Hyperon-Nucleon Interaction Constrained by Light Hypernuclei

Marco Knöll

Hypernuclei in Ab Initio Calculations

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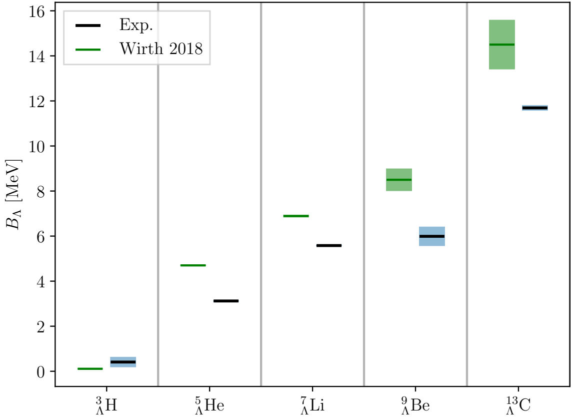
- hypernuclei are systems with strangeness composed of nucleons and hyperons
- CI methods like the (IT-)NCSM have been modified to describe hypernuclei
- calculations require the inclusion of hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions
- first YN interactions from chiral EFT have recently been derived up to N^2LO
  Polinder et. al. 2006, Haidenbauer et. al. 2019, 2023, Haidenbauer et. al. 2023
- LO and NLO interactions have been studied in IT-NCSM and Jacobi NCSM applications
  Wirth 2018, Le et. al. 2020

Optimization of YN Interaction

Motivation for Optimization

- systematic overbinding of hyperon
- hyperon-nucleon interaction is poorly constraint
- LEC optimization successful for regular nuclei
  Hüther et. al. 2020
- ansatz: optimize LECs based on hypernuclear structure observables



Hypernuclear Hamiltonian from Chiral EFT

- hypernuclear Hamiltonian $H = \Delta M + T + V_{\mathrm{NN}} + V_{\mathrm{NNN}} + \ldots + V_{\mathrm{YN}} + V_{\mathrm{YNN}} + \ldots$

- V_{NN}: non-local nucleon-nucleon from chiral EFT at $\mathrm{N^3LO}$ *EMN 2017*
- V_{NNN}: non-local three-body interactions at $\mathrm{N^3LO}$ *Hüther et. al. 2020*

- V_{YN}: hyperon-nucleon interaction in SU(3) chiral EFT at LO *Polinder et. al. 2006*
- V_{YNN}: no initial interaction available but SRG induced forces $V_{\mathrm{YNN}}^{\mathrm{ind}}$ are considered

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Possibilities for Improvement

starting point

- YN interaction at LO
  Polinder et. al. 2006
- 5 LECs
- constrained on 35 YN data points

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higher orders

- extension to NLO (and N^2LO)
  Haidenbauer et. al. 2019 (2023)
- 23 LECs – can be reduced assuming strict SU(3) symmetry
- 36 YN data for S-waves, practically no data for P-waves

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more data

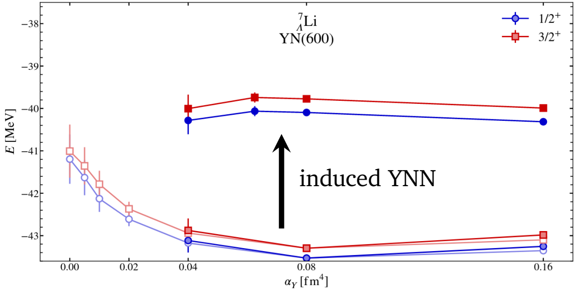
- hypernuclear structure observables as additional constraints
- potentially 33 B_Λ and 11 spectra

Hypernuclear IT-NCSM

- expand Hamiltonian on finite Slater Determinant basis and diagonalize
- include strangeness $\mathcal{S}$ in single-particle basis $|n(ls)jm_j, \mathcal{S}\, tm_t\rangle$
- constituents: $n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+$

Wirth 2018

- SRG induces YNN forces
- inclusion of induced YNN forces is key for accurate description
- access larger model spaces through importance measure

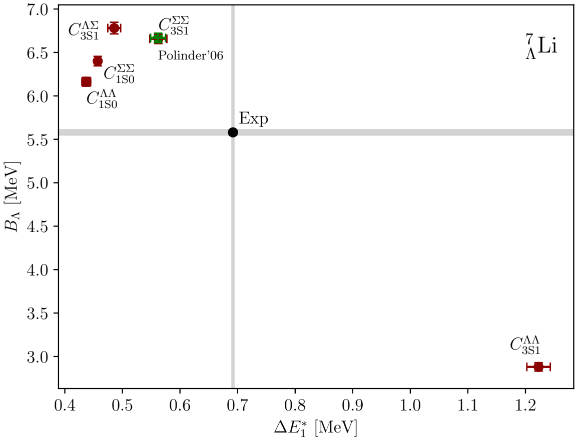


LEC Sensitivity Analysis

- 5 LECs associated with particle species and partial waves:

  $C^{\Lambda\Lambda}_{^1S_0}$, $C^{\Lambda\Lambda}_{^3S_1}$, $C^{\Sigma\Sigma}_{^1S_0}$, $C^{\Sigma\Sigma}_{^3S_1}$, $C^{\Lambda\Sigma}_{^3S_1}$

- vary single LECs by "natural" amounts
- most sensitive to $C^{\Lambda\Lambda}_{^3S_1}$ followed by $C^{\Lambda\Lambda}_{^1S_0}$
- limit optimization to these two LECs



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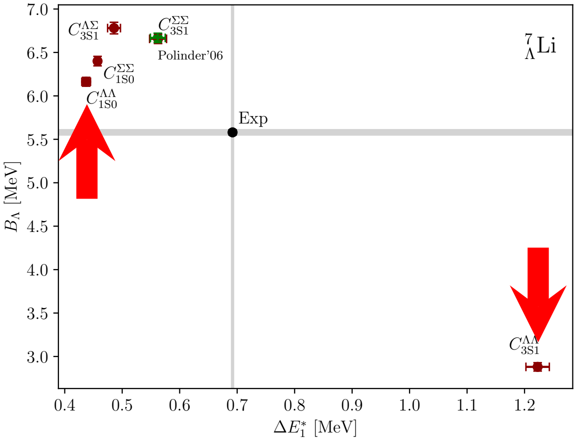


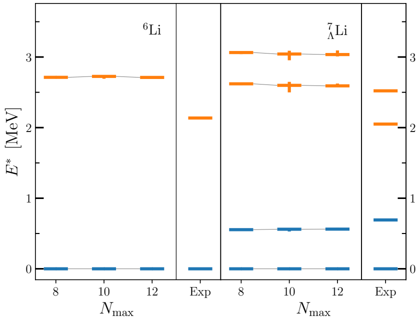
Optimization of LECs

- optimization based on observables predominantly controlled by YN interaction
  $\Rightarrow B_\Lambda$ and hypernuclear splitting ΔE^* of excited energy levels

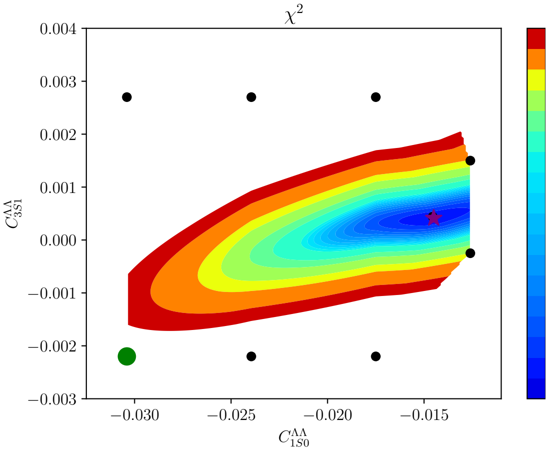
- selected set of experimentally well-known hypernuclei
  $\Rightarrow {}^3_\Lambda\mathrm{H}$, ${}^5_\Lambda\mathrm{He}$, ${}^7_\Lambda\mathrm{Li}$ and ${}^9_\Lambda\mathrm{Be}$

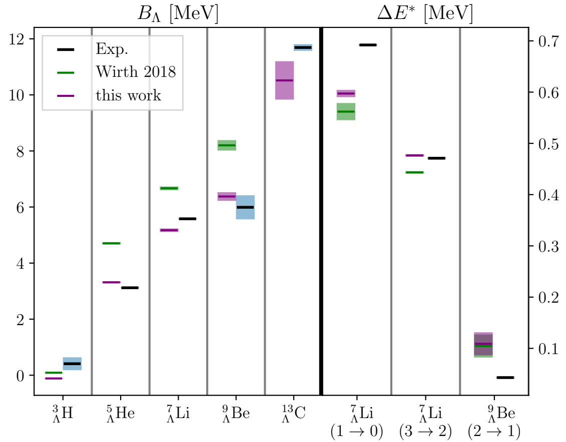
- χ^2-metric including experimental and theoretical errors

$$\chi^2 = \sum \frac{(o - o_{\mathrm{exp}})^2}{\sigma^2_{\mathrm{theo}} + \sigma^2_{\mathrm{exp}}}$$

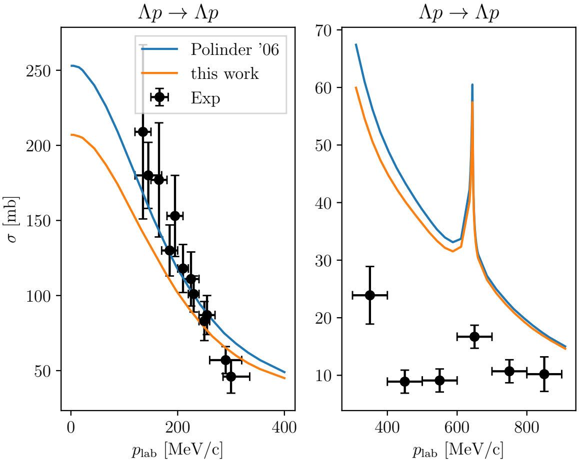


Optimization Results





Description of Scattering Data

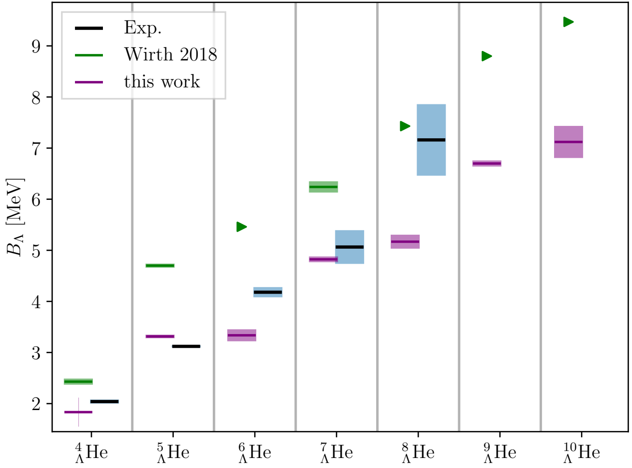


data provided by J. Haidenbauer

- ΛN interaction slightly weakened
- deviation from previous results is reasonably small

$_{\Lambda}$He Isotopic Chain

preliminary



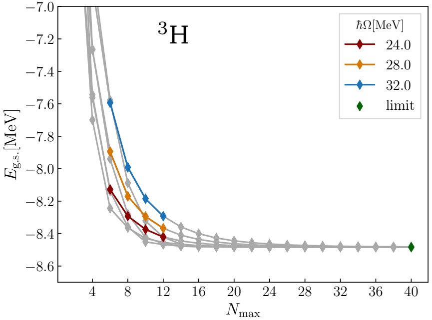
- systematic decrease of B_Λ
- improved agreement with experiment except for $^{8}_{\Lambda}$He
- even isotopes tend to be underbound
- IT-NCSM calculations not converged

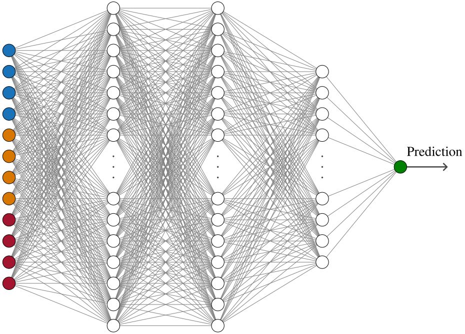
  $\Rightarrow$ require systematic extrapolation and uncertainty estimation

Uncertainty Quantification

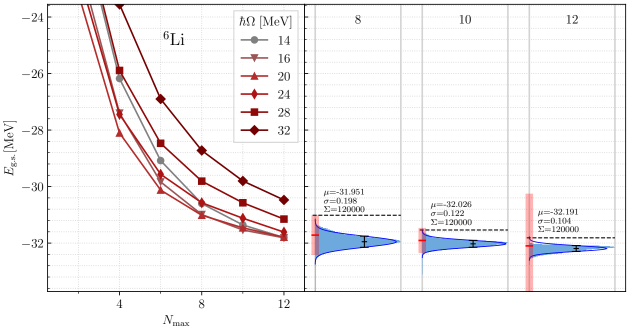
Many-Body Uncertainties – Neural Networks

- prediction and uncertainty from machine learning tool *MK et. al. 2022*
- train 1000 feed-forward ANNs on converged NCSM calculations for ^{2}H, ^{3}H and ^{4}He





Many-Body Uncertainties – Neural Networks



Energies

MK et. al. 2022

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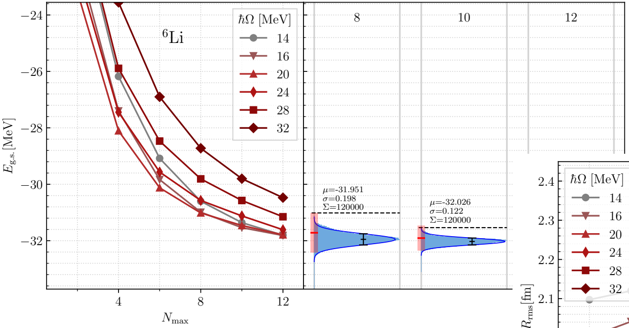

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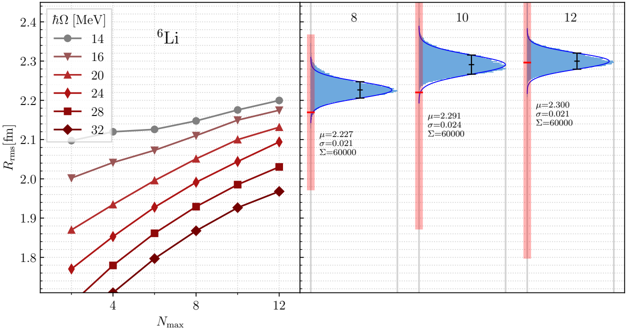

Radii

Wolfgruber, MK, Roth (in prep)

Many-Body Uncertainties – Neural Networks

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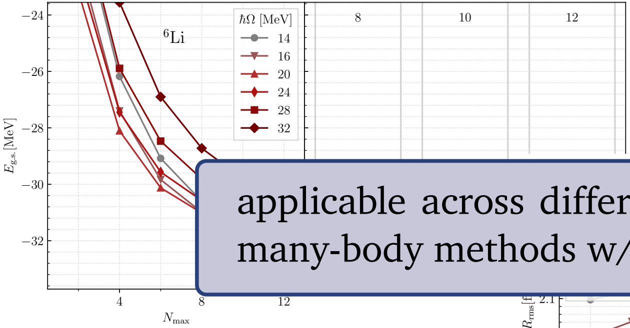

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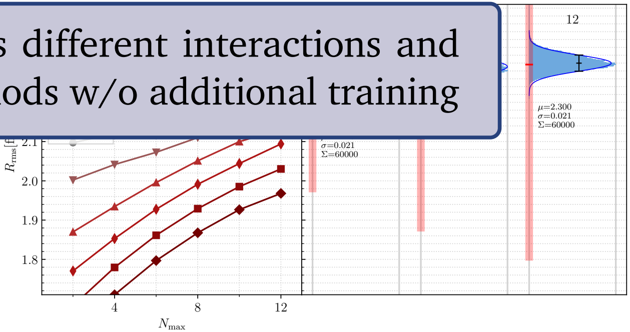
applicable across different interactions and many-body methods w/o additional training



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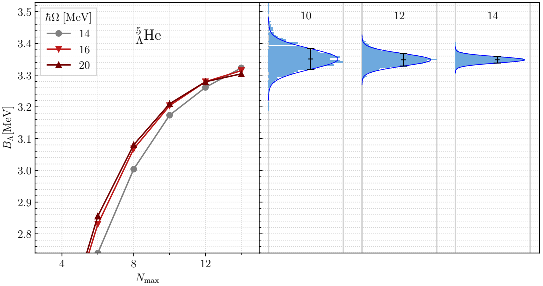
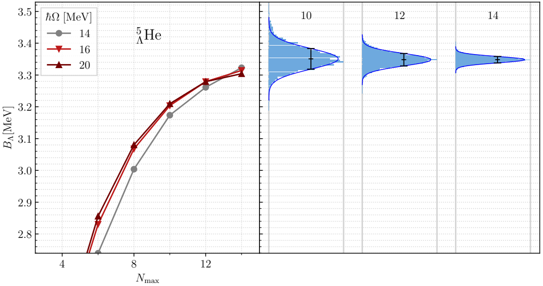
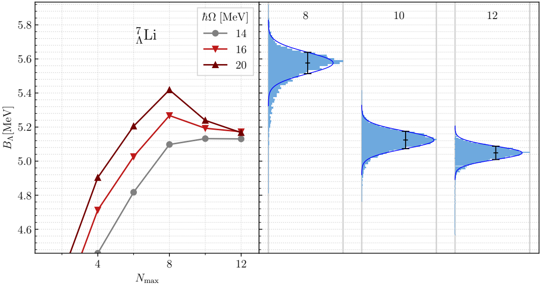


- neural network approach can be applied to hypernuclei including B_Λ
- robust predictions for well-behaved B_Λ

Many-Body Uncertainties – Neural Networks

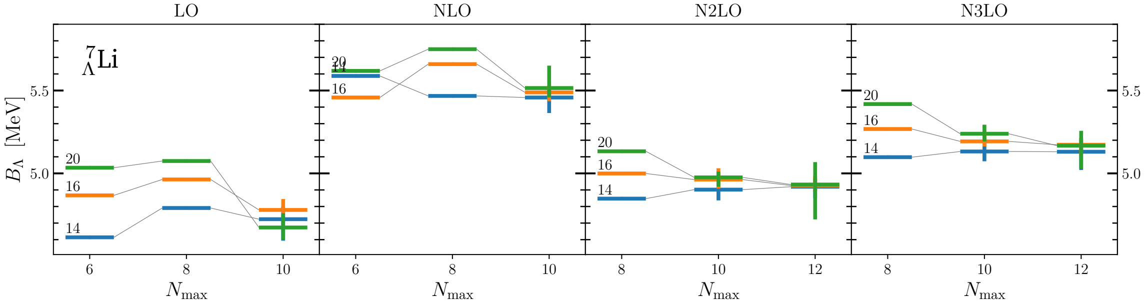




- neural network approach can be applied to hypernuclei including B_Λ
- robust predictions for well-behaved B_Λ

- convergence behavior of B_Λ is not constrained
- sufficiently large model spaces required for accurate prediction

Dependence on Nucleonic Interaction



- non-negligible dependence on nucleonic interaction
- effects decrease with increasing order
- allows for order-by-order uncertainty estimation

Interaction Uncertainties – Bayesian Order by Order

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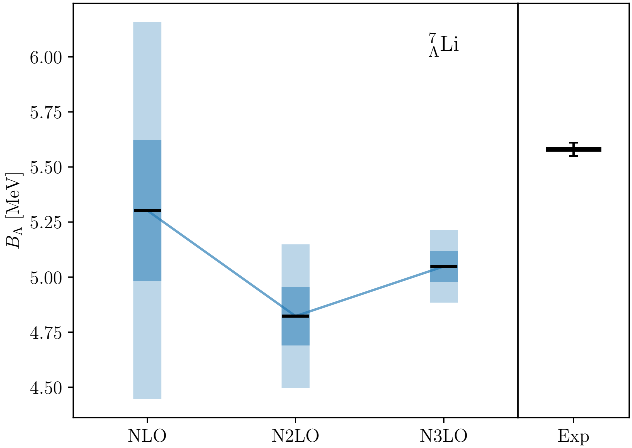

- correlated EFT truncation errors including many-body uncertainties *Melendez et. al. 2019*
- uncertainty estimation regarding the nucleonic sector only
- YN interaction at LO is insufficient for more accurate description

Conclusions

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- significant improvement of the description of hyperon separation energies B_Λ
- p-shell hypernuclei provide a useful source for experimental data
  in addition to the very little scattering data
- uncertainty quantification accessible
- higher accuracy requires the inclusion of higher orders
- next step: LEC optimization based on p-shell hypernuclei for NLO or N^2LO

Thank you for your attention!

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Bundesministerium für Bildung und Forschung

SFB 1245

DFG

Hessisches Kompetenzzentrum für Hochleistungsrechnen

computing time