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Implications of the unitarity limit for neutron spectra of halo nuclei

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SFB 1245

Outline

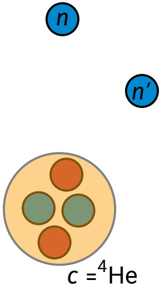
The ^{6}He$(p, p'\alpha)nn$ reaction

Motivation & approach

Ground-state results

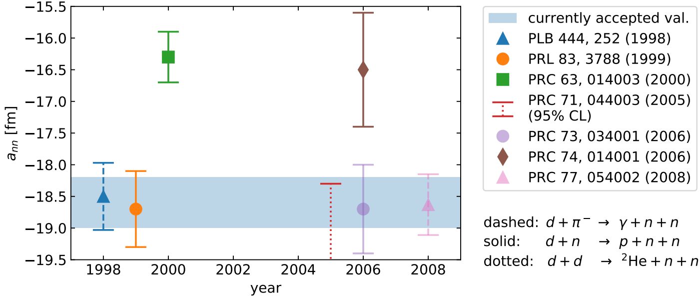
Final-state results

Conclusion & Outlook



Application for use of E_{nn} distribution: measuring the *nn* scattering length

- **motivation:** no high-precision value for the *nn* scattering length available



dashed: $d + \pi^- \rightarrow \gamma + n + n$
solid: $d + n \rightarrow p + n + n$
dotted: $d + d \rightarrow {}^2\text{He} + n + n$

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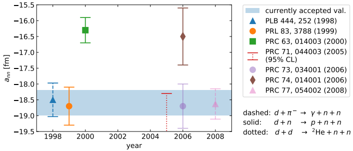

➔ use the reaction ^{6}He$(p, p\alpha)nn$ to determine the scattering length from the final E_{nn} spectrum

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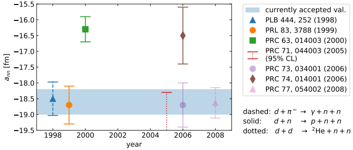

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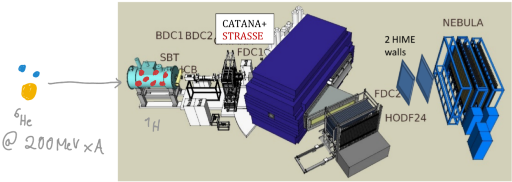

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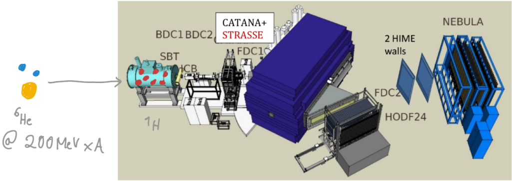
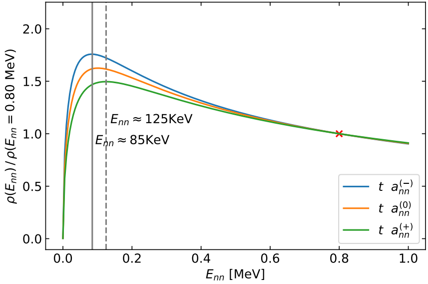


- advantages of this approach
  - different from the previous methods → other systematics
  - final *nn* pair has high center-of-mass velocity in the lab system → avoids problems with detection efficiency
- experiment proposal from Aumann & SAMURAI collaboration approved by RIKEN RIBF NP2012-SAMURAI55R1 (2020)

Application for use of E_{nn} distribution: measuring the *nn* scattering length

- ^{6}He well suitable to an Halo EFT description: S_{2n} = 0.975 MeV < $E_\alpha^* \approx$ 20 MeV
- calculation
  - obtain ground-state wave function
  - apply *nn* final-state interaction (FSI)
  - obtain final distribution



Göbel, Aumann, Bertulani, Frederico, Hammer, Phillips, PRC 104, 024001

-> significant sensitivity of the shape on a_{nn}

Motivation & approach

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- nn relative-energy distributions as an interesting observable
  - $\rho(E_{nn})$ of ^{6}He$(p, p'\alpha)nn$ for a_{nn}
    Göbel, Aumann, Bertulani, Frederico, Hammer, Phillips, PRC 104, 024001
  - $\rho(E_{nn})$ of ^{6}He$(p, p'\alpha)nn$ for calibration run of tetraneutron experiment by Duer et al.
    Duer *et al.*, Nature 606, 678
- investigate universality of this system
  -> could be then used to predict spectra for other halos
  - check how well this works
  - check for *a priori* indicators
- Halo EFT as an ideal tool
  - $\not{\pi}$EFT
  - core & valence nucleons as degrees of freedom
  - results are expanded in k/M_{hi}
    → systematic improvement possible

A few s-wave 2n halos

nucleus	core	$J\pi$	S_{2n} [keV]	E^* [keV]
Li-11		3/2 -	369.3 (6)	
	Li-9	3/2 -		2691 (5)
Be-14		0+	1270 (13)	
	Be-12	0+		2109 (1)
B-17		3/2 -	1380 (210)	
	B-15	?		
B-19		3/2 -	90 (560)	
	B-17	3/2 -		
C-22		0+	100	
	C-20	0+		1618 (11)

data from https://www.nndc.bnl.gov/nudat3/ [except S_{2n} of C-22]

- all can be described via *s*-wave interactions
- core spin = total spin -> core spin can be neglected
  (as long as V_{nc} is the same in $s_c - 1/2$ and $s_c + 1/2$)
- all have a separation of scales between S_{2n} and $E^*(c)$

Obtaining the E_{nn} spectrum

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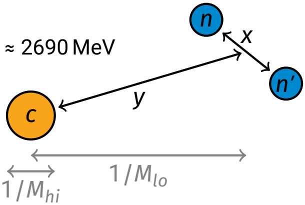
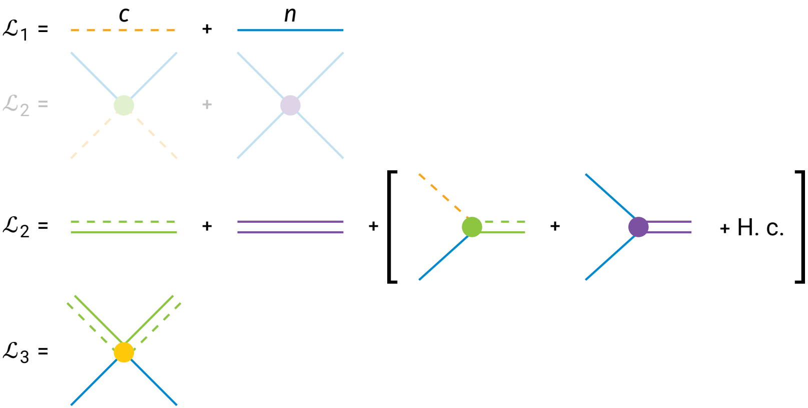
- **approach:** $2n$ s-wave halos in Halo EFT
  1. calculate wave function $\Psi_c(p,q)$
  2. take final state interaction (FSI) into account
  3. calculate the probability distribution for E_{nn}

- **example: properties of ^{11}Li**
  - Borromean $2n$ halo
  - separation of scales: $S_{2n} = 0.269\,\text{MeV} \;<\; E^*(^9\text{Li}) \approx 2690\,\text{MeV}$
  - quantum numbers: $J^\pi = 3/2^-$ $\;(^9\text{Li}: J^\pi = 3/2^-)$
  - leading-order (LO) Halo EFT interaction channels:
    - nn: $^1S_0$
    - nc: $^5S_2$ and $^3S_1$ at LO



Halo EFT for ^{11}Li formulated in Göbel, Acharya, Hammer, Phillips, PRC in press; arXiv:2207.14281
review of Halo EFT in Hammer, Ji, Phillips, JPG 44 (2017)

Lagrangian



Faddeev equations

use EFT in dimer formalism

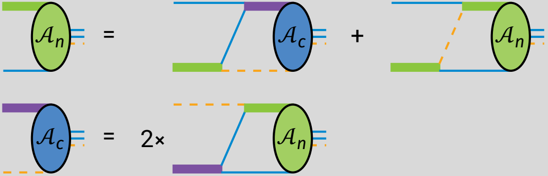
1. step: obtain dressed dimer propagators

& renormalize using **input values**

$a_{nc} \quad \left(a_{nc}^{-1} = \frac{2\pi\Delta_{nc}}{\mu_{nc} g_{nc}^2} + \Lambda\right)$

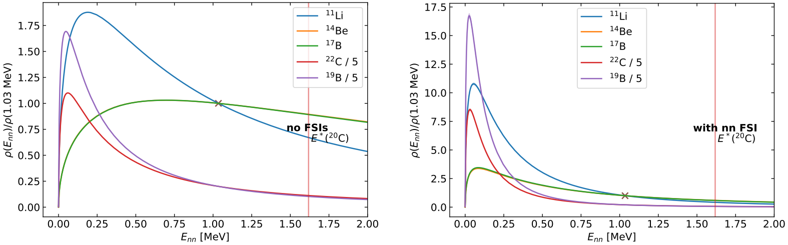
$a_{nn} \quad \left(a_{nn}^{-1} = \frac{2\pi\Delta_{nn}}{\mu_{nn} g_{nn}^2} + \Lambda\right)$

2. step: set up equations for Faddeev transition amplitudes



Results: Different nuclei in comparison

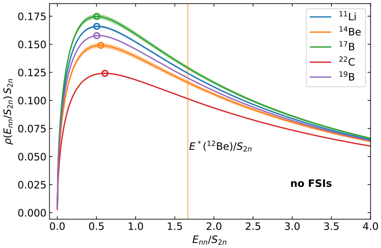
normalization scheme: normalize to certain value @ some position
(experimentally useful)



- hierarchy of S_{2n} becomes clearly visible:
  $S_{2n}(^{19}\mathrm{B}) < S_{2n}(^{22}\mathrm{C}) < S_{2n}(^{11}\mathrm{Li}) < S_{2n}(^{14}\mathrm{Be}) \approx S_{2n}(^{17}\mathrm{B})$
- significant influence of *nn* FSI

Attempts towards an universal distribution

- express distribution in terms of E_{nn}/S_{2n}
- normalization integral: $\int \mathrm{d}x\, \rho(x \coloneqq E_{nn}/S_{2n}) S_{2n} = 1$



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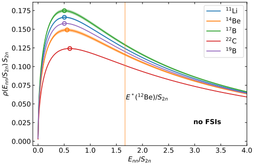


- analysis of parameterization in dimensionless variables:

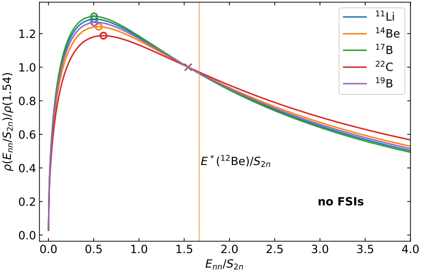
$$\rho(E_{nn}; S_{2n}, a_{nn}, a_{nc}, A) = f(\sqrt{2\mu S_{2n}})\tilde{\rho}\left(\frac{E_{nn}}{S_{2n}}; \bar{a}_{nn}, \bar{a}_{nc}, A\right)$$

with $\bar{a}_{ij} \coloneqq \sqrt{2\mu S_{2n}}a_{ij}$

-> in the case S_{2n} governs distributions, then
  - S_{2n} itself influences only the absolute value
  - the shape is determined by E_{nn}/S_{2n}

Going into the unitarity limit

use different normalization scheme to get rid of the dependence on S_{2n} itself



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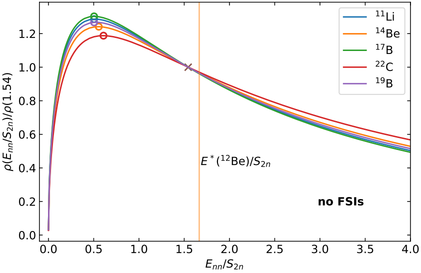

- -> curves almost on top of each other → influence of a_{ij} and A on shape small
- -> test unitarity limit ($t_{ij} \propto (1/a_{ij} + ip)^{-1}$ -> $t_{ij} \propto (ip)^{-1}$)

Unitarity limit in nuclear physics discussed in König, Grießhammer, Hammer, van Kolck, PRL 118, 202501

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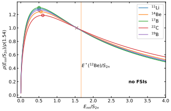

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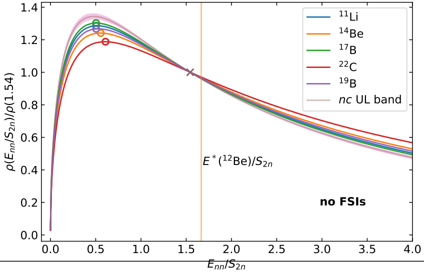

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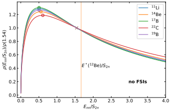

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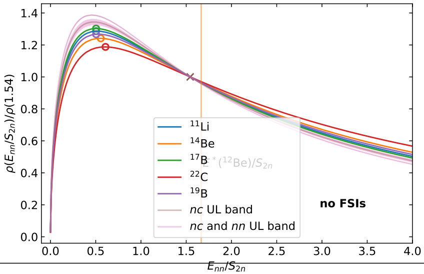

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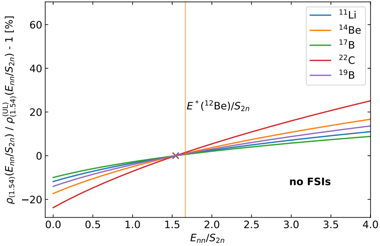


- starting point: leading-order EFT universality; extended by going into the unitarity limit

  $\tilde{\rho}(E_{nn}/S_{2n}; V_{nn}, V_{nc}, S_{2n}, A) \approx \tilde{\rho}\left(E_{nn}/S_{2n}; \sqrt{2\mu S_{2n}}a_{nn}, \sqrt{2\mu S_{2n}}a_{nc}, A\right)$

  $\approx \tilde{\rho}\left(E_{nn}/S_{2n}; \sqrt{2\mu S_{2n}}a_{nn}, A\right)$

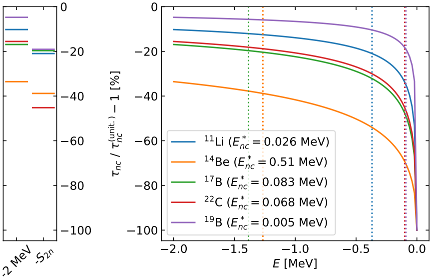
  $\approx \tilde{\rho}(E_{nn}/S_{2n}; A)$

  $\approx \tilde{\rho}(E_{nn}/S_{2n})$

Understanding why the unitarity limit works so well

influence of a_{ij} depends on three points

- size of a_{ij}: the larger a_{ij}, the smaller the influence
- size of S_{2n}: the larger S_{2n}, the smaller the influence of a_{ij}
- interplay of the different forces



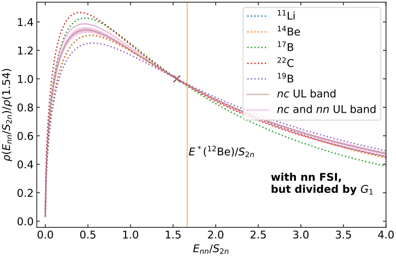
^{22}C t-matrix has largest deviation at S_{2n} and ^{22}C distribution has largest deviation

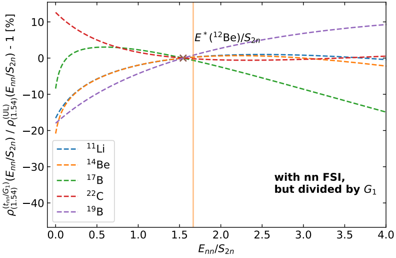
Universality of the final distributions

- build onto the ground-state result and include *nn* FSI using an enhancement factor
  $\tilde{\rho}^{(\mathrm{wFSI})}(E_{nn}/S_{2n}; V_{nn}, V_{nc}, S_{2n}, A) \approx \tilde{\rho}(E_{nn}/S_{2n}; A)\, G(E_{nn}; a_{nn}, r_{nn})$

-> ground state and *nn* FSI have unaligned universalities
univ. driven by S_{2n} vs. "trivial univ." given by a_{nn}

-> $\tilde{\rho}/G_1$ should be an universal function of E_{nn}/S_{2n}





Conclusion & Outlook

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Conclusions

- reliable calculation of *nn* distributions in Halo EFT
- found universality of ground-state distributions and for the final distributions
- *nn* and *nc* interactions can be put in the unitarity limit

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Outlook

- interesting possibilities for comparisons
  - experiments
  - *ab initio* χEFT calculations
- go to NLO to asses the accuracy of the univ. results better
- repeat the calcs. for different kinematics:
  investigate multiple FSIs (use Møller-operator-based approximation scheme)
  (cf. our calculation of $E1$ strength of ^{11}Li)

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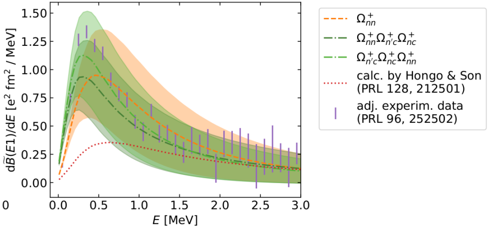

Göbel, Acharya, Hammer, Phillips, PRC in press; arXiv: 2207.14281 [nucl-th] (2022)