_T v_\chi^2} |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 = 16\pi \left(m_T + m_\chi\right)^2 \left[\sigma_0^{\mathrm{SI}} F_{\mathrm{SI}}^2(E_R) + \sigma_0^{\mathrm{SD}} F_{\mathrm{SD}}^2(E_R)\right]$$



Nadler et al., PRL 126

Single-Nucleon Lagrangian

Fitzpatrick et al., 2013;
Hill and Solon, 2015

TRR et al., PRC 106

- Zero derivatives

$$\mathcal{L}_{\chi N}^{(PT)} = \left(\chi^\dagger \chi\right) \left[C_{1,\chi N}^{(PT)} \left(N^\dagger N\right) + C_{4,\chi N}^{(PT)} \left(N^\dagger \tau^3 N\right)\right] + \left(\chi^\dagger \sigma^i \chi\right) \left[C_{3,\chi N}^{(PT)} \left(N^\dagger \sigma^i N\right) + C_{2,\chi N}^{(PT)} \left(N^\dagger \sigma^i \tau^3 N\right)\right]$$

$$C_{1,\chi N}^{(PT)},\, C_{2,\chi N}^{(PT)} \sim O(N_c)$$

$$C_{3,\chi N}^{(PT)},\, C_{4,\chi N}^{(PT)} \sim O(1)$$

- One derivative—16 terms, but only 6 are $O(N_c)$
  - 3 suppressed by WIMP mass

$$\mathcal{L}_{\chi N}^{\mathrm{LO-in-}N_c} \supset \left(\chi^\dagger \chi\right) \left[C_5^{(\not P T)} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^3 N\right) + C_{11}^{(\not P \not T)} \nabla^i \left(N^\dagger \sigma^i \tau^3 N\right)\right] + C_{12}^{(\not P \not T)} \left(\chi^\dagger \sigma^i \chi\right) \nabla^i \left(N^\dagger N\right)$$

Two-Nucleon Lagrangian for WIMP-Deuteron Scattering

- 7 dark matter-two-nucleon couplings

TRR et al., PRC 106

  - 2 $^3S_1$ couplings
  - 1 coupling at $O(N_c)$

$$\mathcal{L}_{\chi NN} = C^{(s)}_{1,\chi NN}\left(\chi^\dagger\chi\right)\left(N^\dagger N\right)^2 + C^{(s)}_{3,\chi NN}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^i N\right)\left(N^\dagger N\right)$$

$$C^{(s)}_{1,\chi NN} \sim O(N_c) \qquad C^{(s)}_{3,\chi NN} \sim O(1)$$

Deuteron Form Factor

- Generic form factors

TRR et al., PRC 106

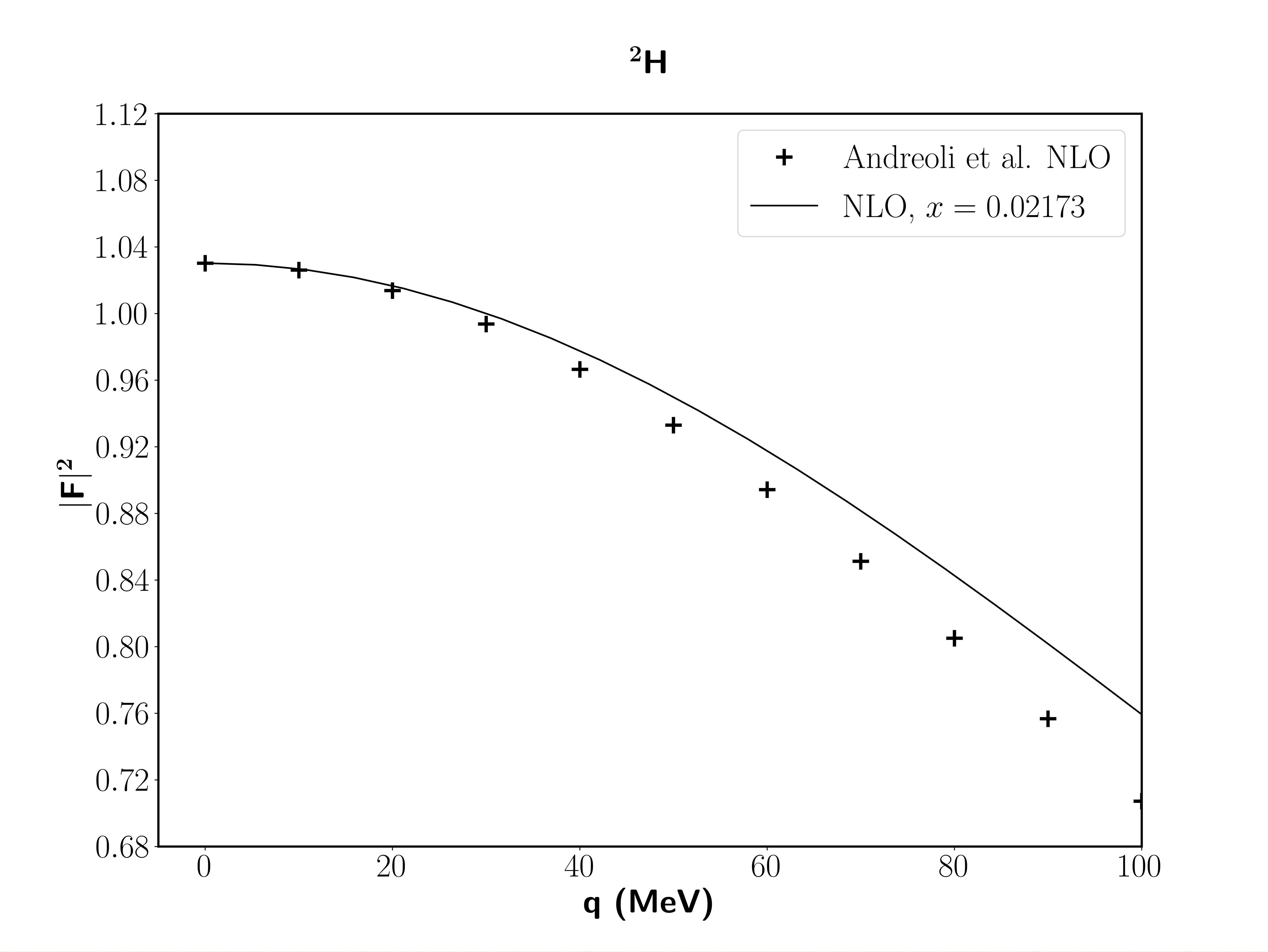
$$F_{\rm SI}(q^2) = \frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right) + \frac{\gamma}{2\pi}\left(\mu-\gamma\right)^2\left[1-\frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)\right]\left[m_N C_2 - \frac{\left(C^{(\mathrm{SI},s)}_{1,\chi NN}+C^{(\mathrm{SI},s)}_{2,\chi NN}\right)}{C^{(PT)}_{1,\chi N}}\right]$$

$$F_{\rm SD}(q^2) = \frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right) + \frac{\gamma}{2\pi}\left(\mu-\gamma\right)^2\left[1-\frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)\right]\left[m_N C_2 - \frac{C^{(\mathrm{SD},\,s)}_{1,\chi NN}}{C^{(PT)}_{3,\chi N}}\right]$$

- Focus on specific model—WIMP coupled to scalar quark current

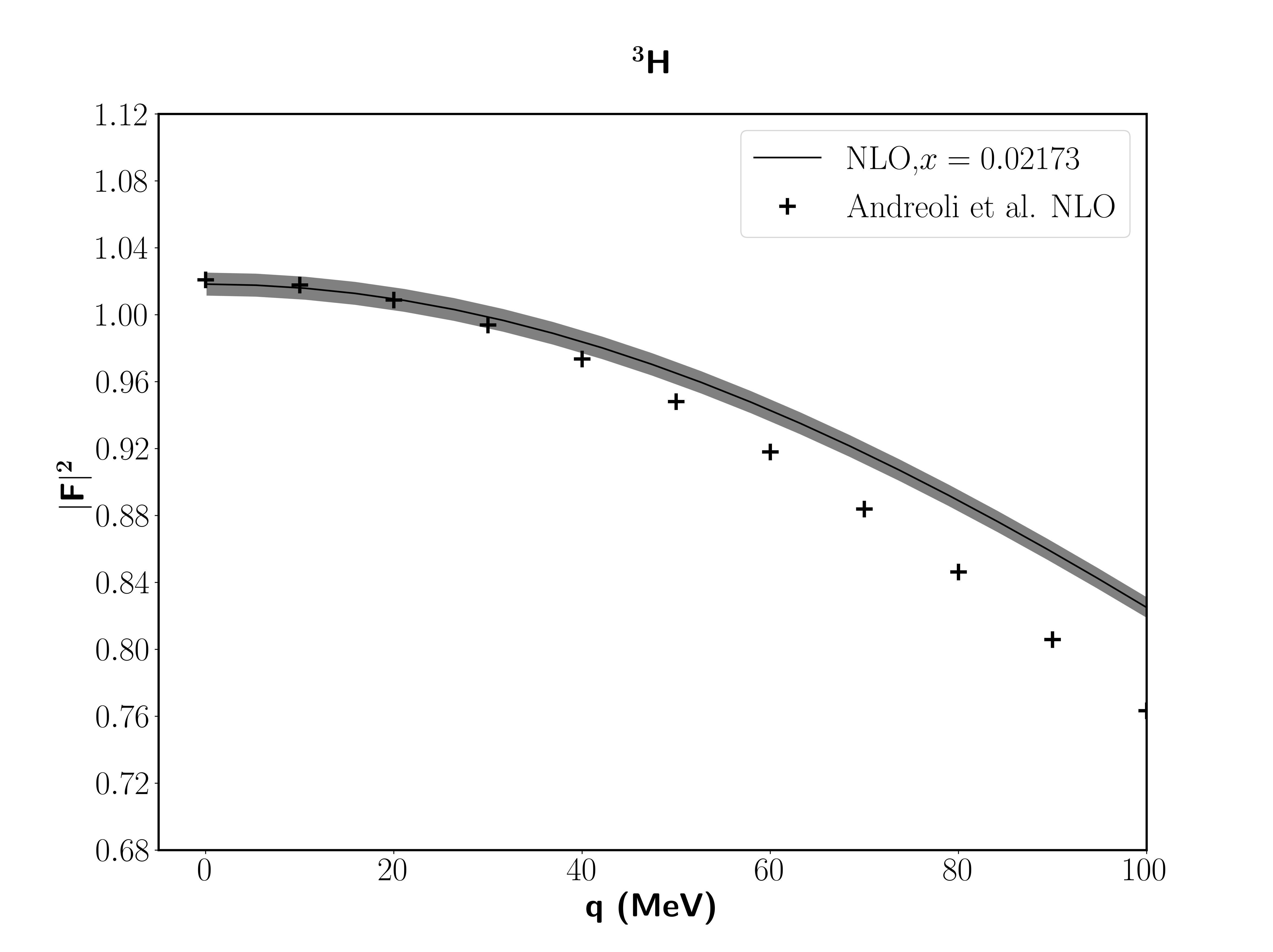
➡Nuclear sigma terms

Scalar Current Response Function: Deuteron

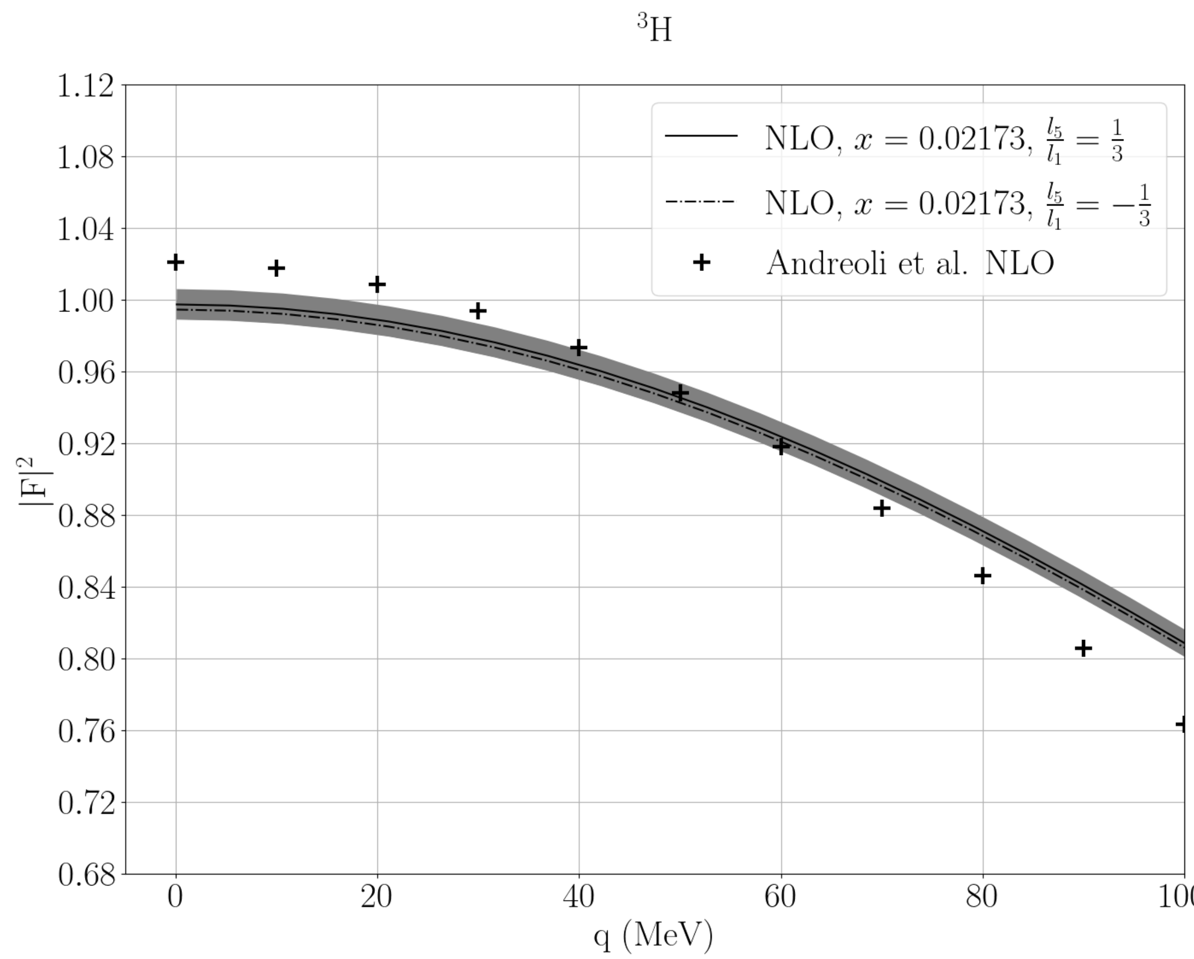


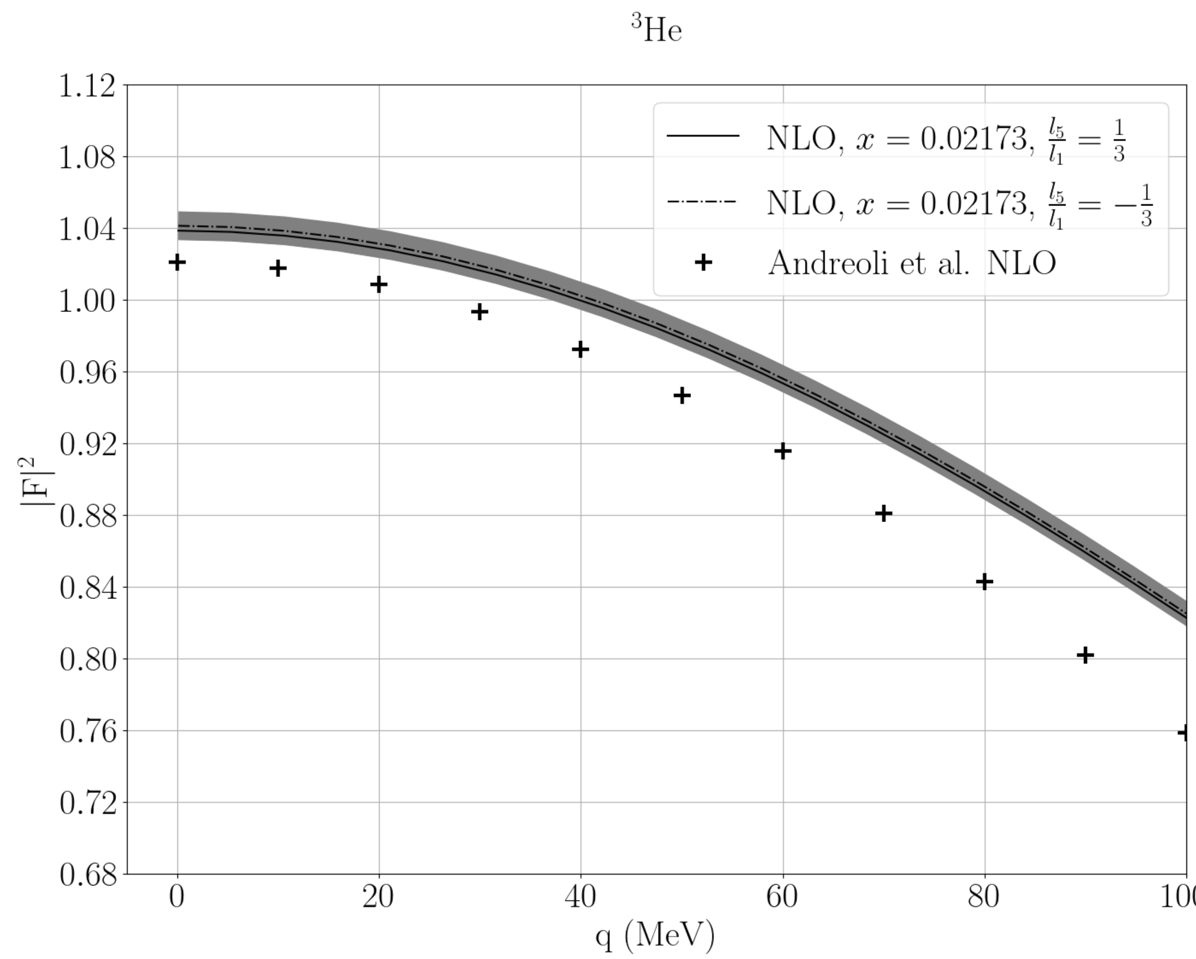
Andreoli et al. PRC 99

Scalar Current Response: Triton



Scalar Current Response Function: Triton vs. 3He





Summary

- Large-N QCD constrains relative sizes of couplings *based on symmetry*
    - No (maybe some) data
    - Constraints for few-nucleon forces agrees with phenomenology
- Inform development Bayesian priors
- Explains sizes of magnetic couplings in pionless EFT
- Novel constraints for BSM processes