Nuclear equation of state for arbitrary proton fraction and temperature

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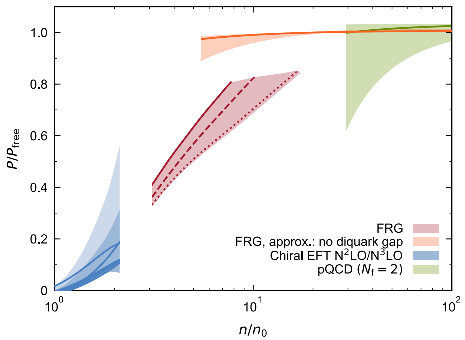
Introduction

- Nuclear matter: idealized system of neutrons and protons in thermodynamic limit (no surface effects, homogeneous, ...)
- Key input for astrophysics
- This talk: nuclear EOS from chiral EFT
- So far EOS studied often for $T = 0$ for PNM ($x = 0$) or SNM ($x = 0.5$)
- Thermal effects matter for astro applications
  e.g. Yasin et al., Phys. Rev. Lett. 124 (2020)
- Astrophysical systems are neutron rich or in β-equilibrium

Leonhardt et at.,
Phys. Rev. Lett. 125 (2020)



JK, Wellenhofer, Hebeler, Schwenk, Phys. Rev. C 103 (2021)
JK, Hebeler, Schwenk, arXiv:2204.14016, PRL in press

$\longrightarrow$ Chiral EFT EOS for finite temperature and asymmetric matter

Method

Nuclear interaction

Chiral EFT: $H = H_0 + V_{NN} + V_{3N}$

↓

Grand-canonical potential

$$\Omega(T, \mu_n, \mu_p) = -\frac{1}{\beta} \ln \mathrm{Tr} \left(e^{-\beta \left(H - \mu_n N_n - \mu_p N_p \right)} \right)$$

↓

Approximation strategy

Many-body perturbation theory, MC integration

↓

Gaussian process emulation for asymmetric matter

$$\{F(x_i, T_j, n_k) + \Delta_{ijk}\}_{ijk} \xrightarrow{\text{GP}} F(x, T, n)$$

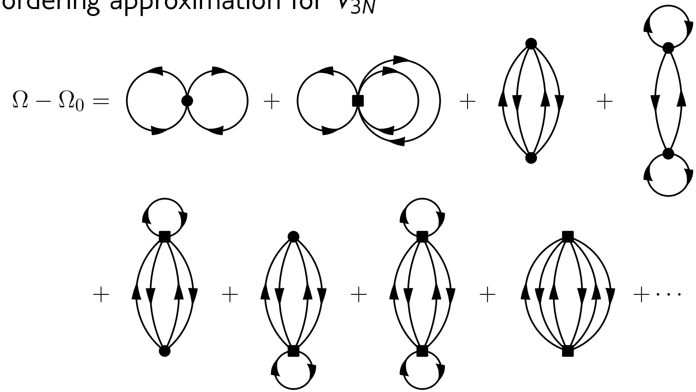
↓

Equation of state (EOS)

$F(x, T, n), P(x, T, n), \ldots$

Many-body perturbation theory (MBPT)

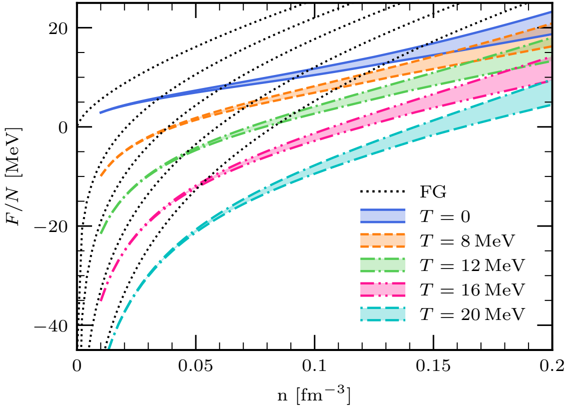
- Expansion in V_{NN} and V_{3N} vertices
- No normal ordering approximation for V_{3N}



- Each line represents spin sums and momentum integral
- Relevant potential is free energy $F(x, T, n)$

Neutron matter free energy

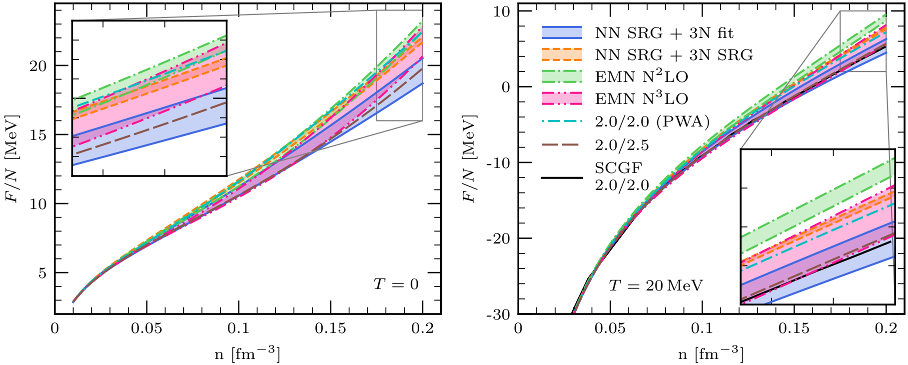
- Systematic study based on different nuclear interactions (bands)
- Large part of temperature dependence from free Fermi gas (FG)



JK, Wellenhofer, Hebeler, Schwenk, Phys. Rev. C 103 (2021)

Neutron matter free energy

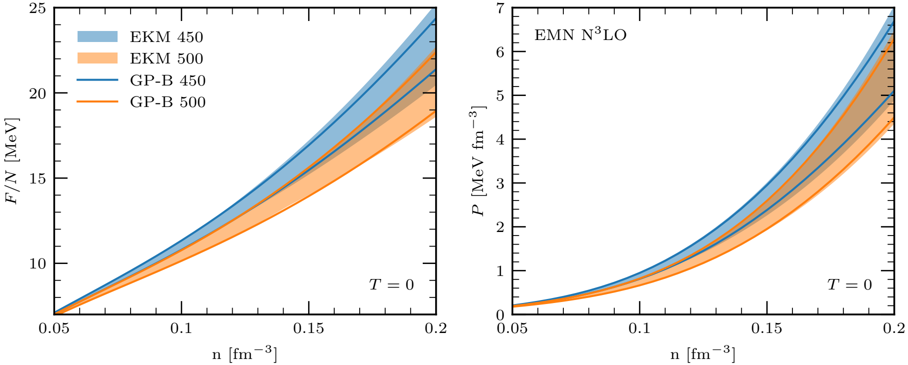
- Bands display SRG ($\lambda_{SRG} = 1.8$ to $2.8\,\mathrm{fm}^{-1}$) or cutoff variations ($\Lambda = 450$ to $500\,\mathrm{MeV}$)
- Consistent with non-perturbative SCGF calculations
  Carbone, Schwenk, Phys. Rev. C 100 (2019)



JK, Wellenhofer, Hebeler, Schwenk, Phys. Rev. C 103 (2021)

EFT uncertainties

- Expansion in powers of $Q = \frac{p}{\Lambda_b}$
- Use order-by-order values to obtain uncertainty estimates
  Epelbaum et al., Eur. Phys. J. A 51 (2015), Drischler et al., Phys. Rev. Lett. 125 (2020)



JK, Wellenhofer, Hebeler, Schwenk, Phys. Rev. C 103 (2021)

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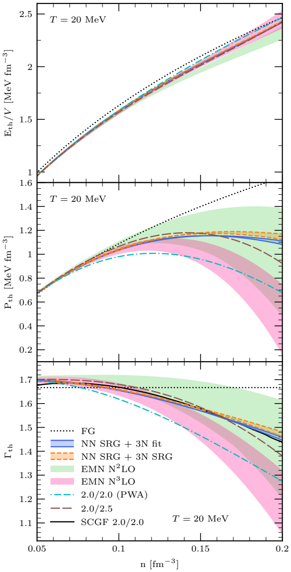
Neutron matter thermal quantities

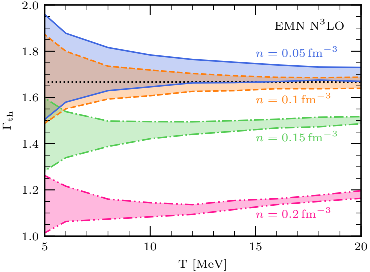


- Thermal part and thermal index

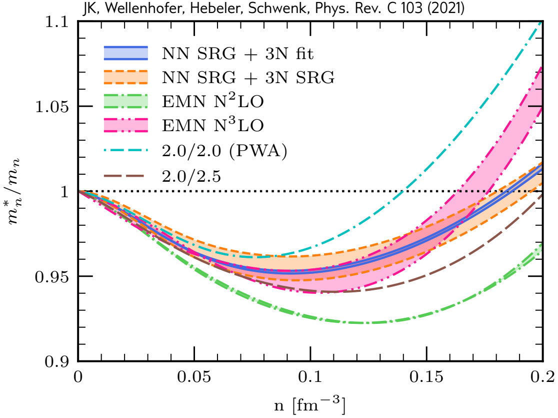
$$X_{th}(T,n) = X(T,n) - X(T=0,n)$$

$$\Gamma_{th}(T,n) = 1 + \frac{P_{th}(T,n)}{E_{th}(T,n)/V}$$

- For free gas: $\Gamma_{FG,th} = \frac{5}{3}$
- Thermal index is used to parameterize temperature dependence see, e.g., Bauswein et al., Phys. Rev. D 82 (2010)
- Weak temperature dependence



Neutron matter effective mass approximation



- For ideal gas with density dependent effective mass $m_n^*(n)$

$$\Gamma_{th}(n) = \frac{5}{3} - \frac{n}{m_n^*}\frac{\partial m_n^*}{\partial n}$$

- Extracted from $\Gamma_{th}(T = 20\,\text{MeV})$
- Could be used in astro applications

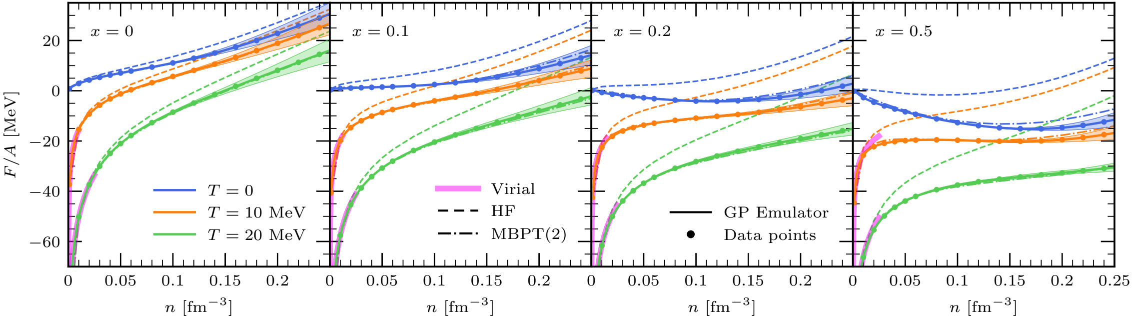
Asymmetric nuclear matter

- EMN interaction at N^3LO ($\Lambda = 450\,\mathrm{MeV}$) Entem, Machleidt, Nosyk, Phys. Rev. C 96 (2017)
- Bands are order-by-order EFT uncertainty estimates Epelbaum et al., Eur. Phys. J. A 51 (2015)

  $$\Delta X^{(j)} = Q \cdot \max\left(\left|X^{(j)} - X^{(j-1)}\right|, \Delta X^{(j-1)}\right)$$

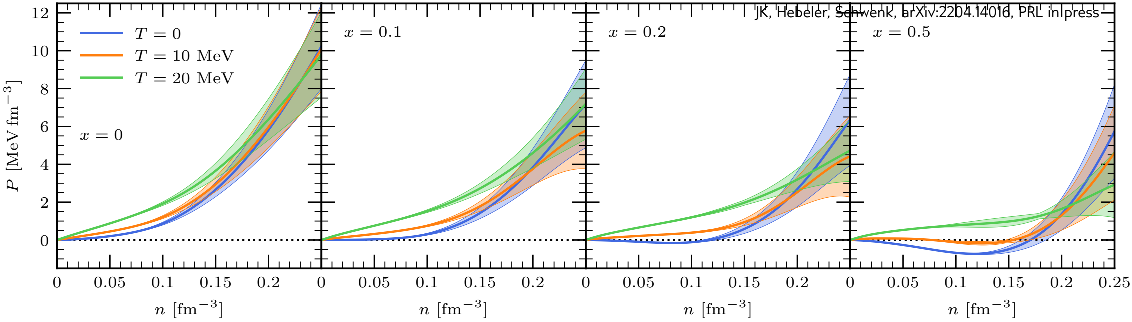
- Excellent reproduction of data by GP, good MBPT convergence, no MC noise
- Virial EOS: model independent fugacity $z_t = e^{\mu_t/T}$ expansion Horowitz, Schwenk, Nucl. Phys. A 776 (2006)

JK, Hebeler, Schwenk, arXiv:2204.14016, PRL in press



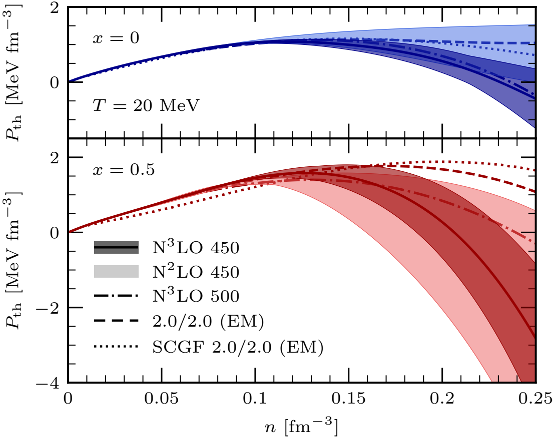
Pressure

- Calculate $P = n^2 \partial_n F/A$ using GP emulator
- For very neutron-rich conditions depends weakly on temperature for $n \gtrsim n_0$
- Pressure isothermals cross at higher density



Thermal pressure

JK, Hebeler, Schwenk, arXiv:2204.14016, PRL in press



- $P_{\rm th}(T) = P(T) - P(T=0)$
- Pressure isothermals cross if $P_{\rm th}(T) = 0$
- For NM associated with increasing effective neutron mass m_n^* (three-nucleon interactions)

Carbone, Schwenk, Phys. Rev. C 100 (2019)

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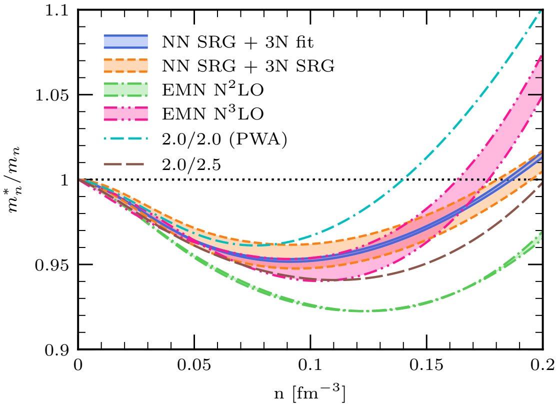

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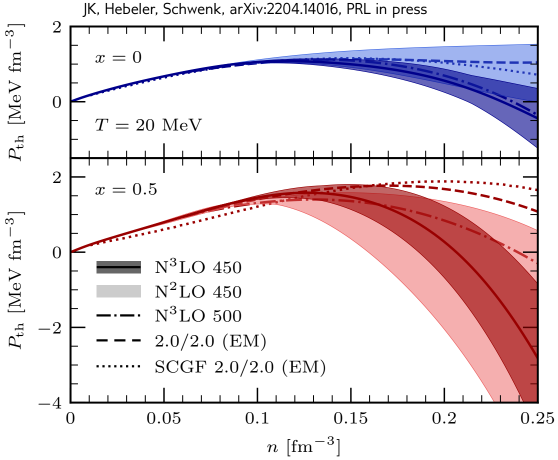

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- For 2.0/2.0 interactions consistent with non-perturbative SCGF calculations
  Carbone, Schwenk, Phys. Rev. C 100 (2019)

Thermal pressure

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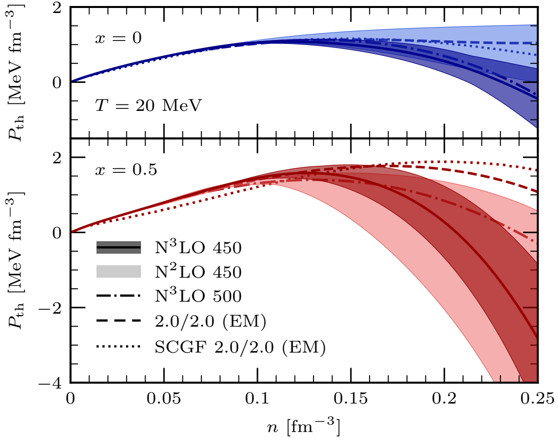

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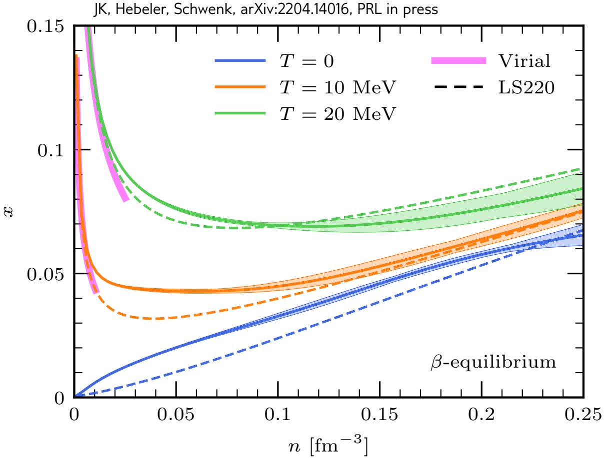
- For 2.0/2.0 interactions consistent with non-perturbative SCGF calculations
  Carbone, Schwenk, Phys. Rev. C 100 (2019)

- Decreasing $P_{\rm th}$ for different chiral orders, cutoffs, and interactions

Neutron star matter



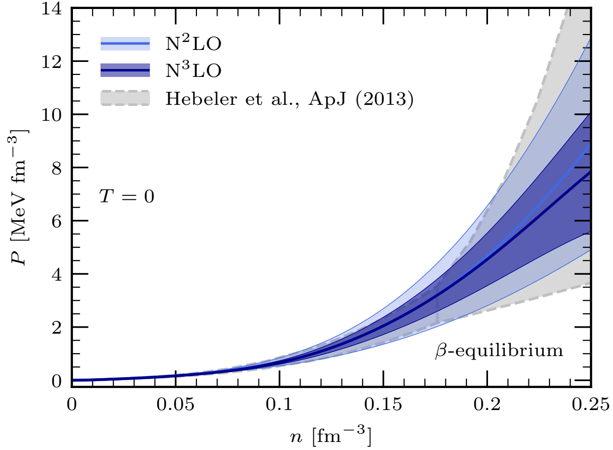
- Determine x by β-eq.

$$m_n + \mu_n = (m_p + \mu_p) + (m_e + \mu_e)$$

- Ultra-rel. e^- with $n_e = n_p$
- Key input $\hat{\mu} = \mu_n - \mu_p = -\frac{\partial}{\partial x}\frac{F}{A}$
- Use GP emulator for derivatives
- Reasonable agreement with LS EOS

Neutron star pressure

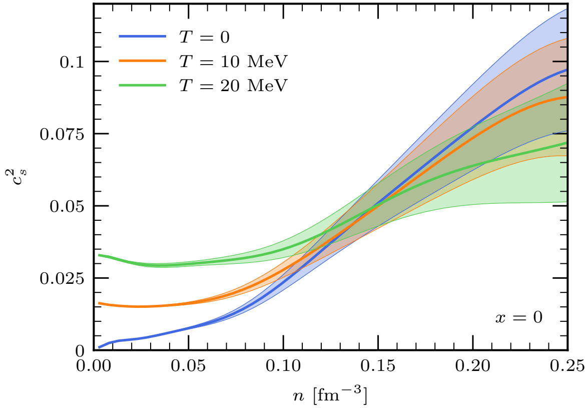
JK, Hebeler, Schwenk, arXiv:2204.14016, PRL in press



- In beta-eq. $P(n, x_{\beta\text{-eq.}}(n,T), T=0)$
- Improvement over older calculations that use parametrization for beta eq.
  Hebeler et al., Astrophys. J. 773 (2013)
- Higher pressure around saturation density
- Compatible, although older band has different meaning (not EFT uncertainty estimates)
- Natural behavior of EFT uncertainty bands

Speed of sound

JK, Hebeler, Schwenk, arXiv:2204.14016, PRL in press



- Pressure derivative at constant entropy

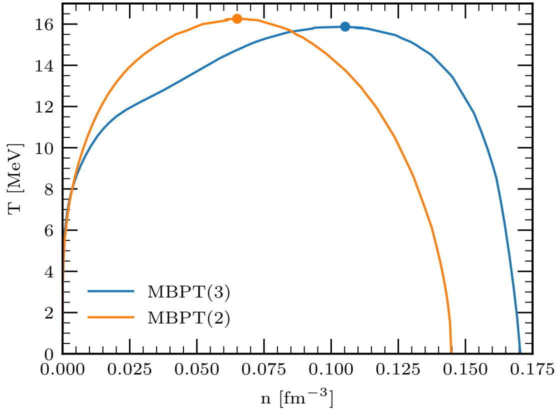
$$c_s^2 = \left.\frac{\partial P}{\partial \epsilon}\right|_{S,x}$$

- With internal energy density

$$\epsilon = n\left(\frac{E}{A} + m_n\right)$$

- At $T = 0$ monotonic increase, increase is weaker at finite T
- Decreases at higher densities with increasing T (like P)

Phase diagram of symmetric nuclear matter



- Preliminary results
- Liquid-gas like phase coexistence

$$T_c = 15.9 - 16.3\,\mathrm{MeV}$$
$$n_c = 0.07 - 0.11\,\mathrm{fm}^{-3}$$

- Want to calculate theoretical EFT uncertainties

Summary

- Calculations of EOS around saturation density using chiral EFT
- Developed calculations for $T > 0$ and arbitrary x
- Investigated thermal behavior using neutron effective mass
- Constructed emulator for free energy
- EFT dominates over MBPT uncertainties for neutron rich matter
- Pressure at higher densities increases with decreasing T (crossing pressure isothermals)
- EOS in beta equilibrium directly without parameterizations between PNM and SNM
- Application: speed of sound and liquid-gas phase transition