Evgeny Epelbaum, RUB

Hirschegg 2023, Januar 15 - 21, 2023

New insights into
Renormalization & regularization of nuclear forces

in collaboration with Ashot Gasparyan and Hermann Krebs

Is the „RG invariant“ χEFT really RG invariant?

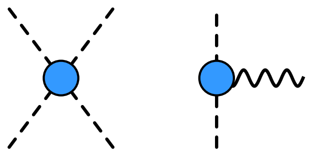
Is the finite-Λ EFT approach renormalizable?

Is mixing DimReg & CutoffReg consistent?

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

Chiral Effective Field Theory

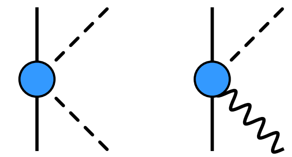
GB dynamics
Weinberg, Gasser, Leutwyler, ...



Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi}{\text{breakdown scale } \Lambda_b} \sim \frac{1}{4} \cdots \frac{1}{3}$$

πN dynamics
Bernard-Kaiser-Meißner et al.



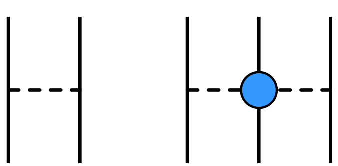
Effective Lagrangian:

$$\mathcal{L}_\pi = \frac{F^2}{4} \mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots ,$$

$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A u \cdot S)N + \ldots ,$$

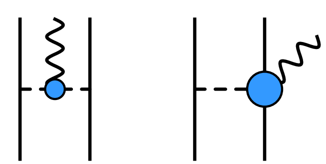
$$\mathcal{L}_{NN} = -\frac{1}{2} C_S (\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \ldots$$

Nuclear forces
Weinberg, van Kolck, Kaiser, EGM, ...



Resummed non-perturbatively
by solving the many-body Schrödinger equation

Nuclear currents
Park et al, Bochum-Bonn, JLab-Pisa



How to renormalize nuclear chiral EFT?

— in any EFT, results of the calculations must be regulator-independent
— pion exchange contributions need to be resummed to all orders

Renormalization: Exact Λ-independence at every order, $\Lambda \gg \Lambda_b$
→ dictates power counting
van Kolck, Long, Yang, Valderrama …

— consistent in the EFT sense?
EE, Gegelia, Meißner, Gasparyan

— achievable at all?
Gasparyan, EE, 2210.16225

Renormalization: Exact Λ-independence only at ∞ order, $\Lambda \sim \Lambda_b$
Lepage, EE, Gegelia, Meißner, …

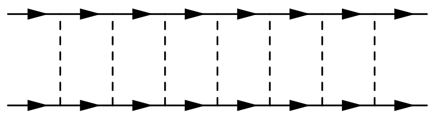
— cutoff independence proven?
Towards a formal proof:
Gasparyan, EE, PRC 105 (22) 024001

Is the „RG-invariant EFT“ cutoff-independent?

Ashot Gasparyan, EE, e-Print: 2210.16225 [nucl-th]

The „RG-invariant EFT“ in a nutshell

OPEP is non-renormalizable in the usual sense, i.e. ∞ many c.t.'s at finite orders

E.g.:  $\propto \frac{1}{d-4} p^6 m^6$ (in spin-triplet channels)

The „RG invariant“ approach with $\Lambda \gg \Lambda_b$: $T \sim 1 + \Lambda + \Lambda^2 + \ldots = (1-\Lambda)^{-1}$ van Kolck, Long, Yang, …

— UV finite, but no loop expansion of the „renormalized“ amplitude

— really an EFT? EE, Gegelia, EPJA 41 (09) 341; EE, Gasparyan, Gegelia, Meißner, EPJA 54 (18) 186

— believed to yield RG-invariant results (if treating corrections perturbatively)

How is the RG invariance understood?

“RGI requires not only independence of observables on the numerical value of the cutoff Λ but also independence on the form of the regulator function itself“ Song, Lazauskas, van Kolck, PRC 96 (2017) 024002

— the amplitude should have a well-defined limit $\{\Lambda_1, \Lambda_2, \ldots\} \to \infty$ at $\forall$ order
— this limit should be unique

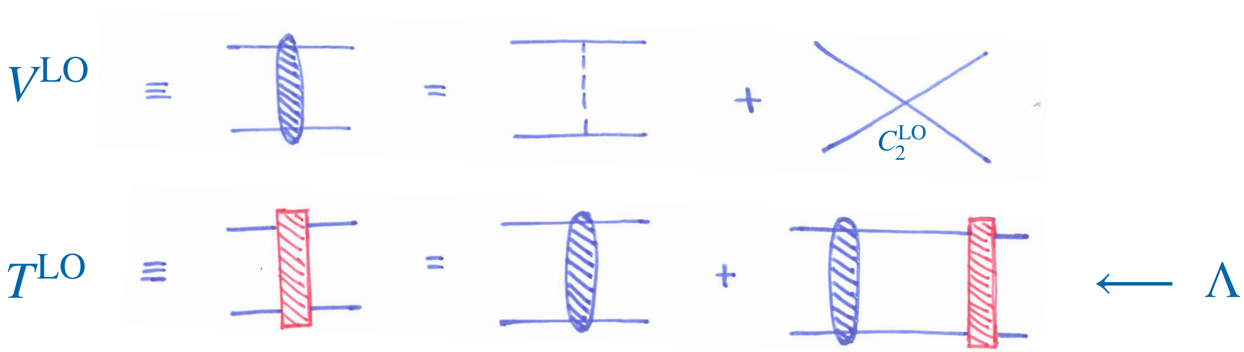
Is the „RG-invariant EFT“ cutoff-independent?

Ashot Gasparyan, EE, e-Print: 2210.16225 [nucl-th]

Consider $^3P_0$ as an example of attractive spin-triplet channel with non-perturbative OPEP

LO: Self-adjoint extension of the singular OPEP ($\sim 1/r^3$) by promoting $C_2 p' p$ to LO

Nogga, Timmermans, van Kolck, PRC 72 (2005) 054006



Renormalization condition:

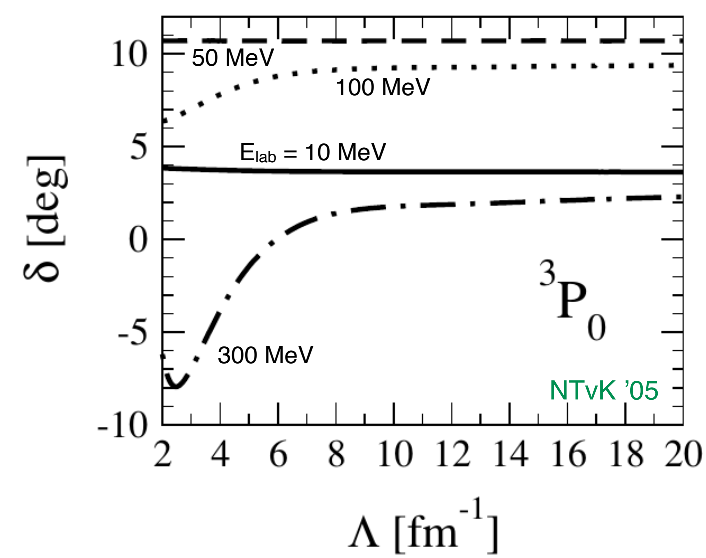
$$\delta_{\rm LO}(k_1) = \delta_{\rm exp}(k_1) \quad \Rightarrow \quad C_2^{\rm LO}(\Lambda)$$

on-shell mom.

(limit-cycle-like
for nonlocal reg.)

The resulting phase shifts at finite energies $\delta_{\rm LO}(k)$ approach a finite limit when $\Lambda \to \infty$

$\Rightarrow$ RG invariance fulfilled



Is the „RG-invariant EFT“ cutoff-independent?

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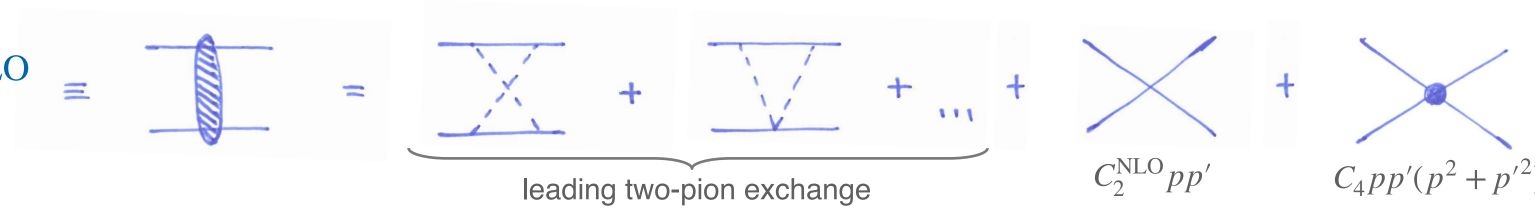
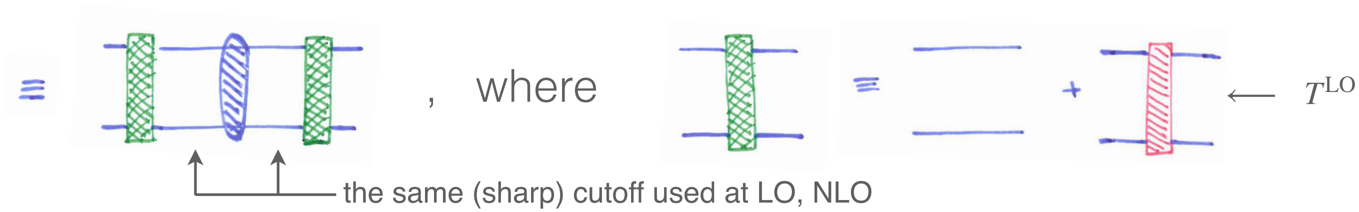
NLO: Distorted-Wave Born approximation

Long, Yang, PRC 84 (2011) 057001

$V^{\rm NLO}$

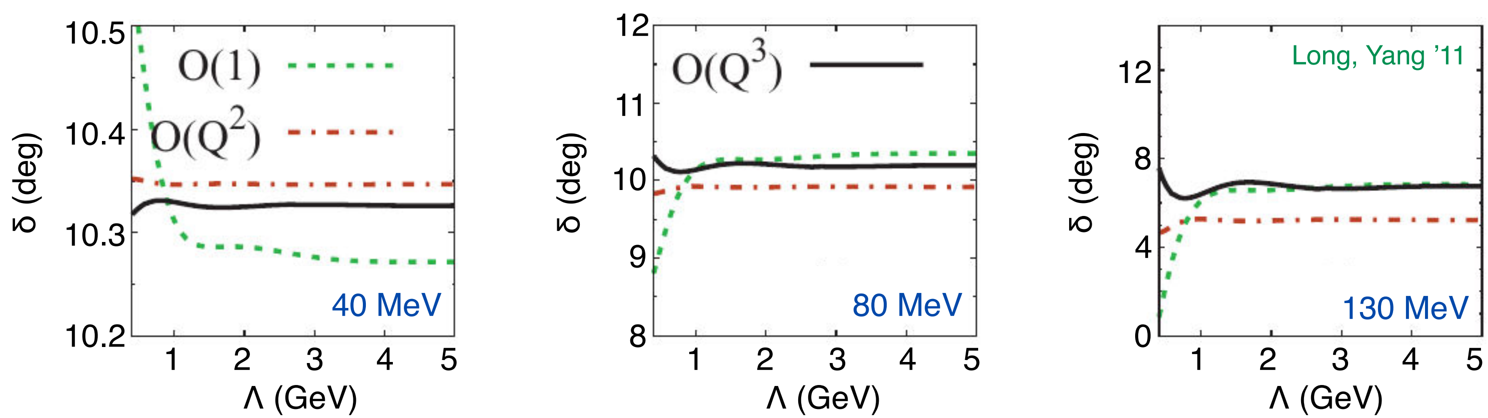


$\Delta T^{\rm NLO}$



Renorm. conditions: $\delta_{\rm NLO}(k_1) = 0 \;\Rightarrow\; \left.\begin{array}{l} [\delta_{\rm LO} + \delta_{\rm NLO}]_{k_1} = \delta_{\rm exp}(k_1) \\ [\delta_{\rm LO} + \delta_{\rm NLO}]_{k_2} = \delta_{\rm exp}(k_2) \end{array}\right\} \Rightarrow \; C_2^{\rm NLO}(\Lambda),\; C_4(\Lambda)$

⇒ resulting phase shifts seem RG invariant



Is the „RG-invariant EFT“ cutoff-independent?

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What can possibly go wrong?

Suppose, for simplicity, that $V_{2\pi}$ is less singular so that no C_4-term is needed at NLO (e.g., $V_{2\pi} \;\to\; V_{2\pi}\,(3M_\pi)^2\left((3M_\pi)^2+q^2\right)^{-1}$)

$$\Delta T^{\rm NLO}_{\Lambda}(k) \;=\; \left[1 \;+\; T^{\rm LO}_{\Lambda_0}\,G_0\right]\left(V_{2\pi,\,\Lambda_1} \;+\; C_2^{\rm NLO}\,\underbrace{V_{{\rm ct},\,\Lambda_1}}_{pp'}\right)\left[1 \;+\; G_0\,T^{\rm LO}_{\Lambda_0}\right]$$

Using the N/D representation of the amplitude + dispersion theory:

$$\Delta T^{\rm NLO}_{\Lambda}(k) \;=\; \frac{k^2}{[D_\infty(k)]^2}\left[N_{2\pi,\infty}(k) \;+\; C_{2\pi}(\Lambda) \;+\; \Delta_{2\pi,\Lambda}(k) \;+\; C_2^{\rm NLO}\left(C_{\rm ct}(\Lambda) \;+\; \Delta_{{\rm ct},\Lambda}(k)\right)\right]$$

on-shell mom. depends only on $\delta_{\rm LO}(k)$ vanish for $\Lambda \to \infty$

Renormalization condition: $\Delta T^{\rm NLO}_{\Lambda}(k_1) \;=\; 0 \quad\Longrightarrow\quad C_2^{\rm NLO}(\Lambda) \approx -\dfrac{N_{2\pi,\infty}(k_1) \;+\; C_{2\pi}(\Lambda)}{C_{\rm ct}(\Lambda)}$

large Λ

$$\Longrightarrow \quad \Delta T^{\rm NLO}_{\Lambda}(k) \;\approx\; \frac{k^2}{[D_\infty(k)]^2}\left[N_{2\pi,\infty}(k) \;-\; N_{2\pi,\infty}(k_1)\right] \quad\longleftarrow\quad \text{RG invariant}$$

Problem: if $C_2^{\rm NLO}(\bar\Lambda) \to \infty$, then $\Delta T^{\rm NLO}_{\Lambda}(k) \;\approx\; \dfrac{k^2}{[D_\infty(k)]^2}\left[N_{2\pi,\infty}(k) \;-\; N_{2\pi,\infty}(k_1) \;+ C_2^{\rm NLO}\,\Delta_{{\rm ct},\Lambda}(k)\right]$

Is the „RG-invariant EFT" cutoff-independent?

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„Exceptional" cutoffs $\bar\Lambda$:

$C_{\rm ct}(\bar\Lambda) + \Delta_{{\rm ct},\bar\Lambda}(k_1) = 0$ and $C_{\rm ct}(\bar\Lambda) + \Delta_{{\rm ct},\bar\Lambda}(k) \neq 0$ (i.e., zero is not factorizable)

The oscillating nature of $\psi^{\rm LO}_{k,\Lambda_0}(r)$ that enters $C_{\rm ct}(\Lambda)$ suggests that „exceptional" cutoff values extend to infinity and cannot be avoided in general…

Using the N/D representation of the amplitude + dispersion theory:

$$\Delta T^{\rm NLO}_{\Lambda}(k) = \frac{k^2}{[D_\infty(k)]^2}\Big[N_{2\pi,\infty}(k) + C_{2\pi}(\Lambda) + \Delta_{2\pi,\Lambda}(k) + C_2^{\rm NLO}\big(C_{\rm ct}(\Lambda) + \Delta_{{\rm ct},\Lambda}(k)\big)\Big]$$

on-shell mom. depends only on $\delta_{\rm LO}(k)$ vanish for $\Lambda \to \infty$

Renormalization condition: $\Delta T^{\rm NLO}_{\Lambda}(k_1) = 0 \quad \Longrightarrow \quad C_2^{\rm NLO}(\Lambda) \approx -\dfrac{N_{2\pi,\infty}(k_1) + C_{2\pi}(\Lambda)}{C_{\rm ct}(\Lambda)}$

large Λ

$$\Longrightarrow \quad \Delta T^{\rm NLO}_{\Lambda}(k) \approx \frac{k^2}{[D_\infty(k)]^2}\Big[N_{2\pi,\infty}(k) - N_{2\pi,\infty}(k_1)\Big] \longleftarrow \text{RG invariant}$$

Problem: if $C_2^{\rm NLO}(\bar\Lambda) \to \infty$, then $\Delta T^{\rm NLO}_{\Lambda}(k) \approx \dfrac{k^2}{[D_\infty(k)]^2}\Big[N_{2\pi,\infty}(k) - N_{2\pi,\infty}(k_1) + C_2^{\rm NLO}\,\Delta_{{\rm ct},\Lambda}(k)\Big]$

Is the „RG-invariant EFT" cutoff-independent?

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Back to the NLO analysis of $^3P_0$ by Long and Yang.

NLO potential: $V^{\rm NLO} = V_{2\pi} + C_2^{\rm NLO}(\Lambda)\, p'p + C_4(\Lambda)\, p'p\,(p^2+p'^2)$

DWA for the NLO amplitude:

$$T^{\rm NLO}(k) = \left[1 + T^{\rm LO} G_0\right] V^{\rm NLO} \left[1 + G_0 T^{\rm LO}\right]$$
$$= T_{2\pi}(k) + C_2^{\rm NLO}\, T_{{\rm ct},\,2}(k) + C_4\, T_{{\rm ct},\,4}(k)$$

— same sharp cutoff Λ used at LO and NLO

— NLO renormalization conditions: $\delta_{\rm NLO}(k_1) = 0$ and $\left[\delta_{\rm LO} + \delta_{\rm NLO}\right]_{k_2} = \delta_{\rm exp}(k_2)$

Exceptional cutoffs $\bar\Lambda$:

$$\begin{vmatrix} T_{{\rm ct},\,2}(k_1) & T_{{\rm ct},\,4}(k_1) \\ T_{{\rm ct},\,2}(k_2) & T_{{\rm ct},\,4}(k_2) \end{vmatrix} = 0 \quad \Longrightarrow \quad C_2^{\rm NLO}(\bar\Lambda),\, C_4(\bar\Lambda) \to \infty$$

$\bar\Lambda \approx$ 0.6 GeV, 2.7 GeV, 6.5 GeV, 12 GeV, …

Is the „RG-invariant EFT" cutoff-independent?

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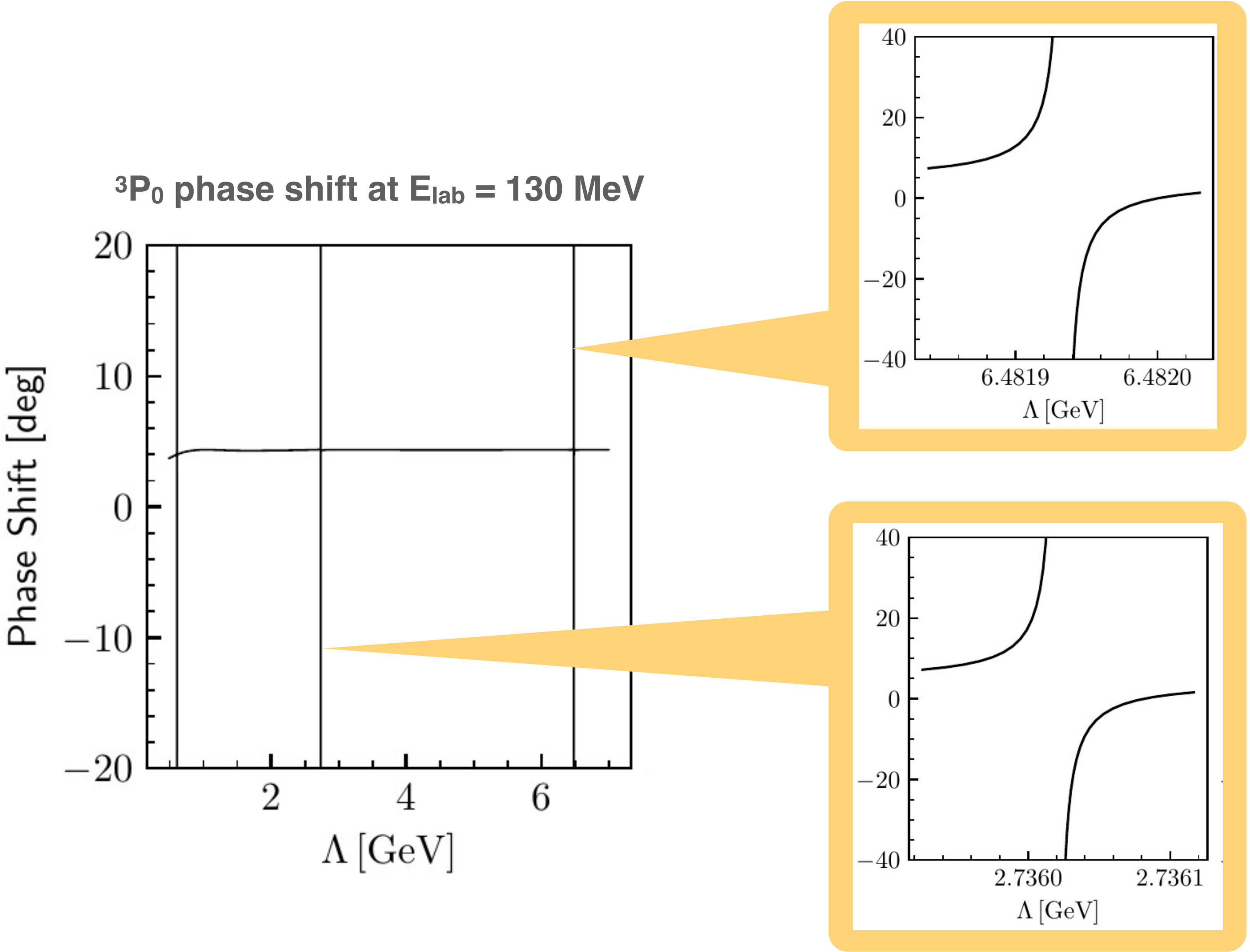

Is the „RG-invariant EFT“ cutoff-independent?

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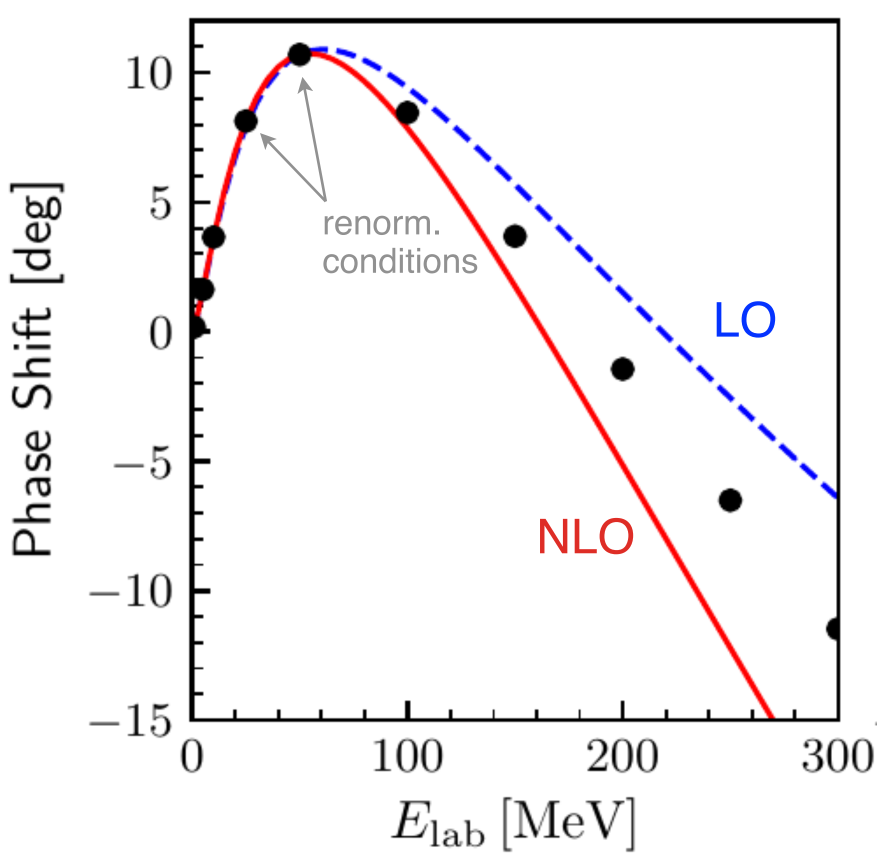
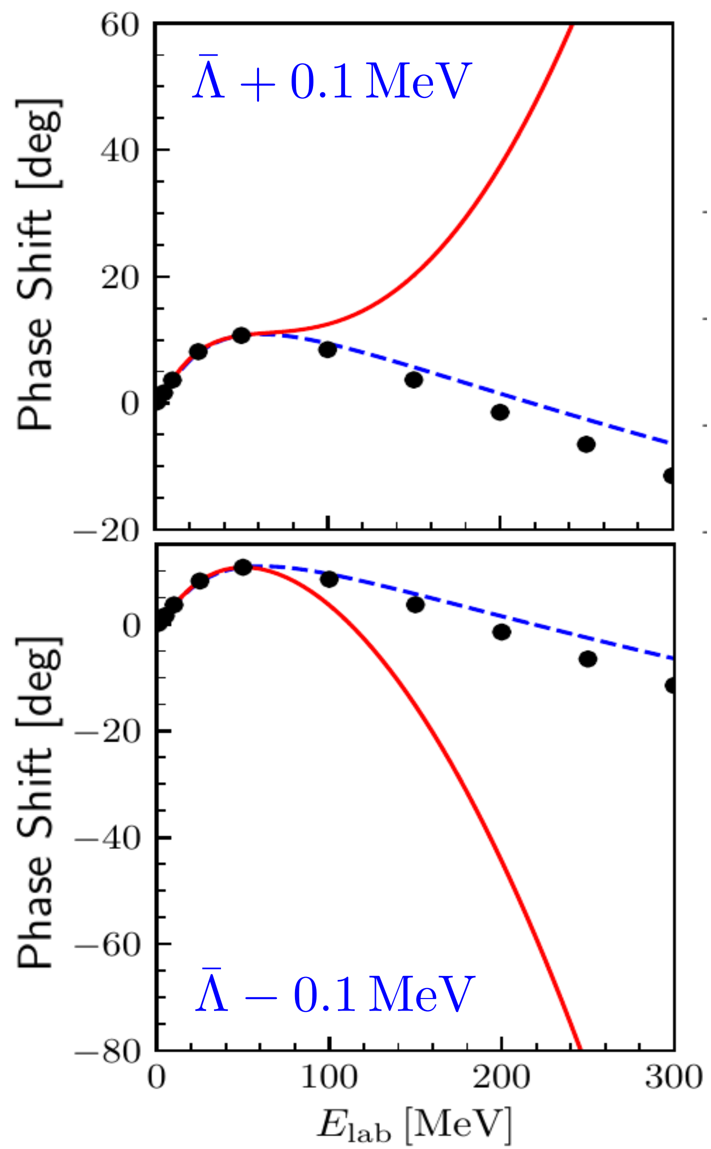
Typical cutoff



„Exceptional“ cutoff $\bar{\Lambda} \sim 12$ GeV



Breakdown of the „RG invariant“ approach!

Renormalizability proof of a finite-cutoff approach to NLO

Ashot Gasparyan, EE, PRC 105 (22) 2, 024001

Renormalizability in the EFT sense: all power-counting breaking terms are absorbable into redefinition of the available counterterms

Renormalizability proof of a finite-Λ approach

Ashot Gasparyan, EE, PRC 105 (22) 2, 024001

Use $\Lambda \sim \Lambda_b$ and assume that the series in V_0 is convergent

Leading order

$$T_0^{[n]} = V_0 (G V_0)^n \sim \mathcal{O}(Q^0)$$

Expect: $\displaystyle \int \frac{p^{n-1} dp}{(\Lambda_V)^n} \sim \left(\frac{\Lambda}{\Lambda_V}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$ $\longleftarrow$ verified explicitly + obtained upper bounds

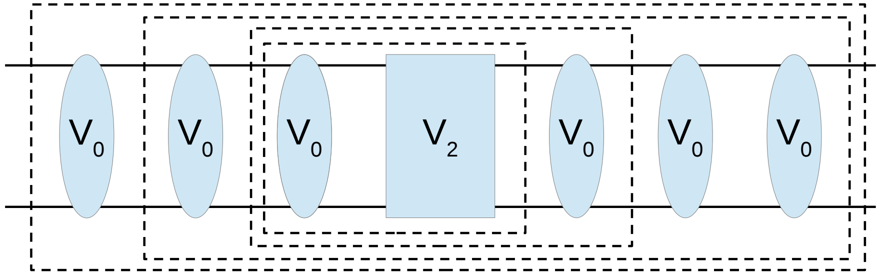
Next-to-leading order

$$T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting breaking terms emerge from momenta $p \sim \Lambda, \;\; p' \sim \Lambda$ in $V_2(p', p)$

Applied the BPHZ subtraction scheme (the forest formula) to prove recursively

$\forall\, m, n$ that $\mathbb{R}\left[T_2^{[m,n]}(k)\right] \sim \mathcal{O}(Q^2)$ and obtained explicit upper bounds.



Generalization to non-perturbative cases is underway Ashot Gasparyan, EE, to appear

Regularization of nuclear interactions and the chiral symmetry

SMS NN chiral potentials Reinert, Krebs, EE, EPJA (18) 85

— local regularization of long-range terms in momentum space

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\, e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\mu \frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2}\, e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

No long-range finite-cutoff artifacts, keeps pion physics intact

— non-local angle-independent Gaussian regulator for contacts:
Simplifies the determination of LECs

The SMS NN potentials

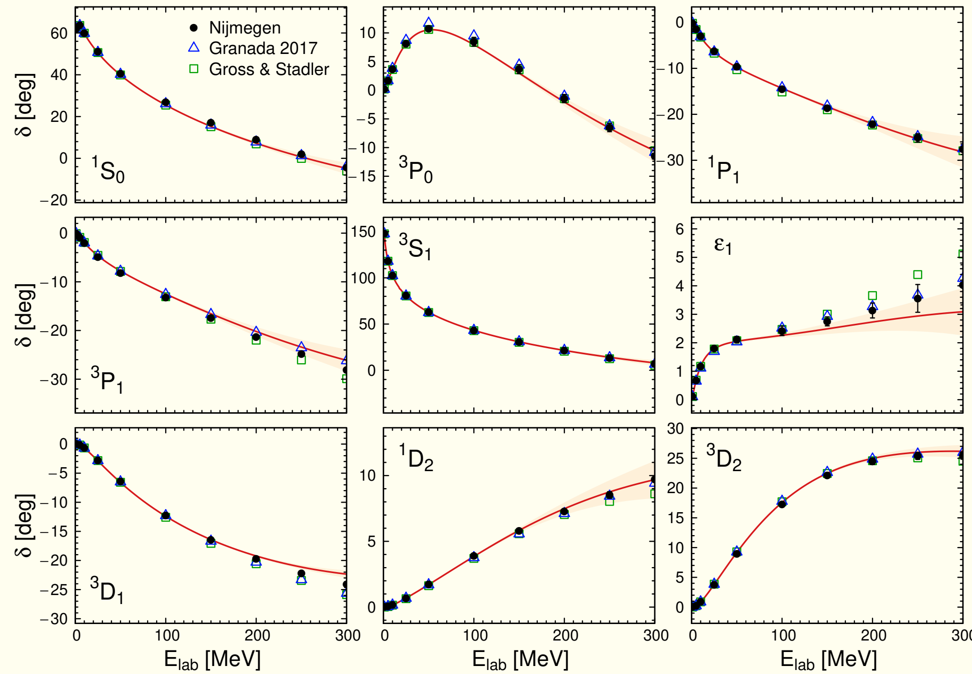
Performance in the 2N sector:

— Statistically perfect description of mutually consistent np + pp data below the inelastic channel (N⁴LO⁺, ~ 30 LECs)

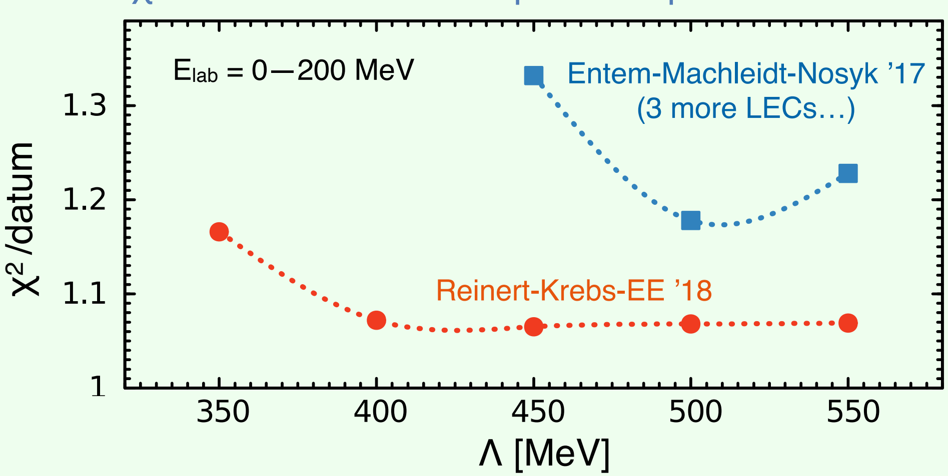
— Most accurate & precise interactions to date EE, Krebs, Reinert, '20,'22

— E.g., electromagnetic structure of the deuteron Filin et al. '20,'21

$$r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}\ {\rm fm}, \quad Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}\ {\rm fm}^2$$



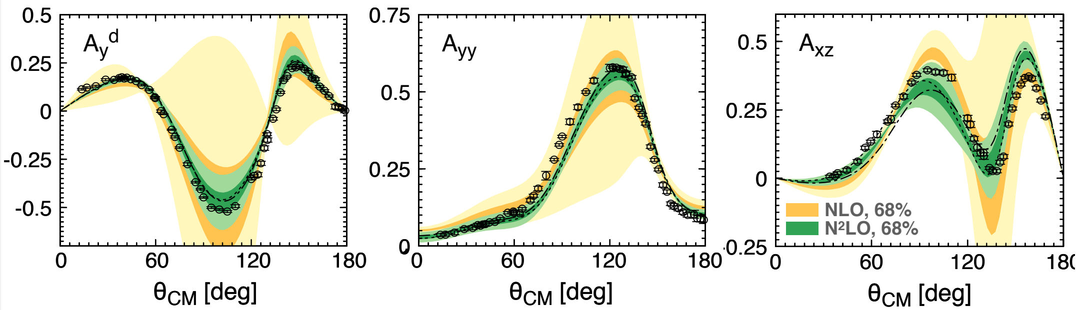
χ² /datum for the description of np data at N⁴LO⁺



⟵ approximate cutoff independence

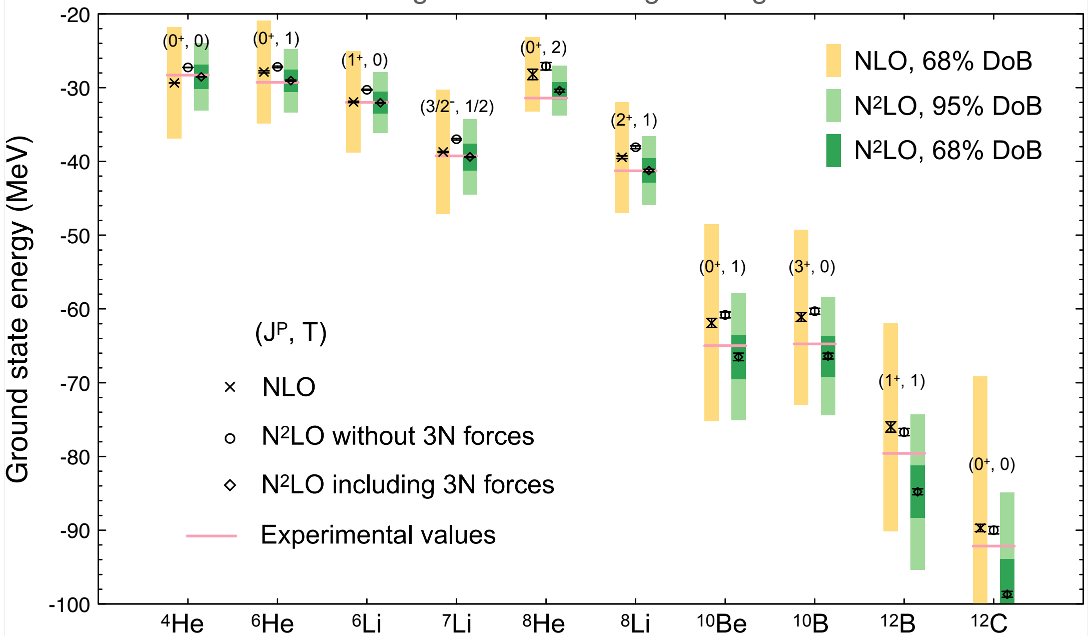
Predictions for Nd scattering and light nuclei

Tensor analyzing powers in Nd elastic scattering at 70 MeV

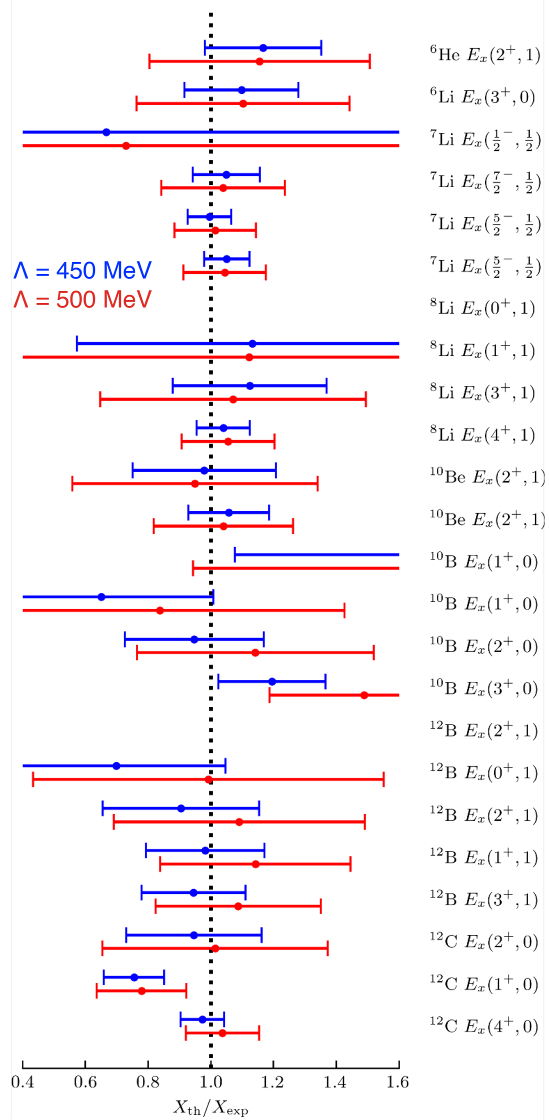


from: P. Maris et al. (LENPIC), Phys. Rev. C 103 (2021) 5, 054001

Predictions for ground-state energies of light nuclei

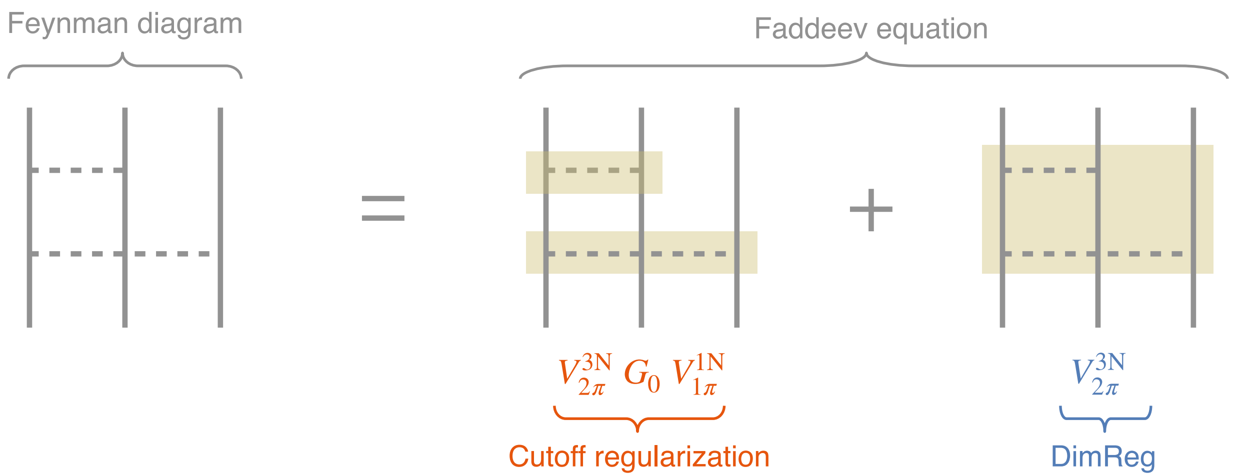


Excitation energies



Regularization and the chiral symmetry

Is using DimReg to derive nuclear potentials + Cutoff in the Schrödinger equation consistent?

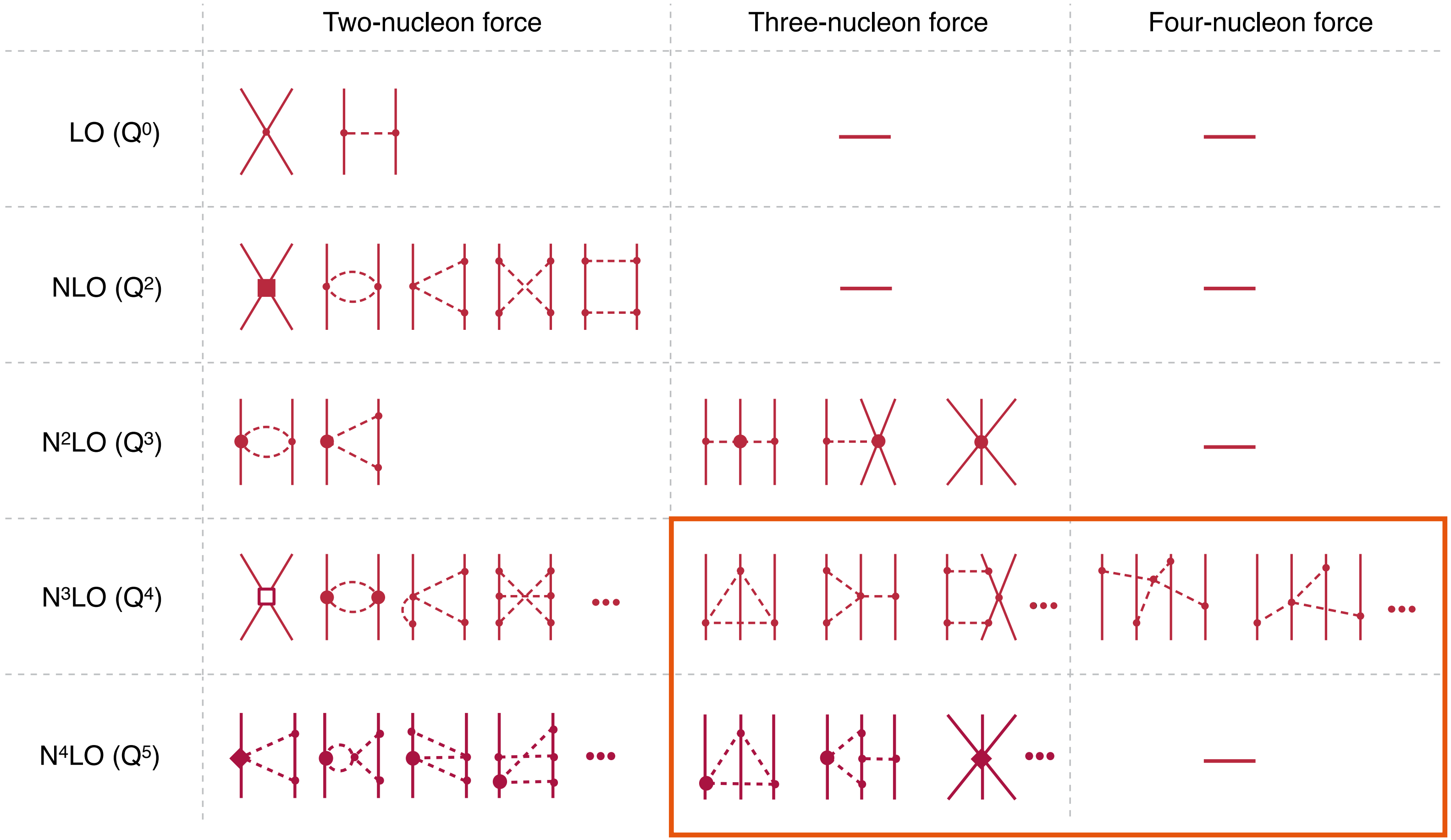


- The Feynman diagram is linearly divergent — c_D term:  $\propto\ \tau_1\cdot\tau_3\,\dfrac{\vec{q}_3\cdot\vec{\sigma}_1\ \vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2}$

- The iterative diagram on the r.h.s.:

$$V_{2\pi}^{3N}\,G_0\,V_{1\pi}^{1N} = -\Lambda\frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\Big[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\Big]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ \dots$$

Regularization and the chiral symmetry



- The issue affects all loop contributions (i.e., from N^{3}LO) to 3NF and currents.
  In contrast, NN forces are not affected (at a fixed pion mass).

Gradient flow regularization

Hermann Krebs, EE, in preparation

⇒ Re-derive nuclear forces & currents using **SYMMETRY PRESERVING** cutoff regularization

An attractive option is the gradient flow method:

— successfully applied to Yang-Mills theories (QCD) Martin Lüscher '14

— proposed as a regulator for chiral EFT in several talks by David Kaplan

Idea: let pion fields evolve in the flow „time" τ by replacing the pion field U by the smoothened one $W(\tau)$, $W(0) = U$, which fulfills the (covariant) gradient flow equation:

$$\partial_\tau W = i w \, \mathrm{EOM}(\tau) \, w, \quad \text{where } w = \sqrt{W} \quad \text{and} \quad \mathrm{EOM} = [D_\mu, \underbrace{w_\mu}_{w_\mu|_{\tau=0} \, = \, u_\mu}] + \frac{i}{2}\chi_i - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

The flow „time" τ acts as a regulator (smearing), the choice $\tau = (2\Lambda)^{-1}$ matches the employed regularization of the OPEP:

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}}$$

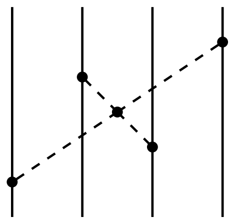
Complication: the regularized Lagrangian involves arbitrary powers of time derivatives

⇒ cannot use Hamiltonian-based methods (like MUT) to derive nuclear forces/currents

⇒ new path-integral method to derive nuclear interactions **Hermann Krebs**, EE, in preparation

Example: gradient flow reg. of the 4NF

Consider e.g. the contribution to the 4NF at N³LO involving a 4π-vertex:



Unregularized expression:

EE, PLB 639 (2006) 456; EPJA 34 (2007) 197

$$V_{4\mathrm{N}} = \frac{g_A^4}{2(2F_\pi)^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{\sigma}_3 \cdot \vec{q}_3 \; \vec{\sigma}_4 \cdot \vec{q}_4}{(\vec{q}_1^{\,2} + M_\pi^2)\,(\vec{q}_2^{\,2} + M_\pi^2)\,(\vec{q}_3^{\,2} + M_\pi^2)\,(\vec{q}_4^{\,2} + M_\pi^2)} \Big[(\vec{q}_1 + \vec{q}_2)^2 + M_\pi^2 \Big]$$

$$+ \text{ 3-pole terms } + \text{ all permutations}$$

Applying the gradient flow regularization method consistent with the 2NF yields:

Hermann Krebs, EE, preliminary

$$V_{4\mathrm{N}} = \frac{g_A^4}{2(2F_\pi)^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{\sigma}_3 \cdot \vec{q}_3 \; \vec{\sigma}_4 \cdot \vec{q}_4}{(\vec{q}_1^{\,2} + M_\pi^2)\,(\vec{q}_2^{\,2} + M_\pi^2)\,(\vec{q}_3^{\,2} + M_\pi^2)\,(\vec{q}_4^{\,2} + M_\pi^2)} \Big[(\vec{q}_1 + \vec{q}_2)^2 + M_\pi^2 \Big]$$

$$\times \left(4\, e^{-\frac{\vec{q}_2^{\,2} + M_\pi^2}{\Lambda^2}}\, e^{-\frac{\vec{q}_3^{\,2} + M_\pi^2}{\Lambda^2}}\, e^{-\frac{\vec{q}_4^{\,2} + M_\pi^2}{\Lambda^2}} - 3\, e^{-\frac{\vec{q}_1^{\,2} + M_\pi^2}{2\Lambda^2}}\, e^{-\frac{\vec{q}_2^{\,2} + M_\pi^2}{2\Lambda^2}}\, e^{-\frac{\vec{q}_3^{\,2} + M_\pi^2}{2\Lambda^2}}\, e^{-\frac{\vec{q}_4^{\,2} + M_\pi^2}{2\Lambda^2}} \right)$$

$$+ \text{ 3-pole terms } + \text{ all permutations}$$

Summary

- Is the „RG invariant χEFT“ really RG invariant? NO (except at LO)
  - — discrete „exceptional“ cutoffs start at $\bar{\Lambda} \approx 0.6$ GeV and extend to infinity
  - — can be easily overlooked in numerical calculations
  - — same issues expected to affect π-less EFT applications to few-body systems

- Is the finite-cutoff χEFT renormalizable? YES (at least to NLO)
  - — explicit renormalizability proof (in the EFT sense) was given to NLO for not genuine non-perturbative channels
  - — generalization to non-perturbative channels underway

- Is mixing DimReg & CutoffReg consistent with χ-symmetry? Generally NO

  $\Rightarrow$ 3NF, 4NF and currents must be re-derived using CutoffReg

  - — Gradient flow: A rigorous, symmetry-preserving regularization method