

Recent progress on IMSRG calculations with 3-body operators

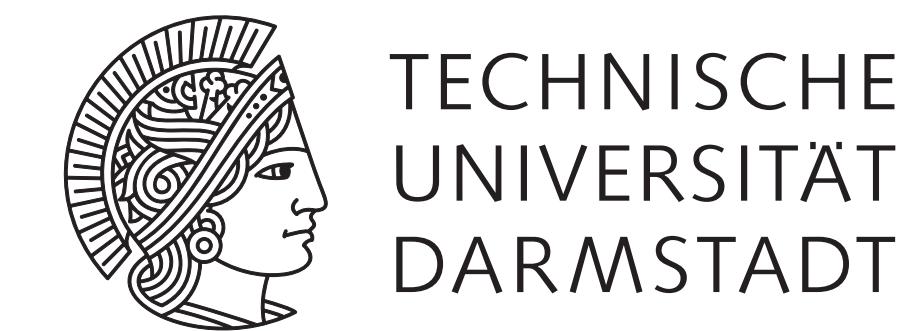
Matthias Heinz

*with Jan Hoppe, Takayuki Miyagi, Alexander Tichai,
Ragnar Stroberg, Kai Hebeler, Achim Schwenk*



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Setting the stage...

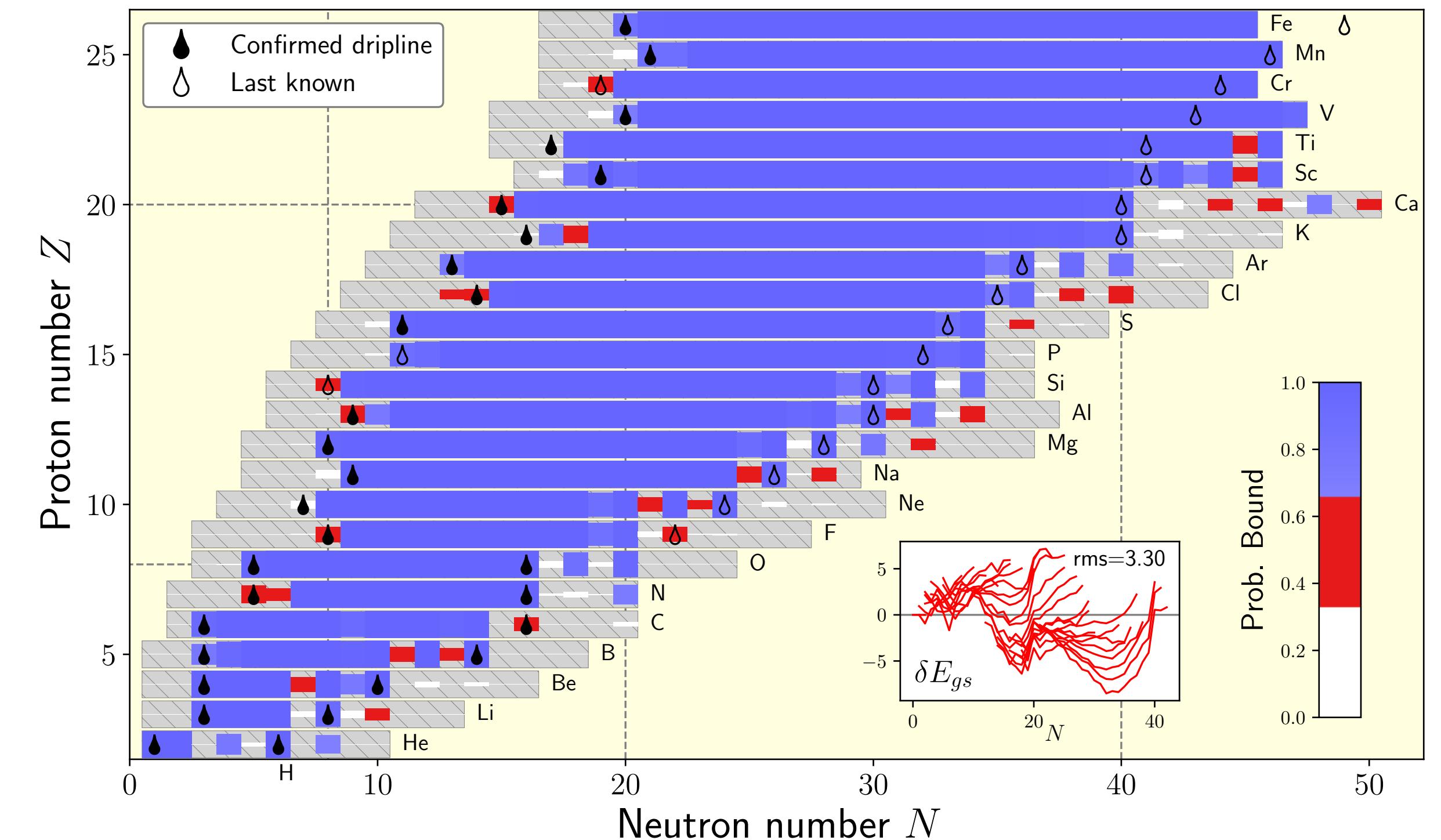
Ab initio ingredients:

- Hamiltonian with 2B and 3B forces
- Systematically improvable many-body method

IMSRG:

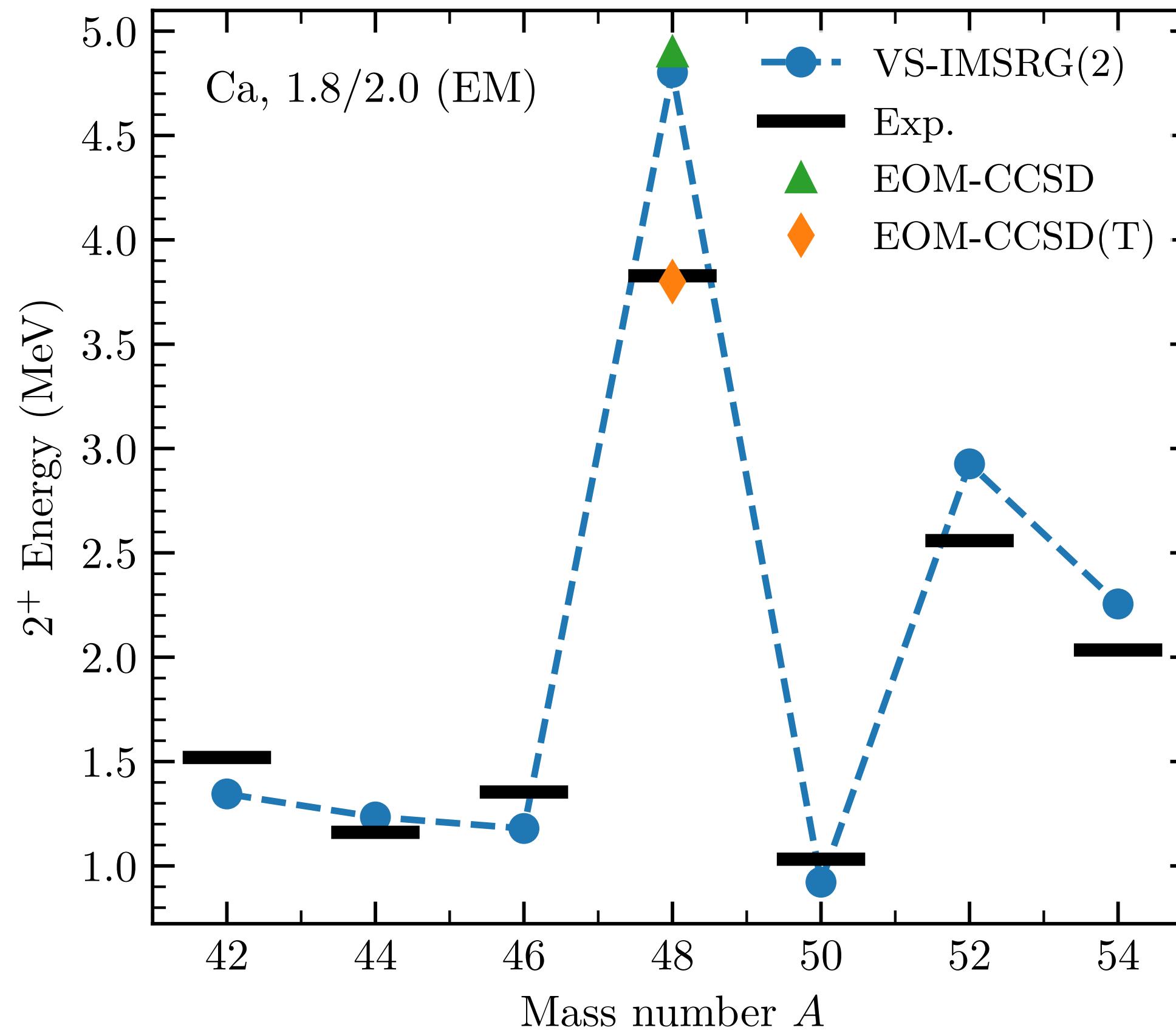
- Polynomially scaling many-body method
- Open-shell systems via MR-IMSRG and **VS-IMSRG**

- Standard: truncate at IMSRG(2) level



Stroberg et al., PRL 126 (2021)

The problem: 2^+ energy of calcium-48

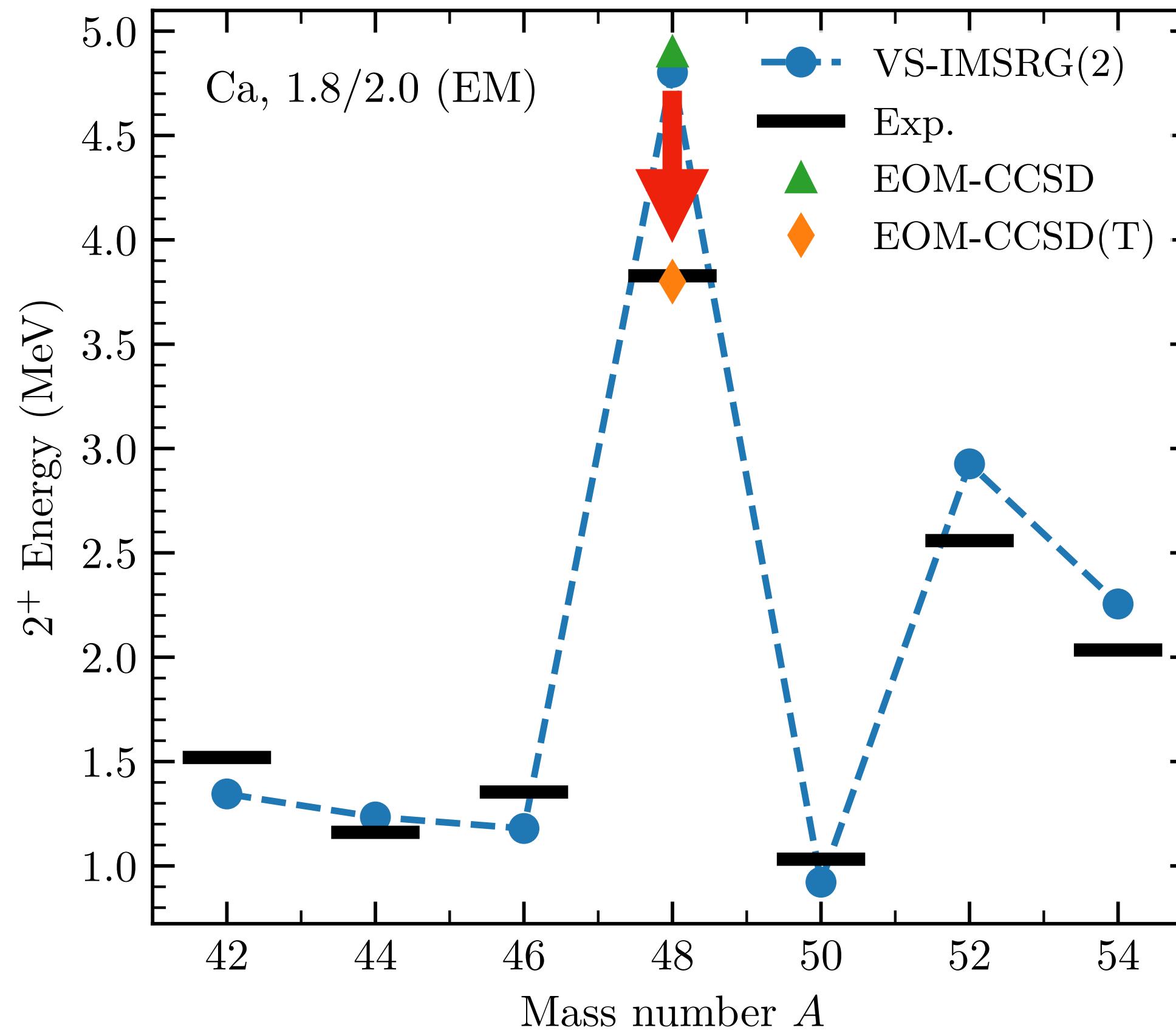


Hagen et al., PRL 117 (2016)

Simonis et al., PRC 96 (2017)

- Success: Description of 2^+ energies **across isotopic chain**
- Failure: **Overprediction** of closed-shell structure at calcium-48
- **Can improvements in the IMSRG bring this down into better agreement with experiment and other theories?**

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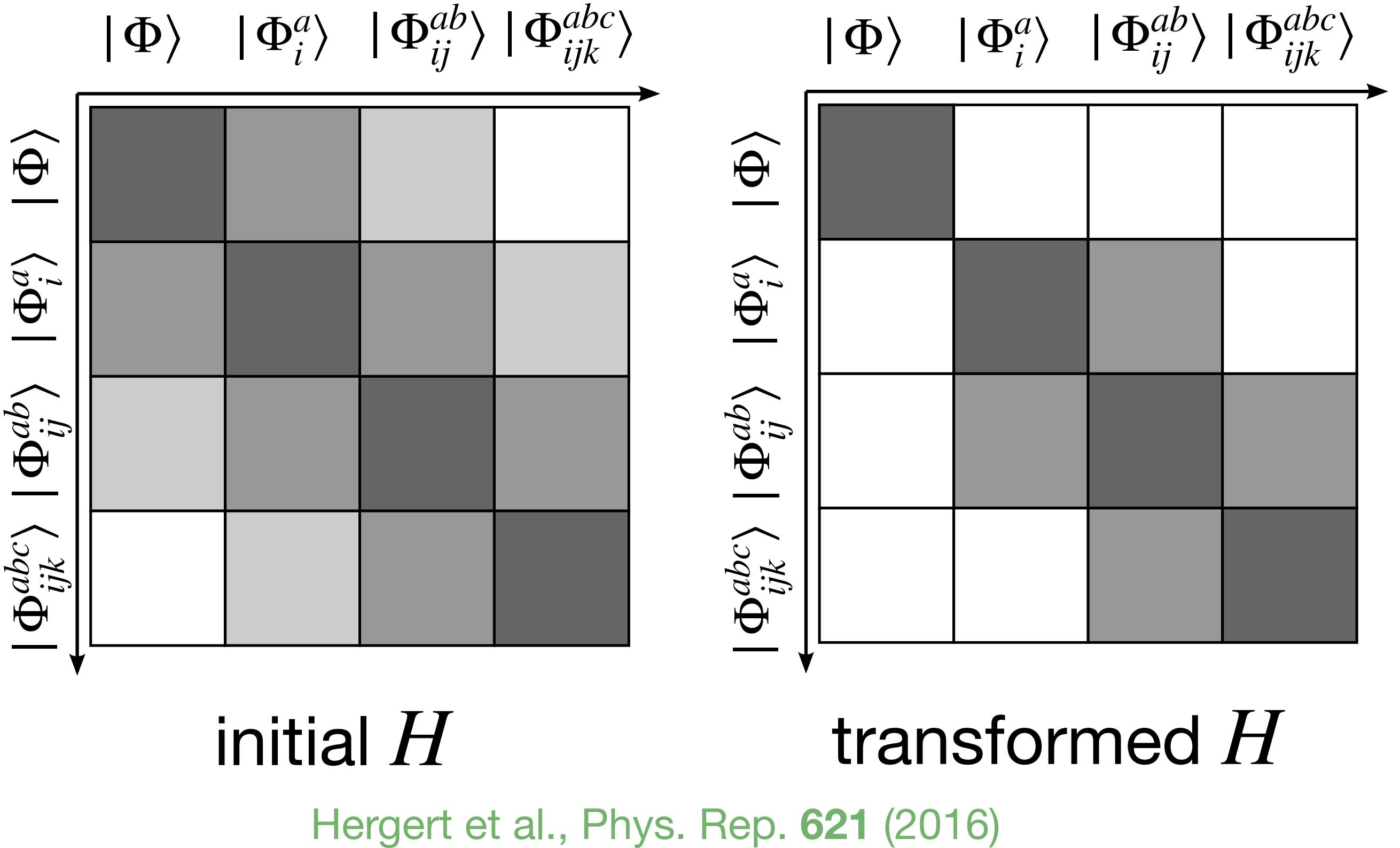
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The IMSRG

- IMSRG generates unitary transformation of Hamiltonian

$$\frac{dH}{ds} = [\eta, H]$$

- Normal order with respect to $|\Phi\rangle$ approximately handles **induced many-body forces**



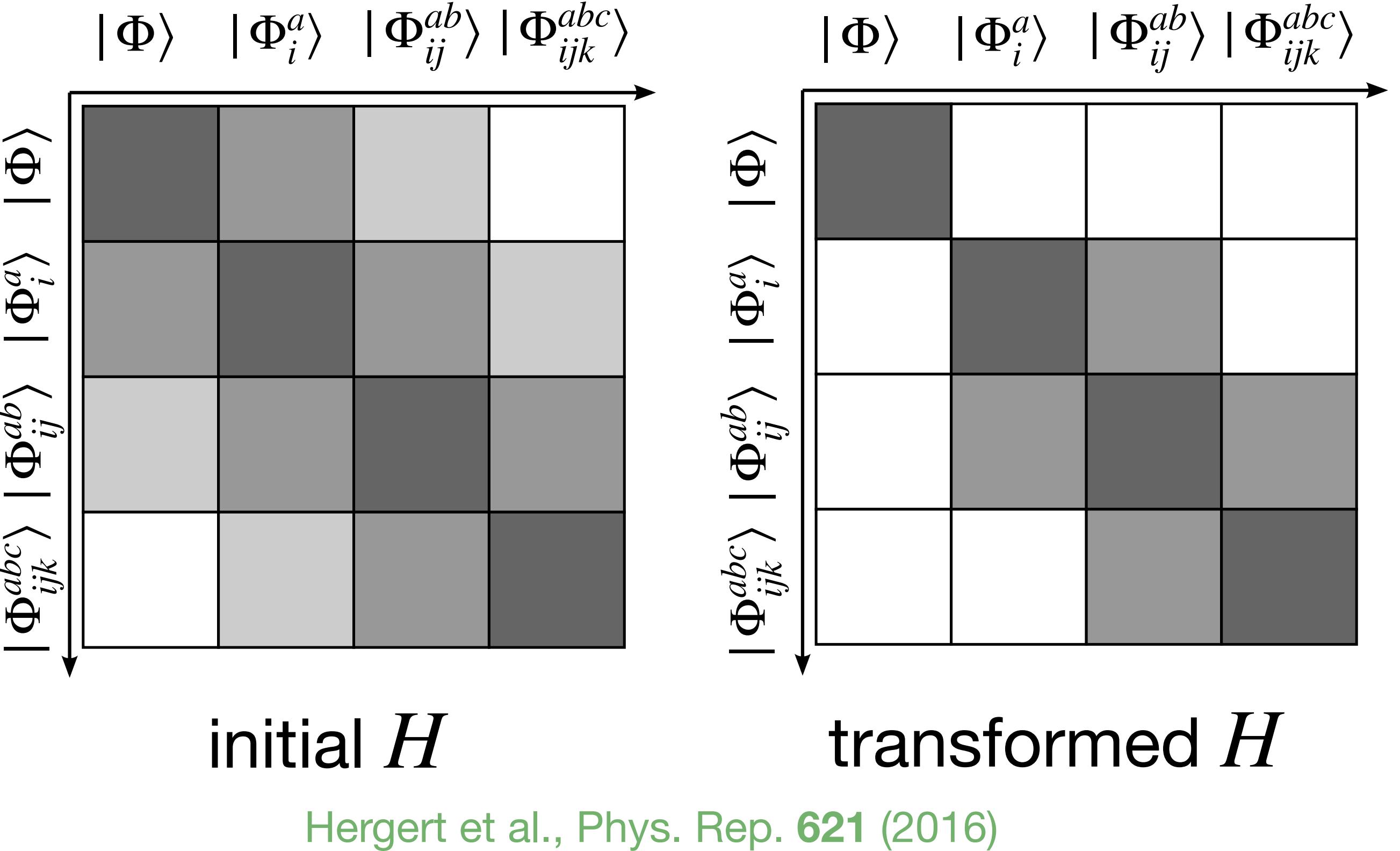
The IMSRG

Tsukiyama et al., PRL 106 (2011)
Hergert et al., Phys. Rep. 621 (2016)

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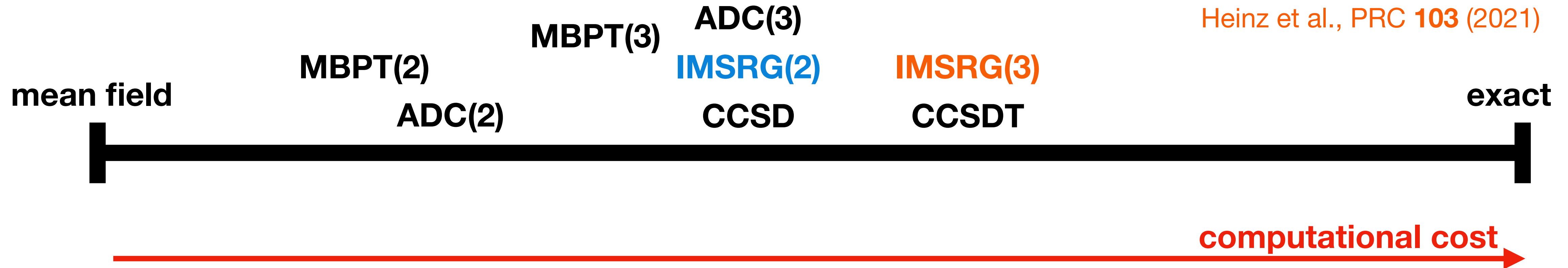


Truncation necessary!

- Standard = IMSRG(2) (2-particle excitations, scales like $(o + u)^6 \equiv N^6$)
- More refined = **IMSRG(3)** ($N^7 - N^9$)

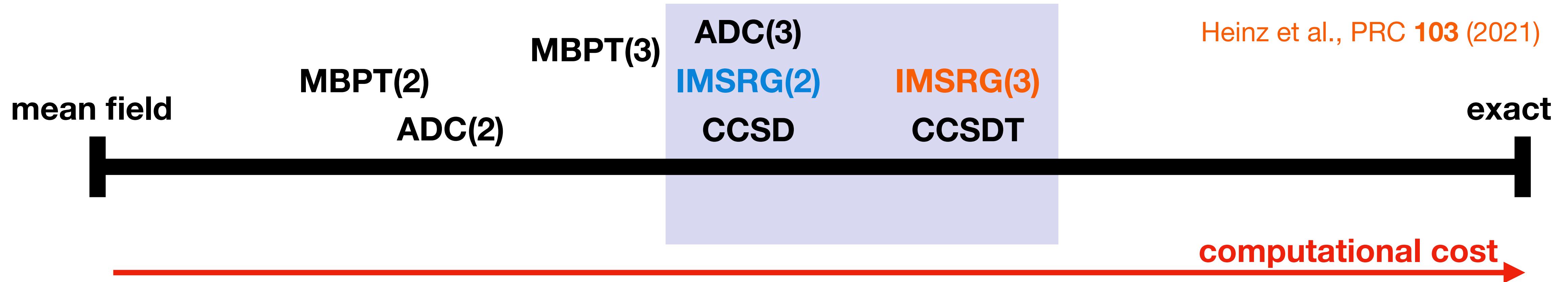
Heinz et al., PRC 103 (2021)

Many-body expansion (ab initio picture)



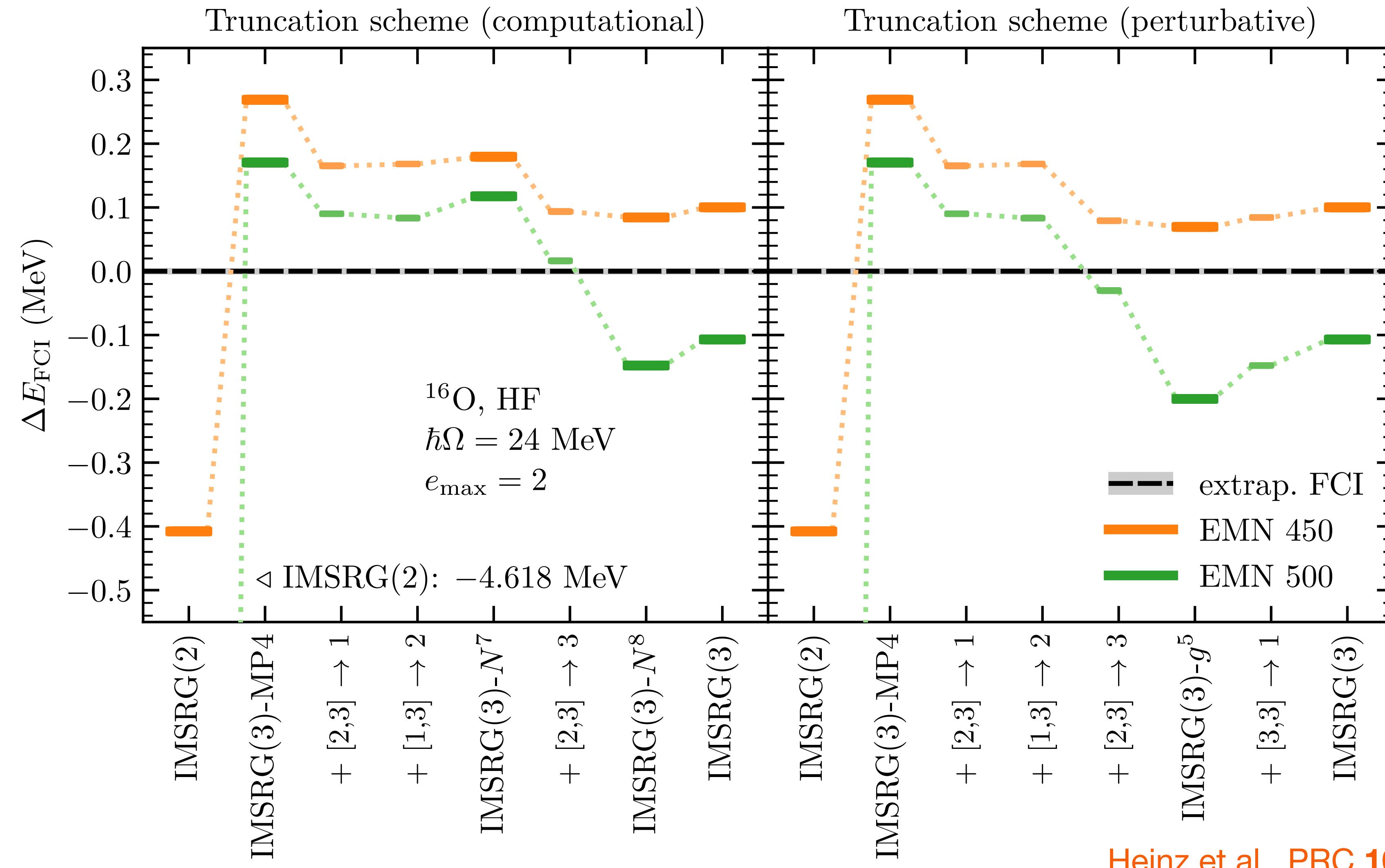
- Systematic expansion to exact result in some limit
- Probe many-body uncertainty by varying many-body truncation
- **IMSRG(3) is a key ingredient here!**

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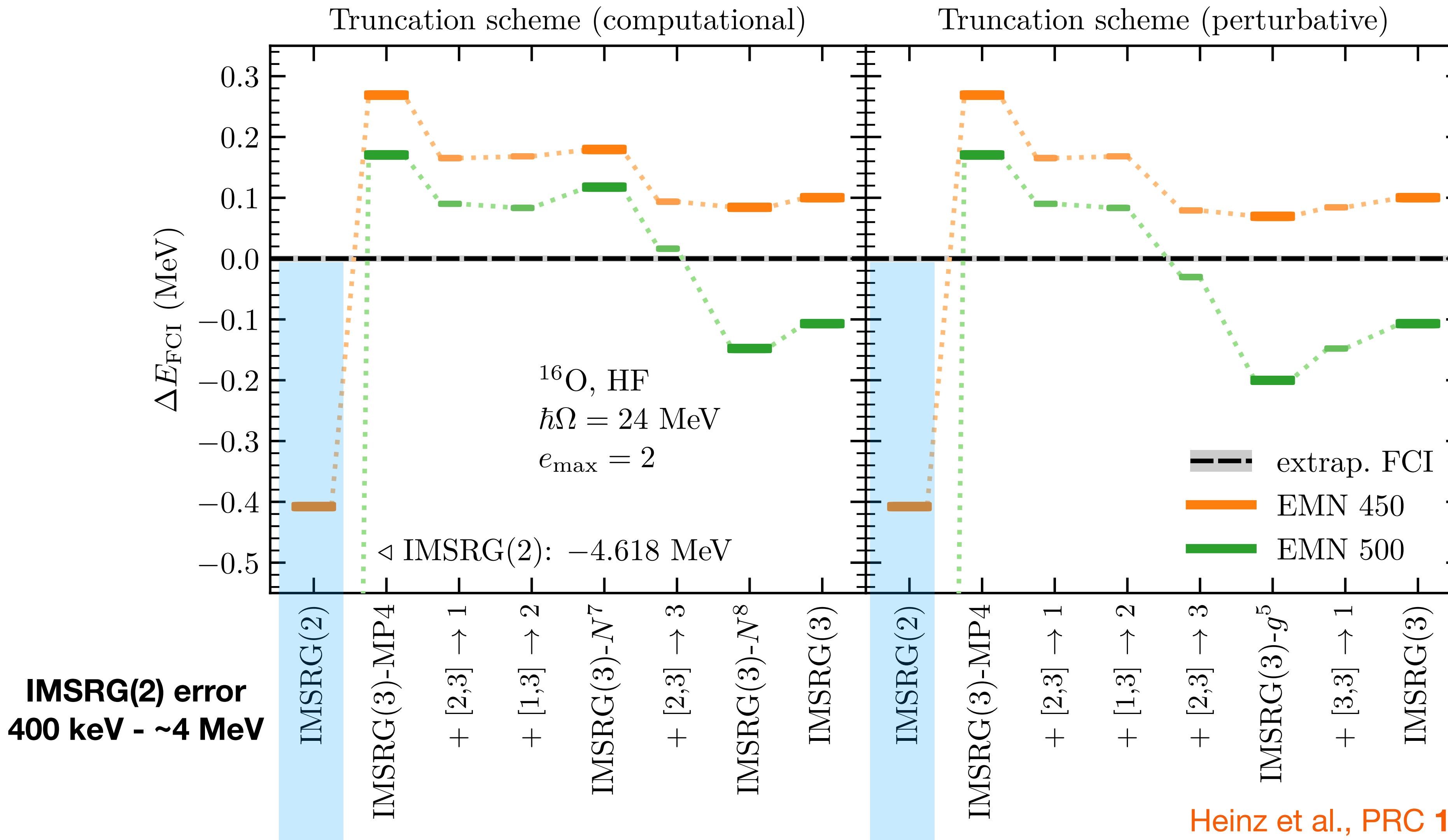


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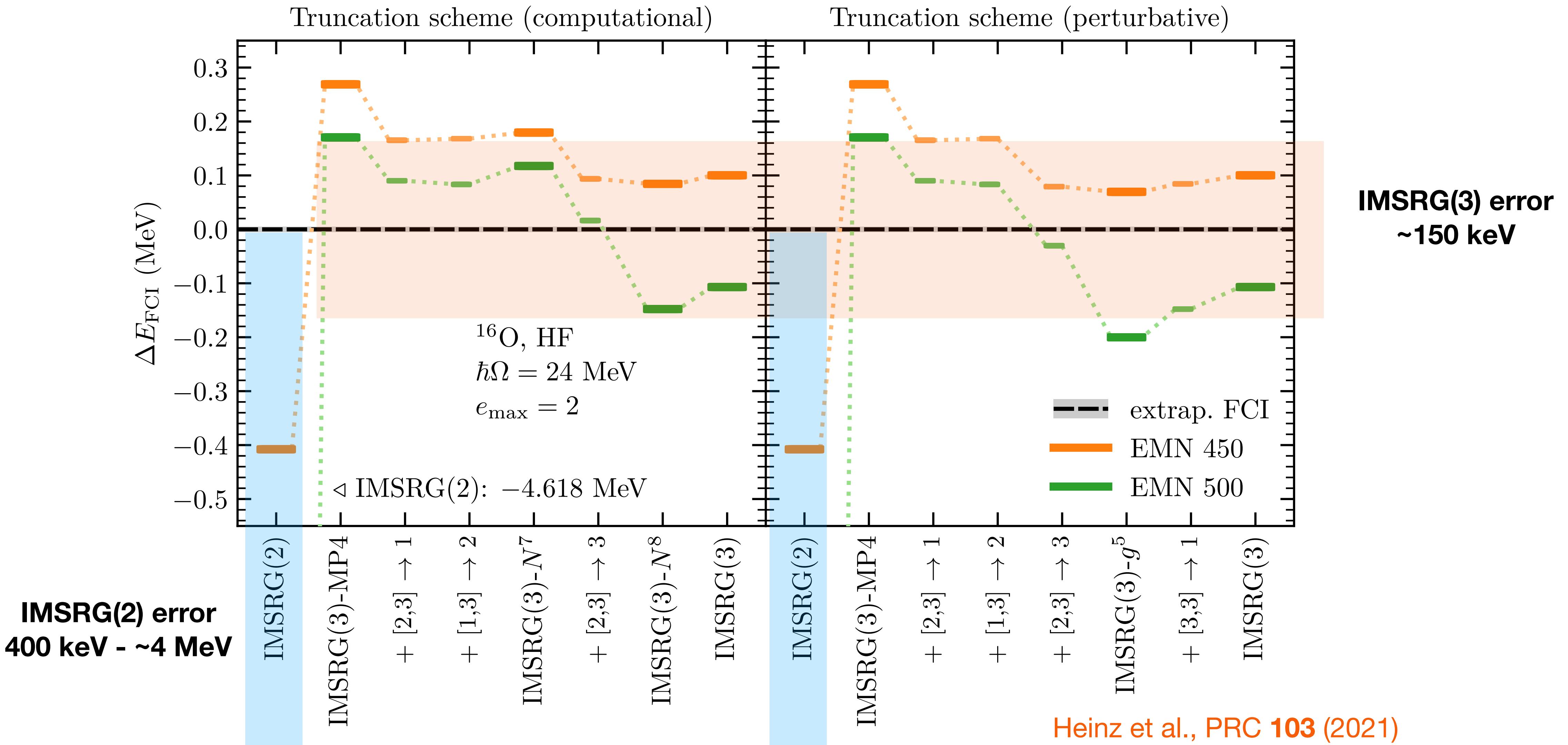
The IMSRG(3) difference



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The technical details...

- Hamiltonian: EM 1.8/2.0
- NAT basis: $e_{\max}^{\text{NAT}} = 16, E_{3\max}^{\text{NAT}} = 22$
- Truncate to smaller e_{\max} for IMSRG
- NO2B initial Hamiltonian
- Capture induced 3B interactions
via IMSRG(3): $e_{\max,3b}, E_{3\max}$
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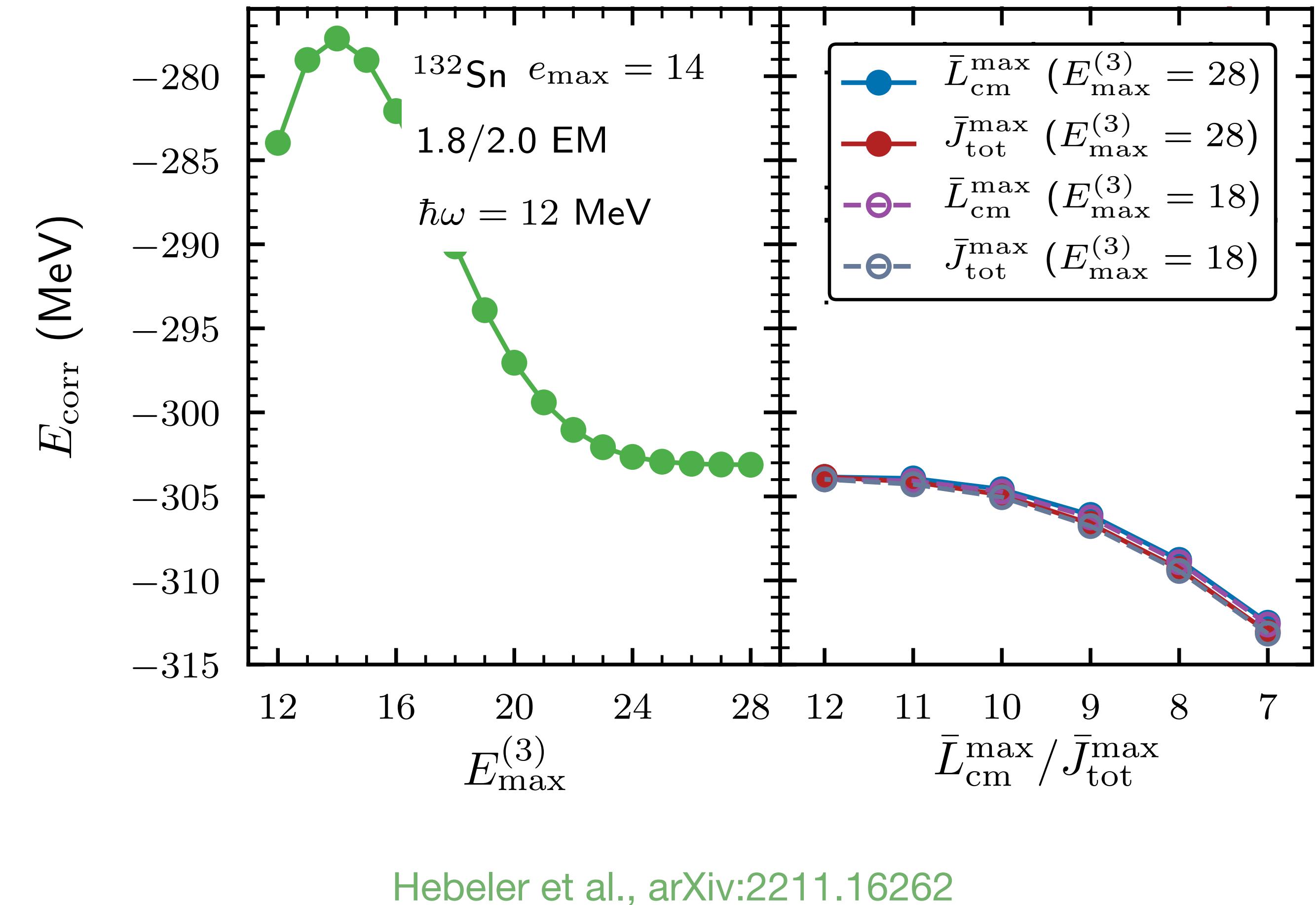
Pragmatic approach:

Work with converged IMSRG(2) and try to capture 3B effects at minimal computational cost

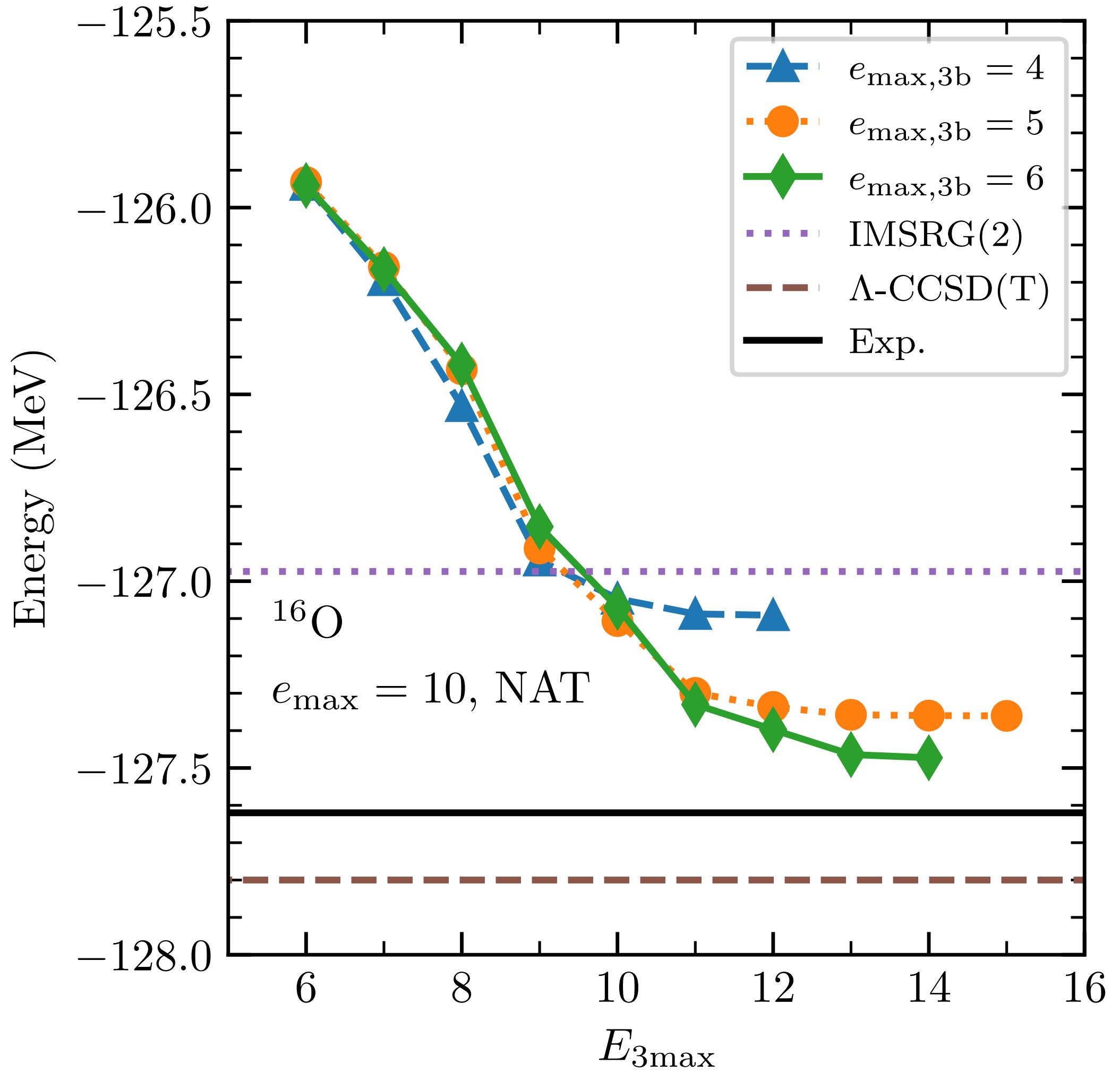
Goal: Converge 3B w.r.t. $e_{\max,3b}, E_{3\max}$

IMSRG(2) as a solid NO2B base

- NAT basis = efficient computational basis
 - Tichai et al., PRL **99** (2019)
 - Hoppe et al., PRC **103** (2021)
 - Novario et al., PRC **102** (2020)
- New treatment of 3N forces in NO2B approximation allow large/no $E_{3\max}$ cut
 - Miyagi et al., PRC **105** (2022)
 - Hebeler et al., arXiv:2211.16262

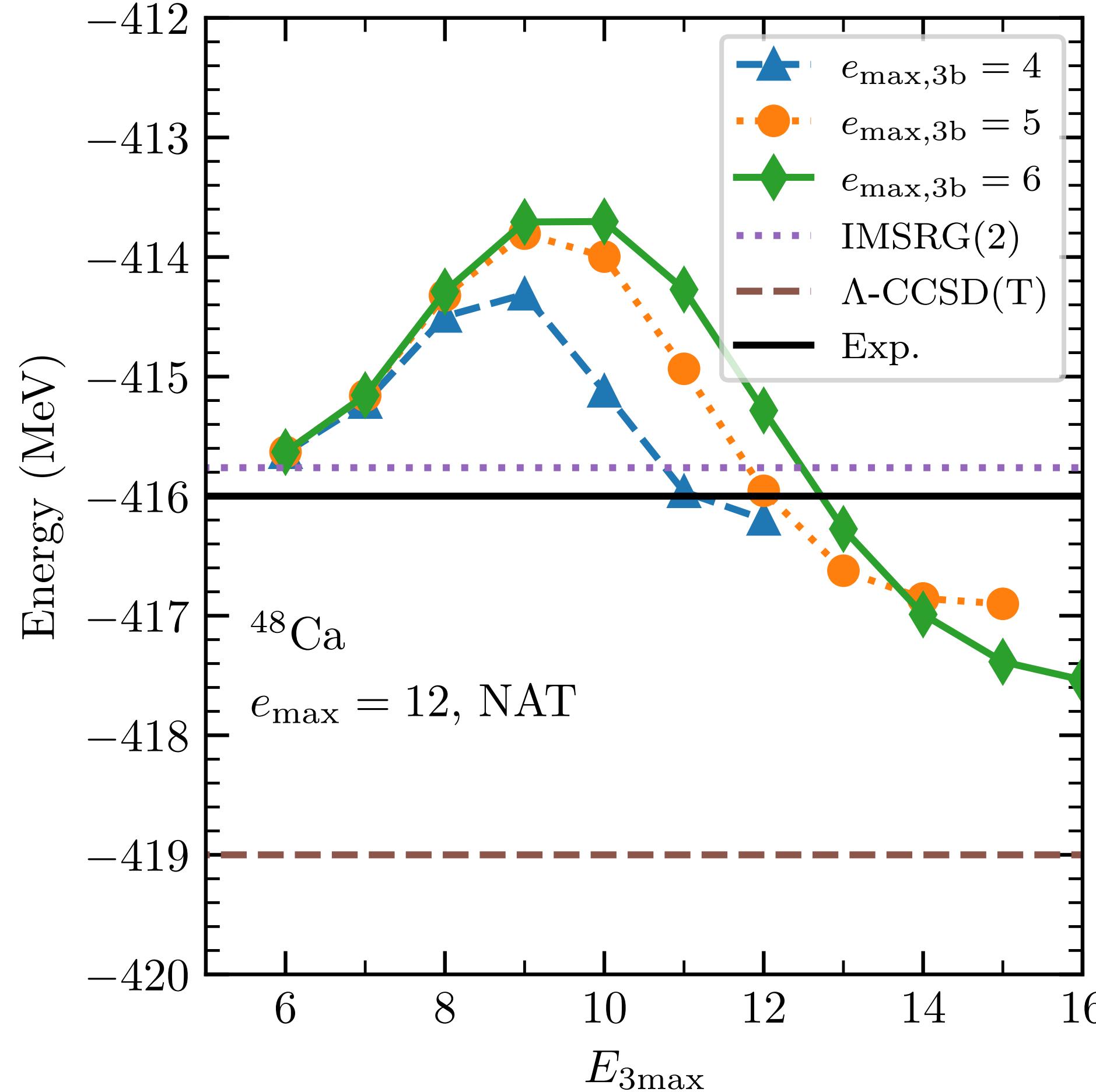


Oxygen-16: The easy case



- IMSRG(2) converged at $e_{\text{max}} = 10$
- Nice convergence in $E_{3\text{max}}$
- $e_{\text{max},3\text{b}}$ seems converged to ~ 100
- Apparent consistency with $\Lambda\text{-CCSD(T)}$

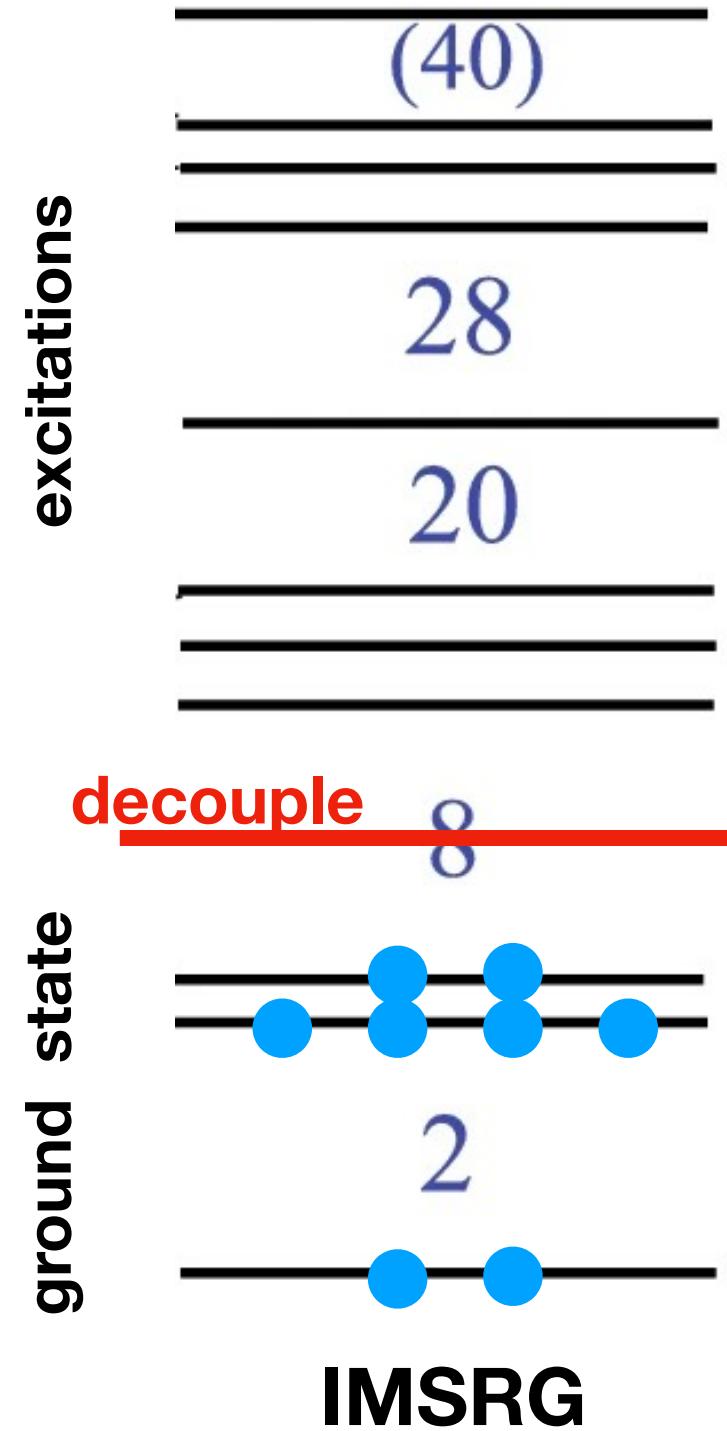
Calcium-48: Not so easy...



- Slow convergence in $e_{\text{max},3\text{b}}$ and $E_{3\text{max}}$
- $E_{3\text{max}} \sim 3e_{\text{max},3\text{b}}$ seems to be required
- Direction of IMSRG(3) corrections consistent with $\Lambda\text{-CCSD(T)}$

The VS-IMSRG

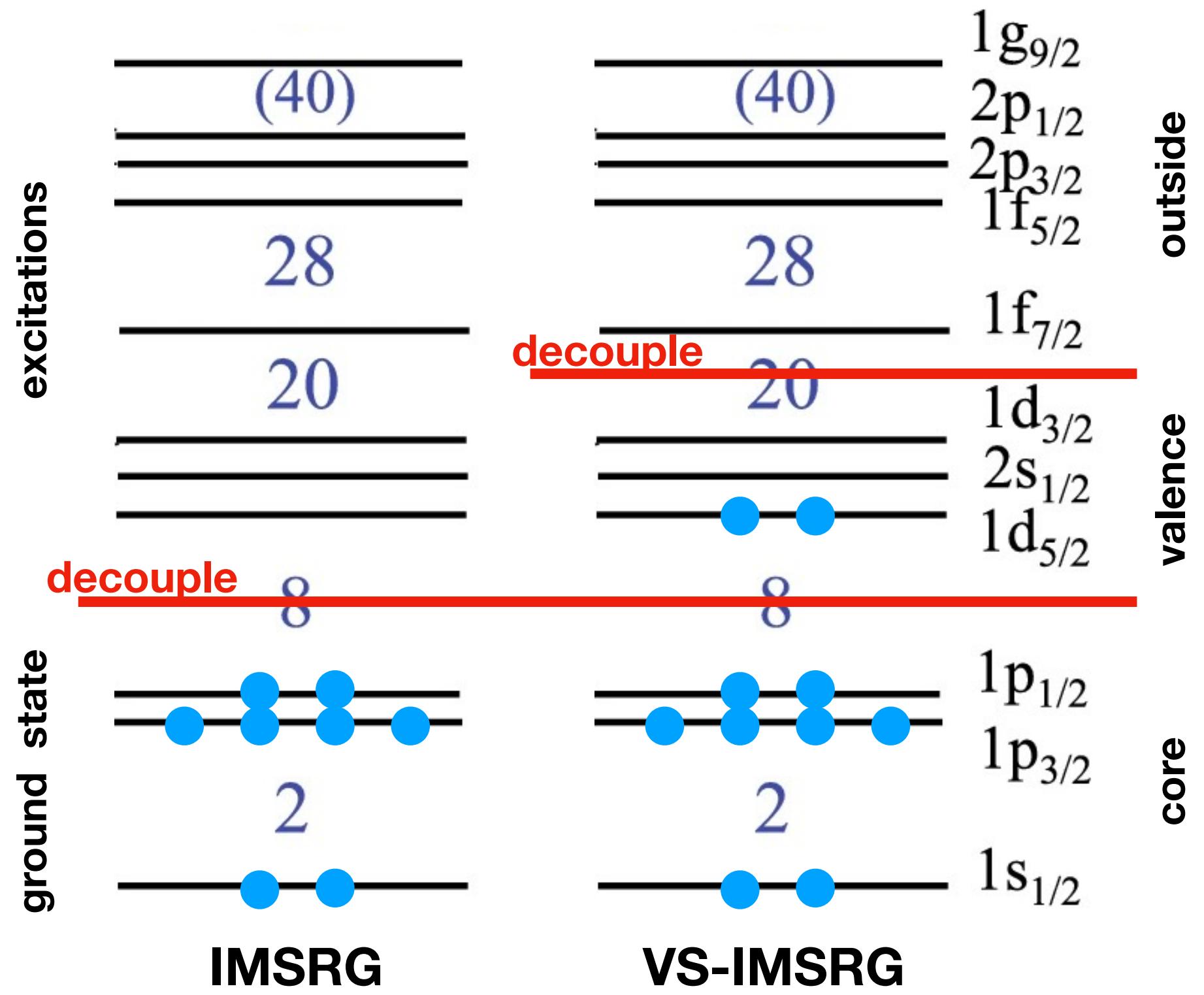
Stroberg et al., PRL 118 (2017)
Stroberg et al., ARNPS 69 (2019)



Hagino et al., Found. Chem. 22 (2020)

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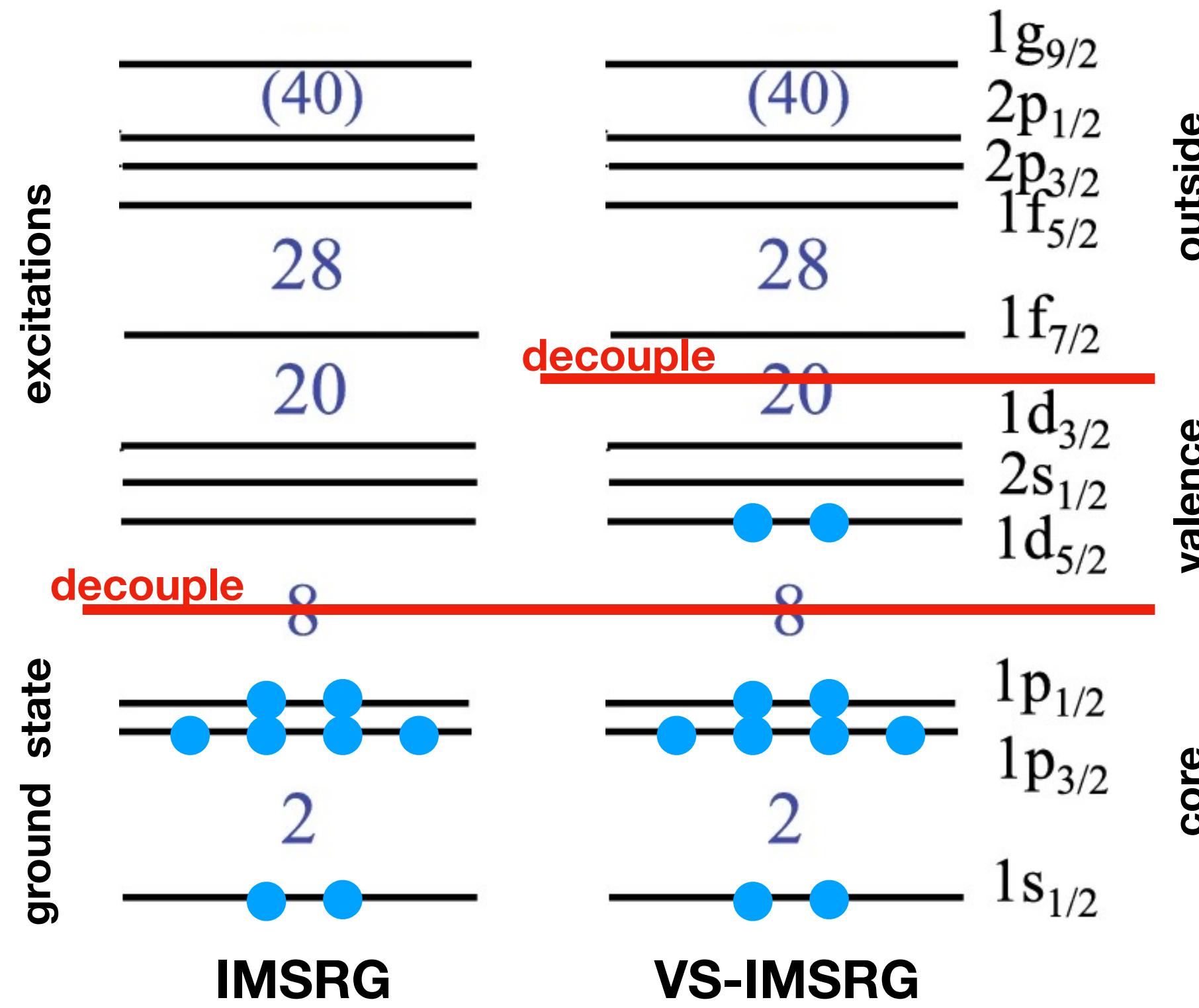
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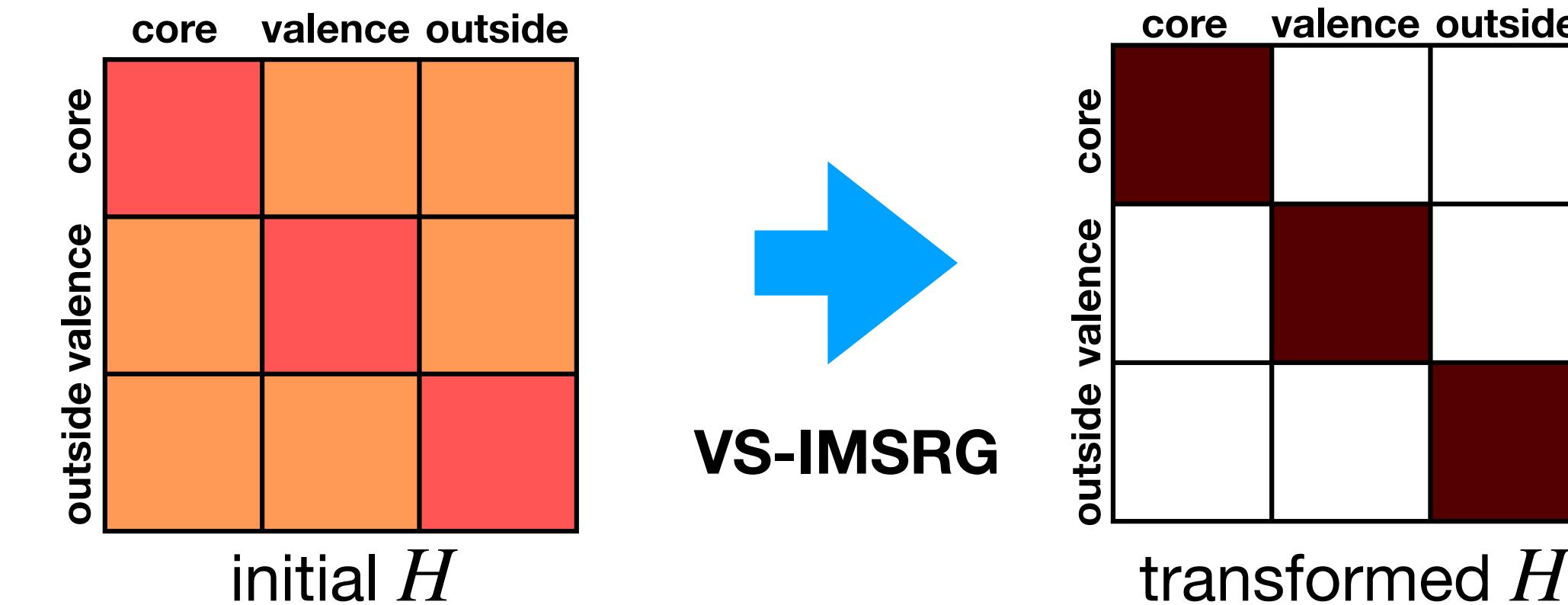
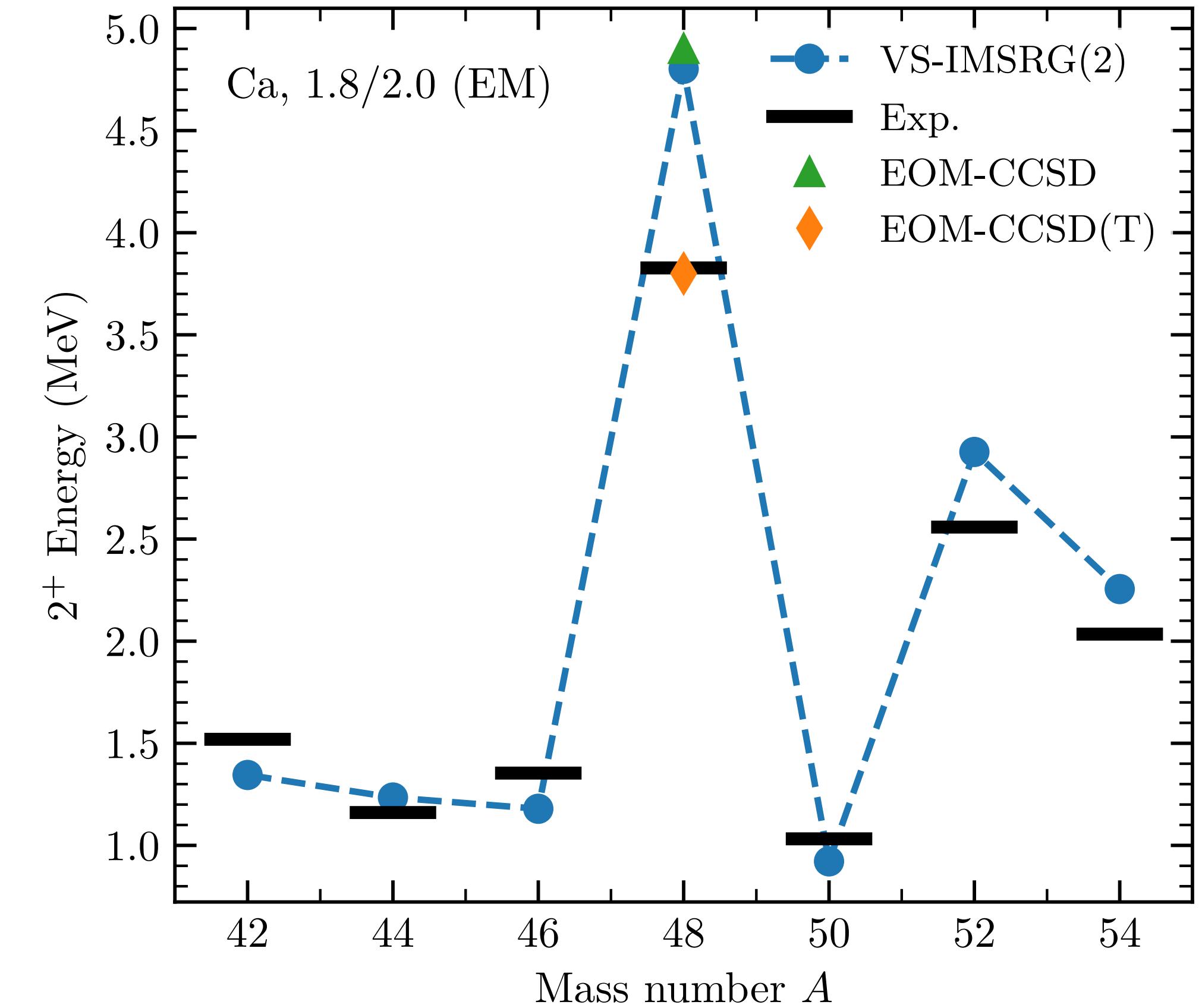


Figure: T. Miyagi

- Decouple core, valence, and outside spaces
- Obtain nucleus-dependent ab initio shell model Hamiltonian
- Can use existing shell model machinery

2^+ energies of calcium-48

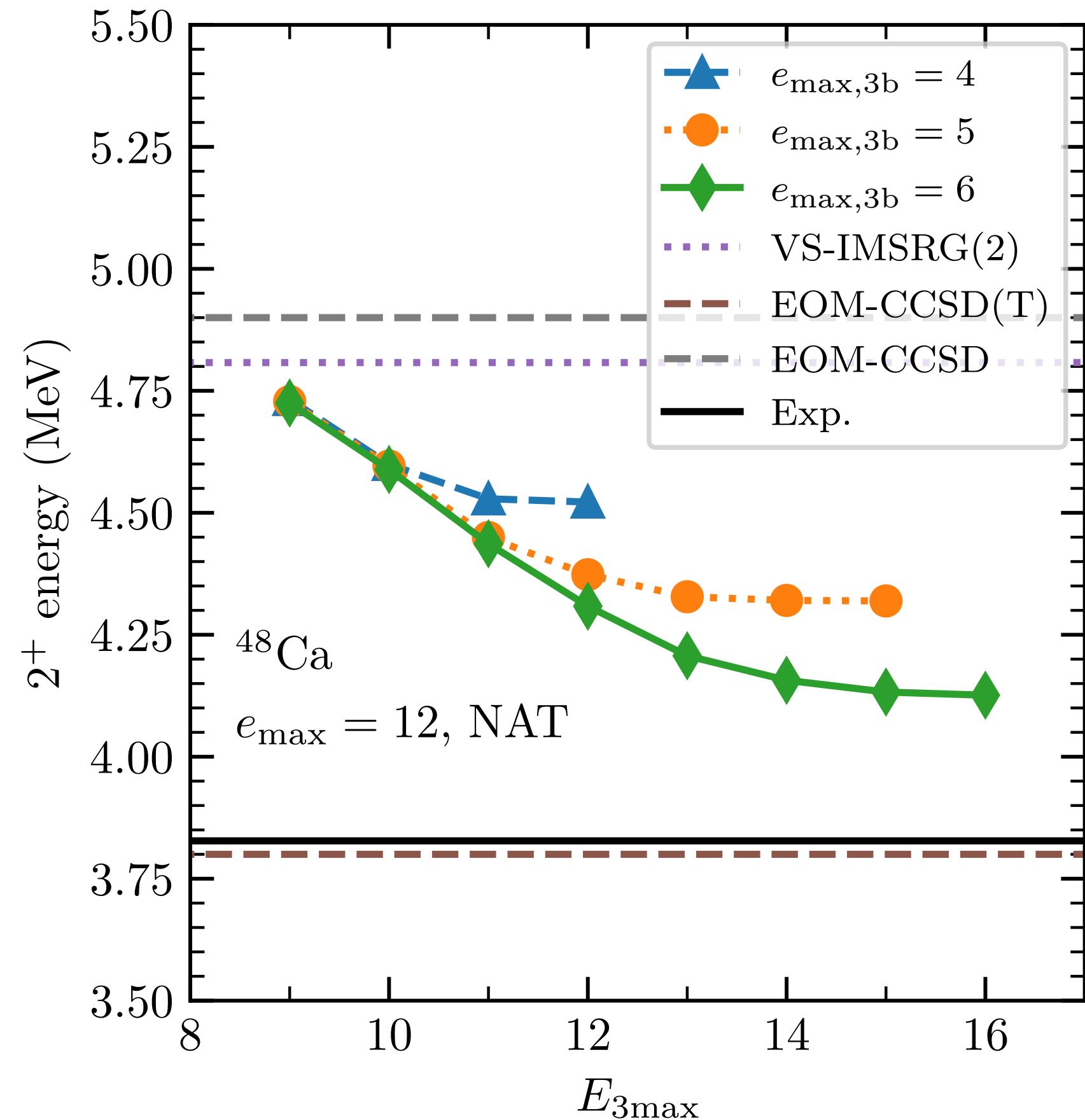
- VS-IMSRG(2) overestimates 2^+ energy
- Does VS-IMSRG(3) solve the problem?



Hagen et al., PRL 117 (2016)

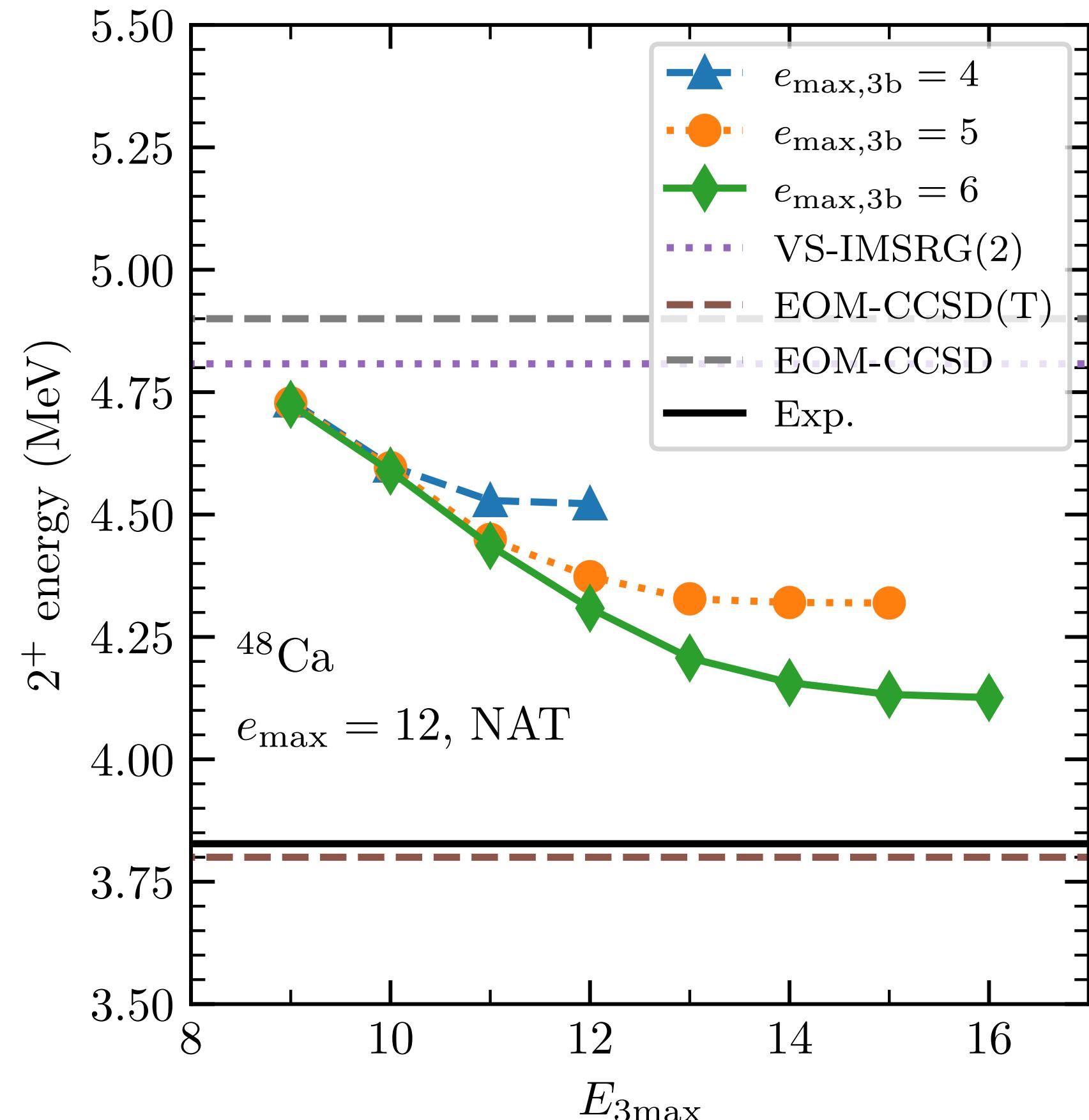
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VS-IMSRG(3) convergence: calcium-48



- Corrections improve description of 2⁺ energy
- But slow convergence in $e_{\text{max},3\text{b}}$

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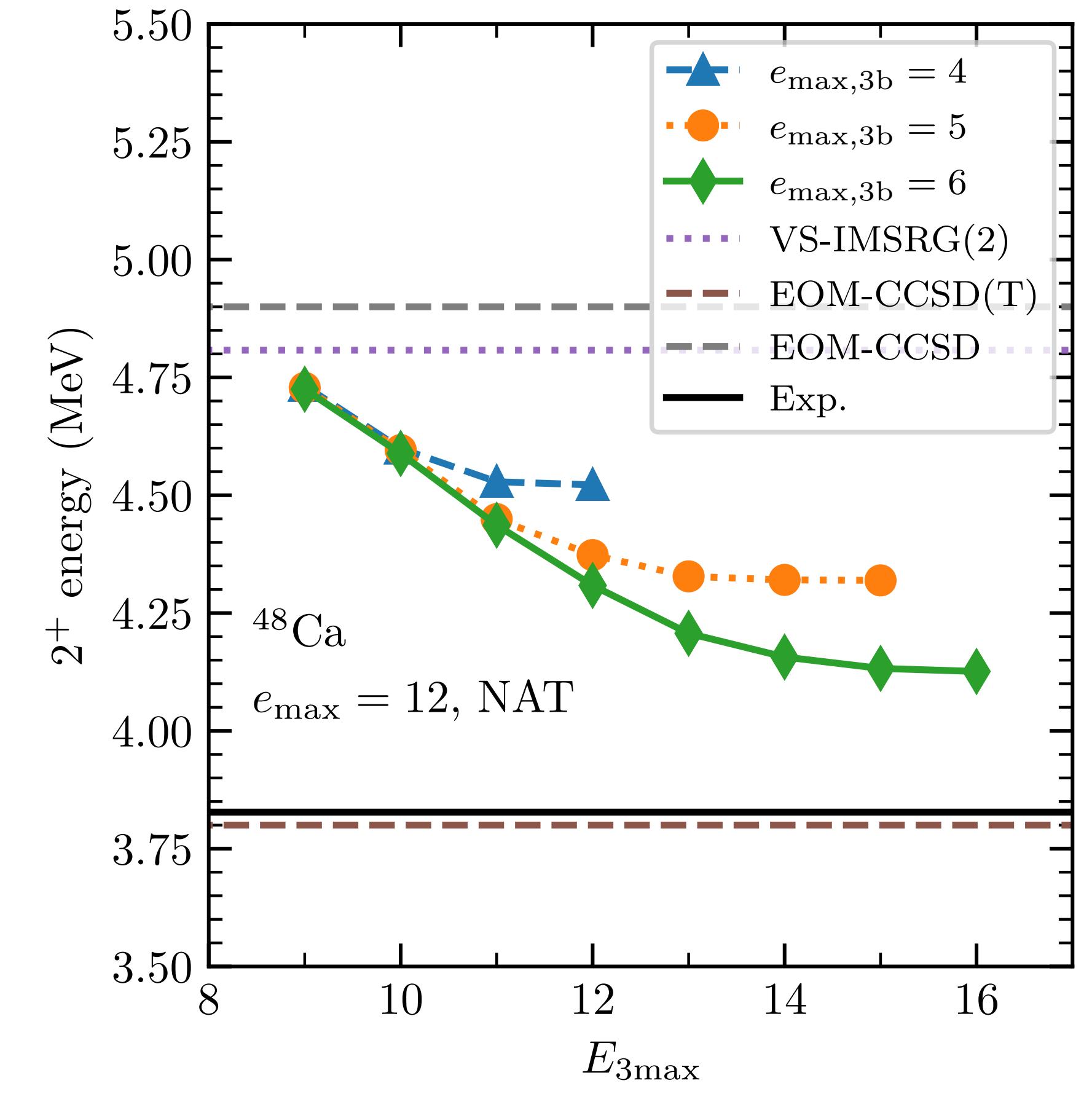
$e_{\text{max},3\text{b}}, E_{3\text{max}}$ seem to be inefficient.

Investigate improved bases
for 3-body operators

Novario et al., PRC 102 (2020)

Conclusions and outlook

- Approaching **realistic IMSRG(3) calculations** of medium-mass nuclei
- Small corrections for ground-state energies;
Larger corrections for 2^+ energies
- Further **optimization** needed
(basis, numerical implementation)
- Impact of IMSRG(3) in **neutron-rich isotopes?**



Acknowledgments

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- TU Darmstadt "STRONGINT" group
- ORNL Nuclear Theory
- ... and all of you for your attention



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