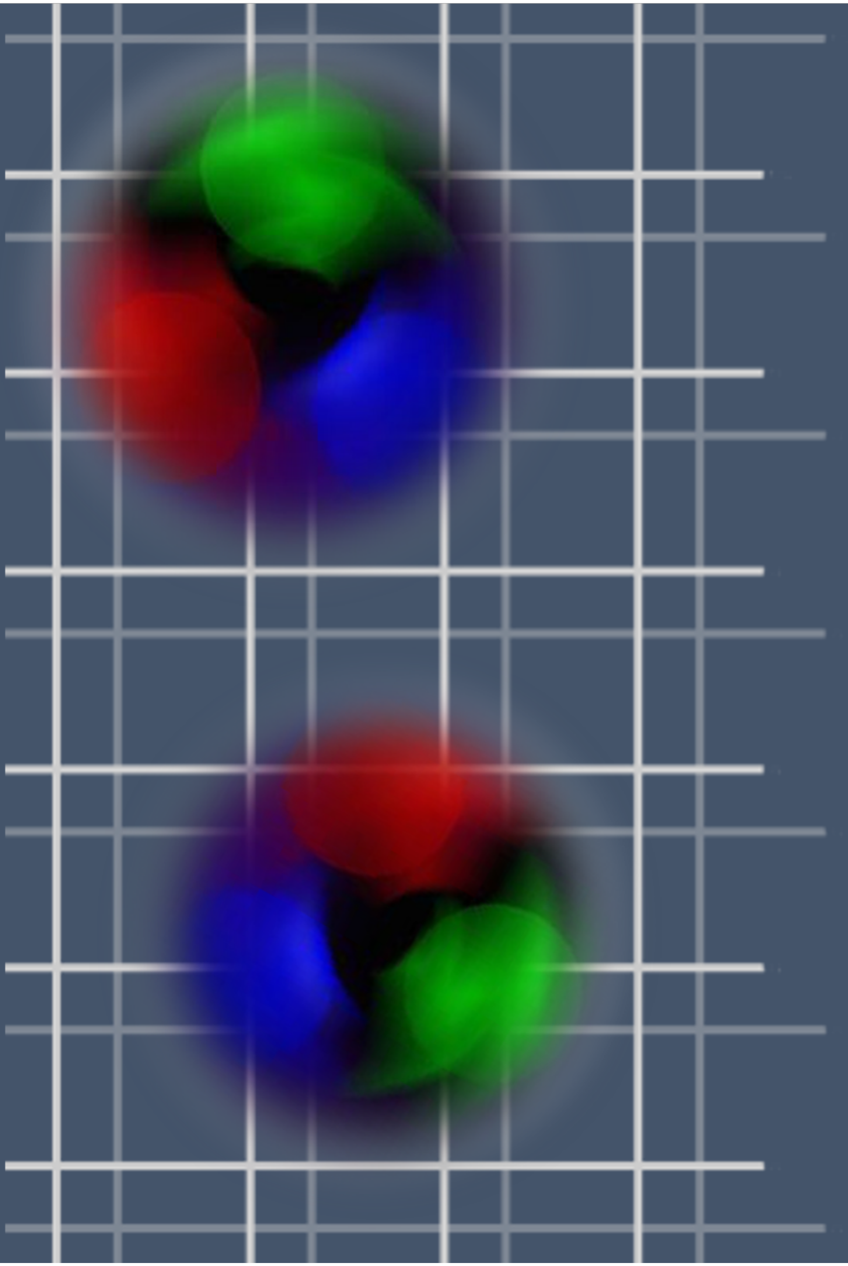


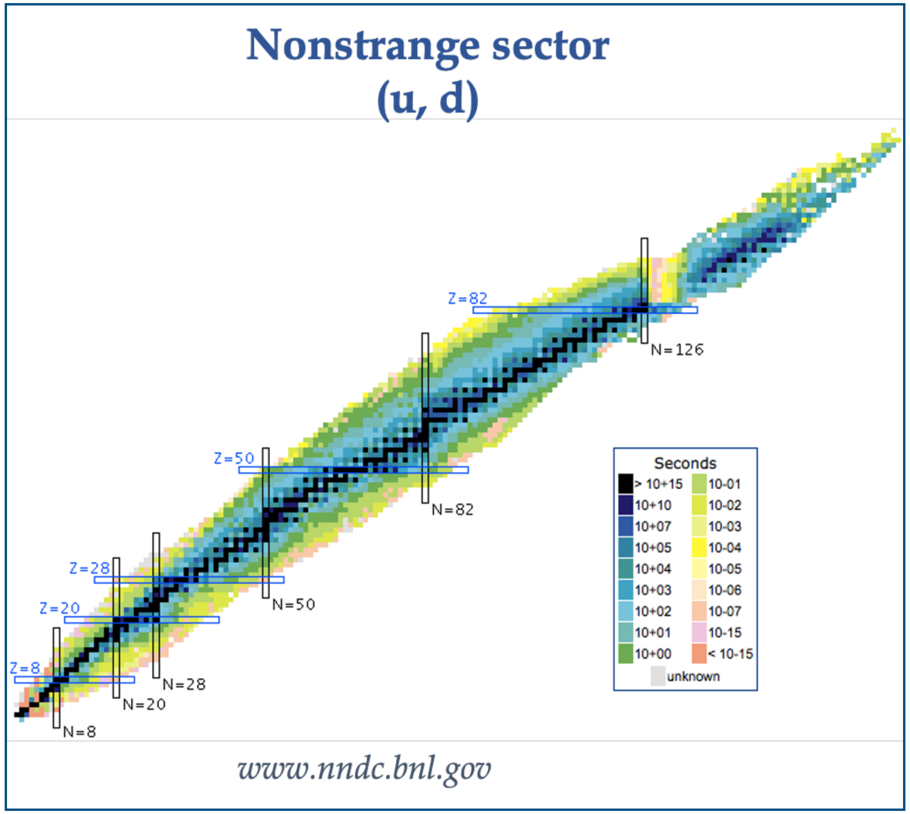
Lattice QCD calculations of Nuclear Physics at unphysical quark masses

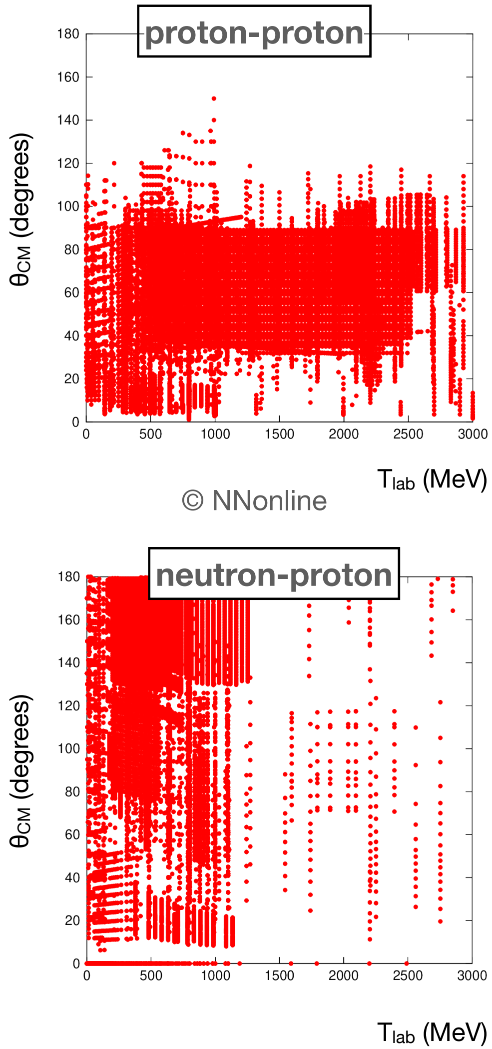
Assumpta Parreño
Universitat de Barcelona

NPLQCD Collaboration
www.ub.edu/nplqcd

Hirschegg 2023
Effective field theories for nuclei and nuclear matter
January 16-20, 2023

UNIVERSITAT DE BARCELONA
ICCUB
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA
EXCELENCIA MARÍA DE MAEZTU



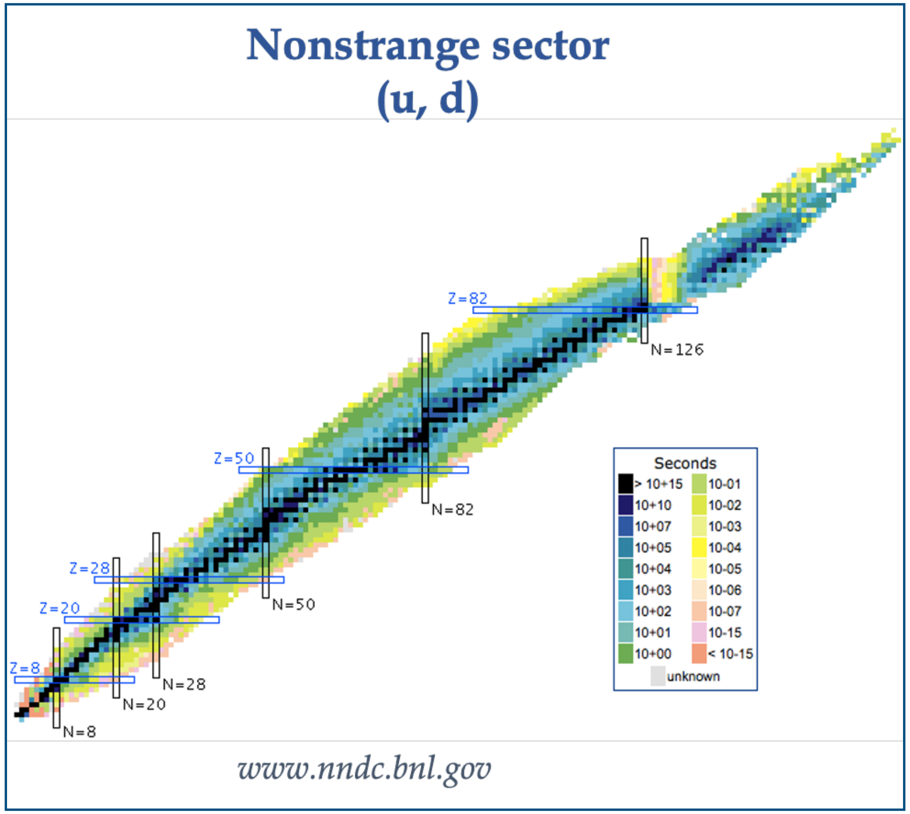


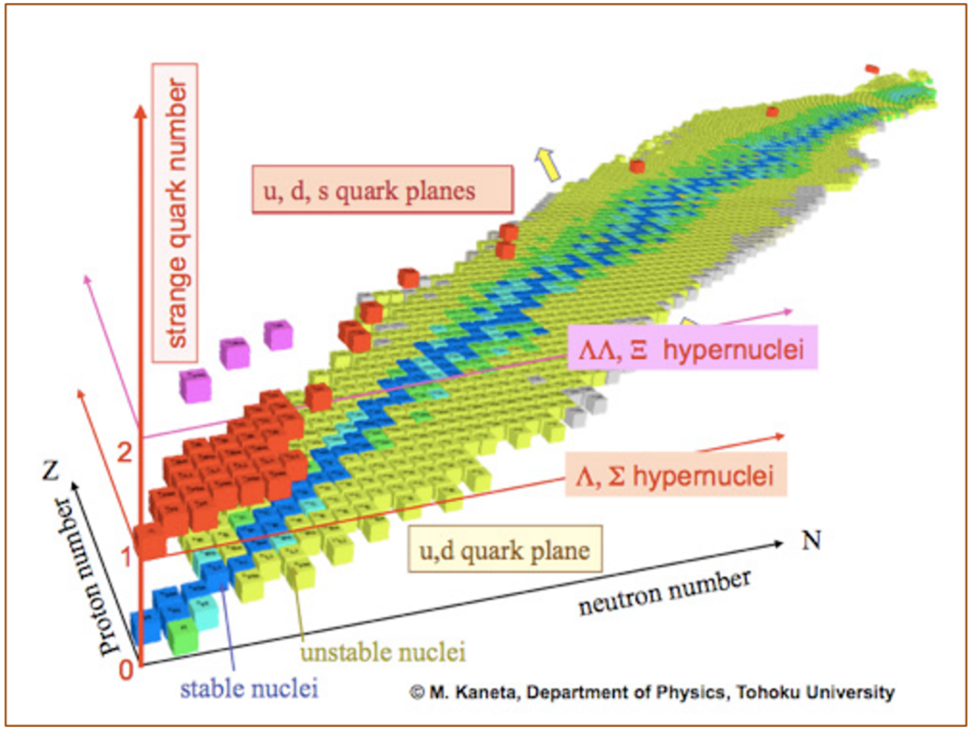
Lattice QCD:

Connection to QCD

Systematically improve the calculation

Control the uncertainties





Lattice QCD:

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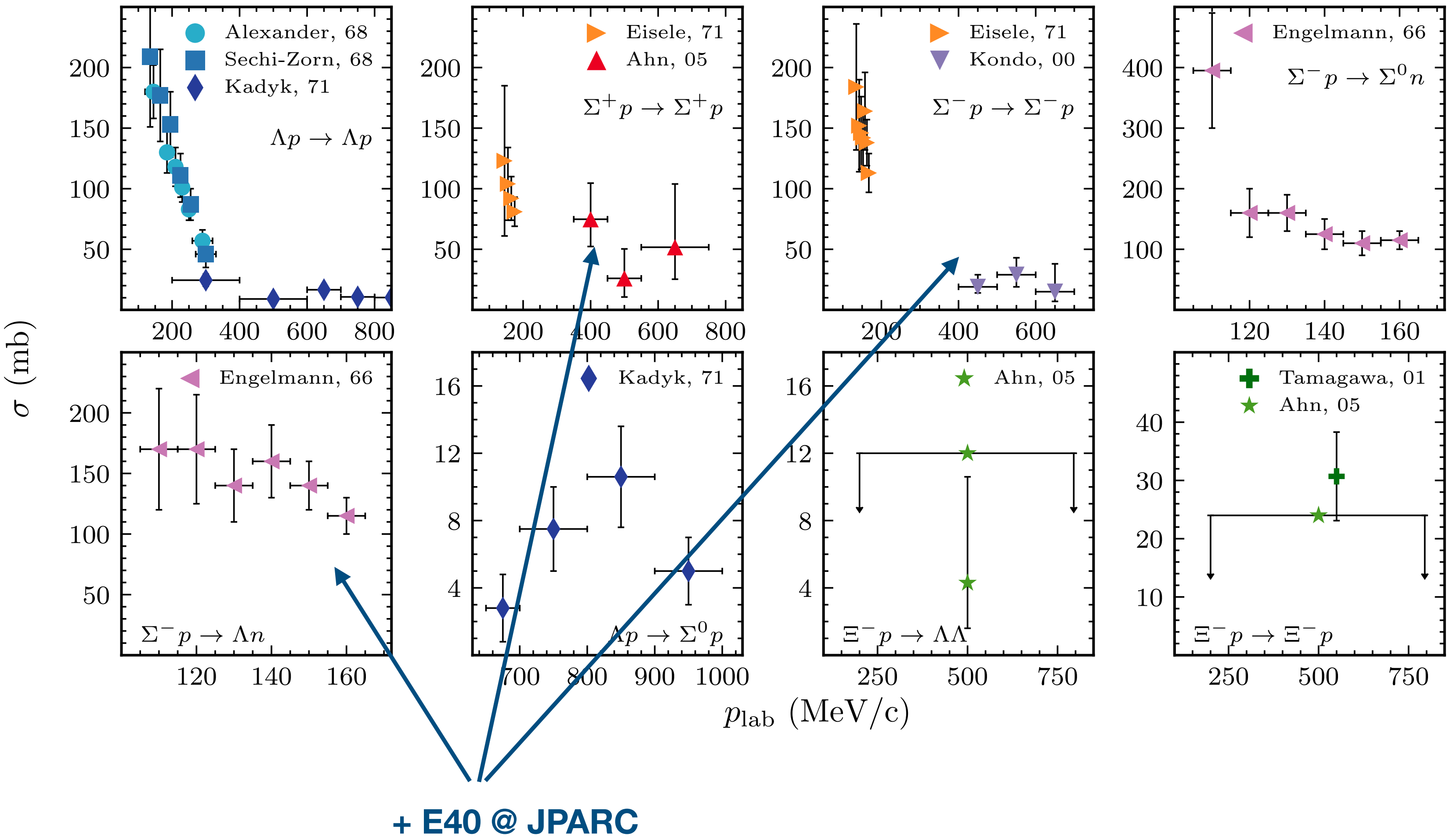
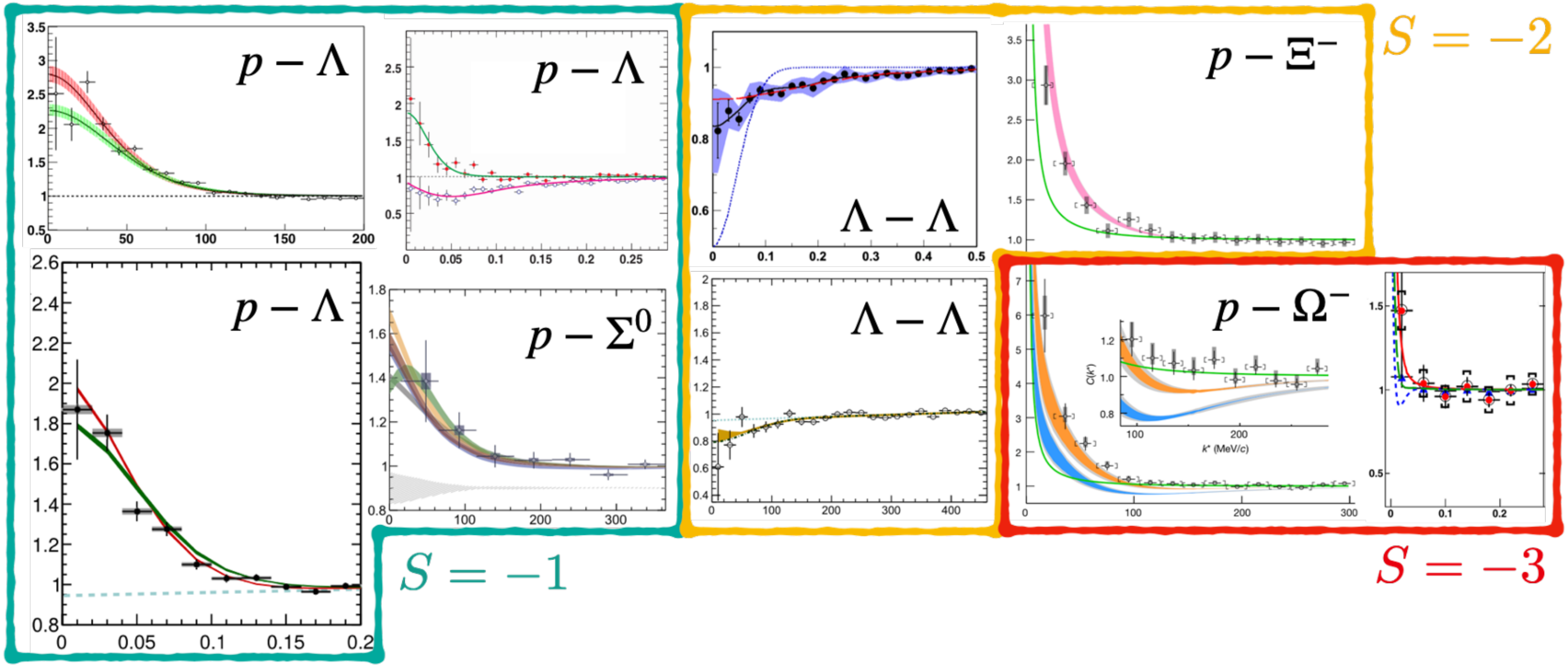


First collected in
Dover and Feschback,
Ann. Phys. 198 (1990)

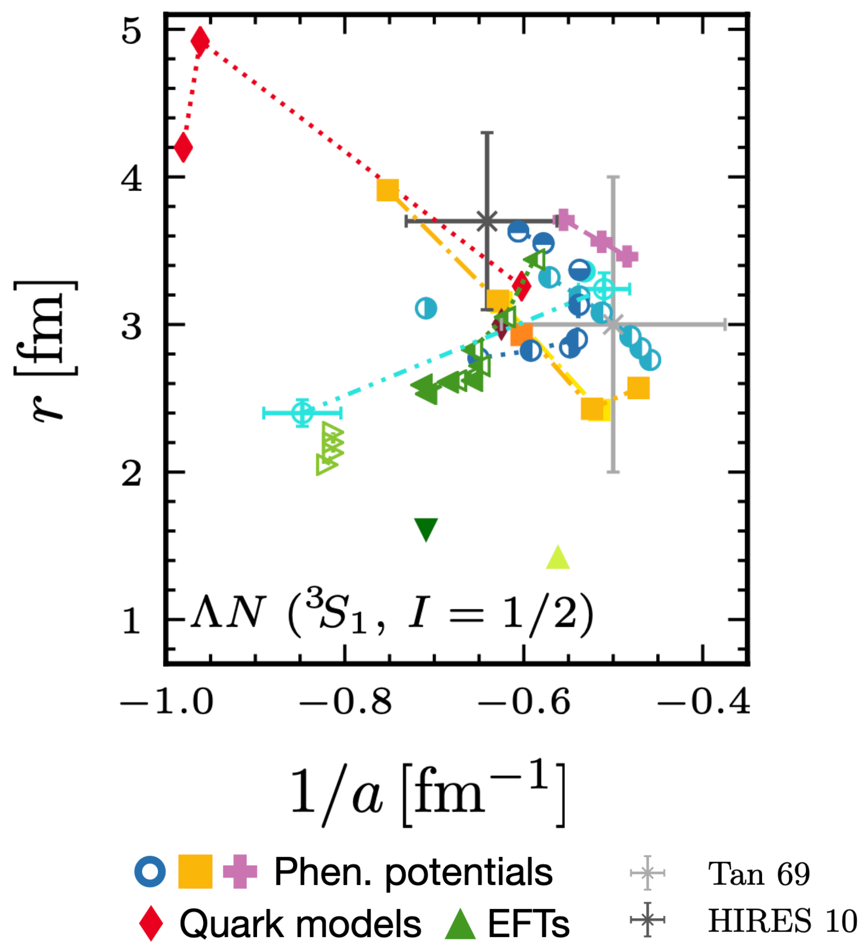
UPDATED BY MARC ILLA, UW
e-Print: 2109.10068 [hep-lat]

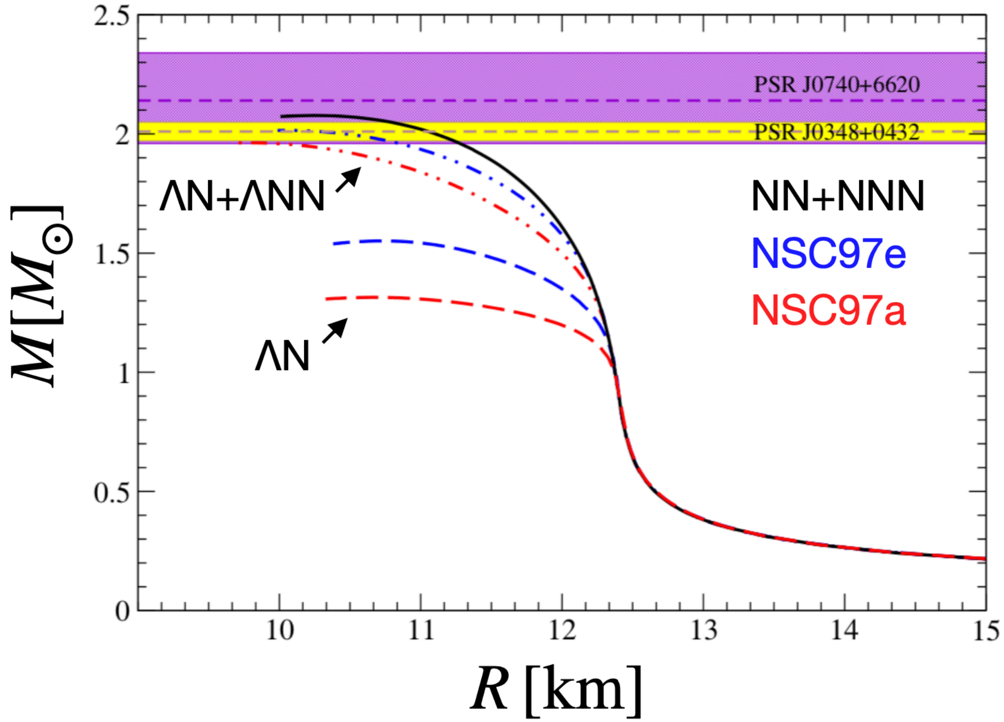
Femtoscopy: correlation function $C(k)$ as a function of relative momentum k



STAR, HADES and ALICE Collaborations

@ Marc Illa thesis,
e-Print: 2109.10068 [hep-lat]





D. Logoteta, I. Vidaña, I. Bombaci,
Eur. Phys. J.A 55 (2019)

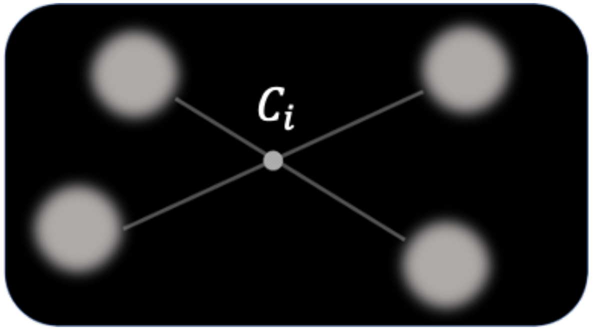
Nuclear physics, the non-perturbative regime of QCD

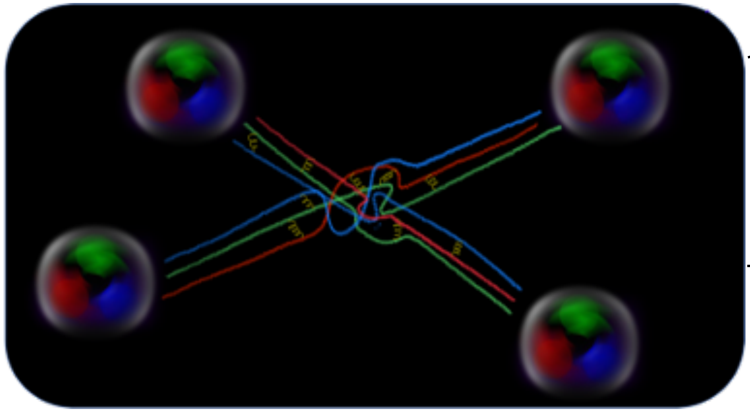
$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F_{uv}^{a}F_{a}^{uv}$$

$$i = r,g,b \quad j = u,d,c,s,t,b$$

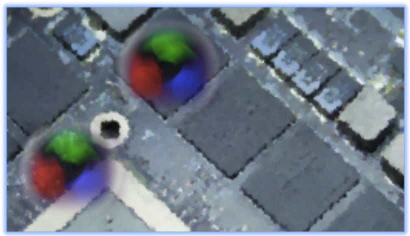
$$\mathcal{L}_{\mathrm{EFT}}\left[\pi, N, \ldots; m_{\pi}, m_{N}, \ldots; C_{i}\right]$$

LECs





The quantum propagation is expressed as a weighted sum over paths

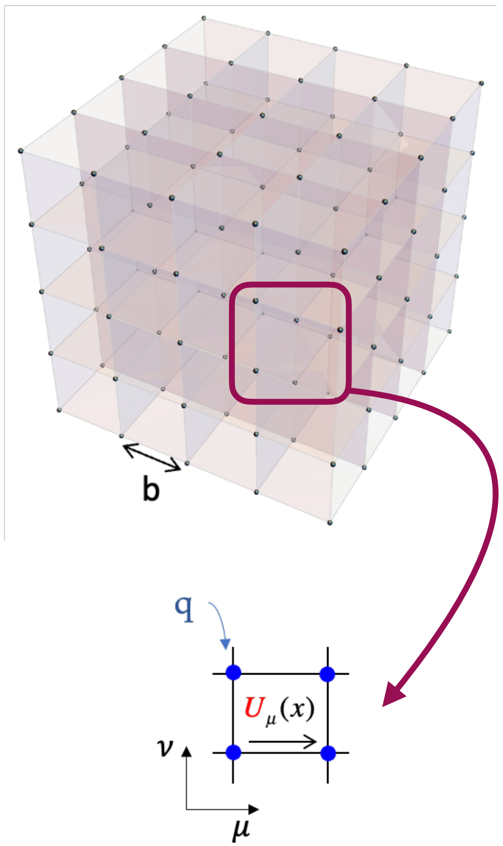


PATH INTEGRAL
Feynman, 1948

$$A = \int D(q)\,\exp\left(i\int_{i}^{f} dt\, L(q(t))\right)$$

go to **Euclidean space**
(numerical methods/important sampling)

$L_x \times L_y \times L_z \times T$



$L >> m_\pi^{-1}$ $\qquad$ $b << \Lambda_{QCD}^{-1}$

Nonperturbative (numerical) solution

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q\, \mathcal{D}\bar{q}\, \mathcal{D}A_\mu\, \hat{\mathcal{O}}[q,\bar{q},A]\, e^{\mathrm{i}S_{QCD}}$$

$$\Big\downarrow\; t \rightarrow \mathrm{i}\,\tau$$

go to **Euclidean space**
numerical methods/important sampling

$$U_\mu^b(x) \equiv e^{ig\int_x^{x+b\mu} A_\mu dx^\mu} = e^{igbA_\mu^b(x)}$$

($SU(3)_c$ matrices, “links”)

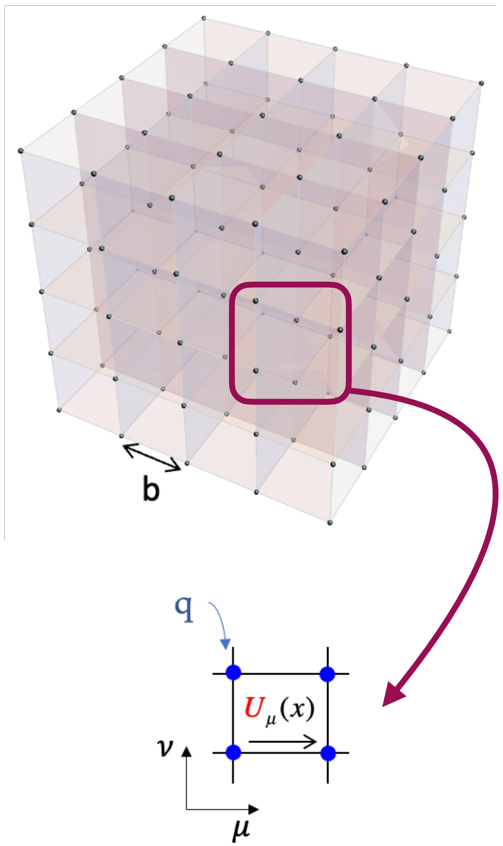
$$\boxed{\left\langle \hat{O} \right\rangle = \frac{1}{Z}\int DU\; D\bar{\psi}\; D\psi\, \hat{O}[\psi,\bar{\psi},U]\, e^{-\bar{\psi}Q(U)\psi - S_g[U]}}$$

$$\downarrow$$

$$\boxed{\left\langle \hat{O} \right\rangle = \frac{1}{Z}\int DU\; \hat{O}[Q(U)^{-1}]\, \det(Q(U))\, e^{-S_g[U]}}$$

propagators $\qquad$ configurations $(\sim P(U))$

$L_x \times L_y \times L_z \times T$



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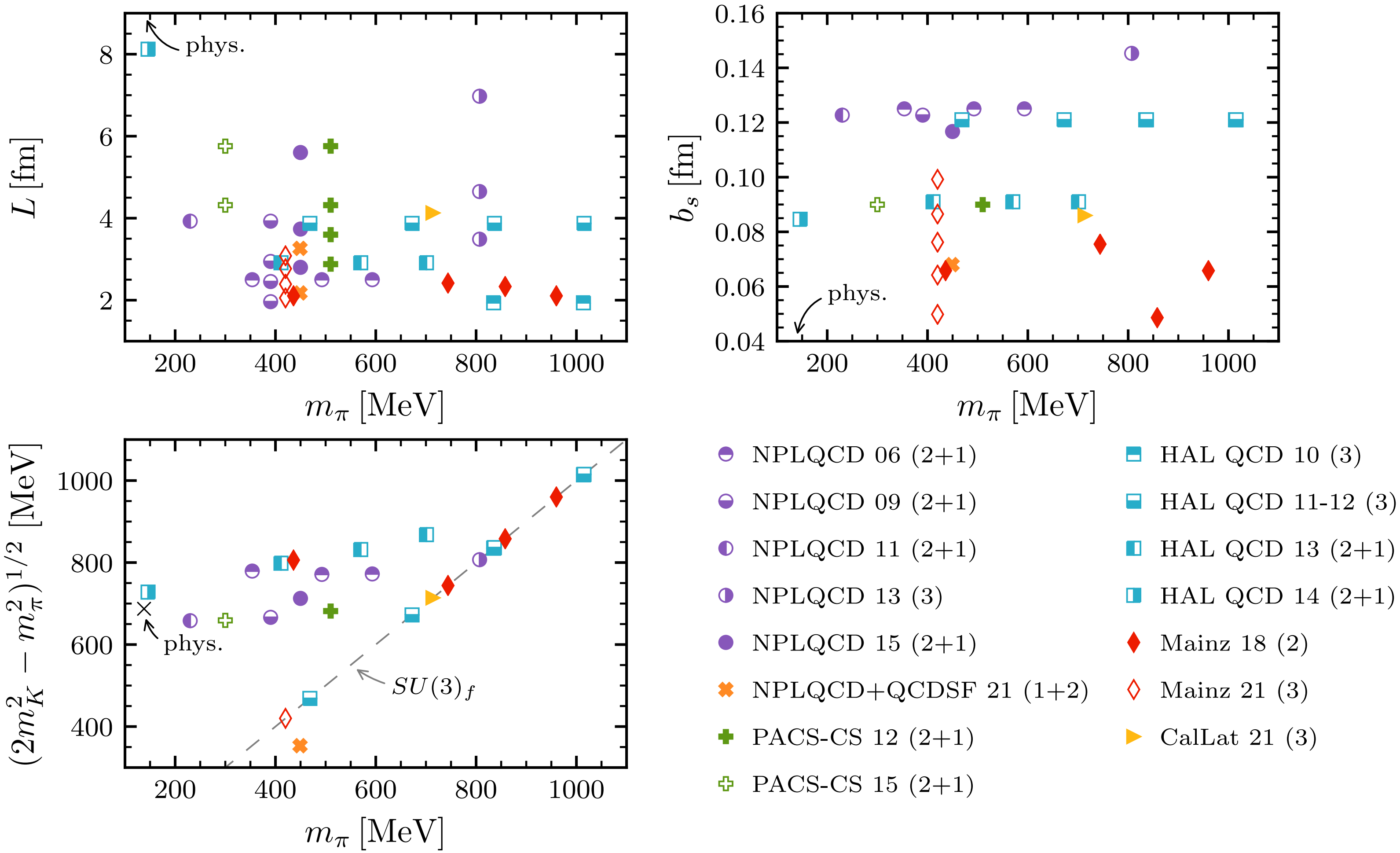
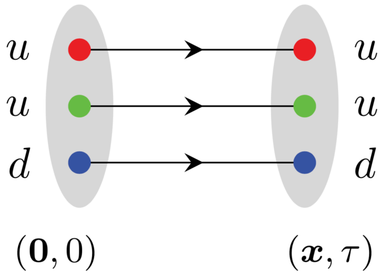


M. Illa, e-Print: 2109.10068 [hep-lat]

Energy levels

$$C_{2pt}(\tau,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\,\Gamma_{\beta\alpha}\langle \mathcal{X}_\alpha(\boldsymbol{x},\tau)\bar{\mathcal{X}}_\beta(\mathbf{0},0)\rangle$$

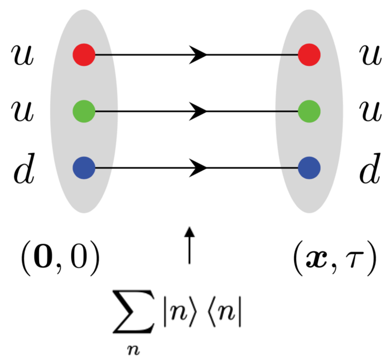


Energy levels

$$C_{2pt}(\tau, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\, \Gamma_{\beta\alpha} \langle \mathcal{X}_\alpha(\boldsymbol{x}, \tau) \bar{\mathcal{X}}_\beta(\mathbf{0}, 0) \rangle$$

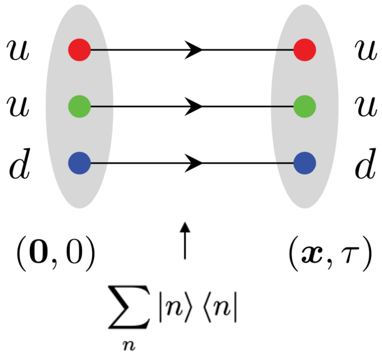
$$= Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)}t} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \ldots$$

Tower of energy eigenstates
of the system
in the finite volume



LQCD DIRECT METHOD: FV Energy levels from two-point correlation functions

Energy levels



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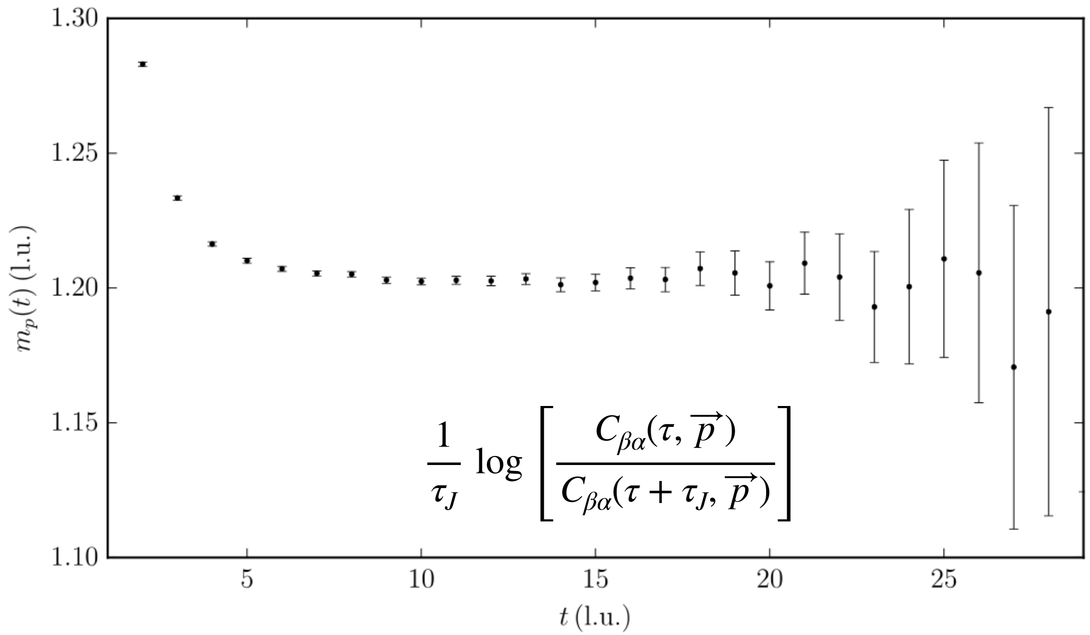
$$= \underbrace{Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)}t}}_{\text{dominates at large t}} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \dots$$

Tower of energy eigenstates of the system in the finite volume

$$E_{\mathfrak{n}}$$

$$p_\alpha(\mathbf{x}, t) = \epsilon^{ijk} u_\alpha^i(\mathbf{x}, t) \left(u^{j\mathsf{T}}(\mathbf{x}, t) C \gamma_5 d^k(\mathbf{x}, t) \right)$$



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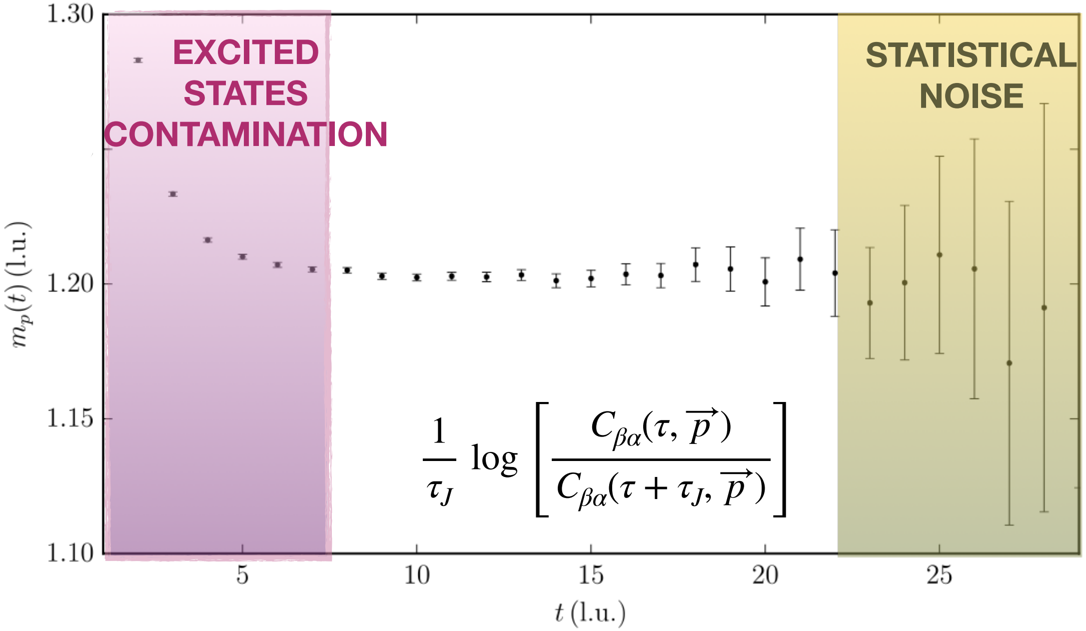
signal-to-noise degradation

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x},t)\left(u^{j\mathsf{T}}(\mathbf{x},t) C\gamma_5 d^k(\mathbf{x},t)\right)$$

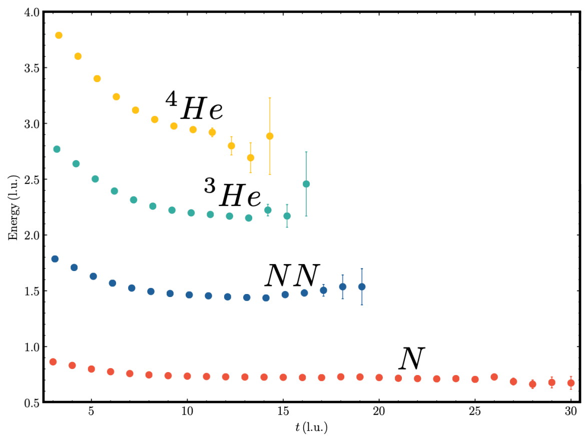


Expectation is that for A nucleons:

$$\frac{\sigma}{\langle C \rangle} \sim \frac{\exp\left[A \left(M_N - \frac{3m_\pi}{2} \right) t \right]}{\sqrt{N}}$$

G. Parisi, Phys.Rept. 103 (1984)
G.P. Lepage, Boulder TASI (1989)
M.L. Wagman, M.J. Savage, Phys.Rev.D 96 (2017)

signal-to-noise degradation



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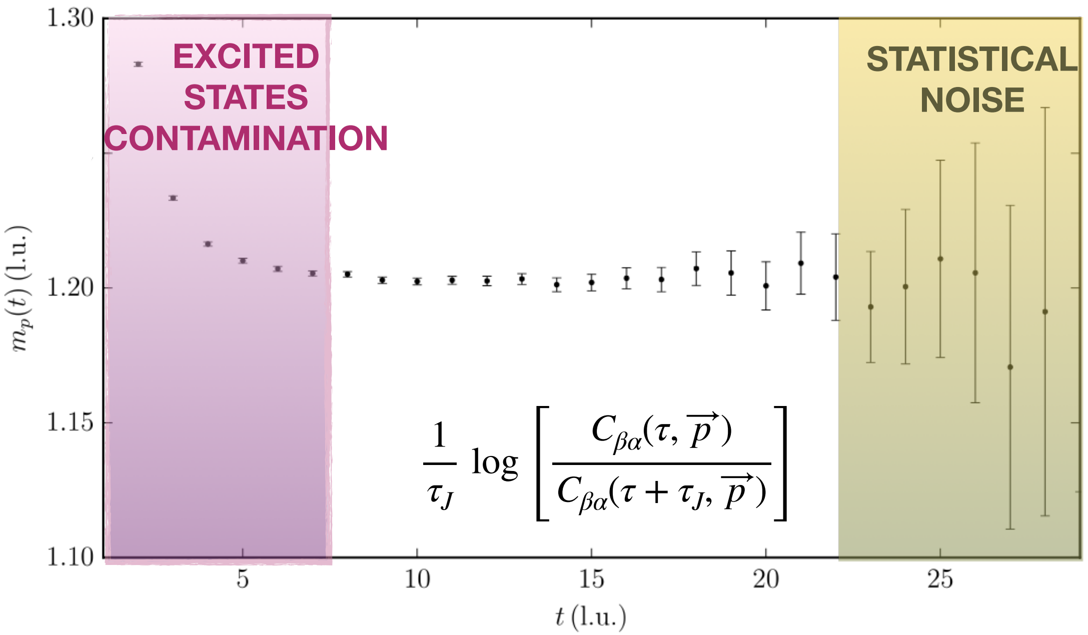

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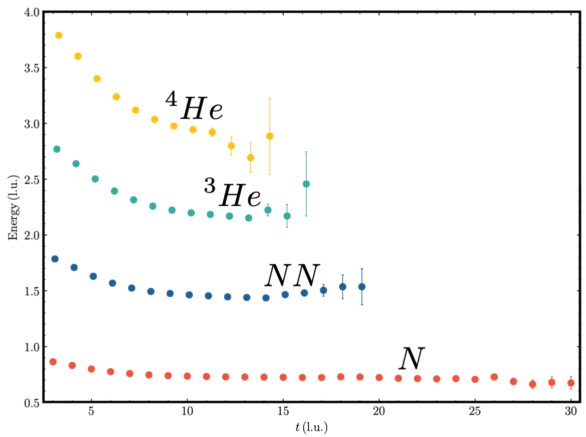
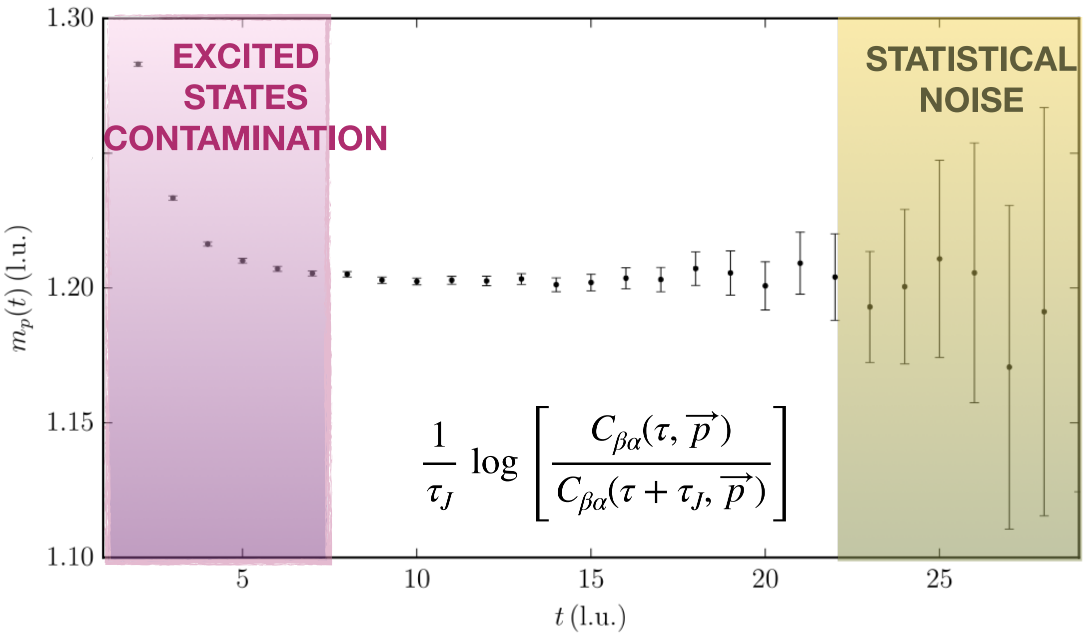


Increase the statistics / Increase the pion mass

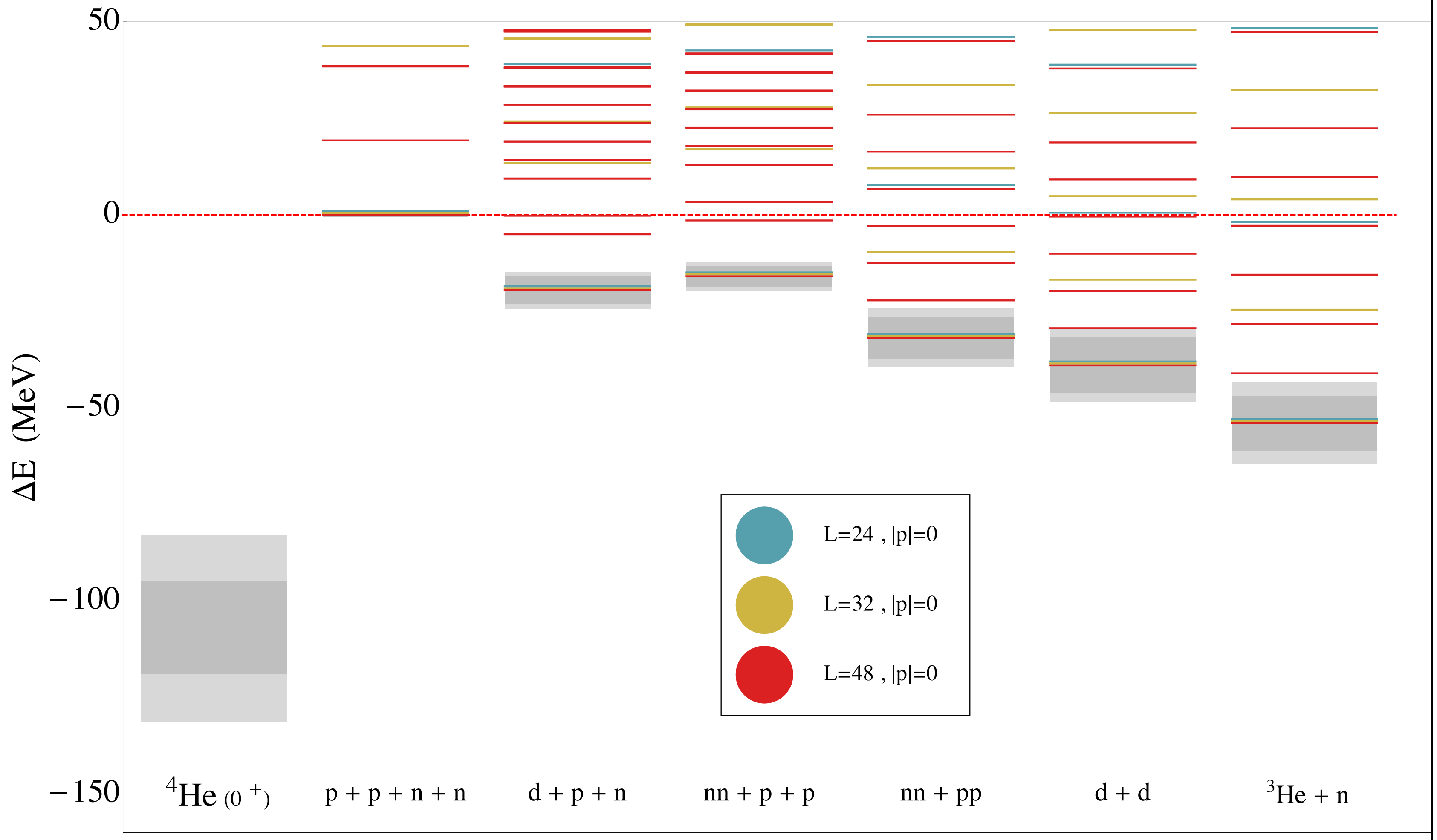
Construct operators with a better overlap with the ground state

signal-to-noise degradation

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x},t)\left(u^{j\mathsf{T}}(\mathbf{x},t) C\gamma_5 d^k(\mathbf{x},t)\right)$$



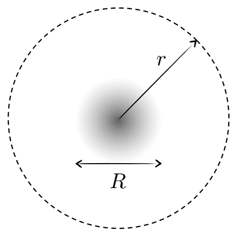
Small excited-state gaps may lead to incorrect identification of ground-state energy



S.R. Beane et al. [NPLQCD], Phys.Rev.D 87 (2013)

M. Lüscher, Comm. Math. Phys. 105 (1986), Nuc. Phys. B 354 (1991)

Infinite volume

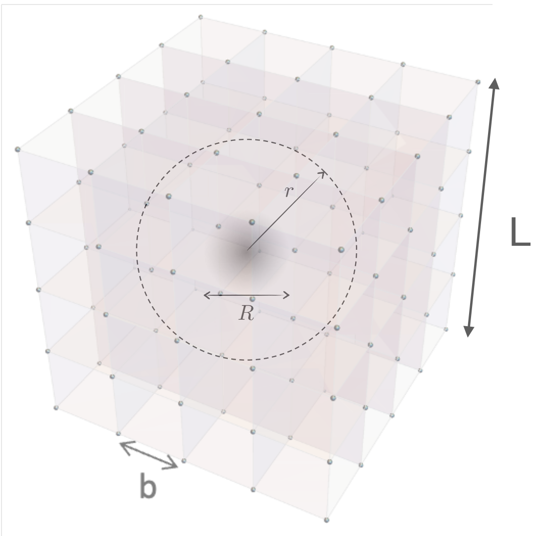


$$u_l(r;k) = \alpha_l(k) j_l(kr) + \beta_l(k) n_l(kr)$$

$$e^{2i\delta_l(k)} = \frac{\alpha_l(k) + i\beta_l(k)}{\alpha_l(k) - i\beta_l(k)}$$

M. Lüscher, Comm. Math. Phys. 105 (1986), Nuc. Phys. B 354 (1991)

finite volume

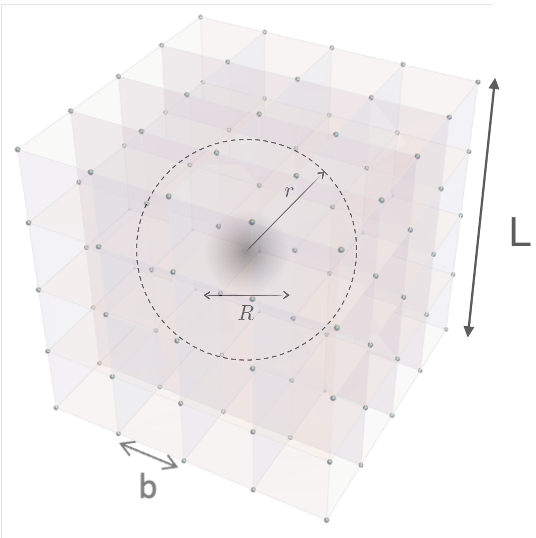


$$\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

$(L \gg R)$

M. Lüscher, Comm. Math. Phys. 105 (1986), Nuc. Phys. B 354 (1991)

finite volume



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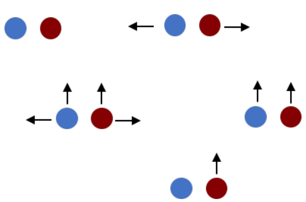


$$\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

$\Delta E \to k*$

kinematic function

Generalized $\mathcal{Z}$ function

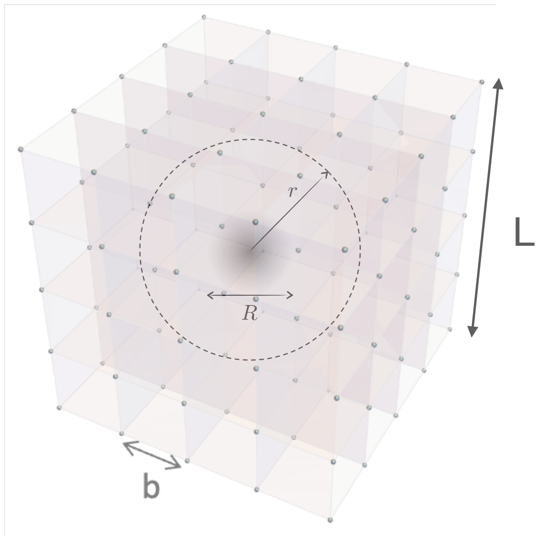
$$\mathcal{Z}^{\boldsymbol{d}}_{lm}(s;q^2) = \sum_{\boldsymbol{r}} \frac{r^l Y_{lm}(\hat{\boldsymbol{r}})}{(|\boldsymbol{r}|^2 - q^2)^s}$$

$$\boldsymbol{r} = \hat{\gamma}^{-1}(\boldsymbol{n} - \alpha\boldsymbol{d})\,,\ \boldsymbol{n} \in \mathbb{Z}^3$$

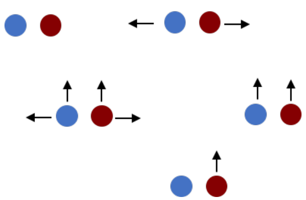
$$\alpha = \frac{1}{2}\left(1 + \frac{m_1^2 - m_2^2}{E^{*2}}\right)$$

M. Lüscher, Comm. Math. Phys. 105 (1986), Nuc. Phys. B 354 (1991)

finite volume



$(L \gg R)$



$$\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right] = 0 \longrightarrow k^{*}\cot\delta = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}\left(1;\left(\frac{k^{*}L}{2\pi}\right)^{2}\right)$$

$\Delta E \rightarrow k^{*}$

kinematic function

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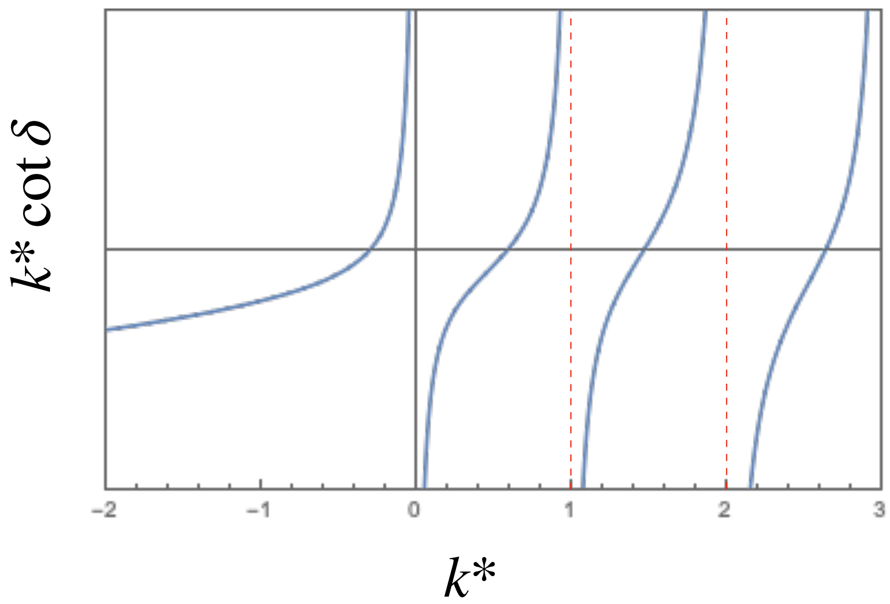

M. Lüscher, Comm. Math. Phys. 105 (1986), Nuc. Phys. B 354 (1991)

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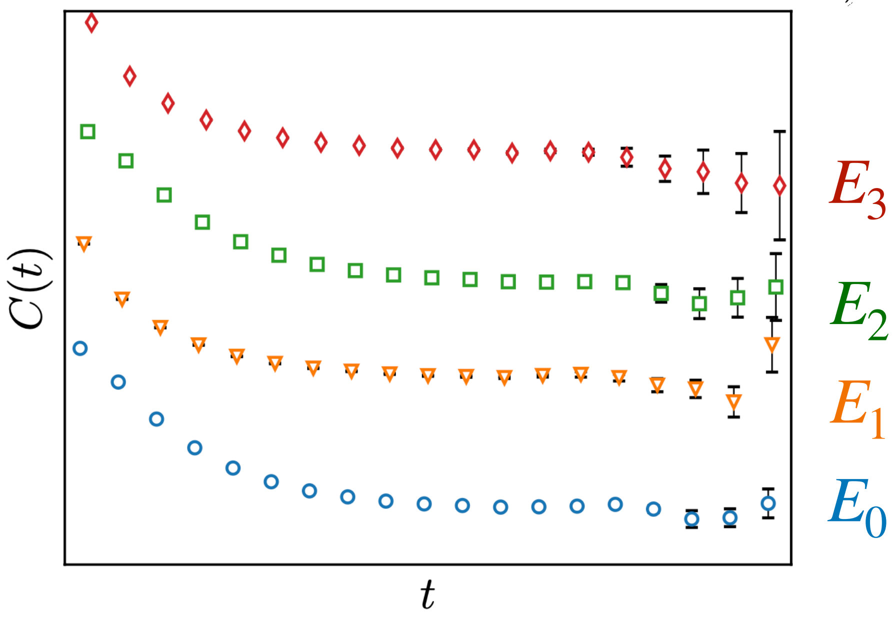
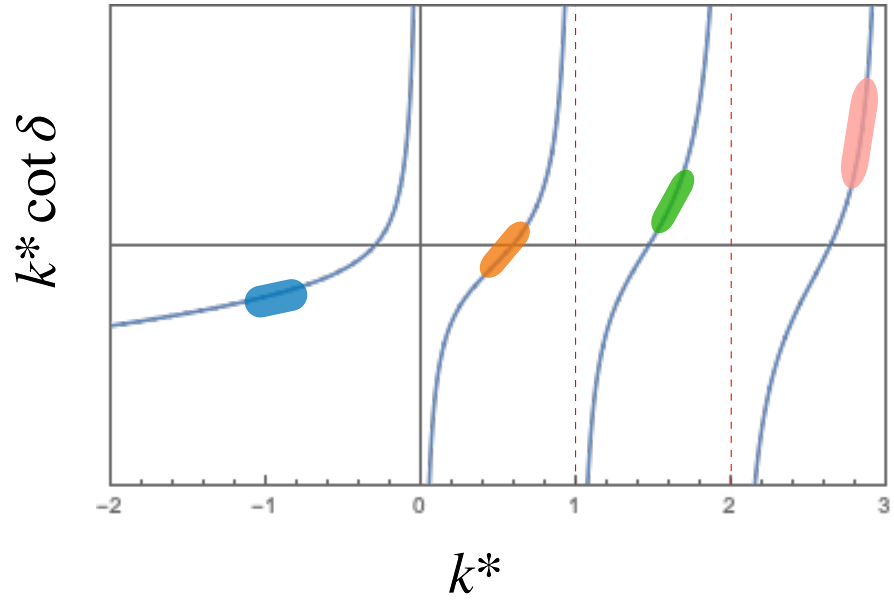
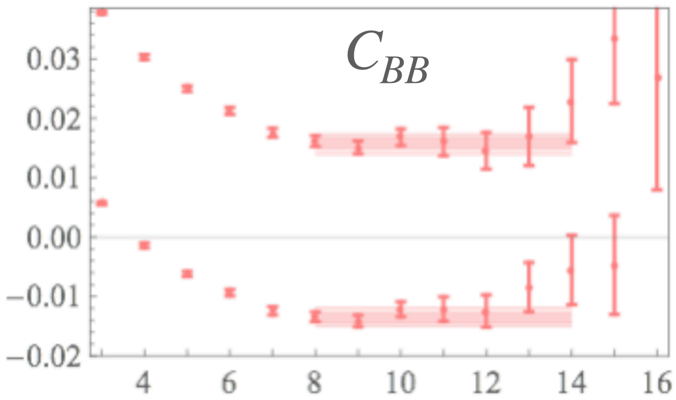
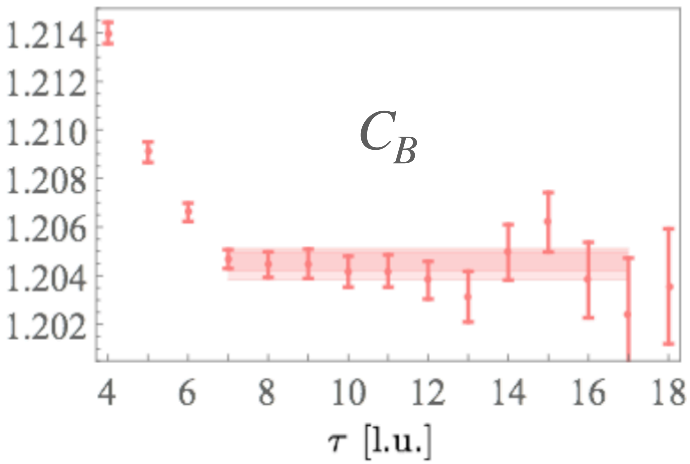




$\Delta E \to k^*$





$$R(\tau, \boldsymbol{p}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{p})}{C_{B_1}(\tau, \boldsymbol{p}) C_{B_2}(\tau, \boldsymbol{p})}$$

$$\downarrow$$

$$\Delta E_{\mathfrak{n}} \longrightarrow k^*$$

Lüscher's method

$$\downarrow$$

$$k^* \cot \delta$$

$$k^* \cot \delta = \frac{2}{\sqrt{\pi} L} \mathcal{Z}_{00}(1; (\tfrac{k^* L}{2\pi})^2)$$

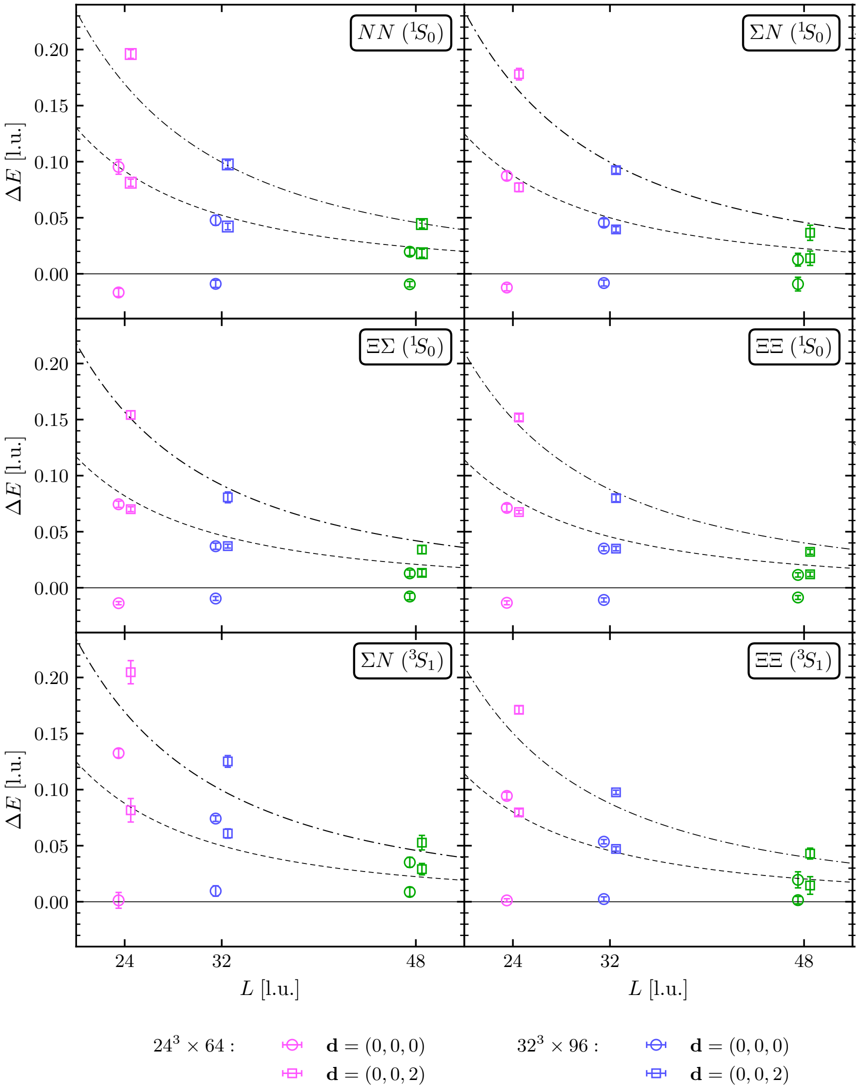
$k^{*2} < 0$

Beane, Bedaque, Parreño, Savage, PLB585 (2004)
Davoudi, Savage, PRD84 (2011)

$$\downarrow$$

$$|k^*| = \kappa^{(\infty)} + \frac{Z^2}{L} \left[6 e^{-\kappa^{(\infty)} L} + \cdots \right]$$

B



M. Illa et al. [NPLQCD], Phys.Rev.D 103 (2021)

$m_\pi \sim 450$ MeV

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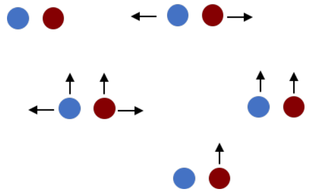

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B

L [fm]	T [fm]
3.4	6.7
4.5	6.7
6.7	9

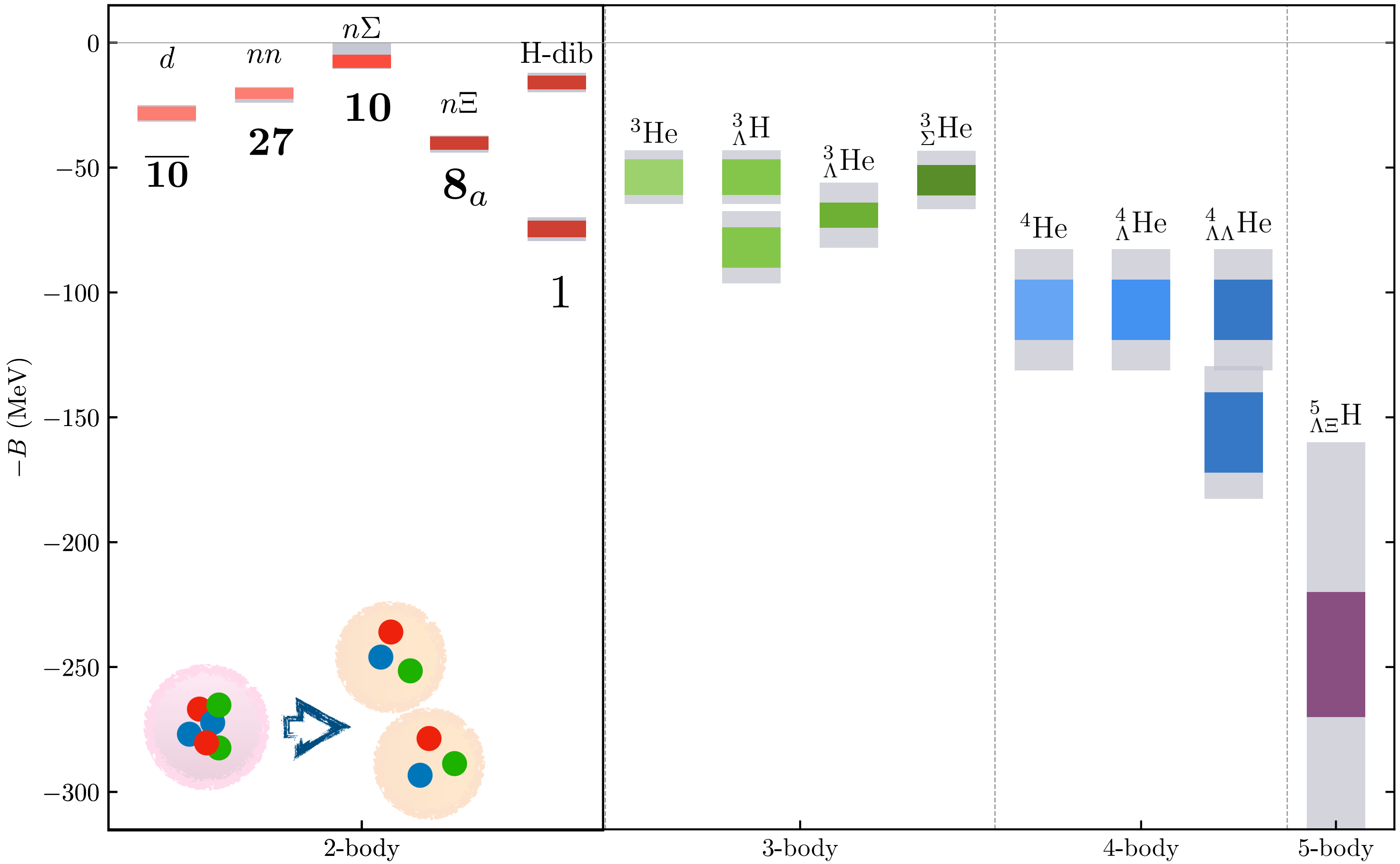
$b[fm] = 0.1453(16)$

$\boxed{SU(3)_f}$

$m_\pi \sim 800$ MeV

no e.m. interactions

NPLQCD, Phys.Rev. D87 (2013) no.3, 034506; Phys.Rev. D96 (2017) no.11, 114510



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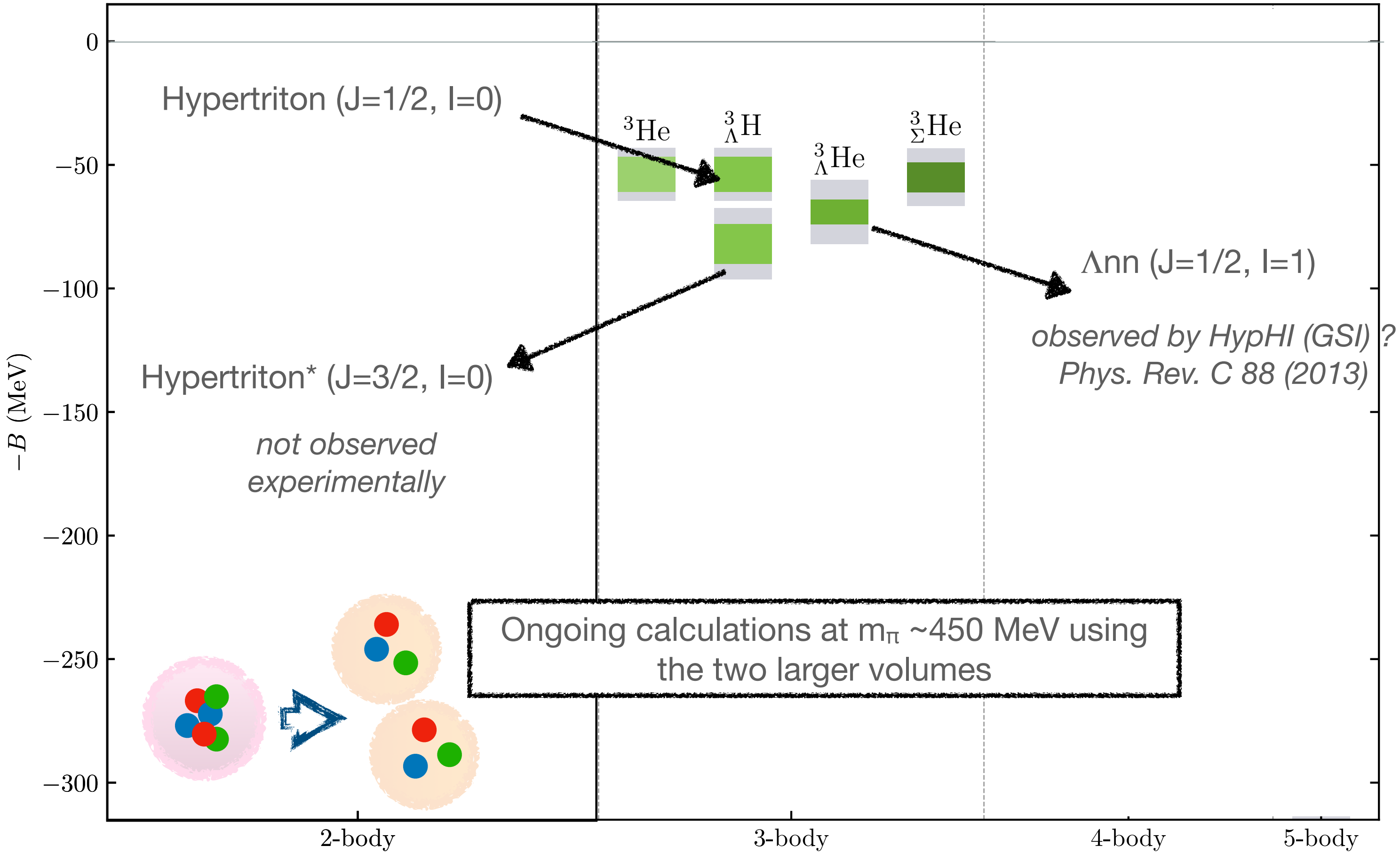
NPLQCD, Phys.Rev. D87 (2013) no.3, 034506;



away from the SU(3)$_f$ limit

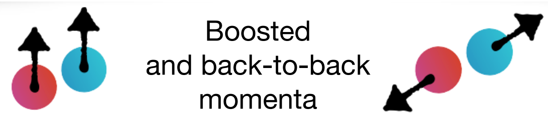
$n_f = 2 + 1$

$m_\pi = 450(5)\text{MeV}$

$b = 0.117(2)$ fm

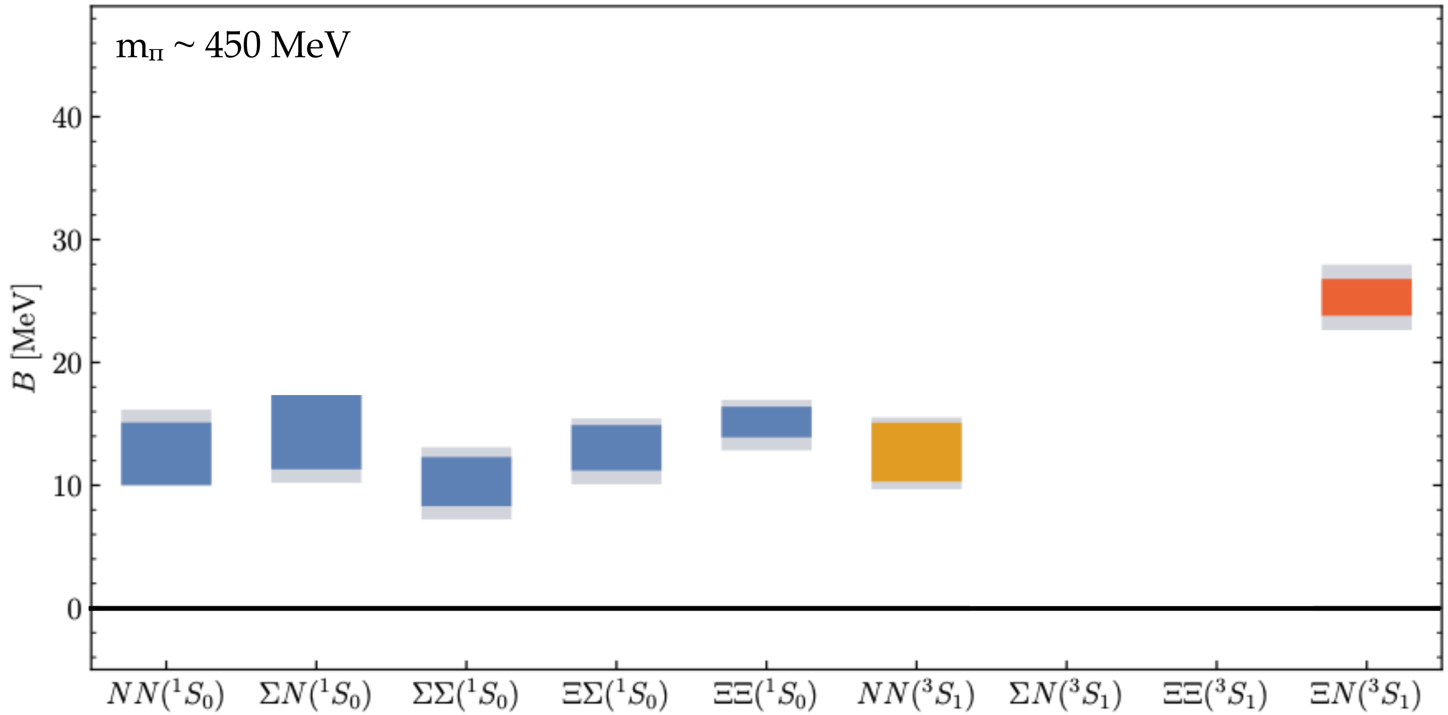
$L = 2.8, 3.7, 5.6$ fm

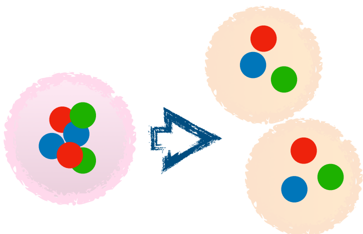
$T = 7.5, 11.2, 11.2$ fm



no e.m. interactions

Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508





away from the SU(3)$_f$ limit

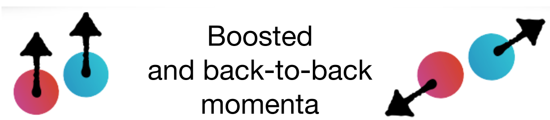
$n_f = 2 + 1$

$m_\pi = 450(5)\mathrm{MeV}$

$b = 0.117(2)$ fm

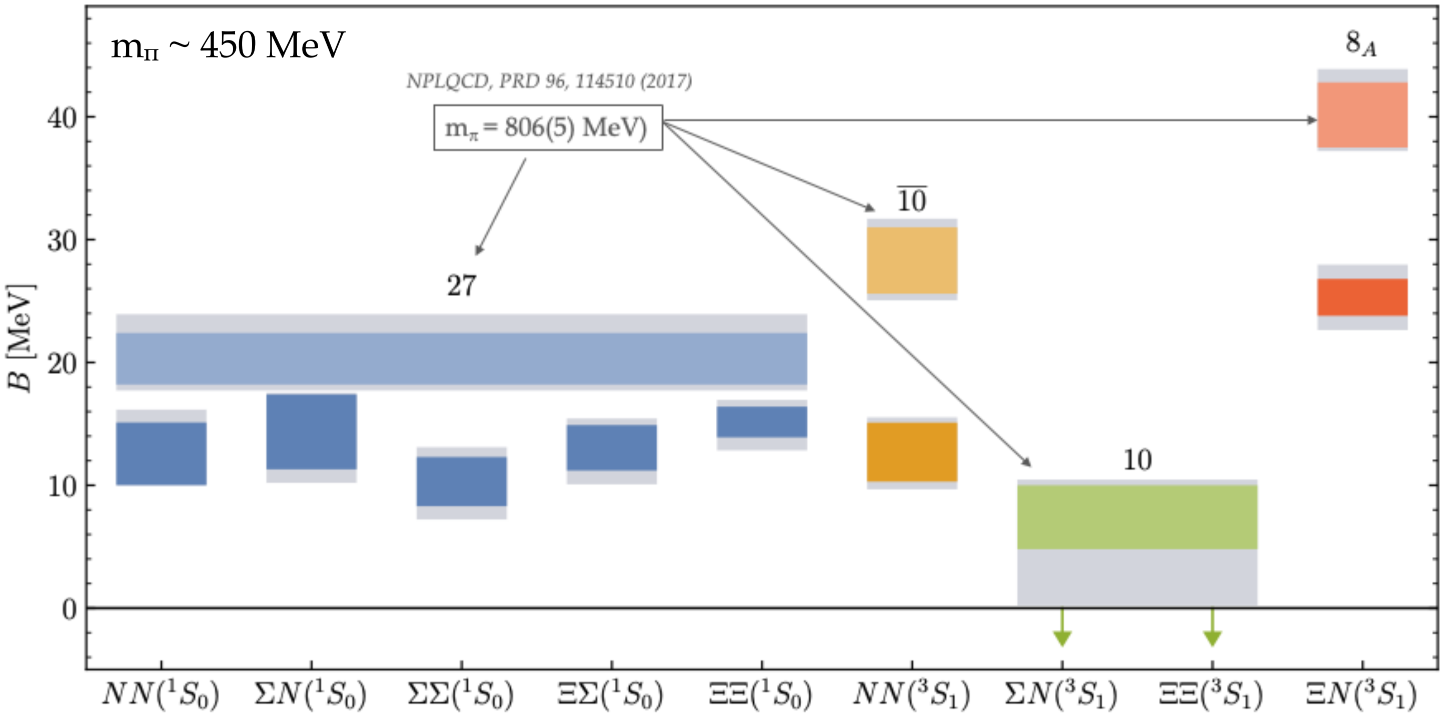
$L = 2.8, 3.7, 5.6$ fm

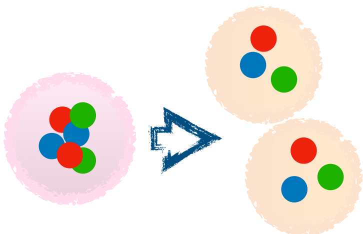
$T = 7.5, 11.2, 11.2$ fm



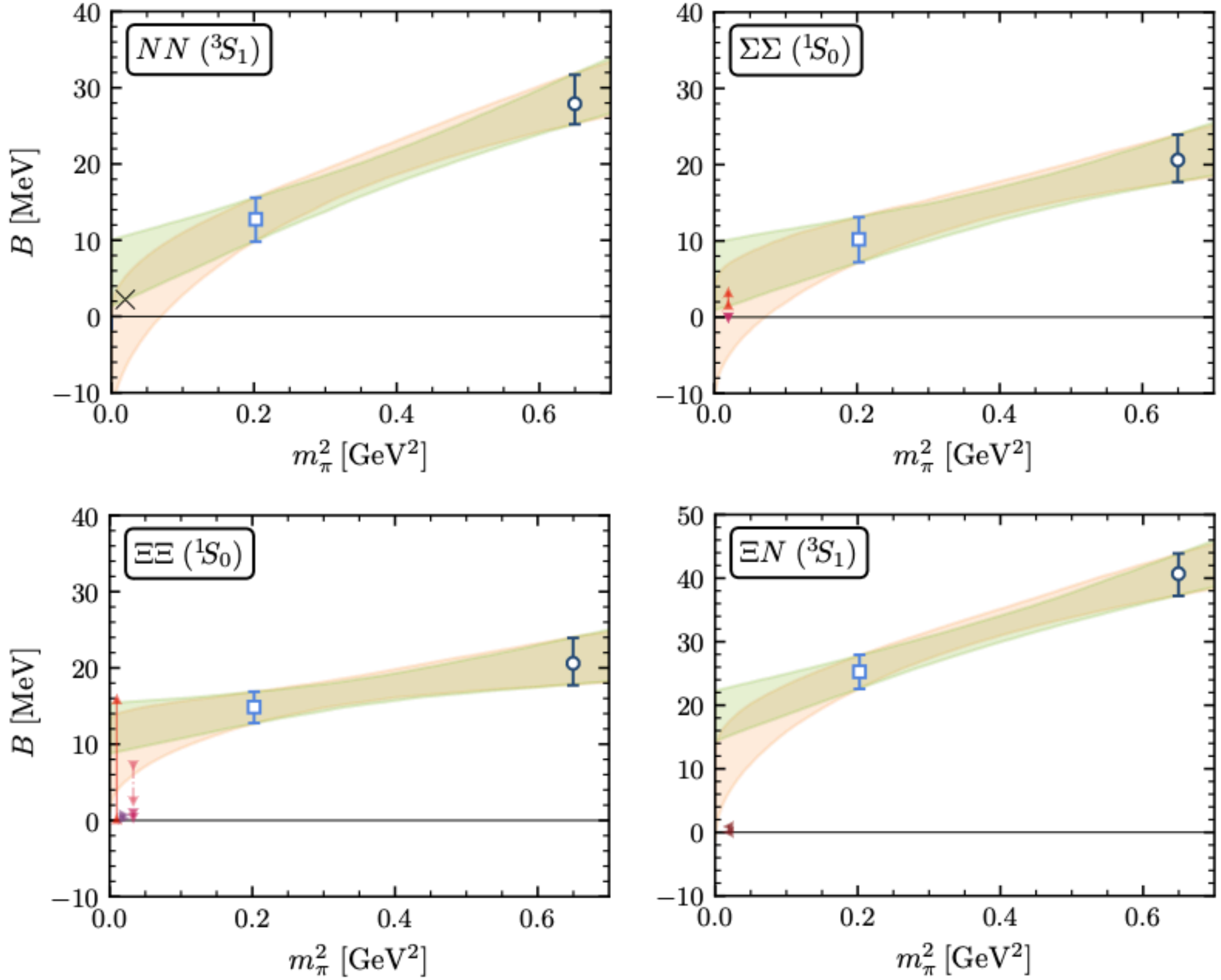
no e.m. interactions

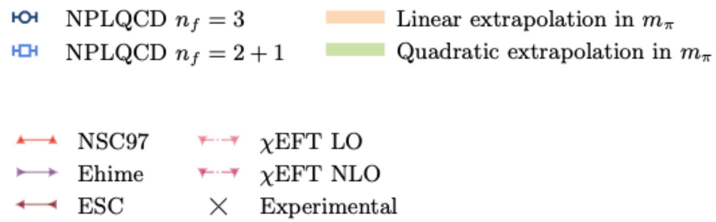
Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508





Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508

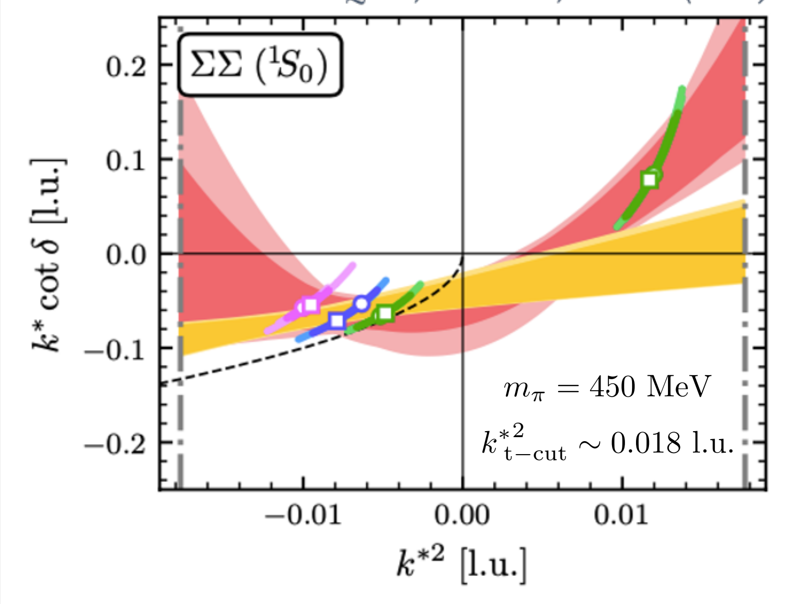




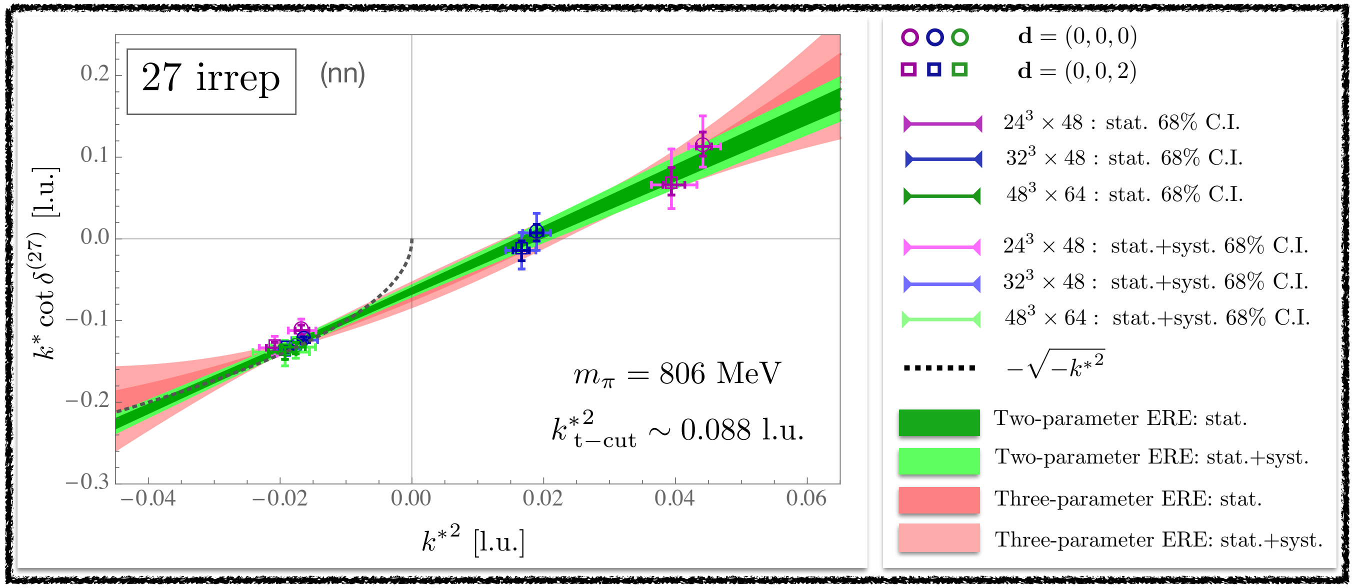
$$\boxed{B_{\mathrm{lin}}(m_\pi) = B^{(0)}_{\mathrm{lin}} + B^{(1)}_{\mathrm{lin}} m_\pi}$$

$$\boxed{B_{\mathrm{quad}}(m_\pi) = B^{(0)}_{\mathrm{quad}} + B^{(1)}_{\mathrm{quad}} m_\pi^2}$$

NPLQCD, PRD 103, 054508 (2021)



NPLQCD, PRD 96, 114510 (2017)



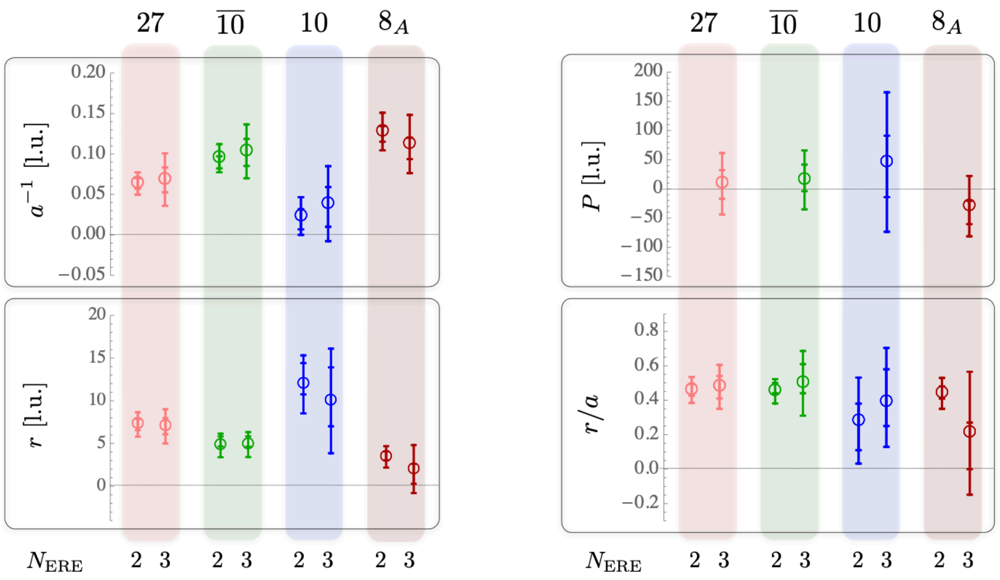
$$k^* \cot\delta = \boxed{\boxed{-\frac{1}{a} + \frac{1}{2} r k^{*2}} + P k^{*4}} + \mathcal{O}(k^{*6})$$

$$k^* \cot\delta = \boxed{\boxed{-\frac{1}{a} + \frac{1}{2} r k^{*2}} + P k^{*4}} + \mathcal{O}(k^{*6})$$

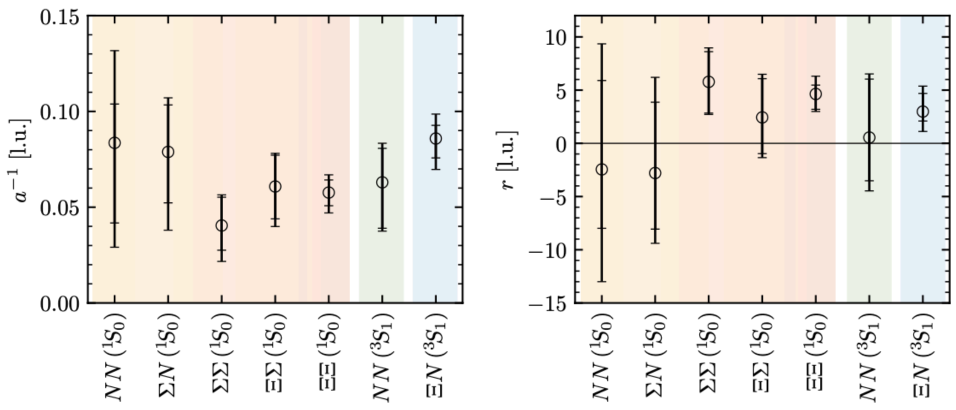
NPLQCD, PRD 96, 114510 (2017)

$\boxed{SU(3)_f}$

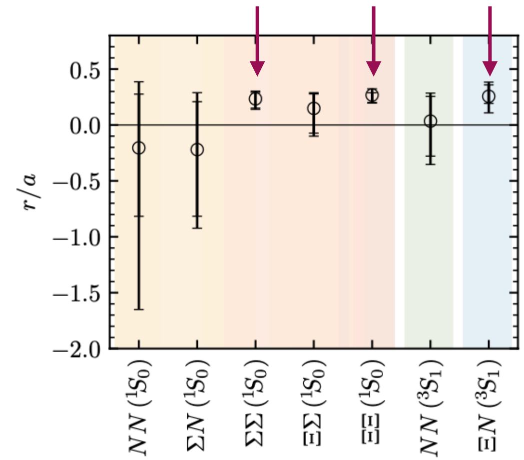
$m_\pi \sim 800$ MeV



NPLQCD, PRD 103, 054508 (2021) $m_\pi = 450$ MeV

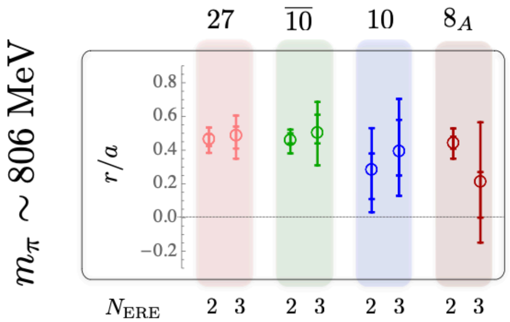


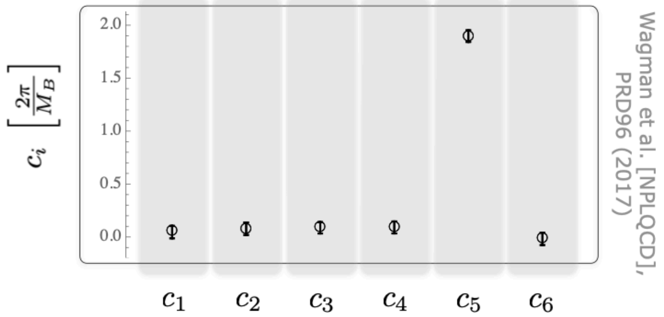
most constrained values show unnaturalness (r/a ~0.2 - 0.3)

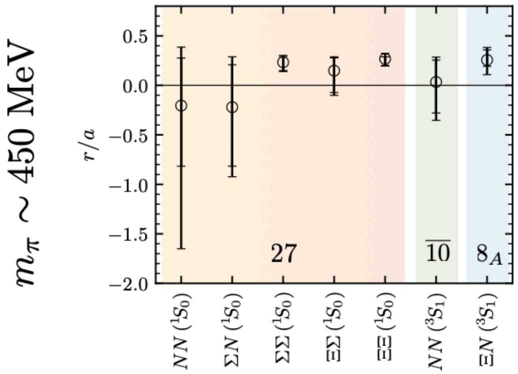


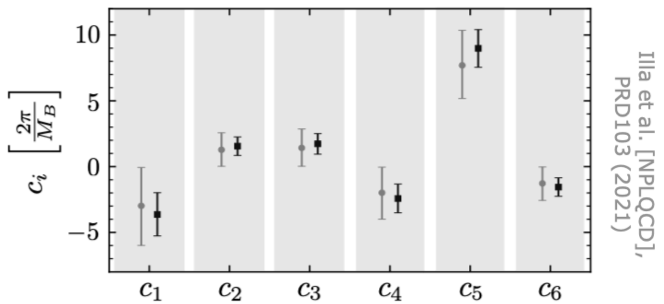
Extracting scattering information

Scattering parameters $\longrightarrow$ EFT matching









Assuming $SU(3)_f$, at leading order we have

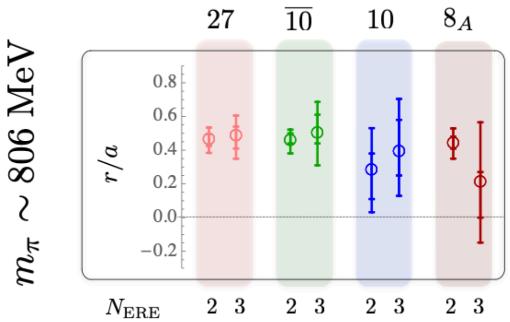
M.J. Savage, M. Wise, Phys.Rev.D 53 (1996)

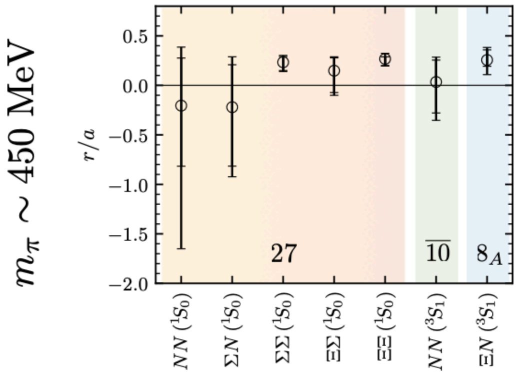
$$\mathcal{L}^{(0),\,SU(3)}_{BB} = -\,c_1 \mathrm{Tr}(B_i^\dagger B_i B_j^\dagger B_j) - c_2 \mathrm{Tr}(B_i^\dagger B_j B_j^\dagger B_i) - c_3 \mathrm{Tr}(B_i^\dagger B_j^\dagger B_i B_j)$$
$$- c_4 \mathrm{Tr}(B_i^\dagger B_j^\dagger B_j B_i) - c_5 \mathrm{Tr}(B_i^\dagger B_i)\mathrm{Tr}(B_j^\dagger B_j) - c_6 \mathrm{Tr}(B_i^\dagger B_j)\mathrm{Tr}(B_j^\dagger B_i)$$

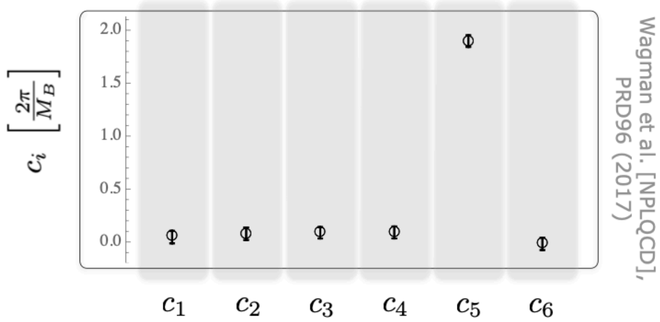
$$c_1, \ldots, c_6 \quad \longrightarrow \quad c^{(27)}, \ldots, c^{(8_a)}$$

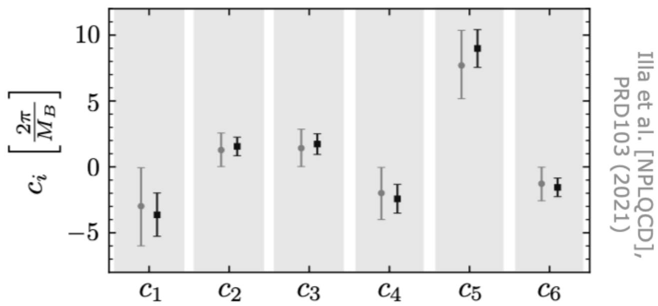
Extracting scattering information

Scattering parameters $\longrightarrow$ EFT matching









Agreement with the large-Nc prediction of an SU(6) symmetry

D. Kaplan, M.J. Savage, Phys.Lett.B 365 (1996)

Assuming $SU(3)_f$, at leading order we have

M.J. Savage, M. Wise, Phys.Rev.D 53 (1996)

$$\mathcal{L}^{(0),\,SU(3)}_{BB} = -\,c_1\mathrm{Tr}(B_i^\dagger B_i B_j^\dagger B_j) - c_2\mathrm{Tr}(B_i^\dagger B_j B_j^\dagger B_i) - c_3\mathrm{Tr}(B_i^\dagger B_j^\dagger B_i B_j) - c_4\mathrm{Tr}(B_i^\dagger B_j^\dagger B_j B_i) - c_5\mathrm{Tr}(B_i^\dagger B_i)\mathrm{Tr}(B_j^\dagger B_j) - c_6\mathrm{Tr}(B_i^\dagger B_j)\mathrm{Tr}(B_j^\dagger B_i)$$

$$c_1, \ldots, c_6 \longrightarrow c^{(27)}, \ldots, c^{(8_a)}$$

extension to larger systems in $SU(3)_f$

EFT ($\not{\pi}$) at LO

$$V^{LO} = \begin{cases} C_0 + C_1 \vec{\sigma}_1 \vec{\sigma}_2 & \text{2B(attractive)} \\ +D_0 & \text{3B(repulsive)} \end{cases}$$

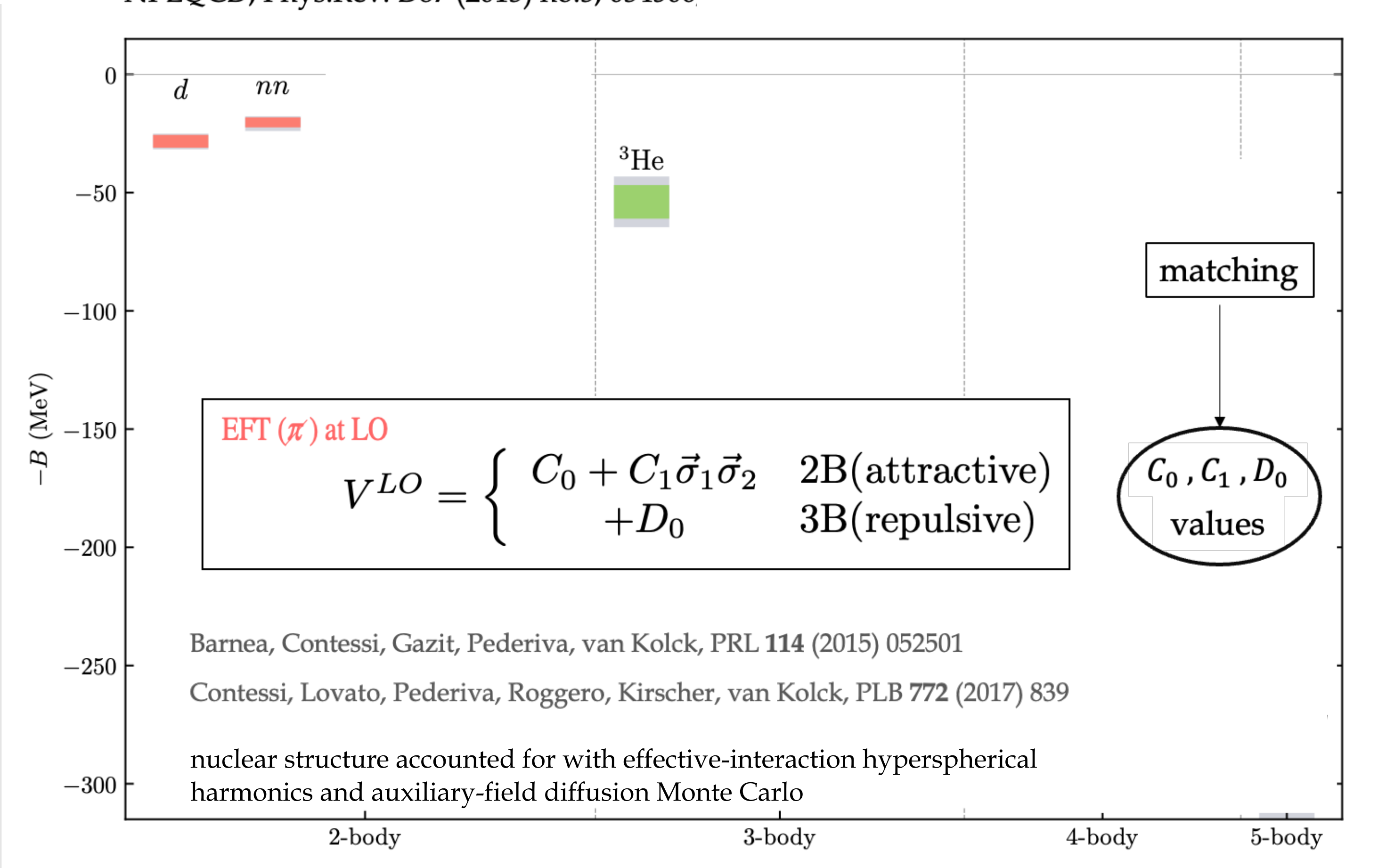
Barnea, Contessi, Gazit, Pederiva, van Kolck, PRL **114** (2015) 052501

Contessi, Lovato, Pederiva, Roggero, Kirscher, van Kolck, PLB **772** (2017) 839

nuclear structure accounted for with effective-interaction hyperspherical harmonics and auxiliary-field diffusion Monte Carlo

EFT. Extension to larger systems in $SU(3)_f$

NPLQCD, Phys.Rev. D87 (2013) no.3, 034506



EFT ($\not\pi$) at LO

$$V^{LO} = \begin{cases} C_0 + C_1 \vec{\sigma}_1 \vec{\sigma}_2 & \text{2B(attractive)} \\ +D_0 & \text{3B(repulsive)} \end{cases}$$

Barnea, Contessi, Gazit, Pederiva, van Kolck, PRL **114** (2015) 052501

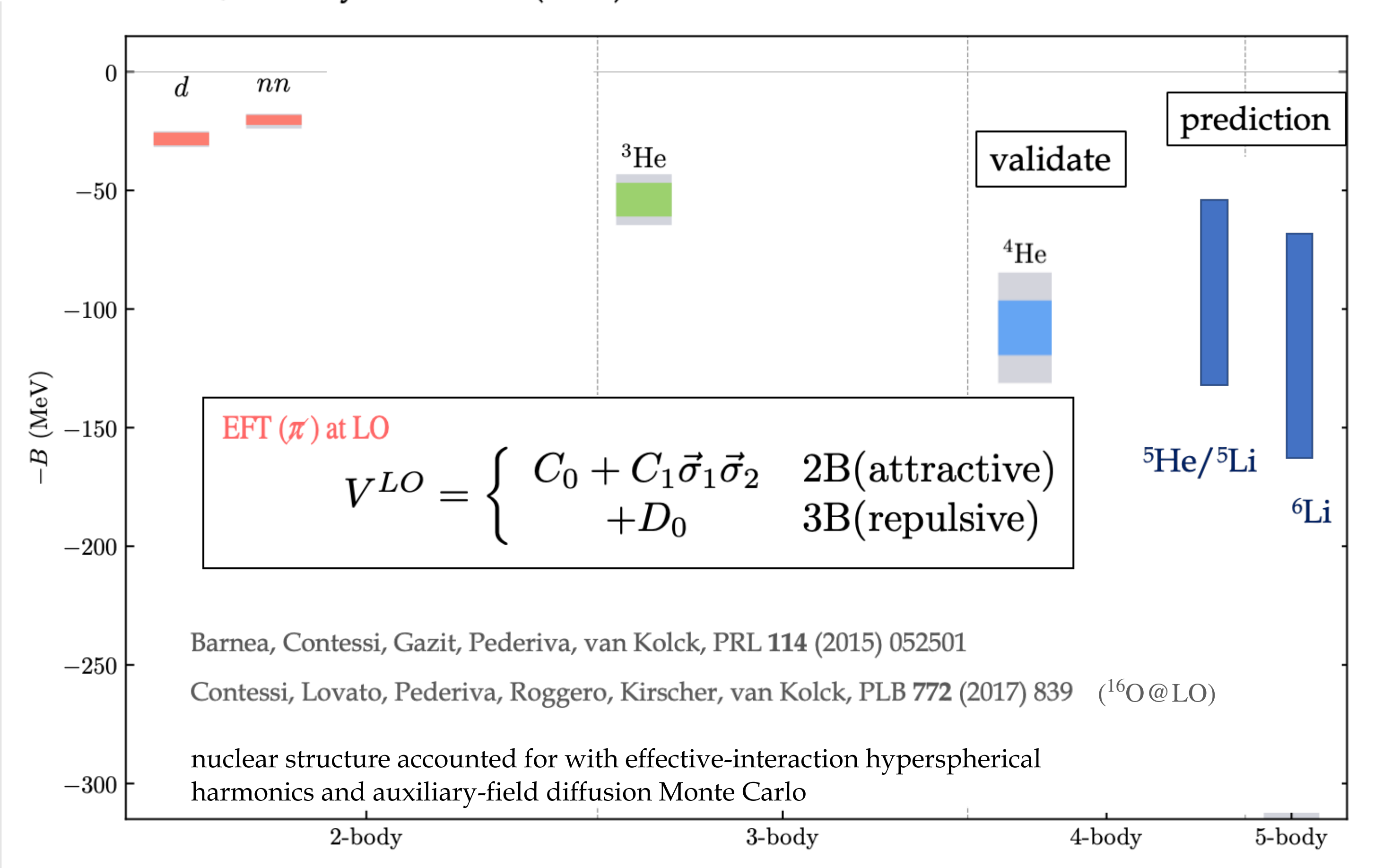
Contessi, Lovato, Pederiva, Roggero, Kirscher, van Kolck, PLB **772** (2017) 839

nuclear structure accounted for with effective-interaction hyperspherical harmonics and auxiliary-field diffusion Monte Carlo

no e.m. interactions

LQCD → FEW-BODY CALCS → NUCLEAR MANY-BODY CALCS

NPLQCD, Phys.Rev. D87 (2013) no.3, 034506;



no e.m. interactions

LQCD → FEW-BODY CALCS → NUCLEAR MANY-BODY CALCS

PHYSICAL REVIEW C **98**, 054301 (2018)

Pion-less effective field theory for atomic nuclei and lattice nuclei

A. Bansal,[1] S. Binder,[1,2] A. Ekström,[3,2] G. Hagen,[2,1] G. R. Jansen,[4,2] and T. Papenbrock[1,2]
[1]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[2]*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[3]*Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden*
[4]*National Center for Computational Sciences, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*

(Received 29 December 2017; revised manuscript received 29 June 2018; published 1 November 2018)

We compute the medium-mass nuclei ^{16}O and ^{40}Ca using pion-less effective field theory (EFT) at next-to-leading order (NLO). The low-energy coefficients of the EFT Hamiltonian are adjusted to experimental data for nuclei with mass numbers $A = 2$ and 3, or alternatively to results from lattice quantum chromodynamics at an unphysical pion mass of 806 MeV. The EFT is implemented through a discrete variable representation in the harmonic oscillator basis. This approach ensures rapid convergence with respect to the size of the model space and facilitates the computation of medium-mass nuclei. At NLO the nuclei ^{16}O and ^{40}Ca are bound with respect to decay into alpha particles. Binding energies per nucleon are 9–10 MeV and 21–40 MeV at pion masses of 140 and 806 MeV, respectively.

DOI: 10.1103/PhysRevC.98.054301

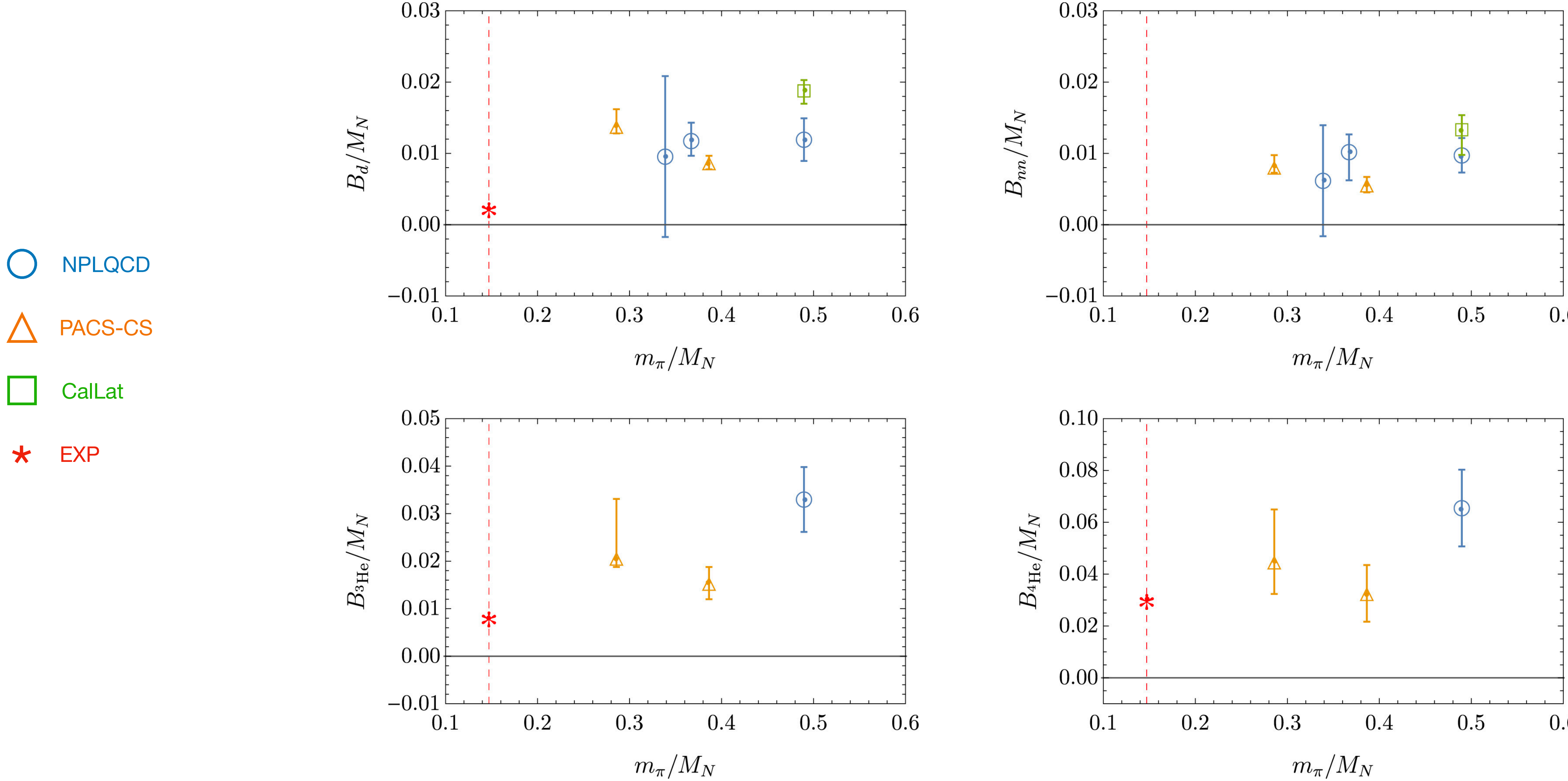
$$
\begin{aligned}
V_{NN}^{\mathrm{LO}}(^1S_0) &= \tilde{C}_{^1S_0} = C_S - 3C_T, & V_{NN}^{\mathrm{NLO}}(^1S_0) &= C_{^1S_0}(p^2 + p'^2),\\
V_{NN}^{\mathrm{LO}}(^3S_1) &= \tilde{C}_{^3S_1} = C_S + C_T, & V_{NN}^{\mathrm{NLO}}(^3S_1) &= C_{^3S_1}(p^2 + p'^2).
\end{aligned}
$$

Extension to ^{16}O and ^{40}Ca

Our results also suggest that medium-mass nuclei can be connected to lattice QCD input. Further progress in this direction, however, requires a resolution of the controversy between the different lattice QCD approaches to light nuclei, increasing the precision of the lattice QCD results that are input to EFTs, and finally, moving towards the physical pion mass.

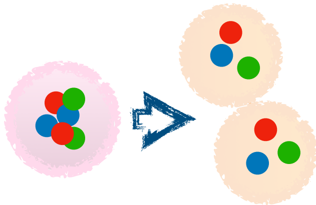
Nuclear physics with LQCD - Controversy

Davoudi, Detmold, Shanahan, Orginos, Parreño, Savage, Wagman, Physics Reports 900 (2021) 1–74

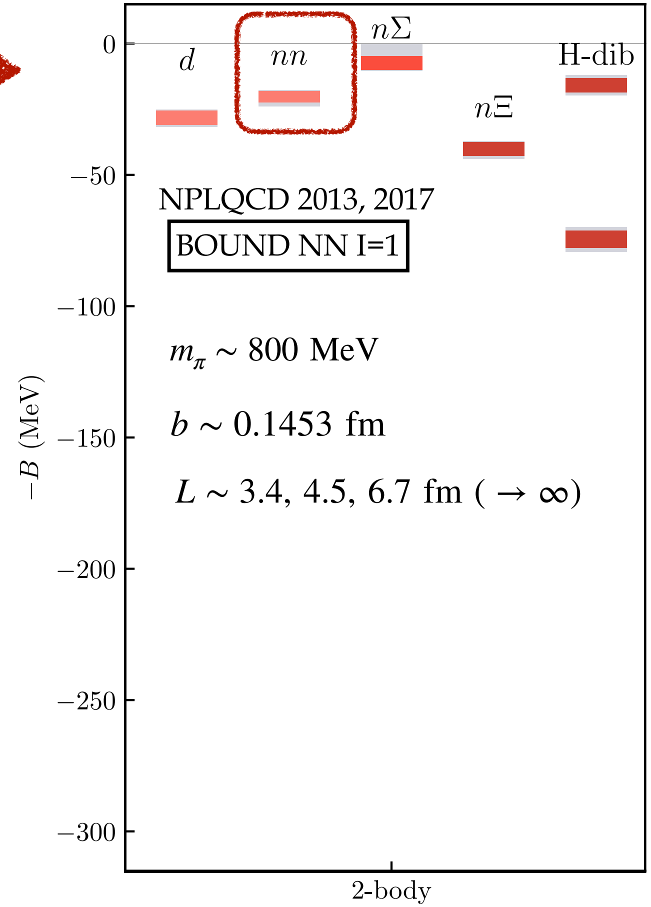


Misidentification of the plateau?

Deep bound states



E. Berkowitz et al. [CalLat], Phys.Lett.B 765 (2017)
S.R. Beane et al. [NPLQCD], arXiv:1705.09239 [hep-lat]
T. Yamazaki et al. [PACS], EPJ Web Conf. 175 (2018)



Small excited-state gaps may lead to incorrect identification of the ground-state energy

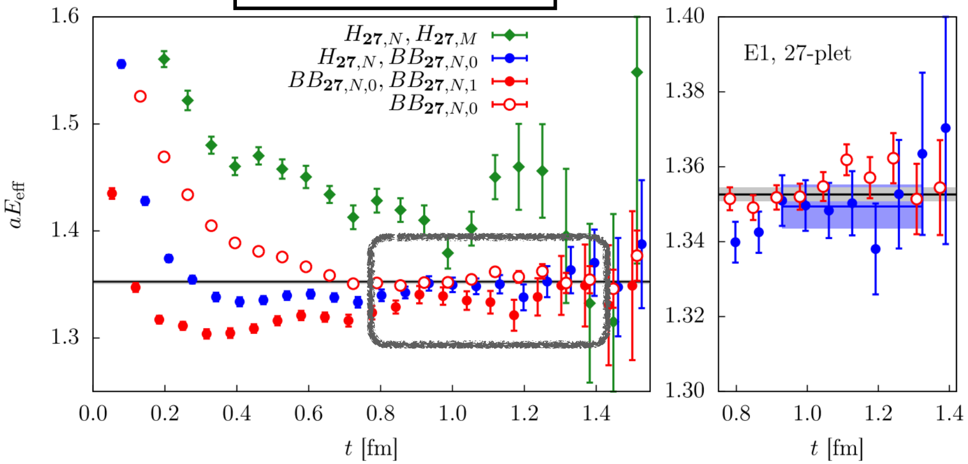
- Is the fitting interval correctly identified?
- Are we missing excited state contributions?
- Is there an operator dependence on the energy levels extracted?

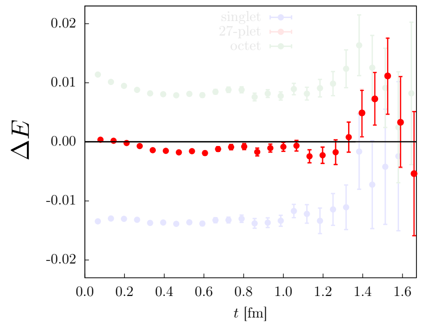
Reduce uncertainty at small time: GPoF, matrix Prony, variational

First variational calculation A. Francis et al., Phys.Rev.D 99 (2019)

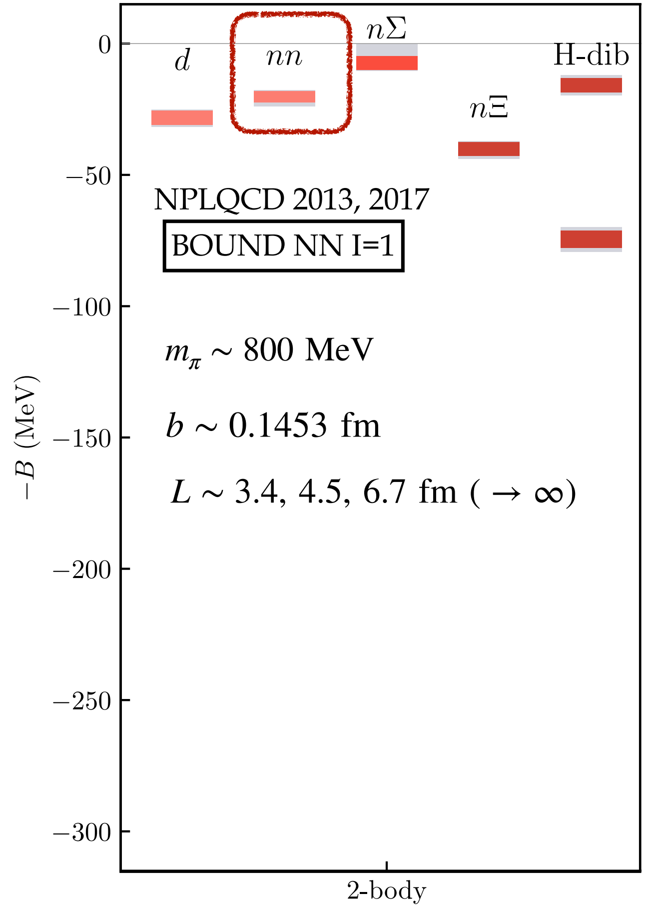
$m_\pi \sim 960$ MeV

UNBOUND NN I=1

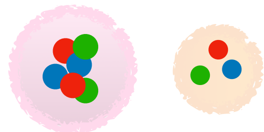




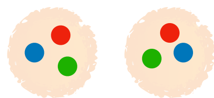
Ground-state energy close to threshold



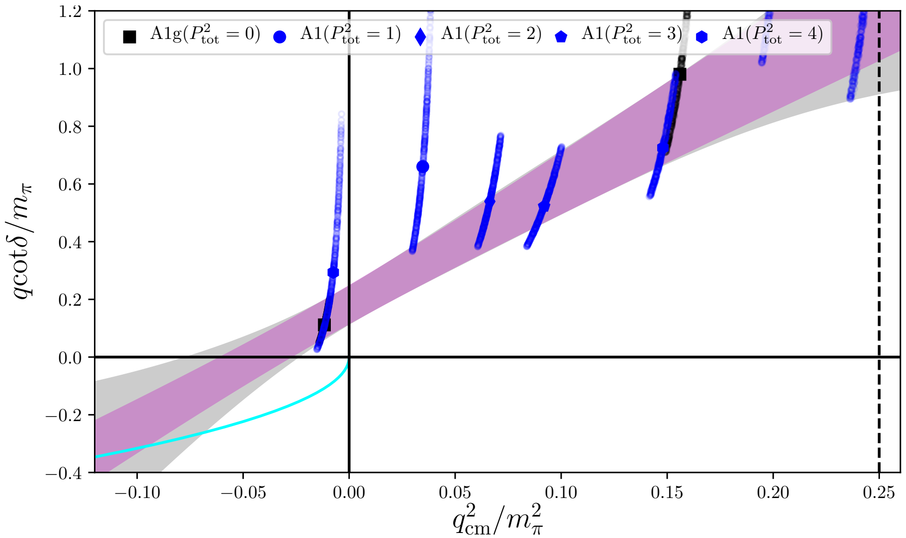
Hermitian 2x2 matrix with hexaquark and dibaryon-like operators



Non-hermitian 2x2 matrix with dibaryon-like operators



CalLat B. Hörz et al., Phys.Rev.C 103 (2021)



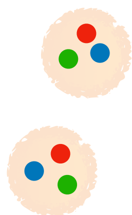
$m_\pi \sim 714$ MeV

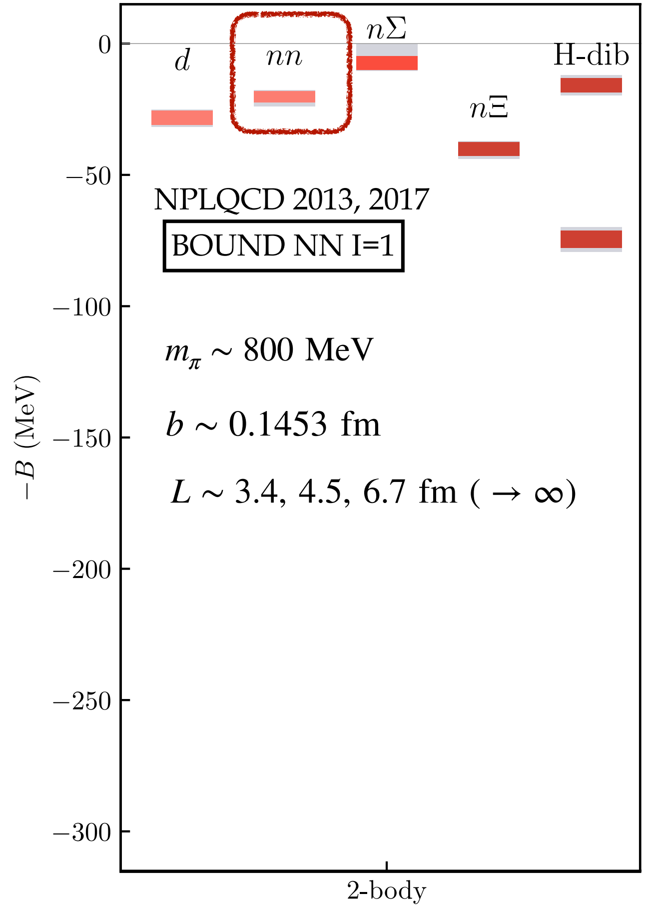
UNBOUND NN I=1

$b \sim 0.086$ fm

$L = 48\, b \sim 4.1$ fm

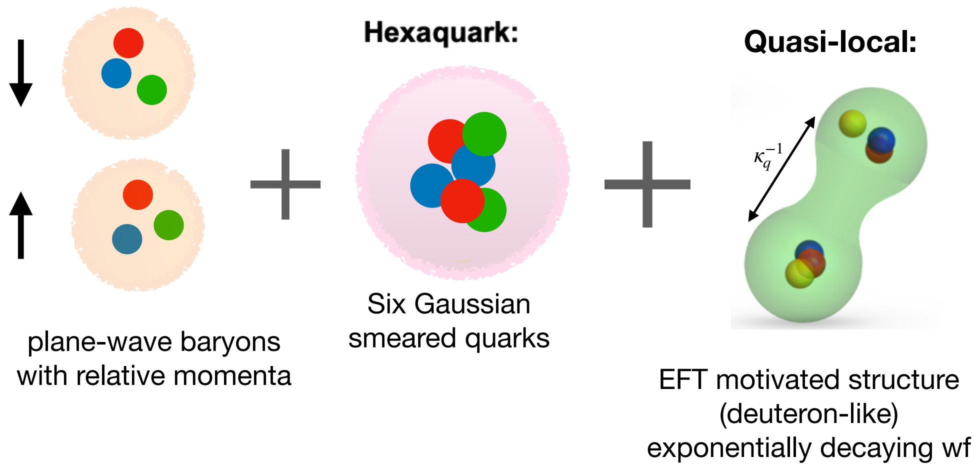
Hermitian 2x2 matrix with dibaryon-like operators





Nuclear physics with LQCD - Variational calculation

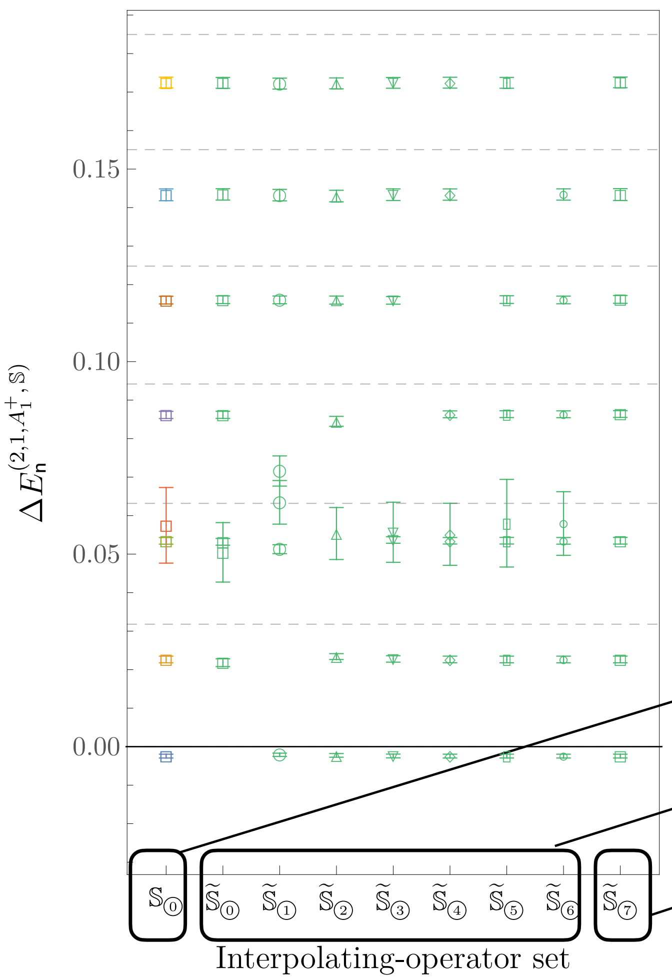
S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



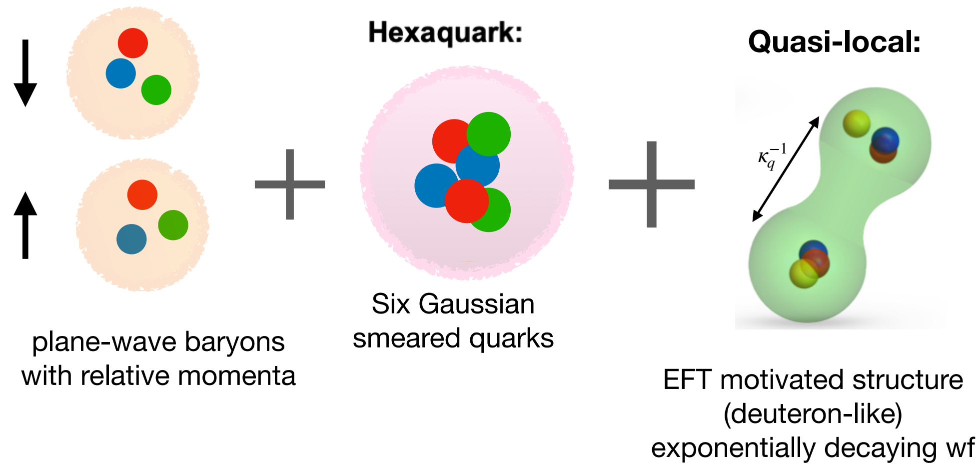
Largest set of operators to date
(ongoing work)

b=0.145 fm , L/b=32 (4.7 fm aprox)

NN (I=1)



S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



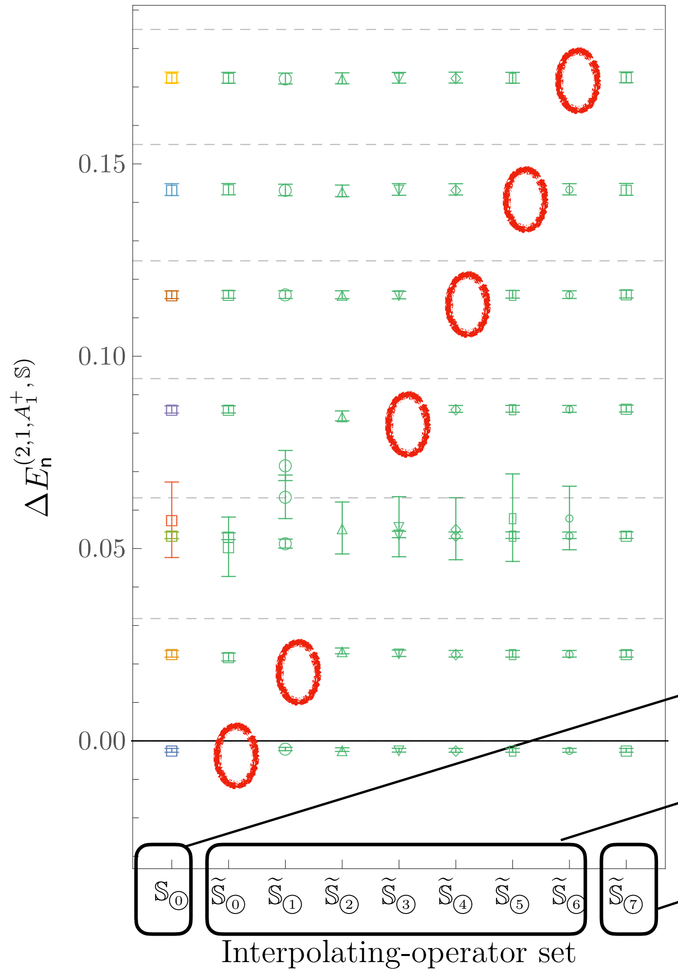
S_0 contains all operators except the quasi-locals
(hexaquark and dibaryons ops with different relative momentum)

Set without a particular dibaryon operator
(taking out a dibaryon op with a given value of the relative momentum)

Set with only the whole set of dibaryon operators (NO hexaquark)

S. Amarasinghe et al (NPLQCD), arXiv:2108.10835

NN (I=1)



Similarly with what happens in the meson sector, removing the operator structure with maximum overlap on to a given energy level leads to missing energy levels

Importance of using an interpolating-operator set with significant overlap onto all energy levels of interest.

Having a large interpolating-operator set is not sufficient to guarantee that a set will have good overlap onto the ground state or a particular excited state

NEEDS MORE WORK

S_0 contains all operators except the quasi-locals

(hexaquark and dibaryons ops with different relative momentum)

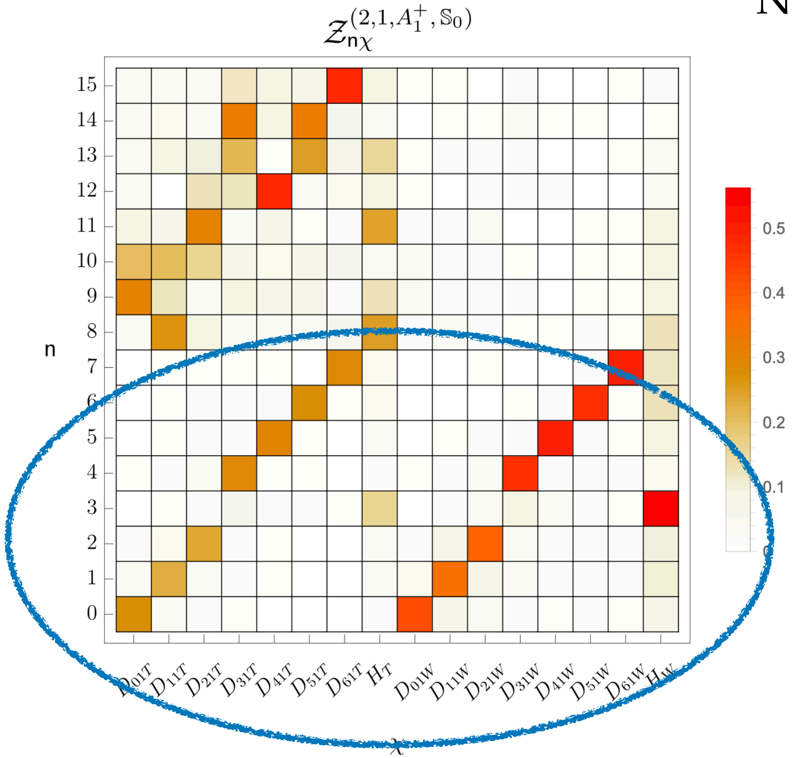
Set without a particular dibaryon operator

(taking out a dibaryon op with a given value of the relative momentum)

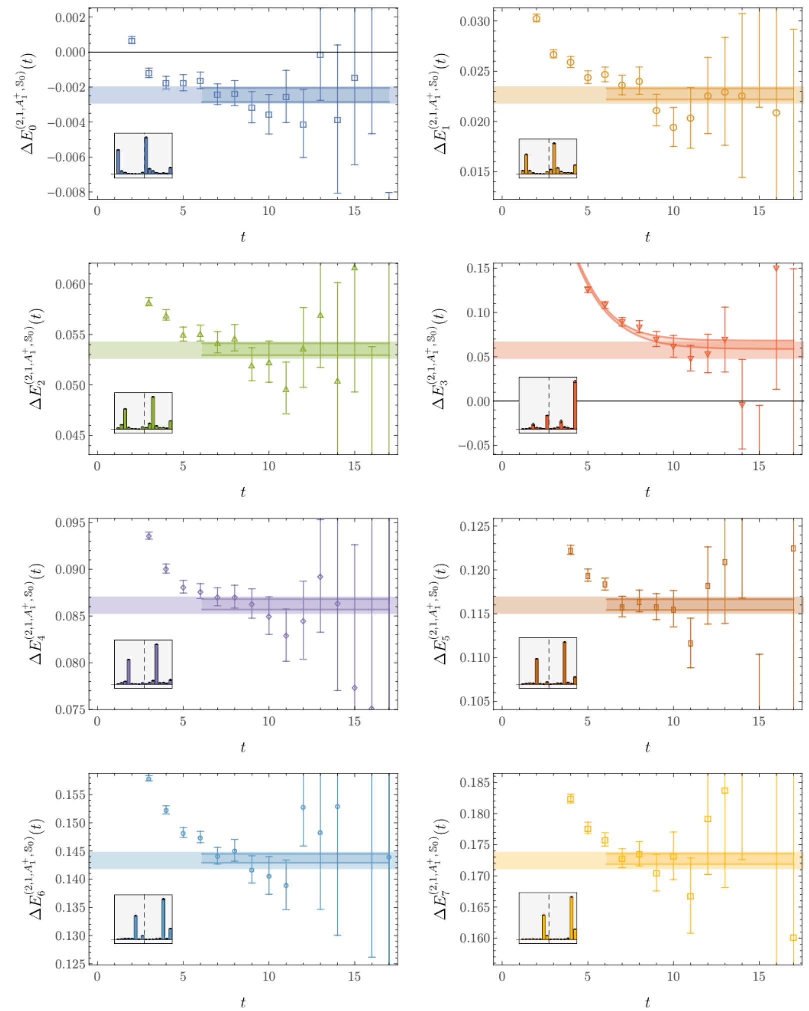
Set with only the whole set of dibaryon operators

NN, I=1

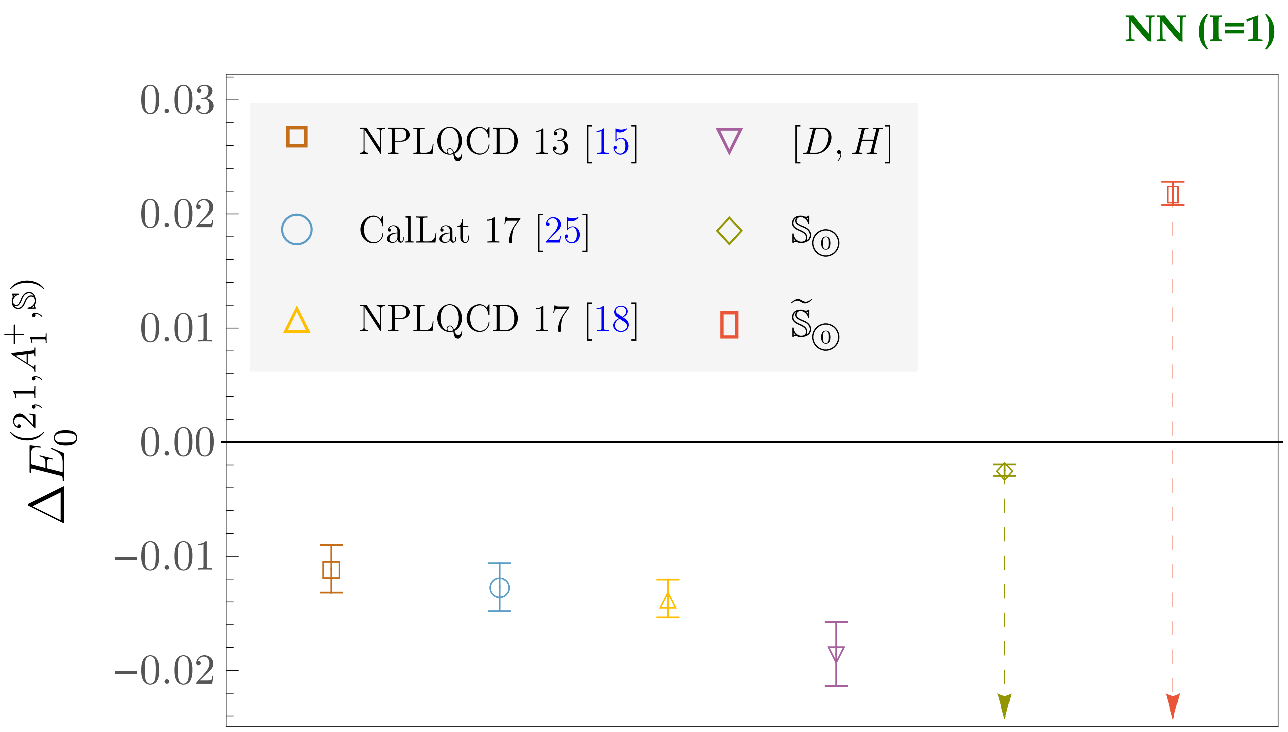
S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



strong overlap with a particular operator

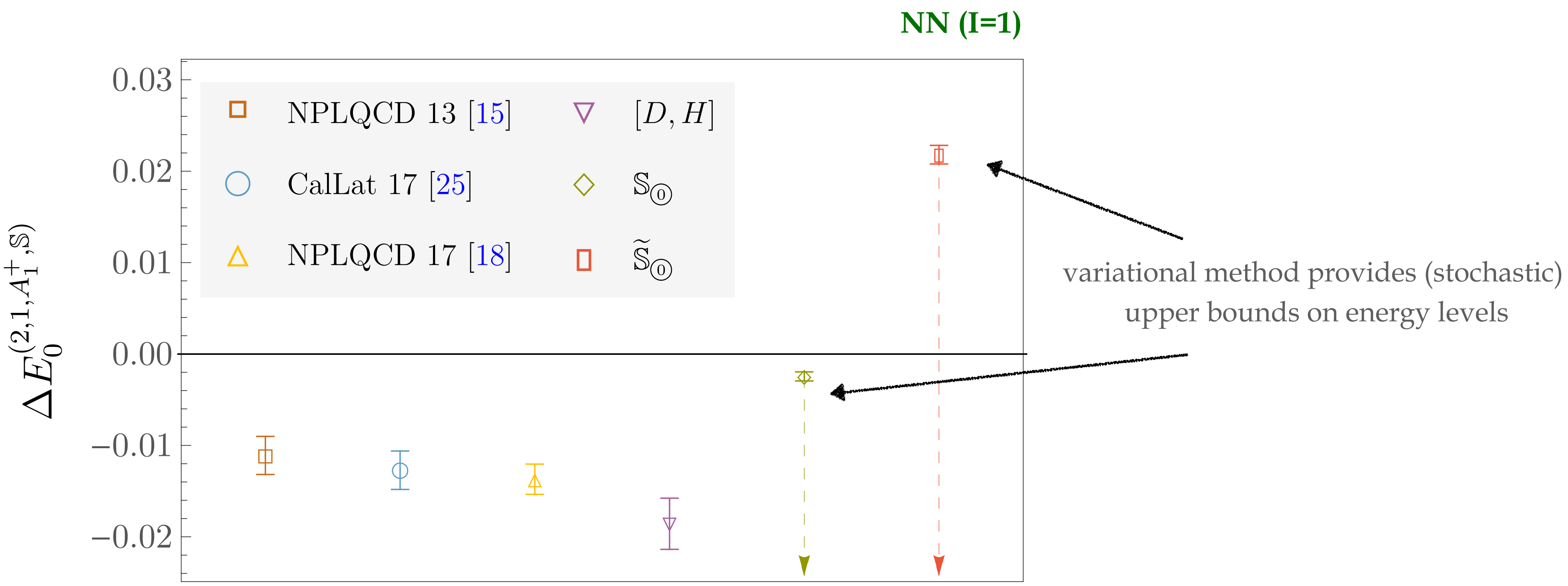


S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



calculations made on the same set of configurations

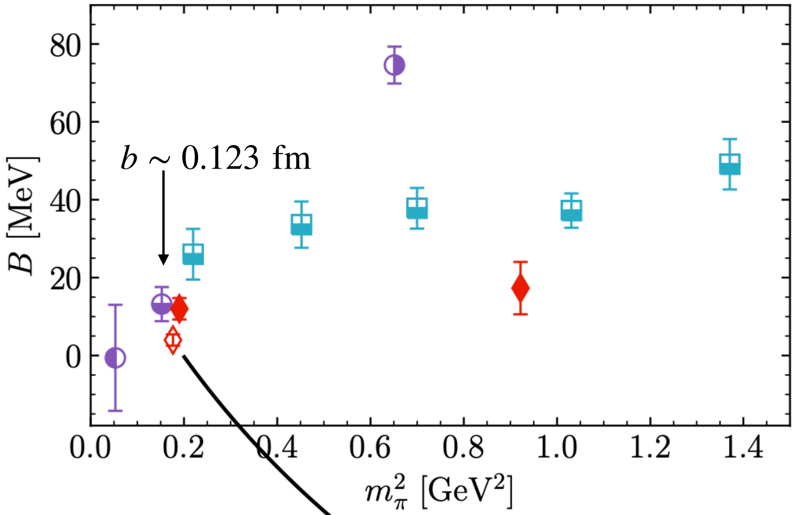
S. Amarasinghe et al (NPLQCD), arXiv:2108.10835



calculations made on the same set of configurations

Lattice artifacts?

Potentially important



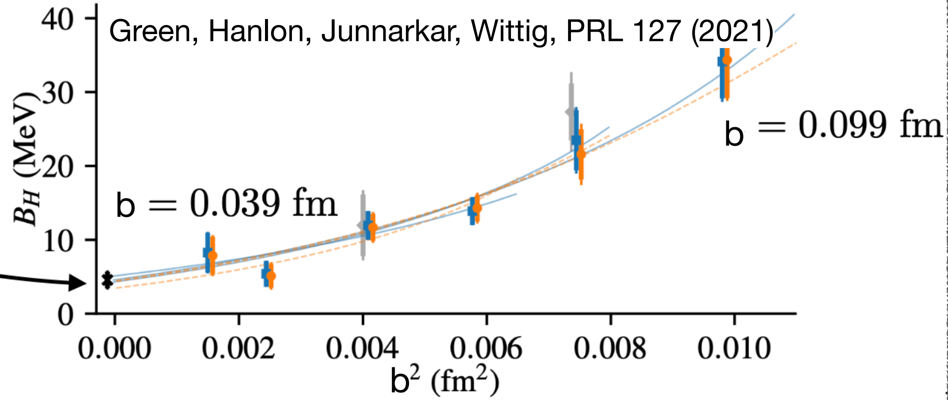
NPLQCD

HAL: Hadrons to Atomic nuclei from Lattice QCD

JG|U → first application of variational approach to BB (with only dibaryon-like operators)

$m_\pi \sim m_K \sim 420$ MeV

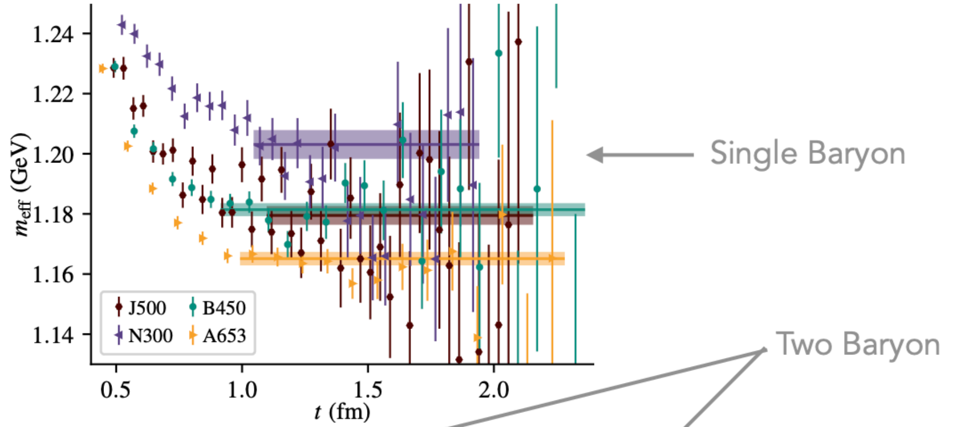
first extrapolation to the continuum for BB systems

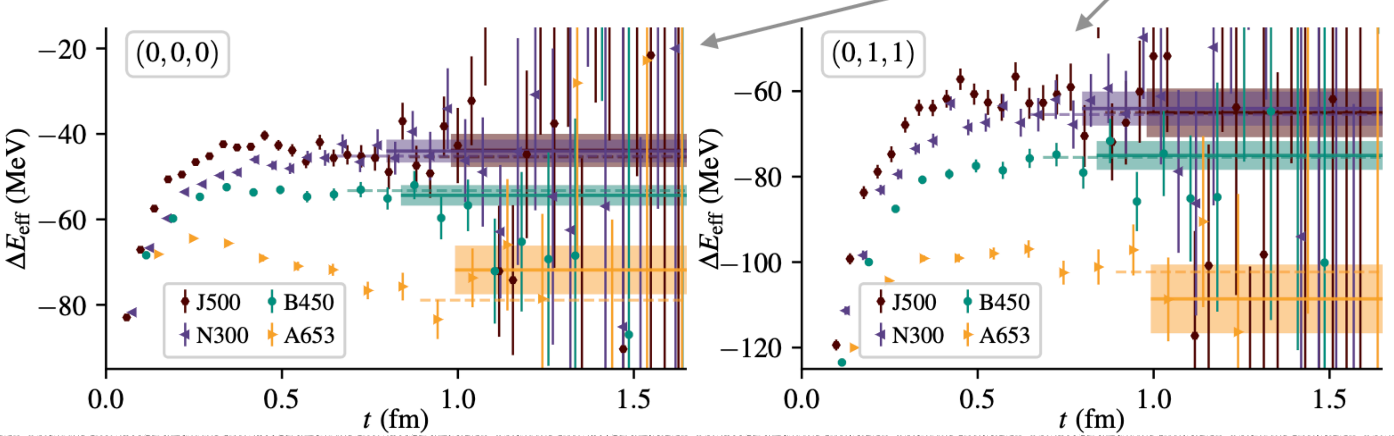


Lattice artifacts?

Potentially important

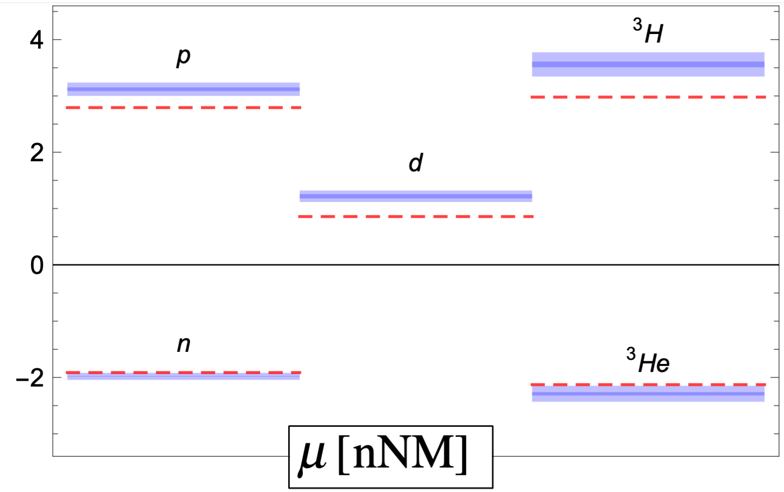
Green et al.,
"Weakly bound H dibaryon from SU(3)-flavor-symmetric QCD"
Phys.Rev.Lett. **127** (2021) 24, 242003





Are we really missing a deep bound state?

NPLQCD, Phys. Rev .Lett. 113 (2014)



$$\mathrm{nNM} = \frac{e}{2M_N^{\mathrm{latt}}} = \frac{e}{2M_N(m_\pi^{\mathrm{latt}})}$$

LQCD @ $m_\pi \sim 800$ MeV

experiment

Shell-model predictions

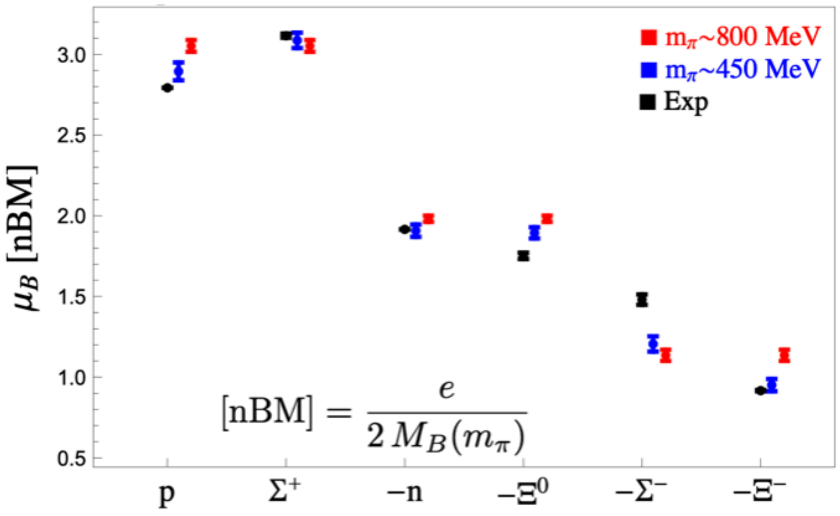
$$\mu(^3H) = \mu_p$$

$$\mu(^3He) = \mu_n$$

$$\mu_d = \mu_n + \mu_p$$

Octet baryon magnetic moments

NPLQCD, PRD95, 114513 (2017)



$(\mu_{-B} \equiv -\mu_B)$

- During the last 2 decades we have witnessed great advances in the calculation of nuclear interactions with Lattice QCD, partly thanks to the technological development but also thanks to the development of algorithms and communication between different communities (computing science, theoretical physics, nuclear).
- Calculations near the physical pion mass (coarse extrapolations at the moment) $\longrightarrow$ are under way
- Are lattice artefacts important? Probably more statistics is needed to answer this question
- Excited state contamination
- Operator dependence: Variational studies have revealed significant interpolating-operator dependence in LQCD calculations of NN energy spectrum with unphysical quark masses
- Variational bounds don't provide conclusive evidence for (or against) bound states.
- Ongoing and future work: include a larger operator set (complete basis of local 6 quark operators), additional volumes, multi-exponential fits vs GEVP
- The analysis of analogous calculations in the strange sector are underway ($\Lambda\Lambda$)