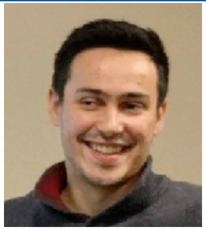
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA
EXCELENCIA MARÍA DE MAEZTU

Nambu Covariant Green's functions
for correlated superfluids

M Drissi C Barbieri




Drissi, Rios & Barbieri, arxiv:2107.09759
Drissi, Rios & Barbieri, arxiv:2107.09763
Drissi & Rios, EPJA 58, 90 (2022)

TRIUMF
UNIVERSITÀ DEGLI STUDI DI MILANO

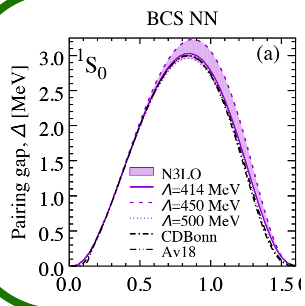
Dr Arnau Rios Huguet
Institute of Cosmos Sciences
Universitat de Barcelona

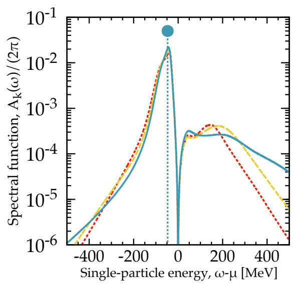
EMMI Hirschegg Workshop
19 January 2023

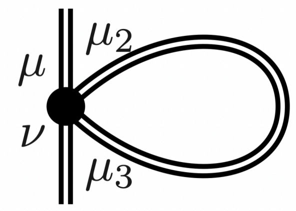
UNIVERSITY OF SURREY
UNIVERSITAT DE BARCELONA
Departament de Física Quàntica i Astrofísica
@HadNucAtUB

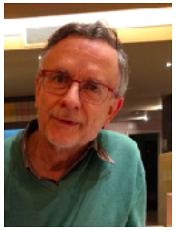
Nuclear Physics in Barcelona

Dense matter

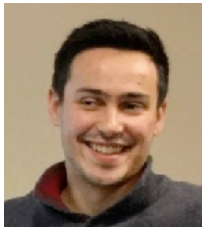




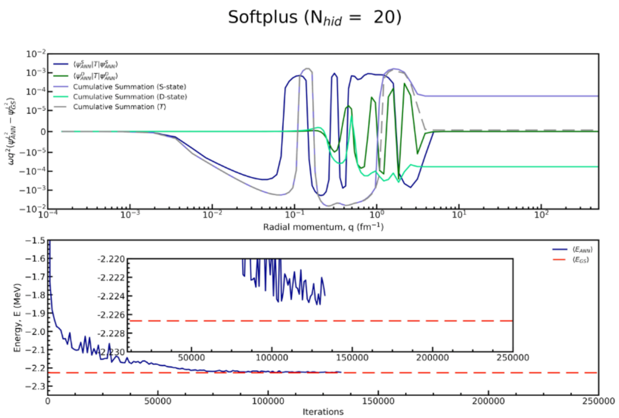


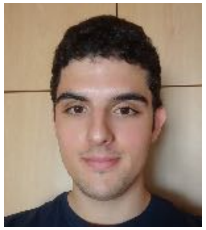


A Polls
A Carbone
W Dickhoff
C Barbieri



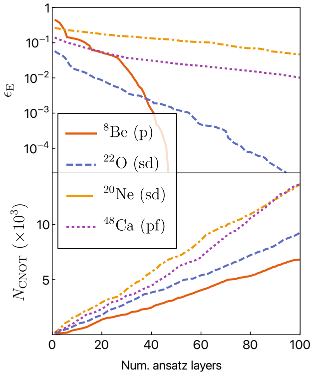
Machine learning

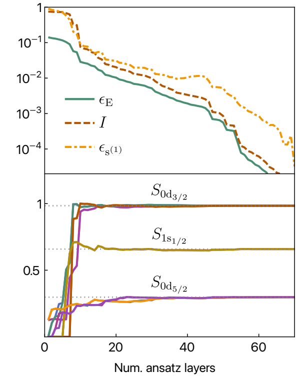


JWT Keeble J Rozalén

Quantum computing






AM Romero J Menéndez

Barcelona Supercomputing Center
Centro Nacional de Supercomputación

Fellowships

Fundación "la Caixa"

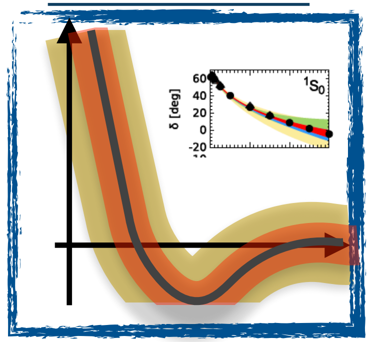
Investigación Programa Ramón y Cajal

bp

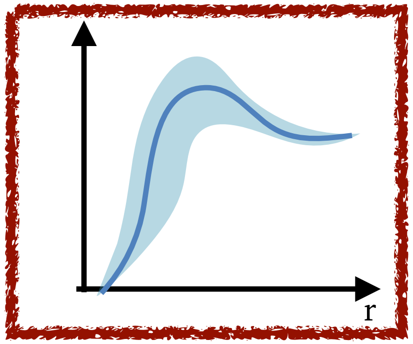
Outline

- Motivation

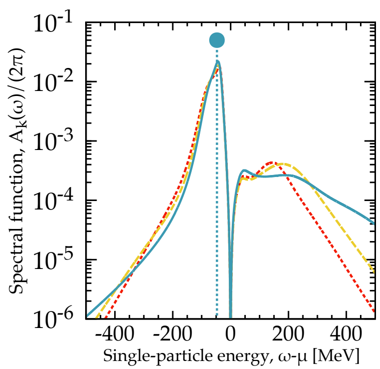
Hamiltonian

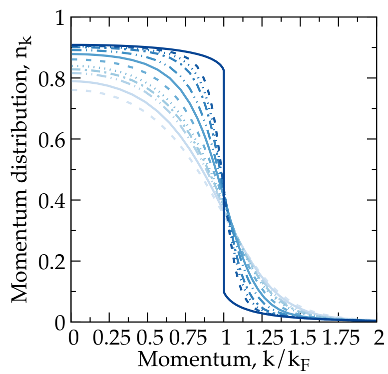


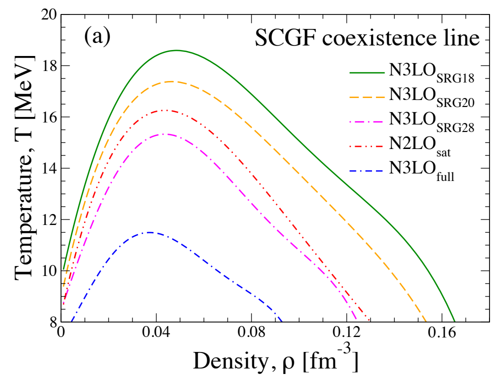
Many-body method



- “Normal” self-consistent Green’s functions

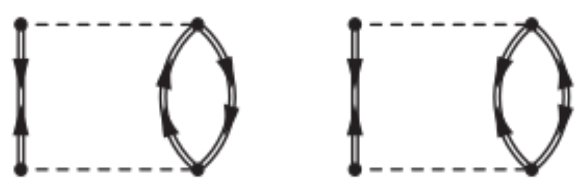


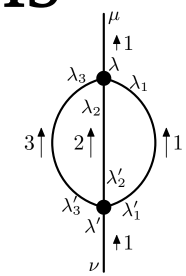




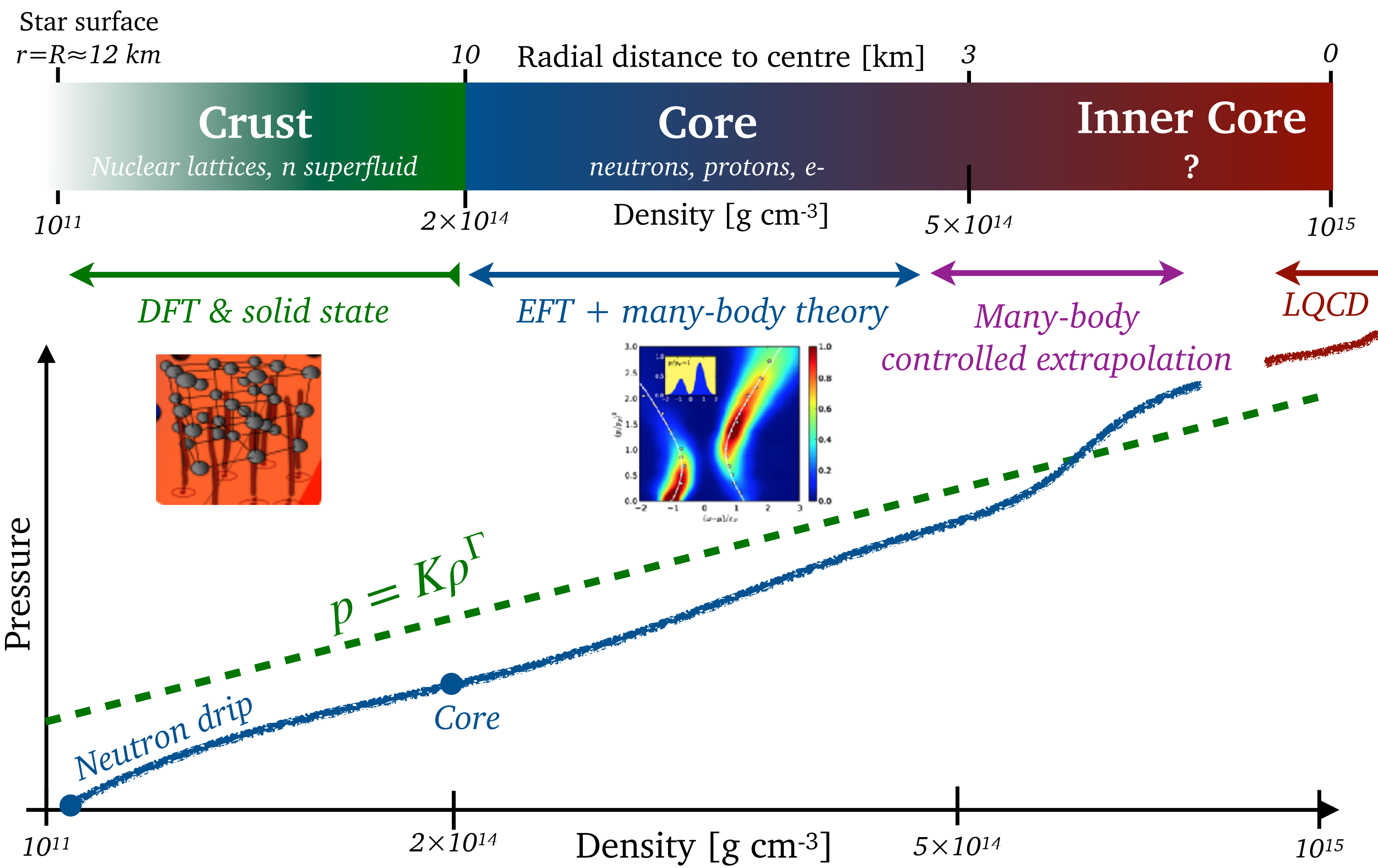
- Nambu-Covariant Green’s functions

$$\Sigma_{ab}^{11\,(2)}(\omega) =$$



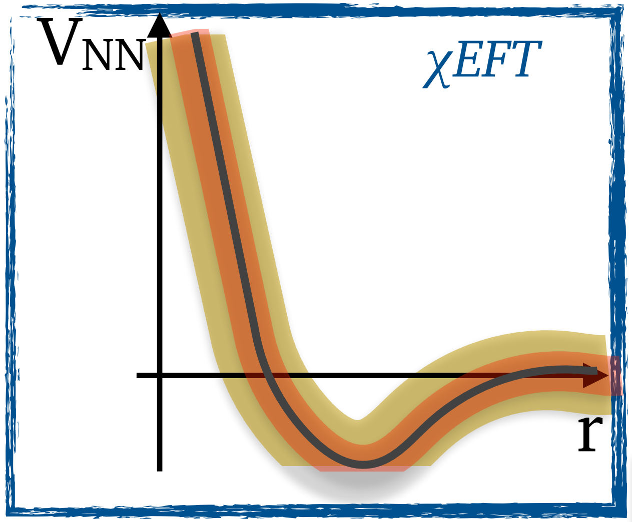


Neutron-star modelling

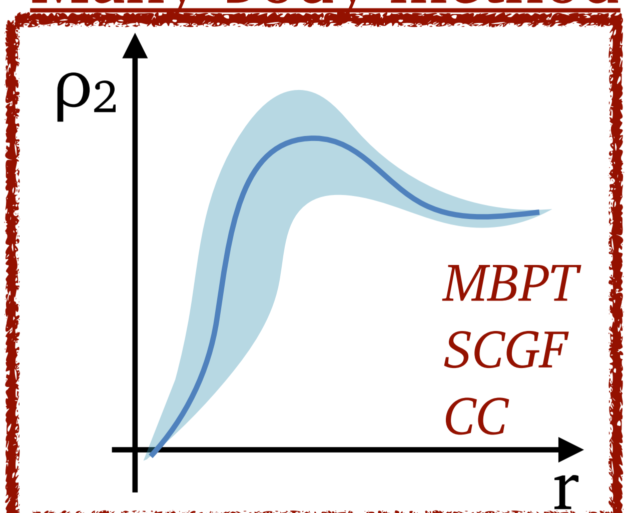


Nuclear error quantification (2022)

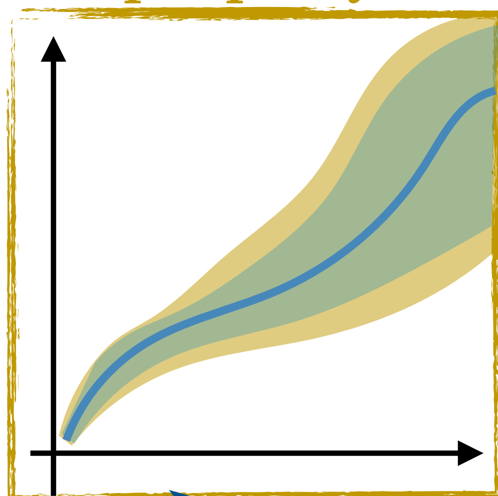
Hamiltonian



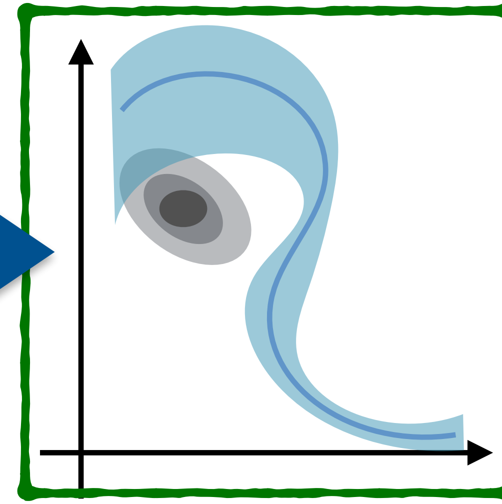
Many-body method



Astronuclear property



Neutron star observations



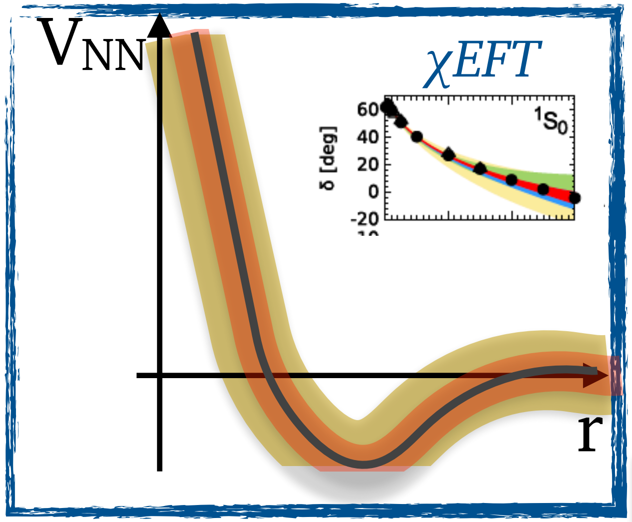
Forwards modelling

- Statistical propagation
- Bayesian analysis
- Emulators

C. Forssen's talk

Nuclear error quantification (2022)

Hamiltonian



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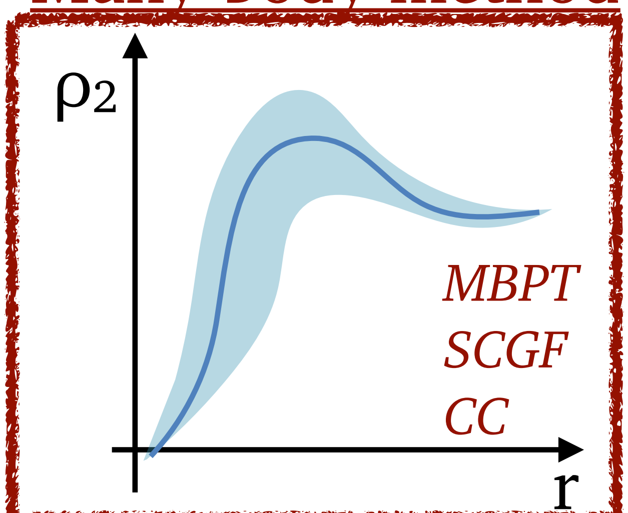

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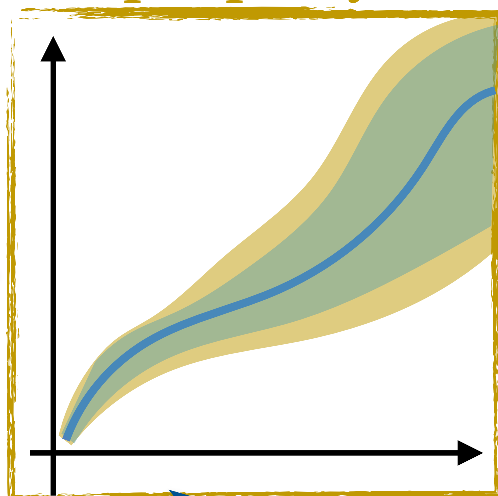

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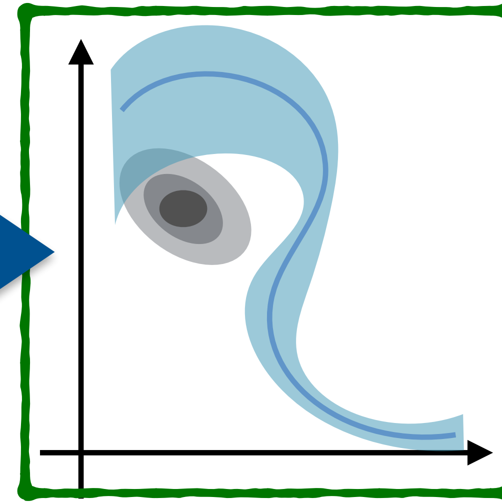

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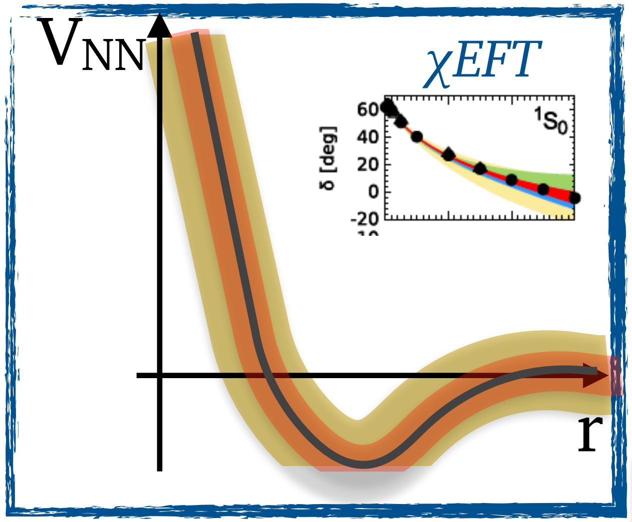

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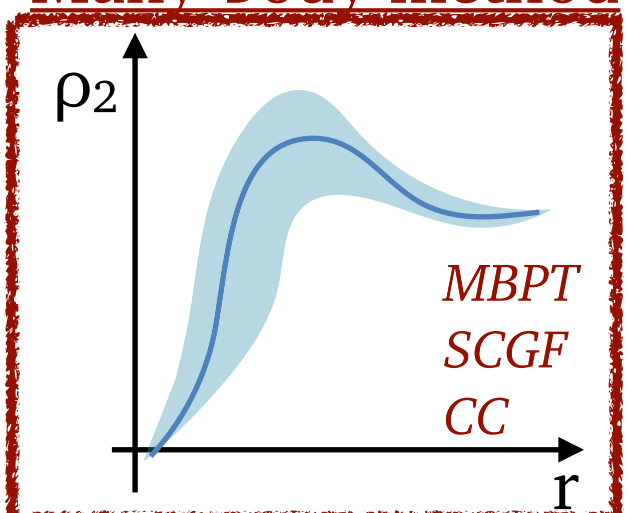
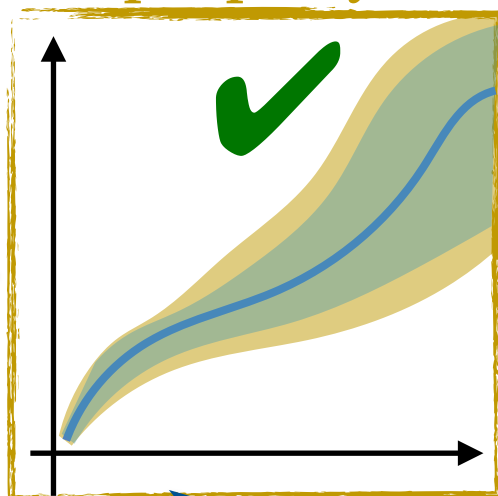
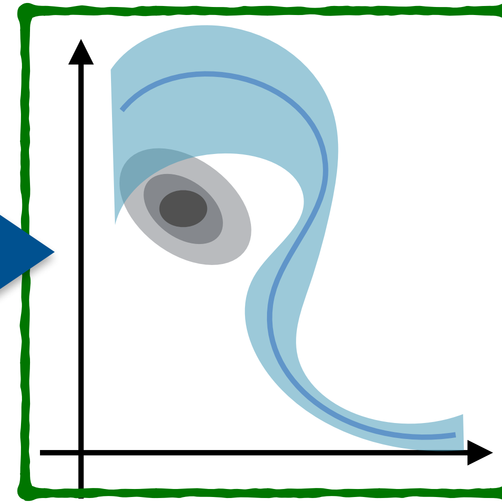


Astronuclear property



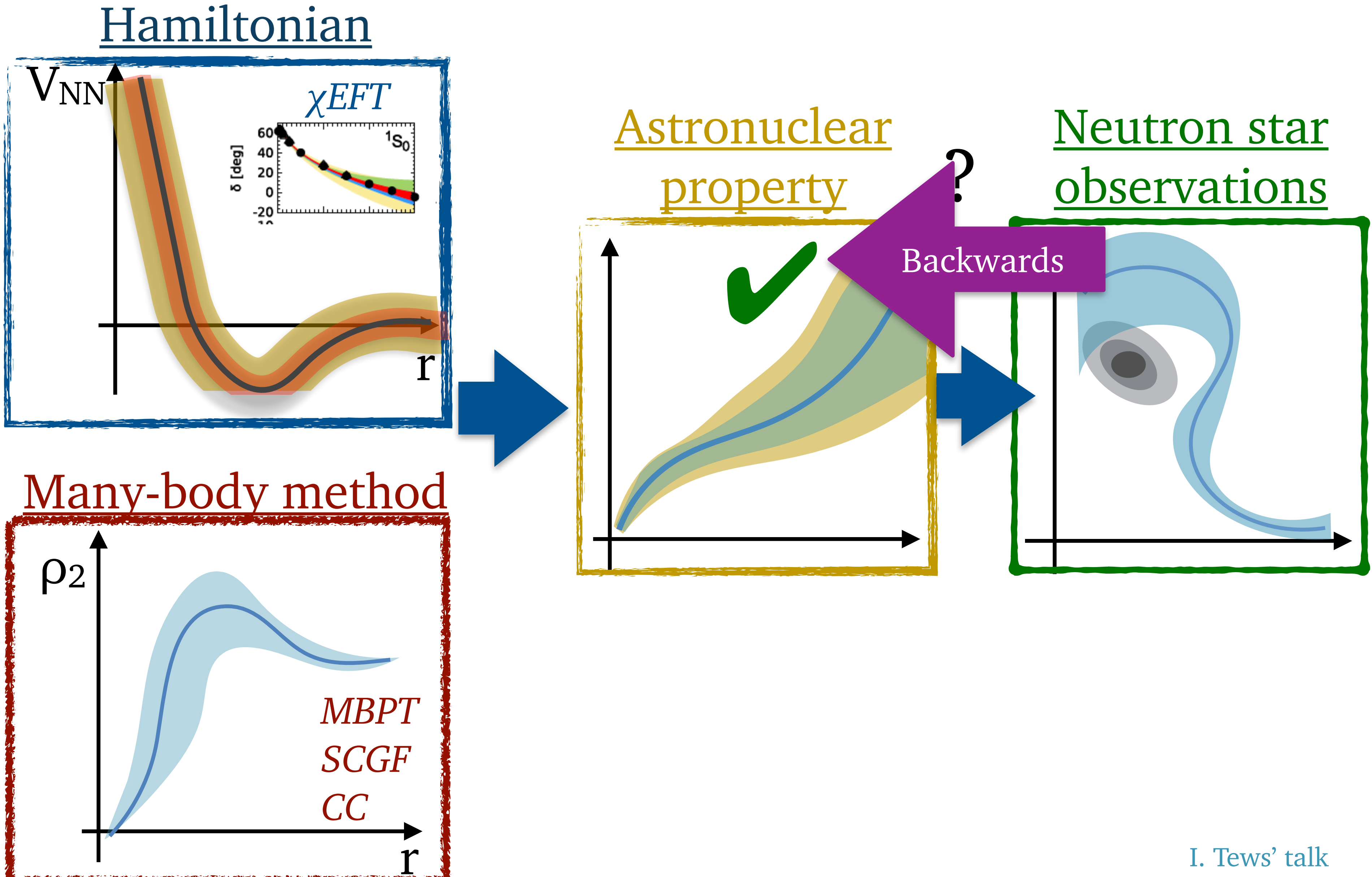
Neutron star observations



Forwards modelling
- Statistical propagation
- Bayesian analysis
- Emulators

C. Forssen's talk

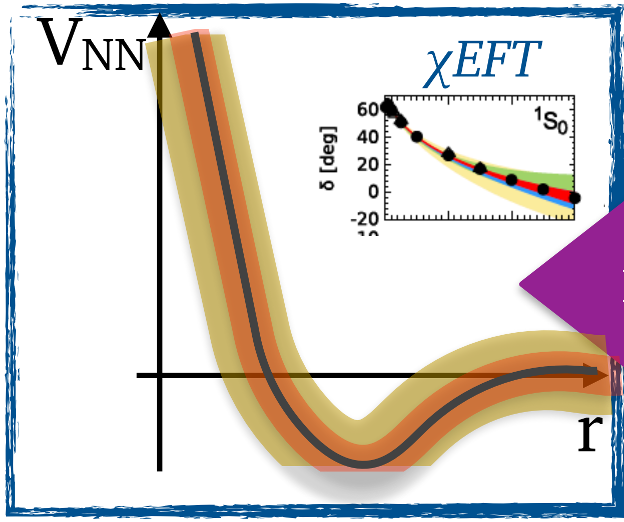
Nuclear error quantification



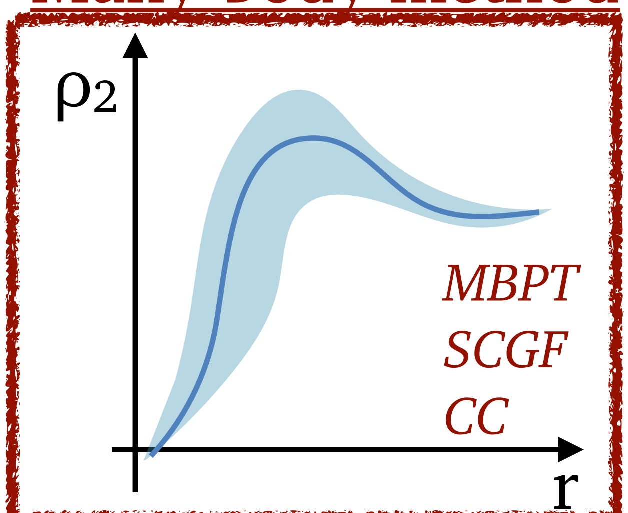
I. Tews' talk

Nuclear error quantification

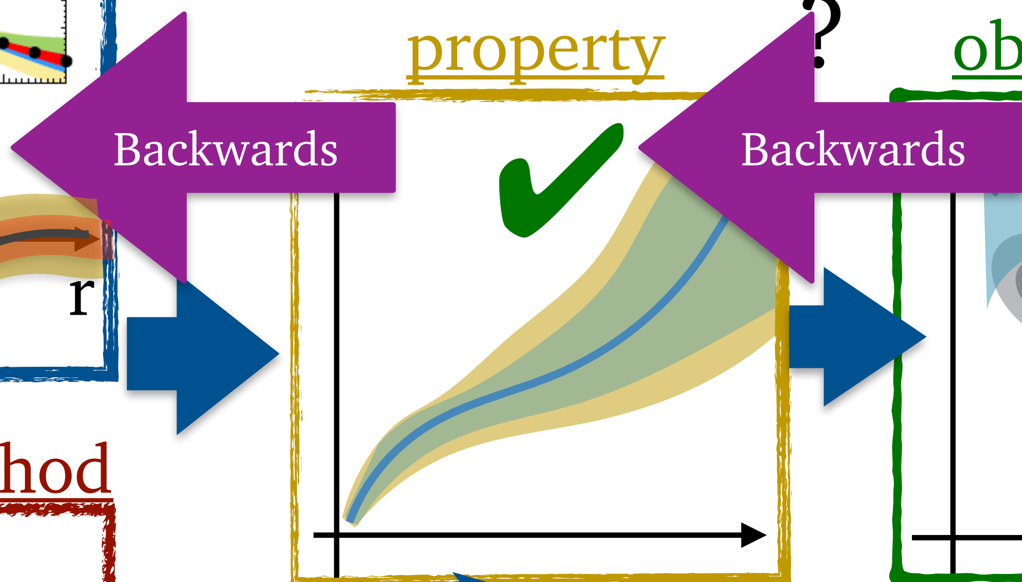
Hamiltonian



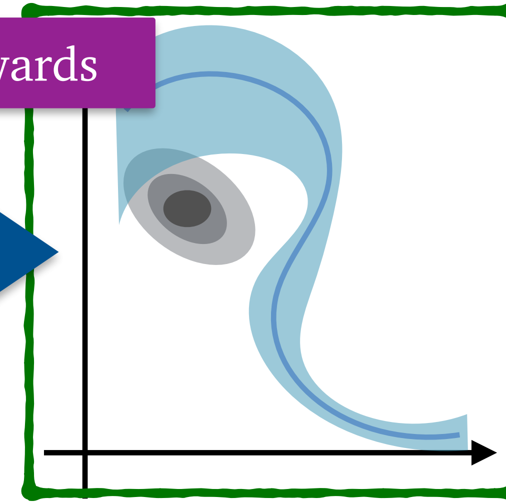
Many-body method



Astronuclear property



Neutron star observations



Requires

- Matter composition
- Regulators & density reach
- Degeneracies?

From M-R to EoS

Tolman-Oppenheimer-Volkov equations

$$\frac{dP(r)}{dr} = -\frac{G}{c^2}\frac{\left[m_<(r) + 4\pi P(r) r^3\right]\left[\epsilon(r) + P(r)\right]}{1 - \frac{2Gm_<(r)}{rc^2}}$$

$$\frac{dm_<(r)}{dr} = 4\pi r^2 \rho(r)$$

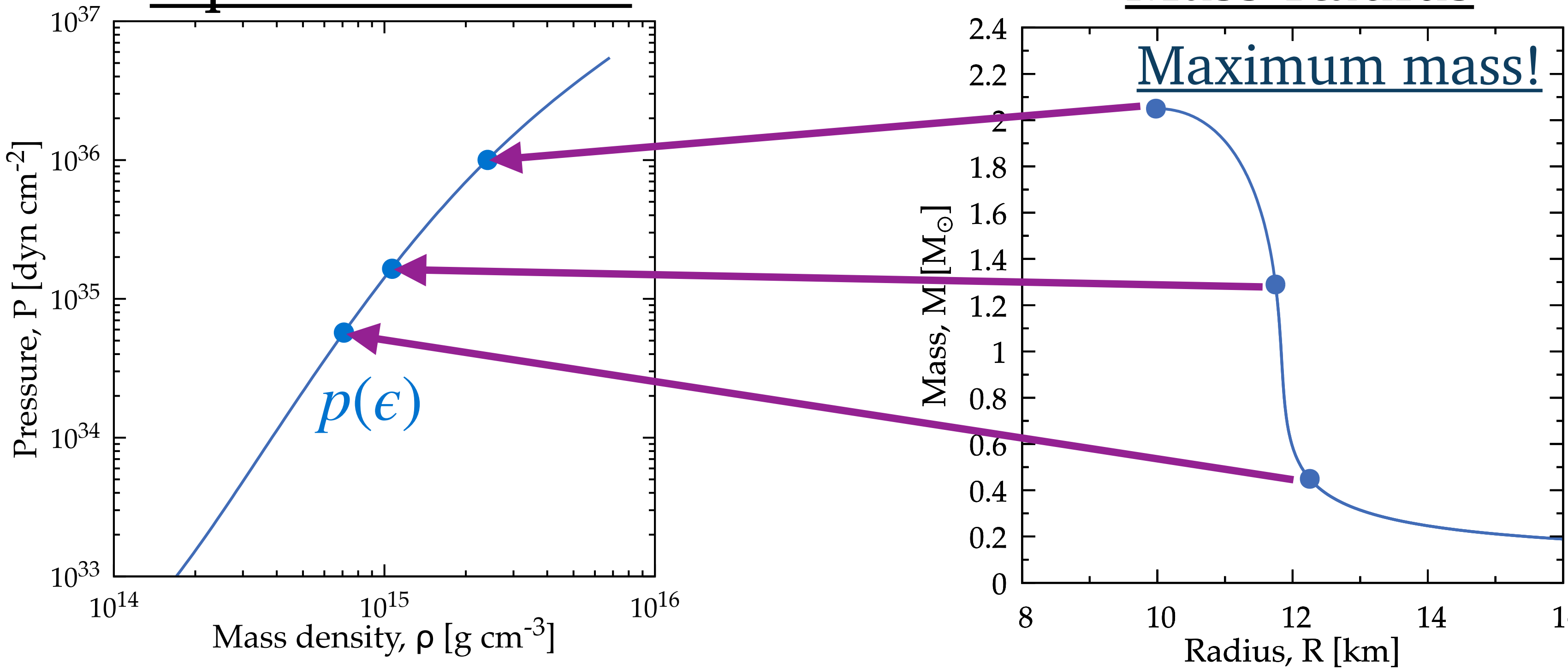
+

EoS

$$P \equiv P(\rho)$$

$$\epsilon = \rho c^2$$

Equation of State **Mass-Radius**



From M-R to EoS

Backwards

Tolman-Oppenheimer-Volkov equations

$$\frac{dP(r)}{dr} = -\frac{G}{c^2}\frac{\left[m_<(r) + 4\pi P(r) r^3\right]\left[\epsilon(r) + P(r)\right]}{1 - \frac{2Gm_<(r)}{rc^2}}$$

$$\frac{dm_<(r)}{dr} = 4\pi r^2 \rho(r)$$

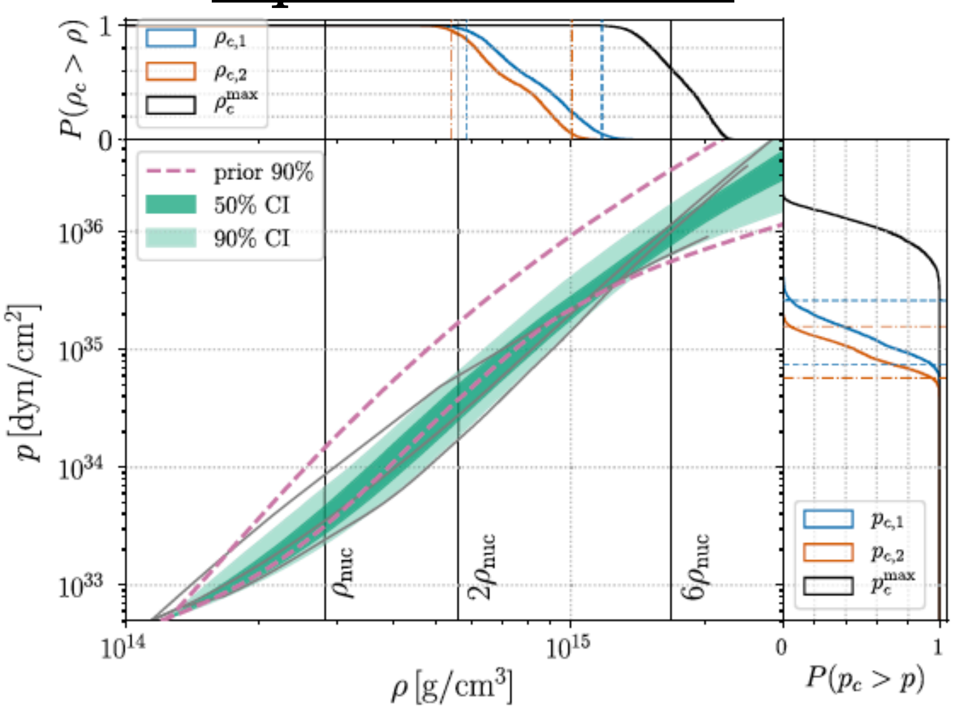
+

EoS

$$P \equiv P(\rho)$$

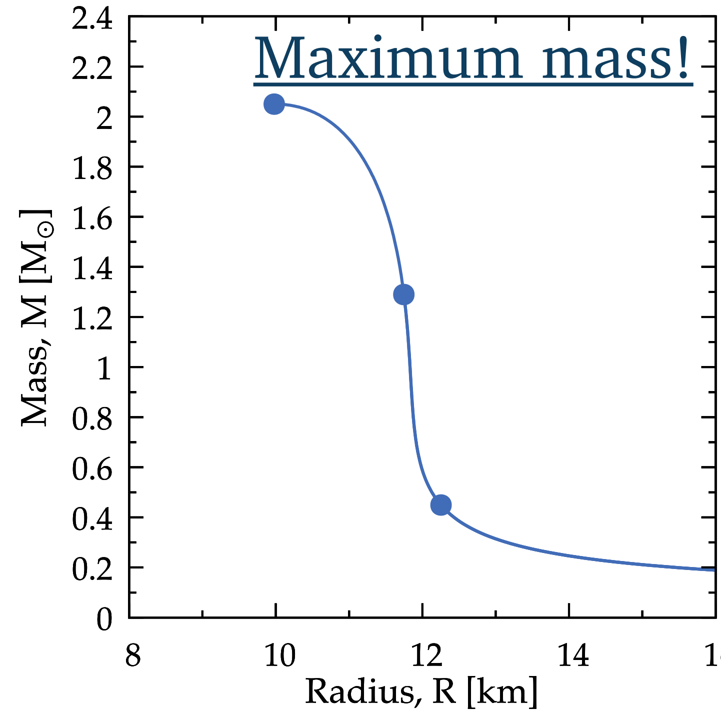
$$\epsilon = \rho c^2$$

Equation of State



LIGO/VIRGO PRL 121 161101 (2018)

Mass-Radius

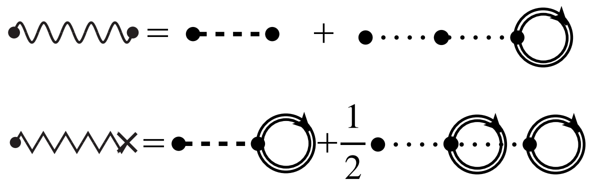


Self-Consistent Green's Functions

Forward

(ρ, T)

2N & 3N forces



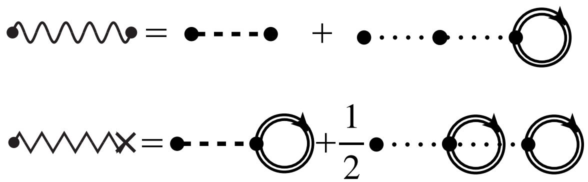
Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis

Self-Consistent Green's Functions

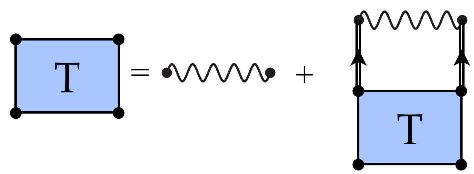
Forward

(ρ, T)

2N & 3N forces

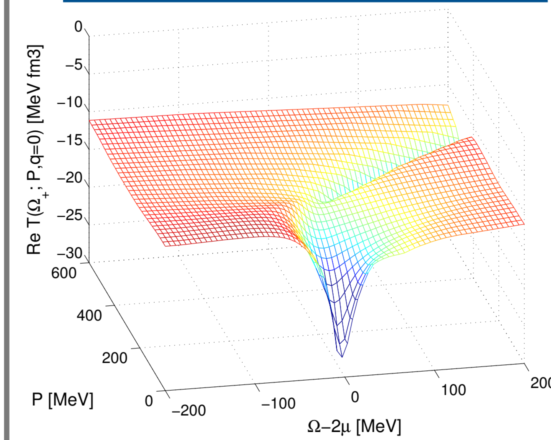


In-medium interaction



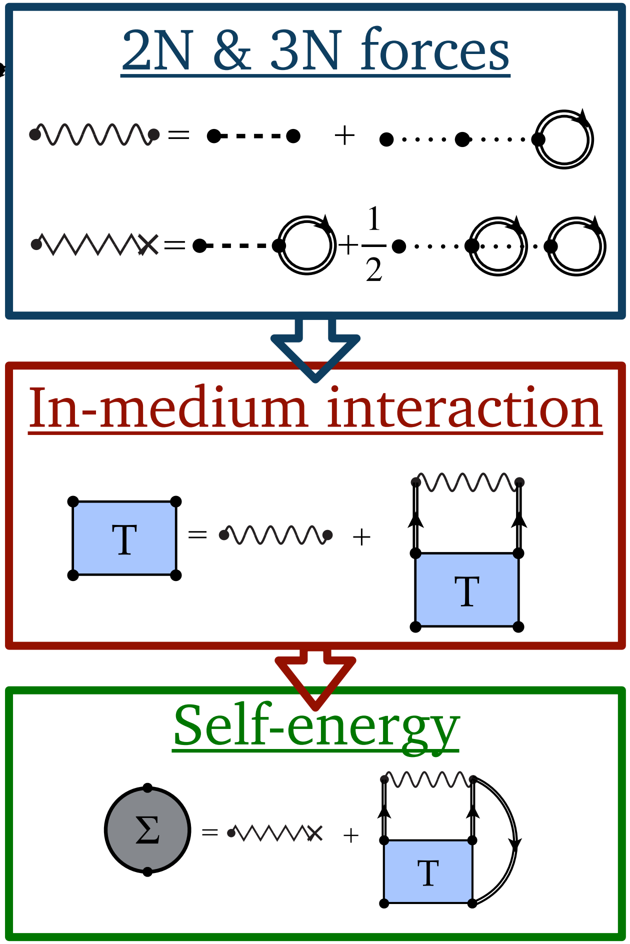
Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis

T-matrix at T=5 MeV



Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

(ρ, T)



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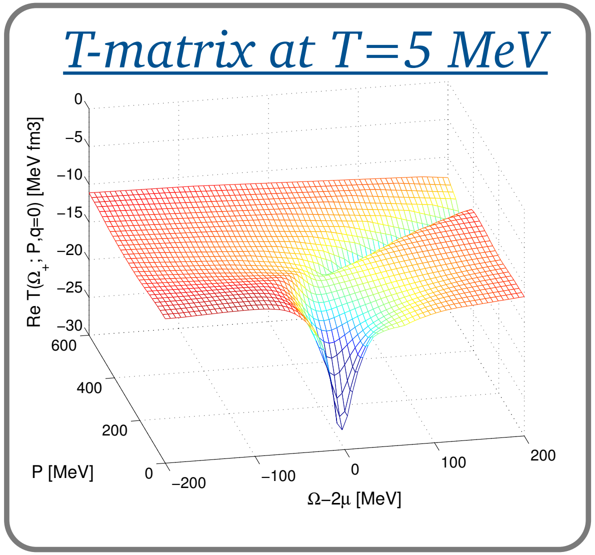

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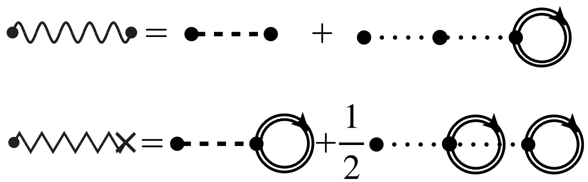
Self-Consistent Green's Functions

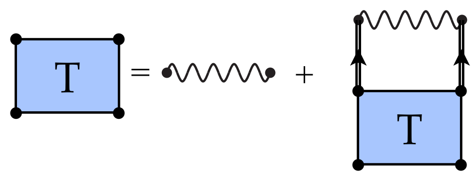
Forward

(ρ, T)

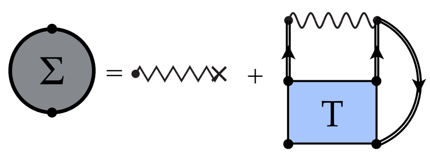
2N & 3N forces



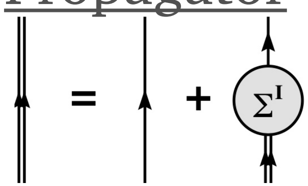
In-medium interaction



Self-energy



Propagator



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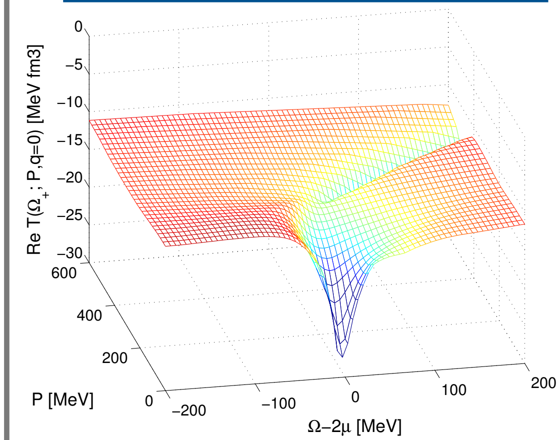

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Self-Consistent Green's Functions

Forward

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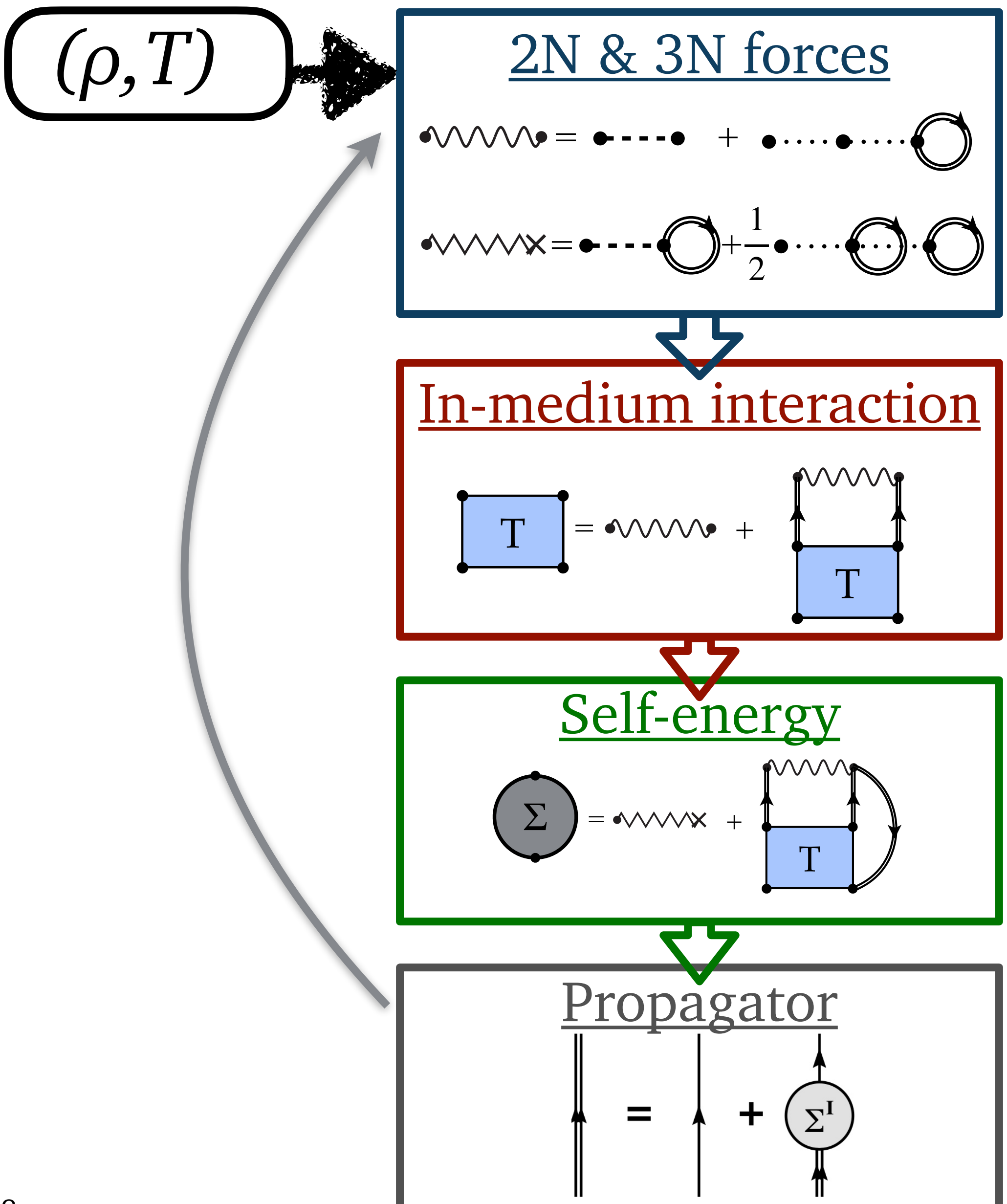

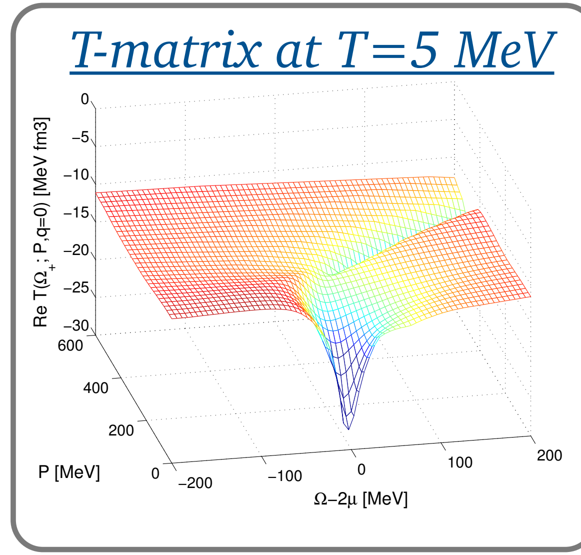

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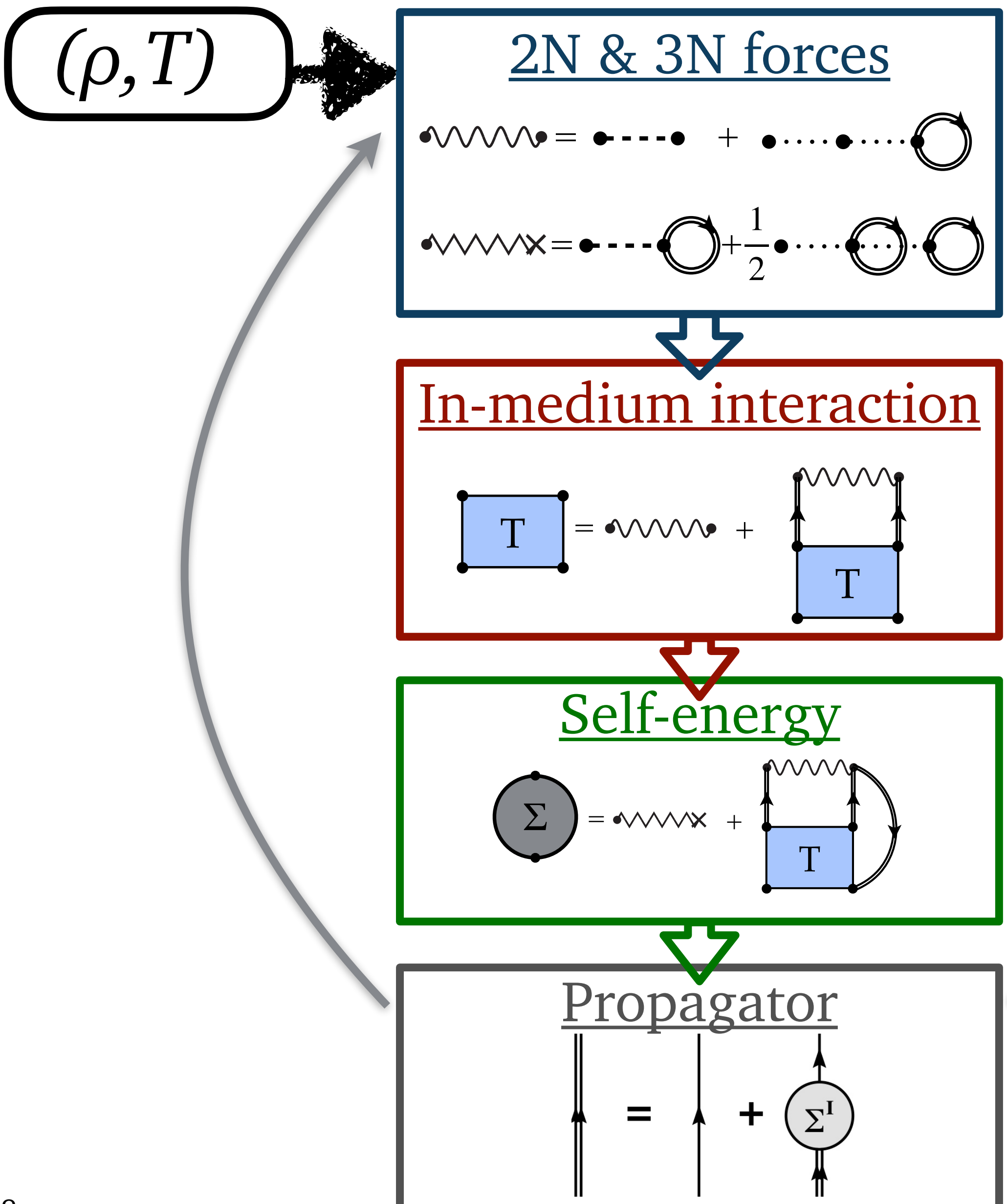
Self-Consistent Green's Functions

Forward



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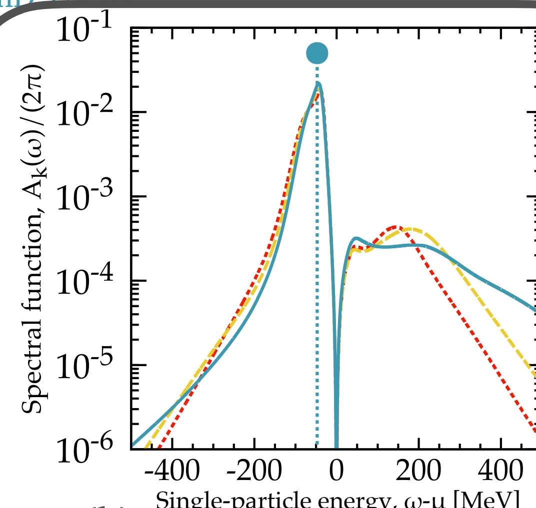

n(k)

Thermodynamics & EoS

Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
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Dewulf *et al.*, PRL **90** 152501 (2003)
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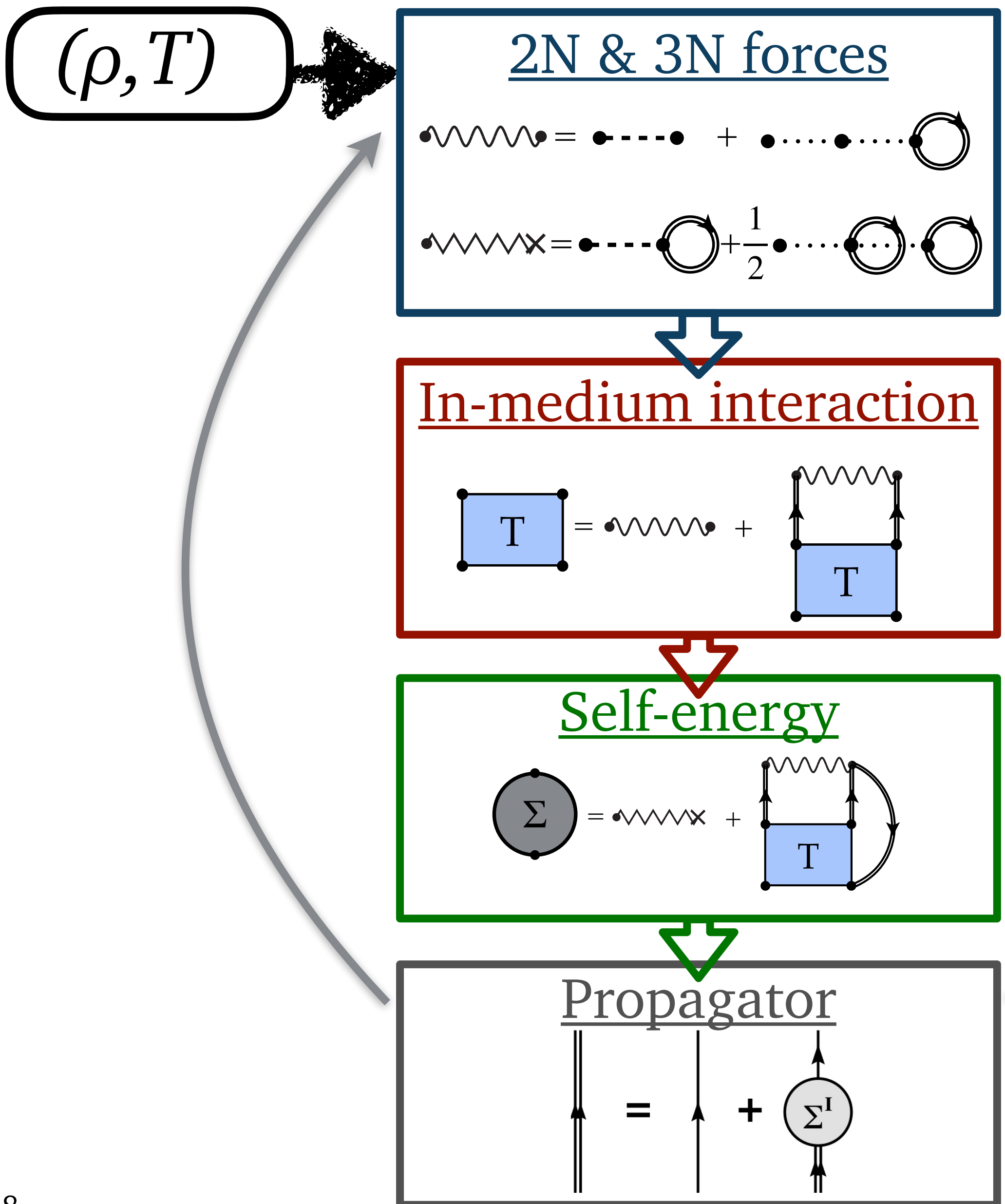
Self-Consistent Green's Functions

Forward



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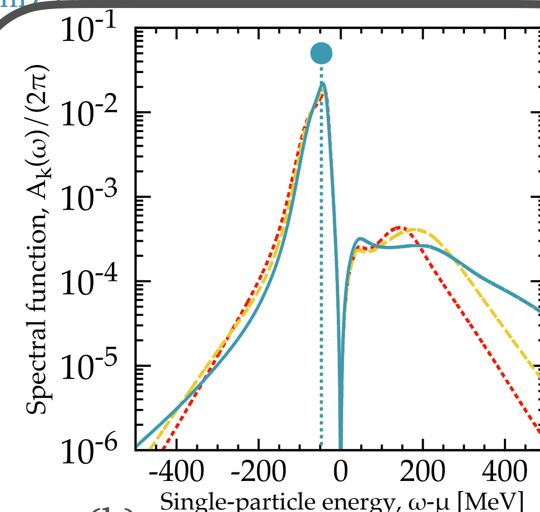

n(k)
Thermodynamics & EoS
Transport

- Off-shell ✔
- Matsubara formalism ✔
- Φ-derivable ✔

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Dewulf *et al.*, PRL **90** 152501 (2003)
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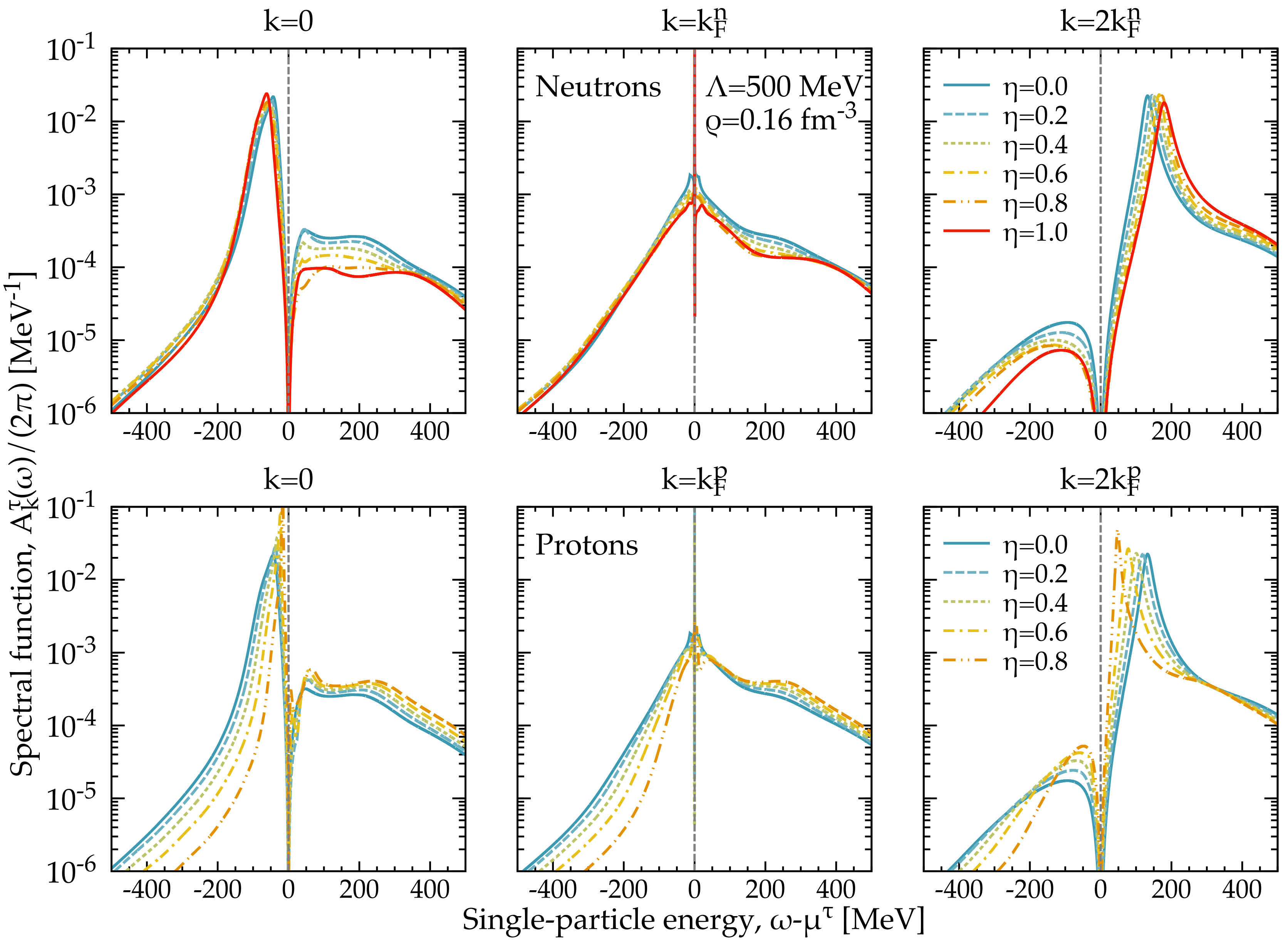
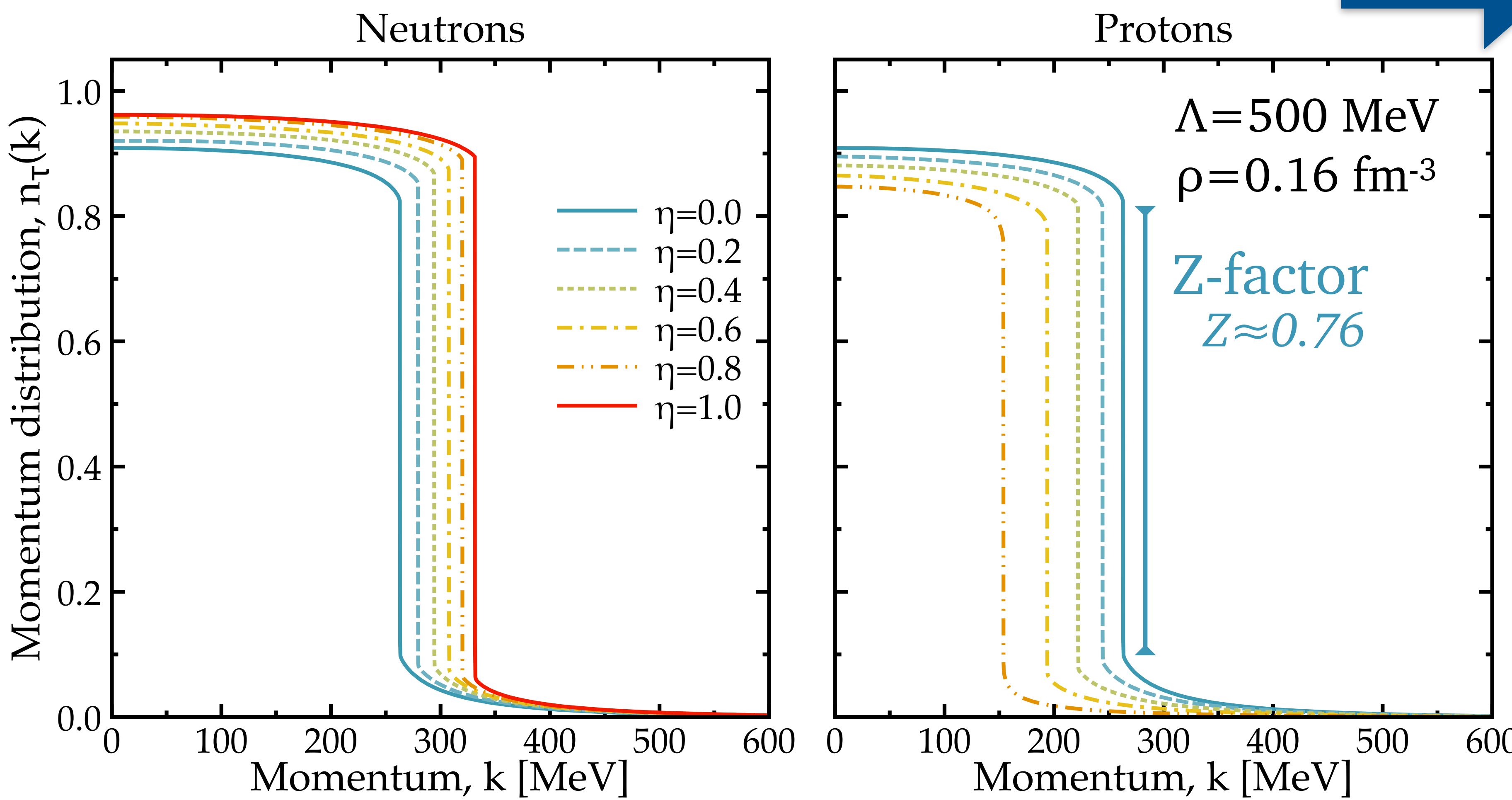
Asymmetry dependence in spectral functions



Momentum distribution

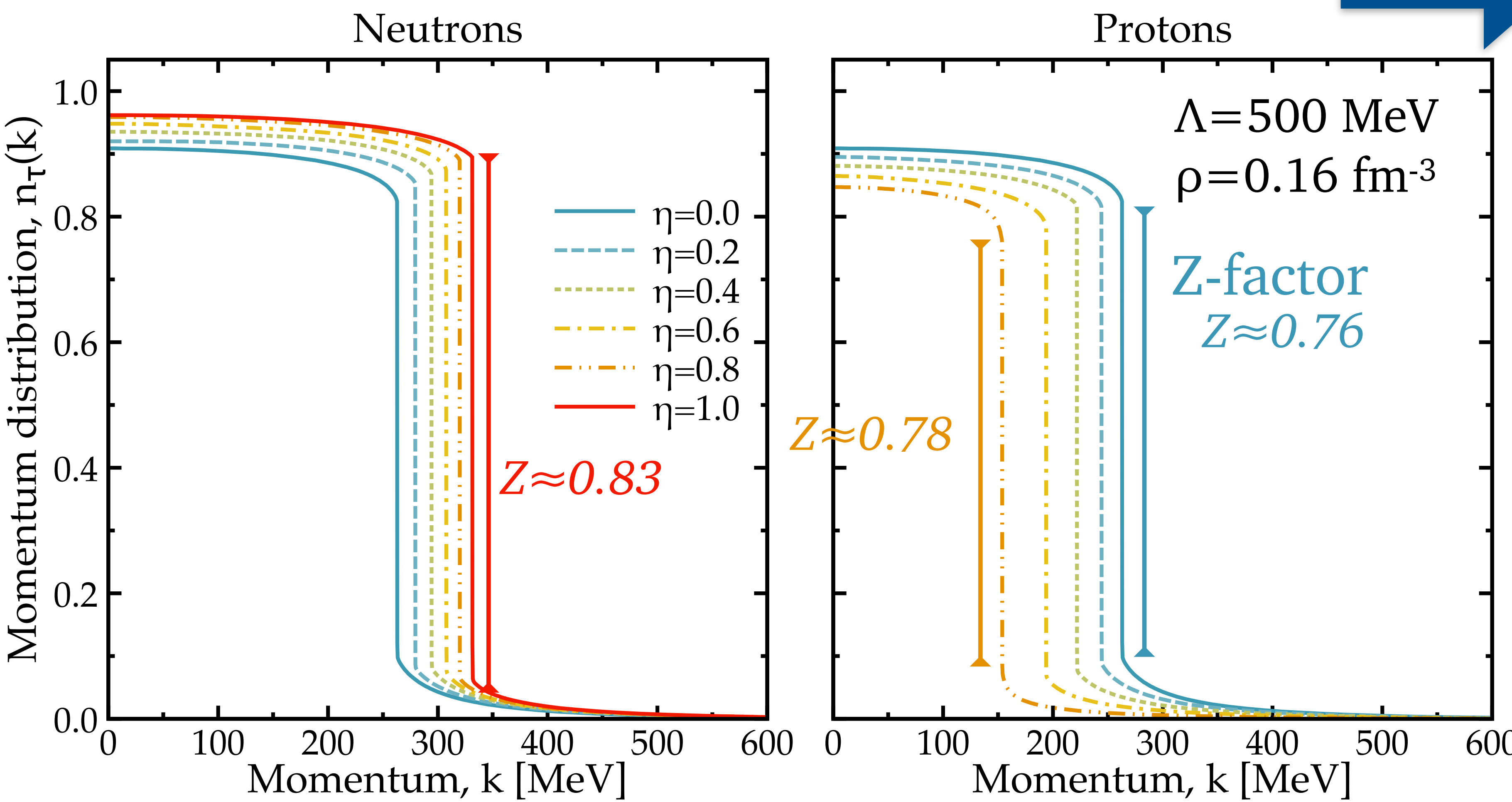
Forward



[Rios, Frontiers Physics fphy.2020.00387 (2020)](#)
[arXiv:2006.10610]

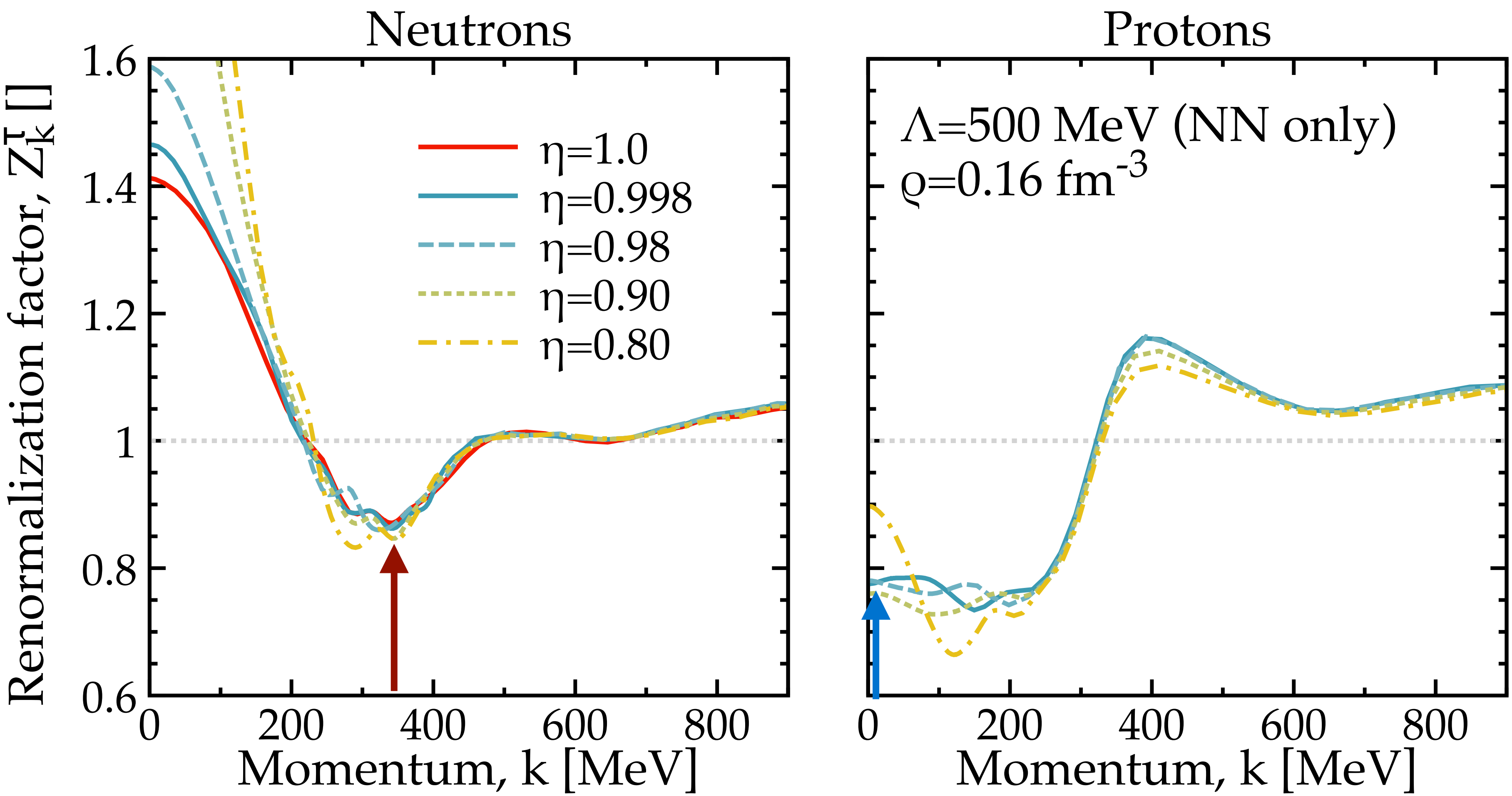
Momentum distribution

Forward



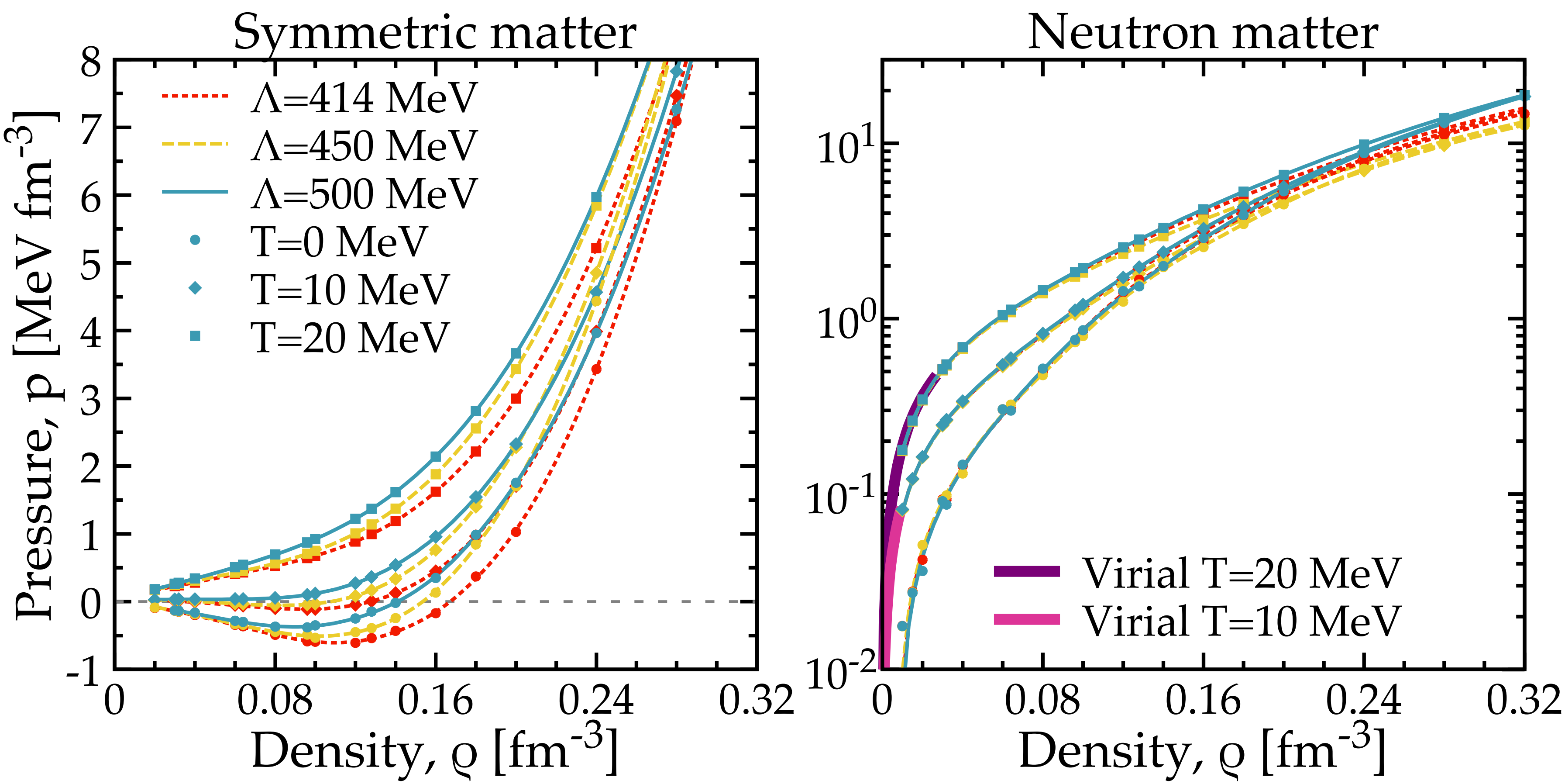
Rios, Frontiers Physics fphy.2020.00387 (2020) [arXiv:2006.10610]

Renormalization factor: towards impurities



Mulkerin et al. *Ann. Phys.* **407** 29 (2019)
Tajima et al. *New J. Phys.* **20** 073048 (2018)
Rios et al. *in preparation*

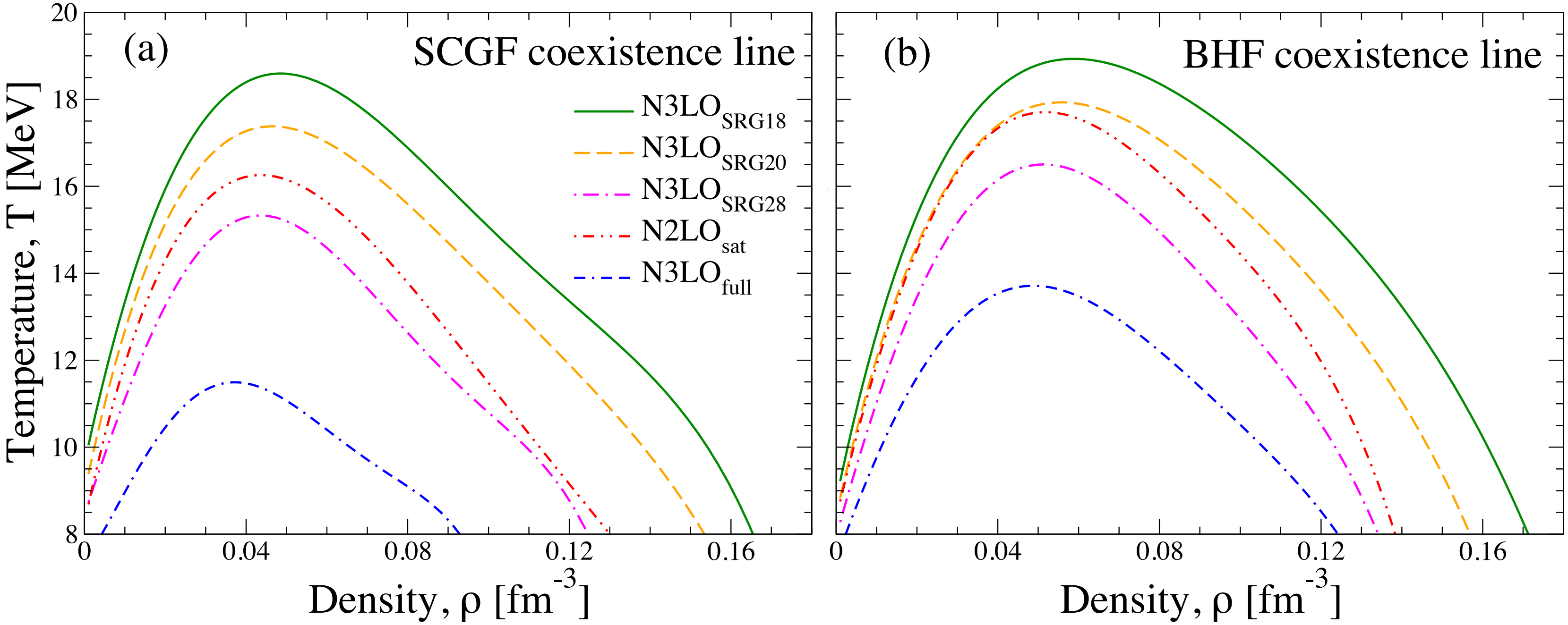
EoS at finite temperature

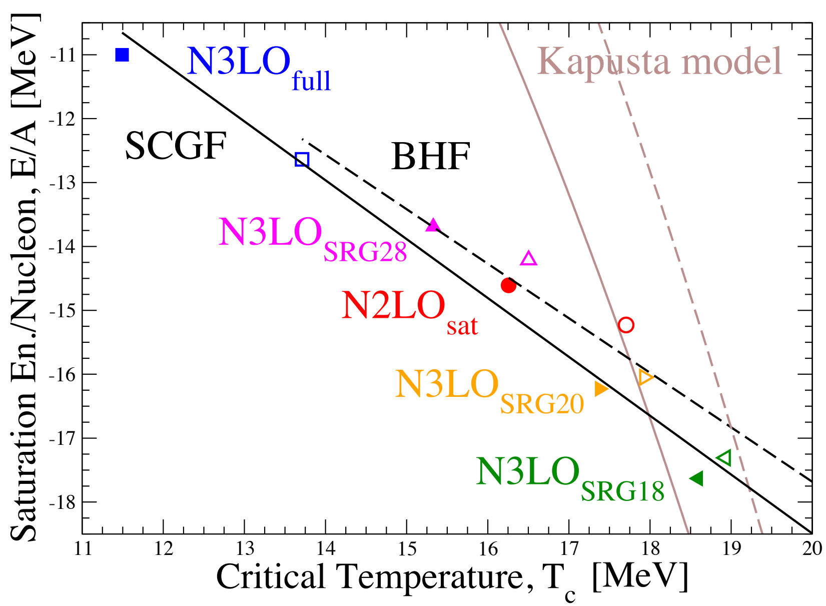


- Relevant & **necessary** for binary NS simulations

[Rios, Frontiers Physics fphy.2020.00387 (2020)]
[[arXiv:2006.10610]]

Liquid-gas phase transition

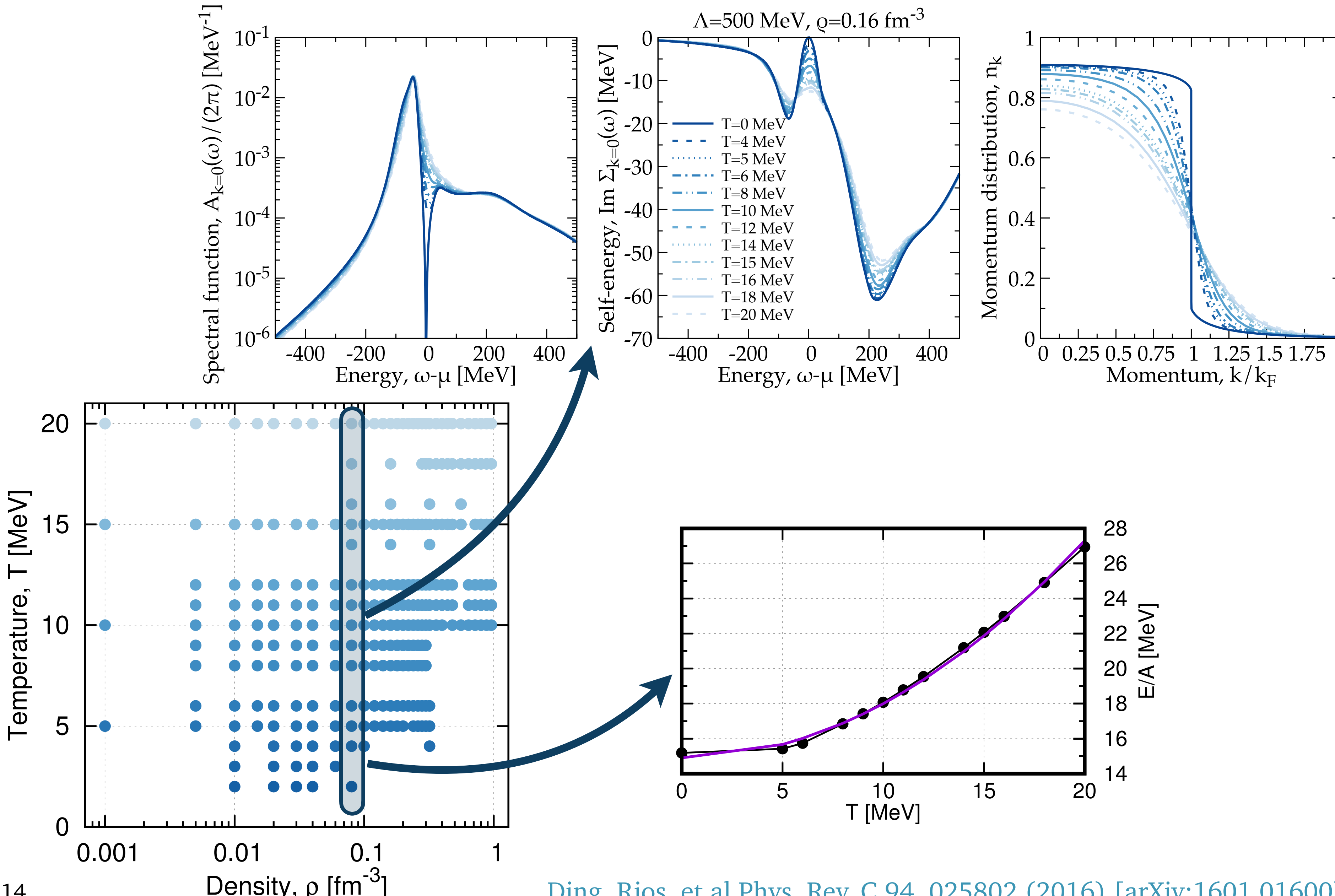




SCGF	ρ_c (fm^{-3})	T_c (MeV)	ρ_0 (fm^{-3})	$\frac{E_0}{A}$ (MeV)	$\frac{m_0^*}{m}$
N3LO$_{SRG18}$	0.048	18.6	0.19	−17.6	0.83
N3LO$_{SRG20}$	0.047	17.4	0.19	−16.2	0.84
N3LO$_{SRG28}$	0.043	15.3	0.16	−13.7	0.88
N2LO$_{sat}$	0.043	16.3	0.15	−14.6	0.90
N3LO$_{full}$	0.038	11.5	0.14	−11.0	0.84
BHF	ρ_c (fm^{-3})	T_c (MeV)	ρ_0 (fm^{-3})	$\frac{E_0}{A}$ (MeV)	$\frac{m_0^*}{m}$
N3LO$_{SRG18}$	0.058	18.9	0.19	−17.3	0.70
N3LO$_{SRG20}$	0.056	17.9	0.18	−16.0	0.73
N3LO$_{SRG28}$	0.051	16.5	0.16	−14.2	0.79
N2LO$_{sat}$	0.051	17.7	0.16	−15.2	0.82
N3LO$_{full}$	0.048	13.7	0.16	−12.6	0.76

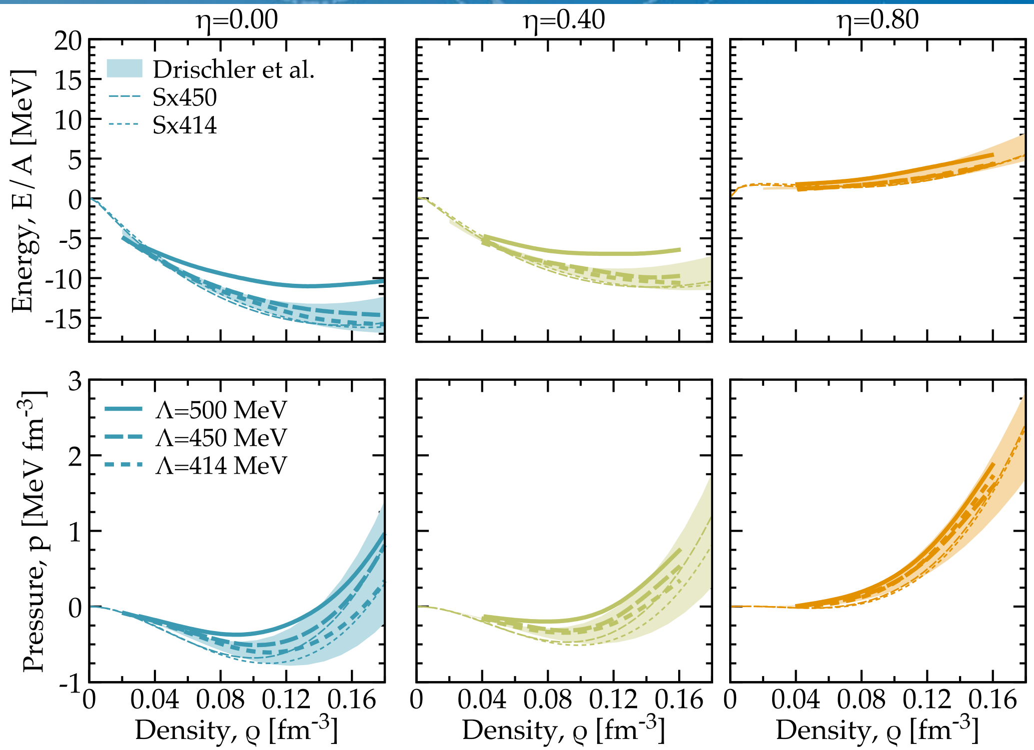
*Carbone, Polls, Rios, PRC **98** 025804 (2018)*
[arXiv:2006.10610]

Zero temperature extrapolation

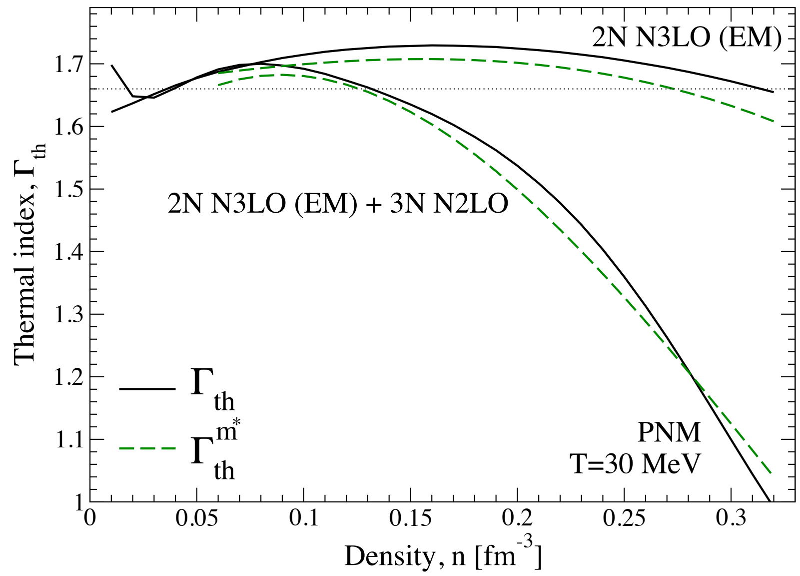


Ding, Rios, et al Phys. Rev. C 94, 025802 (2016) [arXiv:1601.01600]

Self-Consistent Green's Functions



[Rios, Frontiers Physics fphy.2020.00387 (2020) [arXiv:2006.10610]]



[Carbone & Schwenk, PRC **100** 025805 (2019) [arXiv:1904.00924]]

SCGF can treat:

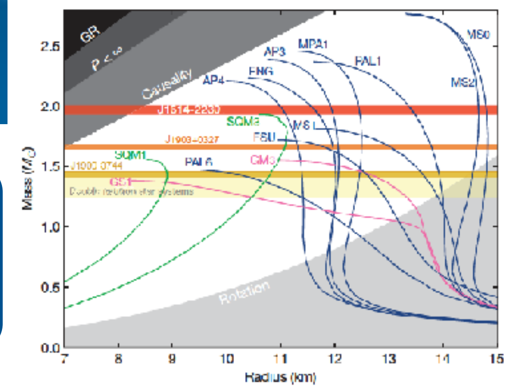
- **Explicitly asymmetric matter ✔**
- Finite temperature ✔
- Systematic expansion ✔
- 3 nucleon forces ✔

but:

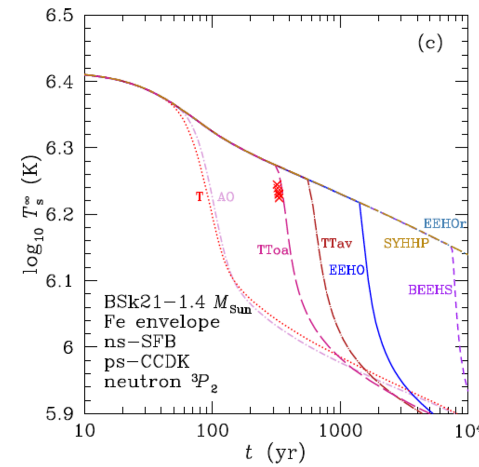
- Numerically intensive
- Pairing?

Neutron star modelling

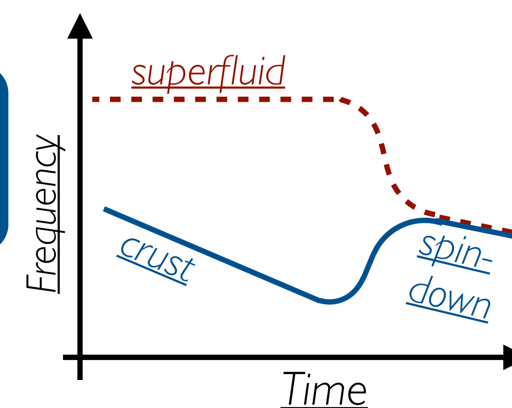
Input #1
EoS
→
Observable #1
Mass-Radius relation



Input #2
Pairing gap
→
Observable #2
Cooling curve



Input #3
Crust-core
→
Observable #3
Glitching



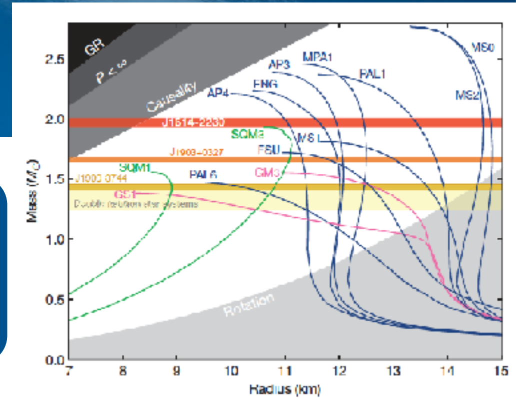
Neutron star modelling

Input

Consistent many-body theory

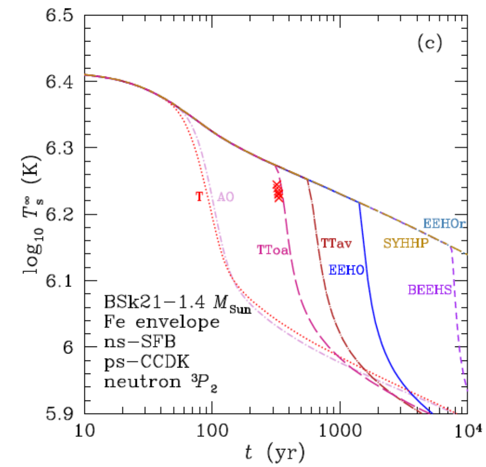
Observable #1

Mass-Radius relation



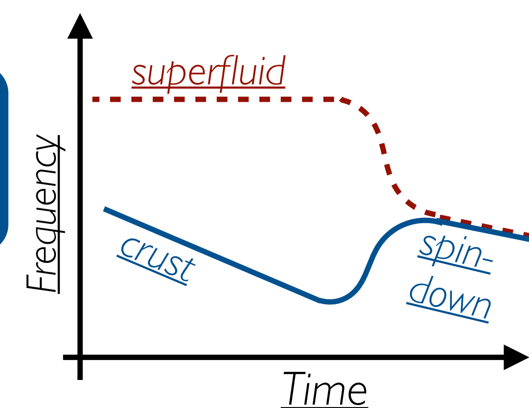
Observable #2

Cooling curve



Observable #3

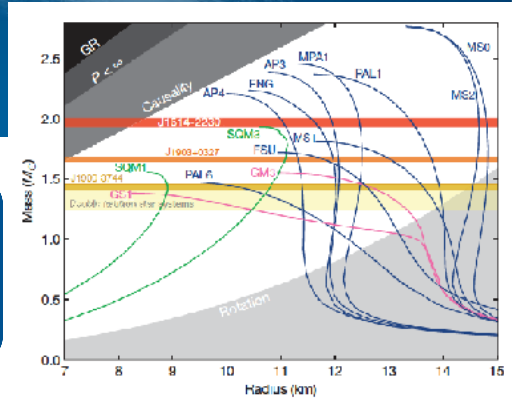
Glitching



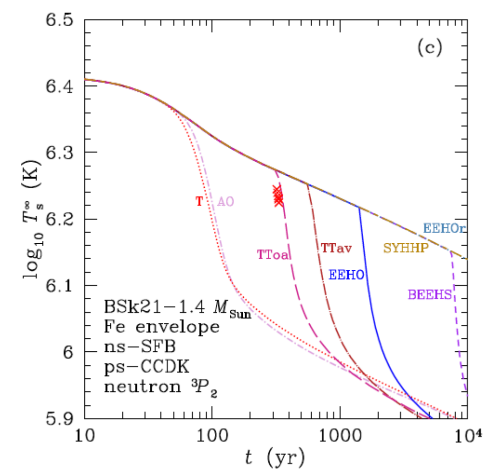
Neutron star modelling

Input
Consistent many-body theory

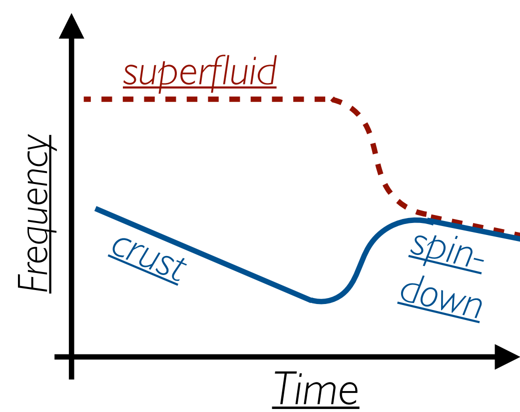
Observable #1
Mass-Radius relation
“Normal”



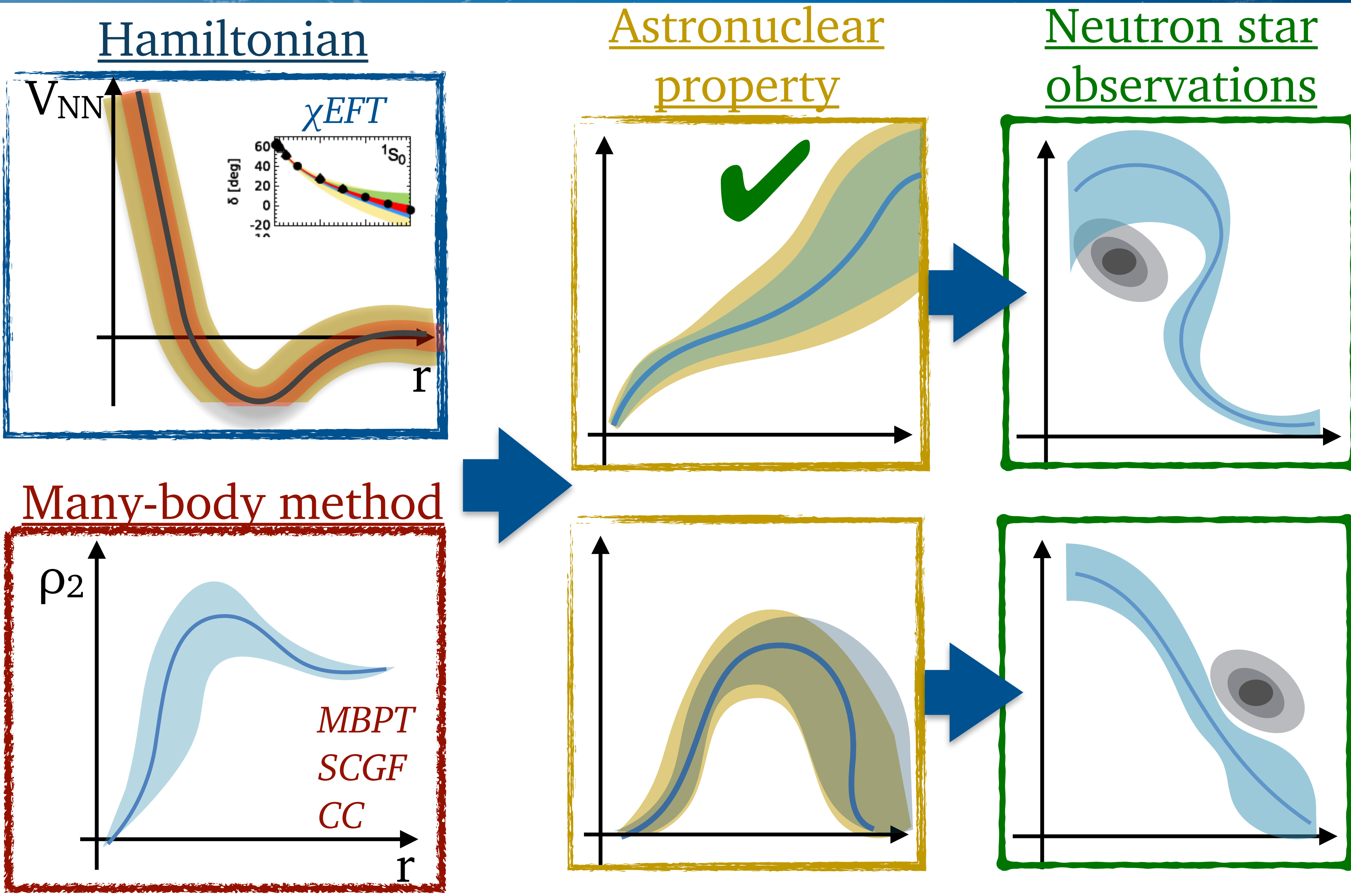
Observable #2
Cooling curve
“Superfluid”



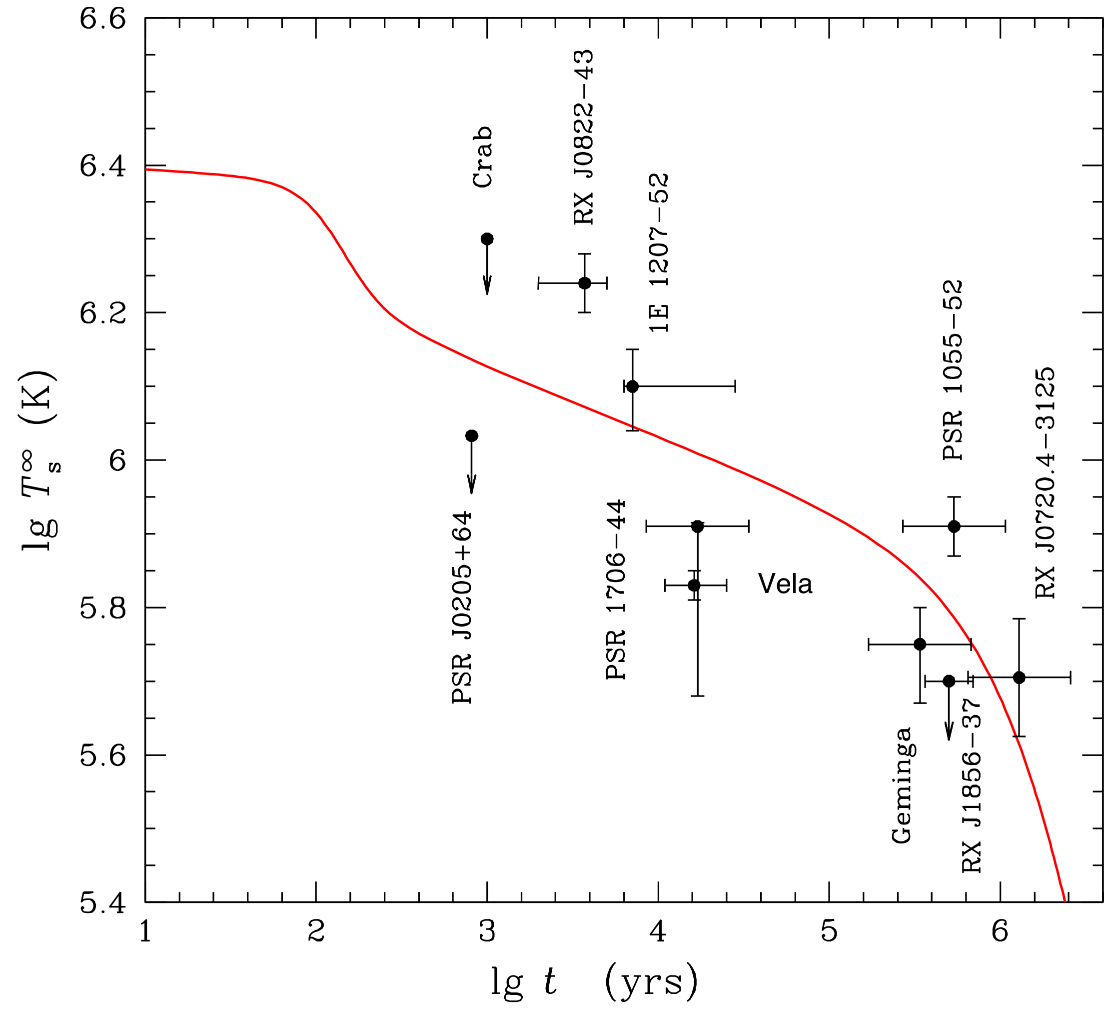
Observable #3
Glitching
“Superfluid”



Nuclear error quantification (2022)



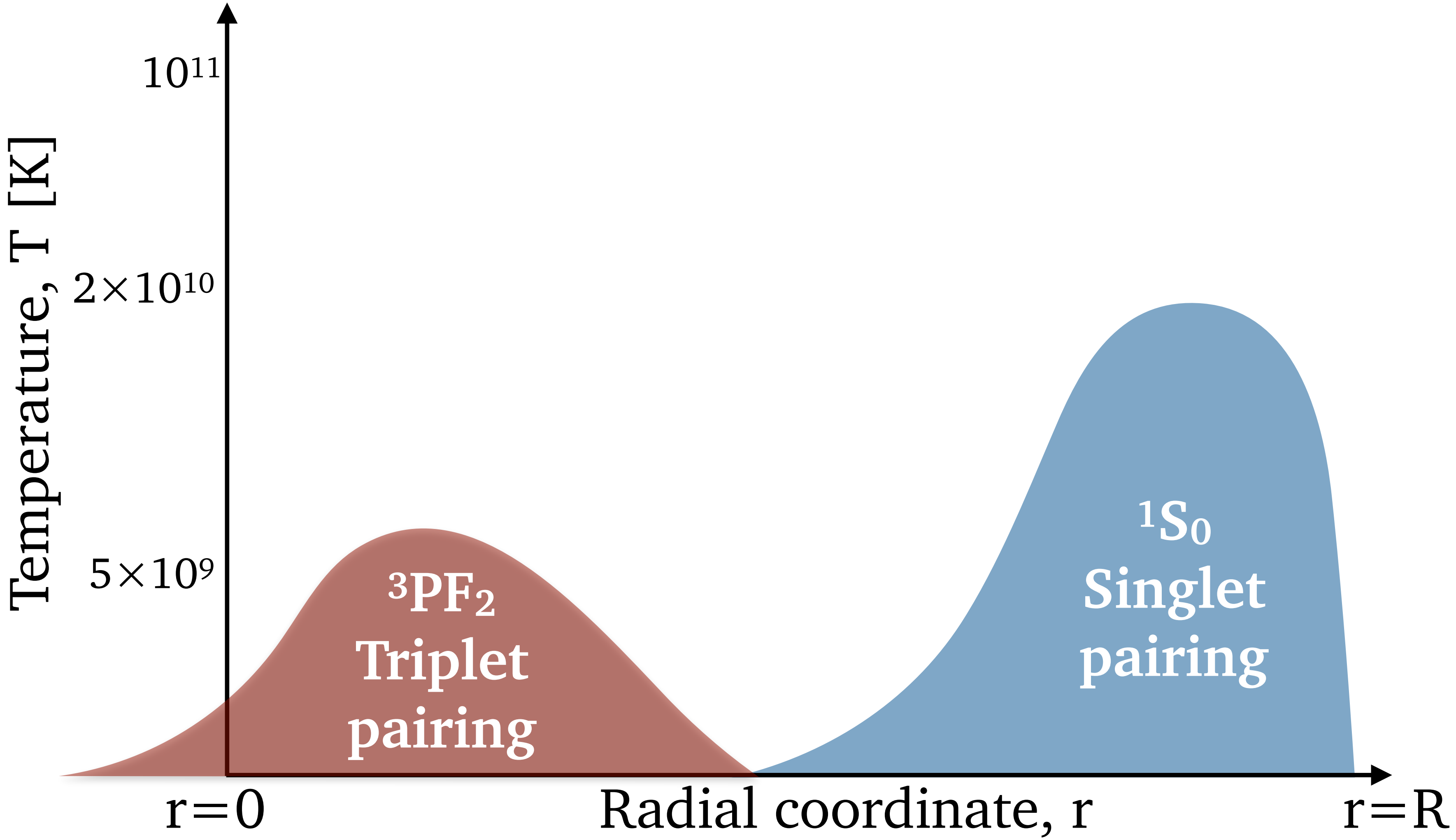
Cooling curve of neutron stars



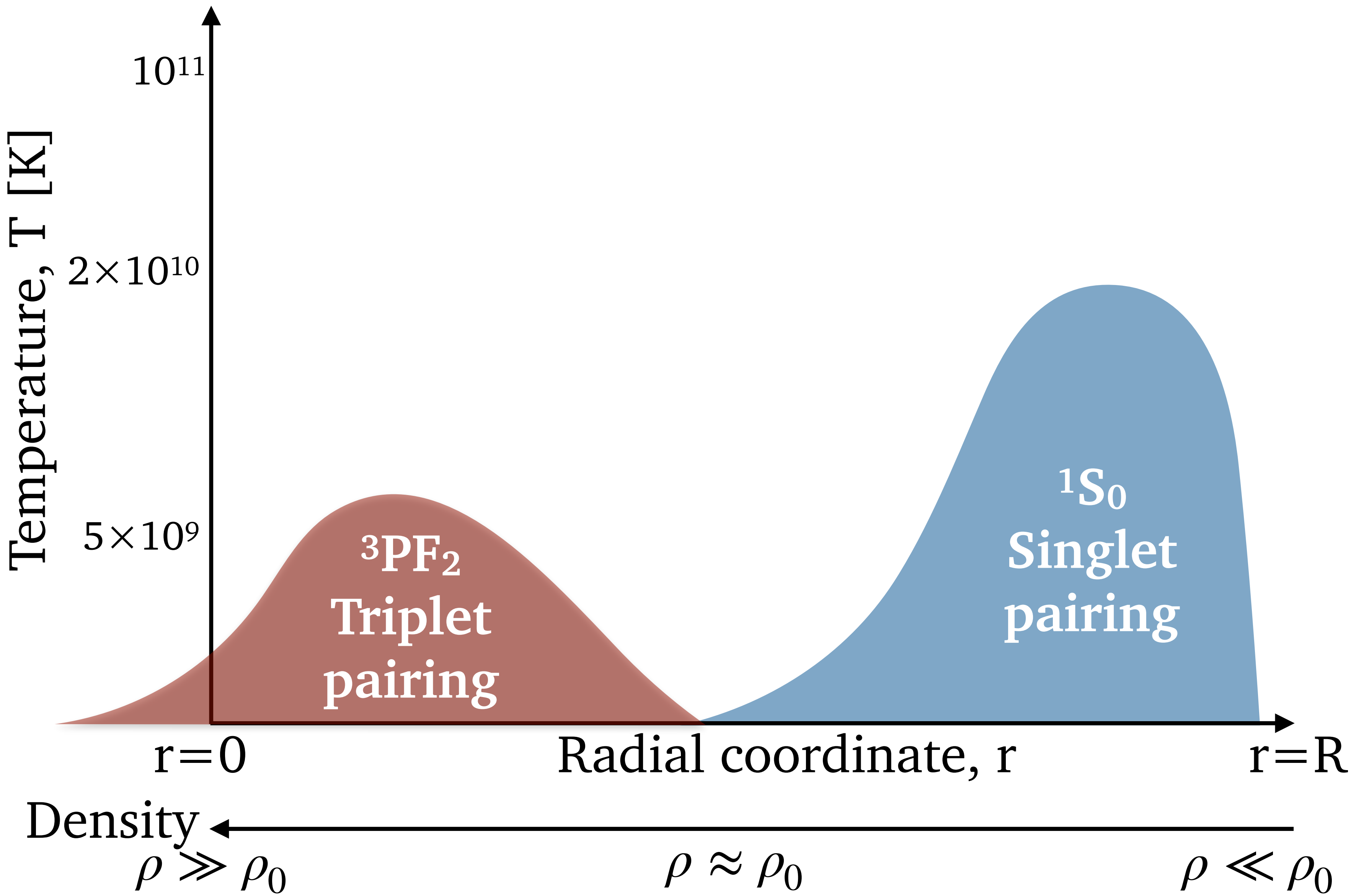
- Observational data available for a handful of NS
- Sensitive to **interior** physics (mostly **pairing**)

Yakovlev & Pethick, ARAA **42** 169 (2004)

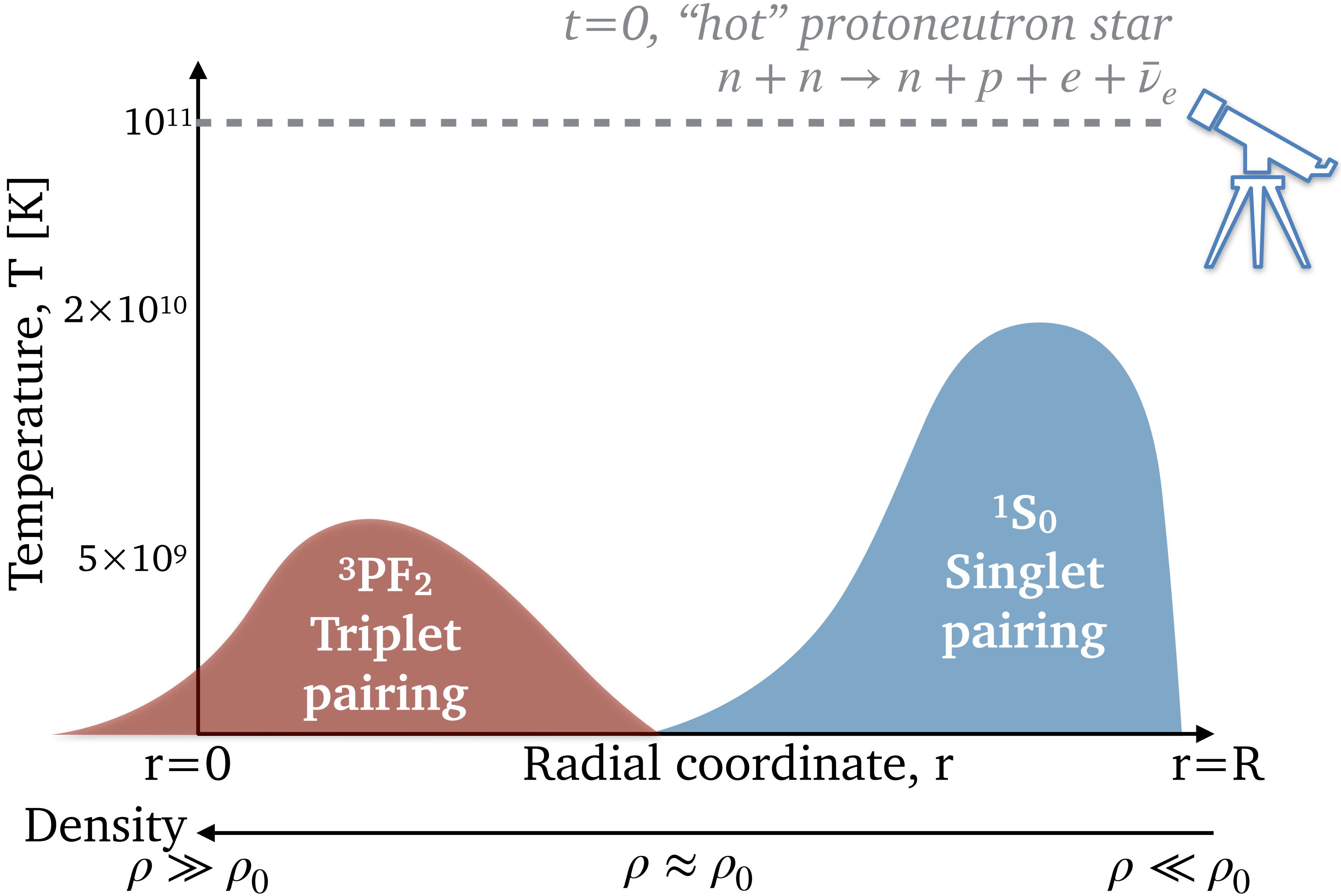
Pairing gaps & cooling



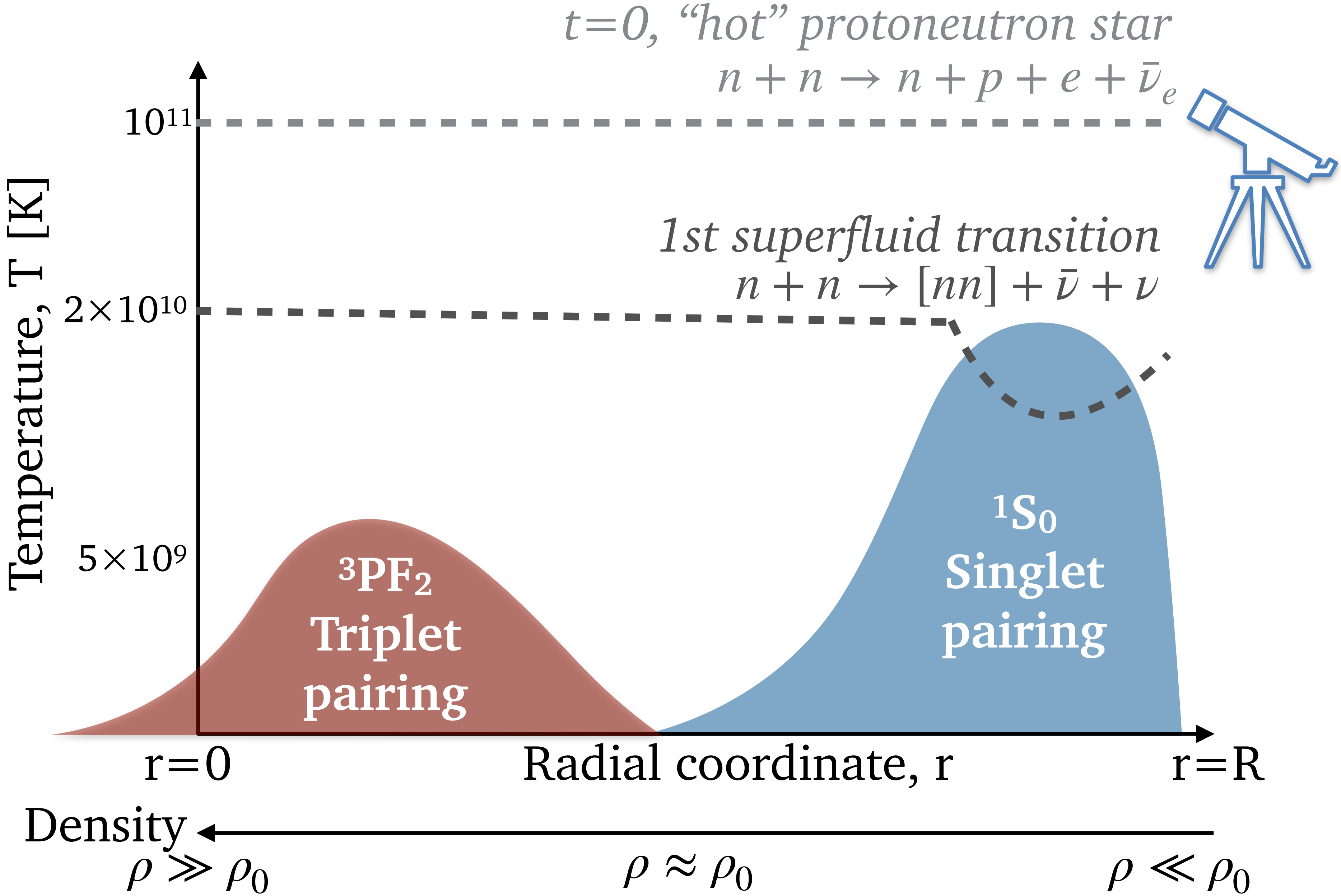
Pairing gaps & cooling



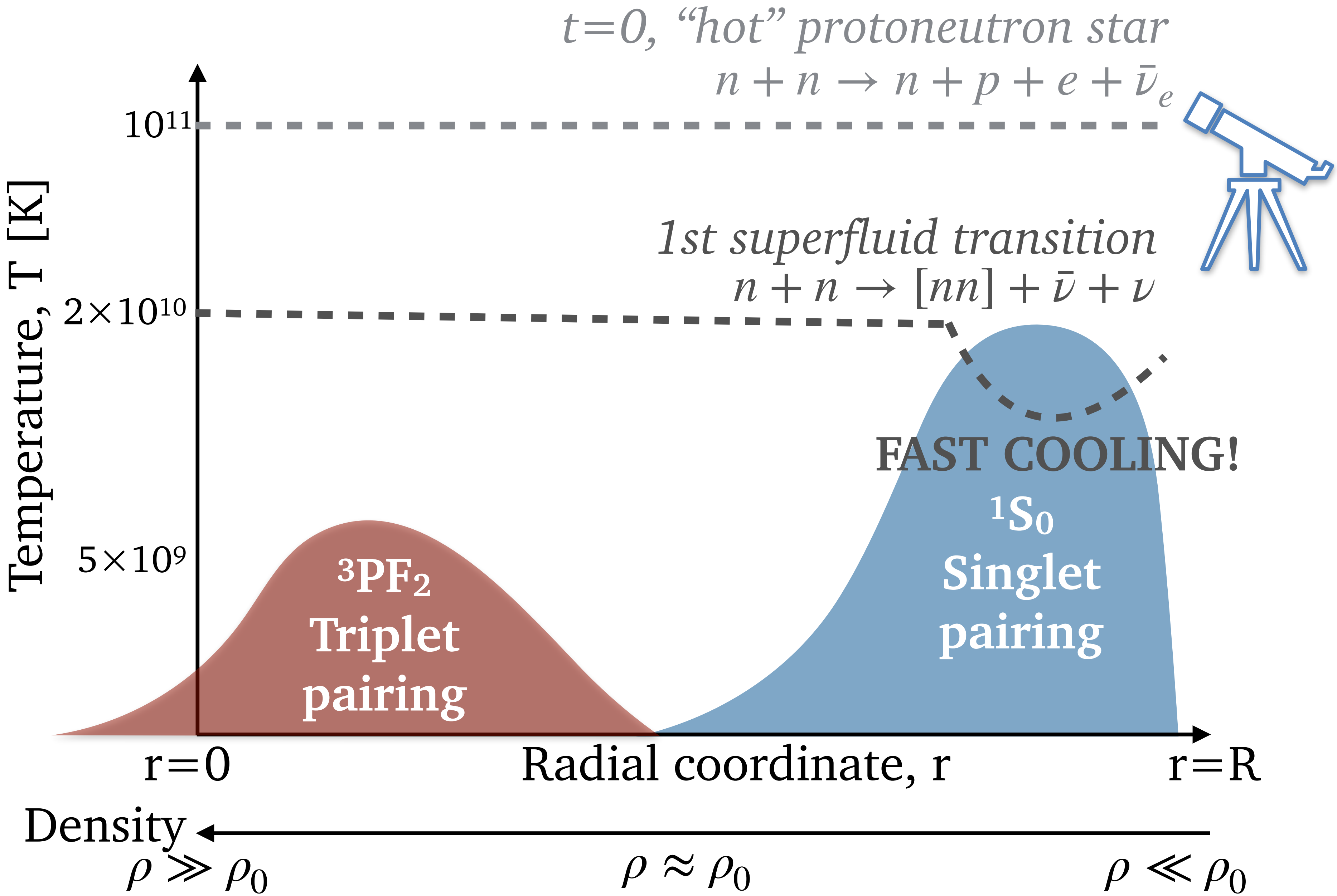
Pairing gaps & cooling



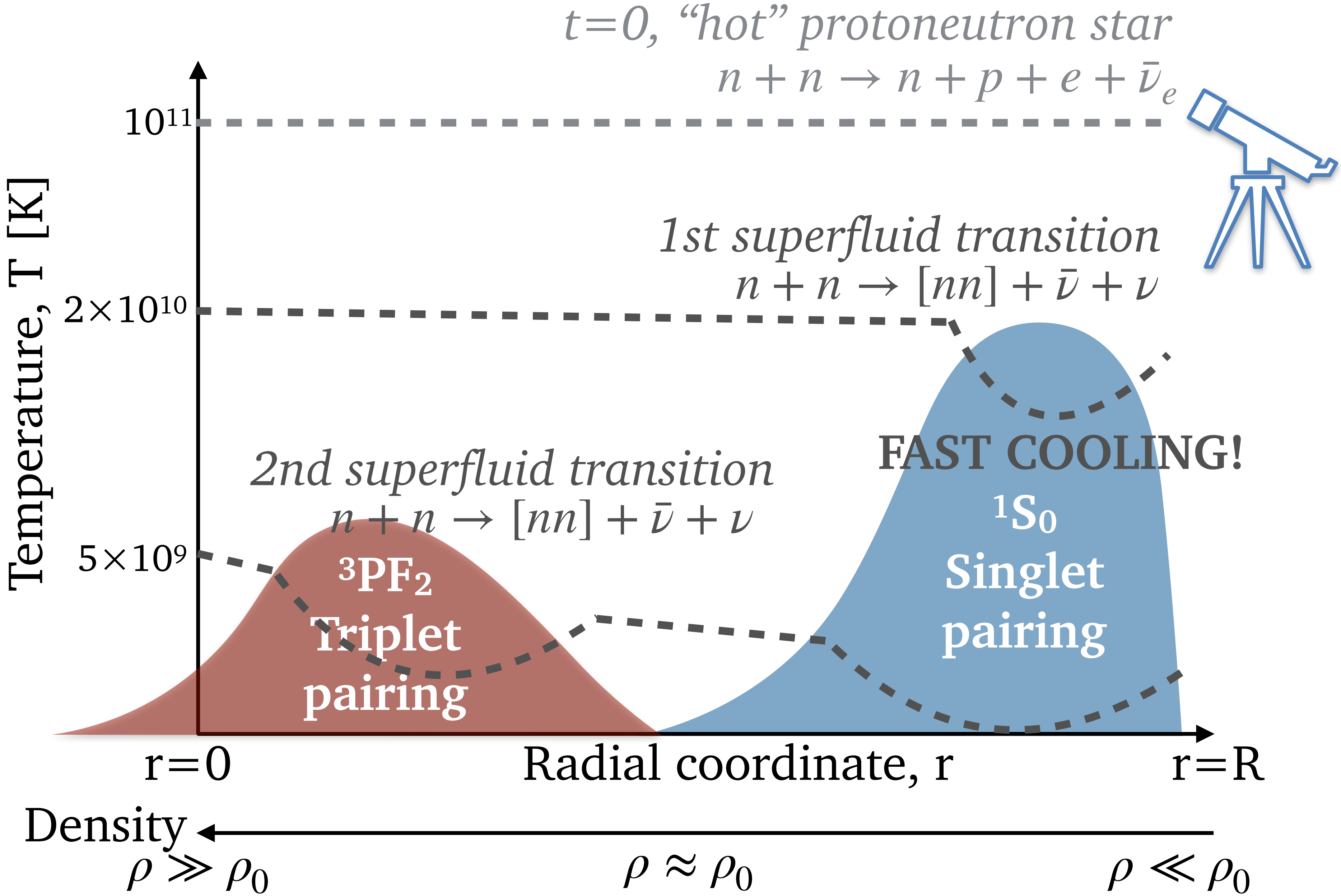
Pairing gaps & cooling



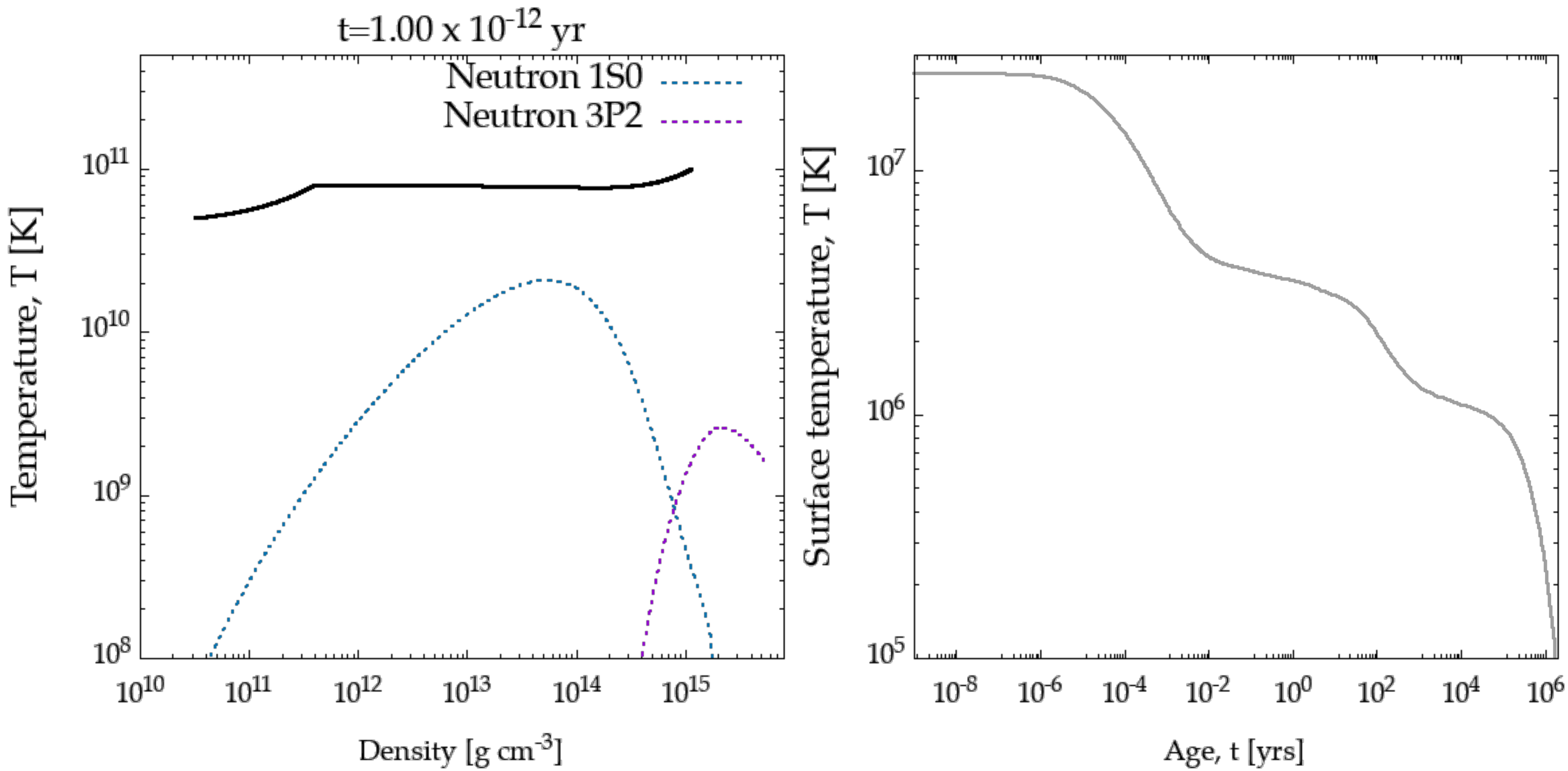
Pairing gaps & cooling



Pairing gaps & cooling

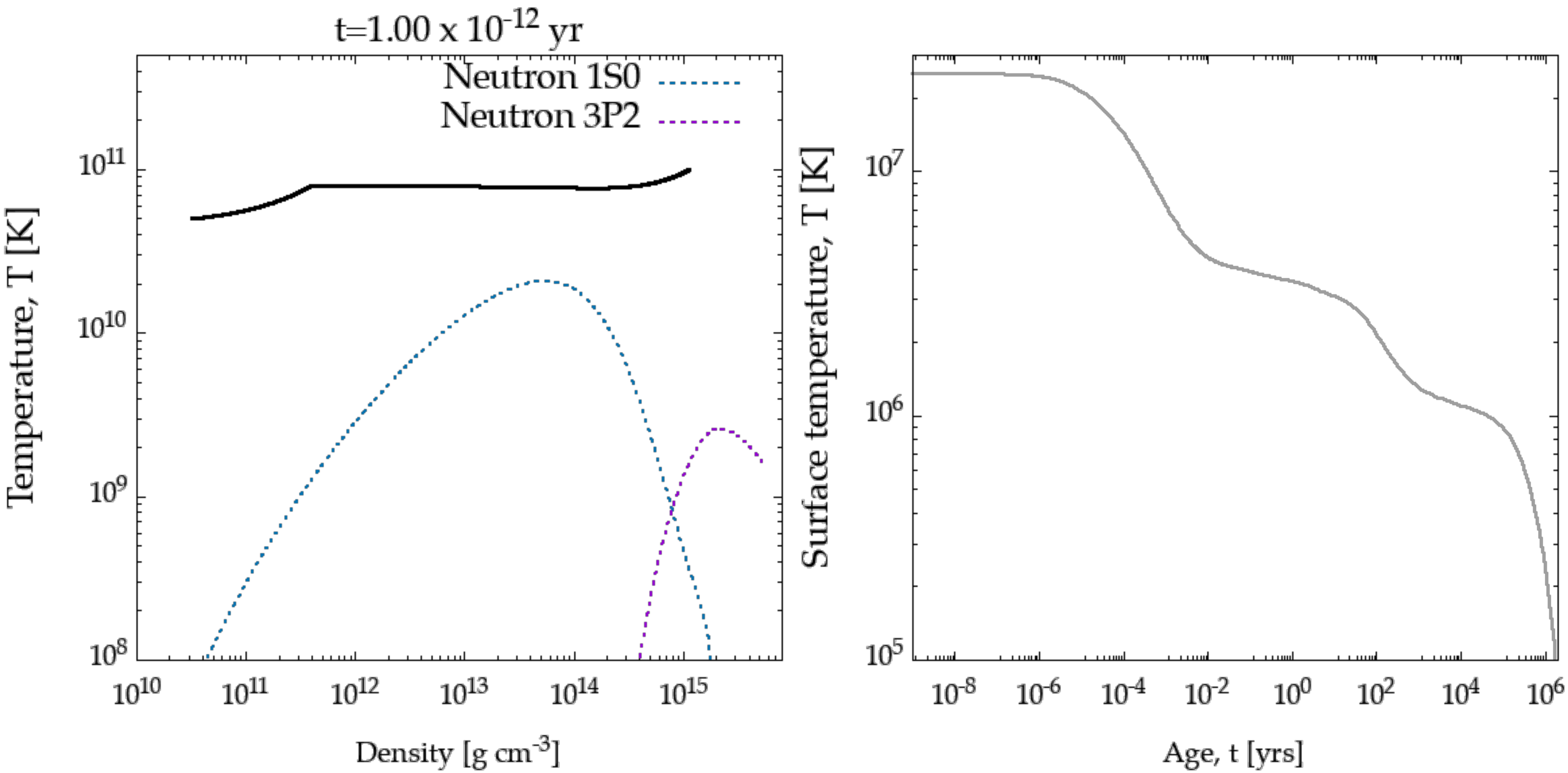


Cooling example



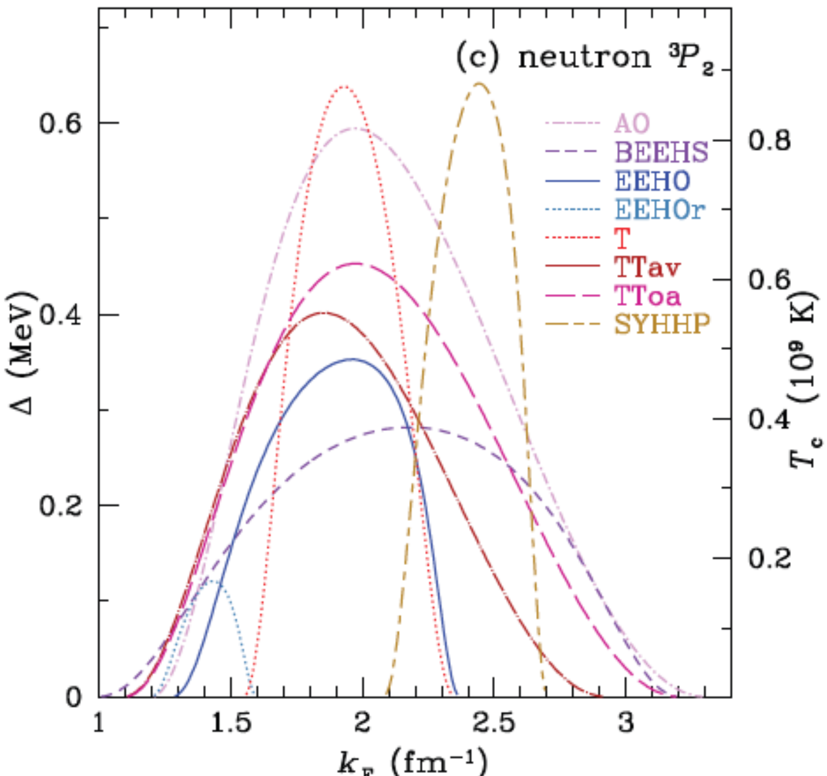
Sellahewa, PhD Thesis (2015)

Cooling example



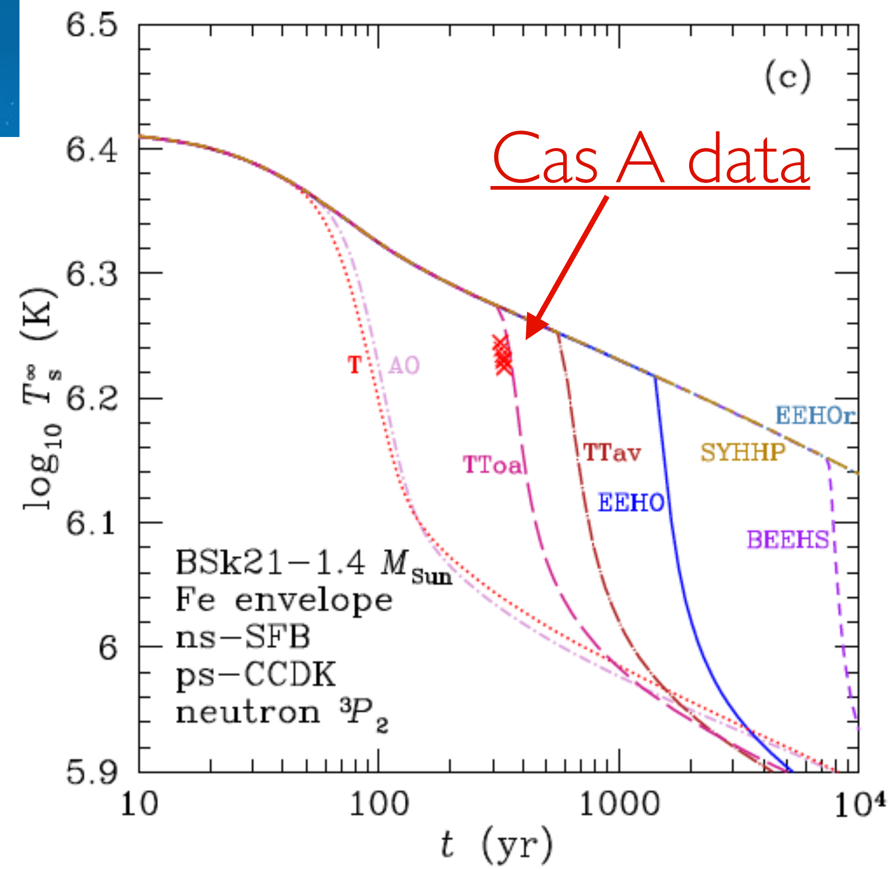
Sellahewa, PhD Thesis (2015)

Cooling of CasA



Ho, *et al.*, PRC **91** 015806 (2015)

Page, *et al.*, *PRL* **106** 081101 (2011)



Ingredients

(a) Mass of pulsar

(b) EoS (determines radius)

(c) Internal composition

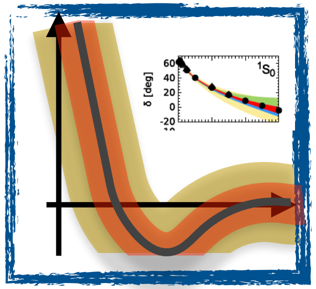
(d) **Pairing gaps** ($^1S_0$ & $^3PF_2$ channels)

(e) Atmosphere composition

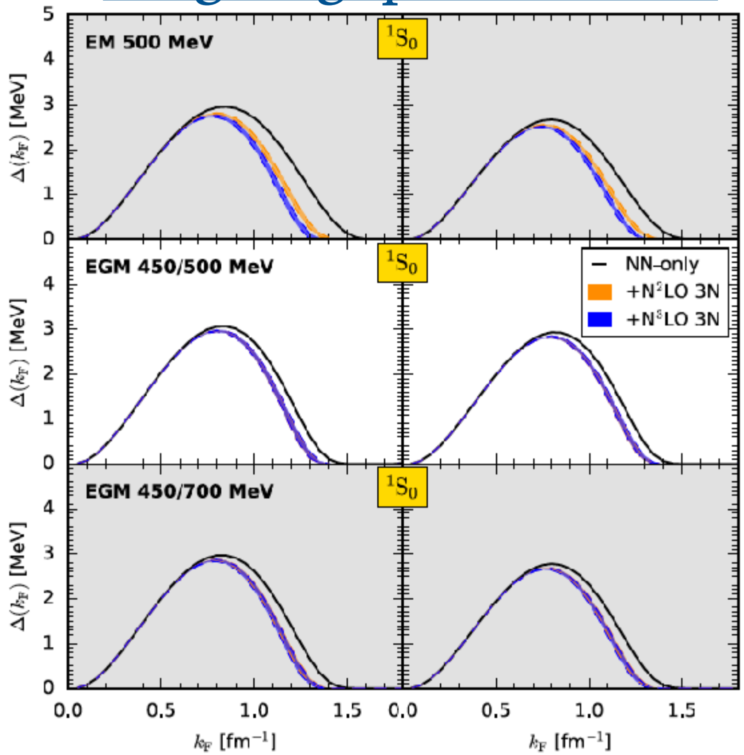
Name	Process	Emissivity (erg cm^{-3} s^{-1})
Modified Urca (neutron branch)	$n+n \to n+p+e^- + \bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2\times10^{21} R T_9^8$
Modified Urca (proton branch)	$p+n \to p+p+e^- + \bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21} R T_9^8$
Bremsstrahlungs	$n+n \to n+n+\nu+\bar{\nu}$ $n+p \to n+p+\nu+\bar{\nu}$ $p+p \to p+p+\nu+\bar{\nu}$	$\sim 10^{19} R T_9^8$
Cooper pair	$n+n \to [nn]+\nu+\bar{\nu}$ $p+p \to [pp]+\nu+\bar{\nu}$	$\sim 5\times10^{21} R T_9^7$ $\sim 5\times10^{19} R T_9^7$
Direct Urca (nucleons)	$n \to p+e^- + \bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27} R T_9^6$

BCS+HF gaps in neutron matter

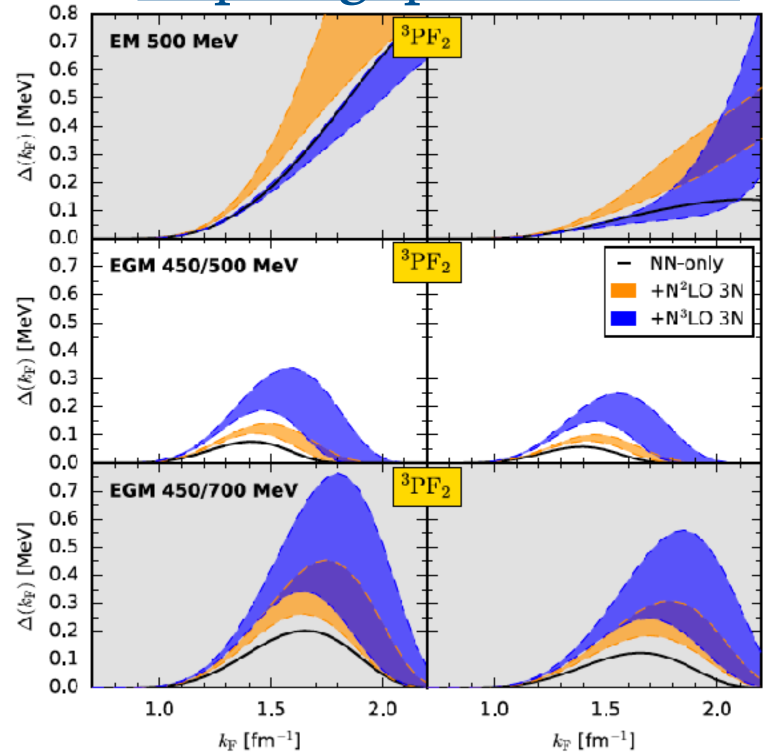
Hamiltonian



Singlet gaps with 3NF



Triplet gaps with 3NF



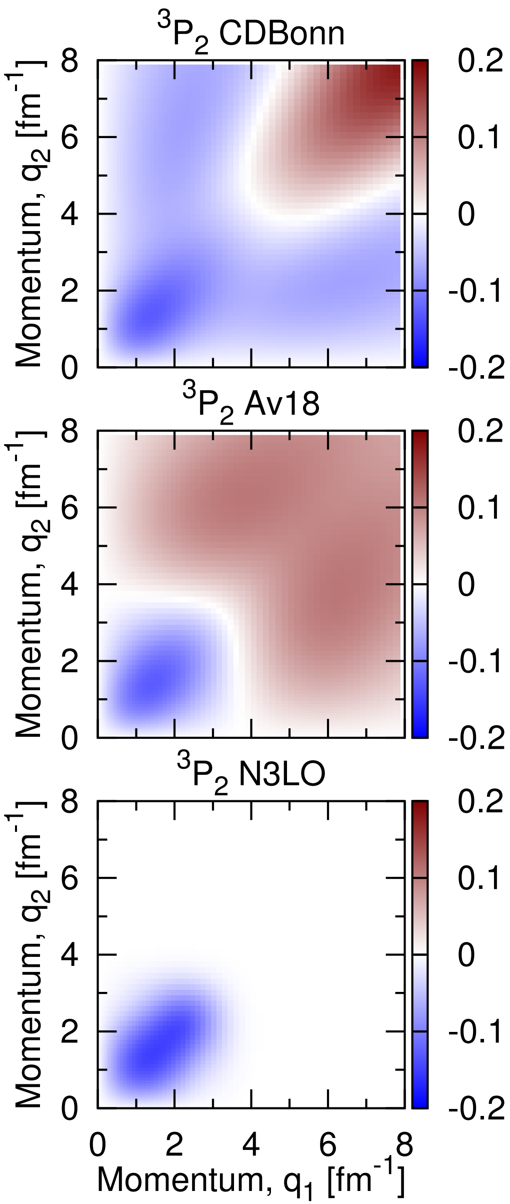
Drischler, Kruger, Hebeler, Schwenk, *Phys Rev C* **95** 024302 (2017) [arXiv:1610.05213]

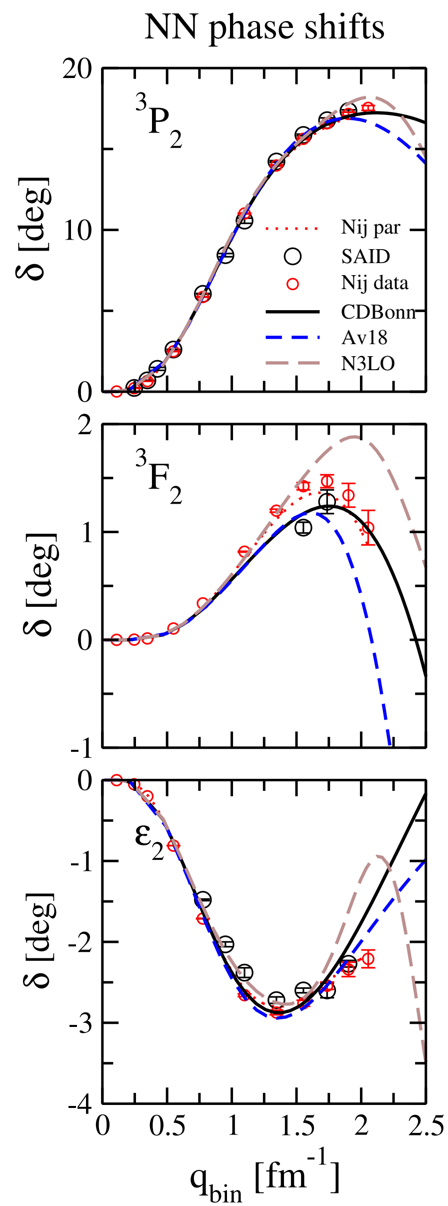
BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu, \quad \varepsilon_k = \frac{k^2}{2m} + U(k) - \mu$$

- **Error estimates** from nuclear force (chiral expansion) ✓
- **Many-body uncertainty?** ✗

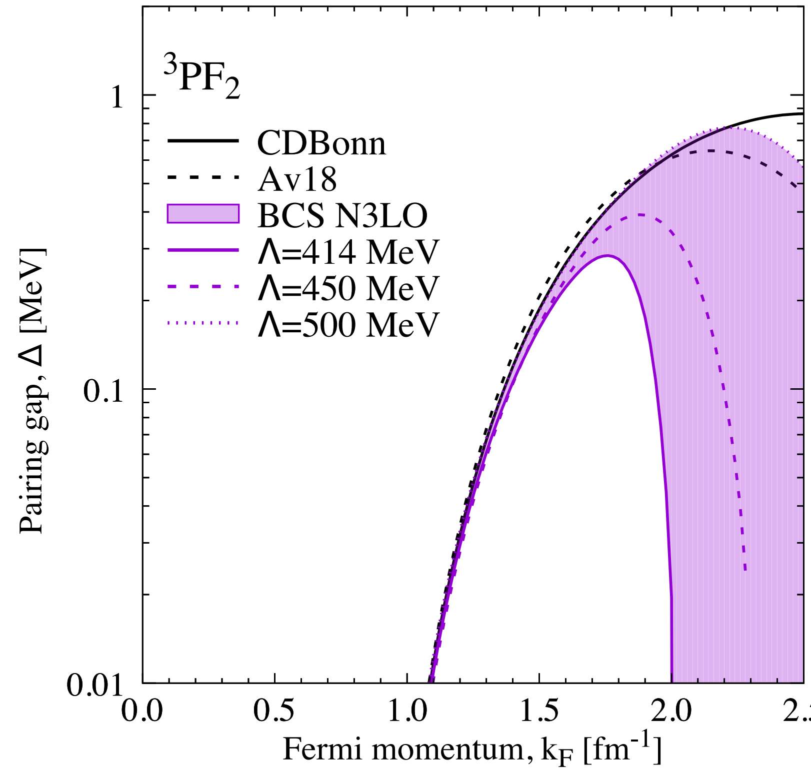
Triplet channel: limits of EFT



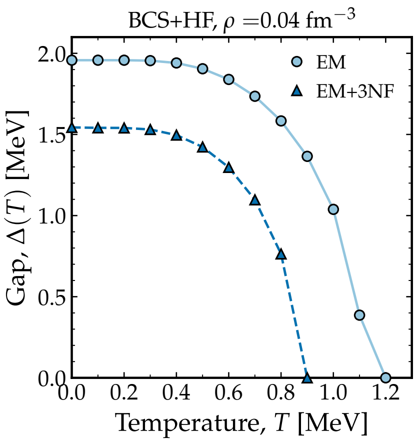


BCS equation

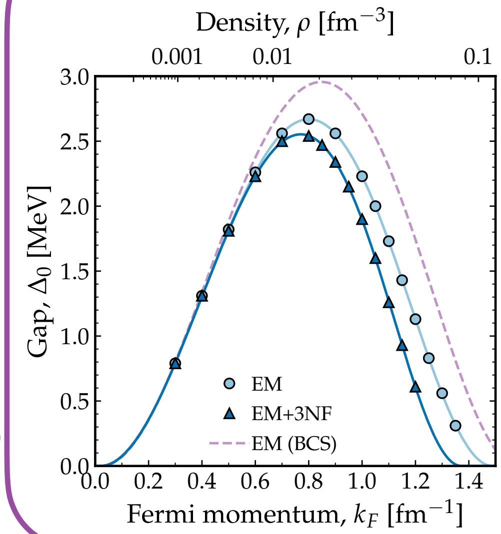
$$\boxed{\Delta^L_k = -\sum_{L'} \int_{k'} \frac{\langle k | V^{LL'}_{nn} | k' \rangle}{2\sqrt{\chi^2_{k'} + |\Delta_{k'}|^2}} \Delta^{L'}_{k'}}$$



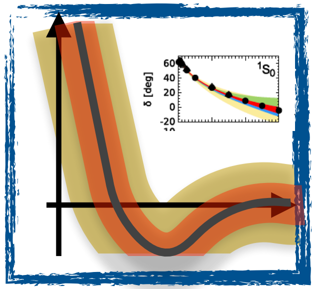
Finite Temperature BCS



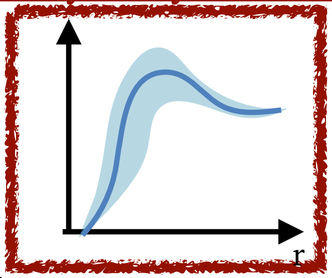
BCS+HF



Hamiltonian

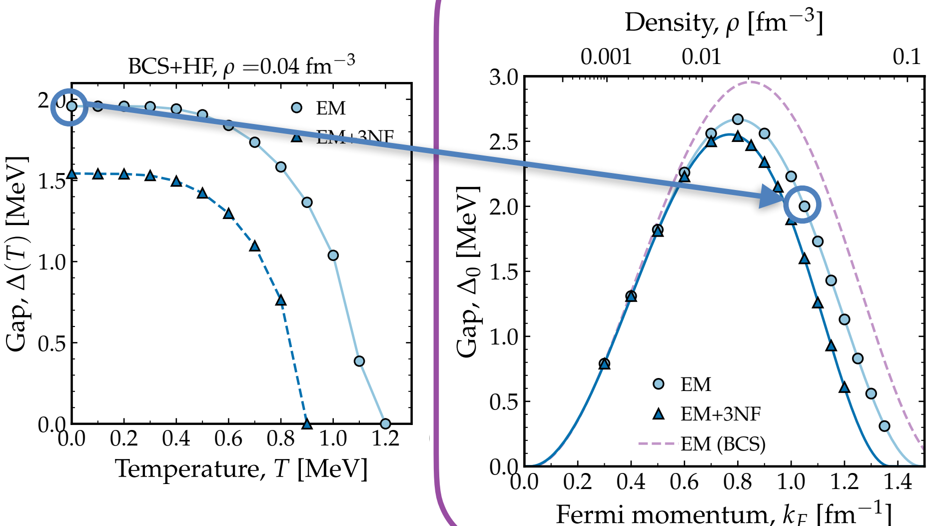


Many-body method

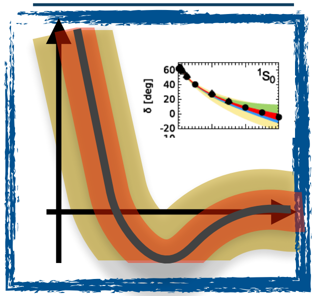


Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

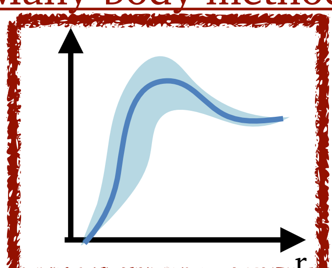
BCS+HF



Hamiltonian

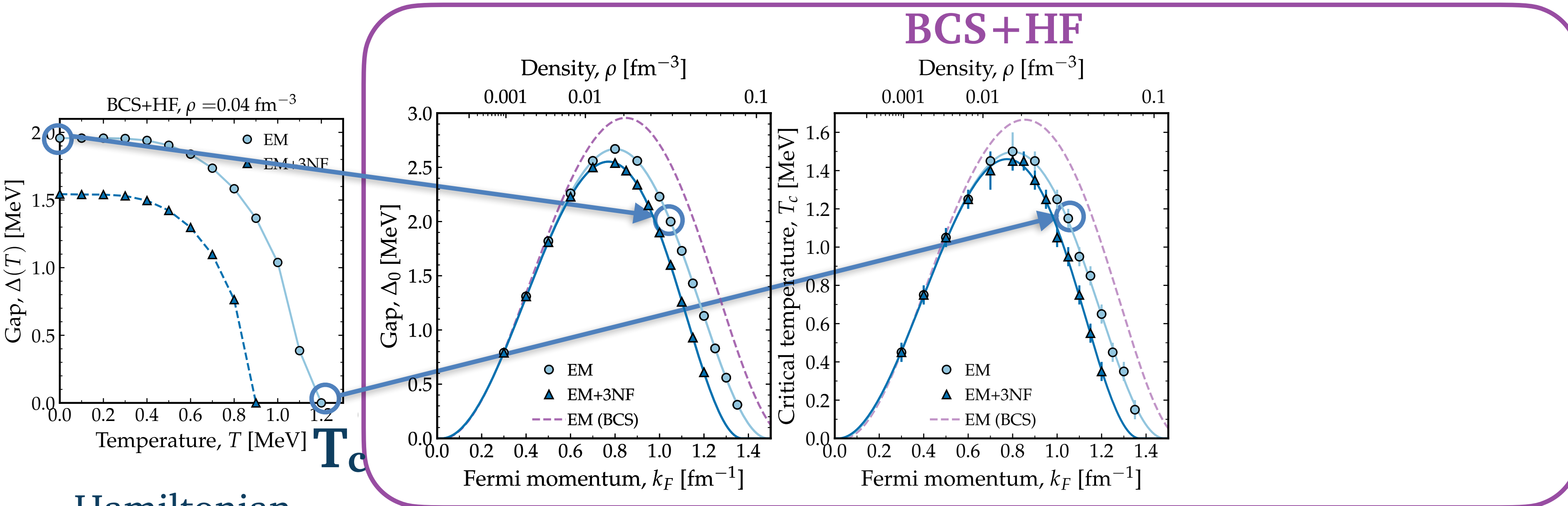


Many-body method

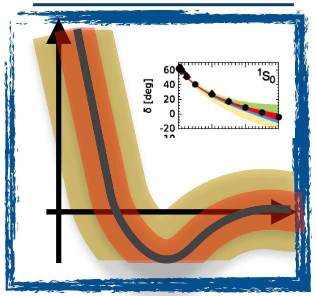


Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

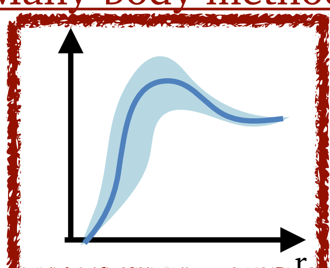
Finite Temperature BCS



Hamiltonian

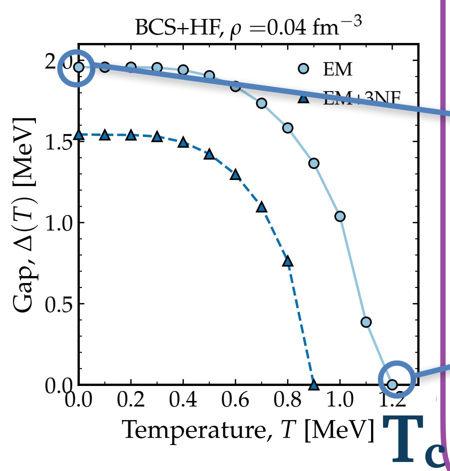


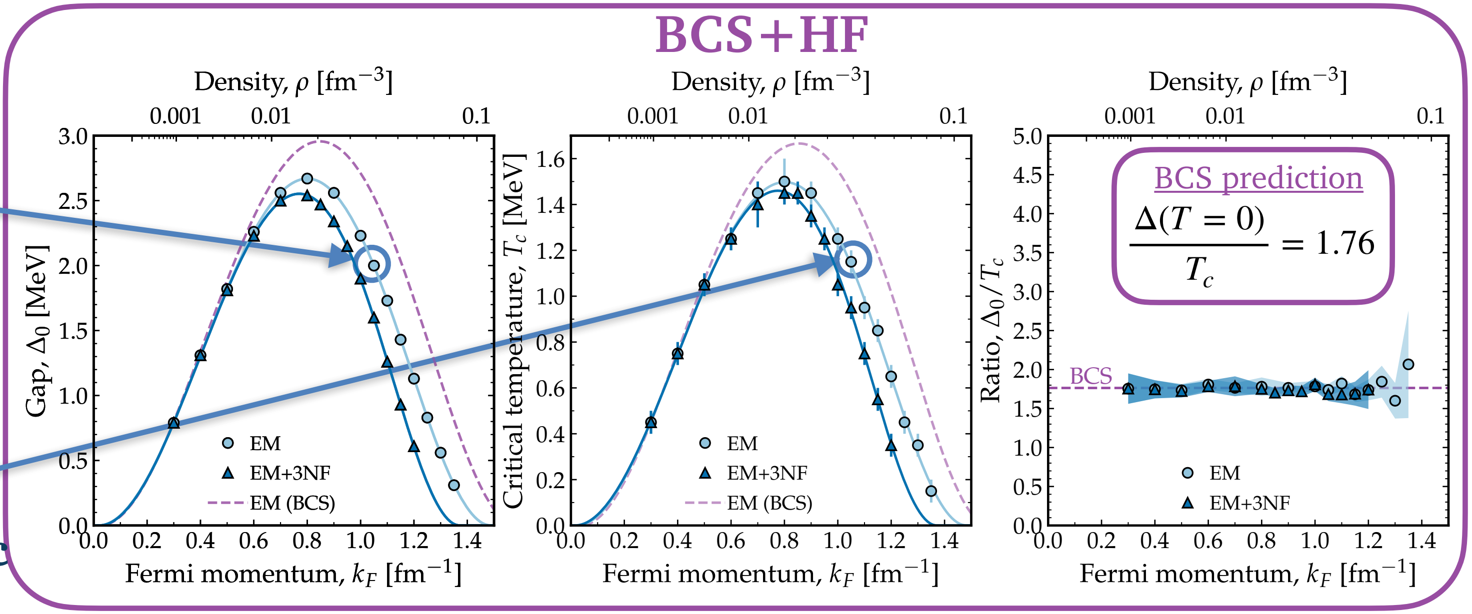
Many-body method



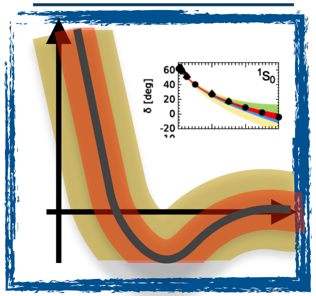
Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

Finite Temperature BCS

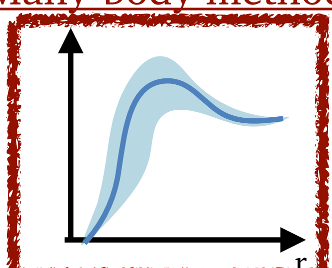




Hamiltonian

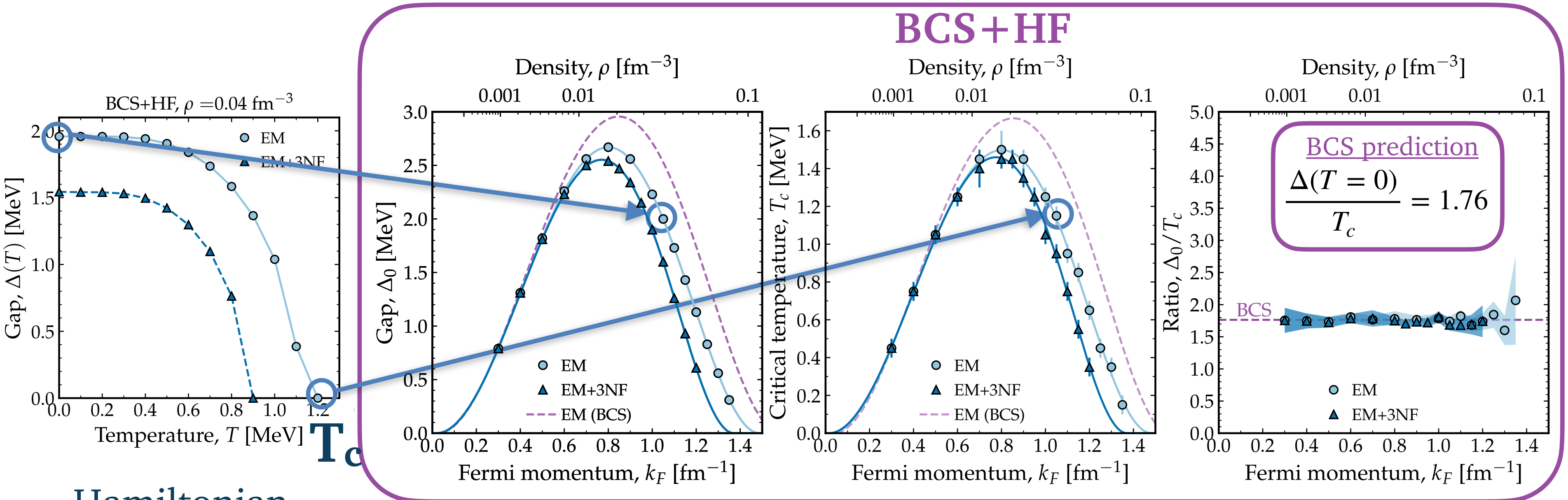


Many-body method

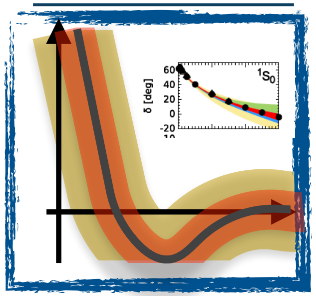


Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

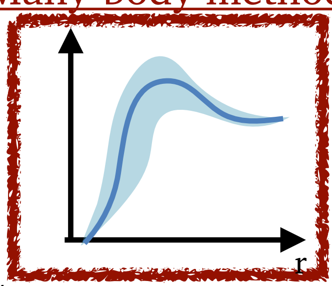
Finite Temperature BCS



Hamiltonian



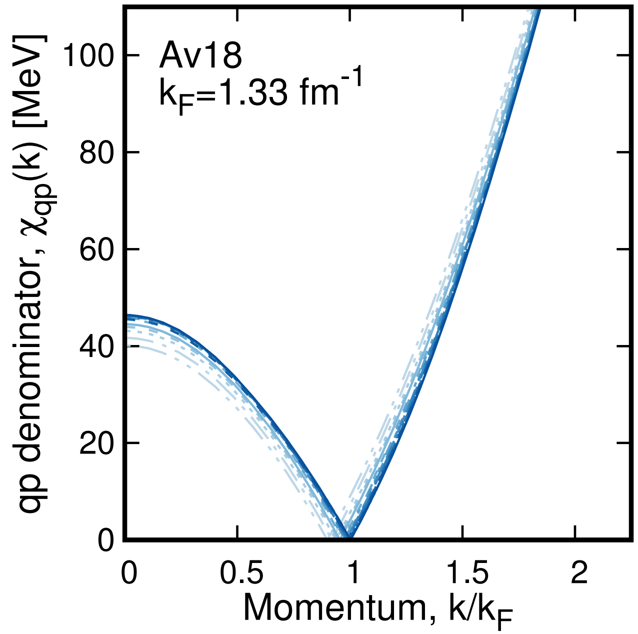
Many-body method

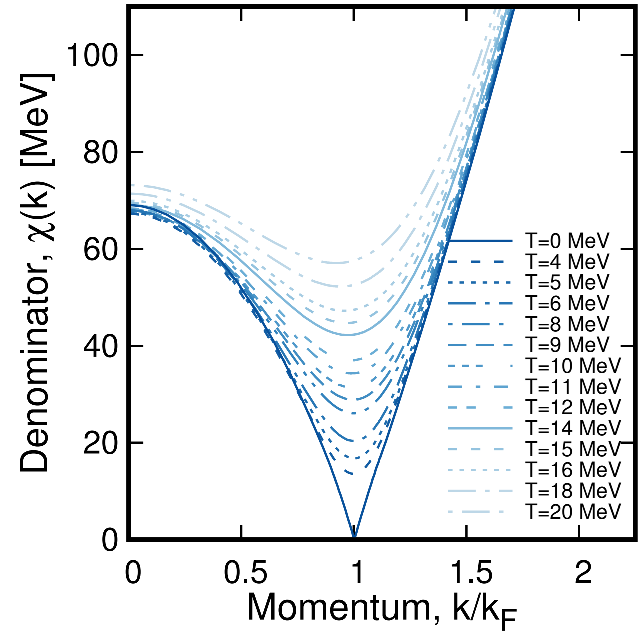


- **BCS** prediction based on **constant V**
- Full chiral **NN** interactions
- Full **HF spectrum**
- 3NFs do not change the ratio!

Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

What equations should we work with?





BCS gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \frac{1}{2\bar{\chi}_k} = \frac{1}{2|\varepsilon_k - \mu|}$$

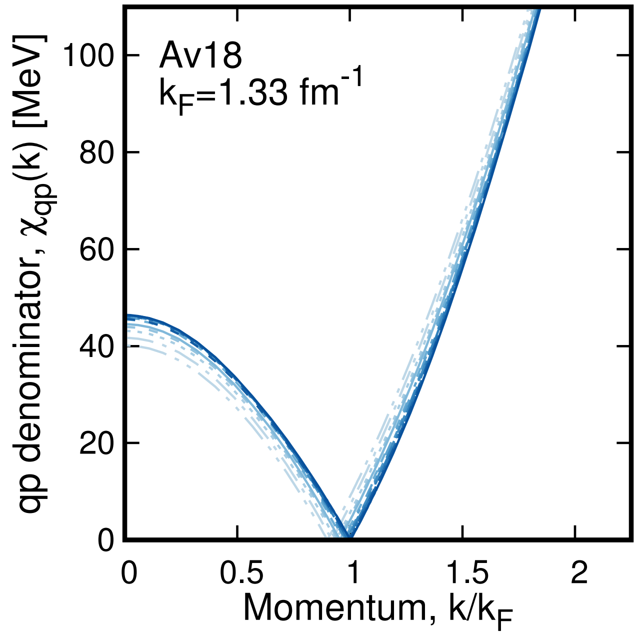
BCS+SRC gap equation

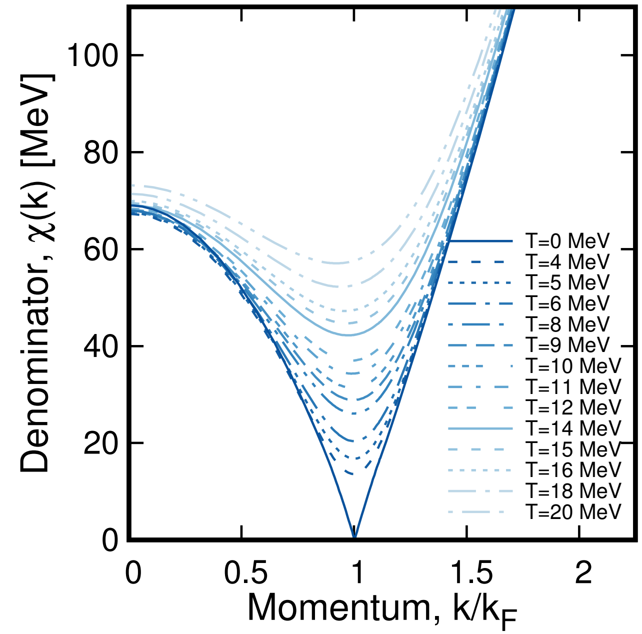
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \frac{1}{2\bar{\chi}_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_s(k,\omega')$$

Muther & Dickhoff, PRC **72** 054313 (2005)
Rios, Polls & Dickhoff, J Low T Phys **189** 234 (2017)

What equations should we work with?





BCS gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \frac{1}{2\bar{\chi}_k} = \frac{1}{2|\varepsilon_k - \mu|}$$

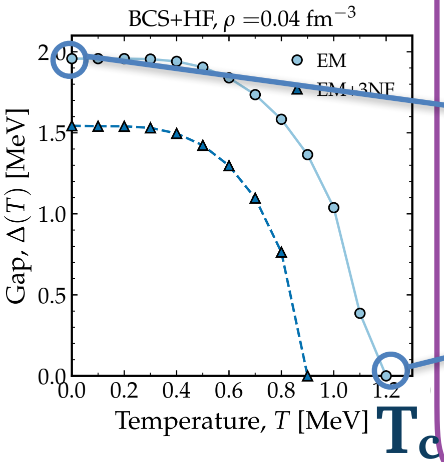
BCS+SRC gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

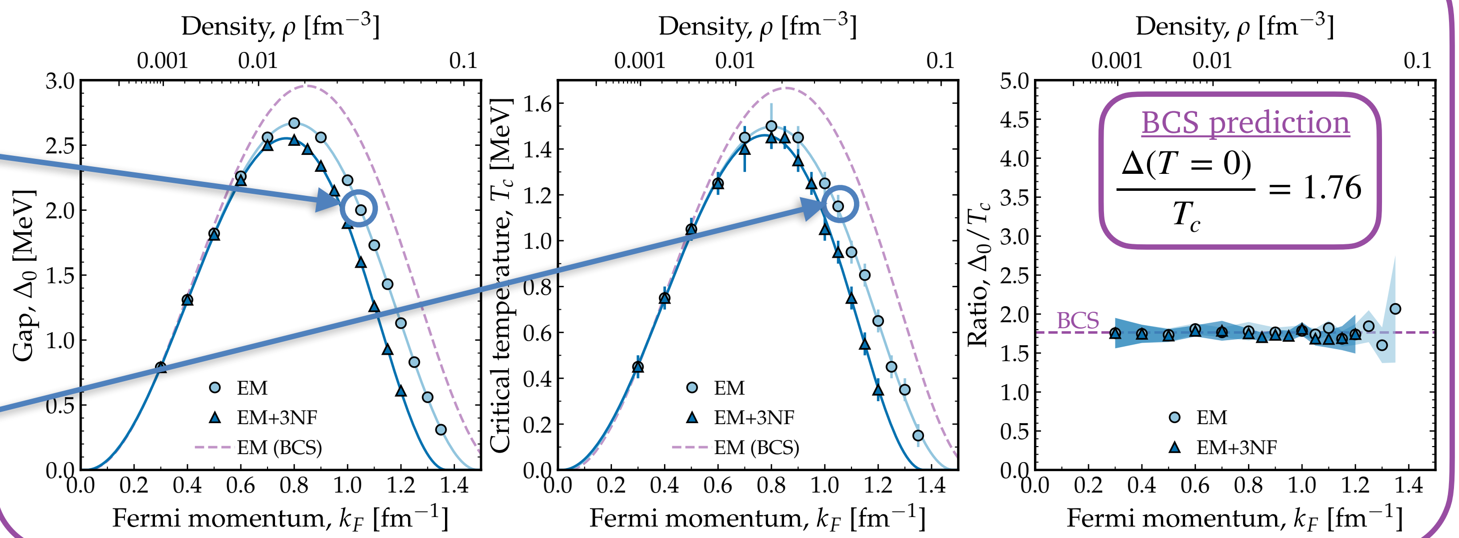
$$+ \quad \frac{1}{2\bar{\chi}_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_{\cancel{s}}(k,\omega')$$

Muther & Dickhoff, PRC **72** 054313 (2005)
Rios, Polls & Dickhoff, J Low T Phys **189** 234 (2017)

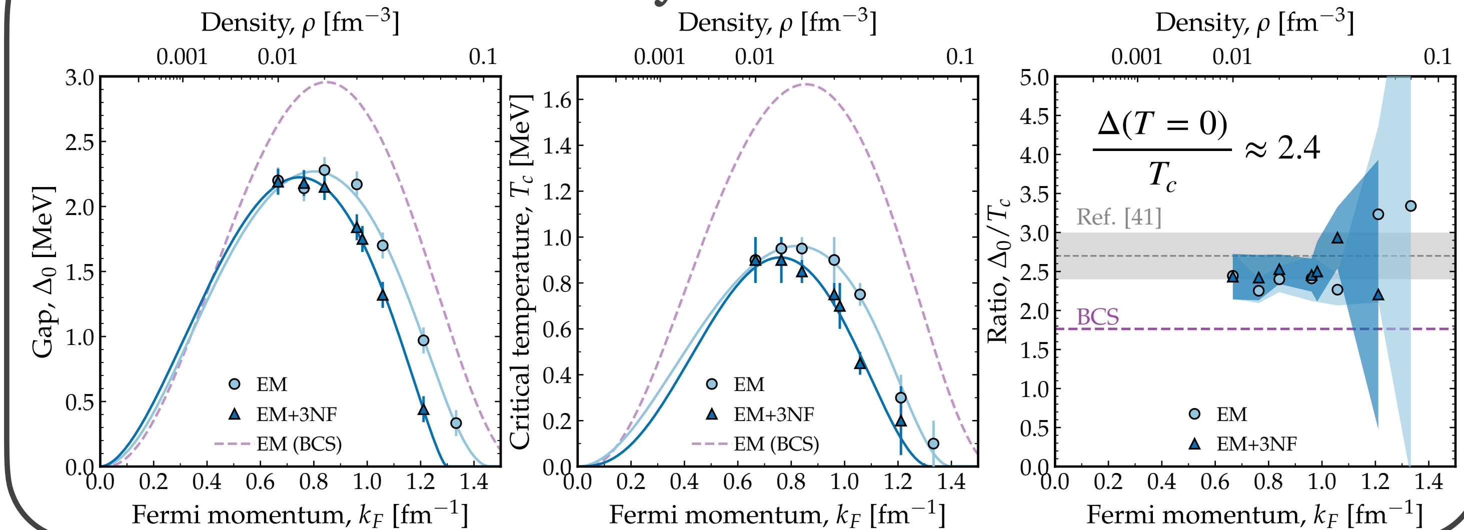
Why does it matter?



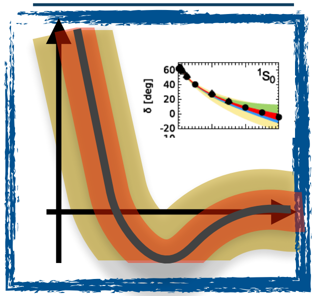
BCS+HF



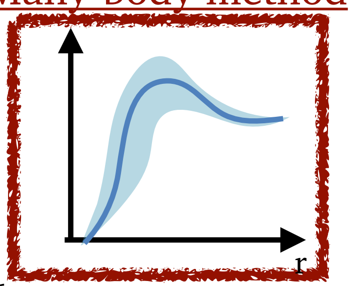
“Beyond”-BCS



Hamiltonian

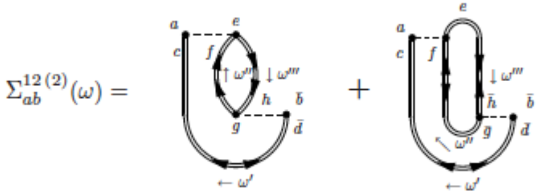


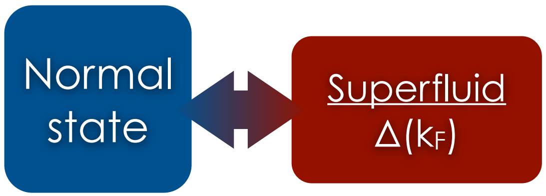
Many-body method



Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

How to go beyond?





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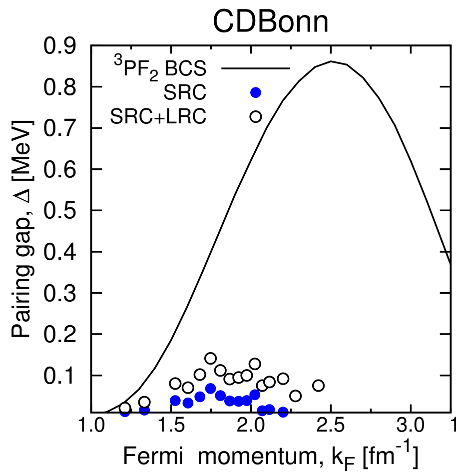

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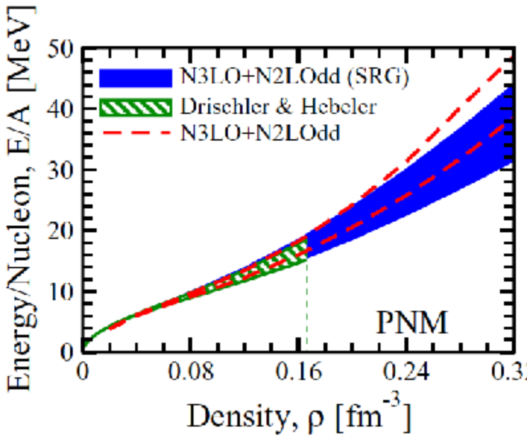
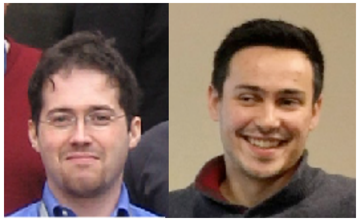
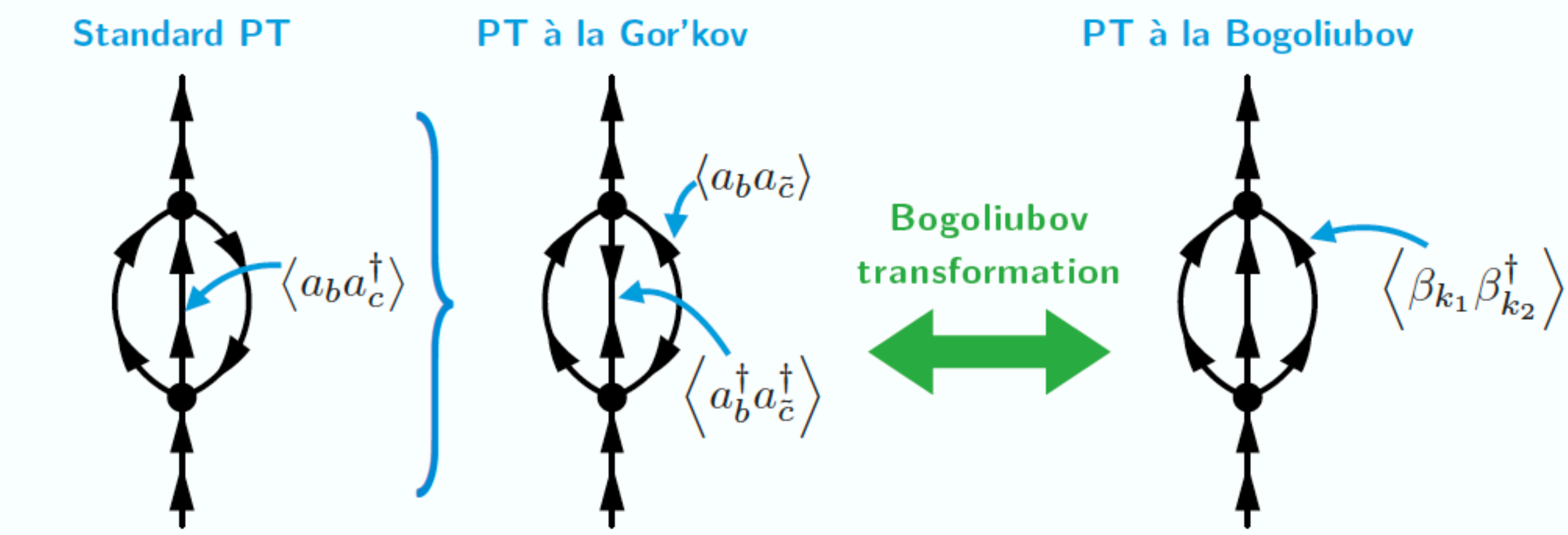


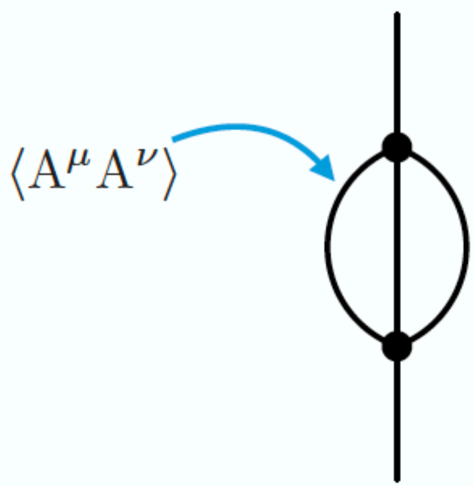
- **Existing frameworks** difficult to generalise
- **Nambu-covariant SCGF technique**
  - Symmetry breaking ✔
  - Finite temperature ✔
  - Systematic expansion w diagrams ✔
  - 3 nucleon forces ✔



Barbieri, Drissi

What was the issue before?





Reformulation

- Based on Nambu fields
- Propagators transform contravariantly
- Vertices transform covariantly
- Un-oriented diagrammatic

Nambu-Covariant Perturbation Theory

Double dimension H space

$$\mathscr{H}_1^e \equiv \mathscr{H}_1 \times \mathscr{H}_1^{\dagger}$$

Basis

$$\mathcal{B}^e \equiv \mathcal{B} \cup \bar{\mathcal{B}}$$

$$|b\rangle$$

$$\langle \bar{b}|$$

Elements

$$\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi_1'| \end{pmatrix}$$

$$\mu = (b, l) \qquad \bar{\cdot} : 1 \mapsto \bar{1} = 2$$

$$\bar{\mu} \equiv (b, \bar{l}) \qquad 2 \mapsto \bar{2} = 1$$

$$|b, 1\rangle \equiv \begin{pmatrix} |b\rangle \\ 0 \end{pmatrix} \qquad |b, 2\rangle \equiv \begin{pmatrix} 0 \\ \langle \bar{b}| \end{pmatrix}$$

Product & metric tensor

$$g\left(\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi_1'| \end{pmatrix}, \begin{pmatrix} |\Psi_2\rangle \\ \langle \Psi_2'| \end{pmatrix}\right) \equiv \langle \Psi_2' \,|\, \Psi_1\rangle + \langle \Psi_1' \,|\, \Psi_2\rangle$$

$$g_{\mu\nu} \equiv g\left(|\mu\rangle\,, |\nu\rangle\right) = \delta_{\mu\bar{\nu}}$$

Nambu fields

Nambu fields

$$\mathrm{A}^{(b,1)} \equiv a_b \ , \qquad \mathrm{A}^{(b,2)} \equiv \bar{a}_b \ , \qquad \bar{\mathrm{A}}_{(b,1)} \equiv \bar{a}_b \ , \qquad \bar{\mathrm{A}}_{(b,2)} \equiv a_b \ .$$

$$\mathrm{A}^{\mu} \equiv A^{(b,g)} = \begin{pmatrix} a_b \\ a^{\dagger}_{\bar{b}} \end{pmatrix}$$

$$\bar{\mathrm{A}}_{\mu} \equiv \bar{A}_{(b,g)} = \begin{pmatrix} a^{\dagger}_{\bar{b}} & a_b \end{pmatrix}$$

$$\boxed{\bar{a}_b = a^{\dagger}_{\bar{b}} \neq a^{\dagger}_{b}}$$

Commutator relations

(On extended indices!)

$$\{\, \mathrm{A}^{\mu}, \mathrm{A}^{\nu} \,\} = g^{\mu\nu} \ ,$$

$$\{\, \mathrm{A}^{\mu}, \bar{\mathrm{A}}_{\nu} \,\} = g^{\mu}{}_{\nu} \ ,$$

$$\{\, \bar{\mathrm{A}}_{\mu}, \mathrm{A}^{\nu} \,\} = g_{\mu}{}^{\nu} \ ,$$

$$\{\, \bar{\mathrm{A}}_{\mu}, \bar{\mathrm{A}}_{\nu} \,\} = g_{\mu\nu}$$

Co- or contravariant

$$\bar{\mathrm{A}}_{\mu} = \sum_{\nu} g_{\mu\nu}\ \mathrm{A}^{\nu} \ ,$$

$$\bar{\mathrm{A}}_{\mu} = \sum_{\nu} g_{\mu}{}^{\nu}\ \bar{\mathrm{A}}_{\nu} \ ,$$

$$\mathrm{A}^{\mu} = \sum_{\nu} g^{\mu\nu}\ \bar{\mathrm{A}}_{\nu} \ ,$$

$$\mathrm{A}^{\mu} = \sum_{\nu} g^{\mu}{}_{\nu}\ \mathrm{A}^{\nu} \ .$$

Tensor

- Let $\mathscr{W}$ a unitary Bogoliubov transformation

$$\mathrm{A}'^{\mu} = \sum_{\nu} (\mathcal{W}^{-1})^{\mu}{}_{\nu}\ \mathrm{A}^{\nu}\ ,$$

$$\bar{\mathrm{A}}'_{\mu} = \sum_{\nu} \mathcal{W}^{\nu}{}_{\mu}\ \bar{\mathrm{A}}_{\nu}\ ,$$

- Definition: ***(p,q)*-tensor** is multi-dim array s.t.

$$t'^{\mu_1\ldots\mu_p}{}_{\nu_1\ldots\nu_q} \equiv \sum_{\substack{\lambda_1\ldots\lambda_p \\ \kappa_1\ldots\kappa_q}} (\mathcal{W}^{-1})^{\mu_1}{}_{\lambda_1} \ldots (\mathcal{W}^{-1})^{\mu_p}{}_{\lambda_p} t^{\lambda_1\ldots\lambda_p}{}_{\kappa_1\ldots\kappa_q}\ \mathcal{W}^{\kappa_1}{}_{\nu_1} \ldots \mathcal{W}^{\kappa_q}{}_{\nu_q}$$

with *p contravariant* & *q covariant* indices

- Operators are polynomials of Nambu fields:

$$O \equiv \sum_{\substack{\mu_1\ldots\mu_k \\ \nu_1\ldots\nu_k}} o^{\mu_1\ldots\mu_k}{}_{\nu_1\ldots\nu_k}\ \bar{\mathrm{A}}_{\mu_1} \ldots \bar{\mathrm{A}}_{\mu_k} \mathrm{A}^{\nu_1} \ldots \mathrm{A}^{\nu_k}$$

Why Bogoliubov tensor algebra?

Tensor product: $r^{\mu_1}{}_{\nu_1}{}^{\mu_2\mu_3} = s^{\mu_1}{}_{\nu_1}\, t^{\mu_2\mu_3}$

Tensor contraction: $r^{\mu}{}_{\nu} = \sum_{\alpha} s^{\mu}{}_{\alpha}\, t^{\alpha}{}_{\nu}$

- Co(contra-)variance under Bogoliubov transforms provide **invariant** expressions **in any basis**
- Potential to **optimise** the extended basis
- **Tensor-network** structure becomes **transparent**
- Leads to **diagrammatic** expansion (à la de Dominicis-Martin or Haussmann)
- Other **formalisms** through specific basis or metric

Perturbative expansion

Hamiltonian partitioning

$$\Omega = \Omega_0 + \Omega_1$$

$$\Omega_0 = \frac{1}{2}\sum_{\mu\nu} U_{\mu\nu} \mathrm{A}^{\mu}\mathrm{A}^{\nu}$$

$$\Omega_1 = \sum_{k=1}^{n} \frac{1}{(2k)!} \sum_{\mu_1\dots\mu_{2k}} v^{(k)}_{\mu_1\dots\mu_{2k}} \mathrm{A}^{\mu_1}\dots\mathrm{A}^{\mu_{2k}}$$

Covariant **k-body** vertices

Fully antisymmetric vertex

- **Definition**

$$v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]} \equiv \frac{1}{(2k)!}\sum_{\sigma\in S_{2k}} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\ \mu_{\sigma(2)}\ \dots\ \mu_{\sigma(2k-1)}\ \mu_{\sigma(2k)}}$$

- **Antisymmetrisation** defines a new *(0,2k)*-tensor
- ***Not*** the case in a *mixed* representation

Green's functions

- Contravariant **k-body** Green's function

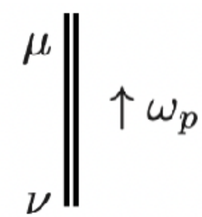
$$(-1)^k\ \mathcal{G}^{\mu_1\dots\mu_{2k}}(\tau_1,\dots,\tau_{2k}) \equiv \left\langle \mathrm{T}\left[\mathrm{A}^{\mu_1}(\tau_1)\ \dots\ \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right]\right\rangle$$

with $\langle\,.\,\rangle = \mathrm{Tr}\left(\,.\,\rho\right)$ and $\rho \equiv \dfrac{e^{-\beta\Omega}}{\mathrm{Tr}\left(e^{-\beta\Omega}\right)}$

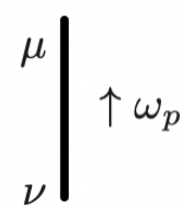
- Unperturbed case: $\Omega \longleftrightarrow \Omega_0$

Propagators

$$-\mathcal{G}^{\mu\nu}(\omega_p) =$$



$$-\left(\mathcal{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$$



Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

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Covariant **k-body** vertices

Fully antisymmetric vertex

- **Definition**

$$v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]} \equiv \frac{1}{(2k)!} \sum_{\sigma \in S_{2k}} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\ \mu_{\sigma(2)}\ \dots\ \mu_{\sigma(2k-1)}\ \mu_{\sigma(2k)}}$$

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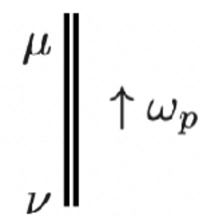
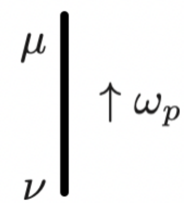
Propagators

$-\mathcal{G}^{\mu\nu}(\omega_p) =$ 

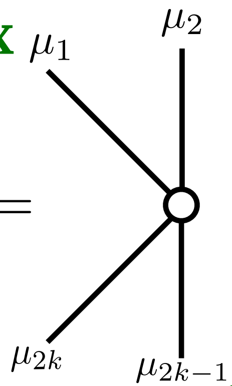
$-\left(\mathcal{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$ 

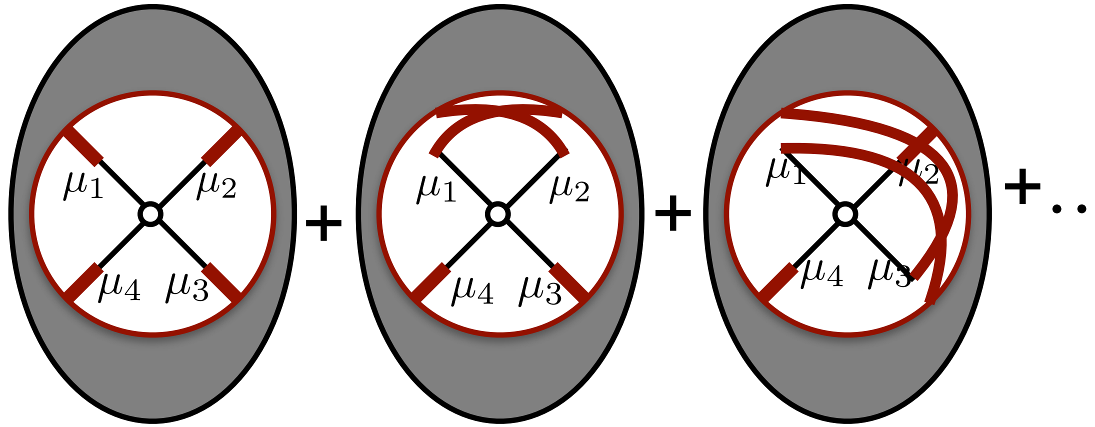
Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Why antisymmetric vertices?

Un-symmetrised vertex

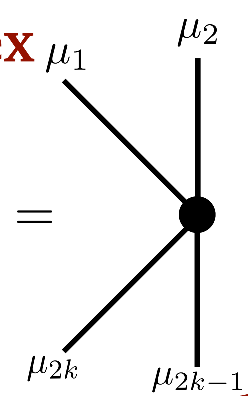
$$v^{(k)}_{\mu_1\mu_2\dots\mu_{2k-1}\mu_{2k}} =$$





Antisymmetrized vertex

$$v^{(k)}_{[\mu_1\mu_2\dots\mu_{2k-1}\mu_{2k}]} =$$



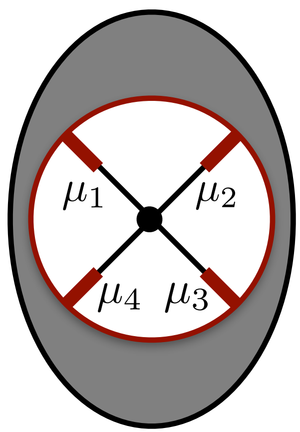


Diagram factorisation

- Derivations rely on
  - Wick theorem ⇒ sum over pairing
  - Sum over single-particle and Nambu indices

➡ **Extends Hugenholtz antisymmetrisation**

- Antisym is a **one-off pre-computing** cost

Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Perturbative expansion

Order n graphical rules

- Draw all topologically distinct **connected unlabelled** diagrams
  - with ***2k* external** legs
  - with ***n* vertices** (for order n contributions)

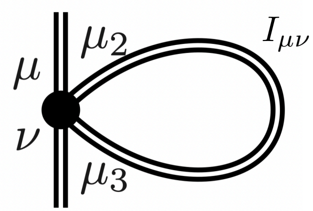
Feynman rules

1. Label vertices from *1* to n
   - S is the number of vertex labels permutations leaving the diagram invariant
2. For each line multiply by $-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_e)$
3. For each k-body vertex multiply by $v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]}$
4. Sum over each internal μ index and each independent ω_e frequency
5. Multiply by $\dfrac{(-1)^{n+L}}{S \times 2^T\ \prod_{l=2}^{l\text{max}} (l!)^m}$

Gaudin rules

- These simplify **Matsubara sums**
- Require **spanning trees**

Tadpoles are exceptional



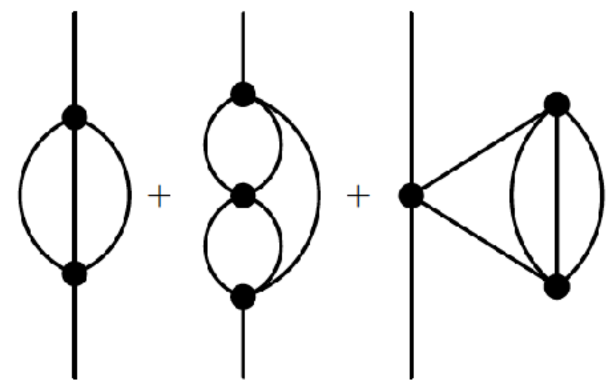
$$I_{\mu\nu} = \sum_{\mu_2\dots\mu_{2k-1}} \frac{(-1)^k}{2^{k-1}(k-1)!}\, v^{(k=2)}_{[\mu\ \dot\mu_2\ \dot\mu_3\ \nu]} \times \frac{1}{\beta}\sum_{\omega_e} -\mathcal{G}^{\mu_2\mu_3}(\omega_e)\, e^{-i\omega_e\eta_p}$$

- **Partially antisymmetrized** vertices needed:

$$v^{(k)}_{[\mu_1\dots\dot\mu_x\dots\dot\mu_y\dots\mu_{2k}]} \equiv \frac{2^p p!}{(2k)!} \sum_{\sigma\in S_{2k}/S_2^p\times S_p} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\dots\dot\mu_x\dots\dot\mu_y\dots\mu_{\sigma(2k)}}$$

- p internal lines are **fixed**
- ***k*-body generalisation** works

HFB partitioning 3rd order



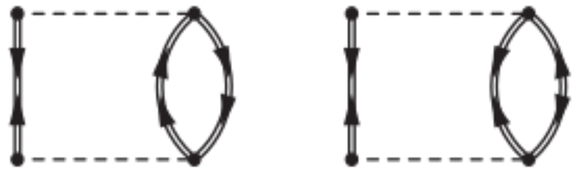
$\mathscr{G}_2$ $\mathscr{G}_3$ $\mathscr{G}_3'$

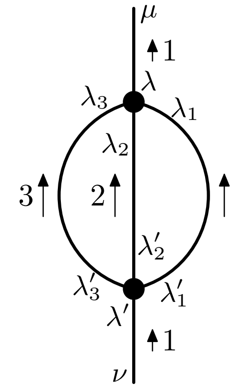
Advantages vs Gorkov

Gorkov GF	NCGF

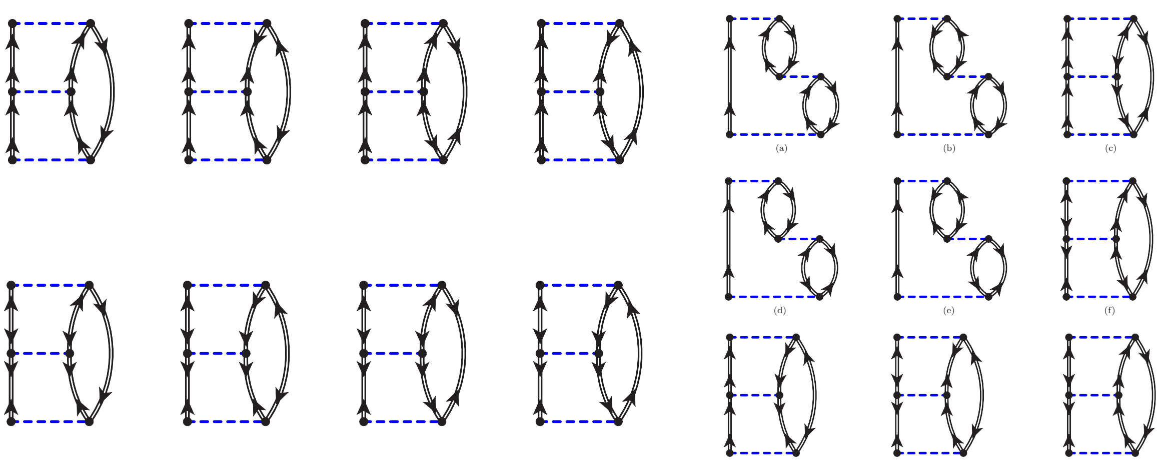
Order 2

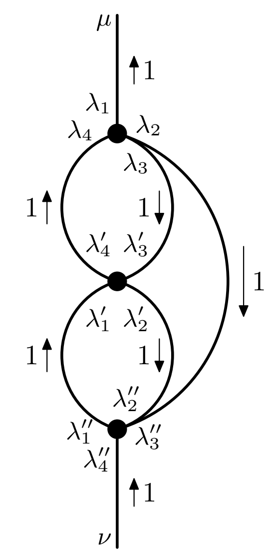
$$\Sigma^{11\,(2)}_{ab}(\omega) =$$



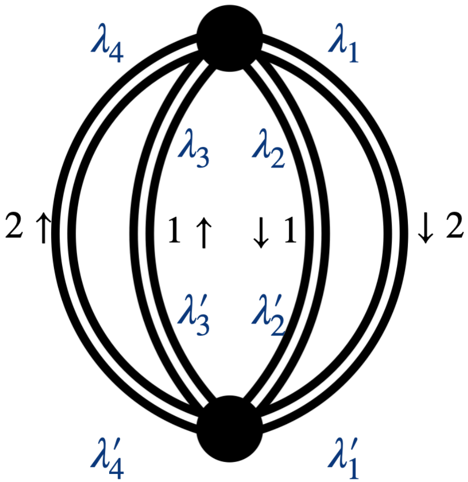


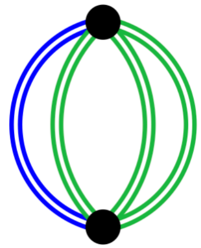
Order 3





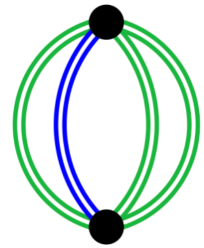
Basic example





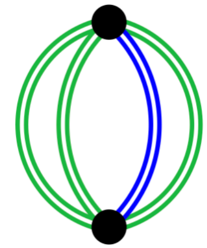
$\mathscr{A}_4$

$$\frac{-f(\epsilon_3)\, f(-\epsilon_2)\, f(-\epsilon_1)}{\epsilon_4+\epsilon_3-\epsilon_2-\epsilon_1}$$



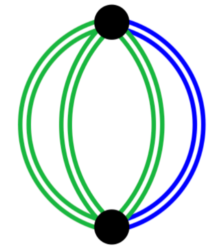
$\mathscr{A}_3$

$$\frac{f(-\epsilon_4)\, f(-\epsilon_2)\, f(-\epsilon_1)}{\epsilon_4+\epsilon_3-\epsilon_2-\epsilon_1}$$



$\mathscr{A}_2$

$$\frac{f(-\epsilon_4)\, f(-\epsilon_3)\, f(-\epsilon_1)}{-\epsilon_4-\epsilon_3+\epsilon_2+\epsilon_1}$$



$\mathscr{A}_1$

$$\frac{f(-\epsilon_4)\, f(-\epsilon_3)\big(-f(\epsilon_2)\big)}{-\epsilon_4-\epsilon_3+\epsilon_2+\epsilon_1}$$

$$I = \frac{1}{48} \sum_{\substack{\lambda_1\lambda_2\lambda_3\lambda_4 \\ \lambda'_1\lambda'_2\lambda'_3\lambda'_4}} v^{(2)}_{[\lambda_1\lambda_2\lambda_3\lambda_4]} v^{(2)}_{[\lambda'_4\lambda'_3\lambda'_2\lambda'_1]} \int_{-\infty}^{+\infty} \frac{d\epsilon_1}{2\pi}\frac{d\epsilon_2}{2\pi}\frac{d\epsilon_3}{2\pi}\frac{d\epsilon_4}{2\pi}\, S^{\lambda'_1\lambda_1}(\epsilon_1)\, S^{\lambda'_2\lambda_2}(\epsilon_2)\, S^{\lambda_3\lambda'_3}(\epsilon_3)\, S^{\lambda_4\lambda'_4}(\epsilon_4)$$

$$\times \frac{f(\epsilon_1)\, f(\epsilon_2)\, f(-\epsilon_3)\, f(-\epsilon_4) \;-\; f(-\epsilon_1)\, f(-\epsilon_2)\, f(\epsilon_3)\, f(\epsilon_4}{\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4}$$

Spectral function

$$\mathscr{G}^{\mu\nu}(\omega_p) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, \frac{S^{\mu\nu}(\omega')}{i\omega_p - \omega'}$$

- Lehmann representation

$$S^{\mu\nu}(\omega) \equiv \frac{1}{Z}\sum_{m,n} \langle \Psi_m | A^\mu | \Psi_n \rangle \langle \Psi_n | A^\nu | \Psi_m \rangle$$
$$\times\, e^{-\beta\Omega_m}\left(1+e^{-\beta\omega}\right)\, (2\pi)\, \delta\left(\Omega_n - \Omega_m - \omega\right)$$

Properties

- Symmetries
  - **Hermiticity:** $\left(S^\mu{}_\nu(\omega)\right)^* = S^\nu{}_\mu(\omega)$
  - **Antisymmetry:** $S^\mu{}_\nu(\omega) = S_\nu{}^\mu(-\omega)\left(\neq S^\nu{}_\mu(-\omega)\right)$
- Positive bounds
  - $S(\omega) \succ 0 \iff$ each principal minor is strictly positive

Dyson equation

- Partitioning considered

$$\Omega = \underbrace{\frac{1}{2!}\sum_{\mu\nu} U_{\mu\nu}\ \mathrm{A}^{\mu}\mathrm{A}^{\nu}}_{\Omega_0} + \underbrace{\frac{1}{4!}\sum_{\alpha\beta\gamma\delta} v^{(2)}_{\alpha\beta\gamma\delta}\ \mathrm{A}^{\alpha}\mathrm{A}^{\beta}\mathrm{A}^{\gamma}\mathrm{A}^{\delta}}_{\Omega_1}$$

- Dyson equation

$$\mathscr{G}^{\mu\nu}(\omega_n) = \mathscr{G}^{(0)\mu\nu}(\omega_n) + \sum_{\lambda_1\lambda_2} \mathscr{G}^{(0)\mu\lambda_1}(\omega_n)\ \Sigma_{\lambda_1\lambda_2}(\omega_n)\ \mathscr{G}^{\lambda_2\nu}(\omega_n)$$

Diagrammatic expansion of $\Sigma_{\mu\nu}(\omega_n)$

- with unperturbed propagators

$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathscr{J}_{\mu\nu}(\omega_n) - \mathscr{J}_{\nu\mu}(-\omega_n)}{2}$$

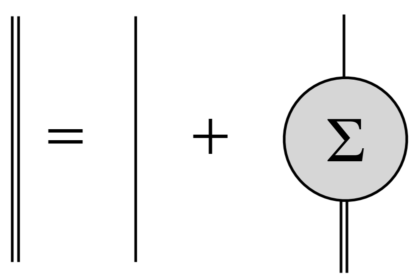
$$\mathscr{J}_{\mu\nu}(\omega_n) = \sum \text{1PI diagrams with } \mathscr{G}^{(0)}$$

- with self-consistent propagators

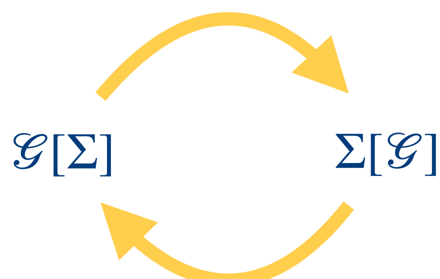
$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathscr{J}_{\mu\nu}(\omega_n) - \mathscr{J}_{\nu\mu}(-\omega_n)}{2}$$

$$\mathscr{J}_{\mu\nu}(\omega_n) = \sum \text{2PI diagrams with } \mathscr{G}\ \Big(= \mathscr{J}_{\mu\nu}(\omega_n)\Big)$$

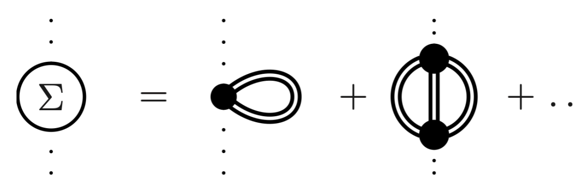
Diagrammatic representation



SCGF cycle



Self-energy expression

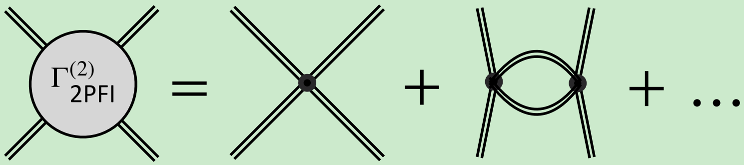


Drissi, Rios & Barbieri, Paper II, arxiv:2107.09763

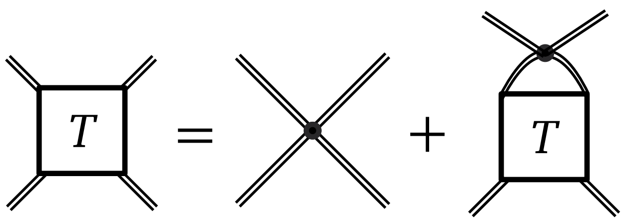
T-matrix: ladders

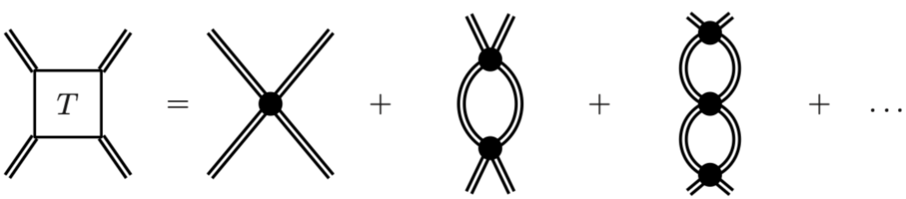
Approximations on $\Gamma^{(2)}_{\mathbf{2PFI}}$

- Sum of all possible rungs



T-matrix $\equiv \Gamma^{(2)}$ in ladder approximation





Ladder approximation

- Analytic/Retarded/Advanced/Sp function $\Rightarrow$ as usual
- T-matrix equation

$$T_{MN}(Z) \; = \; V^{(2)}_{MN} \; + \; \frac{1}{2} \sum_{LL'} V^{(2)}_{ML} \, \Pi^{LL'}(Z) \, T_{L'N}(Z)$$

where $\quad V^{(2)}_{MN} \equiv v^{(2)}_{[\mu_1\mu_2\nu_1\nu_2]}, M \equiv (\mu_1, \mu_2) \;\&\; N \equiv (\nu_1, \nu_2)$

Solving the ladder

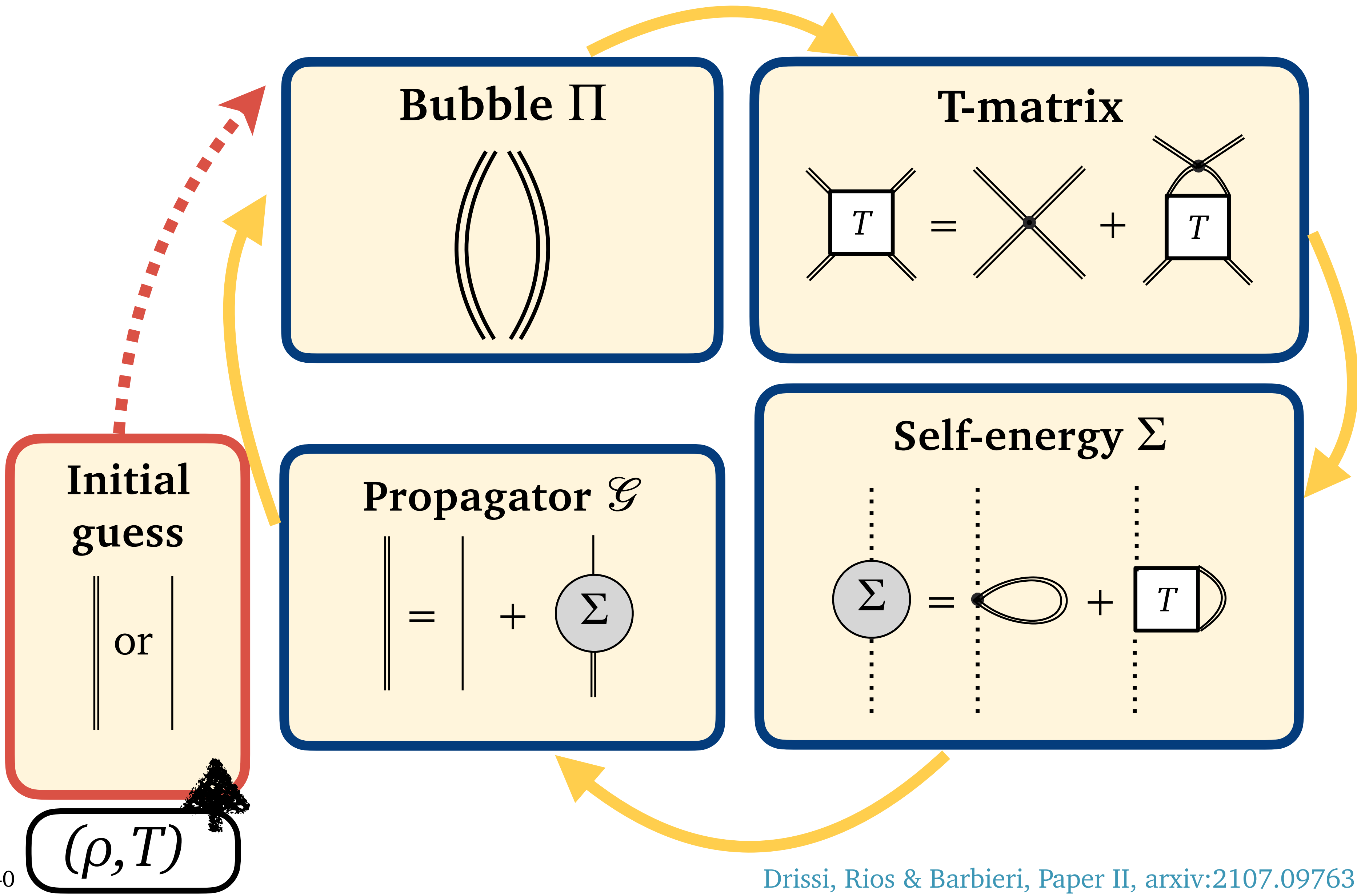
- Spectral representation

$$T_{MN}(Z) \equiv V^{(2)}_{MN} + \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} \frac{\mathcal{T}_{MN}(\Omega)}{Z - \Omega}$$

- Solution

$$\mathcal{T}(\Omega) = iV^{(2)} \left\{ \left(gg - \frac{1}{2}\Pi^R(\Omega)V^{(2)} \right)^{-1} - \left(gg - \frac{1}{2}\Pi^A(\Omega)V^{(2)} \right)^{-1} \right\}$$

Nambu-Covariant Ladders



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Initial guess

Unperturbed $S^{(0)}(\omega)$

/

Refined initial $S(\omega)$

(ρ, T)

Bubble Π

$$P^{MN}(\Omega) = \frac{1}{b(\Omega)}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi} S^{\mu_1\nu_1}(\omega)\, f(\omega) \times S^{\mu_2\nu_2}(\Omega-\omega)\, f(\Omega-\omega)$$

T-matrix

$$\mathscr{T}(\Omega) = i V^{(2)}\left\{\left(gg - \frac{1}{2}\Pi^R(\Omega)V^{(2)}\right)^{-1} - \left(gg - \frac{1}{2}\Pi^A(\Omega)V^{(2)}\right)^{-1}\right\}$$

Self-energy Σ

$$\Gamma_{\mu\nu}(\omega) = -\frac{1}{3}\sum_{\lambda_1\lambda_2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{2\pi}\left[f(\omega') + b(\omega'-\omega)\right] \times \mathscr{T}_{\mu\lambda_1\lambda_2\nu}(\omega-\omega')\, S^{\lambda_1\lambda_2}(\omega')$$

$$\Sigma^{\infty}_{\mu\nu} = \frac{1}{2}\sum_{\mu_2\mu_3} v^{(2)}_{[\mu\dot{\mu}_2\dot{\mu}_3\nu]}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\epsilon}{2\pi} f(-\epsilon)\, S^{\mu_2\mu_3}(\epsilon)$$

Propagator $\mathscr{G}$

$$S(\omega) = i\left(\omega\, g - \left(U + \Sigma^R(\omega)\right)\right)^{-1} - i\left(\omega\, g - \left(U + \Sigma^A(\omega)\right)\right)^{-1}$$

Drissi, Rios & Barbieri, Paper II, arxiv:2107.09763

Thouless' criterion

David J. Thouless

Thouless' criterion

[Thouless, 1960]

- Homogeneous system of fermions + two-body interaction $\bar{V}_{bcde}$
- Finite-temperature: $T > 0$
- Thouless' claim: (in the abstract)

Several assumptions on the potential

> The convergence of the ladder diagrams is suggested as a criterion which the BCS solution must satisfy, and it is shown that this is equivalent to requiring the BCS solution to give a local minimum of the thermodynamic potential.

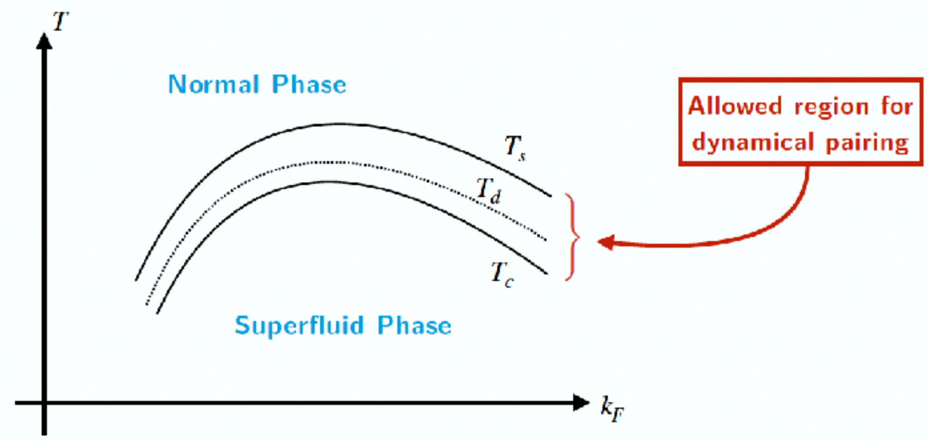
How to extend to all energies?

- Original case considered by Thouless
  - Separable interaction in singlet channel:

    $\bar{V}_{(\vec{k}_1'\uparrow)(\vec{k}_2'\downarrow)(\vec{k}_1\downarrow)(\vec{k}_2\uparrow)} = g\ v(\vec{q}\,')^* \times v(\vec{q}) \times \delta^{(3)}(\vec{P}\,' - \vec{P})$
  - Additional assumption: $\bar{V} = cst \neq 0$ **only** for $||\vec{P}||$ small and $||\vec{q}|| \sim ||\vec{q}\,'|| \sim k_F$

Pairing temperatures

- Critical temperature
  - T_c such that $r\left(\frac{1}{2}\Pi(0)V^{(2)}\right) = 1$
- Dynamical pairing temperature
  - T_d such that $\exists \Omega_p$, $r\left(\frac{1}{2}\Pi(\Omega_p)V^{(2)}\right) = 1$
- Upper-bound on dynamical pairing temperature
  - T_s such that $\left|\left|\frac{1}{2}\Pi(0)V^{(2)}\right|\right|_{S_\infty} = 1$
- Opening of possible regions of interest !
  - In general: $T_c \leq T_d \leq T_s$
  - Recover Thouless' criterion when $T_c = T_s$

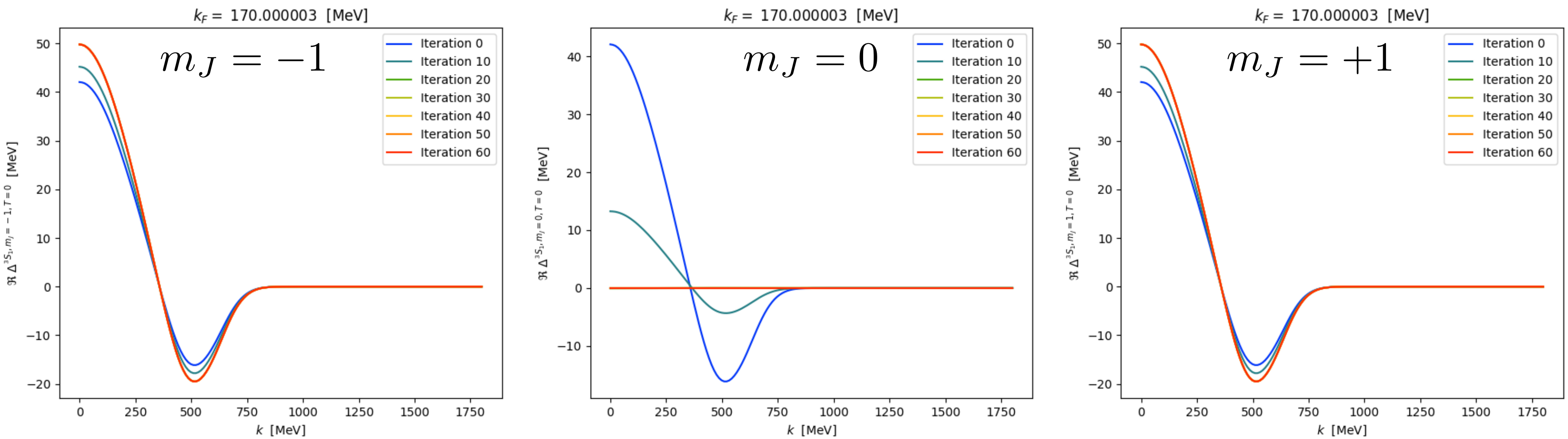


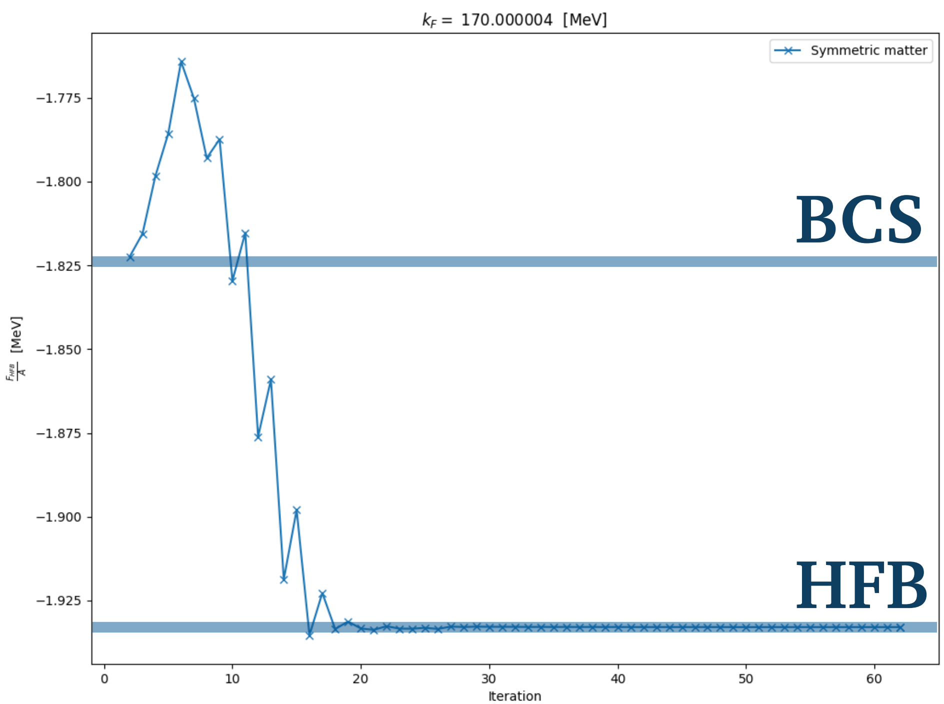
Open questions to be investigated

- ➡ Are T_s and T_d close for relevant physical systems ?
- ➡ What are the characteristic properties when $T_c < T < T_s$?
- ➡ Pre-pairing effects such as pseudo-gap in $S(\omega)$?

Drissi, Rios & Barbieri, Paper II, arxiv:2107.09763

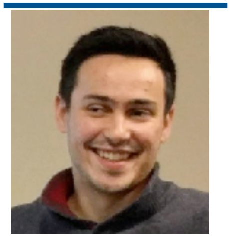
First results





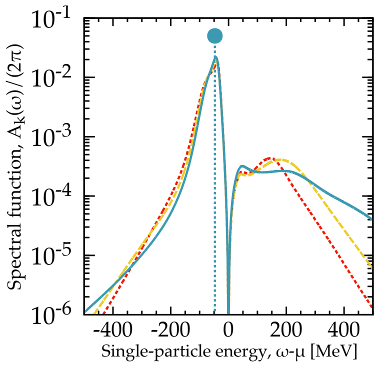
- From BCS to HFB
- 3SD1 channel
- N3LO EM
- T=0.2 MeV

M Drissi



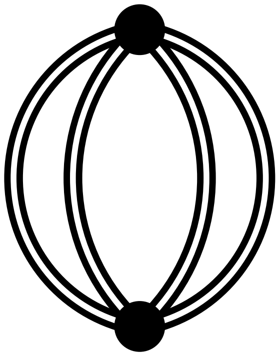
Conclusions

1) Thermal & microscopic properties



2) **Nuclear** uncertainty quantification

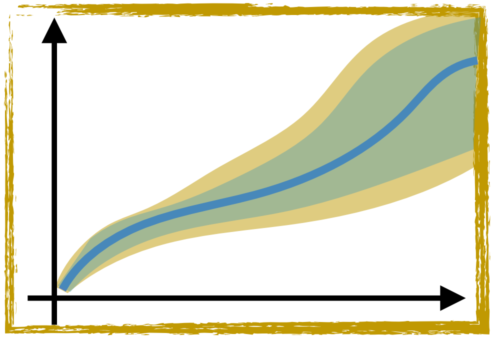
3) New superfluid extensions



Next:

Numerical implementation

Uncertainties in predictions?



Thank you!

Drissi, Rios & Barbieri, arxiv:2107.09759
Drissi, Rios & Barbieri, arxiv:2107.09763
Drissi & Rios, EPJA 58, 90 (2022)

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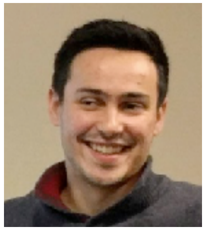

TRIUMF

C Barbieri

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