First Steps Towards QCD Under External Magnetic Fields

- from a Dyson–Schwinger Perspective-

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Outline

1. Introduction

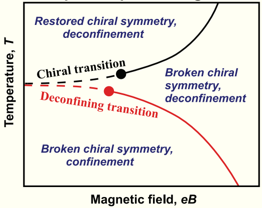
Why are magnetic fields interesting?

- Heavy Ion Collisions
- Cosmological electroweak phase transition
- Neutron Stars
- Condensed Matter Systems
- (Color-) Superconductors

Open Issues

- Modification of the (QCD) vacuum structure ?!
- Chiral Magnetic Effect ?
- Magnetic catalysis ?

Expected phase diagram

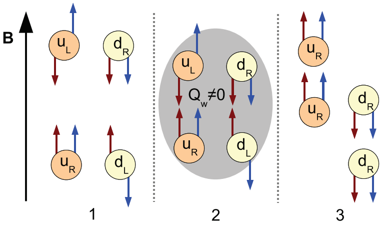


Mizher, Chernodub, Fraga: PRD 82,105016 (2010)

The Chiral Magnetic Effect

Kharzeev, McLerran, Warringa: Nuc. Phys. A 803, 227 (2008),
Fukushima, Kharzeev, Warringa: PRD 78 (2008)

- In non-central collisions: strong magnetic fields are produced
- Magnetic fields induce a (chiral) charge separation



Kharzeev, McLerran, Warringa: Nuc. Phys. A 803, 227 (2008)

- Electric current parallel to external magnetic field

→ Before this study is feasible,
another step in between is in order...

1. Introduction

A First Step: Magnetic Catalysis

Enhancement of chiral symmetry breaking due to an external magnetic field

Gusynin, Miransky, Shovkovy: PRL 73,26 (1994)

Issues...

- Realization of magnetic field ?
- Mechanism driving enhancement ?
- Effect of approximations/truncations ?
- Origin of discrepancies between lattice and effective model results ?

Aim ...

$\Rightarrow$ Obtain complementary information by non-perturbative finite volume study in a Dyson–Schwinger framework

Reminder: Particles in magnetic fields..

- Constant magnetic field in z-direction: $\vec{\mathcal{B}} = (0, 0, \mathcal{B})^{\mathsf{T}}$

Classical Particle

Lorentz force:

$$F(t) = -e\vec{v} \times \vec{\mathcal{B}}$$

- $v = \omega r$
- $\omega = \frac{e\mathcal{B}}{m}$
- No restriction in z-direction
- Circular orbit $\perp \vec{\mathcal{B}}$

QM Particle

Schrödinger equation:

$$-\frac{1}{2m}[\partial_x^2 + (\partial_y + \mathrm{i}e\mathcal{B}x)^2]\psi = E\psi$$

- $E = \omega(n + \frac{1}{2})$
- Shifted harmonic oscillator *eigen*states
- Infinite degeneracy wrt p_y

Implementing a constant magnetic background

- Abelian field in z-direction

$$A_\mu = (0, \mathcal{B}\,z, 0, 0)^{\mathsf{T}}$$

$$\Rightarrow \mathcal{F}_{\mu\nu,ab} = \mathcal{F}_{\mu\nu,ab} + f_{\mu\nu} \cdot \mathbb{1}_{ab}$$

- Principle of minimal coupling

$$\mathrm{D}_\mu = \partial_\mu - ieA_\mu \qquad\qquad \mathcal{D}_\mu = \mathrm{D}_\mu + \mathrm{i}g\,t^a\,\mathcal{A}_{\mu,a}$$

- Leads to the underlying Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\not{\mathcal{D}} - m)\psi + \mathcal{F}_{\mu\nu}\,\mathcal{F}^{\mu\nu}$$

2. The Challenges

Deriving the equations

- Work in the Dyson–Schwinger approach
- Propagator from Green's function identity:

$$(\mathrm{i}\not{\Pi} - m)\, S(x, x') = \delta(x - x')$$

  with $\Pi_\mu = \partial_\mu - eA_\mu$
- 'Standard' approach: expand in plane wave functions find diagonal propagator in momentum space
- Challenge: $[\Pi_\mu, p_\nu] \neq 0$
  $\rightarrow$'standard' not applicable

Following Ritus' method

...to obtain the (inverse) propagator in momentum space

V.I. Ritus: Annals of Phys. 69,555 (1972)

Ritus Method: The Idea

- Observation I:
  S can only depend on scalar structures built from γ^μ contracted with $\Pi_\mu, F_{\mu\nu}, ...$
- Observation II:
  $[(\not\Pi)^2, S(x, x')] = 0$

The Procedure

- Use *eigen*functions of $\not\Pi^2$ to diagonalize propagator
- End up with 'modified' propagator
  diagonal in momentum space
  depending on special subset of momenta

$$
\begin{aligned}
(\mathrm{i}\not\Pi - \mathrm{m})\, S(x, x') &= \int \mathrm{dp}\, \mathbb{E}_p\, (\not\bar{p} - \mathrm{m}) S(p)\, \bar{\mathbb{E}}_p \\
&\stackrel{!}{=} \delta(x - x')
\end{aligned}
$$

3. The Devil In The Details

- Diagonalization procedure shows

$$S(\bar{p}) = (\bar{\not{p}} - \mathrm{m})^{-1}$$

- With momenta given by

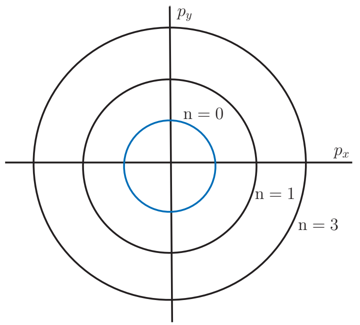
$$\bar{p} = (p_0, 0, \sqrt{k}, p_z)^{\mathrm{T}}$$

- And $\sqrt{k}$ encoding the particles' Landau Levels

$$\sqrt{k} = \sqrt{|e\mathcal{B}|(2\mathrm{n}+1) + \sigma e\mathrm{B}\, sgn(e\mathcal{B})}$$

- States per unit area:
  $\frac{|e\mathcal{B}|}{2\pi}$ for n = 0
  $\frac{|e\mathcal{B}|}{\pi}$ for n $\geqslant 0$

3. The Devil In The Details

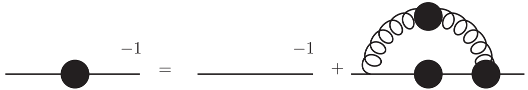


- Lowest Landau level approximation (LLLA) :
  n $= 0$
  (spin polarized state)
- Reliable in range :
  m $\ll \sqrt{|e\mathcal{B}|}$
- Dimensional reduction
  $n = 0 \Rightarrow \sqrt{k} = 0$

$$\bar{p} = (p_0, 0, 0, p_z)^T$$

- NO application of the Mermin–Wagner theorem
  $\rightarrow$ gluons are 4-dimensional
- Drawback/Limit of LLLA: $\mathcal{B} \rightarrow 0$

The Dyson–Schwinger Equations



with the dressed propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\,(A_0(\bar{p})\gamma^0 \bar{p}_0 + A_3(\bar{p})\gamma^3 \bar{p}_3)$$

- $A_1(p)$ and $A_2(p)$ are not accessible in this approximation
- Gluonic input from lattice calculations

  → Fischer, Maas, Pawlowski: Annals Phys.324 (2009)

- Landau gauge
- Modified bare vertex approximation
- Solution in a finite volume → (1+1) torus

3. The Devil In The Details

Magnetic Flux

$$\int dx_\mu A_\mu = \mathcal{B} \cdot \mathrm{F}$$
$$\int dx_\mu A_\mu = \mathcal{B} \cdot (\mathrm{F} - L_x L_y)$$

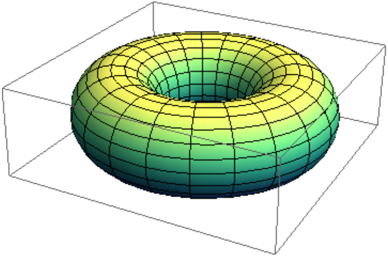
Charged Particles

$$\exp(iq\mathcal{B}\mathrm{F}) \stackrel{!}{=} \exp(iq\mathcal{B}(\mathrm{F} - L_x L_y))$$

$$\Rightarrow q\mathcal{B} = \frac{2\pi}{L_x L_y} b$$

with b= 0,1,2...

Magnetic Field In A Finite Volume



4. My Status Quo

Full Quark Propagator

$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\, A_\mu(\bar{p})\gamma^\mu \bar{p}_\mu$

Quantized B-Field

$|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$

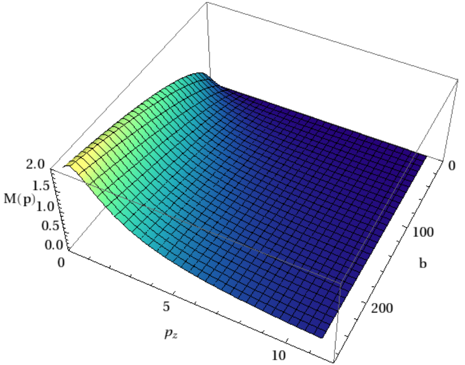
$b \in [0, L_x \cdot L_y]$

Typical Tori

- Box Length: 6 fm
- Mom. points: 4×33
- $\mathcal{B}_{max} \sim 1.8\mathrm{GeV}^2$

Dynamically Generated Mass

$M(p) = \frac{B(p)}{A(p)}$ $\quad [M(p)] = GeV$



Result

- Lowest Landau Level approximation ⇒
  Enhanced mass generation with increasing magnetic field

4. My Status Quo

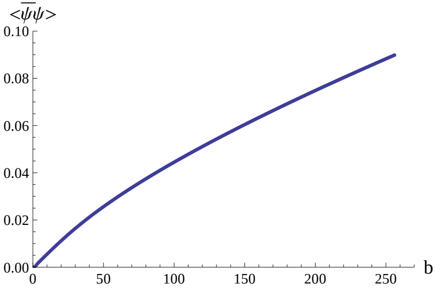
Chiral Condensate

$$\langle\bar{\psi}\psi\rangle_b \sim b \sum_{n_t, n_z} \frac{B_b(p)}{B_b(p)^2 + (A_{0b}\, p_0)^2 + (A_{zb} p_z)^2}$$

Max. B-field

- $|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$
- $\mathcal{B}_{max} \sim 1.8\text{GeV}^2$

- Limit $\mathcal{B} \to 0$ is not reliable in LLLA



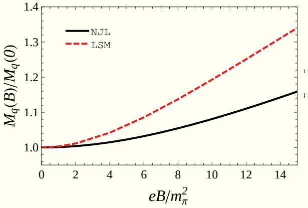
Result

- Lowest Landau Level approximation $\Rightarrow$
  Increasing chiral condensate with increasing magnetic field

4. My Status Quo

Effective Model Approaches

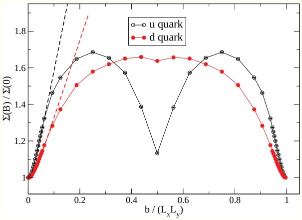
- Mostly LLLA
- Ritus method or Schwinger *eigen*time formalism
- Increasing chiral symmetry breaking with increasing B-field



Ferrari, Garcia, Pinto: arXiv 1207.3714v2

Lattice Gauge Theory

- Beyond LLLA
- Finite volume
- Saturation effects in chiral condensate



D'Elia, Negro: PRD 83 (2011)

5. Down The Road

Wrap up

- Concept of constant external magnetic fields
- Effects on particles' propagators and momenta
- Discussion of Lowest Landau Level approximation

The next steps

- Include to overcome LLLA
- Include finite temperatures
- Volume studies
- Unquenching gluon and adding chemical potential
- ...

Thank you for your attention!

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