# Mean-field calculations with regularized pseudopotentials 

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Effective field theories for nuclei and nuclear matter

## Outline

## Motivation

Two-body and three-body pseudopotentials

Results

## Conclusion and outlooks

## Motivation (in a nutshell ©)

- Effective interactions (pseudopotentials) and/or functionals are the key ingredient for mean-field and beyond-mean-field calculations.


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- To be usable in beyond-mean-field calculations, a functional must be strictly derived from an effective interaction.
M. Anguiano et al., NPA 696 (2001)
J. Dobaczewski et al., PRC 76, 054315 (2007)
D. Lacroix et al., PRC 79, 044318 (2009)


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- A two-body interaction (whatever it is) can not give a satisfying description of infinite nuclear matter (e.g. $m^{*} / m \sim 0.4 *$ ).
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- A two-body interaction (whatever it is) can not give a satisfying description of infinite nuclear matter (e.g. $m^{*} / m \sim 0.4$ ) ).
D. Davesne et al., PRC 97, 044304 (2018)
- A two-body density dependent interaction is fine for mean-field calculations but leads to formal questions and calculation's problems which may (or may not?) be overcome.

| May | $\left\{\begin{array}{l}\text { M. Bender et al., PRC 79, } 044319 \text { (2009) } \\ \text { T.R. Rodríguez, J.L. Egido, PRC 81, 064323 (2010) } \\ \text { G. Hupin et al., PRC 84, 014309 (2011) } \\ \text { W. Satuła, J. Dobaczewski, PRC 90, 054303 (2014) }\end{array}\right.$ |
| :---: | :--- |
| May not | $\left\{\begin{array}{l}\text { T. Duguet et al., PRC 79, 044320 (2009) } \\ \text { L. Robledo, JPG 37, 064020 (2010) }\end{array}\right.$ |

Choice for the effective interaction

Radical solution: no density dependent term

$$
\begin{aligned}
V=V_{2 \text {-body }}+V_{3 \text {-body }} \text { and } \quad E & =\langle\Phi|(T+V)|\Phi\rangle \\
& =E_{H}+E_{F}+E_{P}
\end{aligned}
$$

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$\Rightarrow$ Finite-range (Coulomb has to be treated exactly anyway...)

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- 2-body part: zero-range, finite-range ?
$\Rightarrow$ Finite-range (Coulomb has to be treated exactly anyway...)
- 3-body part: zero-range, finite-range ?

Zero-range: not fully satisfying,
Finite-range: too much time-consuming,
$\Rightarrow$ something between.

Finite-range two-body pseudopotentials ${ }^{1}$

- General idea:
take a Skyrme interaction and replace $\delta(\mathbf{r})$ with $g_{a}(\mathbf{r})=\frac{e^{-\frac{r^{2}}{2^{2}}}}{(a \sqrt{\pi})^{3}}$
- Pseudopotential at "NLO"

$$
\begin{aligned}
v & =\tilde{v}_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2} ; \mathbf{r}_{3}, \mathbf{r}_{4}\right)\left(W_{0} 1_{\sigma q}+B_{0} 1_{q} \hat{P}^{\sigma}-H_{0} 1_{\sigma} \hat{P}^{q}-M_{0} \hat{P}^{\sigma} \hat{P}^{q}\right) \\
& +\tilde{v}_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2} ; \mathbf{r}_{3}, \mathbf{r}_{4}\right)\left(W_{1} 1_{\sigma q}+B_{1} 1_{q} \hat{P}^{\sigma}-H_{1} 1_{\sigma} \hat{P}^{q}-M_{1} \hat{P}^{\sigma} \hat{P}^{q}\right) \\
& +\tilde{v}_{2}\left(\mathbf{r}_{1}, \mathbf{r}_{2} ; \mathbf{r}_{3}, \mathbf{r}_{4}\right)\left(W_{2} 1_{\sigma q}+B_{2} 1_{q} \hat{P}^{\sigma}-H_{2} 1_{\sigma} \hat{P}^{q}-M_{2} \hat{P}^{\sigma} \hat{P}^{q}\right) \\
\text { with } \quad & \tilde{v}_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2} ; \mathbf{r}_{3}, \mathbf{r}_{4}\right)=\delta\left(\mathbf{r}_{1}-\mathbf{r}_{3}\right) \delta\left(\mathbf{r}_{2}-\mathbf{r}_{4}\right) g_{a}\left(\mathbf{r}_{1}-\mathbf{r}_{2}\right) \\
& \tilde{v}_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2} ; \mathbf{r}_{3}, \mathbf{r}_{4}\right)=\delta\left(\mathbf{r}_{1}-\mathbf{r}_{3}\right) \delta\left(\mathbf{r}_{2}-\mathbf{r}_{4}\right) g_{a}\left(\mathbf{r}_{1}-\mathbf{r}_{2}\right) \frac{1}{2}\left[\mathbf{k}_{12}^{* 2}+\mathbf{k}_{34}^{2}\right] \\
& \tilde{v}_{2}\left(\mathbf{r}_{1}, \mathbf{r}_{2} ; \mathbf{r}_{3}, \mathbf{r}_{4}\right)=\delta\left(\mathbf{r}_{1}-\mathbf{r}_{3}\right) \delta\left(\mathbf{r}_{2}-\mathbf{r}_{4}\right) g_{a}\left(\mathbf{r}_{1}-\mathbf{r}_{2}\right) \mathbf{k}_{12}^{*} \cdot \mathbf{k}_{34}
\end{aligned}
$$

- Thanks to the finite range: $\hat{P}^{\sigma} \hat{P}^{q} \equiv-\hat{P}^{x} \neq \pm 1$
- Can be generalized at $\mathrm{N}^{2} \mathrm{LO}, \mathrm{N}^{3} \mathrm{LO}, \ldots$

[^0]
## Finite-range two-body local pseudopotentials

- The conditions

$$
W_{1}=-W_{2}, \quad B_{1}=-B_{2}, \quad H_{1}=-H_{2}, \quad M_{1}=-M_{2}
$$

(and same for higher order terms) make the pseudopotential local

- These are severe restrictions on the flexibility of the functional
- ... but this greatly simplifies the implementation in computer codes
- ... and limits the number of free parameters
- Use of a standard two-body zero-range spin-orbit interaction


## Options for terms beyond two-body

- Contact LO 3- and 4-body terms: SLyMR0 interaction
J. Sadoudi et al., Phys. Scr. T154 (2013) 014013, B. Bally et al., PRL 113, 162501 (2014)
- Contact LO and NLO 3-body terms: SLyMR1 interaction
J. Sadoudi et al., PRC 88 (2013) 064326, R. Jodon, Phys. PhD Thesis, tel-01158085

See recent preprint arXiv:2301.02420 "The shape of gold", by B. Bally, G. Giacolone and M. Bender. Works pretty well in some limited regions of the nuclear chart (e.g. for gold $^{2}$ ).

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- Finite-range 2-body + zero-range 3-body $\Rightarrow$ pathological pairing.

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- Finite-range 2-body + zero-range 3-body $\Rightarrow$ pathological pairing.
- Semi-regularized three-body interaction: symmetrized version of

$$
\begin{aligned}
& V_{3}\left(x_{1}, x_{2}, x_{3} ; x_{4}, x_{5}, x_{6}\right)=W_{3} \overbrace{\delta\left(\mathbf{r}_{14}\right) \delta\left(\mathbf{r}_{25}\right) \delta\left(\mathbf{r}_{36}\right)}^{\text {locality }} \delta_{q_{1} q_{4}}^{\delta_{q_{2} q_{5}} \delta_{q_{3} q_{6}}} \\
& \times \delta_{s_{1} s_{4}} \underbrace{\left(\delta_{s_{2} s_{5}} \delta_{s_{3} s_{6}}+\delta_{s_{2} s_{6}} \delta_{s_{3} s_{5}}\right)}_{=\mathbb{1}_{23}^{\sigma}+P_{23}^{\sigma}} \\
& \underbrace{g_{a}\left(\mathbf{r}_{12}\right)}_{\begin{array}{c}
\text { finite } \\
\text { range }
\end{array}} \underbrace{\delta\left(\mathbf{r}_{23}\right)}_{\begin{array}{c}
\text { zero } \\
\text { range }
\end{array}}
\end{aligned}
$$

with $x \equiv \mathbf{r s q}$ and $\mathbf{r}_{i j}=\mathbf{r}_{j}-\mathbf{r}_{i}$.

[^3]EDF from the semi-regularized three-body term

- Normal part

$$
\begin{aligned}
E & =\frac{W_{3}}{8} \int \mathrm{~d}^{3} r_{1} \mathrm{~d}^{3} r_{2} g_{a}\left(\mathbf{r}_{12}\right)\left\{\rho_{0}\left(\mathbf{r}_{2}\right) \rho_{0}^{2}\left(\mathbf{r}_{1}\right)-\rho_{0}\left(\mathbf{r}_{1}\right) \rho_{1}^{2}\left(\mathbf{r}_{2}\right)+\frac{1}{3} \rho_{0}\left(\mathbf{r}_{2}\right) \mathbf{s}_{0}^{2}\left(\mathbf{r}_{1}\right)-\frac{1}{3} \rho_{0}\left(\mathbf{r}_{2}\right) \mathbf{s}_{1}^{2}\left(\mathbf{r}_{1}\right)\right. \\
- & \frac{1}{4}\left[\rho_{0}\left(\mathbf{r}_{1}\right)+\rho_{0}\left(\mathbf{r}_{2}\right)\right]\left[\rho_{0}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\rho_{1}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right. \\
& \left.+\mathbf{s}_{0}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \cdot \mathbf{s}_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\mathbf{s}_{1}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \cdot \mathbf{s}_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right] \\
& +\frac{1}{2}\left[\rho_{1}\left(\mathbf{r}_{1}\right)+\rho_{1}\left(\mathbf{r}_{2}\right)\right]\left[\rho_{0}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\mathbf{s}_{0}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \cdot \mathbf{s}_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right] \\
& -\frac{1}{6}\left[\mathbf{s}_{0}\left(\mathbf{r}_{1}\right)+\mathbf{s}_{0}\left(\mathbf{r}_{2}\right)\right] \cdot\left[\mathbf{s}_{0}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\mathbf{s}_{1}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right] \\
& \left.+\frac{1}{6}\left[\mathbf{s}_{1}\left(\mathbf{r}_{1}\right)+\mathbf{s}_{1}\left(\mathbf{r}_{2}\right)\right] \cdot\left[\mathbf{s}_{0}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{1}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\mathbf{s}_{1}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \rho_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right]\right\} .
\end{aligned}
$$

- Pairing part

$$
\begin{aligned}
E_{P}=\frac{W_{3}}{8} \int \mathrm{~d}^{3} r_{1} \mathrm{~d}^{3} r_{2} g_{a}\left(\mathbf{r}_{12}\right) & \sum_{q}\left\{\left[\rho_{q}\left(\mathbf{r}_{1}\right)+\rho_{q}\left(\mathbf{r}_{2}\right)\right]\left[\tilde{\rho}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\rho}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\tilde{\mathbf{s}}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \cdot \tilde{\mathbf{s}}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right]\right. \\
+ & \left.\frac{1}{3}\left[\mathbf{s}_{q}\left(\mathbf{r}_{1}\right)-\mathbf{s}_{q}\left(\mathbf{r}_{2}\right)\right] \cdot\left[\tilde{\rho}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\mathbf{s}}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\tilde{\mathbf{s}}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\rho}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right]\right\} .
\end{aligned}
$$

EDF from the semi-regularized three-body term

- Pairing part

$$
\begin{aligned}
E_{P}=\frac{W_{3}}{8} \int & \mathrm{~d}^{3} r_{1} \mathrm{~d}^{3} r_{2} g_{a}\left(\mathbf{r}_{12}\right) \\
& \times \sum_{q}\left\{\left[\rho_{q}\left(\mathbf{r}_{1}\right)+\rho_{q}\left(\mathbf{r}_{2}\right)\right]\left[\tilde{\rho}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\rho}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\tilde{\mathbf{s}}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \cdot \tilde{\mathbf{s}}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right]\right. \\
& \left.+\frac{1}{3}\left[\mathbf{s}_{q}\left(\mathbf{r}_{1}\right)-\mathbf{s}_{q}\left(\mathbf{r}_{2}\right)\right] \cdot\left[\tilde{\rho}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\mathbf{s}}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)+\tilde{\mathbf{s}}_{\bar{q}}^{*}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\rho}_{\bar{q}}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)\right]\right\} .
\end{aligned}
$$

Does not depend on the local pairing densities ! No cut-off needed ! (as long as we don't mix protons and neutrons.)

## Finite-range Gogny pseudopotentials

- Gaussian form factors + zero-range DD term = D1S

$$
\begin{aligned}
& V_{D 1 S}\left(x_{1}, x_{2} ; x_{3}, x_{4}\right)=\left[\sum_{j=1,2} \mathrm{e}^{-\frac{r_{12}^{2}}{\mu_{j}^{2}}}\left(W_{j} \mathbb{1}^{\sigma} \mathbb{1}^{q}+B_{j} P^{\sigma} \mathbb{1}^{q}-H_{j} \mathbb{1}^{\sigma} P^{q}-M_{j} P^{\sigma} P^{q}\right)\right. \\
& +t_{3}\left(\mathbb{1}^{\sigma}+P^{\sigma}\right) \mathbb{1}^{q} \rho_{0}^{\alpha}\left(\mathbf{r}_{1}\right) \delta\left(\mathbf{r}_{12}\right) \\
& \left.+\mathrm{i} W_{0} \mathbb{1}^{q}\left(\delta_{\sigma_{1} \sigma_{3}} \sigma_{\sigma_{2} \sigma_{4}}+\sigma_{\sigma_{1} \sigma_{3}} \delta_{\sigma_{2} \sigma_{4}}\right) \cdot\left(\mathbf{k}_{12}^{*} \times \mathbf{k}_{34}\right)\right] \\
& \text { J.F. Berger et al., CPC } 63 \text { (1991) } 365
\end{aligned}
$$

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& +t_{3}\left(\mathbb{1}^{\sigma}+P^{\sigma}\right) \mathbb{1}^{q} \rho_{0}^{\alpha}\left(\mathbf{r}_{1}\right) \delta\left(\mathbf{r}_{12}\right) \\
& \left.+\mathrm{i} W_{0} \mathbb{1}^{q}\left(\delta_{\sigma_{1} \sigma_{3}} \sigma_{\sigma_{2} \sigma_{4}}+\sigma_{\sigma_{1} \sigma_{3}} \delta_{\sigma_{2} \sigma_{4}}\right) \cdot\left(\mathbf{k}_{12}^{*} \times \mathbf{k}_{34}\right)\right]
\end{aligned}
$$

J.F. Berger et al., CPC 63 (1991) 365

- Gaussian form factors + finite-range DD term = D2

$$
\begin{aligned}
& V_{D 2}\left(x_{1}, x_{2} ; x_{3}, x_{4}\right)=\left[\sum_{j=1,2} \mathrm{e}^{-\frac{r_{12}^{2}}{\mu_{j}^{2}}}\left(W_{j} \mathbb{1}^{\sigma} \mathbb{1}^{q}+B_{j} P^{\sigma} \mathbb{1}^{q}-H_{j} \mathbb{1}^{\sigma} P^{q}-M_{j} P^{\sigma} P^{q}\right)\right. \\
& \quad+\frac{\mathrm{e}^{-\frac{r_{12}^{2}}{\mu_{3}^{2}}}}{\left(\mu_{3} \sqrt{\pi}\right)^{3}} \frac{\rho_{0}^{\alpha}\left(\mathbf{r}_{1}\right)+\rho_{0}^{\alpha}\left(\mathbf{r}_{2}\right)}{2}\left(W_{3} \mathbb{1}^{\sigma} \mathbb{1}^{q}+B_{3} P^{\sigma} \mathbb{1}^{q}-H_{3} \mathbb{1}^{\sigma} P^{q}-M_{3} P^{\sigma} P^{q}\right) \\
& \left.\quad+\mathrm{i} W_{0} \mathbb{1}^{q}\left(\delta_{\sigma_{1} \sigma_{3}} \sigma_{\sigma_{2} \sigma_{4}}+\sigma_{\sigma_{1} \sigma_{3}} \delta_{\sigma_{2} \sigma_{4}}\right) \cdot\left(\mathbf{k}_{12}^{*} \times \mathbf{k}_{34}\right)\right]
\end{aligned}
$$

## Finite-range Gogny pseudopotentials

Density dependent part of D2 effective interaction

$$
\frac{\mathrm{e}^{-\frac{r_{12}^{2}}{\mu_{3}^{2}}}}{\left(\mu_{3} \sqrt{\pi}\right)^{3}} \frac{\rho_{0}^{\alpha}\left(\mathbf{r}_{1}\right)+\rho_{0}^{\alpha}\left(\mathbf{r}_{2}\right)}{2}\left(W_{3} \mathbb{1}^{\sigma} \mathbb{1}^{q}+B_{3} P^{\sigma} \mathbb{1}^{q}-H_{3} \mathbb{1}^{\sigma} P^{q}-M_{3} P^{\sigma} P^{q}\right)
$$

and three-body semi-regularized pseudopotential

$$
W_{3} \delta\left(\mathbf{r}_{14}\right) \delta\left(\mathbf{r}_{25}\right) \delta\left(\mathbf{r}_{36}\right) \delta_{q_{1} q_{4}} \delta_{q_{2} q_{5}} \delta_{q_{3} q_{6}} \delta_{s_{1} s_{4}}\left(\delta_{s_{2} 5_{5}} \delta_{s_{3} s_{6}}+\delta_{s_{2} s_{6}} \delta_{s_{3} s_{5}}\right) g_{a}\left(\mathbf{r}_{12}\right) \delta\left(\mathbf{r}_{23}\right)
$$

lead to similar terms in the functional i.e.

$$
\rho^{\alpha}\left(\mathbf{r}_{1}\right) \rho\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \rho\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right) \quad \text { and } \quad \rho^{\alpha}\left(\mathbf{r}_{1}\right) \tilde{\rho}\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right) \tilde{\rho}\left(\mathbf{r}_{2}, \mathbf{r}_{1}\right)
$$

with $\alpha=\frac{1}{3}$ (Gogny D2) or $\alpha=1$ (semi-regularized).
$\Rightarrow$ doable in deformed (axial) calculations.

## Overview of the fits of the parameters

Many parameters to fit... Two-body up to $\mathrm{N}^{3}$ LO, spin-orbit, three-body.
Minimization of a penalty function built from:

- Infinite nuclear matter properties ( $\rho_{\text {sat }}, E / A, K_{\infty}, m^{*} / m, J, L$ )
- Neutron matter equation of state
- Simple constraints on pairing strengths (strong enough scalar pairing and weak enough vector pairing)
- Binding energies of spherical nuclei
- Single particle energies in ${ }^{208} \mathrm{~Pb}$
- Charge radii
- Finite-size instabilities taken care using constraints on charge density profiles

The result is not a final set of parameters but a proof of principle that such an interaction can give a reasonable description of nuclei.

## Properties of infinite nuclear matter





|  | $\begin{gathered} \rho_{\mathrm{sat}} \\ {\left[\mathrm{fm}^{-3}\right]} \end{gathered}$ | $\begin{gathered} E / A \\ {[\mathrm{MeV}]} \end{gathered}$ | $\begin{gathered} K_{\infty} \\ {[\mathrm{MeV}]} \end{gathered}$ | $\begin{gathered} \mathrm{J} \\ {[\mathrm{MeV}]} \end{gathered}$ | $\begin{gathered} L \\ {[\mathrm{MeV}]} \end{gathered}$ | $m^{*} / m$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D2 | 0.163 | -16.000 | 209.251 | 31.110 | 44.831 | 0.738 |
| D1S | 0.163 | -16.007 | 202.840 | 31.125 | 22.441 | 0.697 |
| RegMR3 | $\begin{gathered} 0.158 \\ \hline: \end{gathered}$ | $\begin{gathered} -16.237 \\ \hline \end{gathered}$ | $\begin{gathered} 285.654 \\ \dot{=} \end{gathered}$ | $31.954$ | $\begin{gathered} 12.798 \\ \because \end{gathered}$ | $0.800$ |

Semi-magic nuclei: binding energy residuals

Comparison with Gogny interactions is not a beauty pageant


## Semi-magic nuclei: charge radii



## Spherical nuclei: binding energy residuals



Set of 214 nuclei with $Z \geqslant 20$ predicted as (quasi-)spherical by D1S

Average neutron and proton gaps


## Mean-field calculations with regularized pseudopotentials

ᄂResults

- Nuclei


## Charge and isovector densities



Single particle energies in ${ }^{208} \mathrm{~Pb}$


Effective mass probably to low near the nucleus surface...

Neutron droplets
S. Gandolfi et al. PRL 106, 012501 (2011)


Pairing in symmetric and neutron matter

- Symmetric matter

|  | Gogny D1S | Gogny D2 | RegMR3 |
| :--- | :---: | :---: | :---: |
| 2-body | $\sum_{q} \tilde{\rho}_{q} \tilde{\rho}_{q}$ <br> attractive | $\sum_{q} \tilde{\rho}_{q} \tilde{\rho}_{q}$ <br> attractive | $\sum_{q} \tilde{\rho}_{q} \tilde{\rho}_{q}$ <br> attractive |
| 3-body or d.d. | - | $\rho_{0}^{\alpha} \sum_{q} \tilde{\rho}_{q} \tilde{\rho}_{q}$ <br> repulsive | $\sum_{q} \rho_{\bar{\rho}} \tilde{\rho}^{\prime} \tilde{\rho}_{q}$ <br> repulsive |

- Neutron matter

|  | Gogny D1S | Gogny D2 | RegMR3 |
| :--- | :---: | :---: | :---: |
| 2-body | $\tilde{\rho}_{n} \tilde{\rho}_{n}$ <br> attractive | $\tilde{\rho}_{n} \tilde{\rho}_{n}$ <br> attractive | $\tilde{\rho}_{n} \tilde{\rho}_{n}$ <br> attractive |
| 3-body or d.d. | - | $\rho_{n}^{\alpha} \tilde{\rho}_{n} \tilde{\rho}_{n}$ | - |
|  | - | repulsive | - |

## Conclusion and outlooks

First density independent effective interaction which gives

- reasonable results at the SR approximation;
- no finite-size instabilities in the $T=1$ channel;
- strong enough pairing in nuclei;
- possibility to do MR calculations without ambiguity.

Outlooks:

- Implementation in 3D codes for SR and MR calculations;
- Minor improvements for the effective mass, slope of the symmetry energy and incompressibility;
- Average gaps in neutron matter too strong... Might be corrected (?) using a slightly modified NLO 3-body term.


## Thank you for your attention

- Main collaborators on this project: Ph. da Costa, J. Dobaczewski, M. Kortelainen.
- Other colleagues involved:
Y. Gao, T. Haverinen, A. Idini, D. Lacroix, M. Martini, A. de Pace, F. Raimondi.


[^0]:    ${ }^{1}$ Cf Jacek's presentation from yesterday

[^1]:    ${ }^{2}$ If I was working for gold, I wouldn't be a physicist.

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