# Equation of state for core collapse supernova simulation and neutron star core

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# Introduction:Core collapse supernova

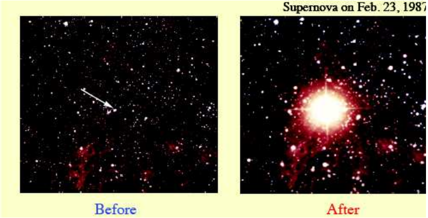


Figure: Explosion at the end of a massive star $\sim 20M_{\odot}$

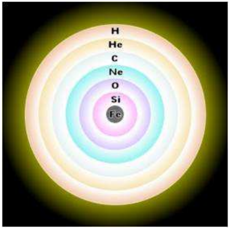
The core collapse supernova explosion (Type II) mechanism is being investigated over the last five decades.
Still, the theory of successful supernova explosion is beyond our reach.

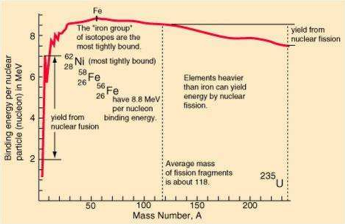
# Supernova Collapse

Gravitational mass of star is supported by energy generated by Nuclear Fusion of elements.

**H -> He -> C/O ->O/Ne/Mg-> Si->Fe**

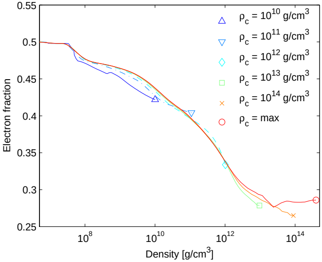
The star fuses increasingly higher mass elements starting with H and He, until a core of Fe is produced.





No further fusion takes place.
Electron degeneracy pressure counters gravity.

# The collapse



[ Ref: Liebendörfer, ApJ. 633 (2005)].

- electron capture and $\nu_e$ emission

$$e^- + p \xrightarrow{W} \nu_e + n$$

- photo disintegration of heavy nuclei

$$\gamma + Fe^{56}_{26} \Leftrightarrow 13\alpha + 4n$$

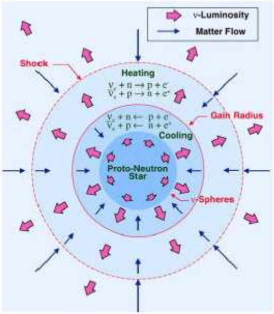
cool the inner core and remove pressure support.

- The star collapses.
- The temperature and central density increase.
- At $\rho > 10^9$ g/cc, $\nu_e$ emission is so large that collapse become supersonic i.e. $v_{max} > c_s$.

[ Ref:H.A. Bethe, Rev. Mod Physics, 62 (1990)]

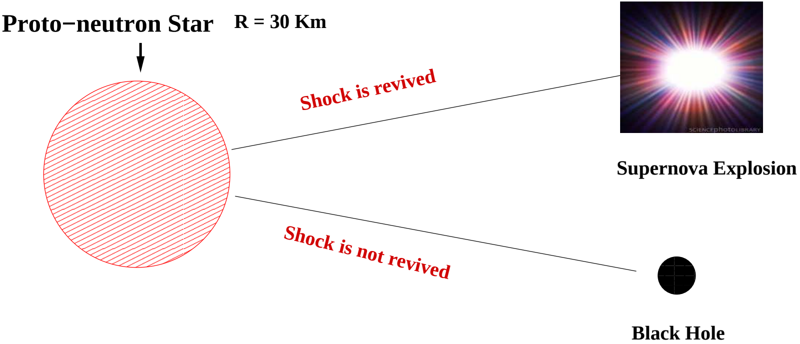
# Introduction:Core collapse supernova

- Gravitational core-collapse of a star with M $> 8M_{\odot}$
- At $\rho_0 = 2x10^{14} g/cc$, the core is converted in nuclear matter.
- n, p are so close that repulsive strong force becomes significant,it haults the collapse.
- It detaches the surrounding stellar material, which continues to fall inwards, $\Leftarrow$ core bounce.
- Shock wave is formed, it crosses neutrinospheres $\Rightarrow$ burst of neutrinos
- The central object formed at core bounce is hot and lepton-rich protoneutron star (PNS).



# The shock stalls: Neutron star or a Black Hole?

- The shock initially races through still collapsing outer core.
- The shock soon loses energy and stalls.
- If revived, a neutron star is born in the core collapse supernova soon after bounce.
- If the shock fails to revive, continued accretion pushes PNS over its maximum mass, a black hole may be formed.



**Proto−neutron Star** **R = 30 Km**

Shock is revived

Shock is not revived

**Supernova Explosion**

**Black Hole**

# How to revive the shock?

- Energy deposition by neutrinos may revive the shock. [ Ref:Bethe & Wilson, 1985]
- This mechanism has been able to to produce explosions only in low mass 8.8$M_\odot$ (O-Ne-Mg cores)[ Ref: Kitaura 2006 & T.Fischer et. al arXiv:1011.3409v2]
- We explore the possibility that a phase transition from hadron to hyperon matter can result in a second shock.
- The shock propagation across the neutrino-sphere releases an additional neutrino bursts:Observational signature.

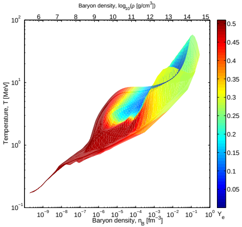
# Phase transition from nuclear to strange matter

- Mainly Nuclear EoS are used.
- Strangeness in the post-bounce phase of a core-collapse supernova is an interesting possibility.
  - Hyperons produced at the cost of the nucleons. $n + p \longrightarrow p + \Lambda + K^0, \;\; n + n \longrightarrow n + \Sigma^+ + K^-$
  - Bose-Einstein condensates of Kaons
  - Quarks
- **Our aim is to study if a 1st order phase transition from hadron to hyperon matter can result in a second shock.**
- A strong signature of quark-hadron phase transition was predicted during the post-bounce phase.[ Ref:I. Sagert et. al.2009]

# Equation of State

Microphysics inputs such as equation of state (EoS) are important for simulations of stellar collapse for a wide range of density, Temperature and composition.

- density ($10^4 - 10^{15} \mathrm{g/cm}^3$),
- temperature ($0 - 100$MeV)
- composition (proton fraction $0 - 0.6$).



Phase space of covered in Core collapse simulation of a $40M_\odot$ progenitor with Shen EoS Fischer, ApJS 194, 39 (2011)

Mainly two sets of EoS are used-
a)Lattimer-Swesty(LS) Lattimer and Swesty, 1991
b)Shen Shen, Toki, Oyamatsu and Sumiyoshi, 1998

However the constituents here are non-strange particles like neutrons, protons, $\alpha$-particles and heavy nuclei.

# EoS continued...

**LS EoS**

- Based on Skyrme type interaction with two and many body terms
- compressible liquid drop model
- Simplified treatment of pasta phases between $0.1 - 0.5\rho_0$.

**Others**

- Parameterised EoS (Baron-Cooperstein, Takahara-Suko,Bruenn, Swesty... 1980)
- Mixture of nuclei (Hempel & Schanffner-Bielich 2011, Hempel 2012)
- Extension of RMF (Shen-Horowitz, Furusawa 2011)

# Shen Nuclear finite temperature EoS

- Shen nuclear EoS is based on a relativistic mean field model at intermediate and high densities ($\rho > 10^{14.2}$ g/cc).
- At low temperature(T<14MeV), and $\rho < 10^{14.2}$ g/cc, Thomas Fermi approximation is used.
- In this case the non-uniform matter is modelled to consist of free nucleons, $\alpha$ particles and heavy nuclei.
- Uniform nucleon gas of n, p, $\alpha$ particles at extremely low density ($\rho < 10^{10}$ g/cc)and finite temperature is considered.
- Leptons are treated as uniform non-interacting relativistic particles and their contributions are added separately.
- Minimisation of free energy is done both for non-uniform matter and uniform nucleon gas at low density.

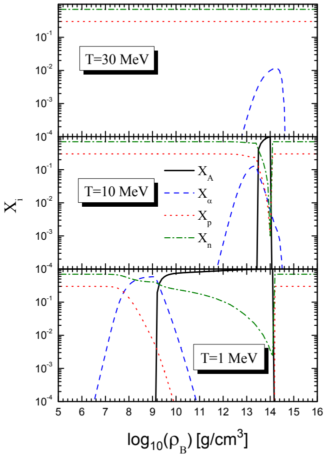
[ Ref:Shen et al.Nuclear Physics A, 637 (1998) 435 ]

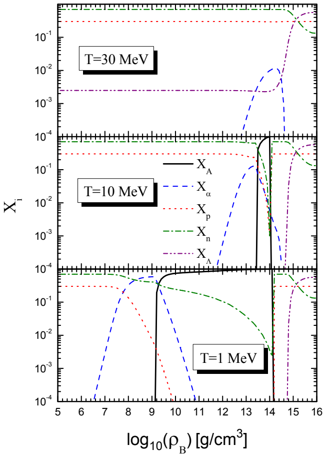
# Shen Hyperon EoS

- Recently Shen et. al extended their nuclear EoS to include hyperons.
- Hyperons produced at the cost of the nucleons.
  $n + p \longrightarrow p + \Lambda + K^0, \;\; n + n \longrightarrow n + \Sigma^+ + K^-$
  Ref:S.B., D. Bandyopadhyay, Phys.Rev.C64:055805, Phy. Rev.C63:035802, Schaffner & Mishustin, PhysRevC:53.1416
- $\Lambda$s, being the lightest hyperons with an attractive potential of $\sim -30$ MeV in nuclear matter, are believed to populate the dense matter among all strange baryons.
- Other hyperons, $\Xi$ & $\Sigma$ are excluded due to their relatively higher threshold and lack of experimental data.

[ Ref:Shen et al.arXiv:1105.1666v1 ]

# Particle fraction as in Shen EoS





[ Ref:Shen et al.arXiv:1105:1666v1]

# Model for high density EoS at finite temperature

- The EoS is computed in the RMF model. The interaction between baryons is mediated by the exchange of scalar ($\sigma$) and vector ($\omega, \rho$) mesons.
- The Lagrangian density for baryons is given by

$$
\begin{aligned}
\mathcal{L}_B \quad = \quad & \sum_{B=n,p} \bar{\Psi}_B \left( i\gamma_\mu \partial^\mu - m_B^* - g_{\omega B}\gamma_\mu \omega^\mu - g_{\rho B}\gamma_\mu \mathbf{t}_B \cdot \boldsymbol{\rho}^\mu \right) \Psi_B \\
& + \frac{1}{2}\left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - U(\sigma) \\
& - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} g_4 \left( \omega_\mu \omega^\mu \right)^2 \\
& - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu \; .
\end{aligned}
$$

where $m_B^* = m_B - g_\sigma \sigma$ is the baryon effective mass &
$U(\sigma) = \frac{1}{3} g_2 m_N \left( g_{\sigma N} \sigma \right)^3 + \frac{1}{4} g_3 \left( g_{\sigma N} \sigma \right)^4$ scalar self-interaction term.

Ref:S.B., D. Bandyopadhyay, Phys.Rev.C64:055805, Phy. Rev.C63:035802, Schaffner & Mishustin, PhysRevC:53.1416, S. Banik, W. Greiner, D. Bandyopadhyay

# EoM

In the mean field approximation, the meson field equations

$$m_\sigma^2 \sigma = -\frac{\partial U}{\partial \sigma} + \sum_B g_{\sigma B} n_B^s$$

$$m_\omega^2 \omega_0 - g_4 \omega_0^3 = \sum_B g_{\omega B} n_B$$

$$m_\rho^2 \rho_{03} = \sum_B g_{\rho B} I_{3B} n_B$$

The scalar density and baryon number density

$$\begin{aligned} n_B^S &= 2 \int \frac{d^3k}{(2\pi)^3} \frac{m_B^*}{E^*} \left( \frac{1}{e^{\beta(E^* - \nu_B)} + 1} + \frac{1}{e^{\beta(E^* + \nu_B)} + 1} \right) , \\ n_B &= 2 \int \frac{d^3k}{(2\pi)^3} \left( \frac{1}{e^{\beta(E^* - \nu_B)} + 1} - \frac{1}{e^{\beta(E^* + \nu_B)} + 1} \right) \end{aligned}$$

with $E* = \sqrt{k^2 + m_B^2}, m_B^* = m_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^*$, $\nu_B$ being the kinetic part of $\mu_B$. (Ref:S. Banik, W. Greiner, D. Bandyopadhyay Phys.Rev.C78:065804,2008)

- Total baryon number

$$n_B = n_n + n_p + n_\Lambda$$

- Threshold Condition for Hyperons $\mu_n = \mu_\Lambda$
- At fixed $n_B$, T & $Y_p$, the coupled equations are solved.
- Constraints: charge neutrality & baryon number conservation.

**However...**

- With the advent of hyperons, EoS becomes too soft → neutron star masses are too low
- More hyperon physics at high densties required.
- High mass neutron stars with hyperons possible: quark-meson coupling model, SU(3) nonlinear sigma model, extended RMF model etc.

# EoS: Pressure & Energy density

The thermodynamic potential per unit volume for nucleons is given by

$$\begin{aligned}\frac{\Omega_B}{V} &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{4}g_4\omega_0^4 - \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &-2T\sum_B\int\frac{d^3k}{(2\pi)^3}[ln(1+e^{-\beta(E^*-\nu_B)}) + ln(1+e^{-\beta(E^*+\nu_B)})] .\end{aligned}$$

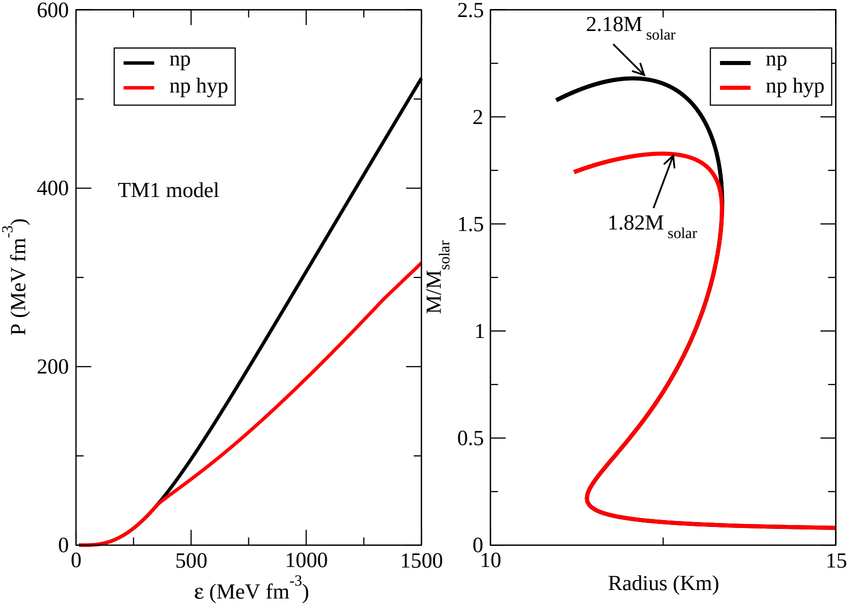
Here, $\beta = 1/T$ and $E^* = \sqrt{(k^2 + m_B^{*2})}$.

$$\boxed{P_B = -\Omega_B/V .}$$

The energy density is given by,

$$\begin{aligned}\epsilon_B &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{3}{4}g_4\omega_0^4 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &+2\sum_B\int\frac{d^3k}{(2\pi)^3}E^*\left(\frac{1}{e^{\beta(E^*-\nu_B)}+1} + \frac{1}{e^{\beta(E^*+\nu_B)}+1}\right) .\end{aligned}$$

(S. Banik, W. Greiner, D. Bandyopadhyay Phys.Rev.C78:065804,2008)


TM1 model
np
np hyp
P (MeV fm$^{-3}$)
ε (MeV fm$^{-3}$)
600
400
200
0
0
500
1000
1500
2.18M solar
1.82M solar
M/M$_{solar}$
Radius (Km)
2.5
2
1.5
1
0.5
0
10
15

# Dynamical core-collapse of a massive star

- We interpolate the original Shen EoS table, to generate more data.
- We get a thermodynamically consistent EoS which covers a finer uniformly-space of
  - $log_{10}\rho$ from $10^{5.1}$ to $10^{16}$ g/cc
  - $log_{10}T$ from 0.1 to $10^{2.6}$MeV
  - $Y_e$ from 0.0-0.65. [ Ref:Shen et al.arXiv:1105.1666v1 ]
- Added $\gamma$ and e after interpolation. [ Timmes & Swesty APJ sup series, 126(2000)]
- At densities below $10^7$ g/cc, Timmes EoS is used assuming the matter is composed of e, p, n, $\gamma$, $\alpha$ particles and heavy nuclei. [ Timmes & Arnnet 1999]
- For various progenitor models of Wooseley et al. we performed the simulations using a spherically symmetric GR hydrodynamics code called $GR1D$ for the Shen EoS.
- GR1D studies systematics stellar collapse to neutron stars and black hole formation. [ Ref:C. D. Ott and E. O'Connor, Class.Quant.Grav.27:114103, 2010]

# GR1D code

The Schwarzschild-like line element

$$ds^2 = -\alpha(r,t)^2 dt^2 + X(r,t)^2 dr^2 + r^2 d\Omega^2 \ ,$$

where $\alpha(r,t) = exp^{(\Phi(r,t))}$ & $X(r,t) = [1 - 2m(r)/r]^{-1/2}$.

In ideal hydrodynamics, the fluid stress-energy tensor & matter current density are

$$\begin{aligned} T^{\mu\nu} &= \rho h u^\mu u^\nu + g^{\mu\nu} P \\ J^\mu &= \rho u^\mu \end{aligned}$$

where $\rho$ is the matter density, P is the fluid pressure, $h = 1 + \epsilon + P/\rho$ is the specific enthalpy, $\epsilon$ the internal energy, $u^\mu$ is the 4-velocity of the fluid.

Fluid evolution equations are derived from local conservation laws

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu J^\mu = 0$$

[ Ref:C. D. Ott and E. O'Connor, Class.Quant.Grav.27:114103, 2010]

# Neutrino Mechanism

- Neutrino effects are very crucial in supernova simulations.
- Source of cooling & heating
  - $\nu$-emission :cooling

$$e^- + p \longrightarrow \nu_e + p$$
$$e^+ + n \longrightarrow \bar{\nu}_e + p$$

  - $\nu$-absorption :heating

$$\nu_e + n \longrightarrow e^- + p$$
$$\bar{\nu}_e + p \longrightarrow e^+ + n$$

- It might be included via Boltzmann transport treatment.

# Neutrino scheme in GR1D

- We choose a computationally more effecient scheme.
- Neutrino emission takes place after electron-capture by free or bound protons leading to fall of $Y_e$ at the core.
- Prebounce: effective $Y_e(\rho)$ approximation. [ Ref: Liebendörfer, Astrophys.J. 633 (2005)].
- Postbounce: 3-flavor, energy-averaged neutrino leakage scheme, which captures the effects of cooling.

$$Q_\nu^- \propto T^{-6}$$

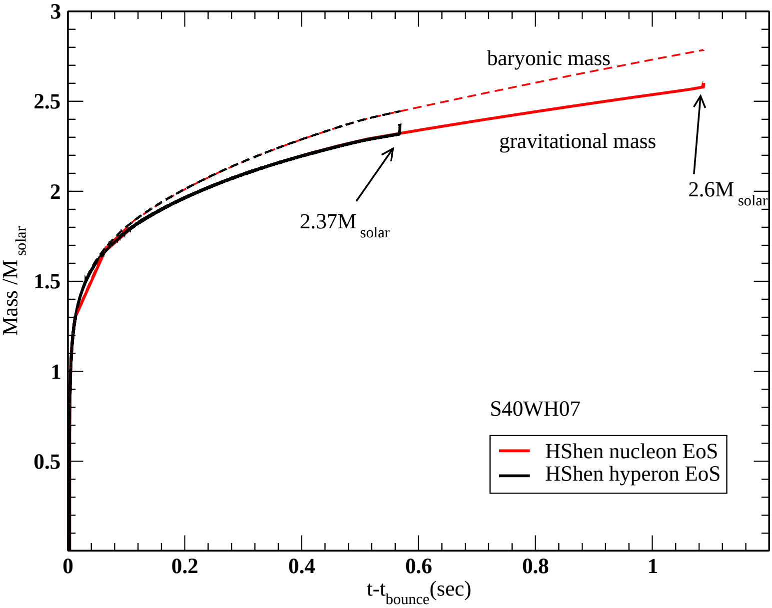
- The leakage scheme provides approximate energy and number emission rates.
- Neutrino heating is included via a parameterized charged-current heating scheme. [ Ref:H. T. Janka, A & A, **368**, 527 (2001)]

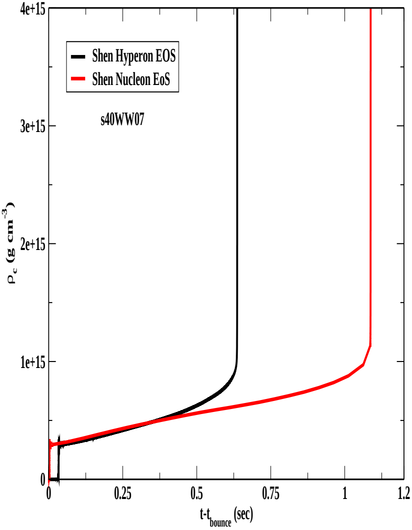
$$Q_\nu^+ \propto L_\nu r^{-2} < \epsilon_\nu^2 >$$

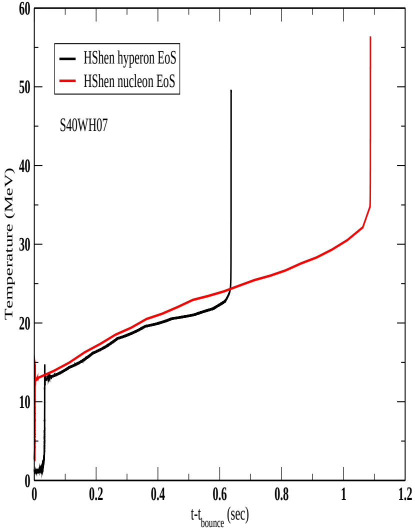
[ Ref:C. D. Ott and E. O'Connor, Astrophys.J.730:70,2011]

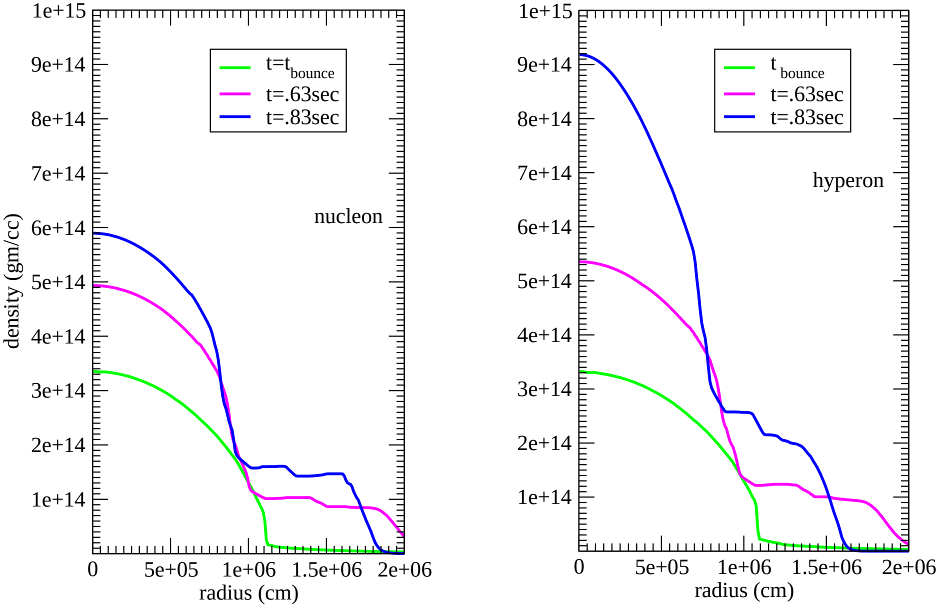
# Results

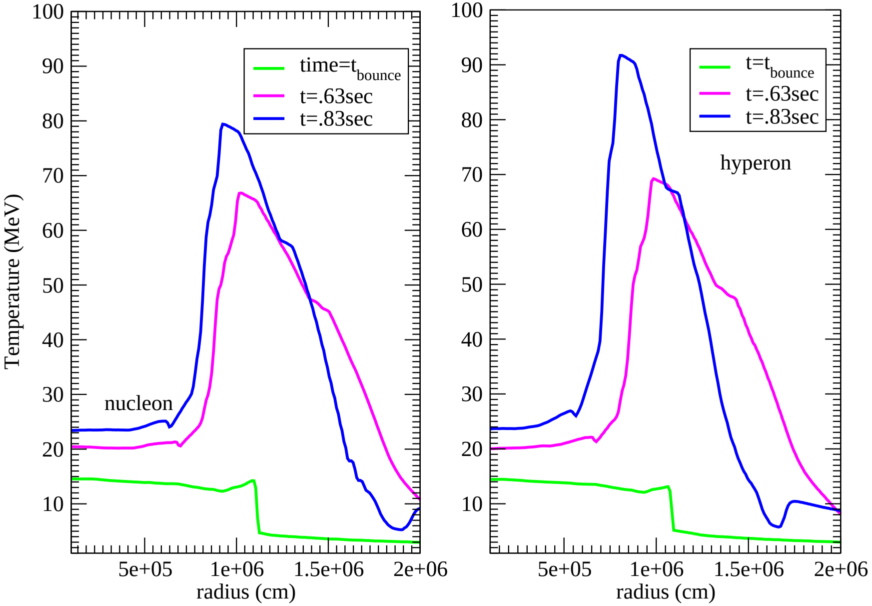
For a $40M_\odot$ progenitor models of Wooseley et al. [ Ref: S. E. Woosley, A. Heger, and T. A. Weaver, Rev. Mod. Phys. **74**, 1015 (2002).] we show our simulation results using GR1D and the Shen EoS(both nucleon and hyperon).

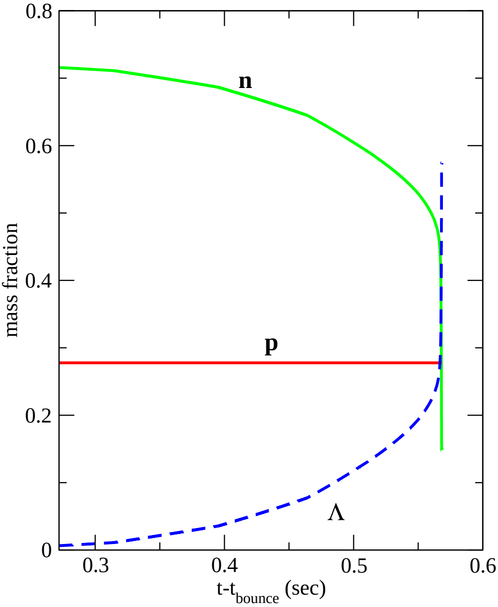

baryonic mass
gravitational mass
2.37M solar
2.6M solar
S40WH07
HShen nucleon EoS
HShen hyperon EoS
Mass /M solar
t-t bounce (sec)
0
0.2
0.4
0.6
0.8
1
0.5
1
1.5
2
2.5
3


Shen Hyperon EOS
Shen Nucleon EoS
s40WW07
$\rho_c$ (g cm$^{-3}$)
4e+15
3e+15
2e+15
1e+15
0
0.25
0.5
0.75
1
1.2
t-t$_{\rm bounce}$ (sec)


HShen hyperon EoS
HShen nucleon EoS
S40WH07
Temperature (MeV)
60
50
40
30
20
10
0
0.2
0.4
0.6
0.8
1
1.2
t-t$_{\rm bounce}$ (sec)


1e+15
9e+14
8e+14
7e+14
6e+14
5e+14
4e+14
3e+14
2e+14
1e+14
density (gm/cc)
0
5e+05
1e+06
1.5e+06
2e+06
radius (cm)
t=t_bounce
t=.63sec
t=.83sec
nucleon
1e+15
9e+14
8e+14
7e+14
6e+14
5e+14
4e+14
3e+14
2e+14
1e+14
0
5e+05
1e+06
1.5e+06
2e+06
radius (cm)
t bounce
t=.63sec
t=.83sec
hyperon


Temperature (MeV)
100
90
80
70
60
50
40
30
20
10
5e+05
1e+06
1.5e+06
2e+06
radius (cm)
time=$t_{bounce}$
t=.63sec
t=.83sec
nucleon
$t=t_{bounce}$
t=.63sec
t=.83sec
hyperon


0.8
0.6
0.4
0.2
0
mass fraction
n
p
Λ
0.3
0.4
0.5
0.6
t-t_bounce (sec)

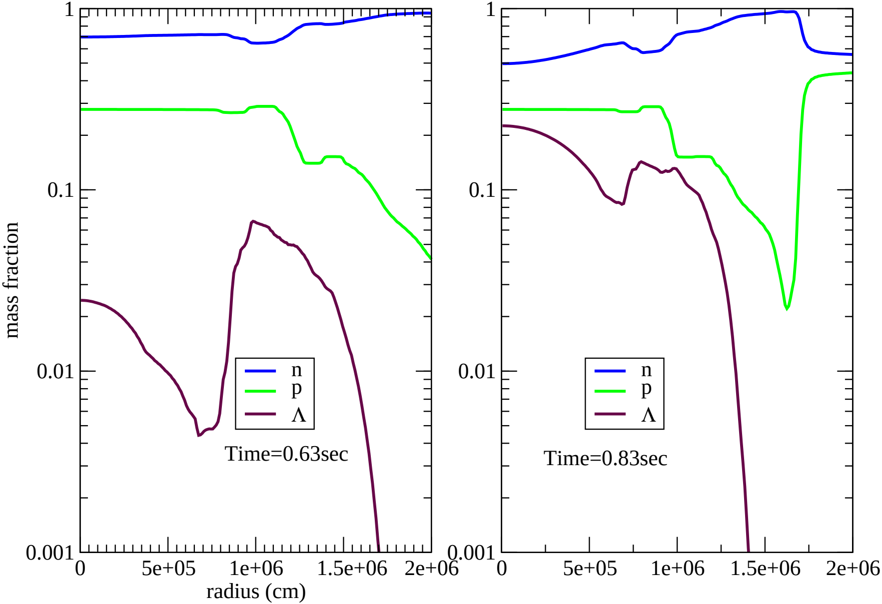
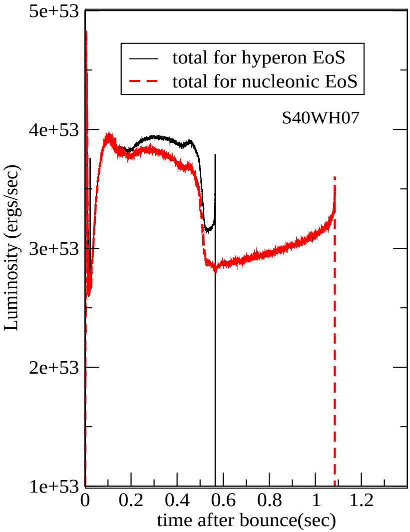


Figure: snapshots of mass fractions as a function of radius



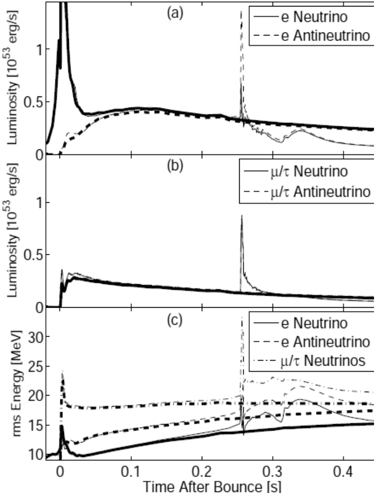


Figure: Ref:I. Sagert et.al, PhysRevLett.102.081101

# Summary

- Hyperons appear just before bounce.
- The hadron-hyperon phase transition is a weak first order phase transition.
- Hyperons triggers black hole formation, fails to generate second shock.
- Hyperon emergence in the collapse produces an intense but short neutrino burst, that may be used as a probe of black hole formation.
- No second neutrino burst is observed as in quark-hadron phase transition: Too much softening of EoS at higher density region.
- Need for input on high density matter from heavy ion experiments (RHIC, FAIR)

# Outlook

- Neutrino effects in principle might be included via Boltzmann transport treatment.
- We would like to investigate the effect of a strong first order phase transition on the core collpase supernova.
- This could generate a second bounce and neutrino burst.
- We are working towards generating a table of EoS for a first order antikaon condensation for a broad range of density, temperature and charge-to-baryon number ratio in hadronic part for numerical simulations of core-collapse supernovae.
- Our aim is to explore the possibility that a hadron-antikaon phase transition during the early post bounce evolution may result in an explosion and its observational consequence in the form of neutrino signatures.

# Thanks to Collaborators

Debades Bandyopadhyay, Saha Institute of Nuclear Physics, Kolkata,
Evan O’Connor & Christian Ott, California Institute of Technology, USA