

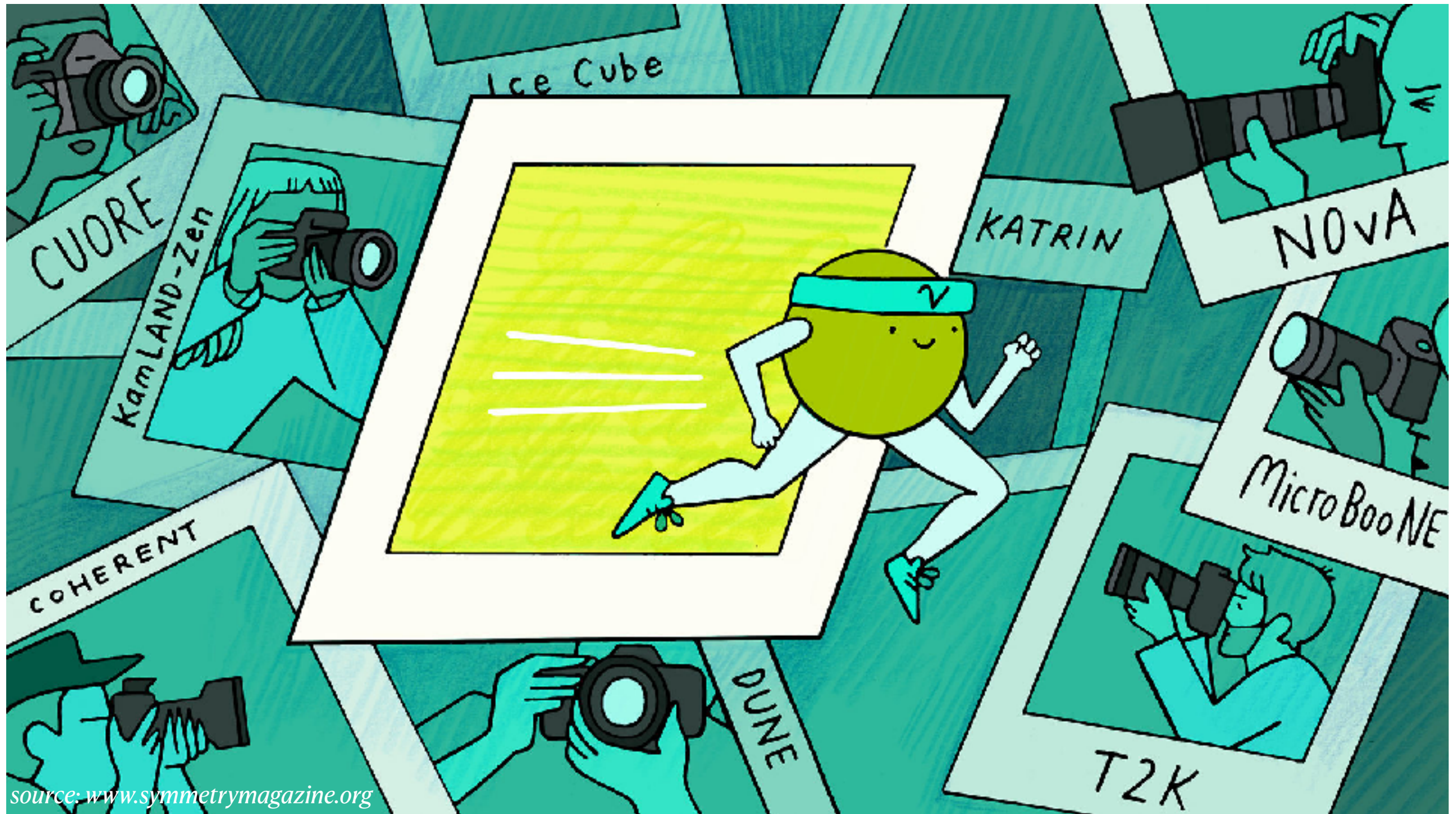
Nuclear ab initio studies for neutrino oscillations

Joanna Sobczyk

Hirschegg, 17 January 2023

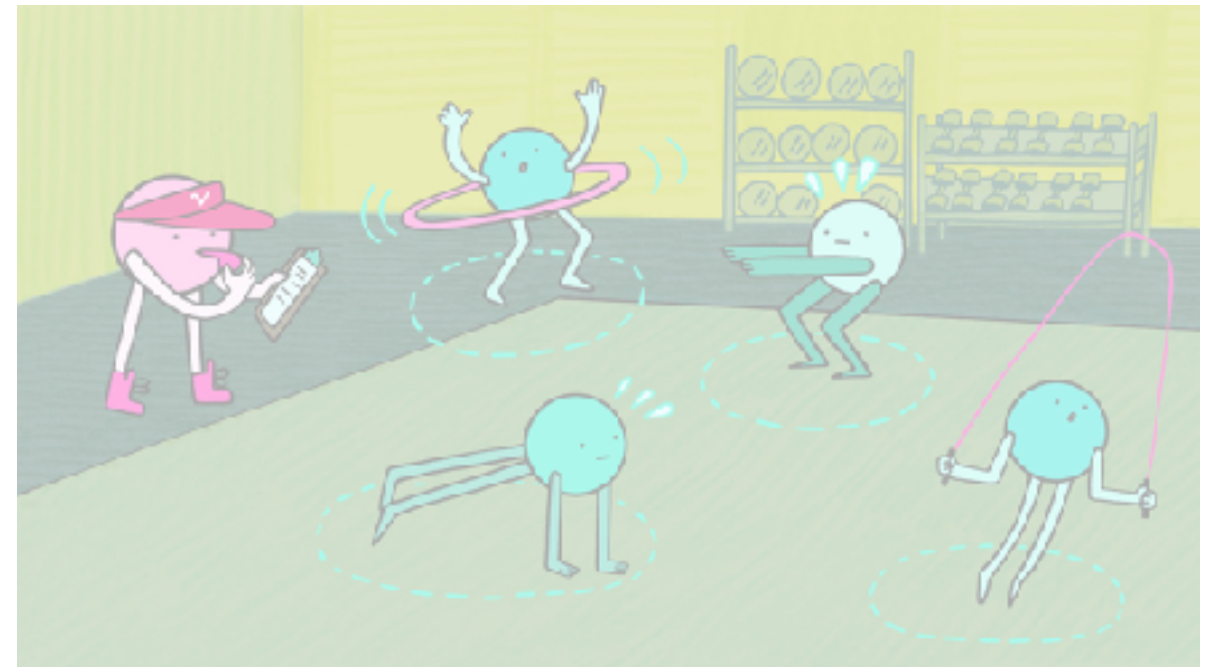
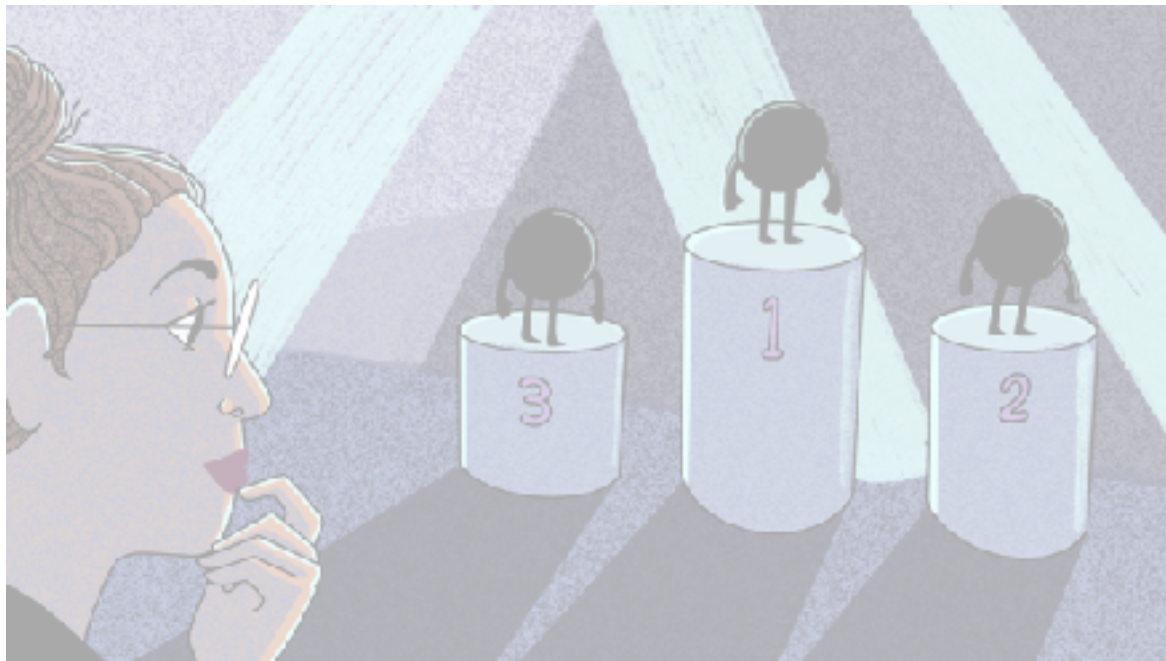


This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 101026014



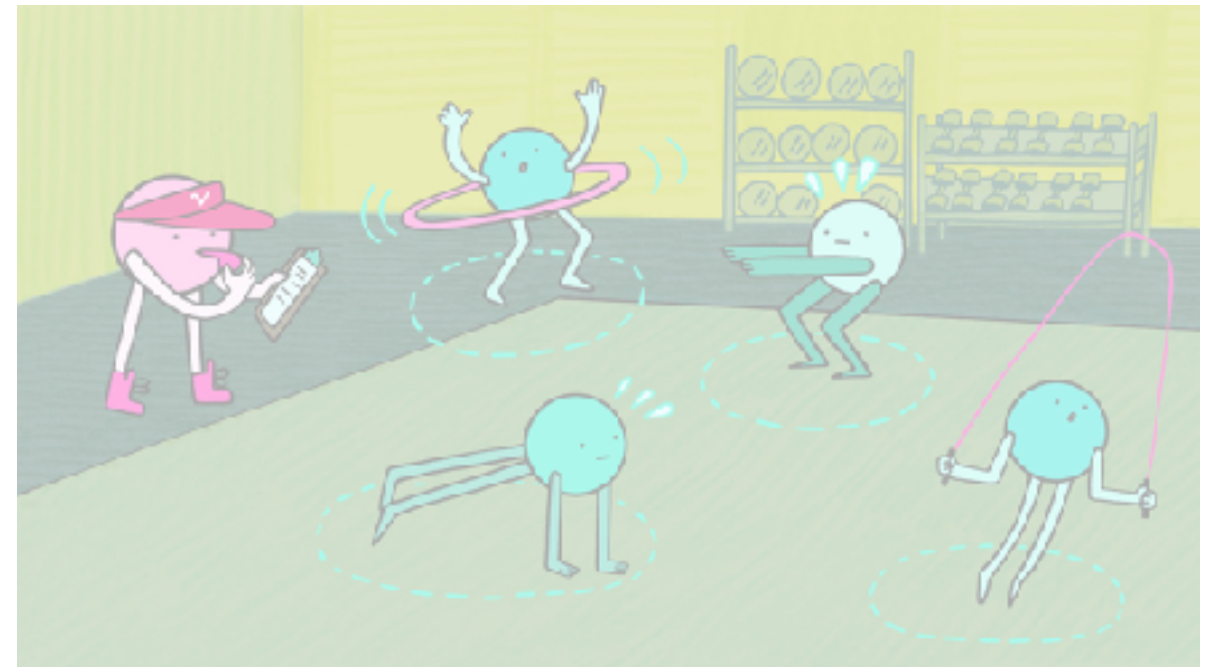
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Neutrino Physics



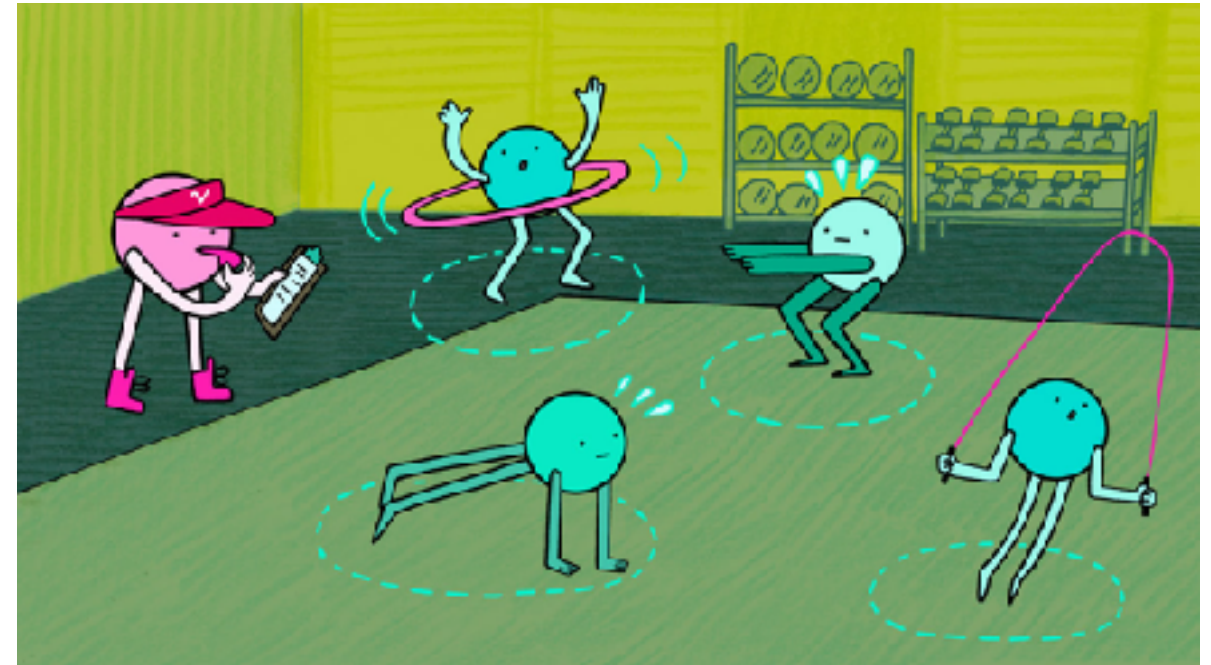
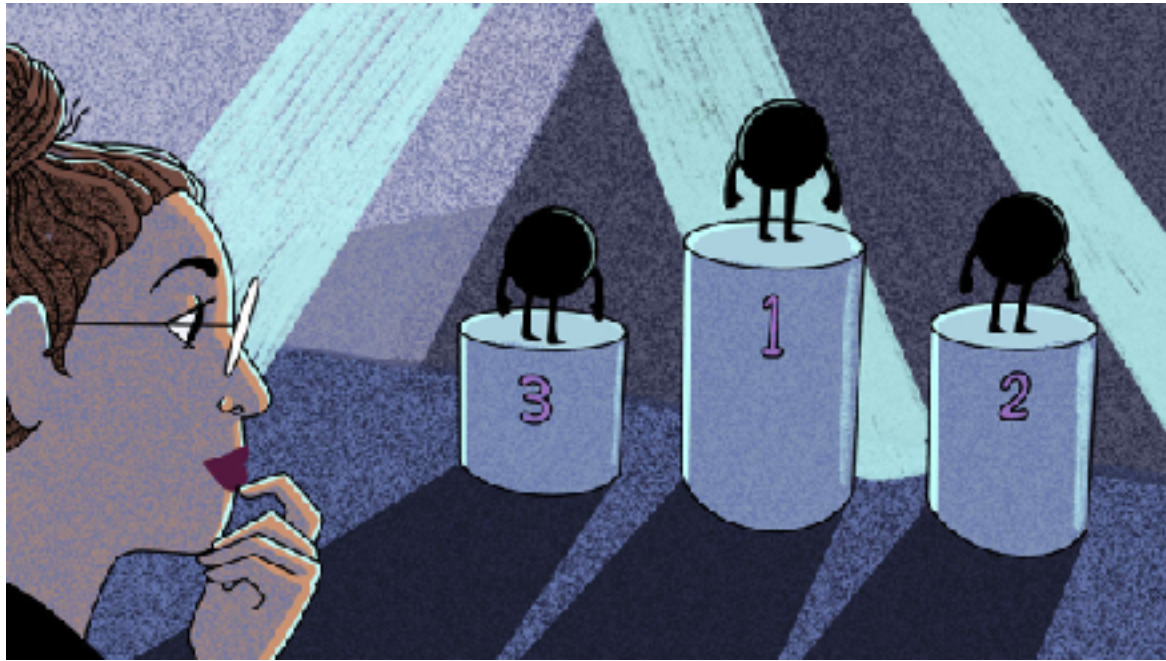
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Neutrino Physics



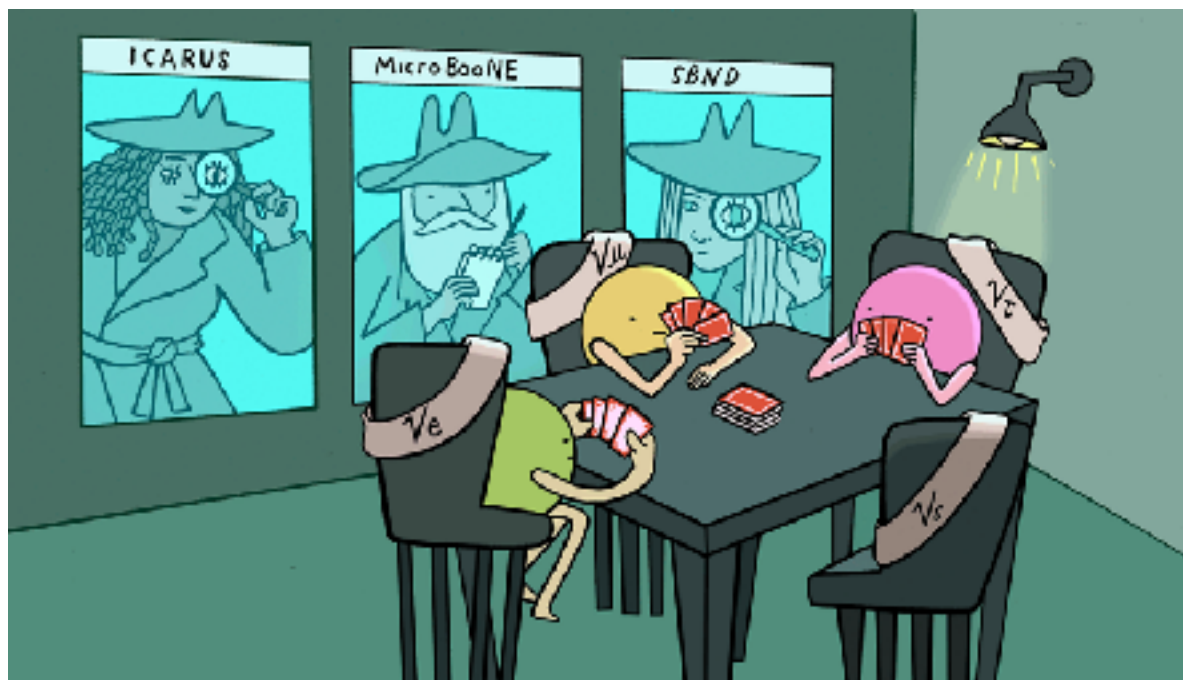
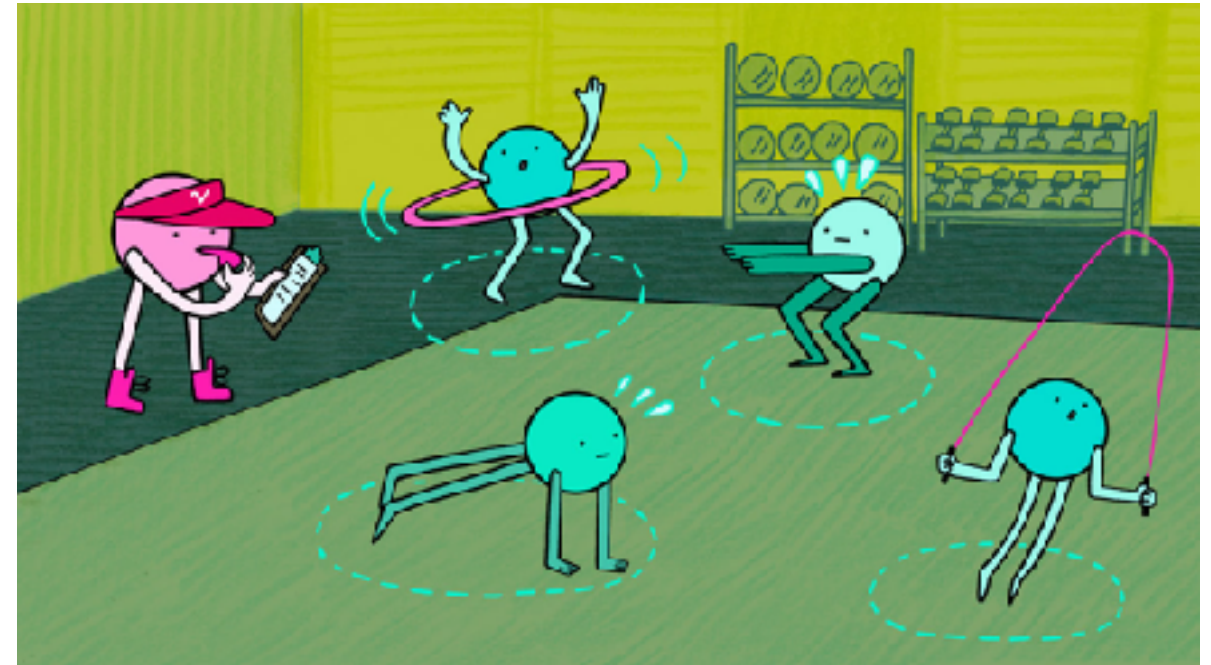
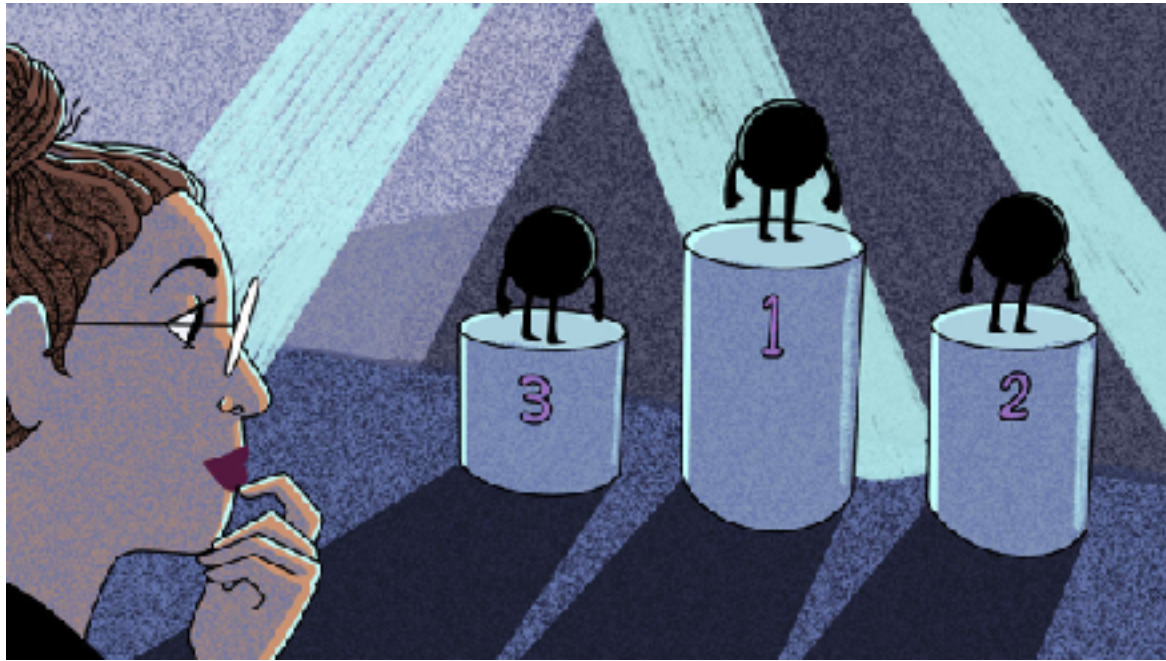
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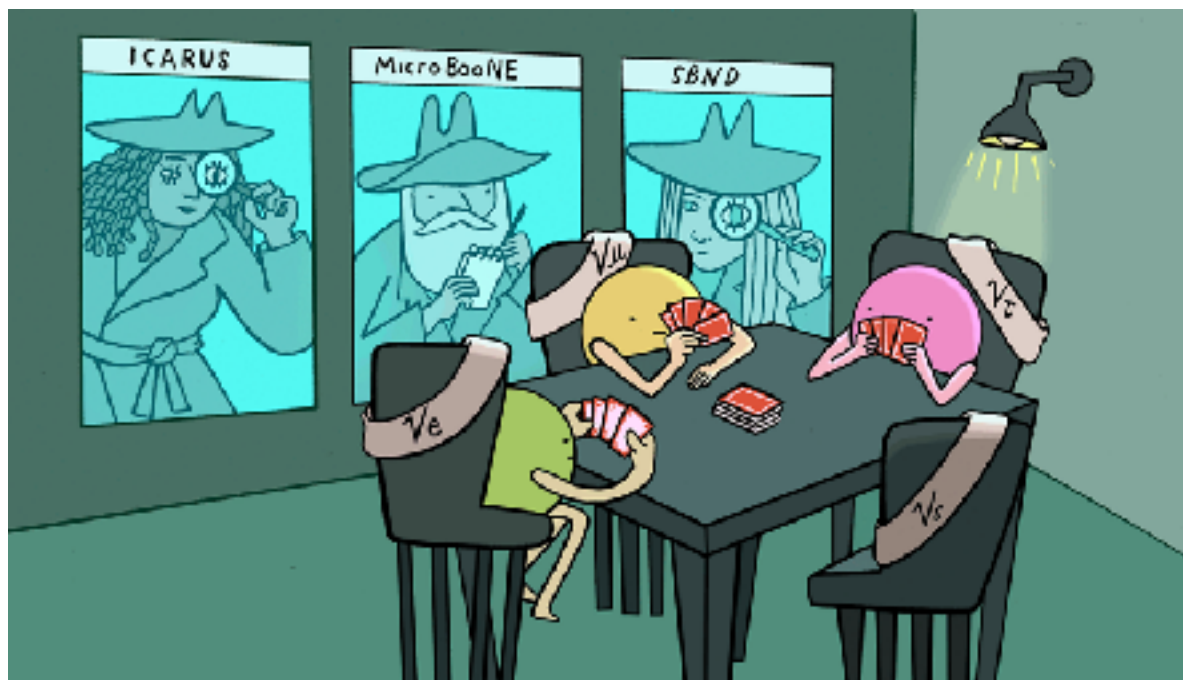
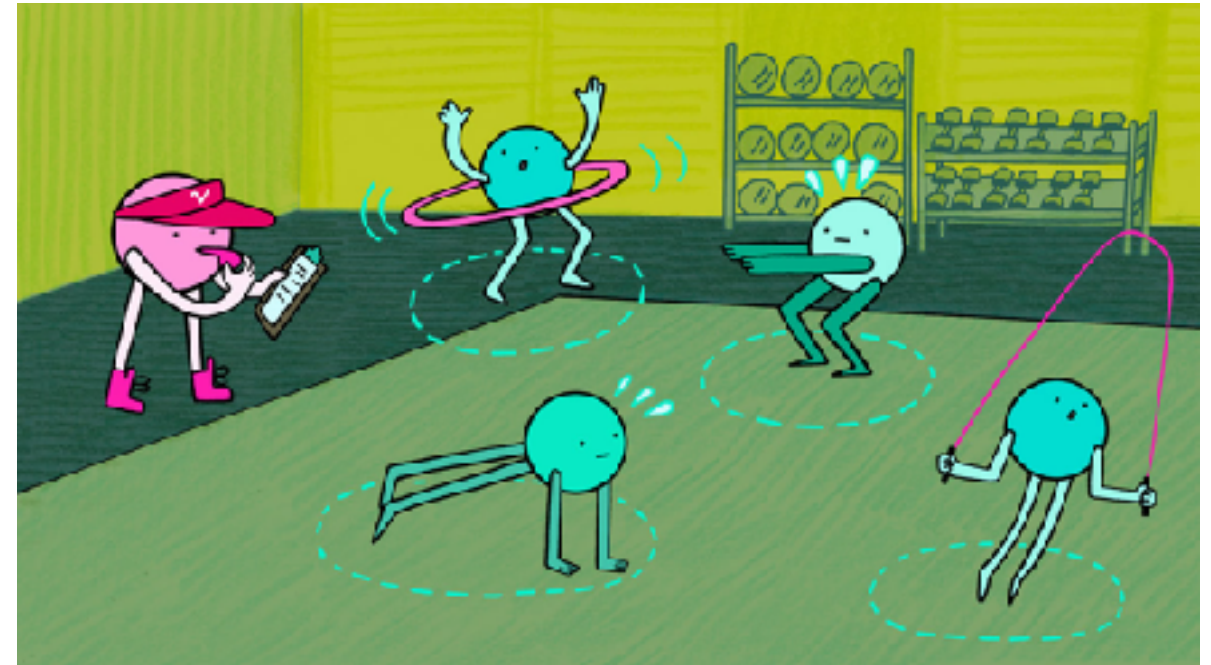
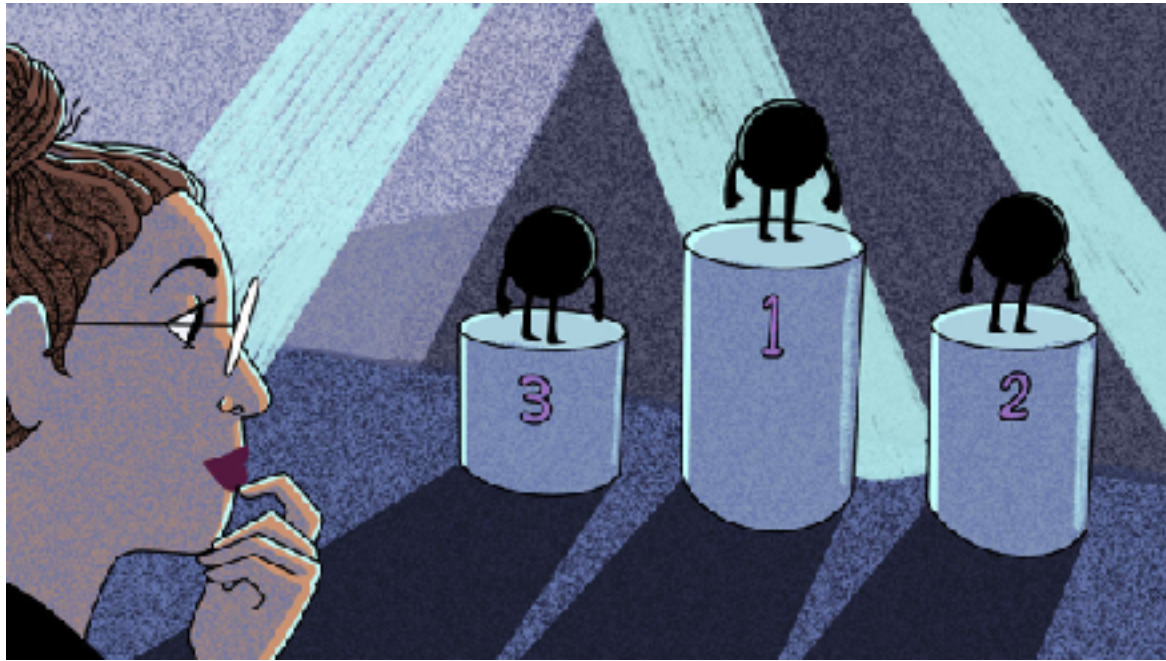
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Neutrino Physics



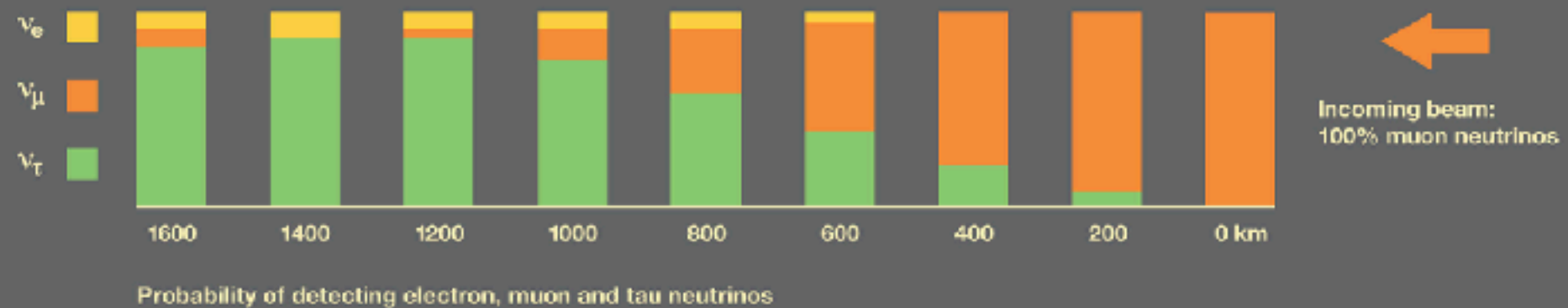
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Neutrino Physics

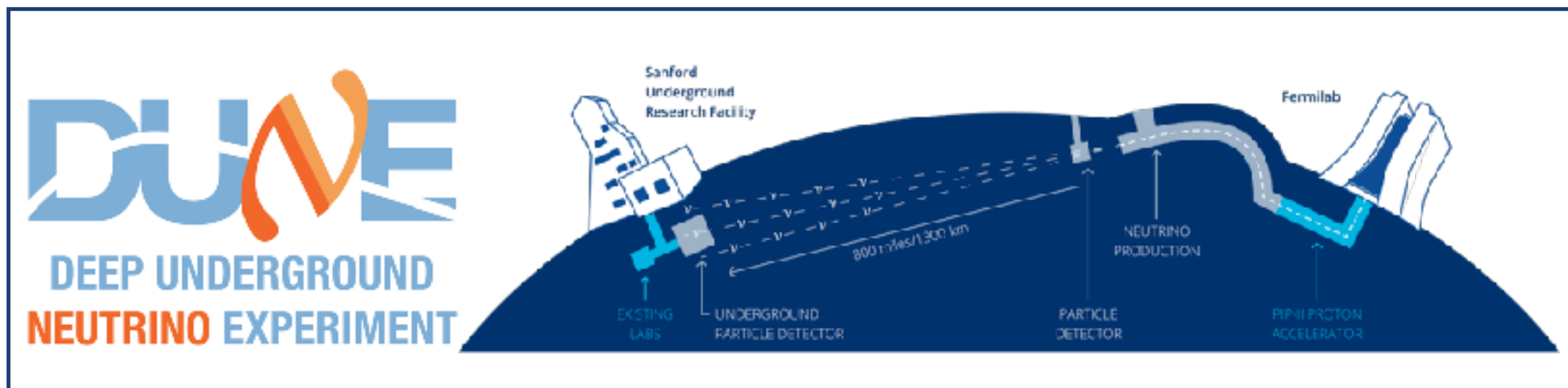
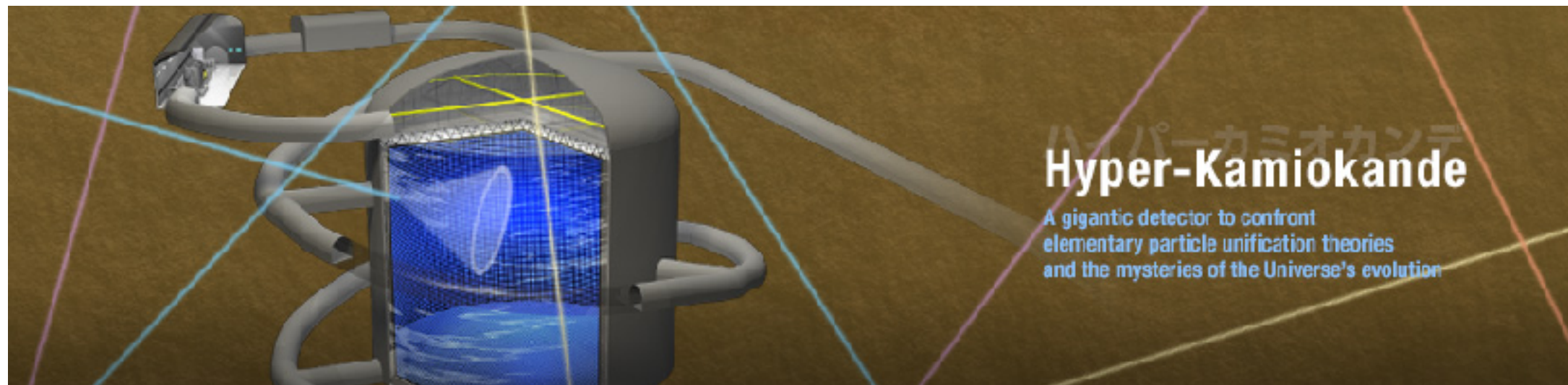


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Neutrino oscillations



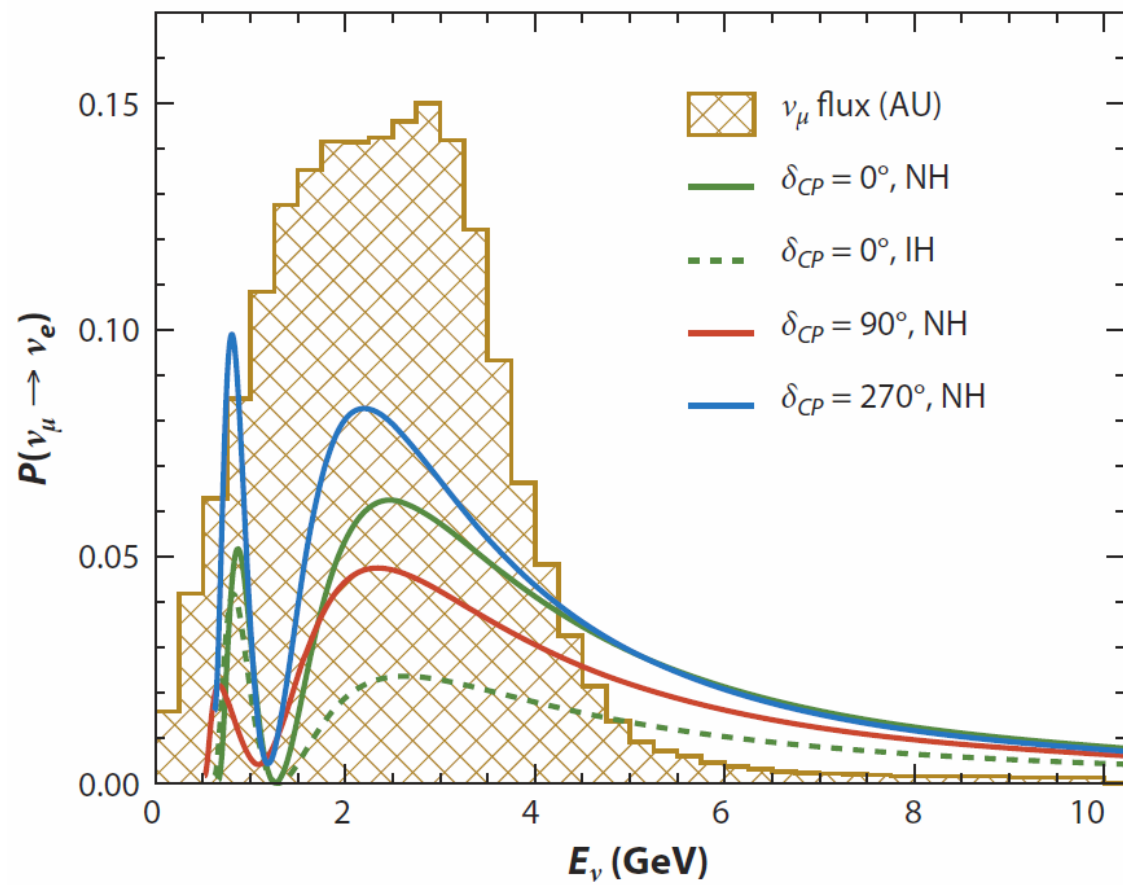
Next generation experiments



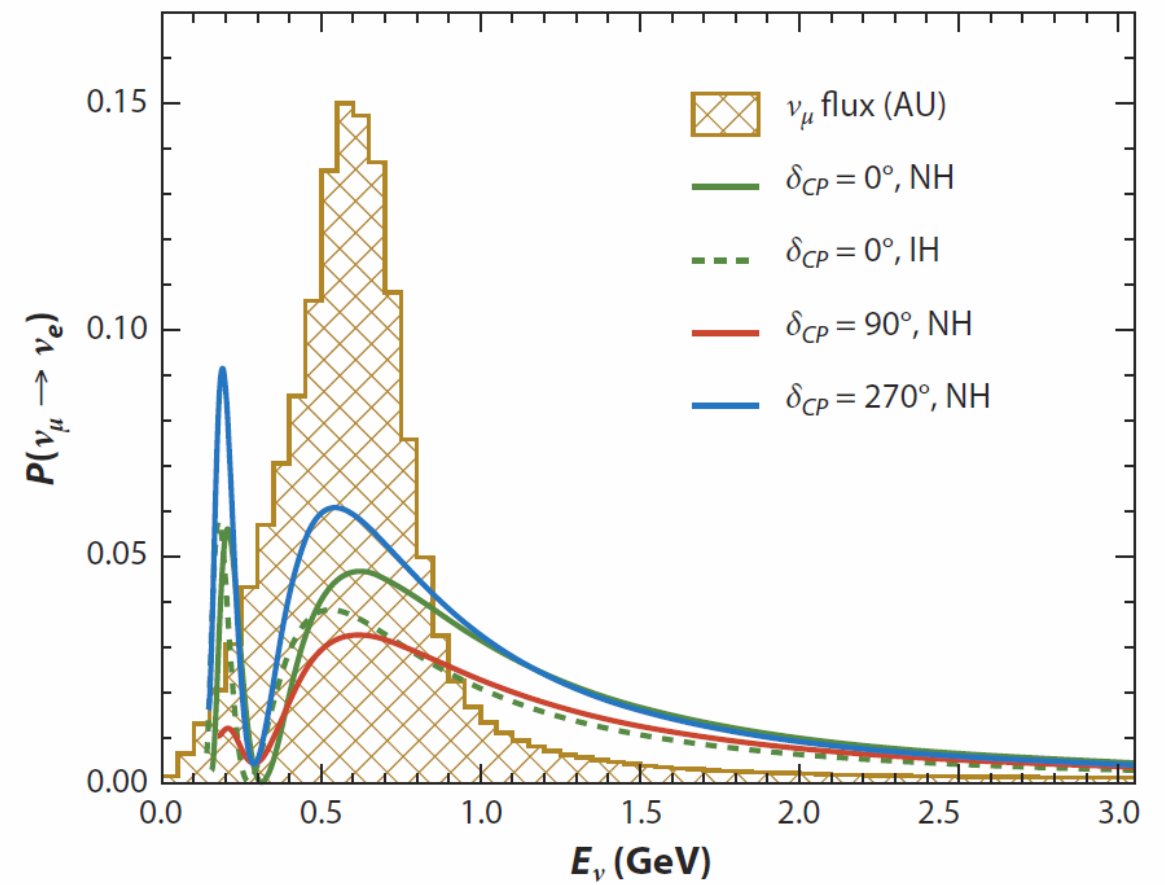
- ✓ CP-violation measurement
- ✓ Determining ν mass ordering
- ✓ Proton decay searches
- ✓ Cosmic neutrino observation

Aims & challenges

DUNE



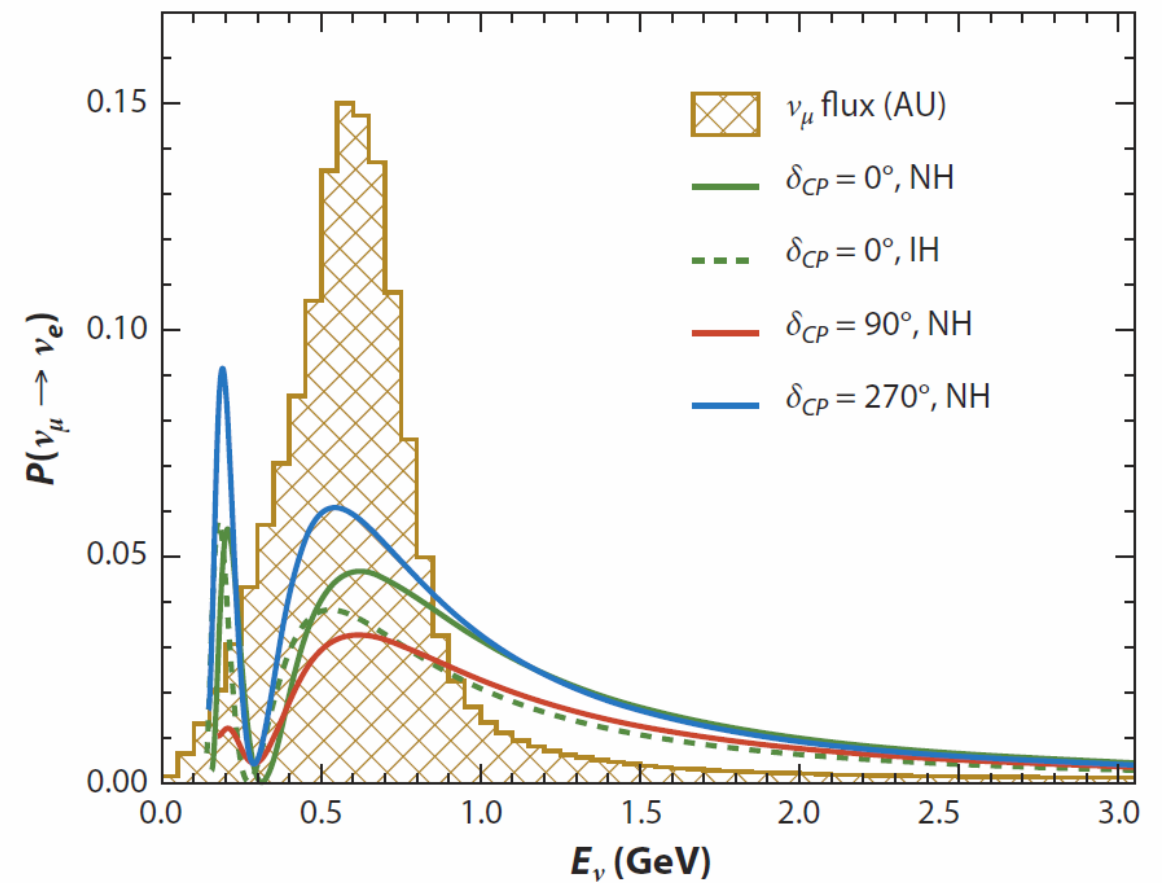
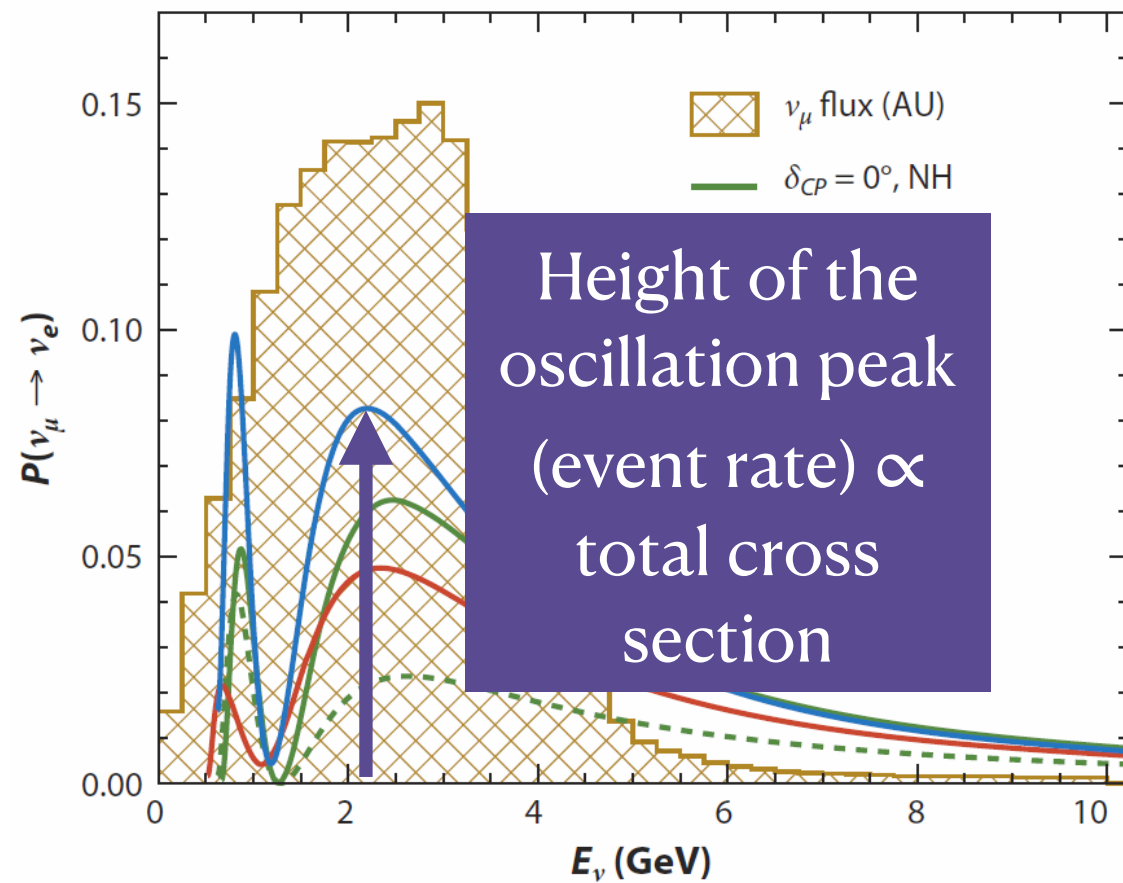
T2HK



Aims & challenges

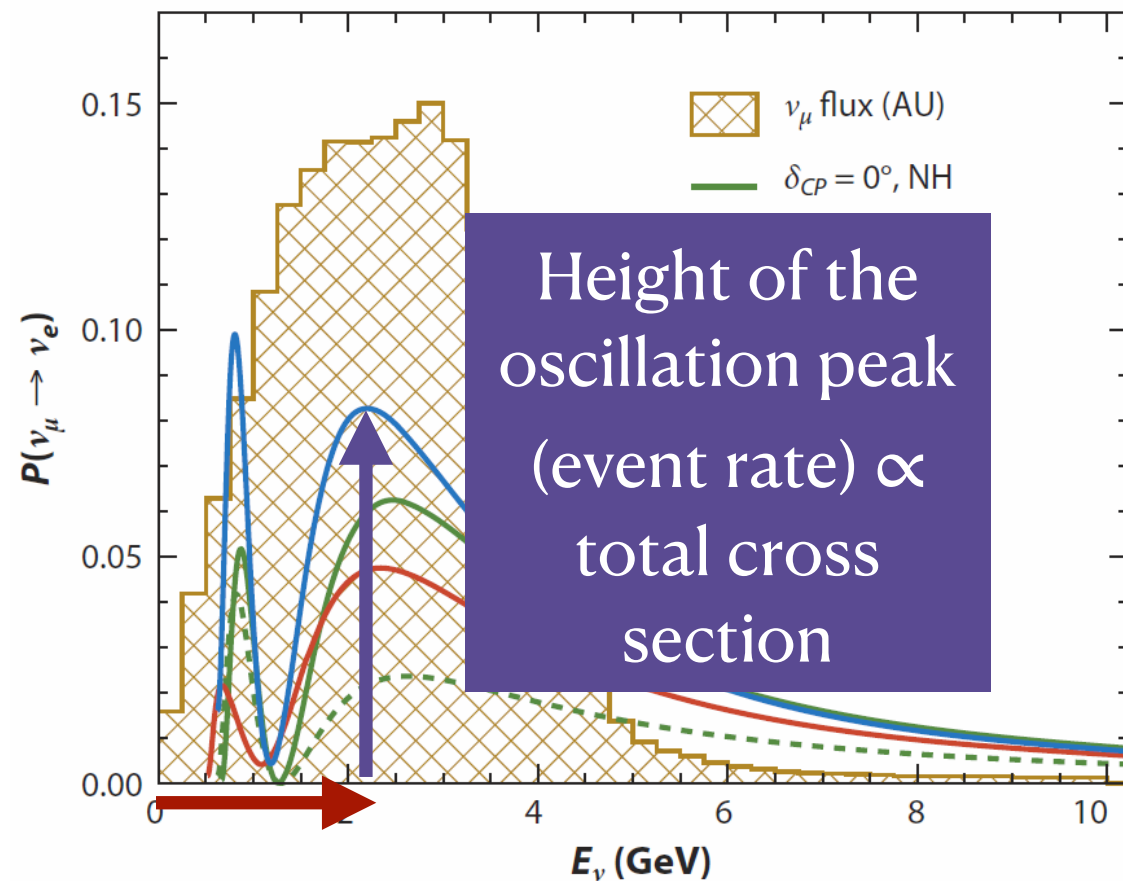
DUNE

T2HK



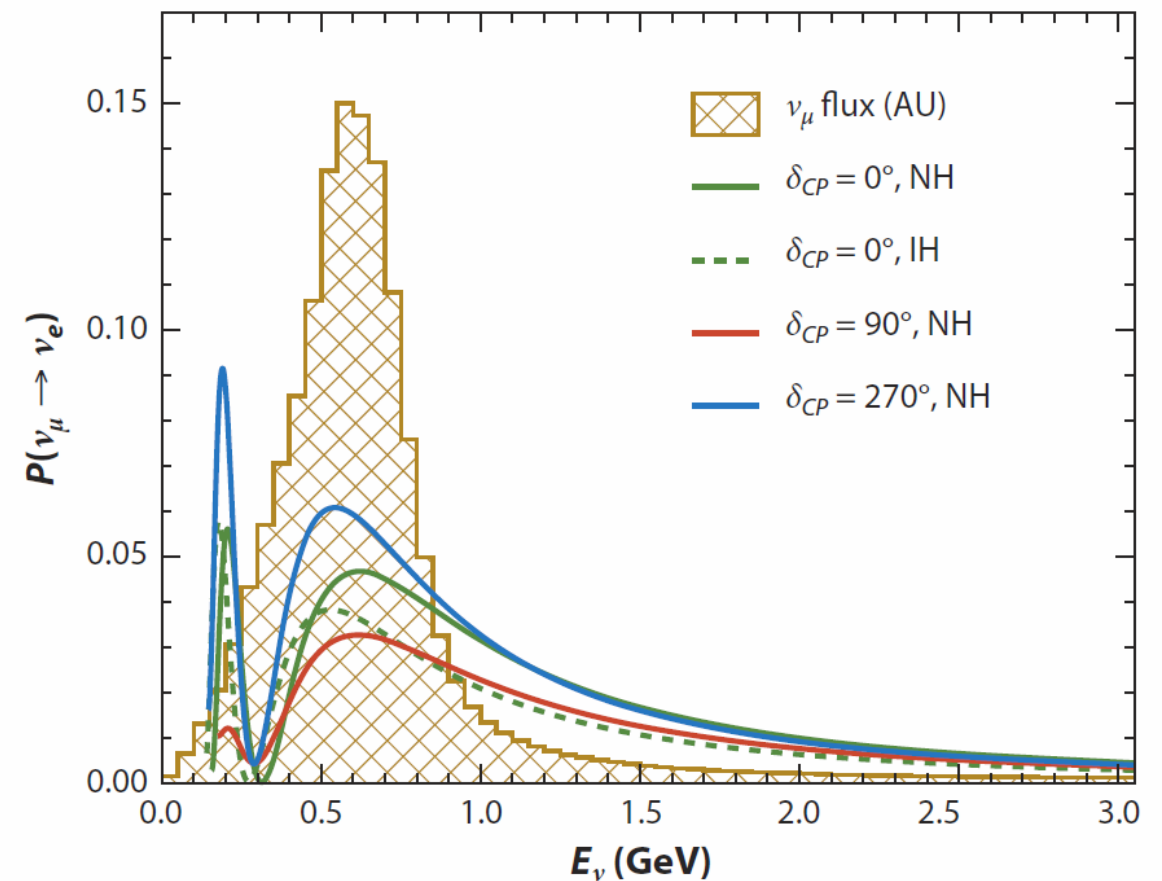
Aims & challenges

DUNE



Position of the oscillation peak depends on energy reconstruction

T2HK

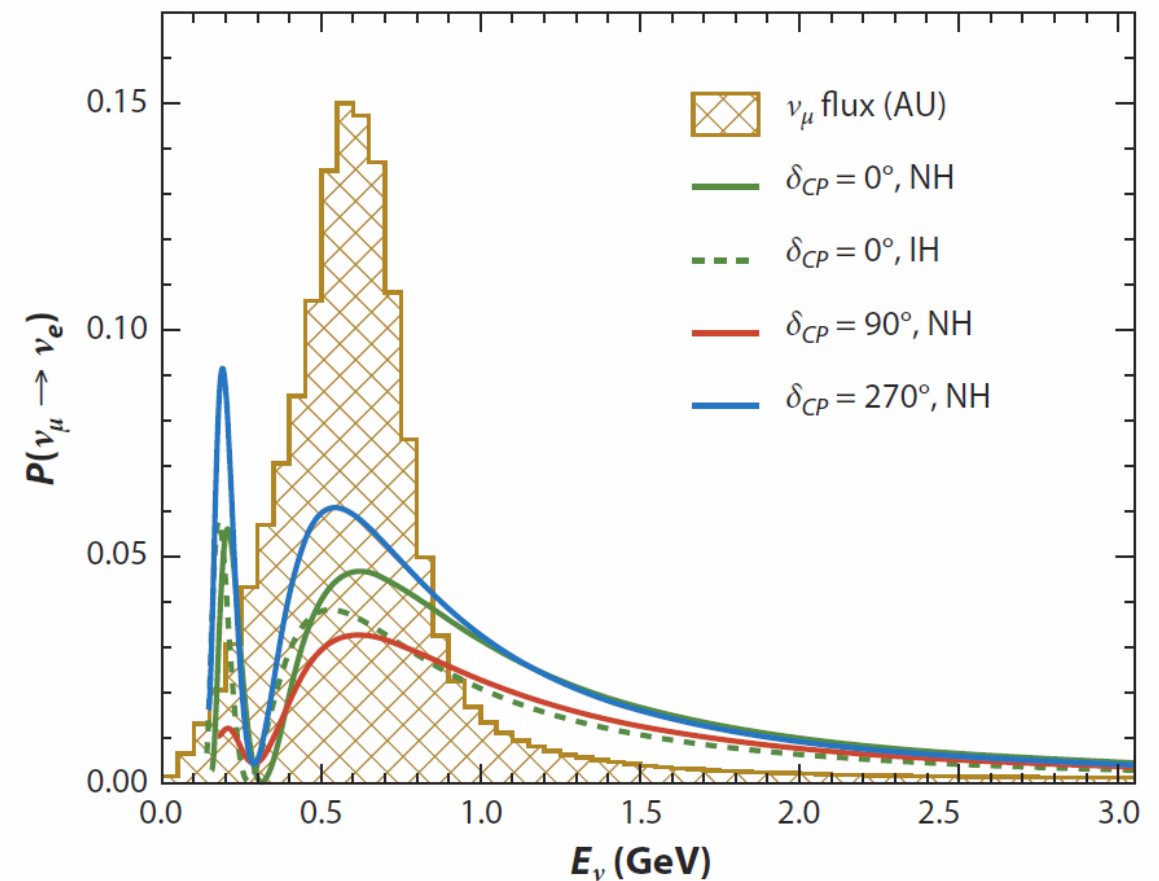
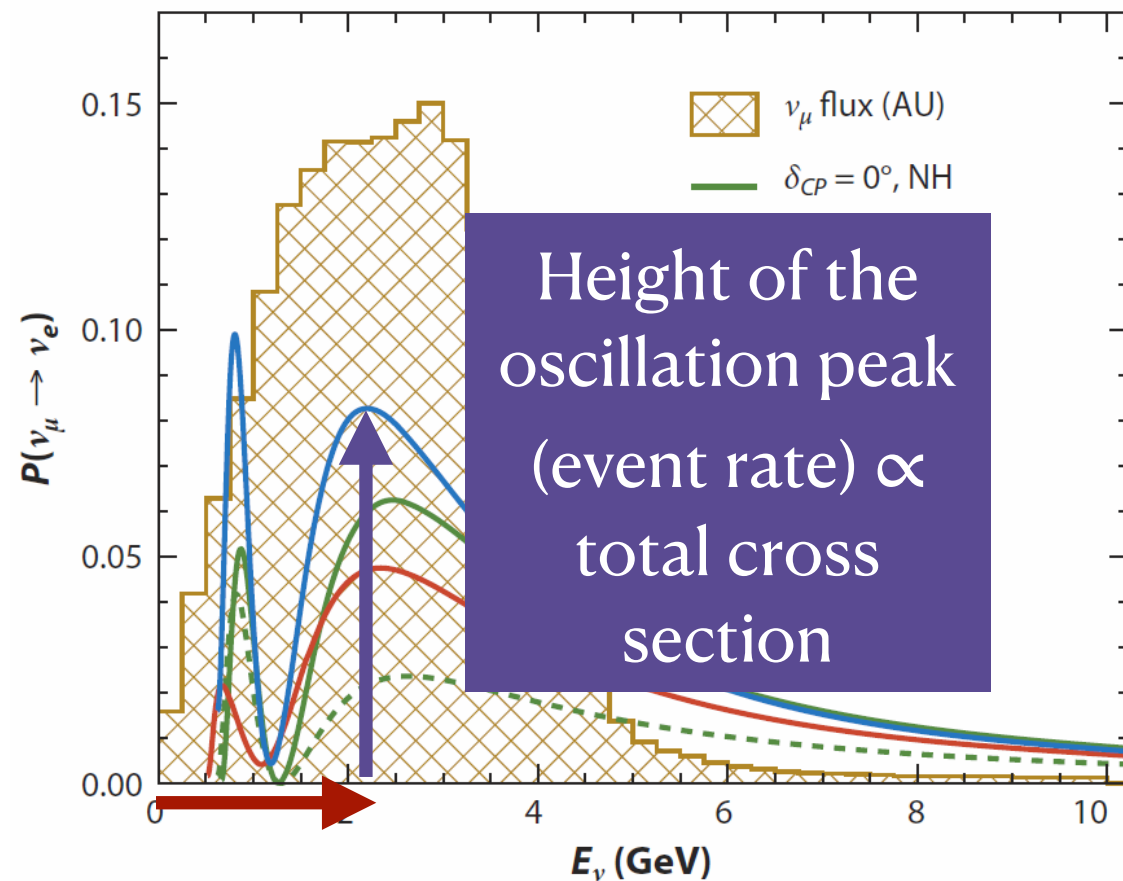


DUNE aims at uncertainties $< 1\%$ meaning $O(25 \text{ MeV})$ precision of energy reconstruction

Aims & challenges

DUNE

T2HK



Position of the oscillation peak depends on energy reconstruction

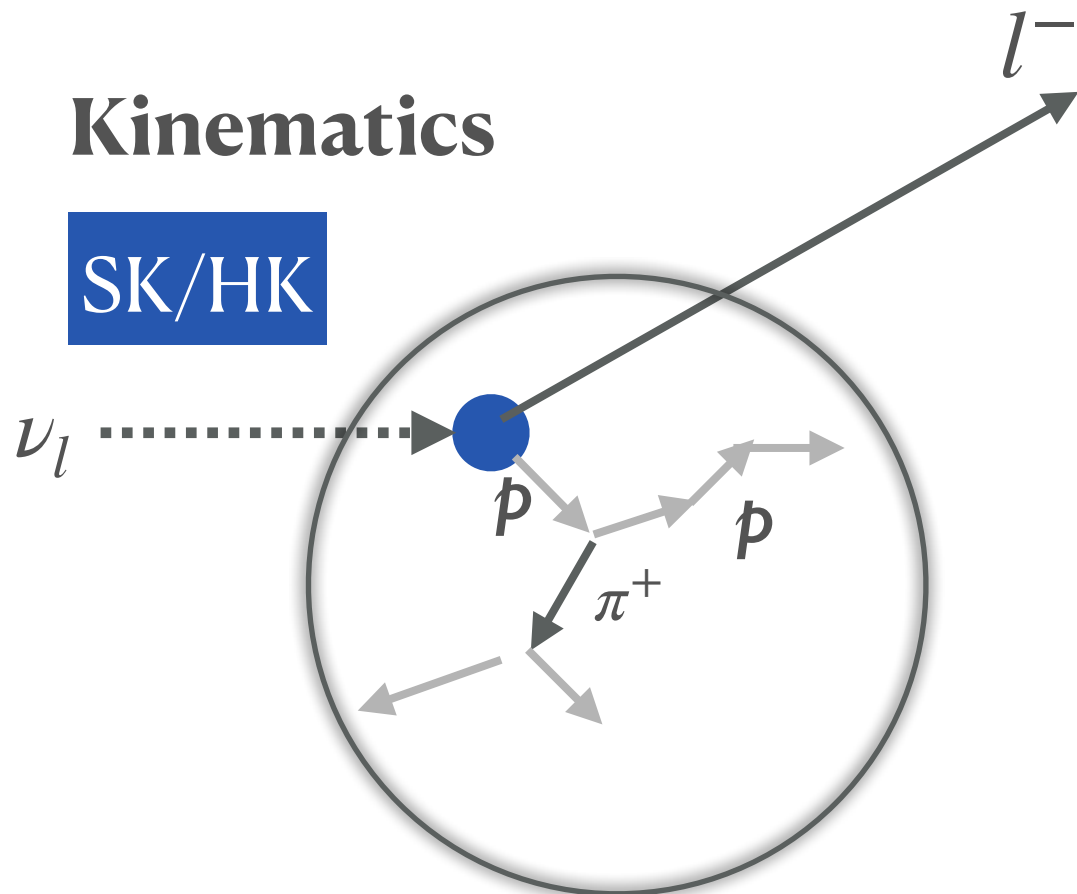
DUNE aims at uncertainties $< 1\%$ meaning $O(25 \text{ MeV})$ precision of energy reconstruction

Systematic errors should be small since statistics will be high.

Energy reconstruction

Kinematics

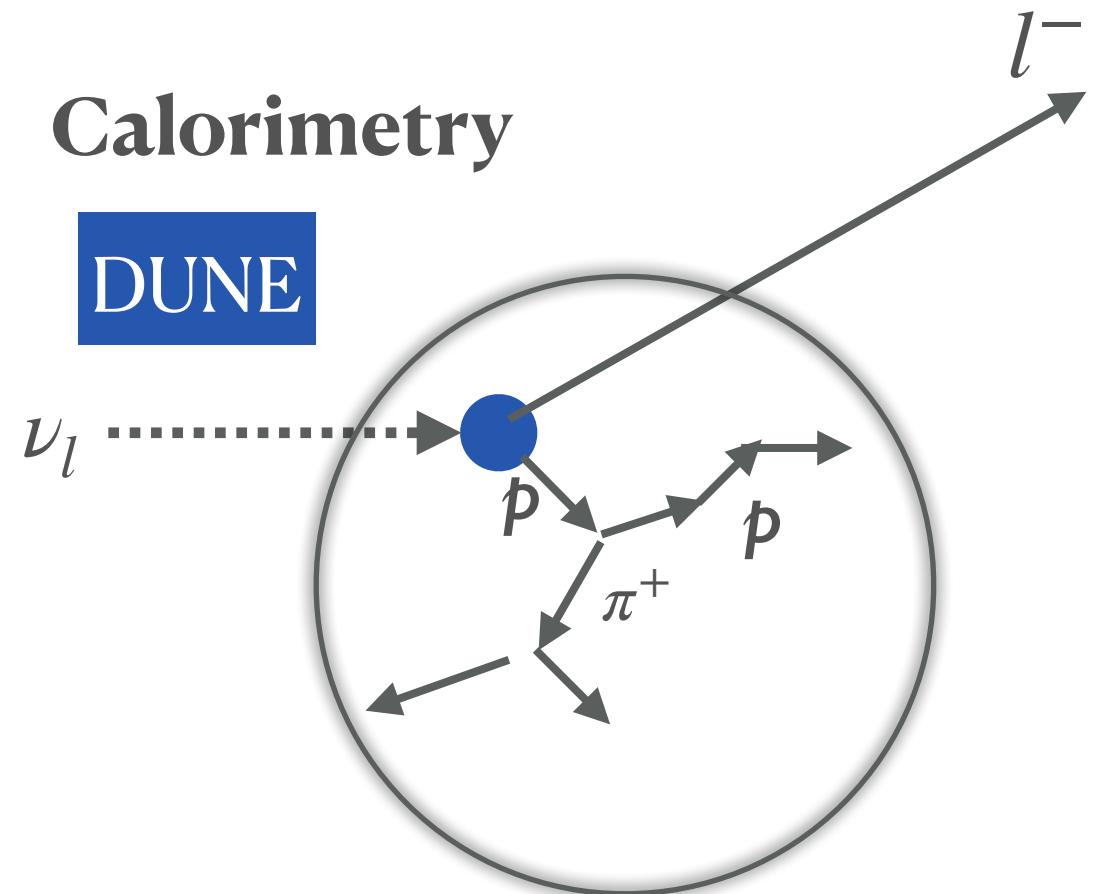
SK/HK



- ✓ depends on lepton reconstruction
- ✓ relies on identification of interaction channel (for quasi-elastic works well)

Calorimetry

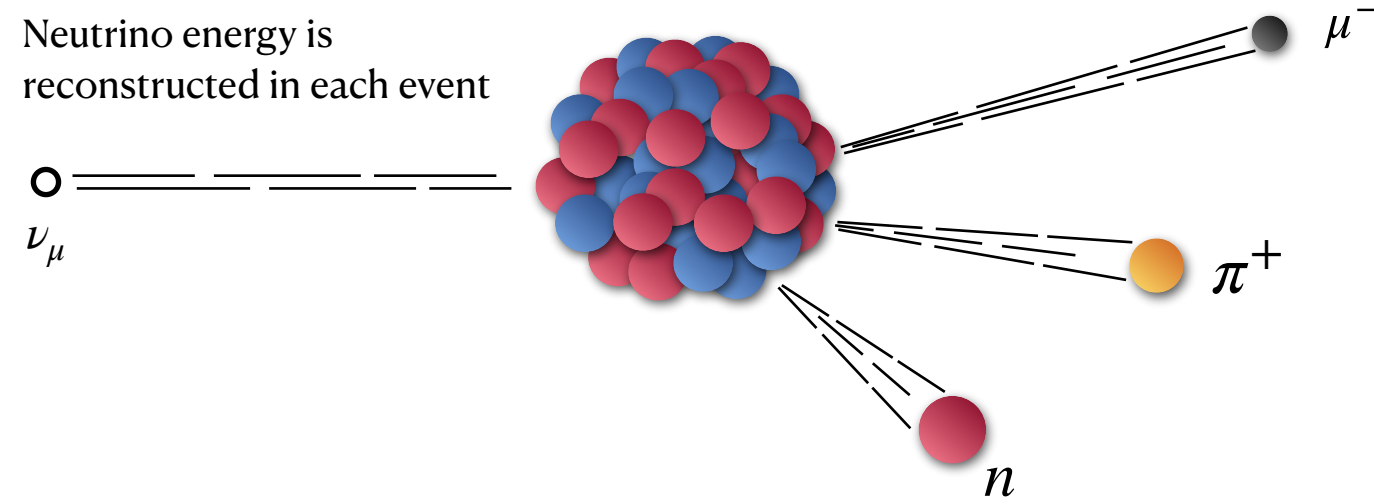
DUNE



- ✓ energy conservation
- ✓ relies on visible energy
- ✓ hadron masses influence the energy balance

Nuclear models implemented in **Monte Carlo** event generators play crucial role.

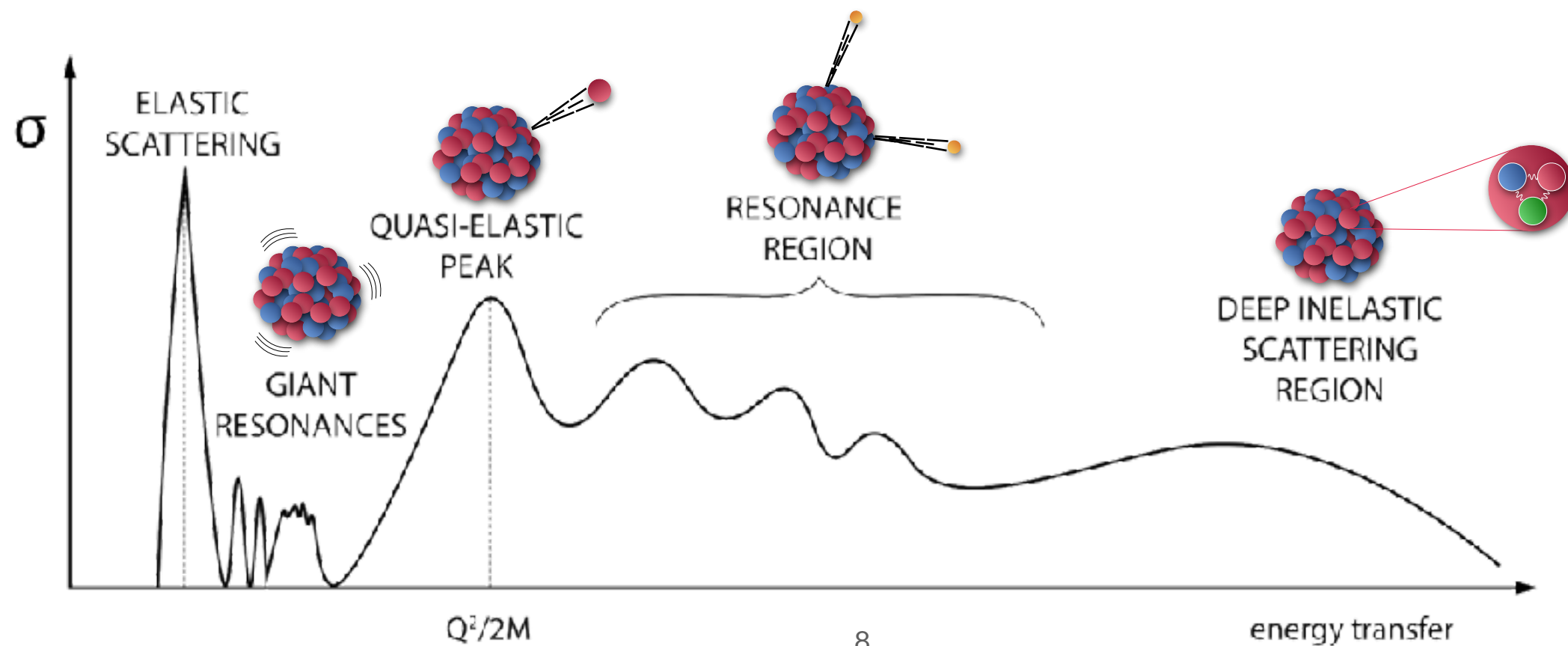
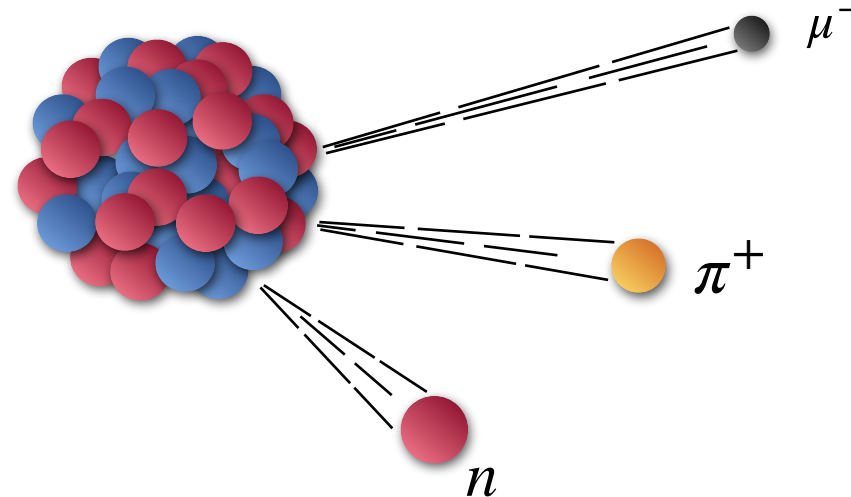
Motivation



Motivation

Neutrino energy is
reconstructed in each event

ν_μ

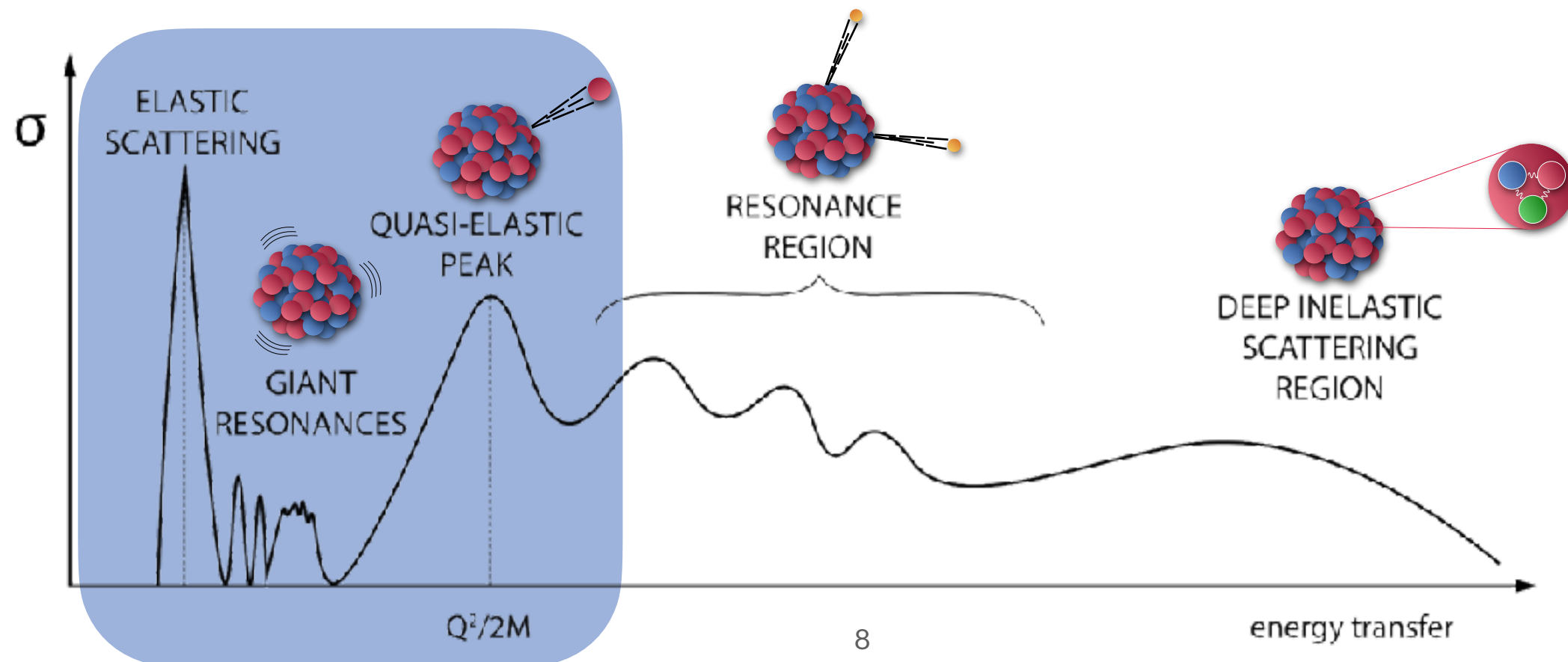
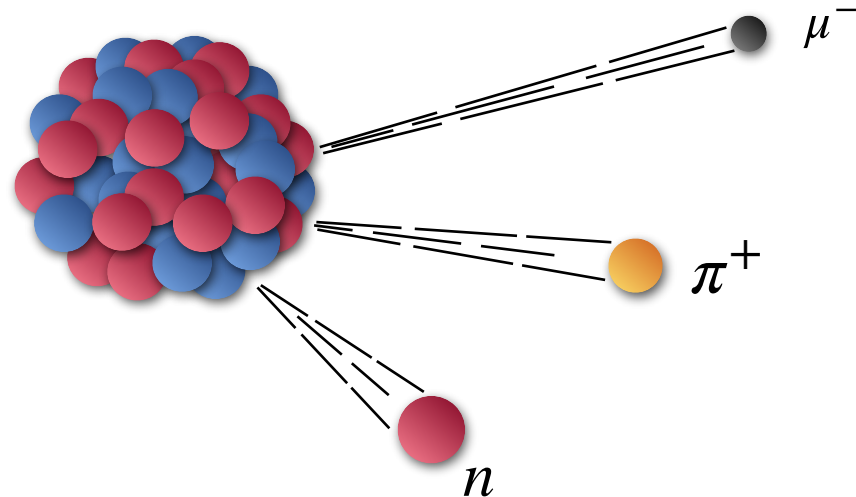


Motivation

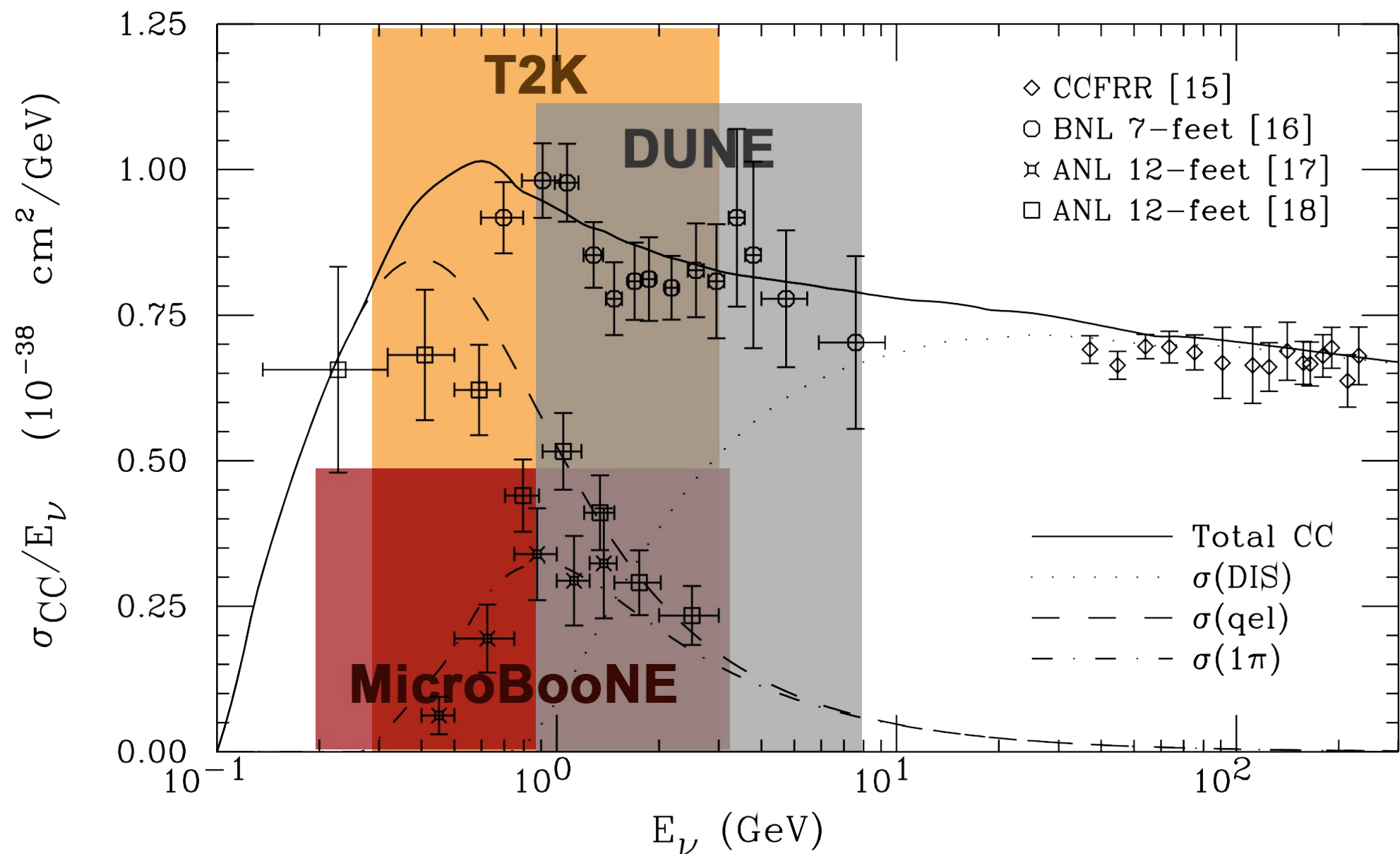
Neutrino energy is
reconstructed in each event

○

ν_μ

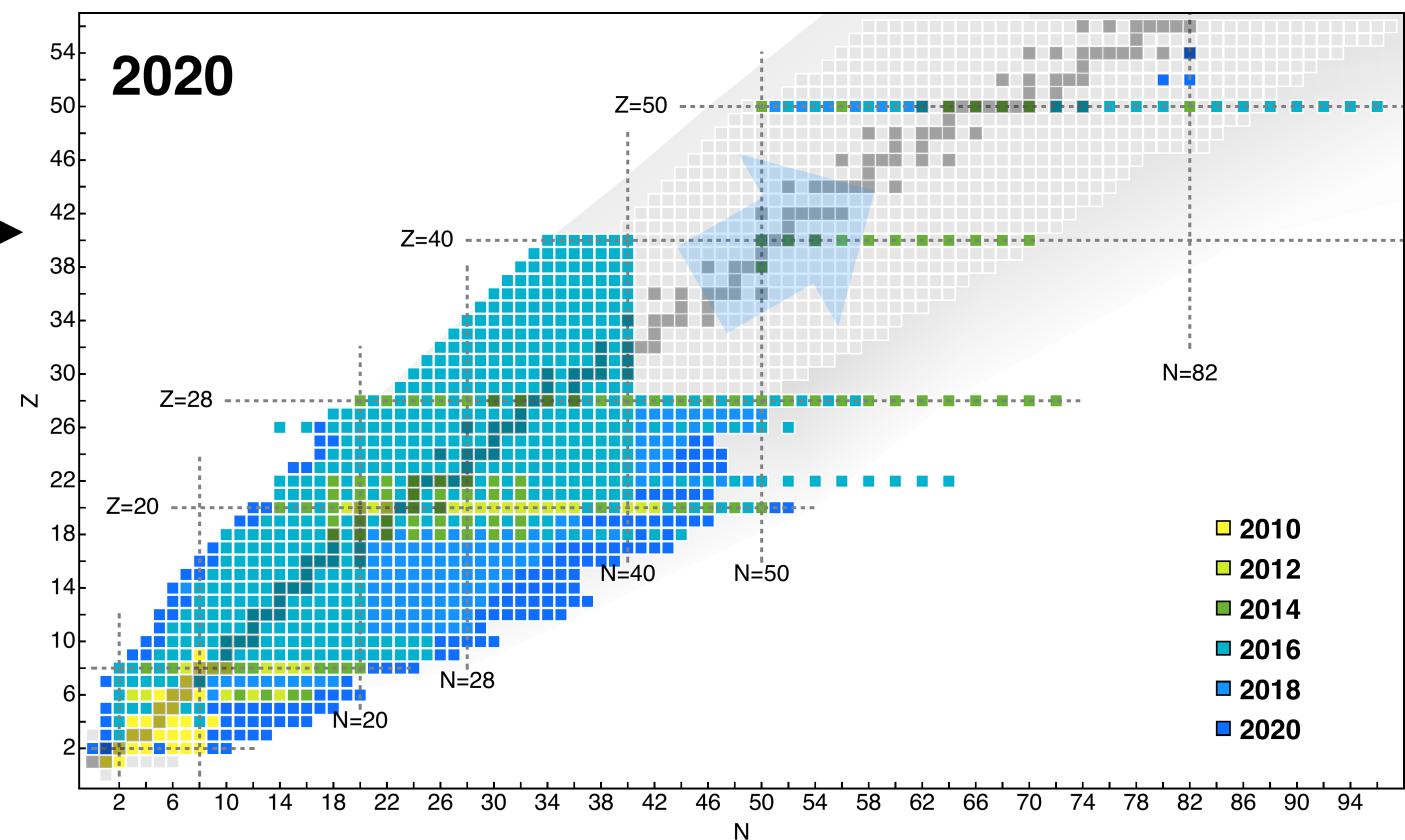
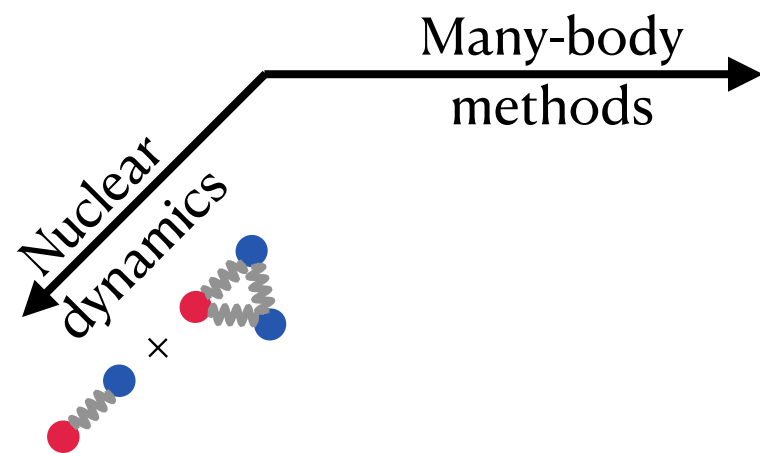


Why is QE important?



From: P. Lipari et. al., PRL 74 (1995)

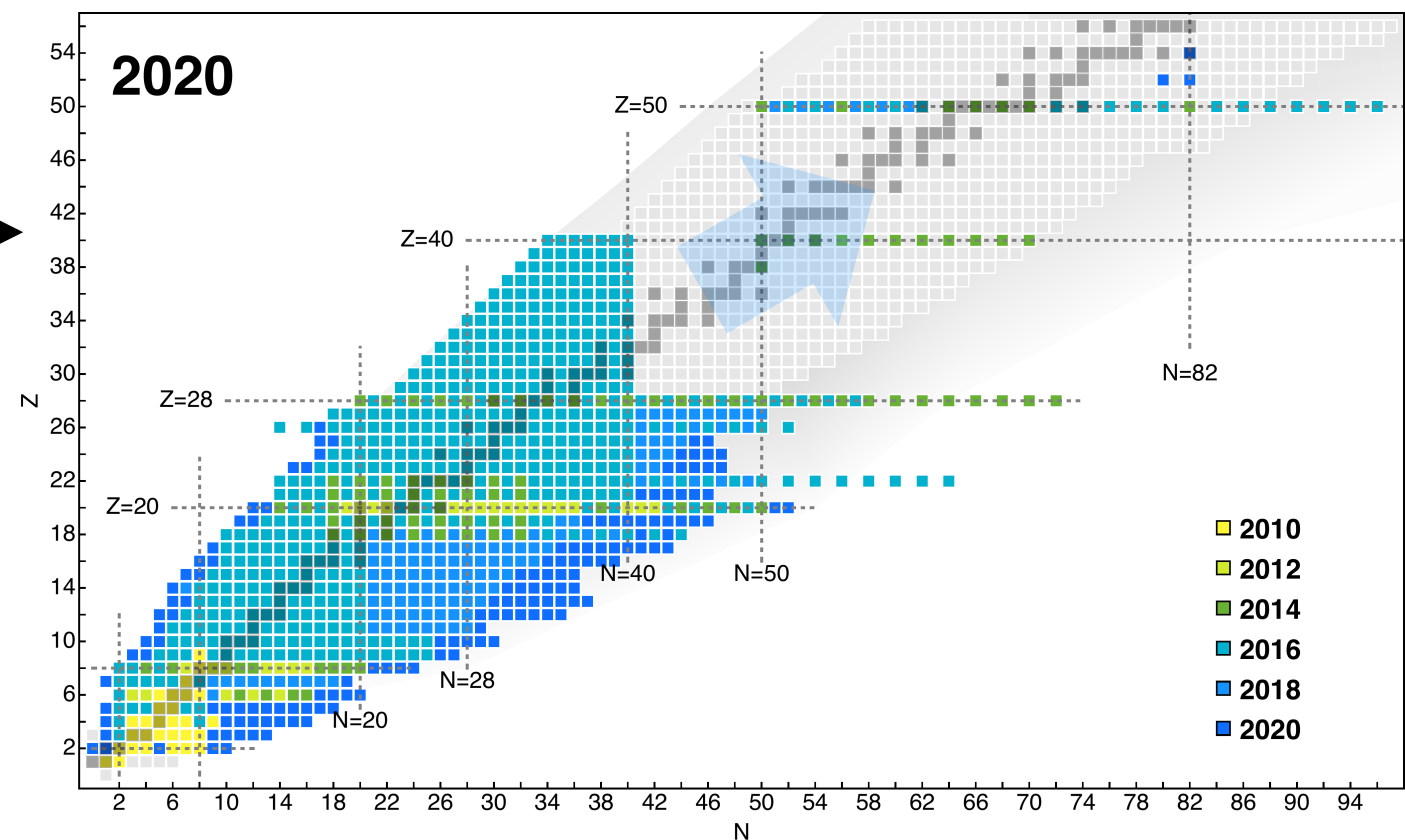
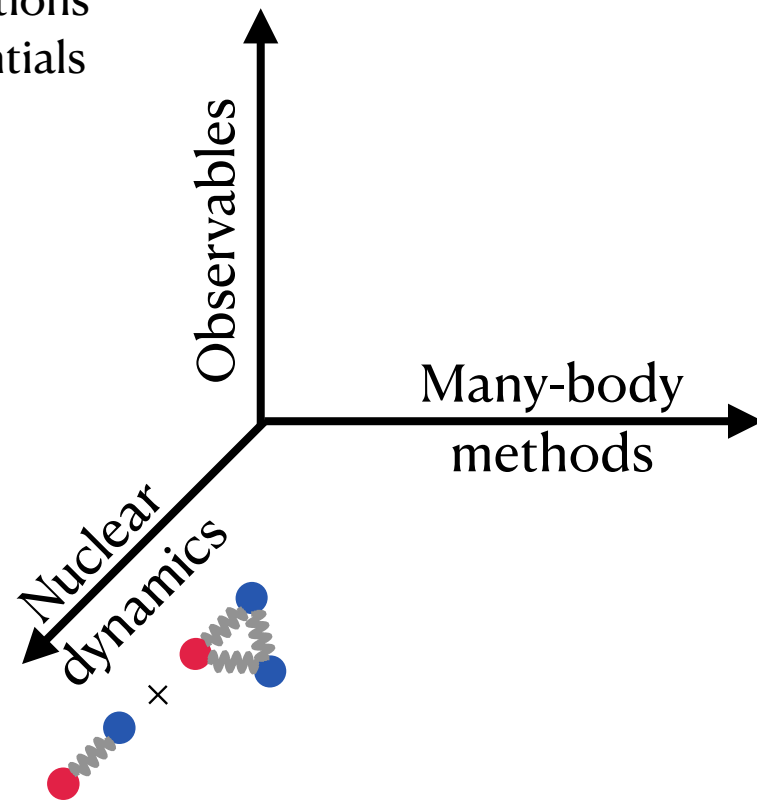
Motivation



H. Hergert, *Front.in Phys.* 8 (2020) 379

Motivation

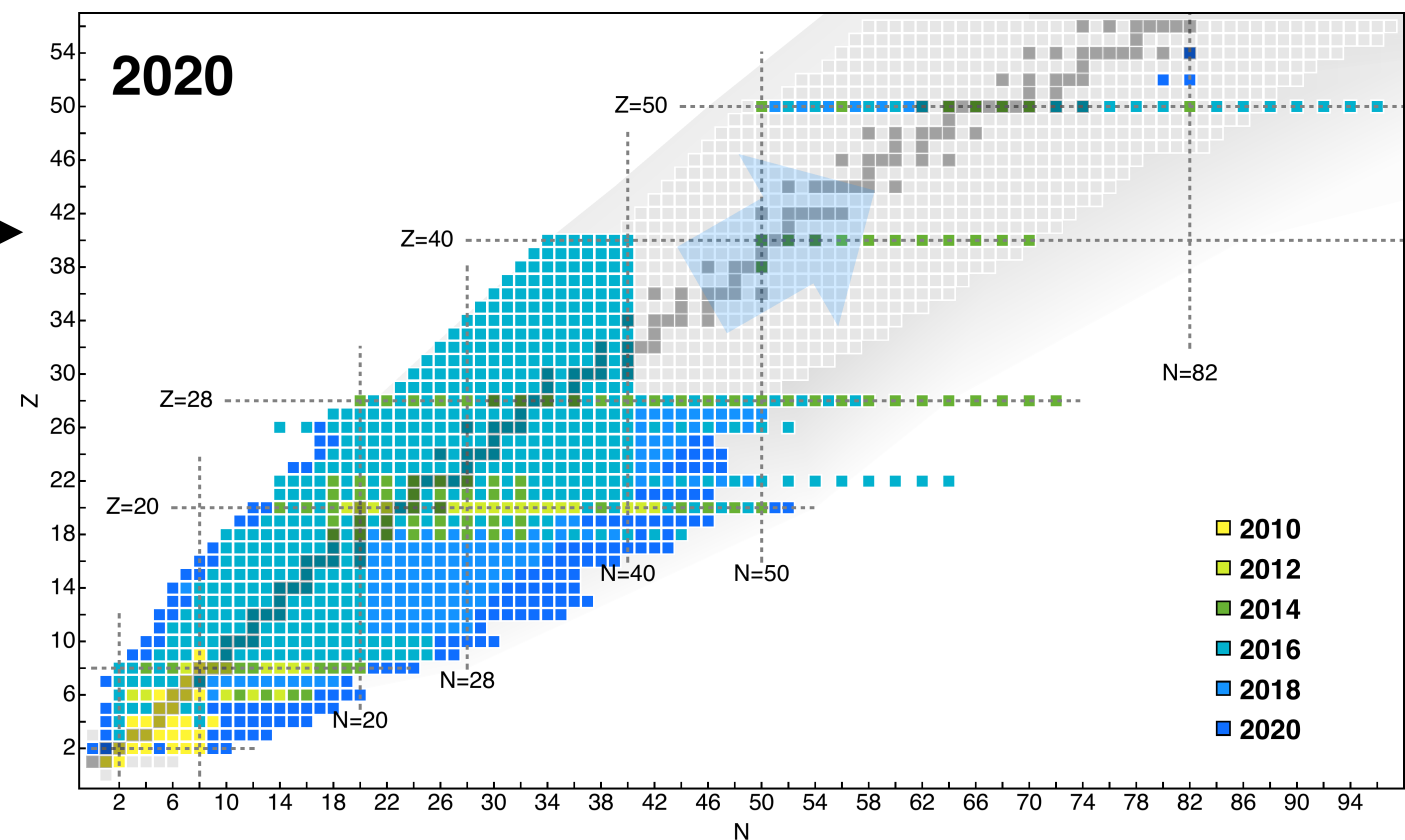
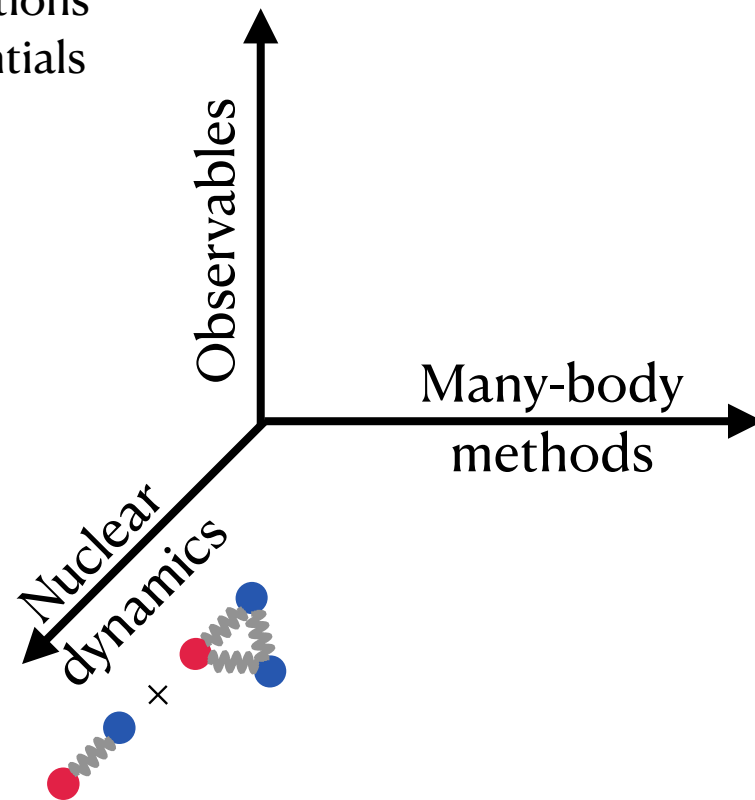
- ➔ Nuclear responses
- ➔ Spectral functions
- ➔ Optical potentials
- ...



H. Hergert, *Front.in Phys.* 8 (2020) 379

Motivation

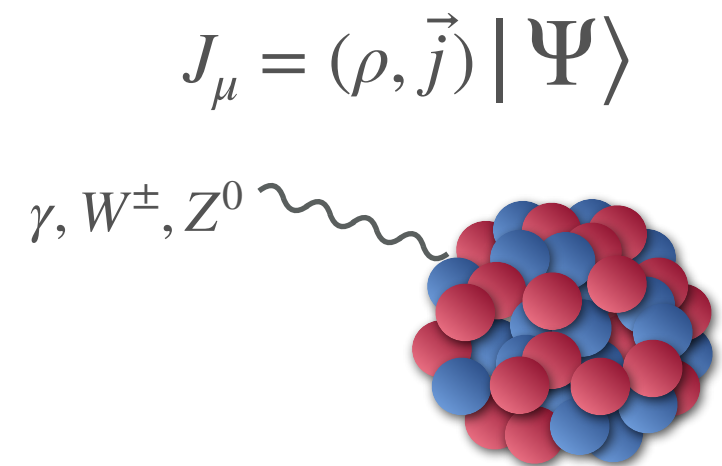
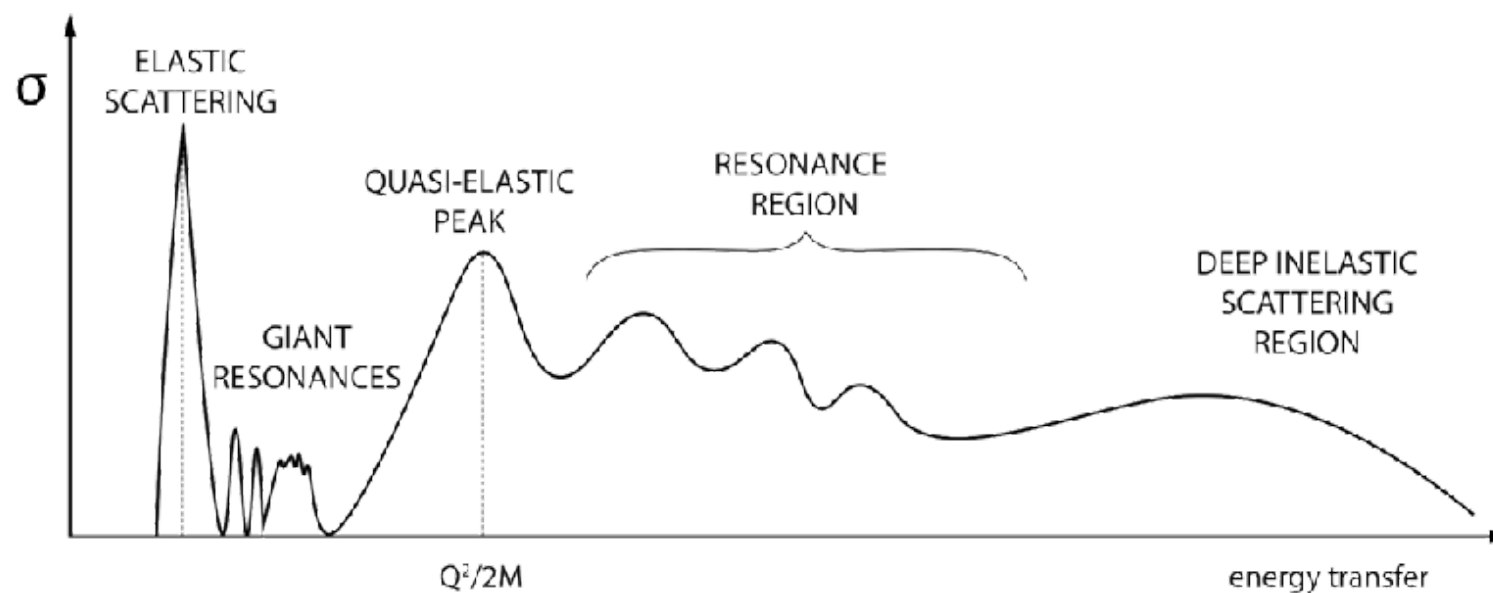
- ➔ Nuclear responses
- ➔ Spectral functions
- ➔ Optical potentials
- ...



H. Hergert, *Front.in Phys.* 8 (2020) 379

- ➔ Neutrinos challenge ab initio nuclear theory
- ➔ Controllable approximations within ab initial nuclear theory

Nuclear response



$$\sigma \propto L^{\mu\nu} R_{\mu\nu}$$

lepton tensor nuclear responses

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^\dagger(q) | \Psi_f \rangle \langle \Psi_f | J_\nu(q) | \Psi \rangle \delta(E_0 + \omega - E_f)$$

Electrons for neutrinos

$$\left. \frac{d\sigma}{d\omega dq} \right|_{\nu/\bar{\nu}} = \sigma_0 \left(v_{CC} R_{CC} + v_{CL} R_{CL} + v_{LL} R_{LL} + v_T R_T \pm v_{T'} R_{T'} \right)$$



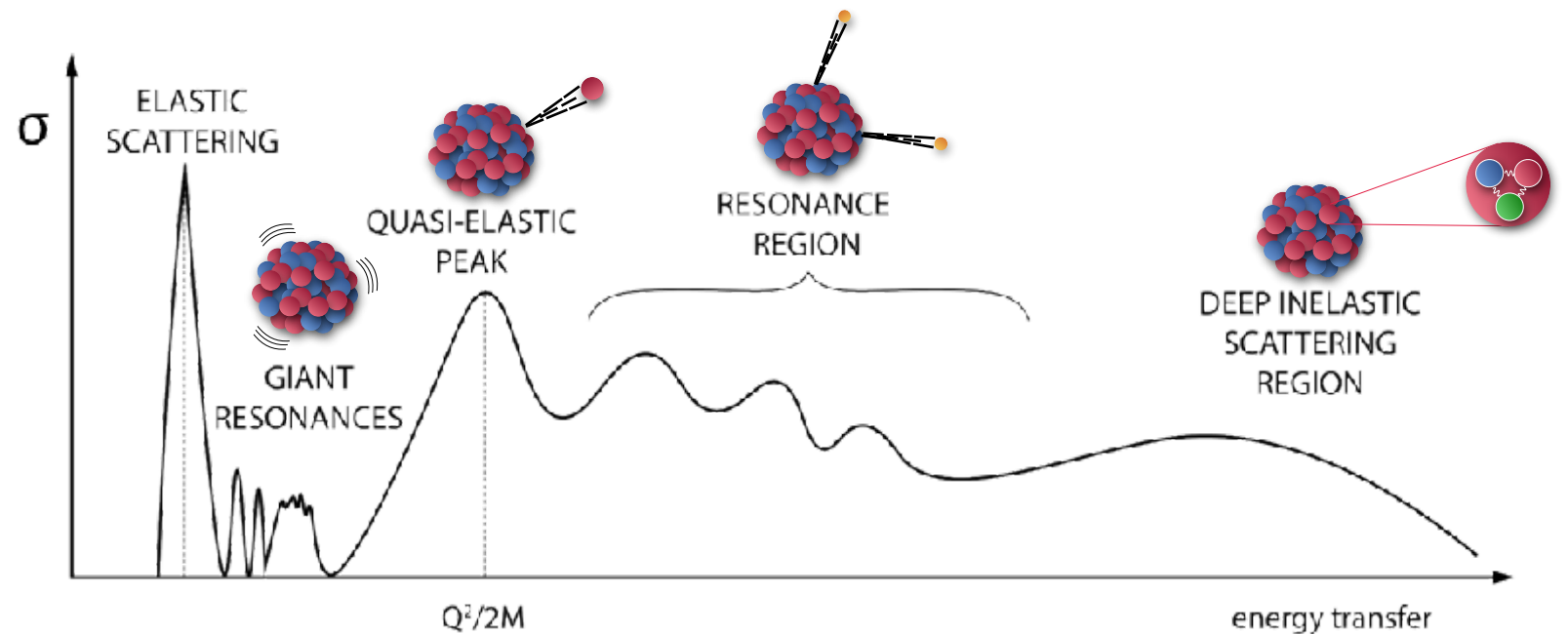
$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M \left(v_L R_L + v_T R_T \right)$$



- ✓ much more precise data
- ✓ we can get access to R_L and R_T separately (Rosenbluth separation)
- ✓ experimental programs of electron scattering in JLab, MAMI, MESA

Quasielastic response

- Momentum transfer ~hundreds MeV
- Upper limit for ab initio methods
- Important mechanism for T2HK, DUNE
- Role of final state interactions
- Role of 1-body and 2-body currents



First step: analyse the longitudinal response

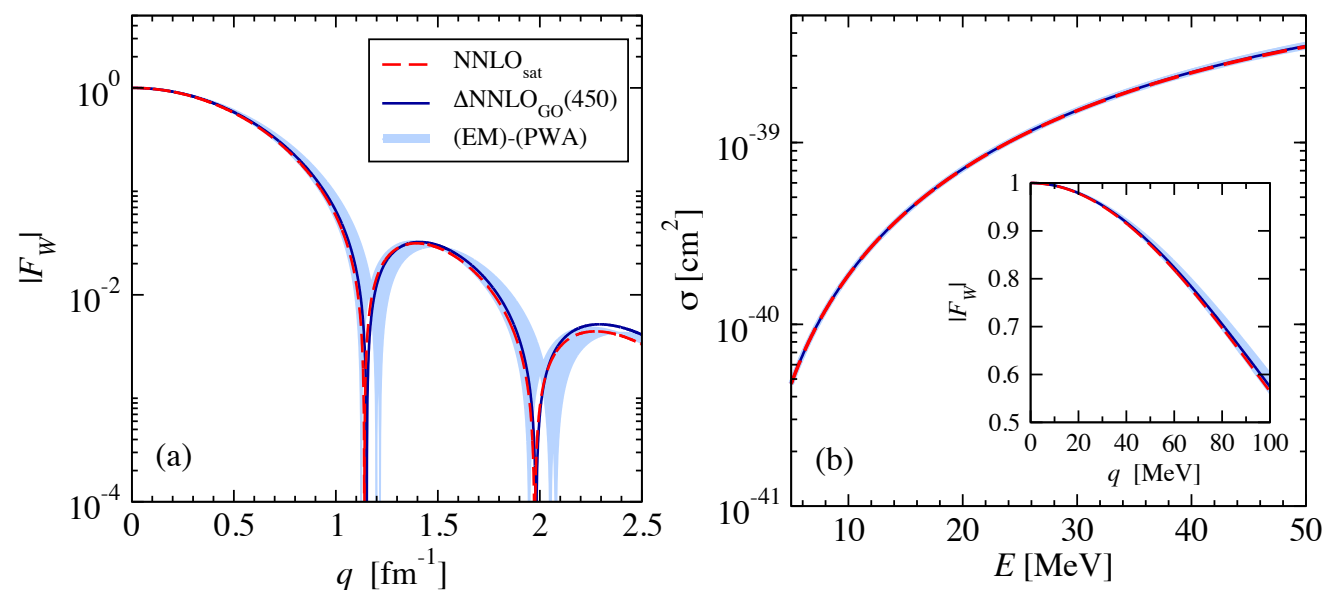
$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M \left(v_L R_L + v_T R_T \right)$$

$$\text{charge operator } \hat{\rho}(q) = \sum_{j=1}^Z e^{iqz'_j}$$

Formalism

✓ Coupled cluster (CCSD)

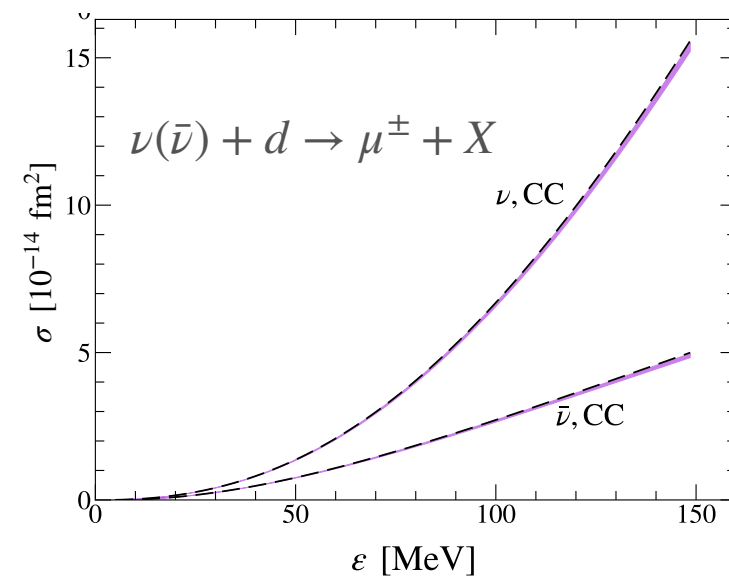
coherent elastic neutrino scattering on ^{40}Ar



C. Payne et al.
Phys.Rev.C 100 (2019) 6, 061304

✓ Electroweak currents

Multipole decomposition for 1- and 2-body EW currents



B. Acharya, S. Bacca
Phys.Rev.C 101 (2020) 1, 015505

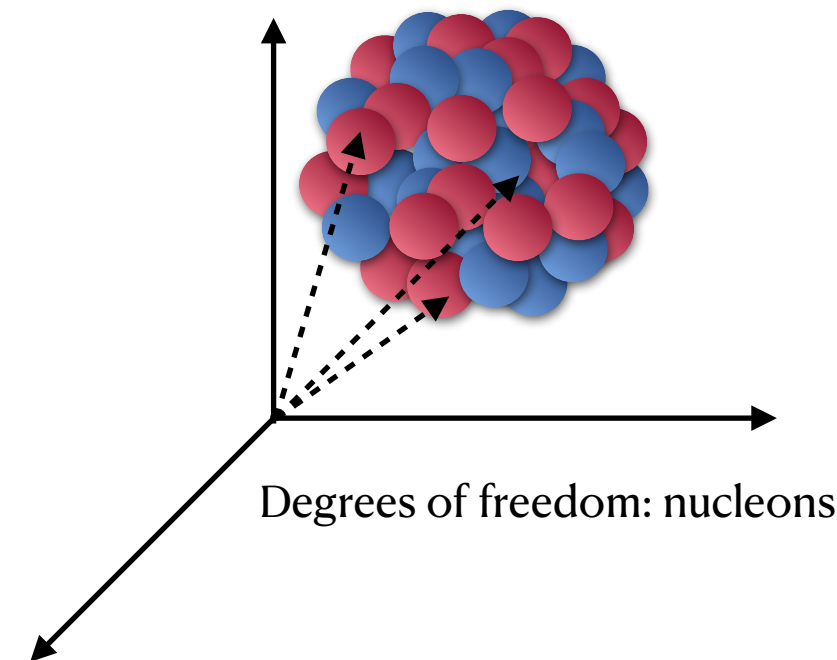
✓ Chiral potentials: NNLO_{sat} and $\Delta\text{NNLO}_{\text{GO}}$

A. Ekström et al. *Phys.Rev.C* 91 (2015) 5, 051301
W. Jiang et al. *Phys.Rev.C* 102 (2020) 5, 054301

Coulomb sum rule

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$

easier to calculate since we do
not need $|\Psi_f\rangle$



Coulomb sum rule

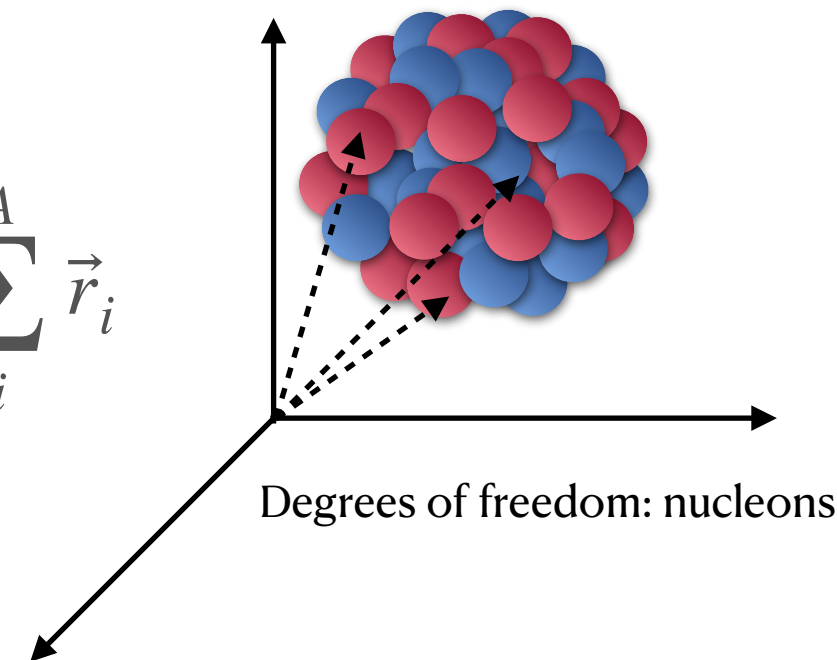
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easier to calculate since we do
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center of mass problem

$|\Psi\rangle$ has $3A$ coordinates \rightarrow $3(A-1)$ ^{intrinsic} coordinates + $\vec{R} = \frac{1}{A} \sum_i^A \vec{r}_i$

With translationally non-invariant operators
we may excite spurious states



Coulomb sum rule

Project out spurious states: $\hat{\rho} |\Psi\rangle = |\Psi_{phys}\rangle + |\Psi_{spur}\rangle$

It has been shown that to good approximation the ground state factorizes:

$$|\Psi\rangle = |\Psi_I\rangle |\Psi_{CoM}\rangle$$

center of mass wave
function is a Gaussian

G. Hagen, T. Papenbrock, D. Dean
Phys.Rev.Lett. 103 (2009) 062503

We follow a similar ansatz for the excited states:

$$\hat{\rho} |\Psi\rangle = |\Psi_I^{exc}\rangle |\Psi_{CoM}\rangle + |\Psi_I\rangle |\Psi_{CoM}^{exc}\rangle$$

Coulomb sum rule

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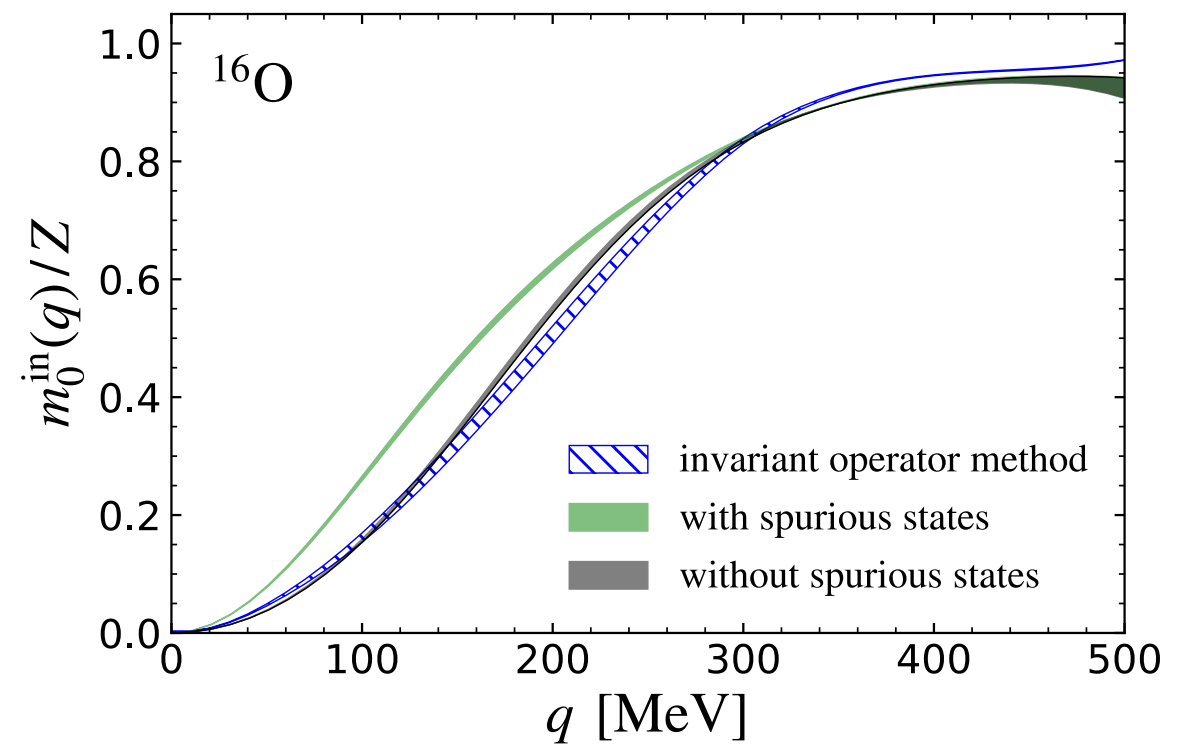
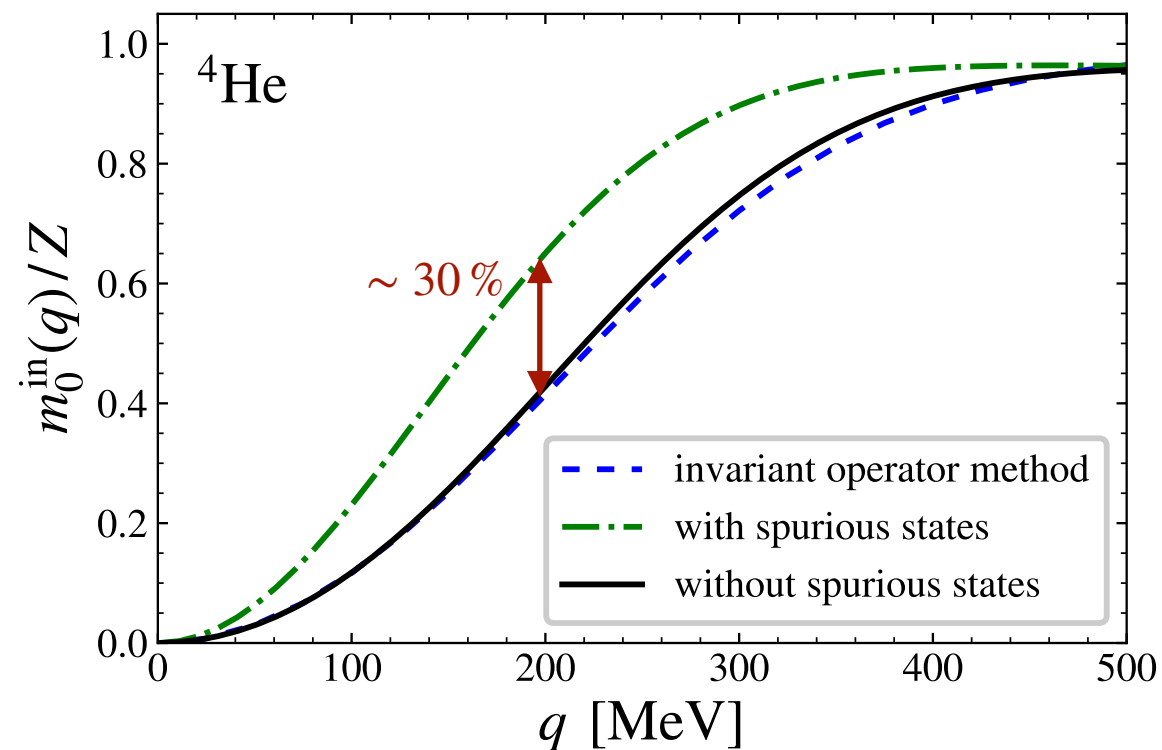
G. Hagen, T. Papenbrock, D. Dean
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spurious

Coulomb sum rule

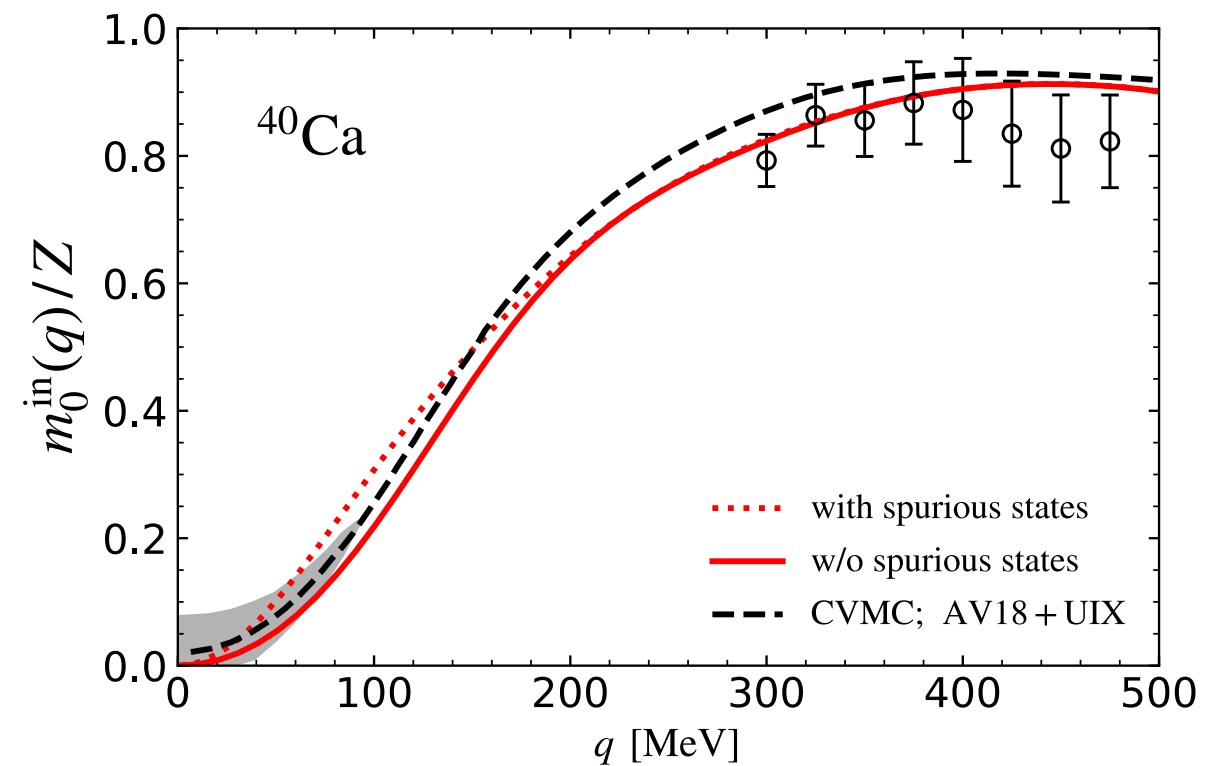
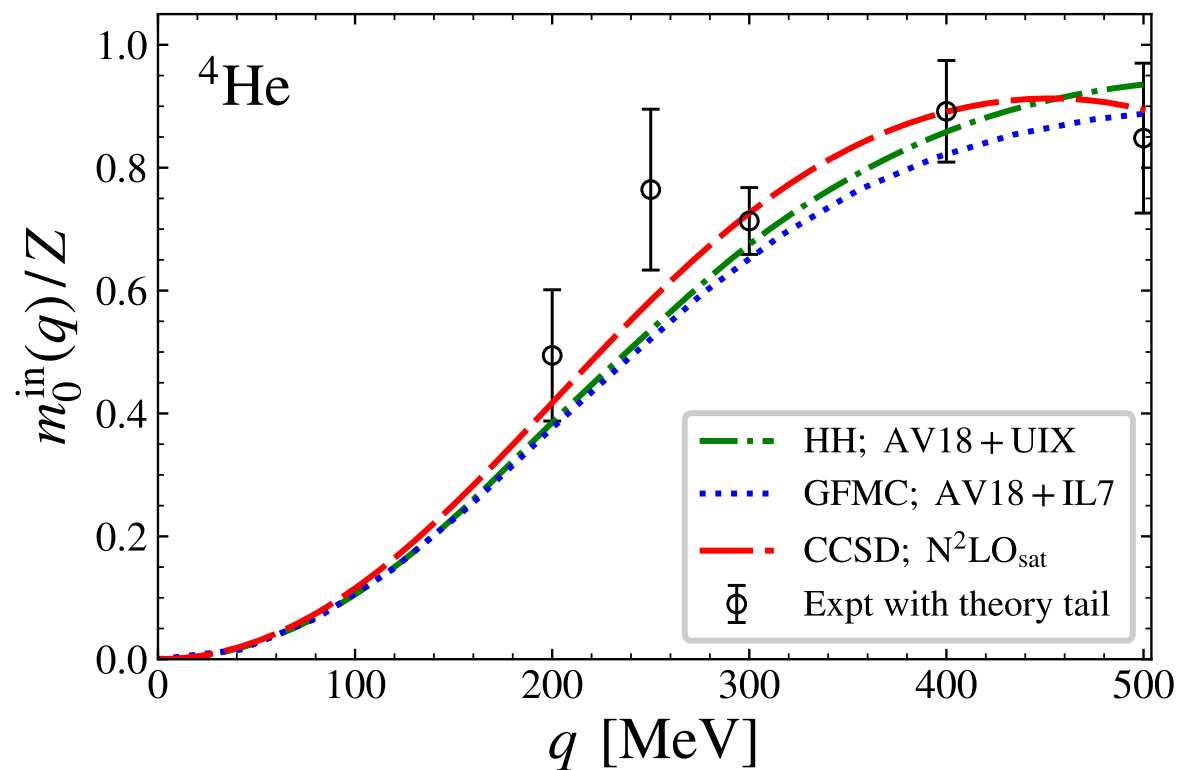


J.E.S. B. Acharya, S. Bacca, G. Hagen
Phys. Rev. C 102 (2020) 064312

CoM spurious states dominate for light nuclei

Coulomb sum rule

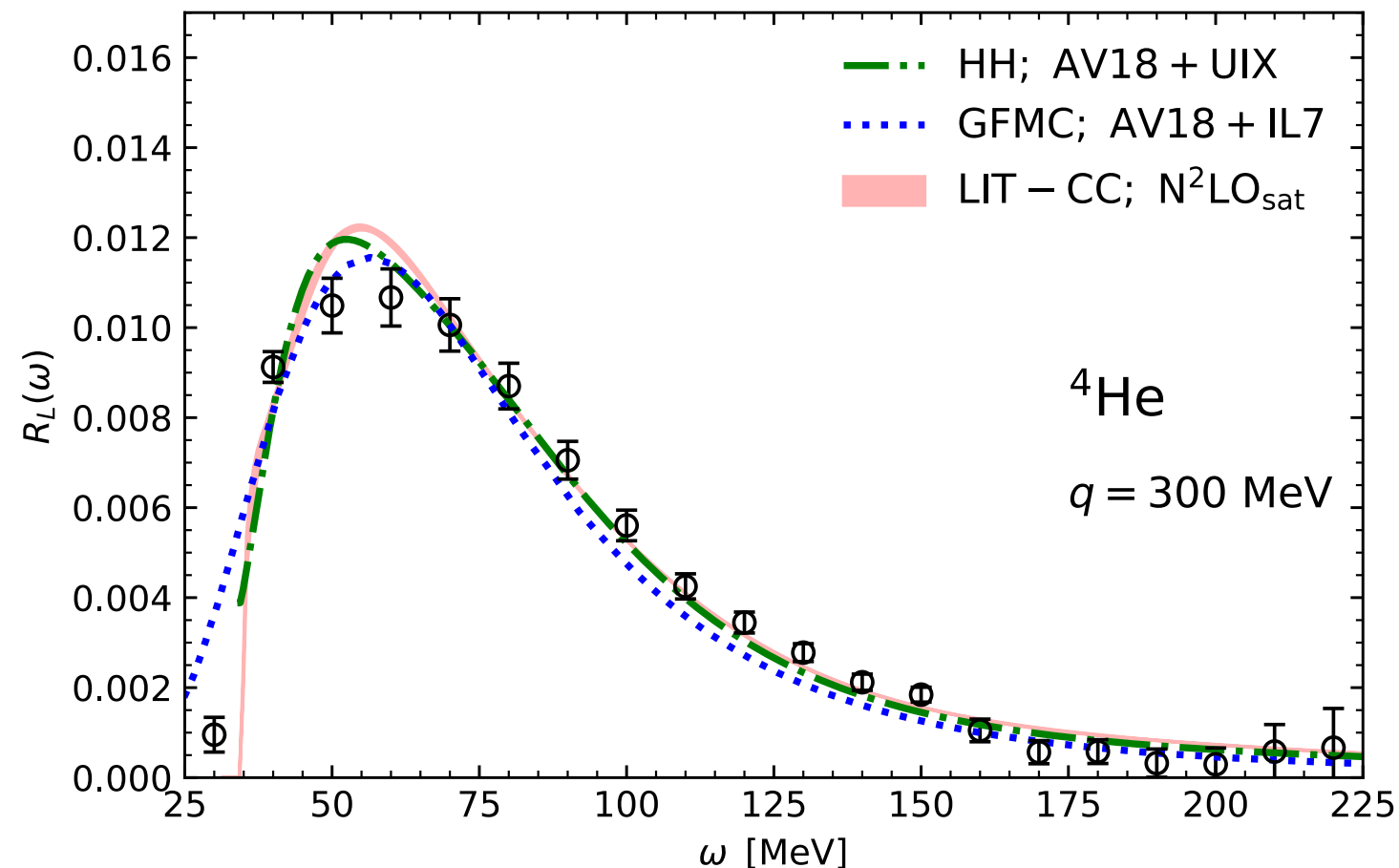
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Nuclear responses

Longitudinal response

Lorentz Integral Transform + Coupled Cluster



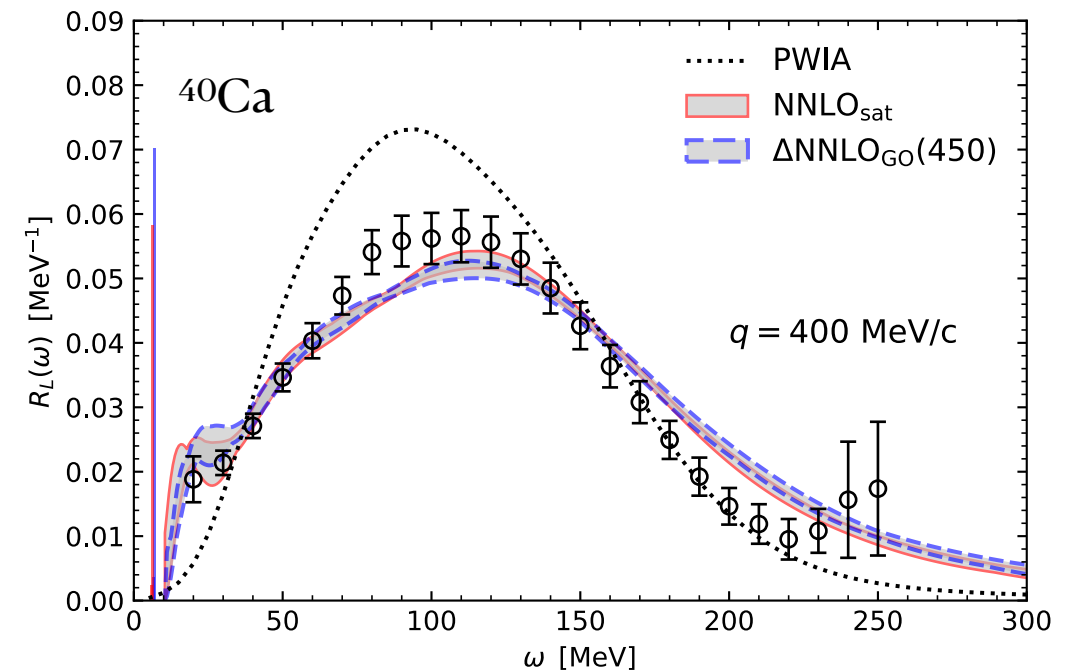
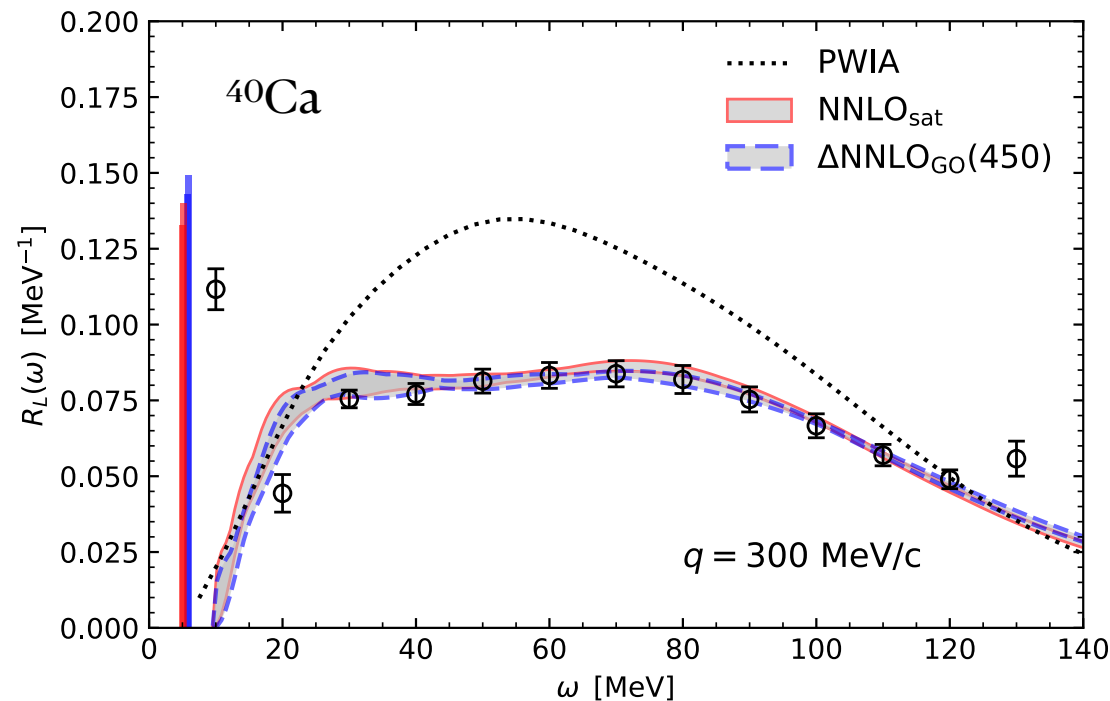
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

Uncertainty band: inversion procedure

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

Longitudinal response ^{40}Ca

Lorentz Integral Transform + Coupled Cluster



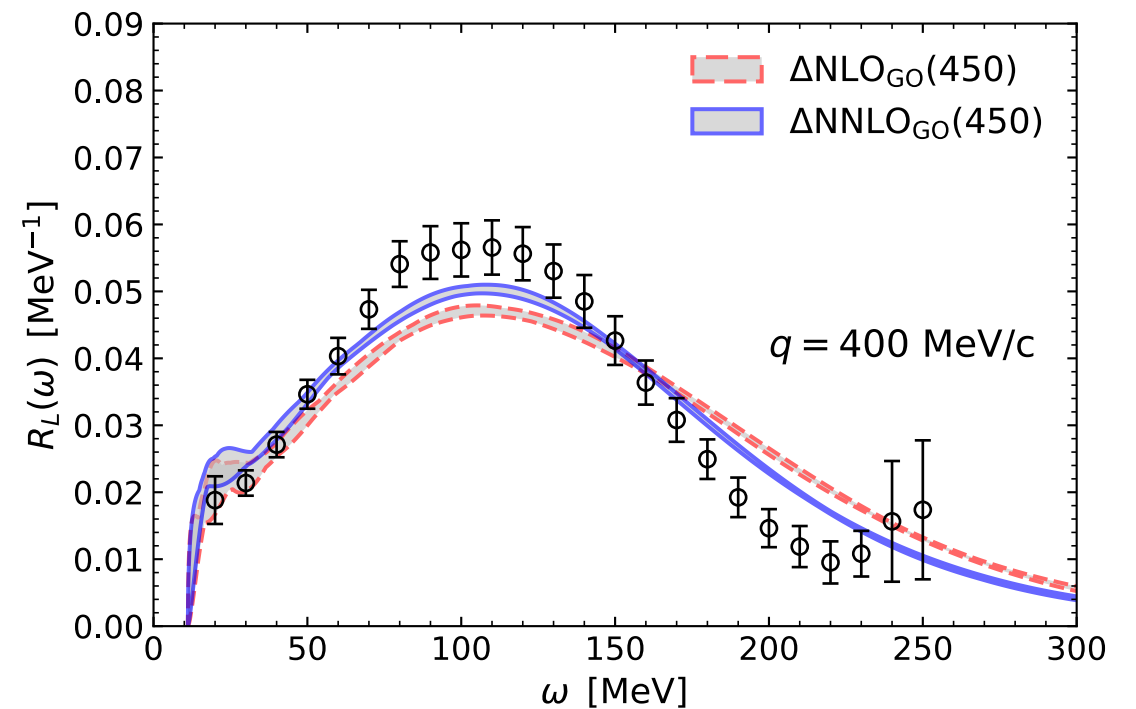
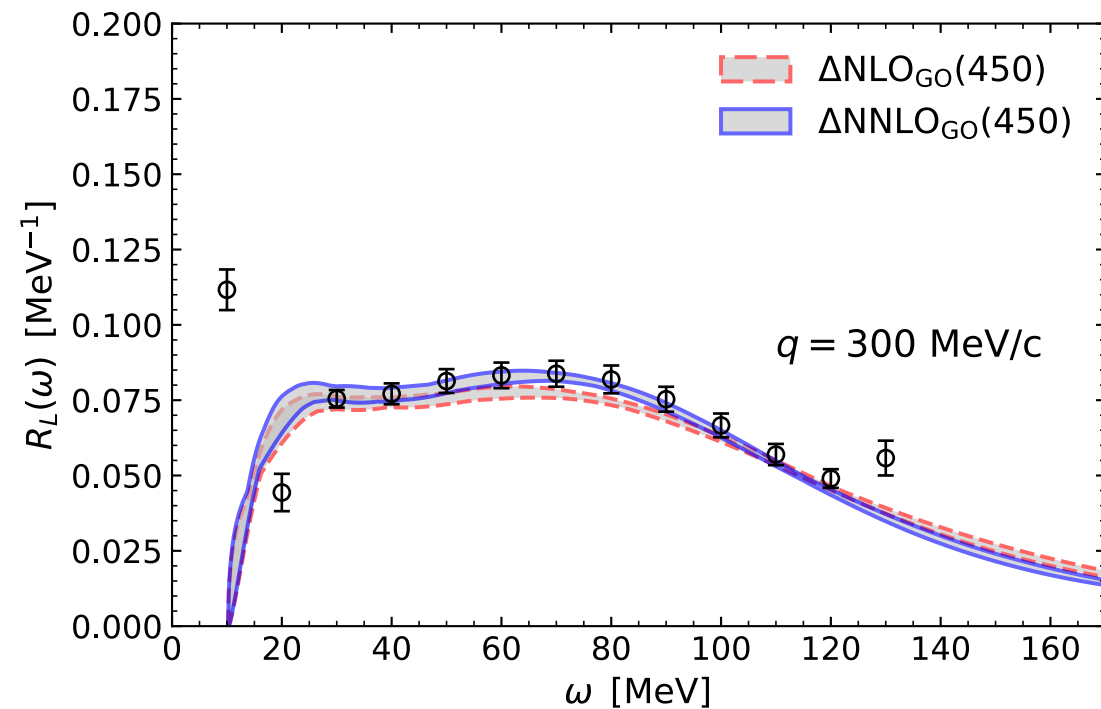
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

- ✓ CC singles & doubles
- ✓ varying underlying harmonic oscillator frequency
- ✓ two different chiral Hamiltonians
- ✓ *inversion procedure*

First ab-initio results for
many-body system of
40 nucleons

Chiral expansion for ^{40}Ca

(Longitudinal response)

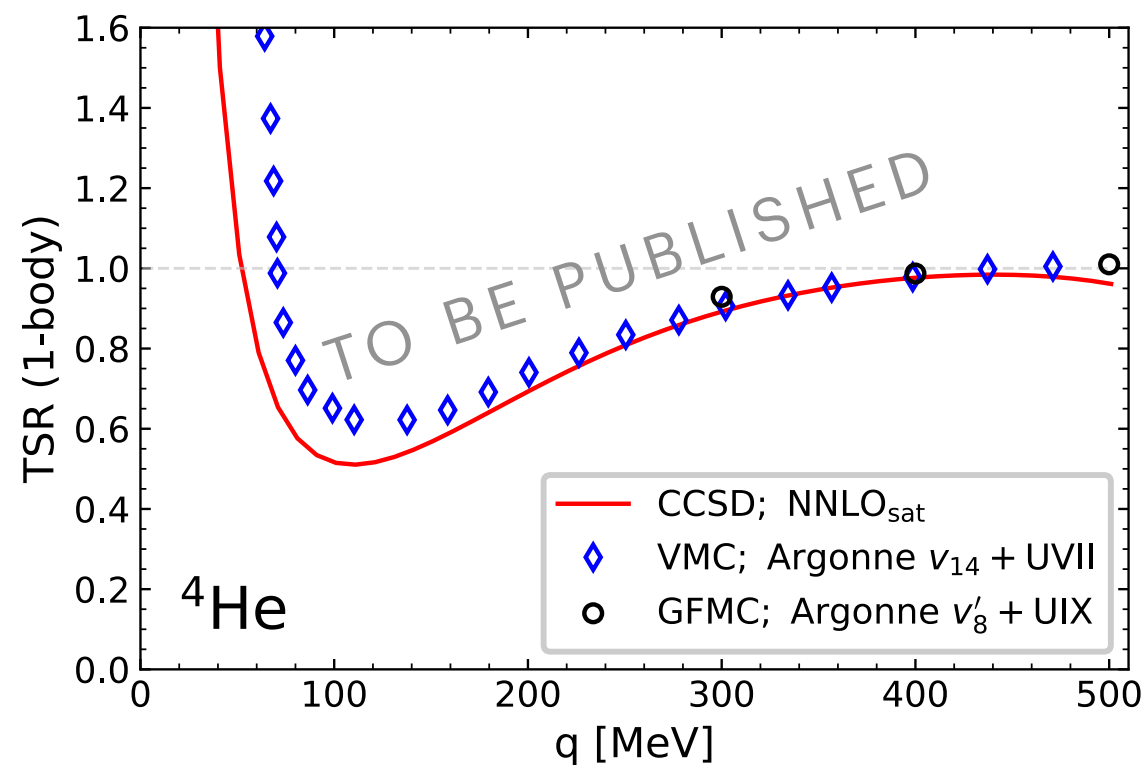


B. Acharya, S. Bacca, JES et al. Front. Phys. 1066035(2022)

- ✓ Two orders of chiral expansion
- ✓ Convergence better for lower q (as expected)
- ✓ Higher order brings results closer to the data

Transverse response

$$\text{TSR}(q) = \frac{2m^2}{Z\mu_p^2 + N\mu_n^2} \frac{1}{q^2} \left(\langle \Psi | \hat{j}^\dagger \hat{j} | \Psi \rangle - |\langle \Psi | \hat{j} | \Psi \rangle|^2 \right)$$



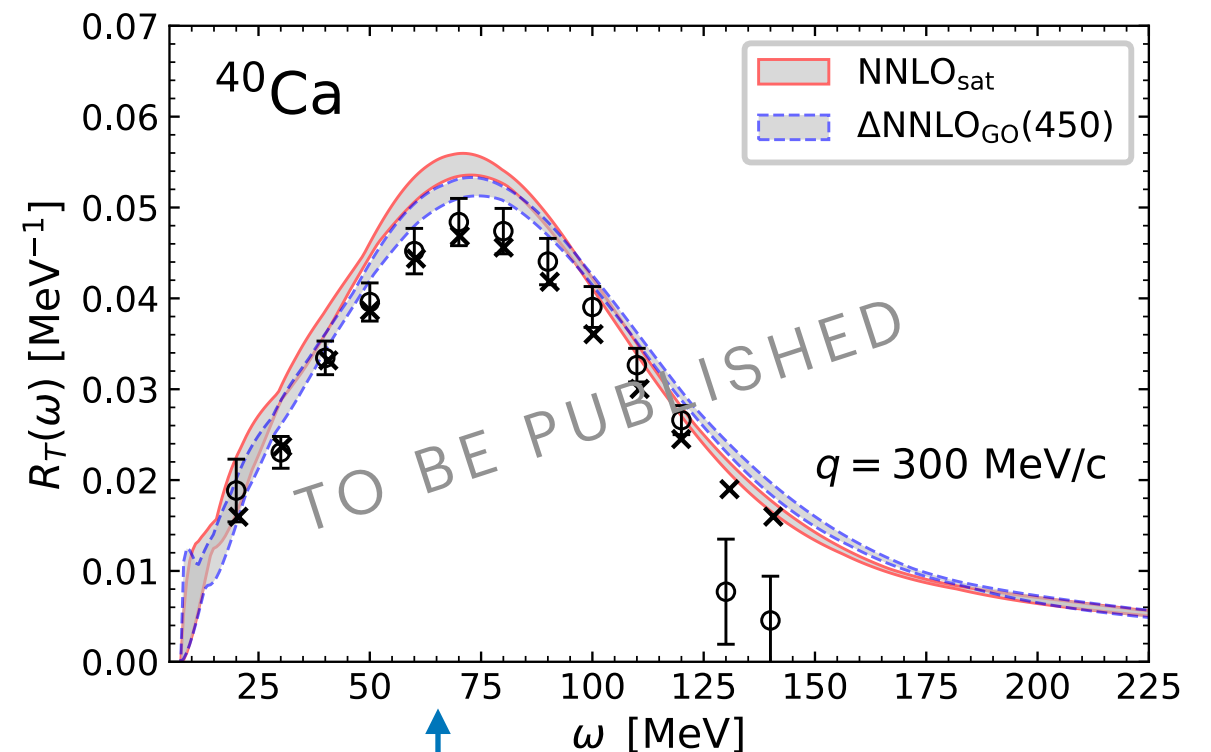
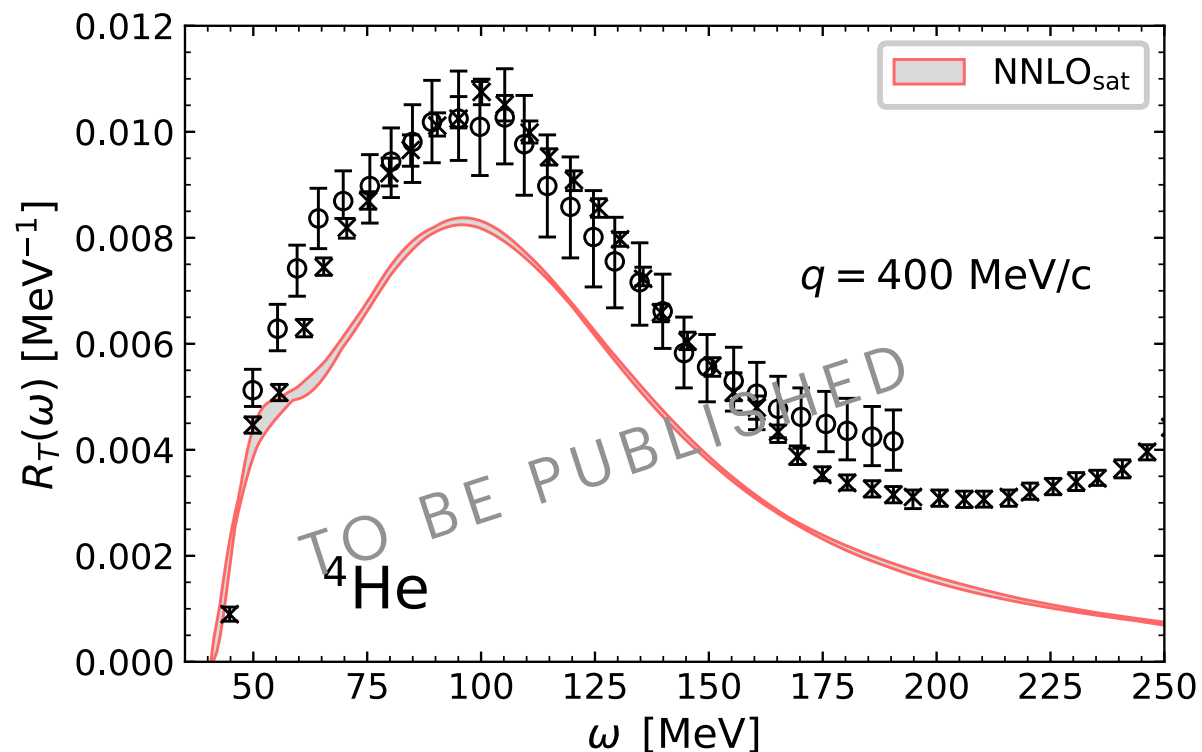
TSR($q \rightarrow 0$) \propto kinetic energy



TSR($q \rightarrow \infty$) = 1

$$\mathbf{j}(\mathbf{q}) = \sum_i \frac{1}{2m} \epsilon_i \{ \mathbf{p}_i, e^{i\mathbf{q}\mathbf{r}_i} \} - \frac{i}{2m} \mu_i \mathbf{q} \times \sigma_i e^{i\mathbf{q}\mathbf{r}_i}$$

Transverse response



- ➔ This allows to predict electron-nucleus cross-section
- ➔ Currently only 1-body current

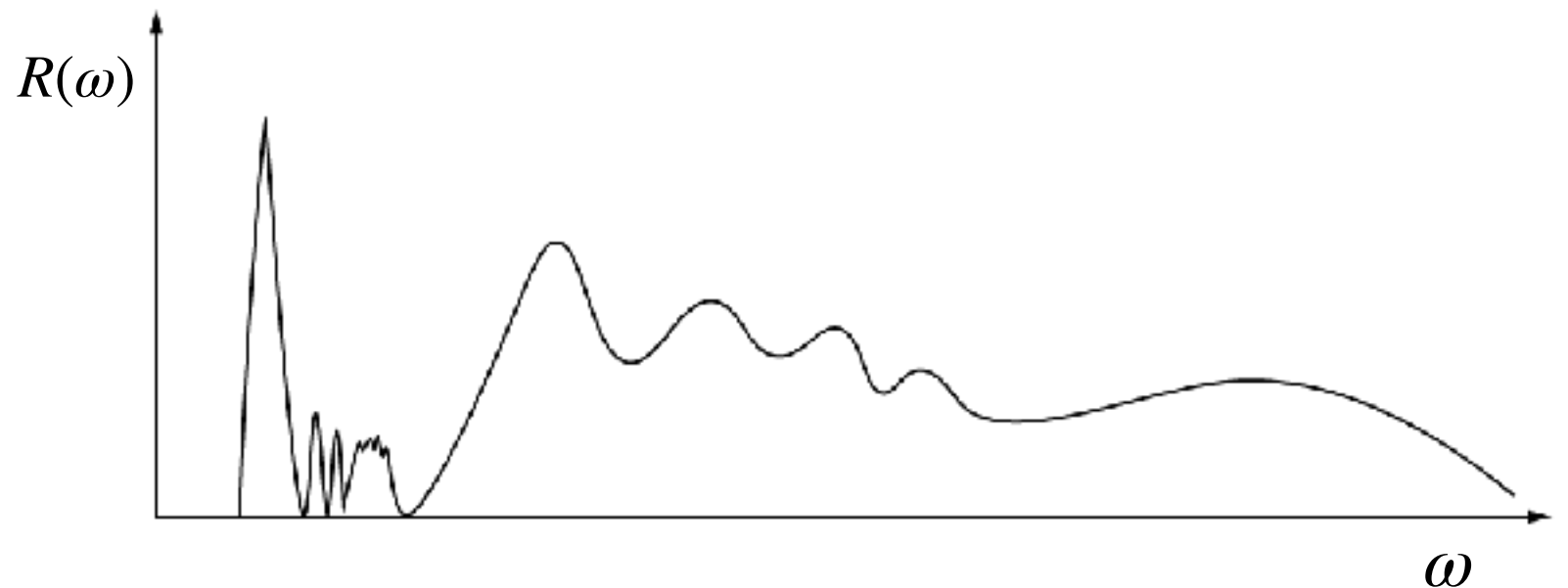
2-body currents important for ^4He
 → more correlations needed?
 → 2-b currents strength depends on nucleus?

ChEK method

Chebyshev Expansion of integral Kernel

$$\Phi = \int f(\omega) R(\omega) d\omega$$

- Sum-rules
- Flux folding
- Histogram
- ...



$$\Phi \approx \tilde{\Phi} = \int f(\omega') \int K(\omega', \omega) R(\omega) d\omega d\omega'$$

expansion in Chebyshev
polynomials

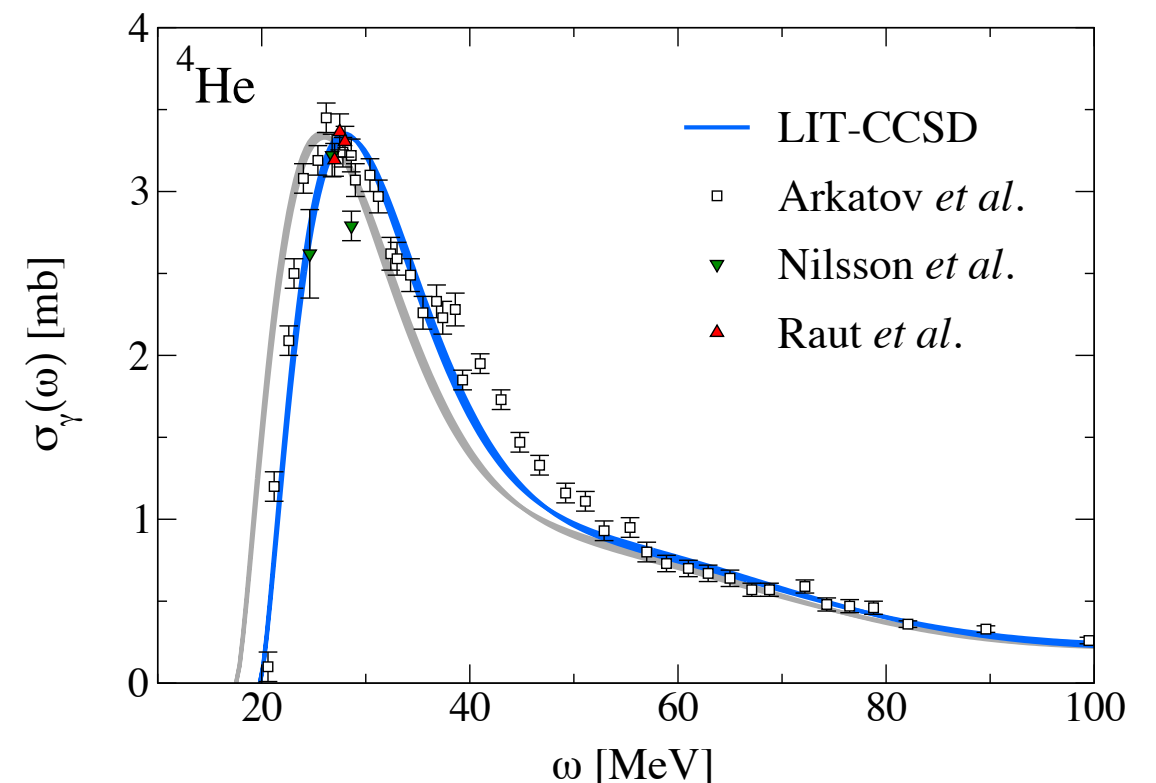
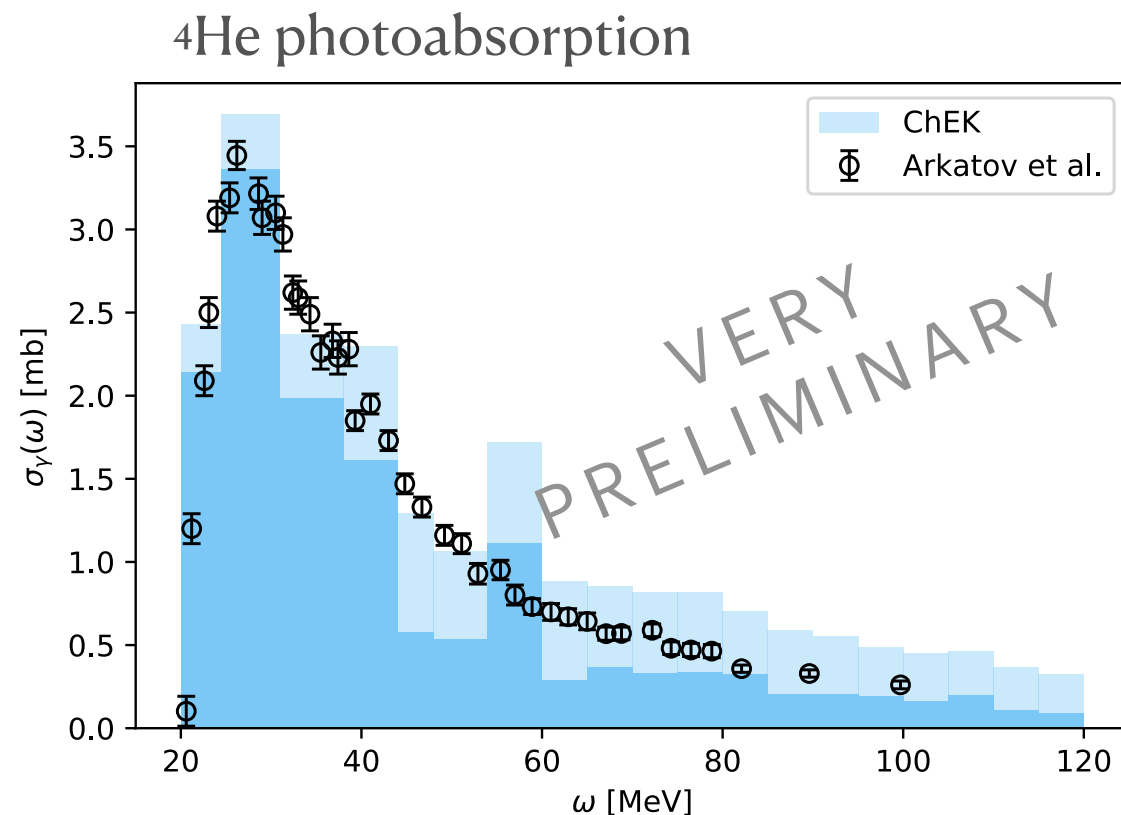
$$K(\omega, \sigma) = \sum_k c_k(\sigma) T_k(\omega)$$

estimated error

$$|\Phi - \tilde{\Phi}| < \epsilon$$

ChEK method

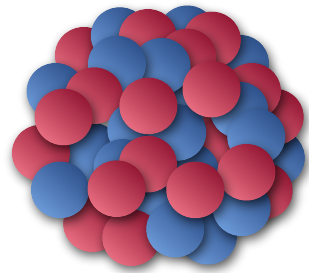
Chebyshev Expansion of integral Kernel



S. Bacca, N. Barnea, G. Hagen, G. Orlandini; *Phys.Rev.C* 90 (2014) 6

- ➔ No assumption about the shape of the response
- ➔ Rigorous error estimation
- ➔ Convenient when the response has a complicated structure

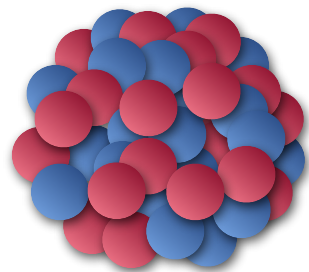
Low/high energies



$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

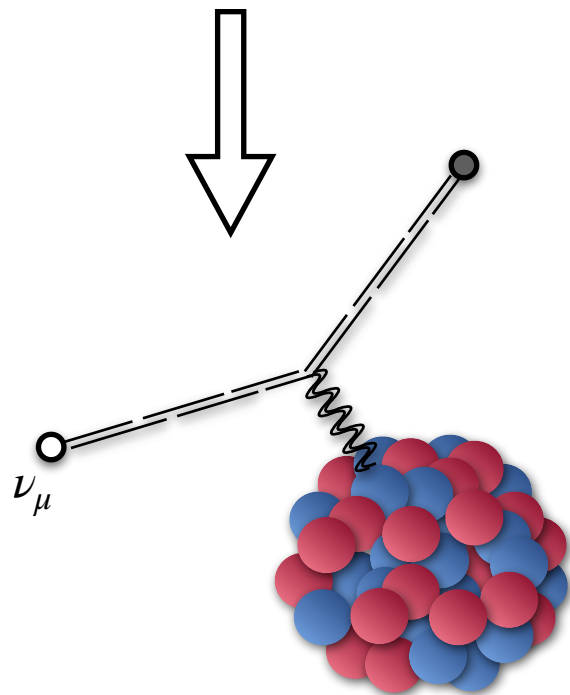
Many-body problem

Low/high energies



$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

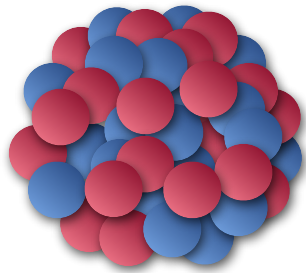
Many-body problem



$$\langle\psi_f|\hat{j}|\psi_A\rangle$$

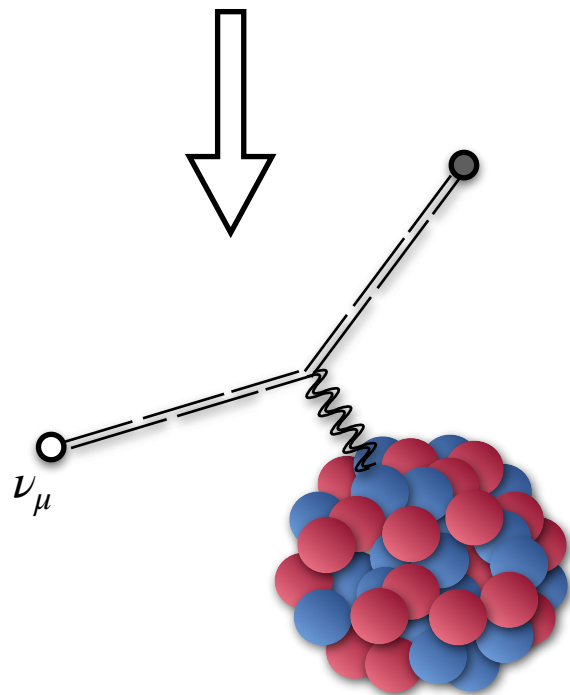
Electroweak responses

Low/high energies



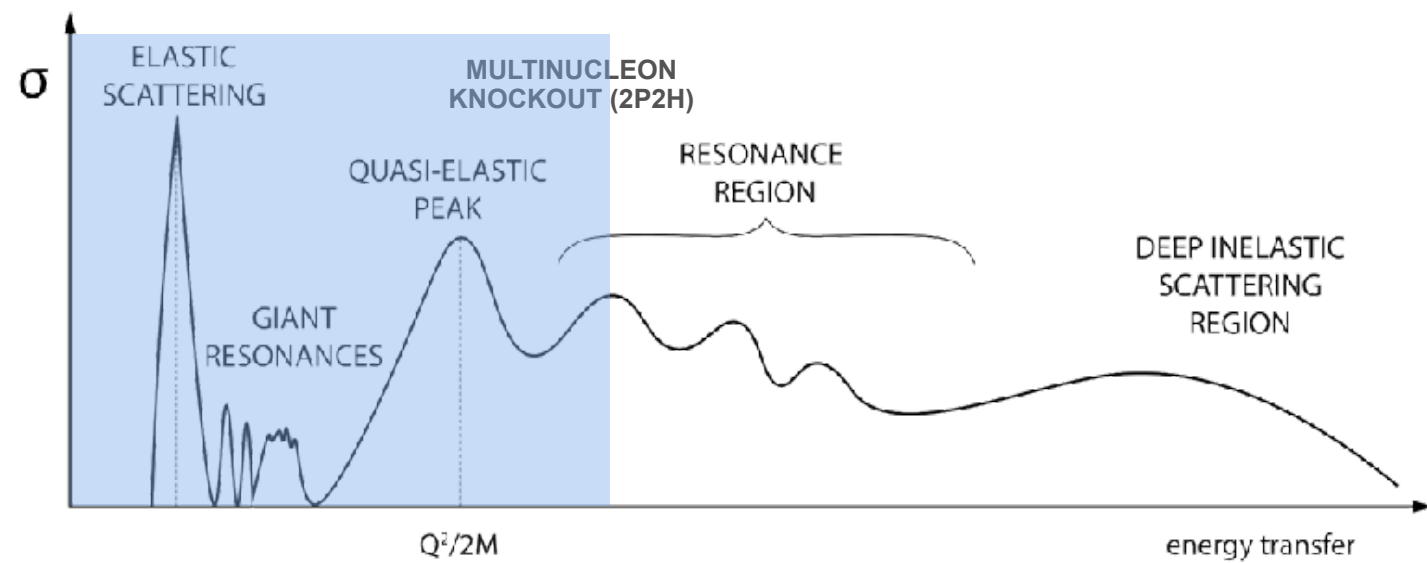
$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

Many-body problem

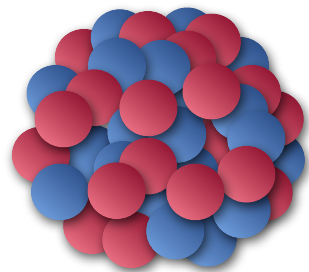


$$\langle\psi_f|\hat{j}|\psi_A\rangle$$

Electroweak responses

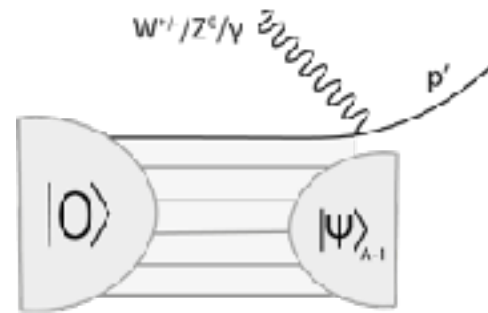
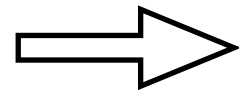


Low/high energies

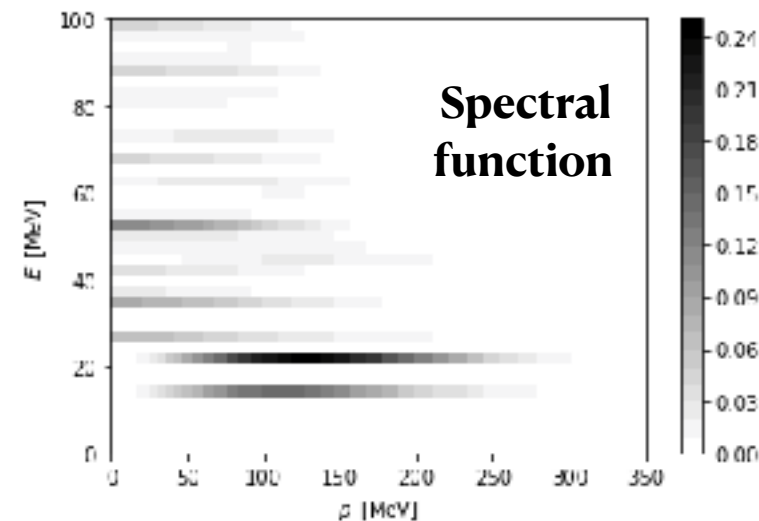


$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

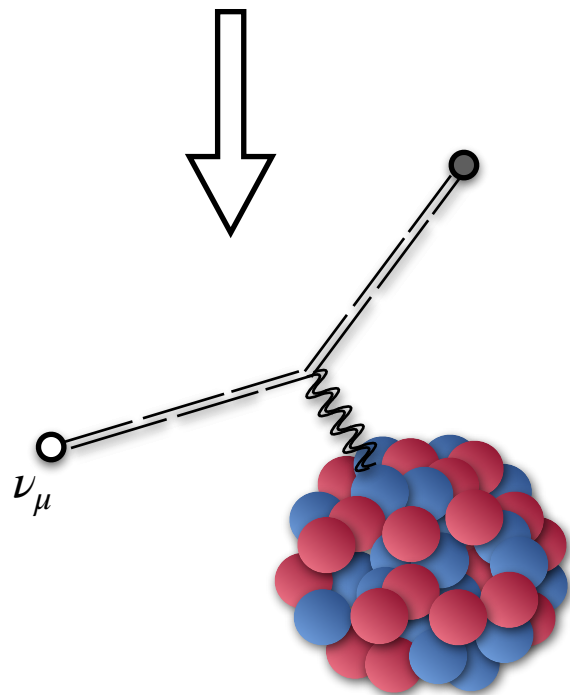
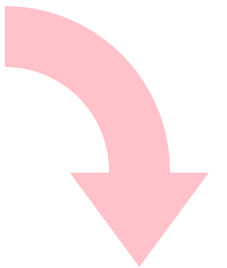
Many-body problem



Impulse Approximation

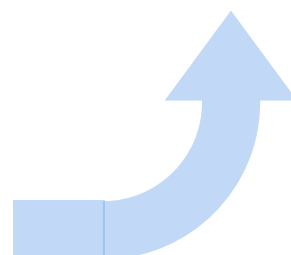
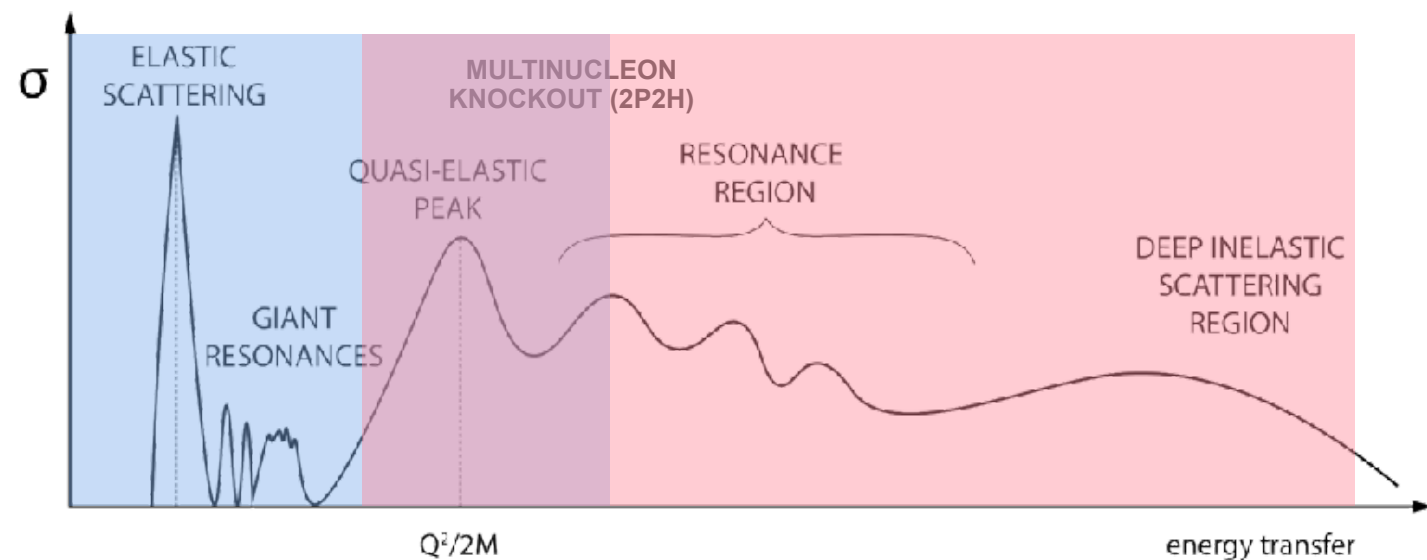


Probability density of finding nucleon
(E, \mathbf{p}) in ground state nucleus



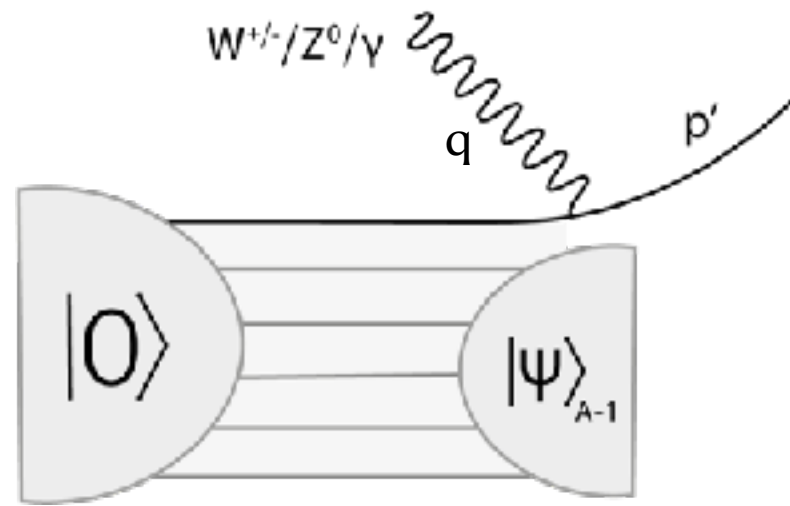
$$\langle \psi_f | \hat{j} | \psi_A \rangle$$

Electroweak responses



Spectral functions

Coupled Cluster + ChEK method

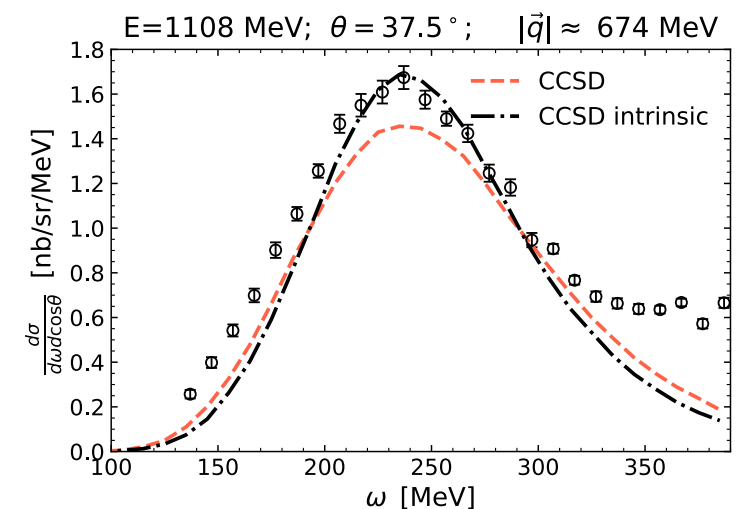
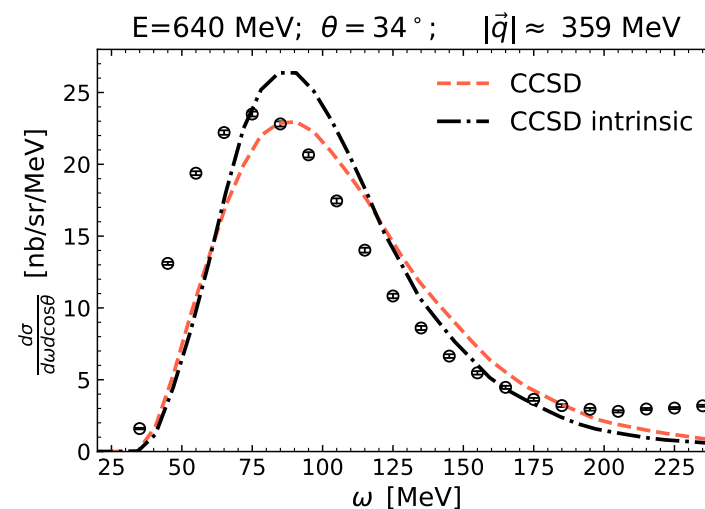
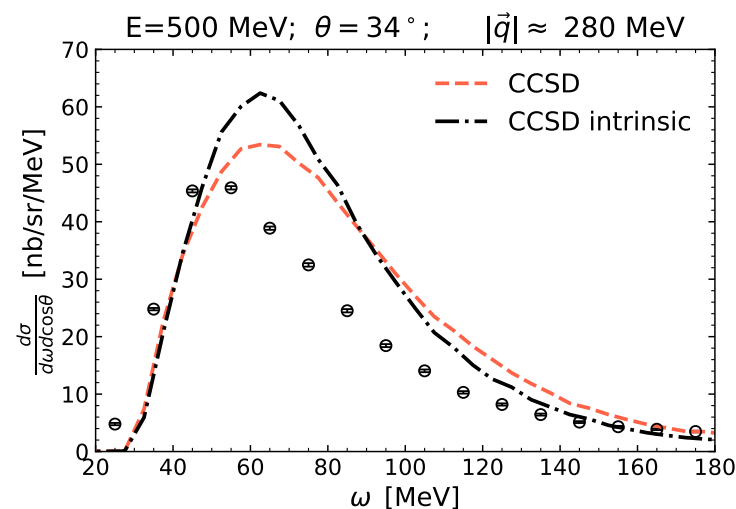


$$\sigma \propto |\mathcal{M}|^2 S(E, p)$$

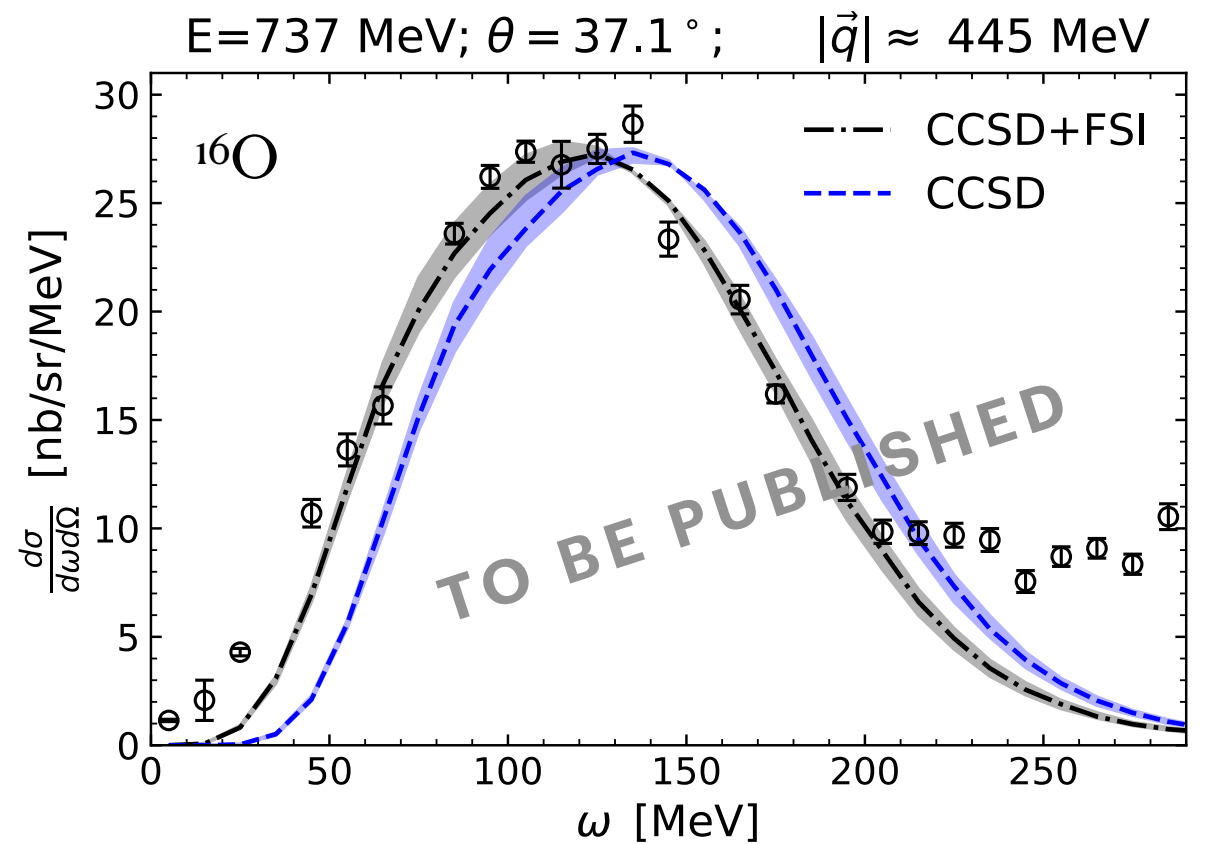
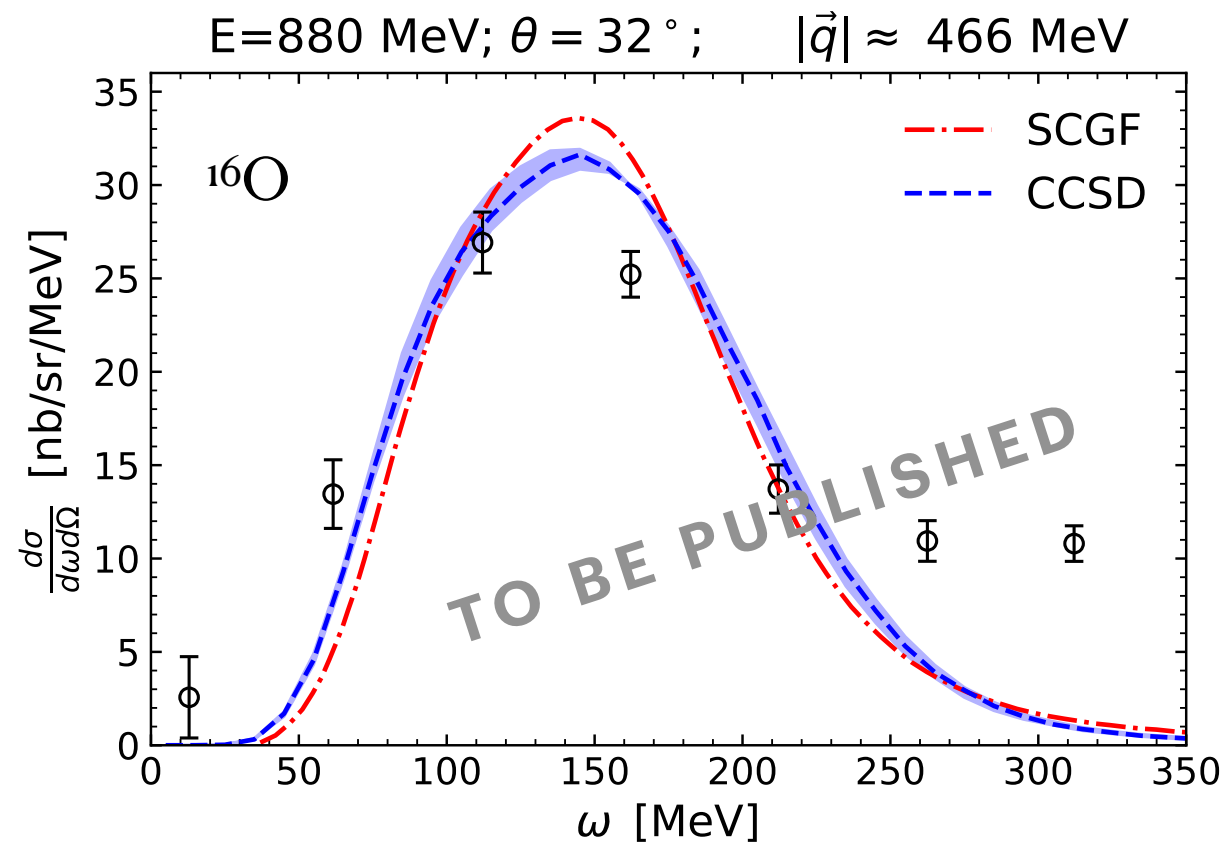
Factorized interaction vertex
(relativistic, pion
production...)

Spectral function -
nuclear information

growing \mathbf{q} momentum transfer \rightarrow final state interactions play minor role



Final state interactions



JES et al, in preparation (2022)

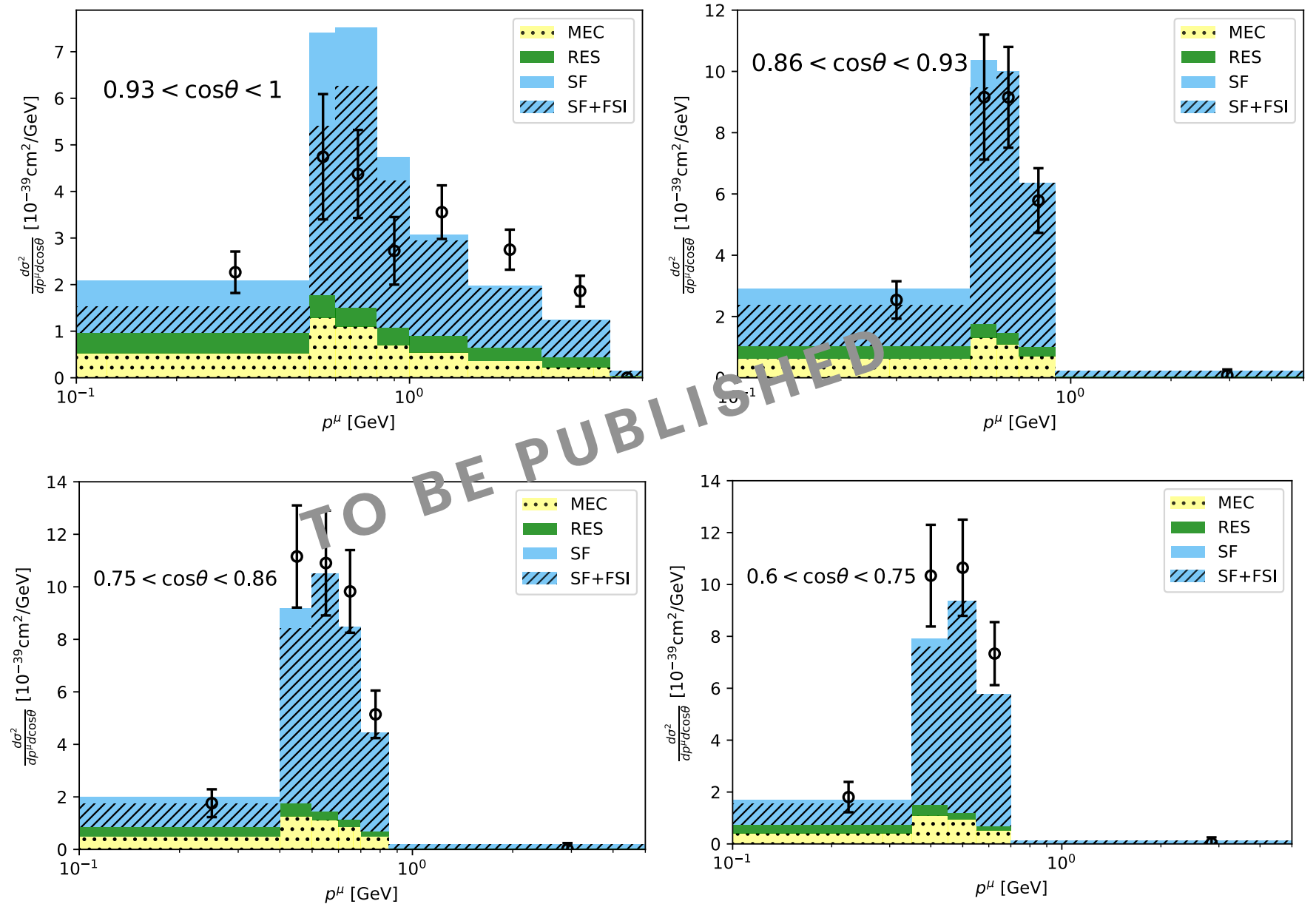
How to account for the FSI? Optical potential for the outgoing nucleon

Spectral function for neutrinos

$$\nu_{\mu} + {}^{16}\text{O} \rightarrow \mu^{-} + X$$

- Comparison with T2K long baseline ν oscillation experiment
- CC0 π events
- Spectral function implemented into NuWro Monte Carlo generator

Data: Phys. Rev. D 101, 112004 (2020)



Outlook

- LIT-CC benchmark for electron scattering → ready for neutrino
- Role of 2-body currents for medium-mass nuclei
- Explore possible applications of the ChEK method
- Spectral functions (within Impulse Approximation):
 - Relativistic regime
 - Semi-inclusive processes
 - Further steps: 2-body spectral functions, accounting for FSI

Thank you for attention

BACKUP

Details on inversion procedure

- Basis functions

$$R_L(\omega) = \sum_{i=1}^N c_i \omega^{n_0} e^{-\frac{\omega}{\beta_i}}$$

- Stability of the inversion procedure:
 - Vary the parameters n_0 , β_i and number of basis functions N (6-9)
 - Use LITs of various width Γ (5, 10, 20 MeV)

Lorentz integral transform

$$L(\sigma) = \int \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} d\omega = \int \frac{R(\omega)}{(\omega + \tilde{\sigma}^*)(\omega + \tilde{\sigma})} d\omega$$

$$L(\sigma) = \int d\omega \sum_f \langle \Psi_0 | \rho^\dagger \frac{1}{\omega + \tilde{\sigma}^*} | \Psi_f \rangle \langle \Psi_f | \frac{1}{\omega + \tilde{\sigma}} \rho | \Psi_0 \rangle \delta(\omega + E_0 - E_f)$$

$$L(\sigma) = \sum_f \langle \Psi_0 | \rho^\dagger \frac{1}{E_f - E_0 + \tilde{\sigma}^*} | \Psi_f \rangle \langle \Psi_f | \frac{1}{E_f - E_0 + \tilde{\sigma}} \rho | \Psi_0 \rangle$$

$$L(\sigma) = \sum_f \langle \Psi_0 | \rho^\dagger \frac{1}{H - E_0 + \tilde{\sigma}^*} | \Psi_f \rangle \langle \Psi_f | \frac{1}{H - E_0 + \tilde{\sigma}} \rho | \Psi_0 \rangle$$

$$\langle \tilde{\Psi} | \quad | \tilde{\Psi} \rangle$$

We need to solve

$$(H - E_0 + \tilde{\sigma}) | \tilde{\Psi} \rangle = \rho | \Psi \rangle \quad \text{Schrodinger-like equation}$$

Lorentz Integral Transform (LIT)

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Integral
transform

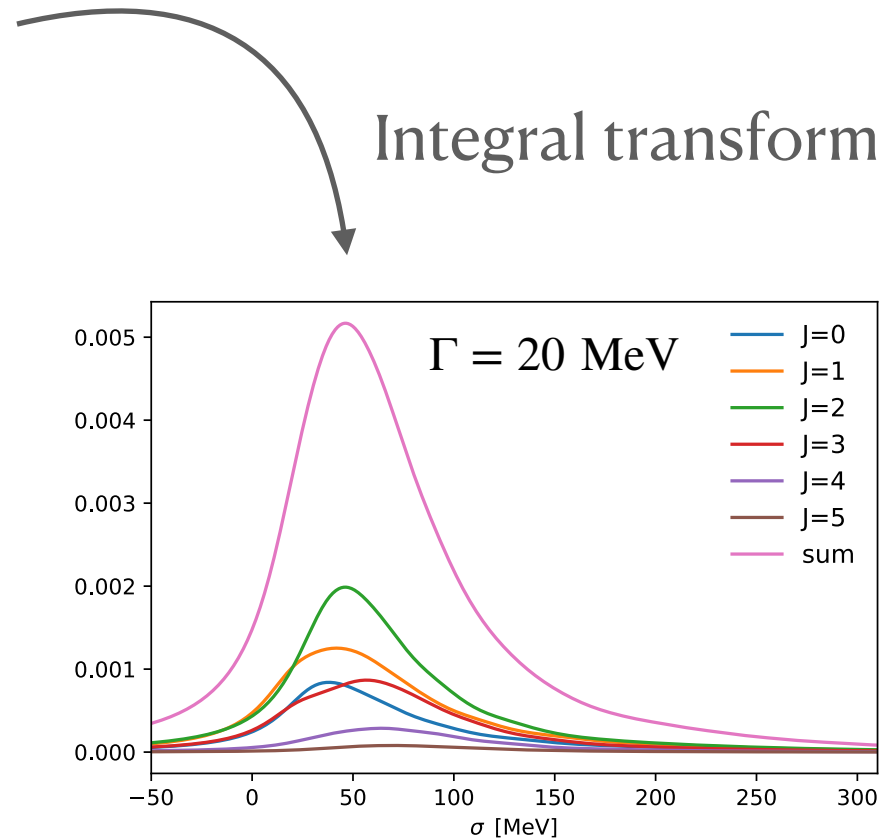
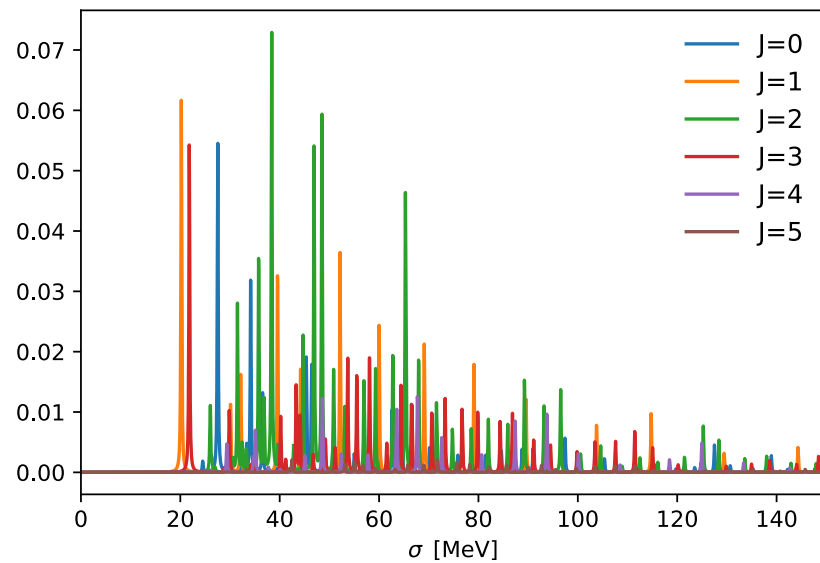
$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger K(\mathcal{H} - E_0, \sigma) J_\nu | \Psi \rangle$$

Lorentzian kernel:

$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$

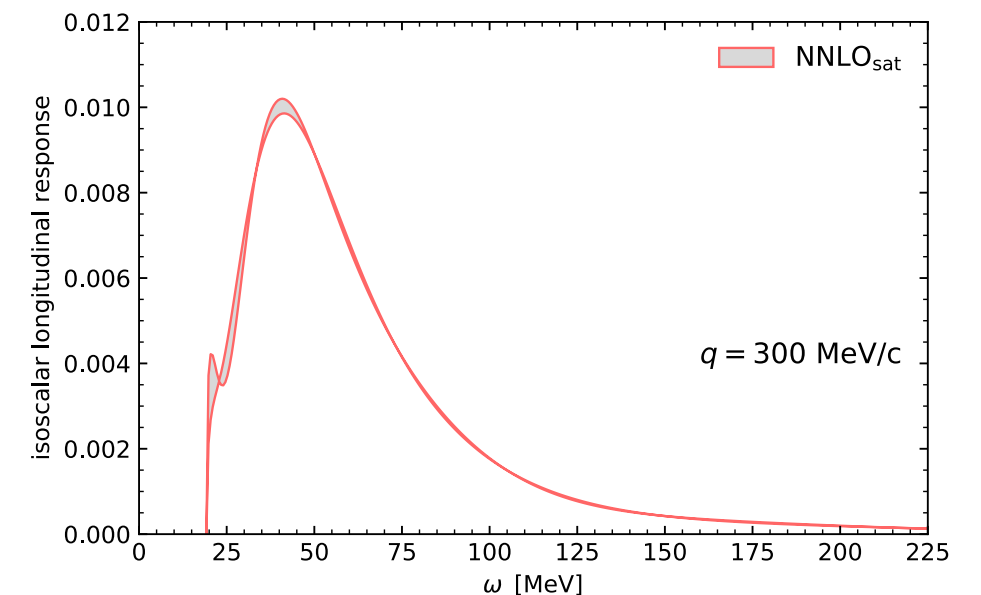
$S_{\mu\nu}$ has to be inverted to get access to $R_{\mu\nu}$

Lorentz Integral Transform

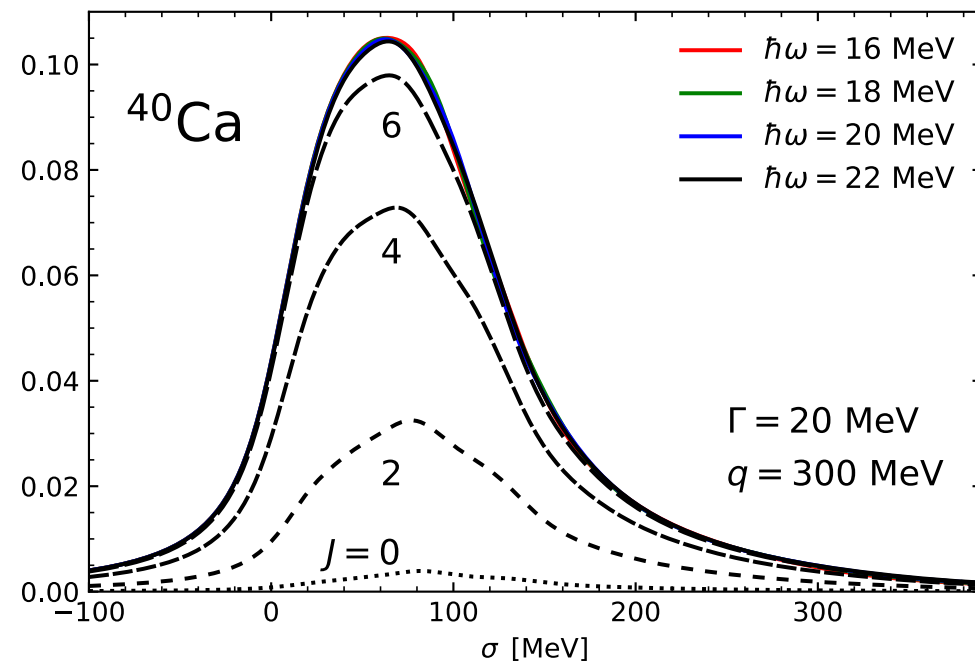


Inversion

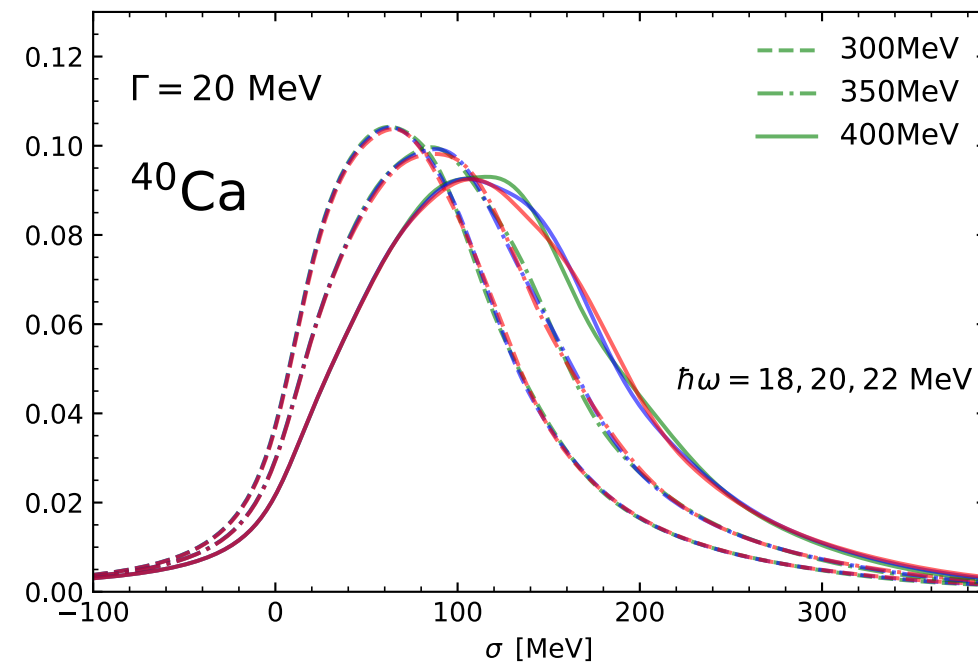
Longitudinal isoscalar
response on ^4He
at $q=300$ MeV



Longitudinal response ^{40}Ca

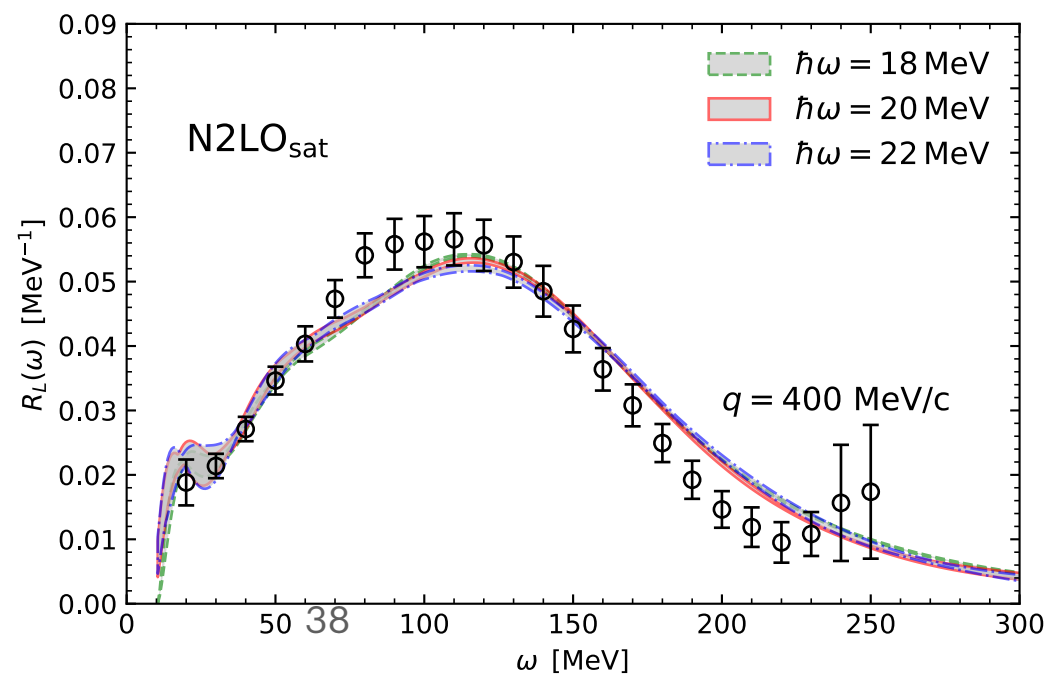


Sum over multipoles



Underlying oscillator frequency

Inversion



ChEK method

$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_{\mu}^{\dagger} K(\mathcal{H}, \sigma) J_{\nu} | \Psi \rangle$$

- Expansion in Chebyshev polynomials

$$K(\mathcal{H}, \sigma) = \sum_{k=0}^N c_k(\sigma) T_k(\mathcal{H})$$

- Recursive relations of Chebyshev polynomials

$$T_0(x) = 1; \quad T_{-1}(x) = T_1(x) = x$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

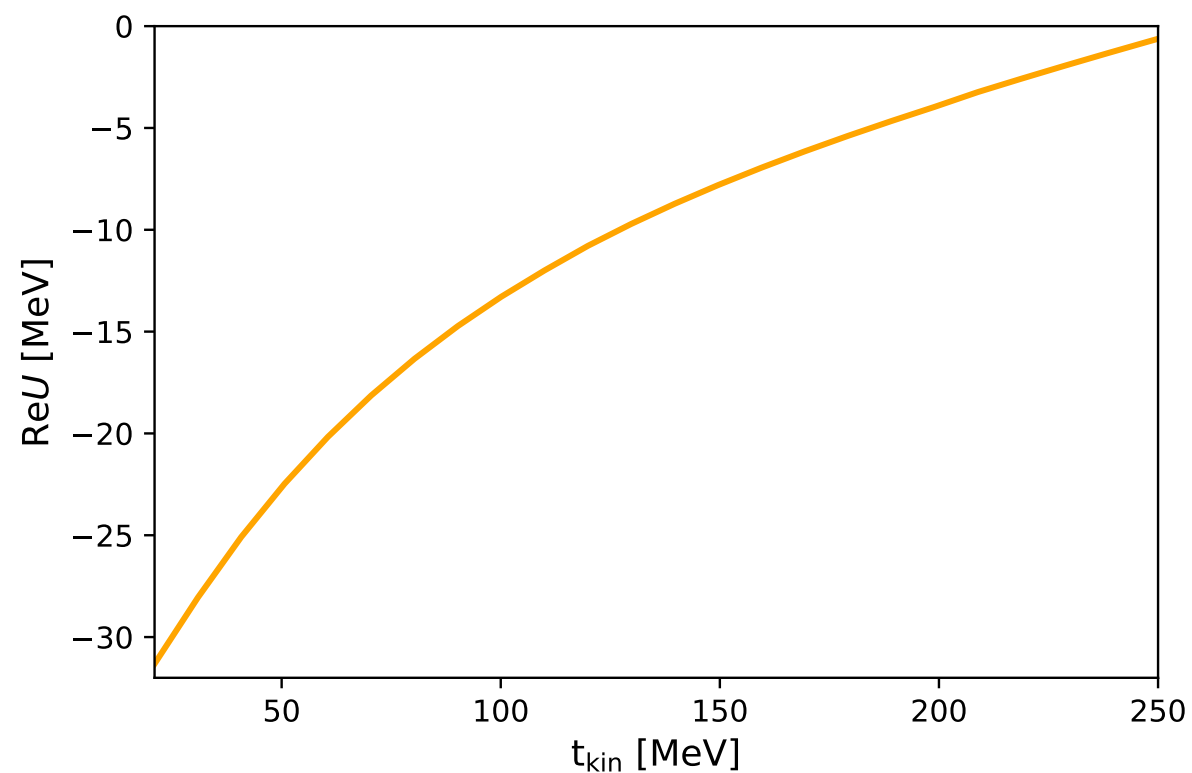
- Gives error estimate of energy integrals of local density of states $R(\omega)$

$$Q(R, f) = \int d\omega R(\omega) f(\omega)$$

Optical potential

$$W_{\text{FSI}}^{\mu\nu}(q) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} dE \frac{m}{E_p} \frac{m}{E_{p+q}} \\ \left[S^n(\mathbf{p}, E) w_n^{\mu\nu}(p, q) + S^p(\mathbf{p}, E) w_p^{\mu\nu}(p, q) \right] \\ \times \theta(\mathbf{p}' - p_F) \delta(\omega + E - E_{p+q} - E_f^{\text{kin}} - \text{Re}U)$$

$$w^{\mu\nu}(p, q) = \langle p + q | j^\mu | p \rangle^\dagger \langle p + q | j^\nu | p \rangle$$



Spectral function

Green's function:

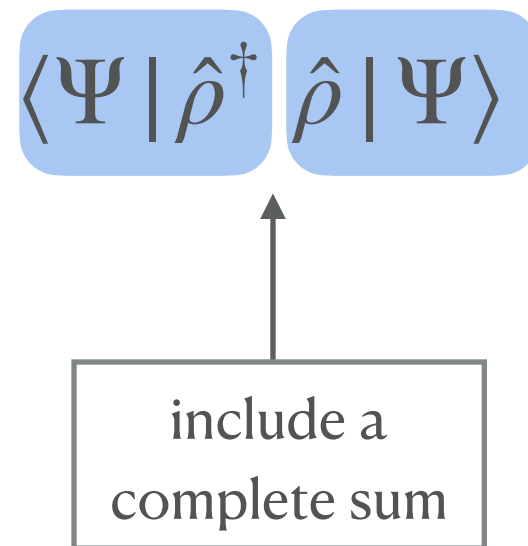
$$G_h(\alpha, \beta, E) = \langle 0 | a_\beta^\dagger \frac{1}{E - (E_0 - \hat{H}) - i\epsilon} a_\alpha | 0 \rangle$$

$$\text{Im}G_h(\alpha, \beta, E) = -\pi \oint_{\Psi_{A-1}} \langle 0 | a_\beta^\dagger | \Psi_{A-1} \rangle \langle \Psi_{A-1} | a_\alpha | 0 \rangle \delta(E - (E_0 - E_\Psi))$$

Spectral function:

$$S(\mathbf{p}, E) = -\frac{1}{\pi} \sum_{\alpha, \beta} \langle \mathbf{p} | \alpha \rangle \langle \mathbf{p} | \beta \rangle^\dagger \text{Im}G_h(\alpha, \beta, E)$$

Analysis of spurious states



Analysis of spurious states

$$\langle \Psi | \hat{\rho}^\dagger \left[\sum_{f_I} |f_I\rangle |\Psi_{CoM}^0\rangle \langle \Psi_{CoM}^0| \langle f_I| + \sum_{f_{CoM}} |\Psi_I^0\rangle |f_{CoM}\rangle \langle f_{CoM}| \langle \Psi_I^0| \right] \hat{\rho} | \Psi \rangle$$

excitation of intrinsic wave
function (physical
spectrum)

excitation of CoM wave
function (spurious)

Analysis of spurious states

$$\langle \Psi | \hat{\rho}^\dagger \left[\sum_{f_I} |f_I\rangle |\Psi_{CoM}^0\rangle \langle \Psi_{CoM}^0| \langle f_I| + \sum_{f_{CoM}} |\Psi_I^0\rangle |f_{CoM}\rangle \langle f_{CoM}| \langle \Psi_I^0| \right] \hat{\rho} | \Psi \rangle$$

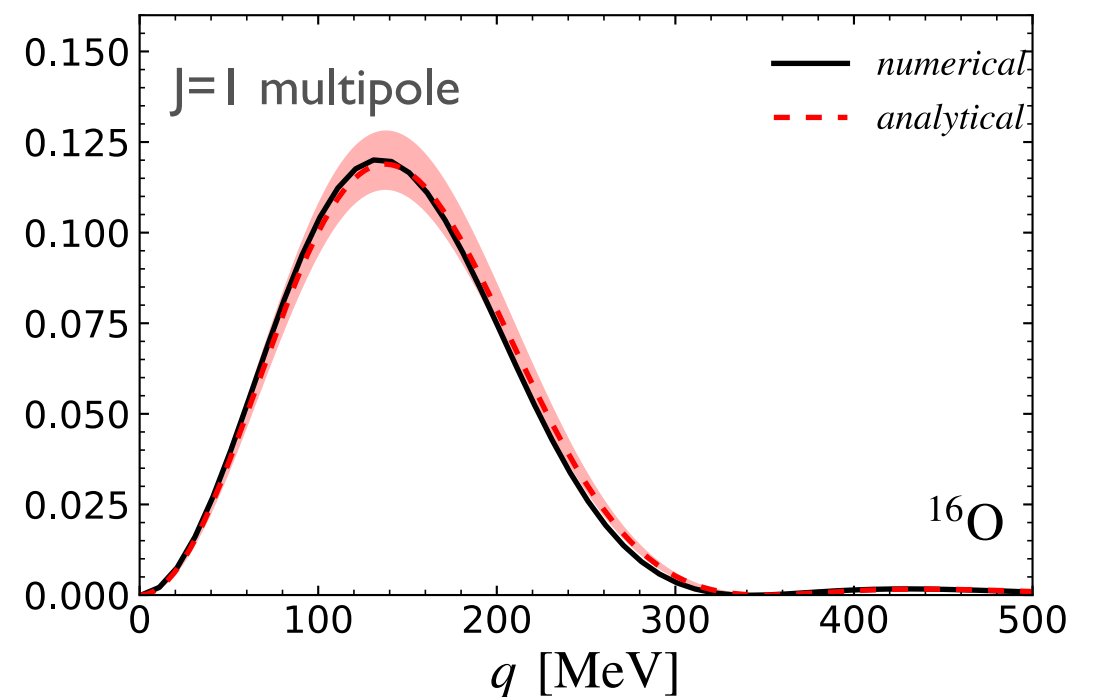
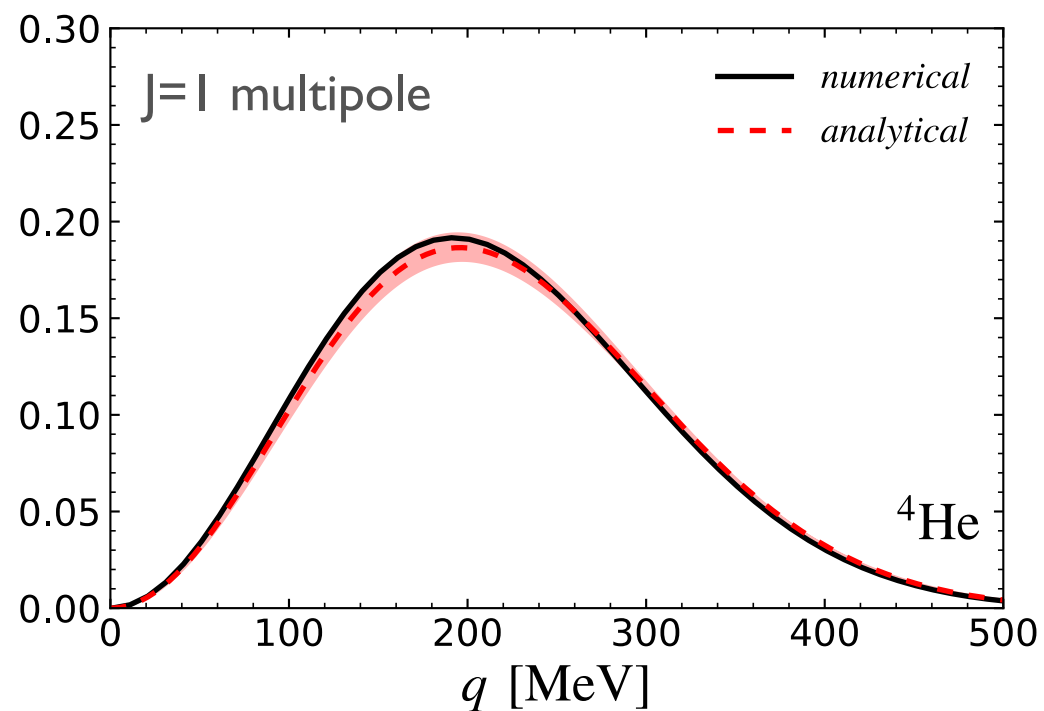
excitation of intrinsic wave function (physical spectrum)

excitation of CoM wave function (spurious)

With our ansatz about $|\Psi_{CoM}\rangle$ (Gaussian) we get analytical function for spurious states

We get numerically spurious states (nonphysical excitations with energy close to 0)

Analysis of spurious states



With our ansatz about $|\Psi_{CoM}\rangle$
(Gaussian) we get analytical
function for spurious states

We get numerically spurious states
(nonphysical excitations with
energy close to 0)

Nuclear Hamiltonian and currents

	H_{NN}	H_{NNN}	V^0	\vec{V}	A^0	\vec{A}
$(Q/\Lambda)^n :$						
$(Q/\Lambda)^{n+1} :$						
$(Q/\Lambda)^{n+2} :$						
$(Q/\Lambda)^{n+3} :$						

$$-\frac{1}{4} \frac{m}{g_A} \frac{c_D}{\Lambda} + \frac{c_3}{3} + \frac{2c_4}{3} + \frac{1}{6} = d_R$$

*Nucleon-structure diagrams and relativistic corrections not shown

Author: Bijaya Acharya

Coupled cluster method

Reference state (Hartree-Fock): $|\Psi\rangle$

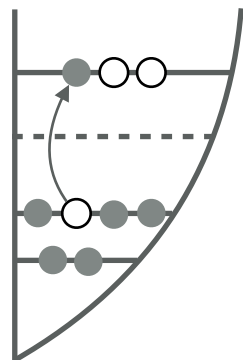
Include correlations through e^T operator

similarity transformed
Hamiltonian (non-Hermitian)

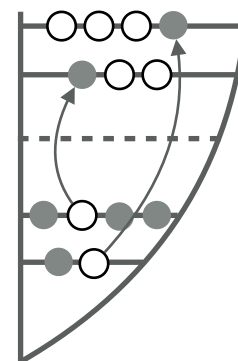
$$e^{-T} \mathcal{H} e^T |\Psi\rangle \equiv \bar{\mathcal{H}} |\Psi\rangle = E |\Psi\rangle$$

Expansion: $T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$

singles



doubles

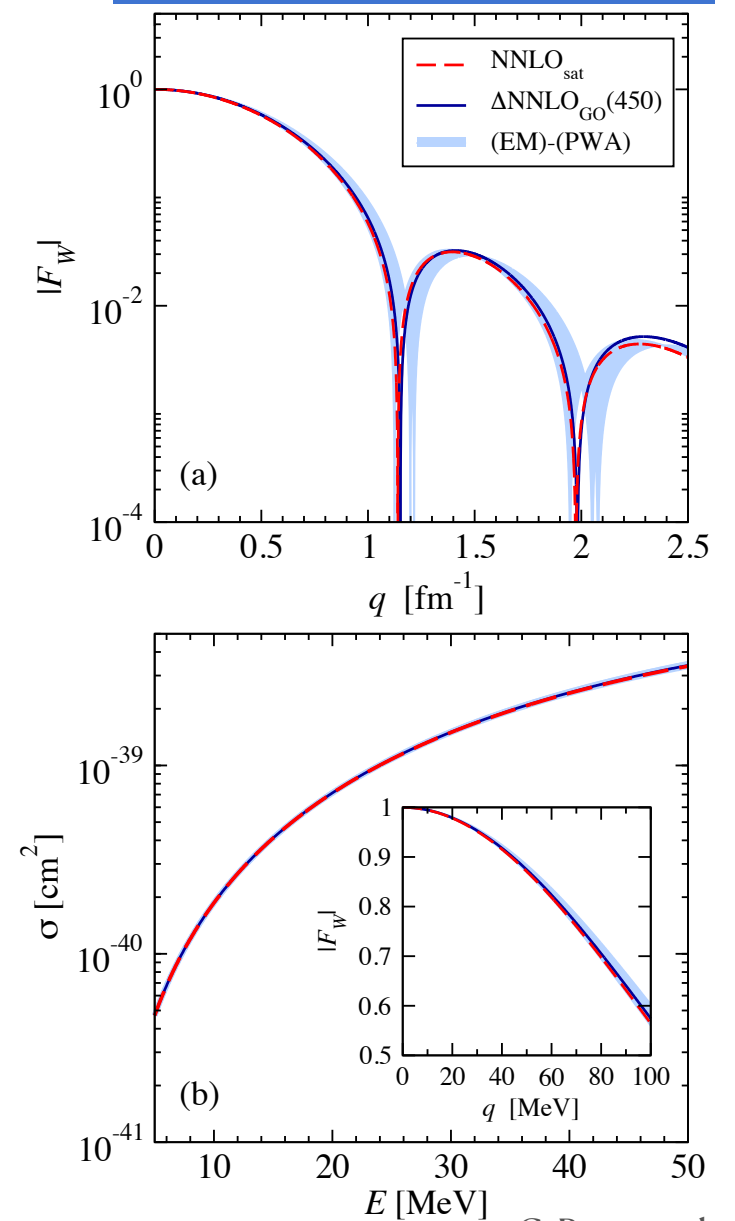


← coefficients obtained
through coupled cluster
equations

Coupled cluster method

- ✓ Controlled approximation through truncation in T
- ✓ Polynomial scaling with A (predictions for ^{100}Sn , ^{208}Pb)
- ✓ Size extensive
- ✓ Works most efficiently for doubly magic nuclei

coherent elastic neutrino scattering on ^{40}Ar

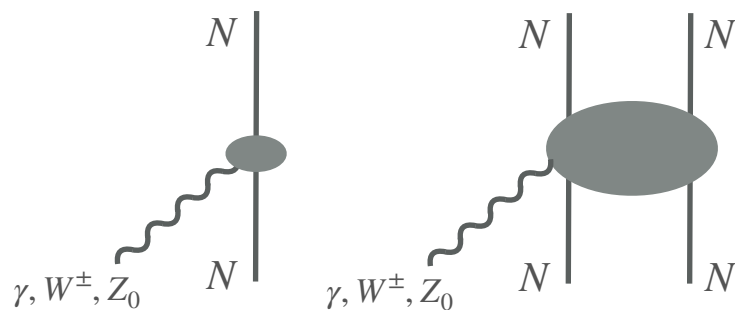


C. Payne et al.

Phys.Rev.C 100 (2019) 6, 061304

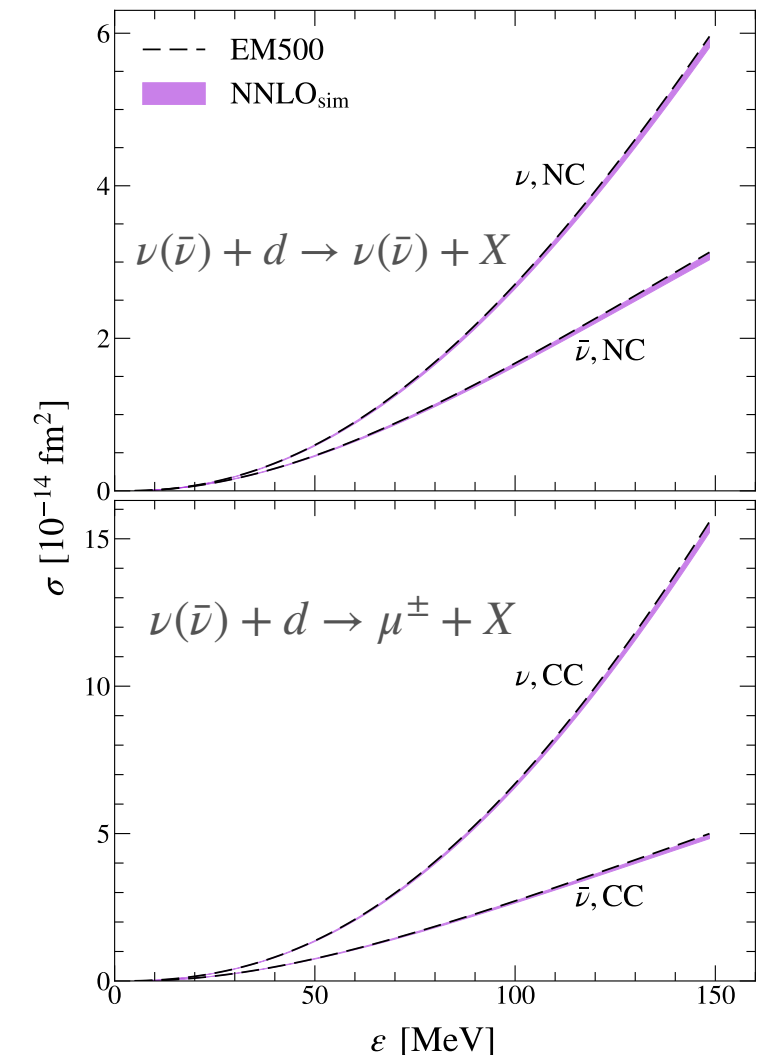
Electroweak currents

$$J = \sum_i j_i + \boxed{\sum_{i < j} j_{ij}} + \dots$$



known to give significant
contribution for neutrino-
nucleus scattering

Current decomposition into multipoles
needed for various *ab initio* methods:
CC, No Core Shell Model, In-Medium
Similarity Renormalization Group



Multipole decomposition for 1-
and 2-body EW currents

B. Acharya, S. Bacca
Phys.Rev.C 101 (2020) 1, 015505