

Signals for the QCD phase transition and critical point in a Langevin dynamical model

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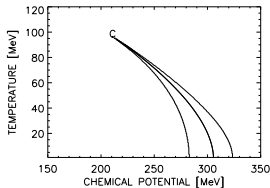
Jan Steinheimer

Thomas Lang



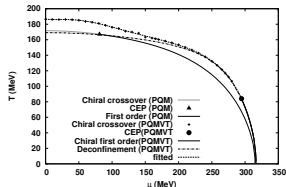
Effective models of QCD

Sigma model



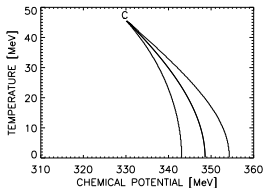
(Scavenius, Mocsy, Mishustin, Rischke, PRC **64** (2001))

Polyakov-quark-meson model



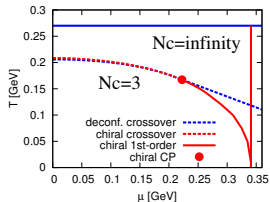
(Gupta, Tawari, arXiv:1107.1312v1 [hep-ph] (2011))

Nambu-Jona-Lasinio model



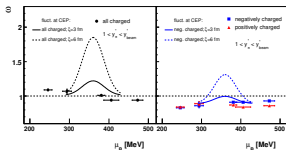
(Scavenius, Mocsy, Mishustin, Rischke, PRC **64** (2001))

Polyakov-NJL model



(C. Sasaki, APPS.3:659-668 (2010))

Signals for the critical point



(NA49 collaboration, Nucl. Phys. A **830** (2009))

Moments/cumulants of particle distribution are sensitive to correlation length

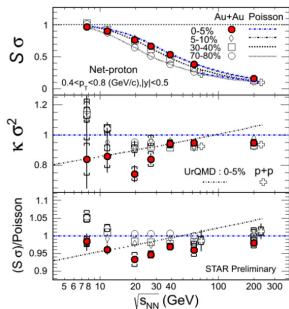
$$\langle (\delta N)^2 \rangle \sim \xi^2$$

$$\langle (\delta N)^3 \rangle \sim \xi^{4.5}$$

$$\langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 \sim \xi^7$$

(Stephanov, PRL **102** (2009))

Further investigations of CP phenomena at FAIR where large μ_B region will be covered

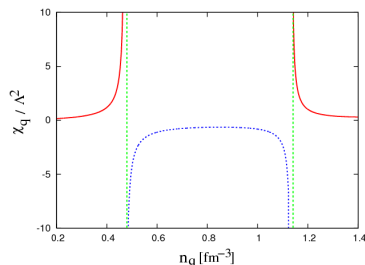
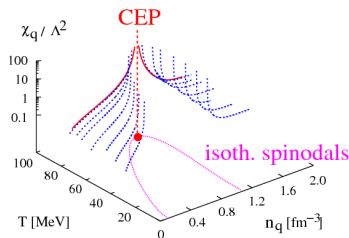


(STAR collaboration, Quark Matter 2012)



Signals for a first order phase transition

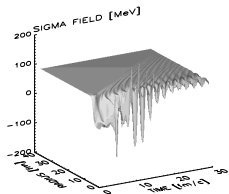
Out of equilibrium: Fluctuations at the first order transition can be as strong as at the critical point



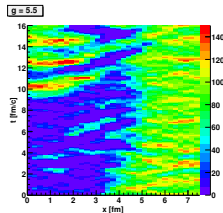
(Sasaki, Friman and Redlich, J. Phys. G **35** (2008))

Goal: study phase transition within fully dynamical model

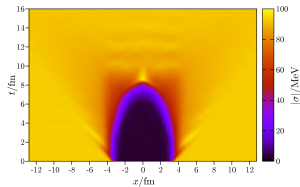
The Chiral fluid dynamics model



(I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999))



(K. Paech, H. Stöcker and A. Dumitru, Phys. Rev. C **68** (2003))



(M. Nahrgang, C. H., S. Leupold, I. N. Mishustin and

M. Bleicher, arXiv:1105.1962v2)

1. quark fluid coupled to chiral fields
2. inclusion of fluctuations
3. dissipation and noise

A dynamical model for the Polyakov loop

Phenomenological kinetic term and biquadratic coupling to chiral fields

$$\mathcal{L} = \mathcal{L}_\phi + \frac{N_c}{g^2} |\partial_\mu \ell|^2 T^2 - \mathcal{U}(\ell) - \frac{h^2}{2} \phi^2 |\ell|^2 T^2$$

(Dumitru, Pisarski, Phys. Lett. B **504** (2001))

Problem when $T = T(x^\mu)$
due to

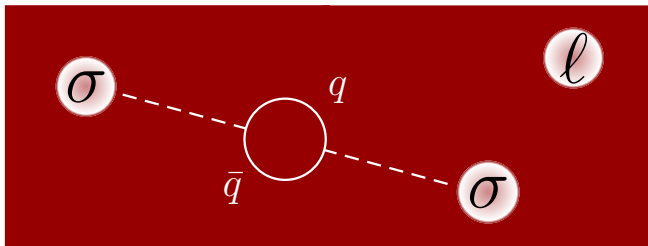
$$\frac{\partial}{\partial x^\mu} \left[\frac{N_c}{g^2} |\partial_\mu \ell|^2 T (x^\mu)^2 \right]$$

in Euler-Lagrange equation

Possible solutions

- ▶ replace T by T_0
- ▶ Polyakov loop always in equilibrium
- ▶ use simple relaxation equation

Chiral fluid dynamics with a Polyakov loop



$$\mathcal{L} = \bar{q} \left[i \left(\gamma^\mu \partial_\mu - i g_{QCD} \gamma^0 A_0 \right) - g \sigma \right] q + 1/2 (\partial_\mu \sigma)^2 - U(\sigma) - \mathcal{U}(\ell, \bar{\ell})$$

- ▶ quarks: heat bath in local thermal equilibrium, interacting with:
- ▶ σ : mesonic field, propagated via Langevin equation
- ▶ ℓ : Polyakov loop, coupled to heat bath

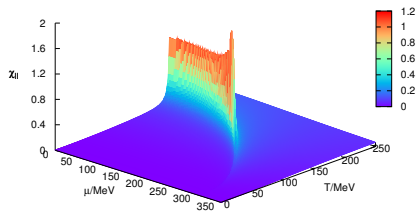
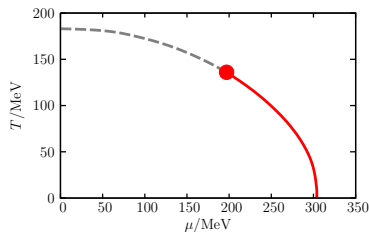
(I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999), K. Paech, H. Stöcker and A. Dumitru, Phys. Rev. C **68** (2003),

M. Nahrgang, S. Leupold, C. H. and M. Bleicher, Phys. Rev. C **84** (2011), In preparation: C. H., M. Nahrgang, I. N. Mishustin and M. Bleicher)

Thermodynamics

Grand canonical potential in the mean field approximation

$$\Omega_{\bar{q}q} = -2N_f T \int \frac{d^3p}{(2\pi)^3} \ln \left[1 + 3\ell e^{-\beta(E-\mu)} + 3\ell e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)} \right] \\ + \ln \left[1 + 3\ell e^{-\beta(E+\mu)} + 3\ell e^{-2\beta(E+\mu)} + e^{-3\beta(E+\mu)} \right]$$



The coupled dynamics of fields and fluid

Langevin equation for the sigma field

$$\partial_\mu \partial^\mu \sigma + \eta_\sigma(T) \partial_t \sigma + \frac{\partial V_{\text{eff}}}{\partial \sigma} = \xi_\sigma, \quad \langle \xi_\sigma(t) \xi_\sigma(t') \rangle = \frac{1}{V} \delta(t-t') m_\sigma \eta_\sigma \coth\left(\frac{m_\sigma}{2T}\right)$$

(Nahrgang, Leupold, C. H., Bleicher, Phys. Rev. C **84** (2011))

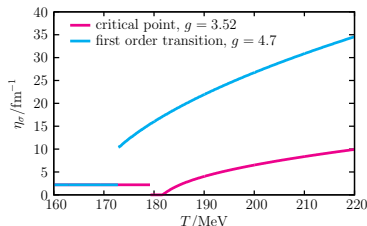
Relaxation equation for the Polyakov loop, constant $\eta_\ell = ?$

$$\eta_\ell \partial_t \ell + \frac{\partial V_{\text{eff}}}{\partial \ell} = \xi_\ell$$

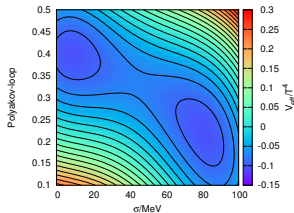
$$\langle \xi_\ell(t) \xi_\ell(t') \rangle T^2 = \frac{1}{V} \delta(t-t') 2\eta_\ell T$$

Energy momentum tensor of ideal fluid for quarks

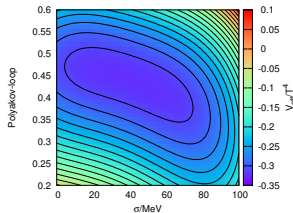
$$\partial_\mu T_q^{\mu\nu} = S_\sigma^\nu + S_\ell^\nu, \quad S_\sigma^\nu = \left(-\frac{\partial \Omega_{q\bar{q}}}{\partial \sigma} - \eta_\sigma \partial_t \sigma \right) \partial^\nu \sigma, \quad S_\ell^\nu = \left(-\frac{\partial \Omega_{q\bar{q}}}{\partial \ell} - \eta_\ell \partial_t \ell T^2 \right) \partial^\nu \ell$$



Box: Relaxation to equilibrium at $\mu = 0$



first order transition,
 $g = 4.7$, $T_c = 172.9 \text{ MeV}$

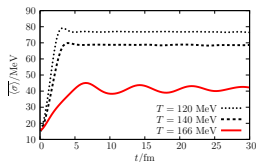


critical point,
 $g = 3.52$, $T_c = 180.5 \text{ MeV}$

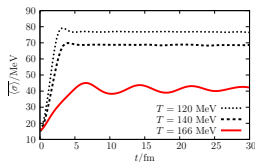
- ▶ study CP and FO by tuning quark meson coupling g
- ▶ initialize a cubic volume with $T > T_c$
- ▶ initialize σ, ℓ with their equilibrium values
- ▶ quench to $T < T_c$
- ▶ initialize energy density and pressure of quark fluid
- ▶ let system evolve and relax

Box: Relaxation to equilibrium $\mu = 0$, critical point

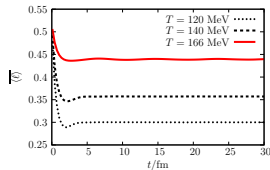
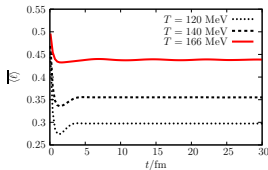
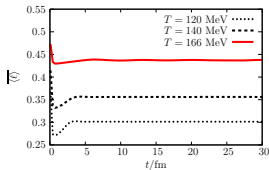
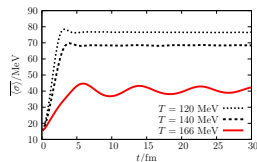
$$\eta = 2.2/\text{fm}$$



$$\eta = 5.0/\text{fm}$$

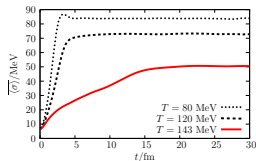


$$\eta = 10.0/\text{fm}$$

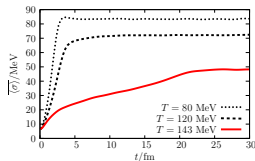


Box: Relaxation to equilibrium $\mu = 0$, first order

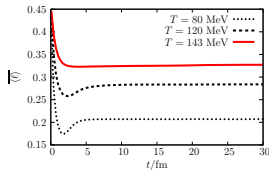
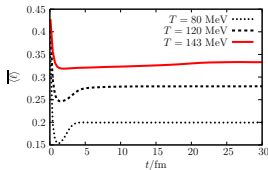
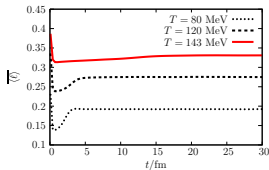
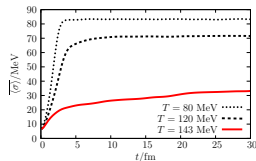
$$\eta = 2.2/\text{fm}$$



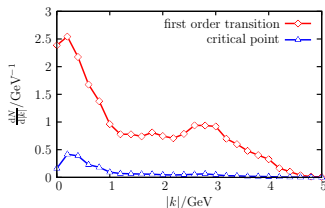
$$\eta = 5.0/\text{fm}$$



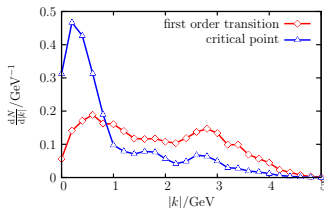
$$\eta = 10.0/\text{fm}$$



Box: Fourier analysis of Polyakov loop fluctuations



$t = 12 \text{ fm}$, during transition

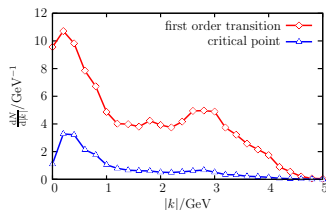


$t = 24 \text{ fm}$, after equilibration

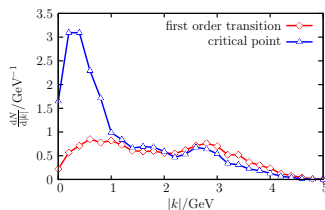
Intensity of Polyakov loop fluctuations:

$$N = \int_{\Delta k} d^3k N_k = \int_{\Delta k} d^3k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} d^3k T^2 \frac{\omega_k^2 |\delta \ell_k|^2 + |\dot{\delta \ell}_k|^2}{(2\pi)^3 2\omega_k}$$

Box: Fourier analysis of sigma fluctuations



$t = 12$ fm, during transition

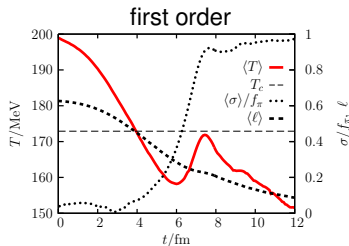


$t = 24$ fm, after equilibration

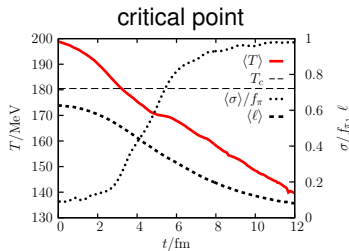
Intensity of sigma fluctuations:

$$N = \int_{\Delta k} d^3k N_k = \int_{\Delta k} d^3k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} d^3k \frac{\omega_k^2 |\delta\sigma_k|^2 + |\delta\dot{\sigma}_k|^2}{(2\pi)^3 2\omega_k}$$

Expanding plasma: first order vs. critical point



- ▶ formation of supercooled phase
- ▶ decay after ~ 2 fm
- ▶ reheating of the quark fluid

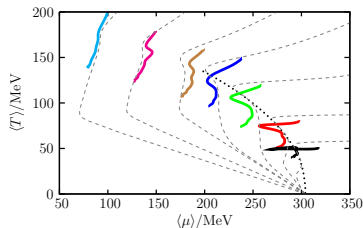


- ▶ smooth transition
- ▶ saddle point in $\langle T \rangle$ near T_c
- ▶ slowing down

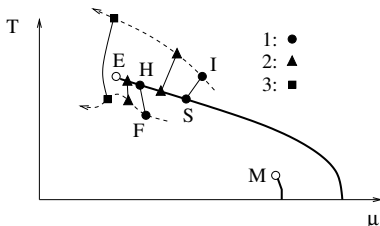
Expanding plasma: trajectories at finite μ

Isentropic trajectories in T - μ -plane are determined by

$$\frac{S}{A} = 3 \frac{e(T, \mu) + p(T, \mu) - \mu n(T, \mu)}{T n(T, \mu)}$$

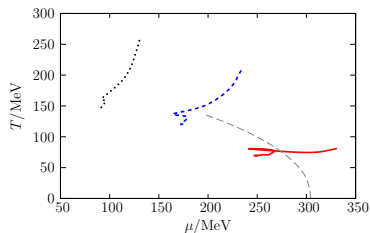


$S/A = 24, 16, 12, 10, 8, 6, 4$



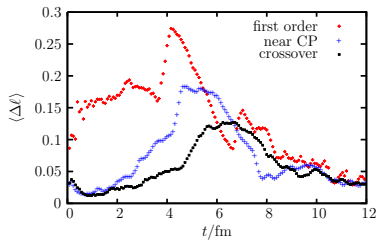
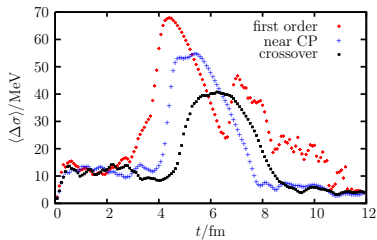
(Stephanov, Rajagopal and Shuryak, PRL **81** (1998))

Expanding plasma: Nonequilibrium fluctuations

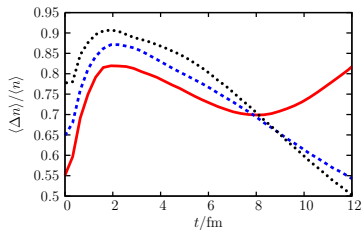
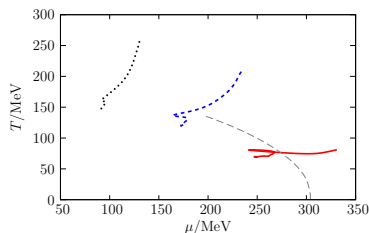


$$\langle \Delta \sigma \rangle = \sqrt{\langle (\sigma(x) - \sigma_{\text{eq}}(x))^2 \rangle}$$

$$\langle \Delta \ell \rangle = \sqrt{\langle (\ell(x) - \ell_{\text{eq}}(x))^2 \rangle}$$



Expanding plasma: Baryon density fluctuations



- ▶ Root mean squared fluctuations $\langle \Delta n \rangle = \sqrt{\langle (n(x) - \langle n(x) \rangle)^2 \rangle}$
- ▶ Enhancement of density fluctuations along first order transition line
- ▶ Density inhomogeneities are pronounced after crossing the phase transition

Expanding plasma: Formation of high density domains

$n(x)/\langle n \rangle$, first order

$n(x)/\langle n \rangle$, critical point

Summary

- ▶ Chiral fluid dynamics with Polyakov loop
- ▶ at zero baryochemical potential:
 - ▶ supercooling and reheating
 - ▶ critical slowing down
 - ▶ out of equilibrium: large fluctuations at first order transition
 - ▶ in equilibrium: enhanced fluctuations of soft modes at critical point
- ▶ at finite baryochemical potential:
 - ▶ trajectories overshoot isentropes due to supercooling
 - ▶ enhanced density fluctuations at first order transition

The Polyakov loop extended linear-sigma-model

The Lagrangian

$$\mathcal{L} = \bar{q} \left[i \left(\gamma^\mu \partial_\mu - i g_{QCD} \gamma^0 A_0 \right) - g\sigma \right] q + 1/2 (\partial_\mu \sigma)^2 - U(\sigma) - \mathcal{U}(\ell, \bar{\ell})$$

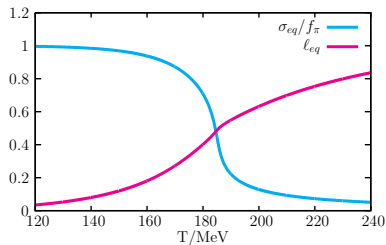
with the mesonic potential

$$U(\sigma) = \frac{\lambda^2}{4} (\sigma^2 - \nu^2)^2 - h_q \sigma - U_0$$

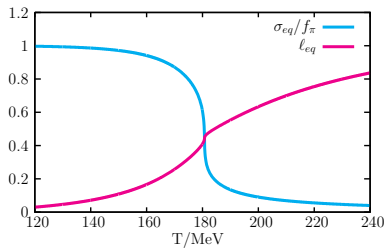
and the Polyakov loop potential

$$\frac{\mathcal{U}}{T^4}(\ell, \bar{\ell}) = -\frac{b_2(T)}{4} (|\ell|^2 + |\bar{\ell}|^2) - \frac{b_3}{6} (\ell^3 + \bar{\ell}^3) + \frac{b_4}{16} (|\ell|^2 + |\bar{\ell}|^2)^2$$

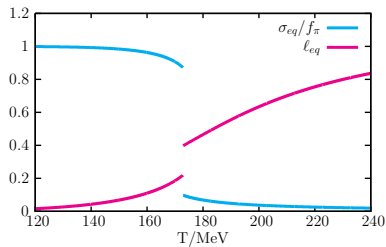
Thermodynamics



crossover



critical point



first order transition

order of the transition at $\mu = 0$
tuned via coupling g
 $g = 3.2$ (physical): crossover
 $g = 3.52$: critical point
 $g = 4.7$: first order

Propagation of the quark fluid: Source terms

For the quarks: $T^{\mu\nu}$ of ideal fluid

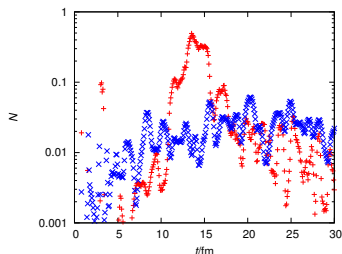
$$\partial_\mu T_q^{\mu\nu} = -\partial_\mu (T_\sigma^{\mu\nu} + T_\ell^{\mu\nu}) = S_\sigma^\nu + S_\ell^\nu$$

Source terms

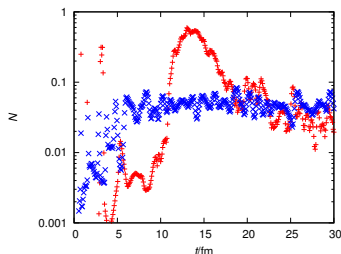
$$\begin{aligned} S_\sigma^\nu &= \left(-\frac{\partial \Omega_{q\bar{q}}}{\partial \sigma} - \eta_\sigma \partial_t \sigma \right) \partial^\nu \sigma \\ S_\ell^\nu &= \left(-\frac{\partial \Omega_{q\bar{q}}}{\partial \ell} - \frac{2N_c}{g_s^2} \eta_\ell \partial_t \ell T^2 \right) \partial^\nu \ell \end{aligned}$$

- ▶ account for energy-momentum transfer only due to damping
- ▶ transfer of stochastic energy is estimated numerically

Box: Fourier analysis of Polyakov loop fluctuations



$0 \leq |k| < 100 \text{ MeV}$

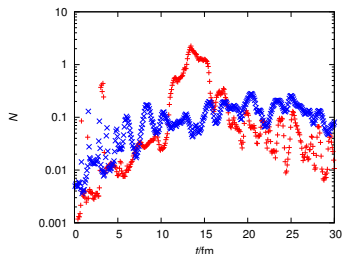


$100 \leq |k| < 200 \text{ MeV}$

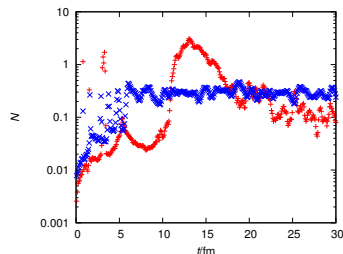
Intensity of Polyakov loop fluctuations:

$$N = \int_{\Delta k} d^3k N_k = \int_{\Delta k} d^3k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} d^3k T^2 \frac{\omega_k^2 |\ell_k|^2 + |\dot{\ell}_k|^2}{(2\pi)^3 2\omega_k}$$

Box: Fourier analysis of sigma fluctuations



$0 \leq |k| < 100 \text{ MeV}$



$100 \leq |k| < 200 \text{ MeV}$

Intensity of sigma fluctuations:

$$N = \int_{\Delta k} d^3k N_k = \int_{\Delta k} d^3k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} d^3k \frac{\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2}{(2\pi)^3 2\omega_k}$$