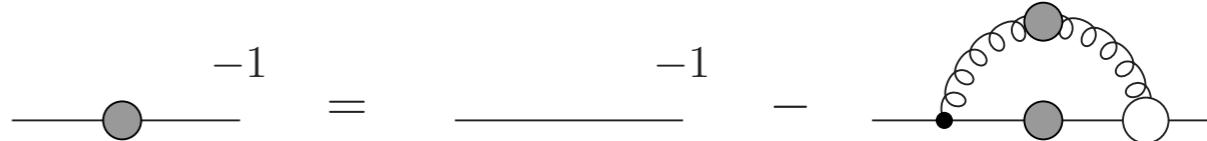




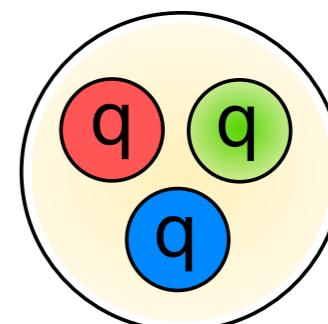
Hadron spectra and properties from functional methods

Overview

I. Mass from nothing

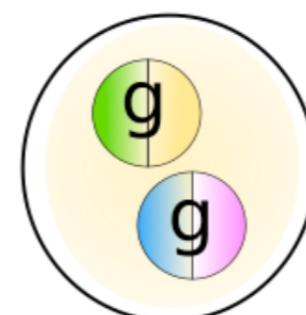


2. Baryons



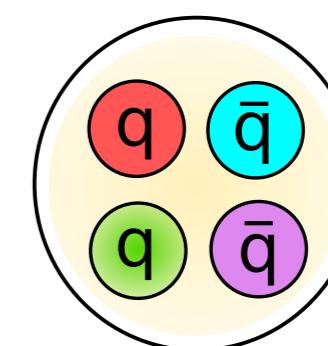
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3. Glueballs



CF, Huber, Sanchis-Alepuz, EPJC 80 11, 1077 (2020), arXiv:2004.00415

4.(Heavy-light) Tetraquarks



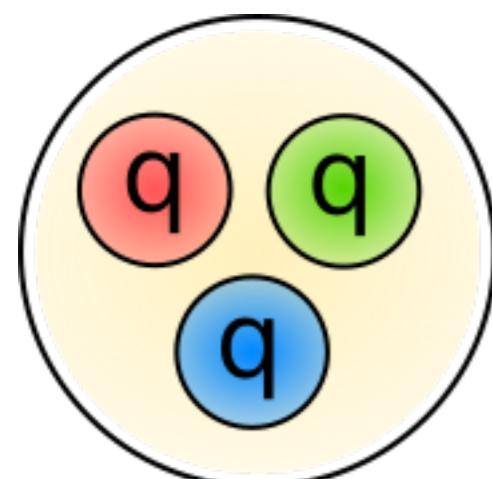
Wallbott, Eichmann and CF, PRD 100 (2019) no.1, 014033, arXiv:1905.02615

Wallbott, Eichmann and CF, PRD 102 (2020) no.5, 051501, arXiv:2003.12407

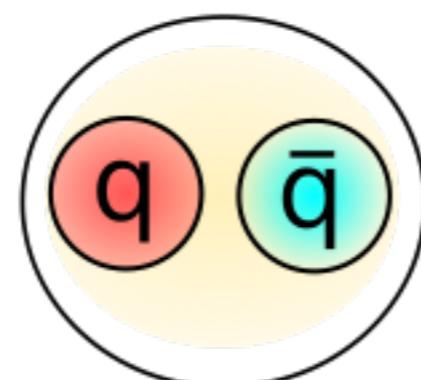
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495

Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4,38, arXiv:2008.10240

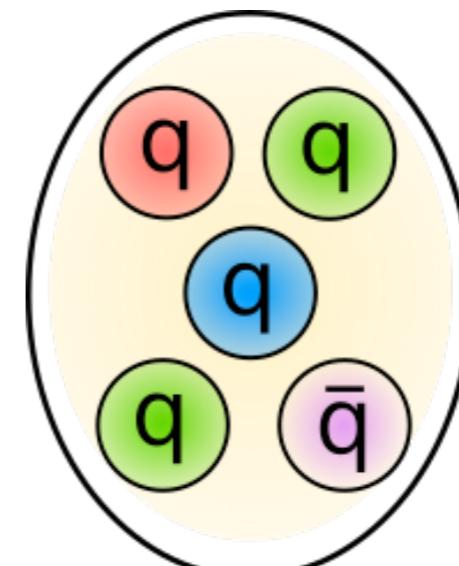
Hadrons: baryons, mesons and ... exotics !



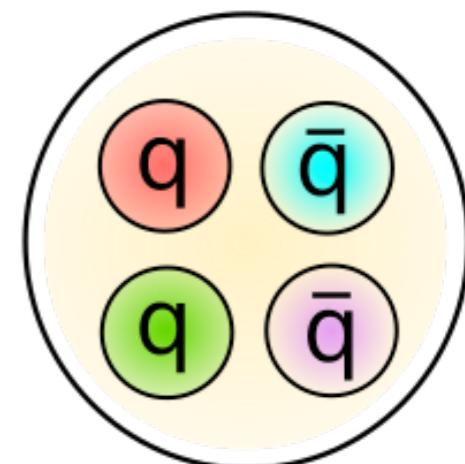
Baryon



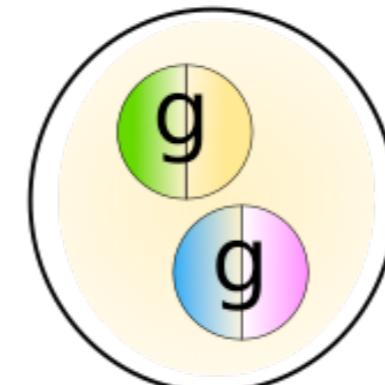
Meson



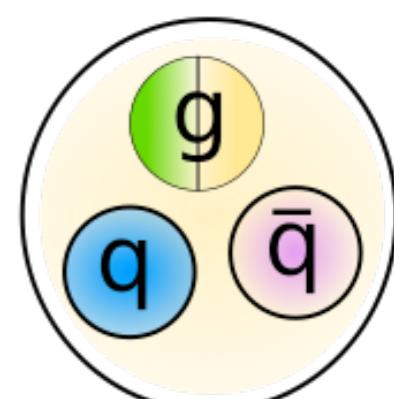
Pentaquark



Tetraquark



Glueball

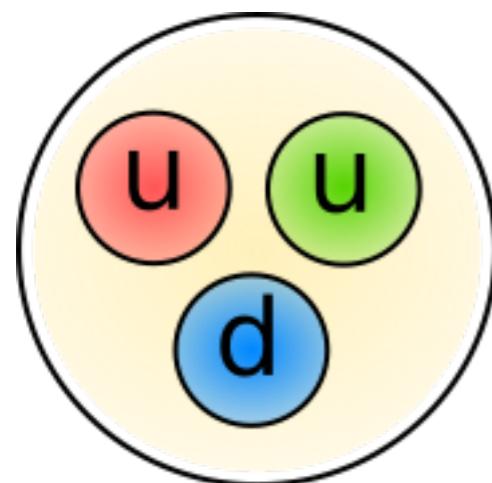


Hybrid

ordinary hadrons

'exotic' hadrons

Properties of QCD: Dynamical mass generation

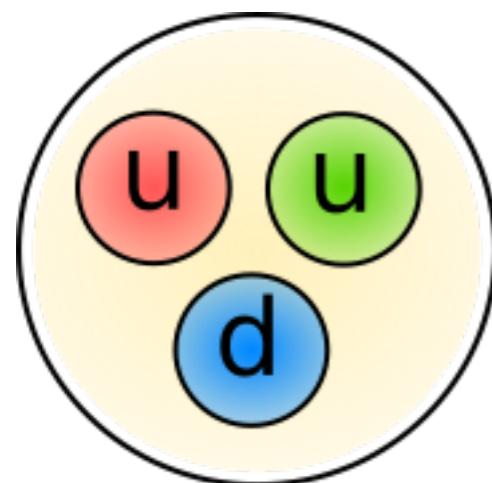


$$m_{\text{proton}} = 938 \text{ MeV}$$

Dynamical quark masses via weak force

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000

Properties of QCD: Dynamical mass generation

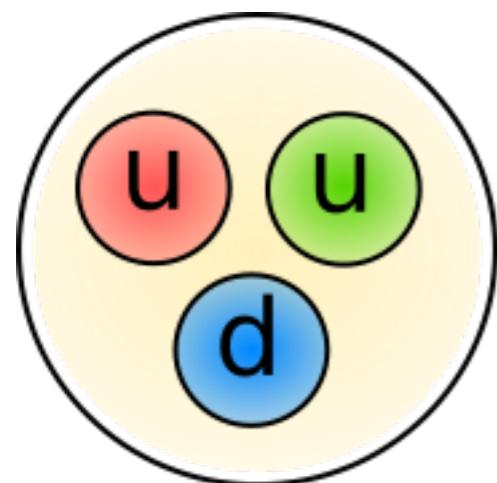


$$m_{\text{proton}} = 938 \text{ MeV}$$

Dynamical quark masses via weak force and strong force:

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000
M_{strong} [MeV]	350	350	350	350	350	350

Properties of QCD: Dynamical mass generation



$$m_{\text{proton}} = 938 \text{ MeV}$$

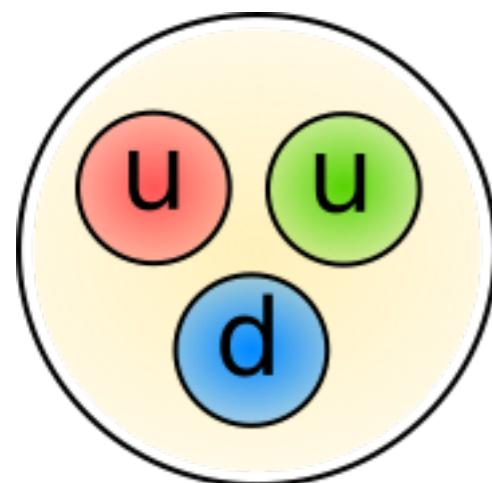


Yoichiro Nambu,
Nobel prize 2008

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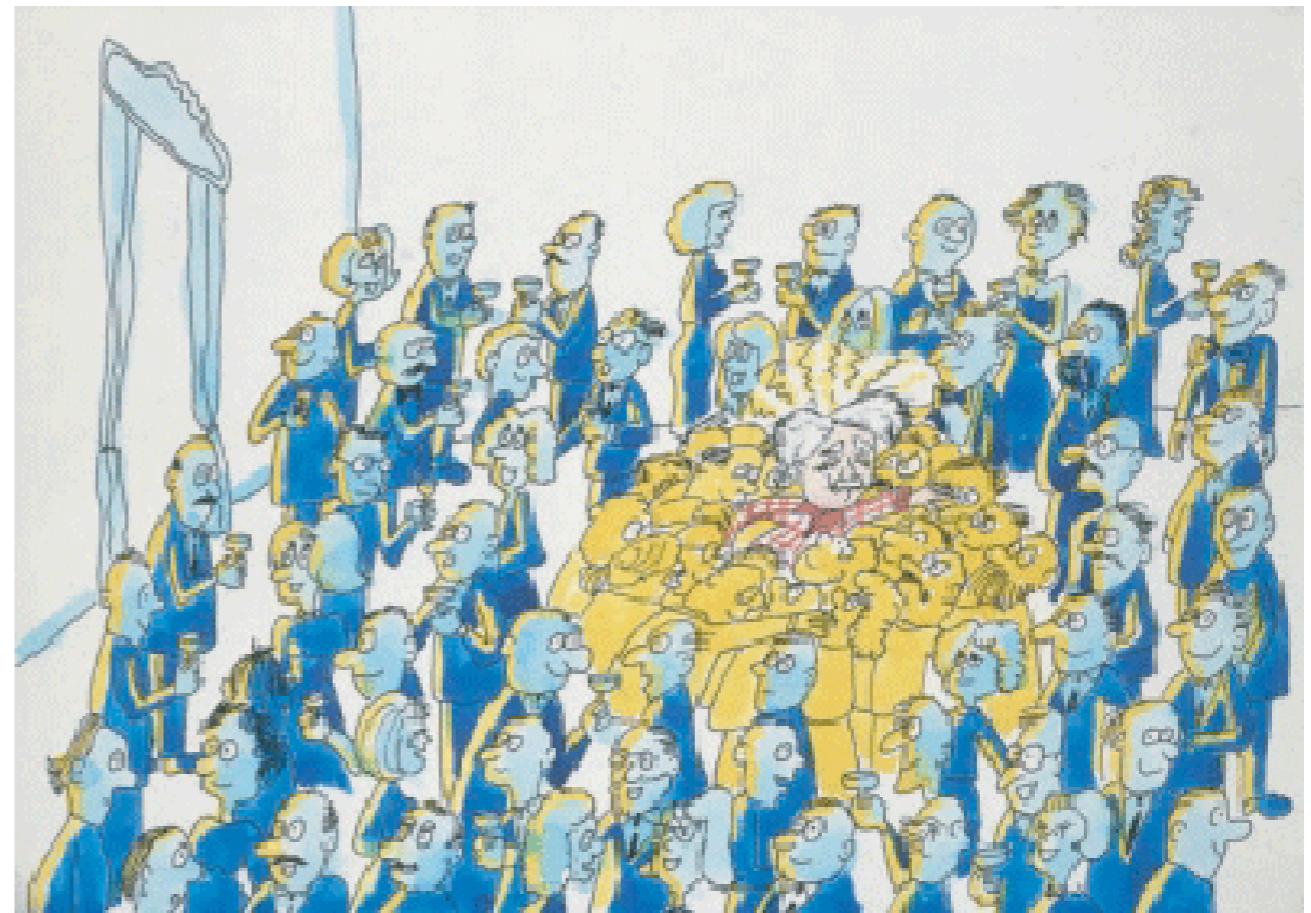


Dynamical mass generation



$$\text{---} \circ -1 = \text{---} -1 - \text{---}$$
A Feynman diagram illustrating the concept of dynamical mass generation. It shows a loop of three vertices connected by a horizontal line. The first vertex is a solid black circle, the second is a grey circle, and the third is an open white circle. The horizontal line connecting the first and second vertices has a minus sign (-) above it. The horizontal line connecting the second and third vertices has a minus sign (-) below it. The label '-1' is placed above the first vertex. The label '-1' is placed above the second vertex. The label '-1' is placed above the third vertex.

Dynamical mass generation



$$\text{---} \circ -1 = \text{---} - \text{---} \circ \text{---}$$

A Feynman diagram equation. On the left, a horizontal line with a grey circle at the right end is labeled -1 . An equals sign follows. On the right, there are two horizontal lines. The top line has a black dot at the left end and a grey circle at the right end, labeled -1 . A minus sign follows. The bottom line has a black dot at the left end and an empty white circle at the right end.

Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber,EPJ C77 (2017) no.11, 733

Dyson-Schwinger equations - “3PI vs RL”

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propagators

$$\begin{array}{c} -1 \\ \text{---} \quad \text{---} \end{array} = \begin{array}{c} -1 \\ \text{---} \quad \rightarrow \end{array} - \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array}$$

$$\begin{array}{c} -1 \\ \text{---} \quad \text{---} \end{array} = \begin{array}{c} -1 \\ \text{---} \quad \text{---} \end{array} - \frac{1}{2} \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array}$$

$$+ \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array} + \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array}$$

$$- \frac{1}{6} \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array} - \frac{1}{2} \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array}$$

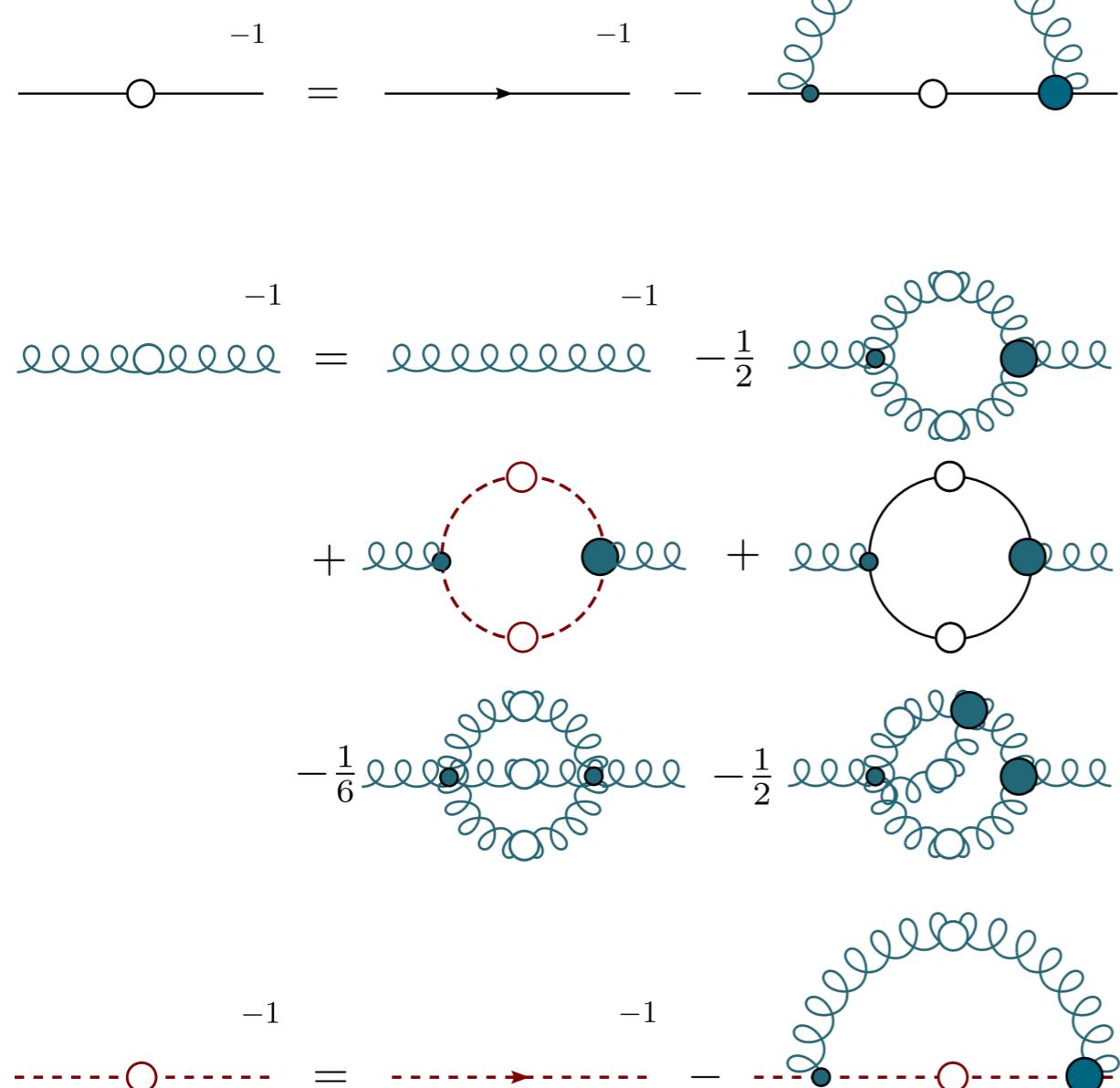
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CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
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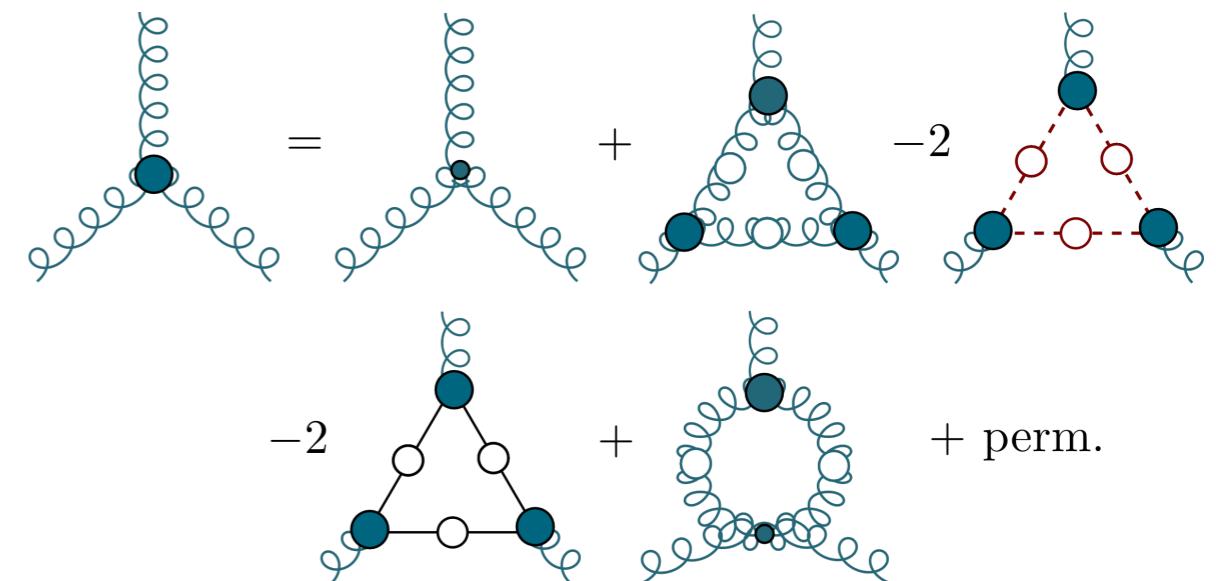
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propagators



vertices

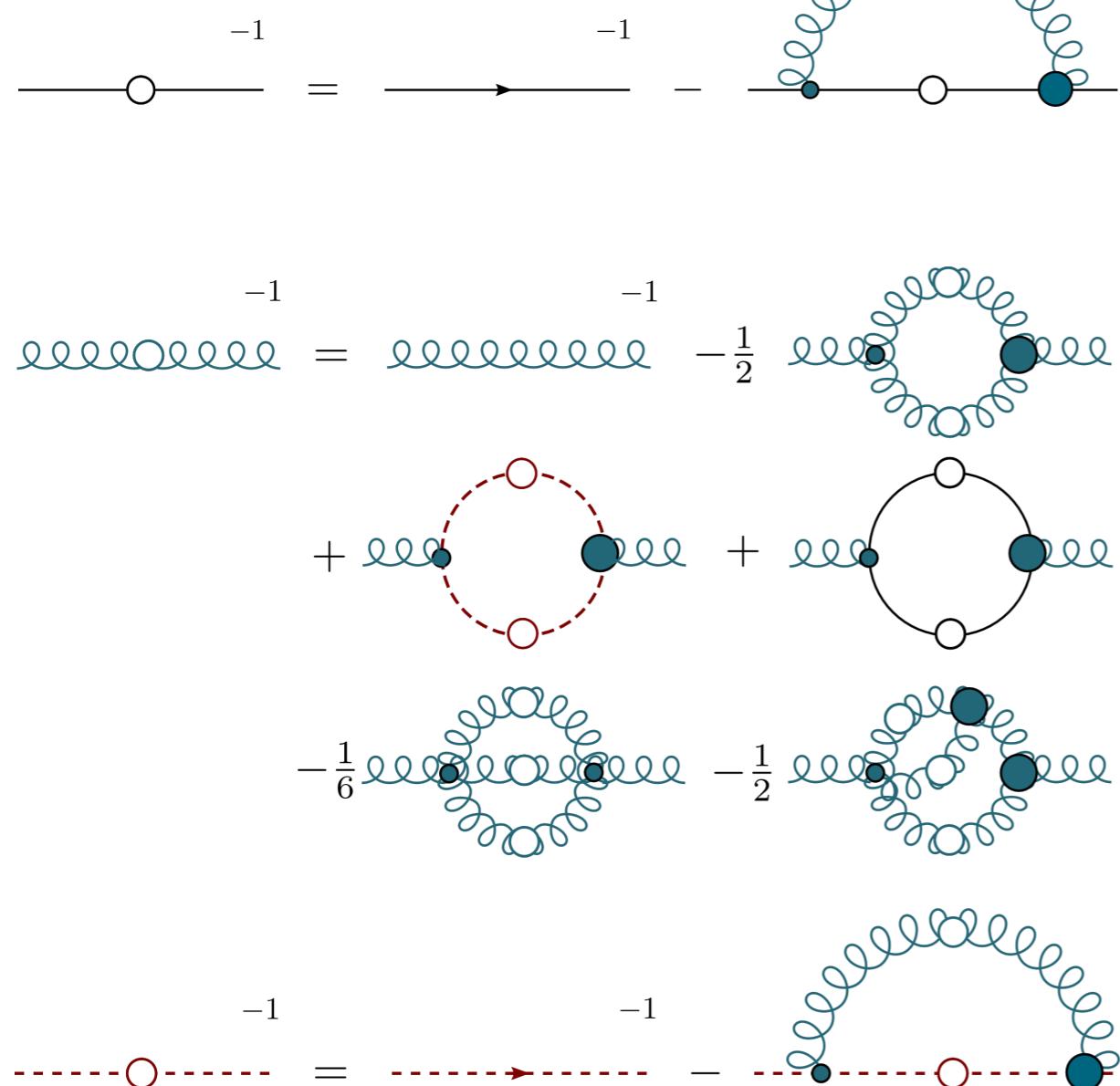


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber,EPJ C77 (2017) no.11, 733

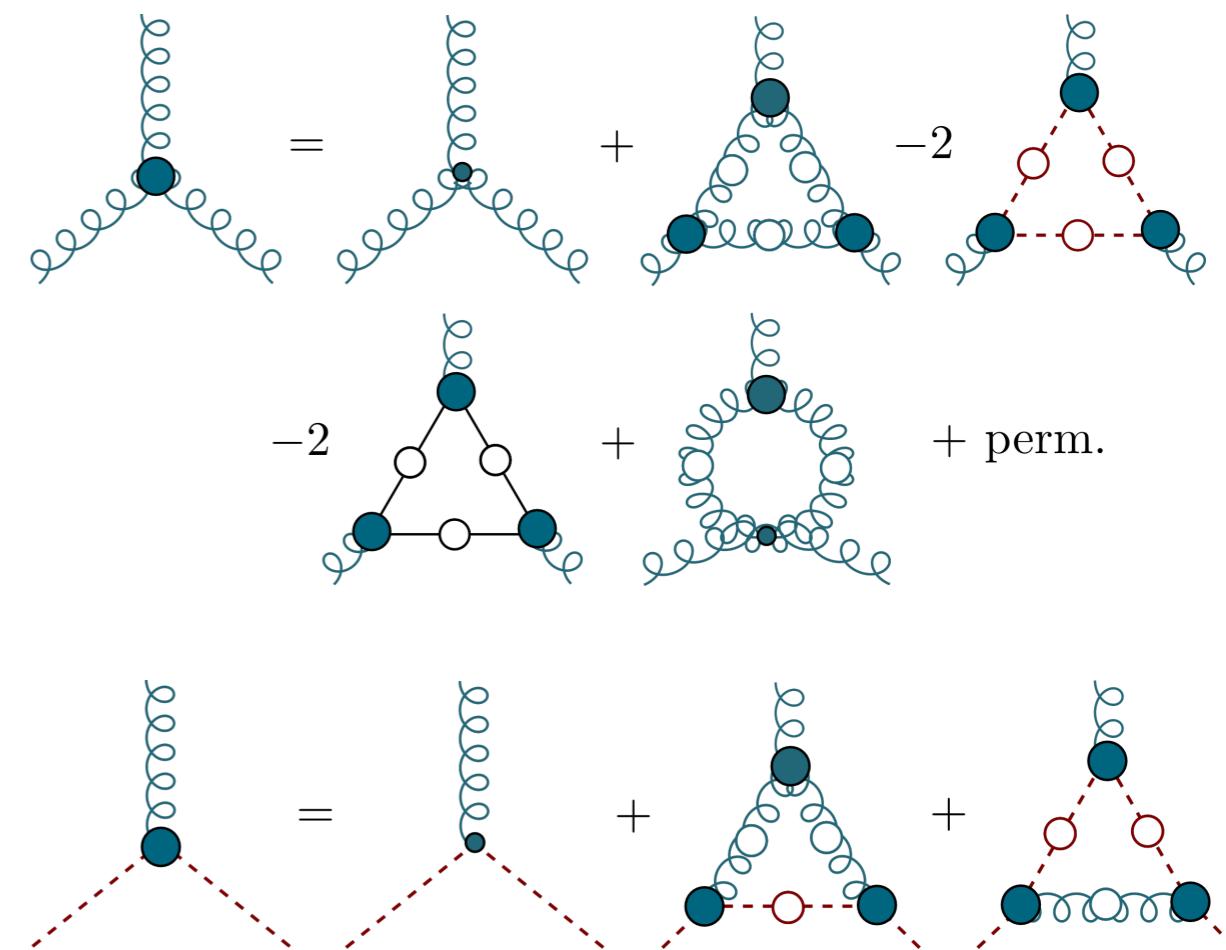
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propagators



vertices

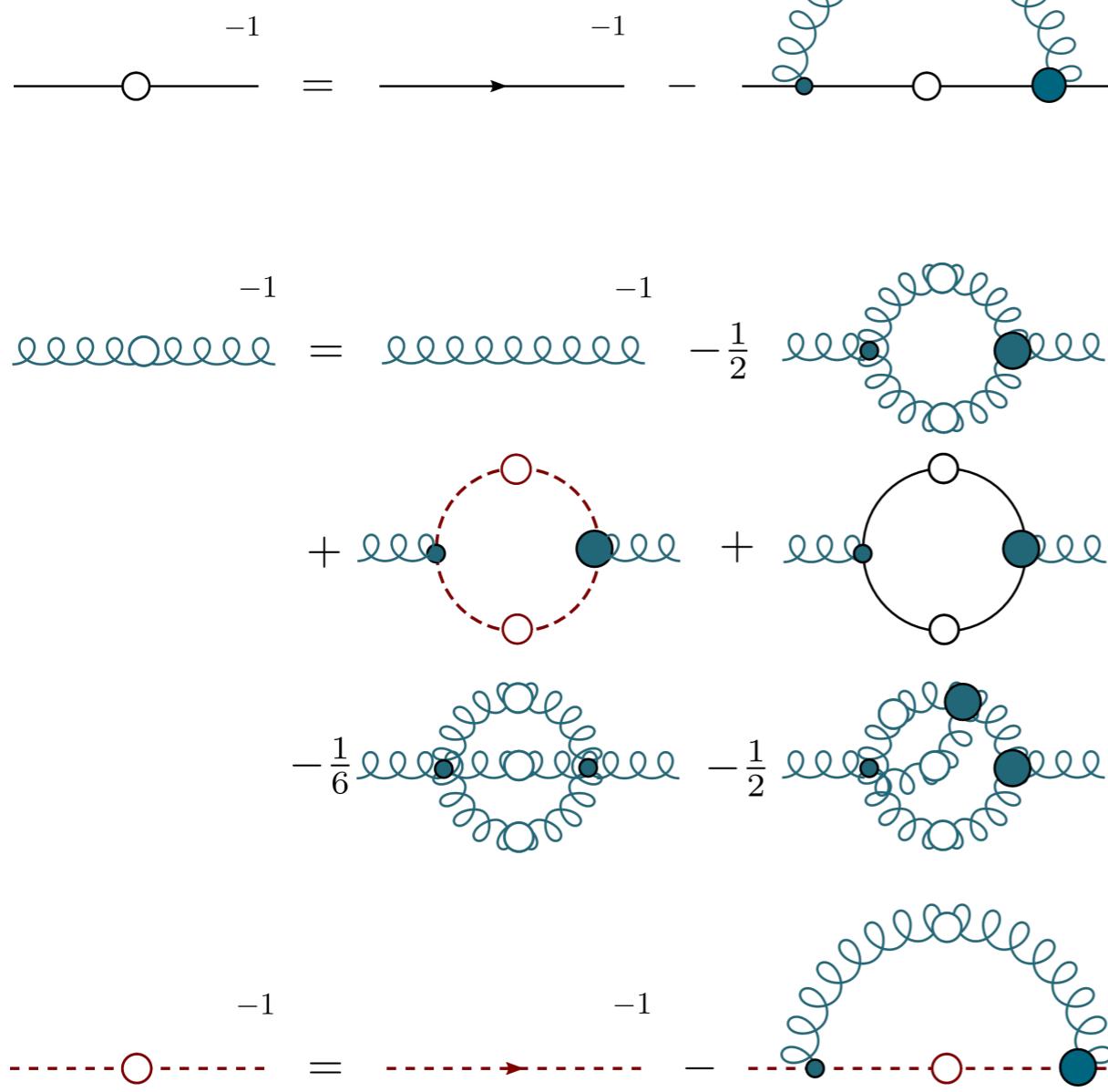


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber,EPJ C77 (2017) no.11, 733

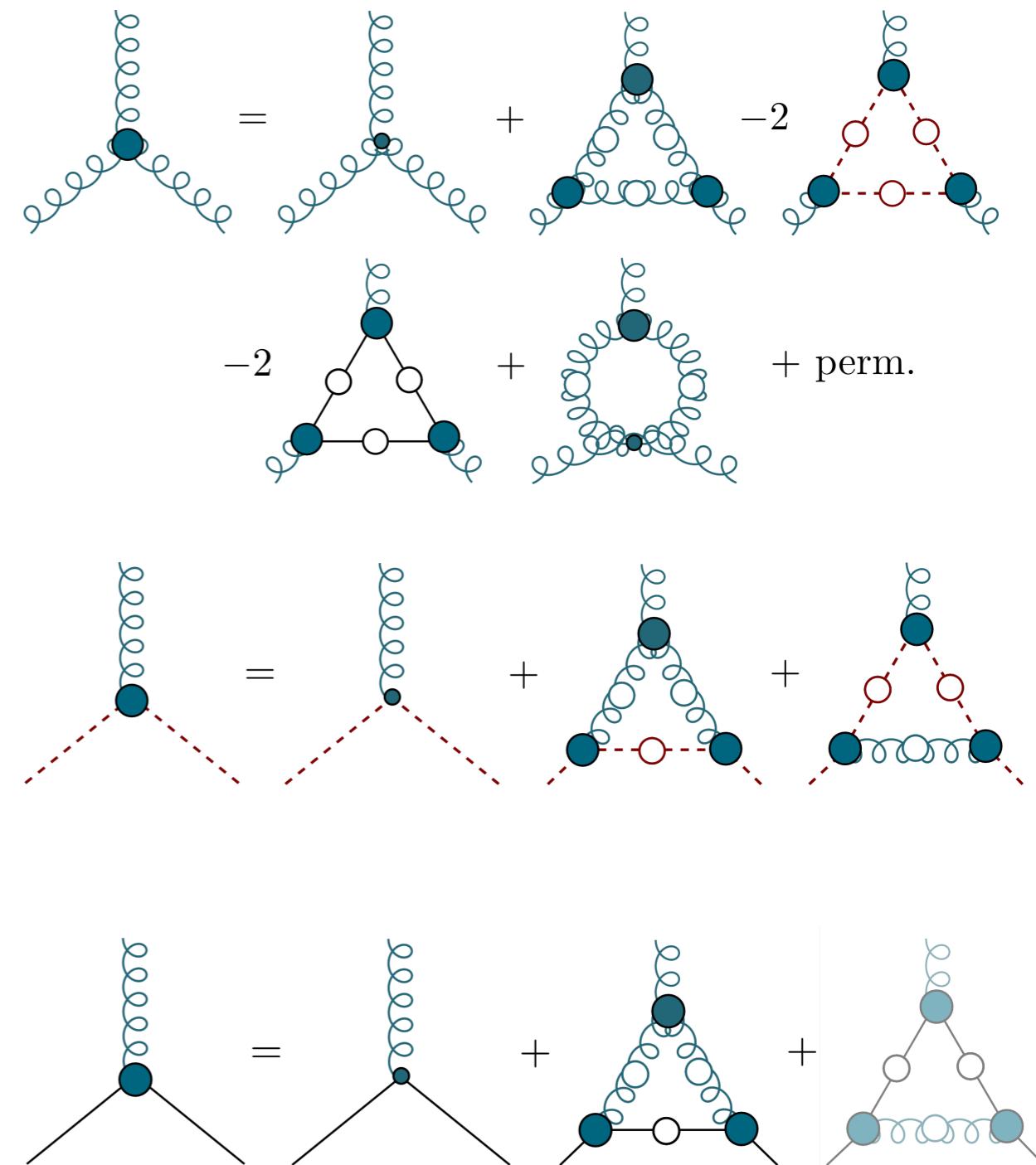
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propagators



vertices



CF,Alkofer, PRD67 (2003) 094020
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propagators

$$-\frac{1}{-1} = \text{---} \rightarrow - \text{---} -$$

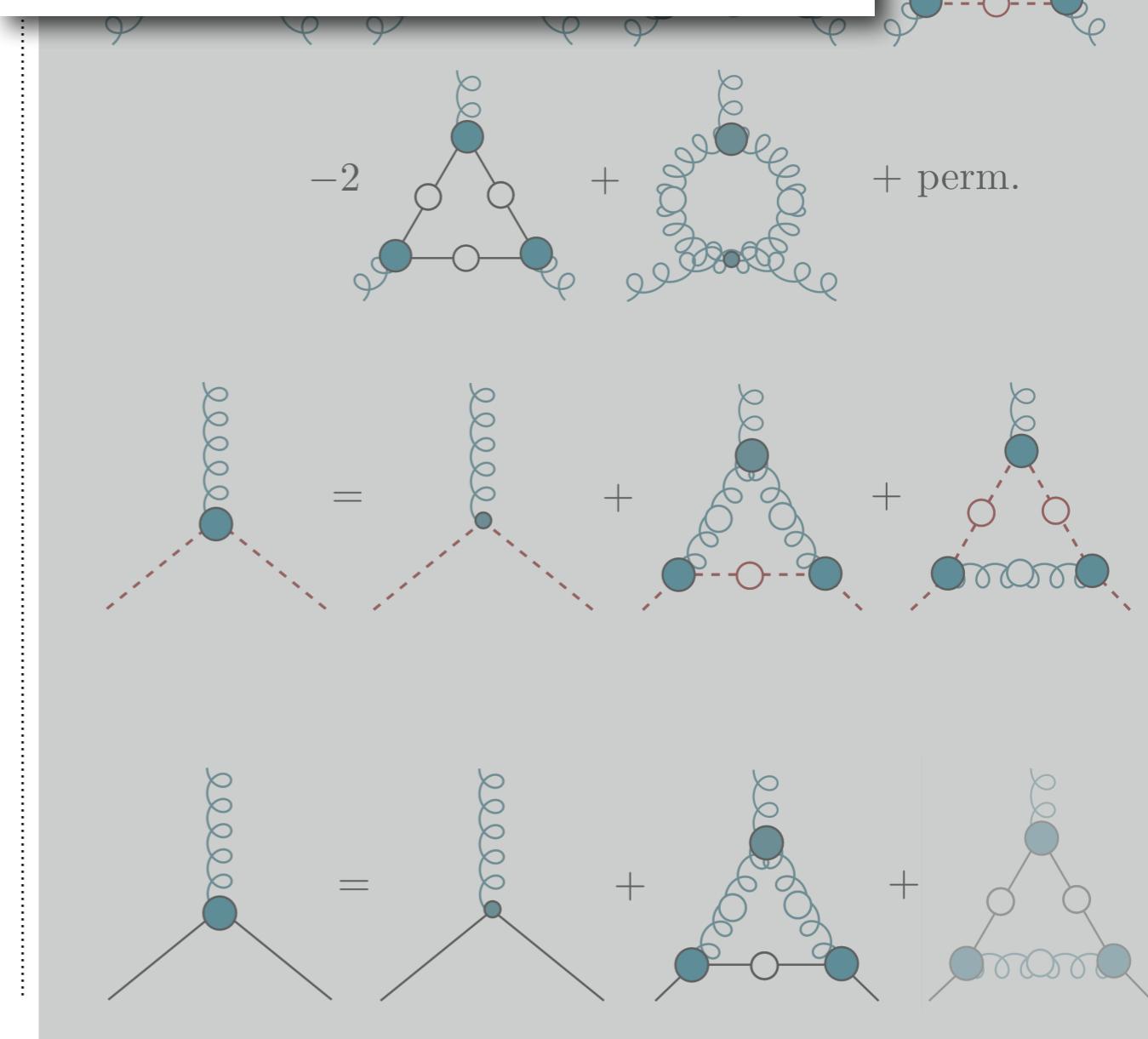
$$-\frac{1}{-1} = \text{---} \rightarrow - \text{---} - \frac{1}{2} \text{---} \text{---} + \text{---} \text{---} - \frac{1}{2} \text{---} \text{---} - \frac{1}{6} \text{---} \text{---} - \frac{1}{2} \text{---} \text{---}$$

$$-\frac{1}{-1} = \text{---} \rightarrow - \text{---} -$$

CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber,EPJ C77 (2017) no.11, 733

vertices

“rainbow-ladder” (RL) :
 model for gluon+vertex



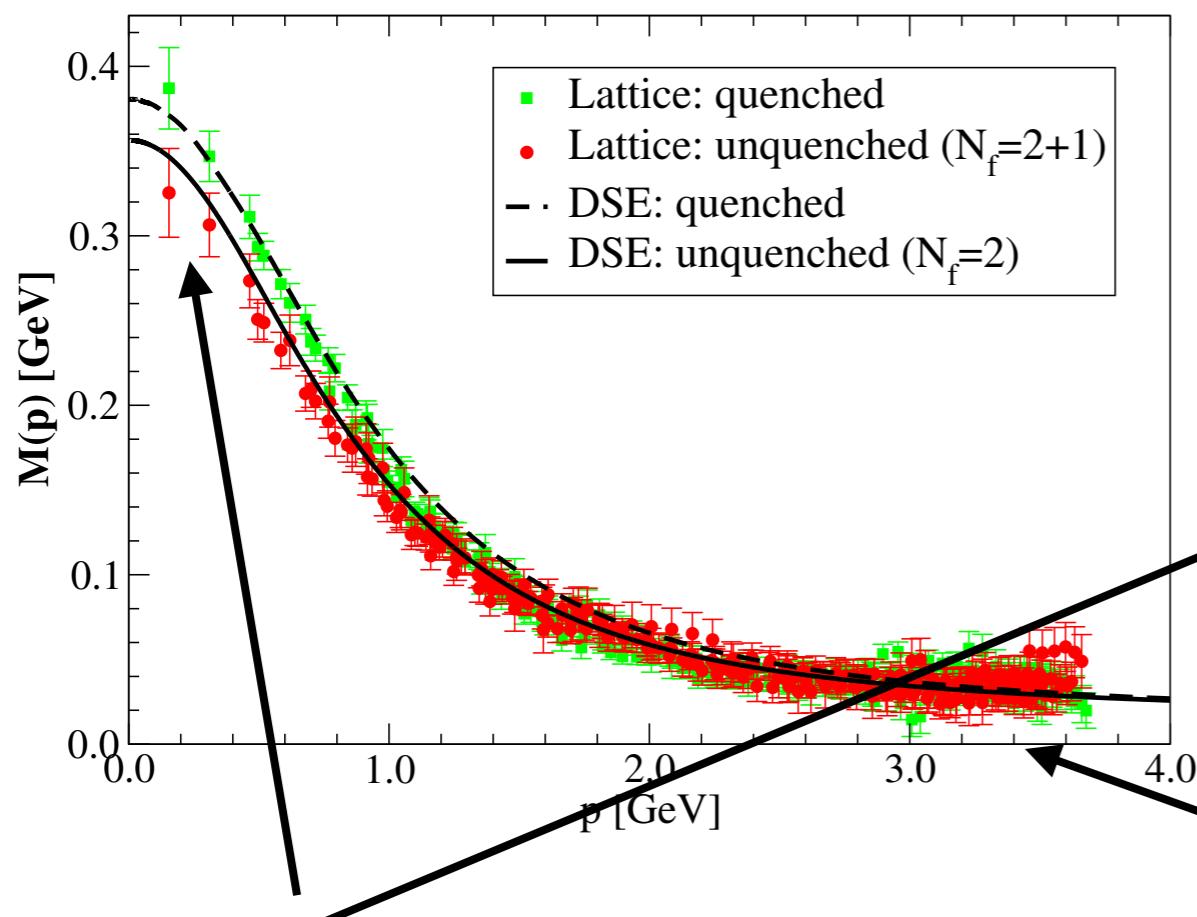
Quarks: mass from interaction

$$-1 = -1 - \text{Diagram}$$

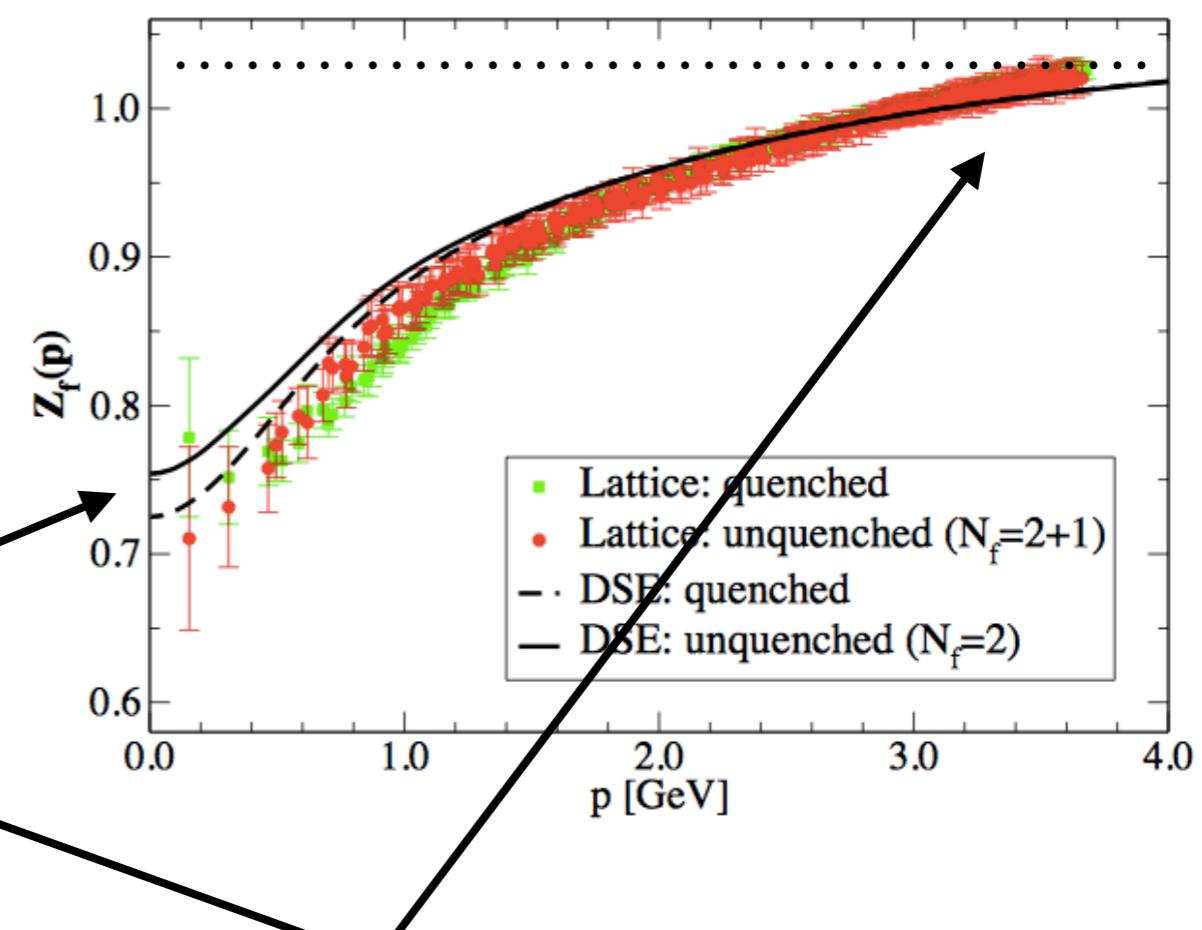
The diagram shows a quark loop with a gluon exchange between two quarks. The quarks are represented by circles, and the gluon by a wavy line.

$$S(p) = Z_f(p^2) \frac{-ip + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
 Williams, CF, Heupel, PRD 93 (2016) 034026
 Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

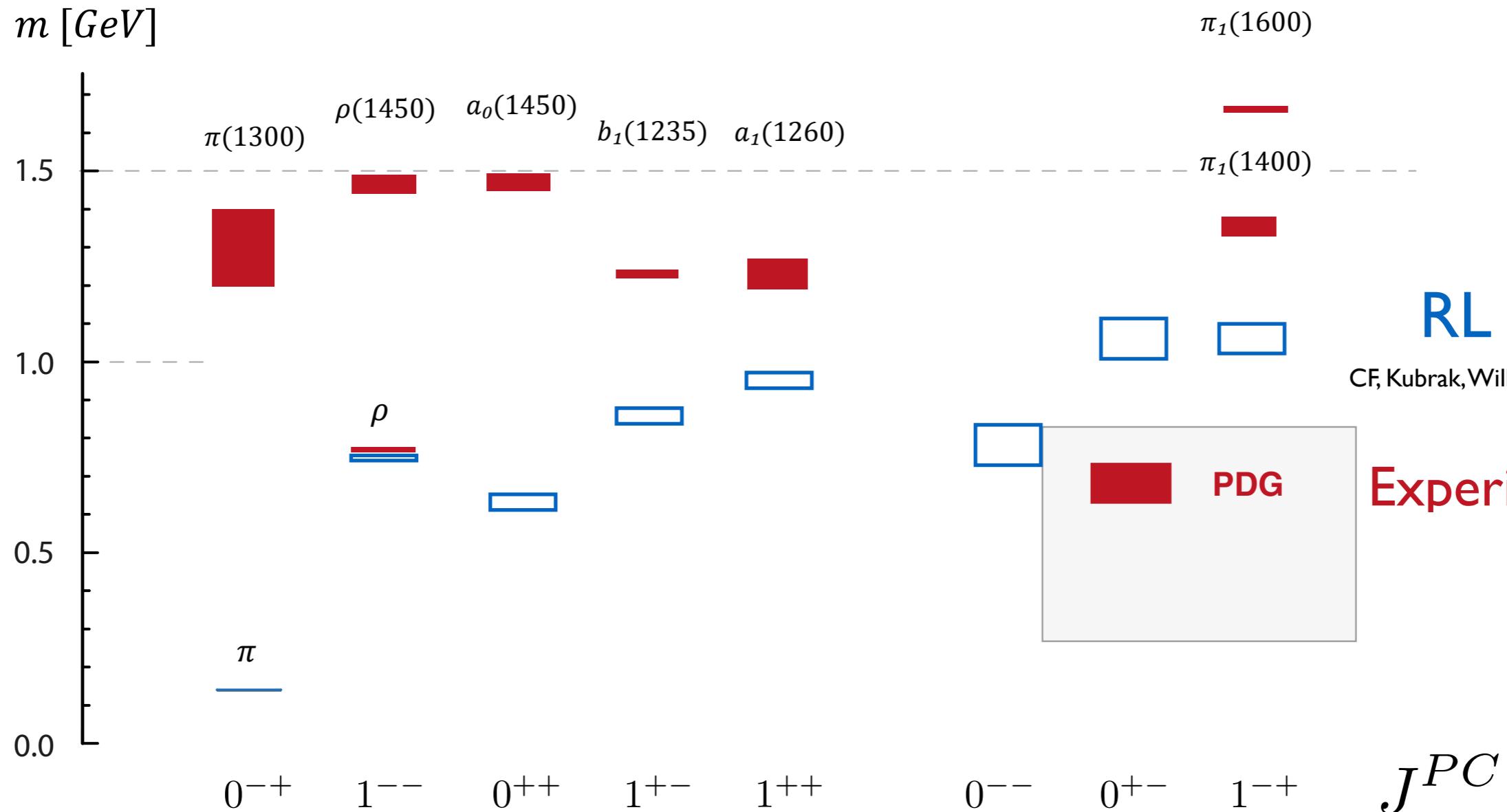
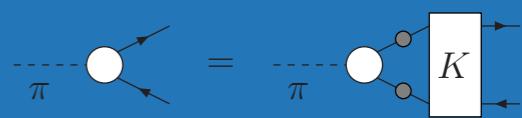


'constituent quark':
 large mass; very composite



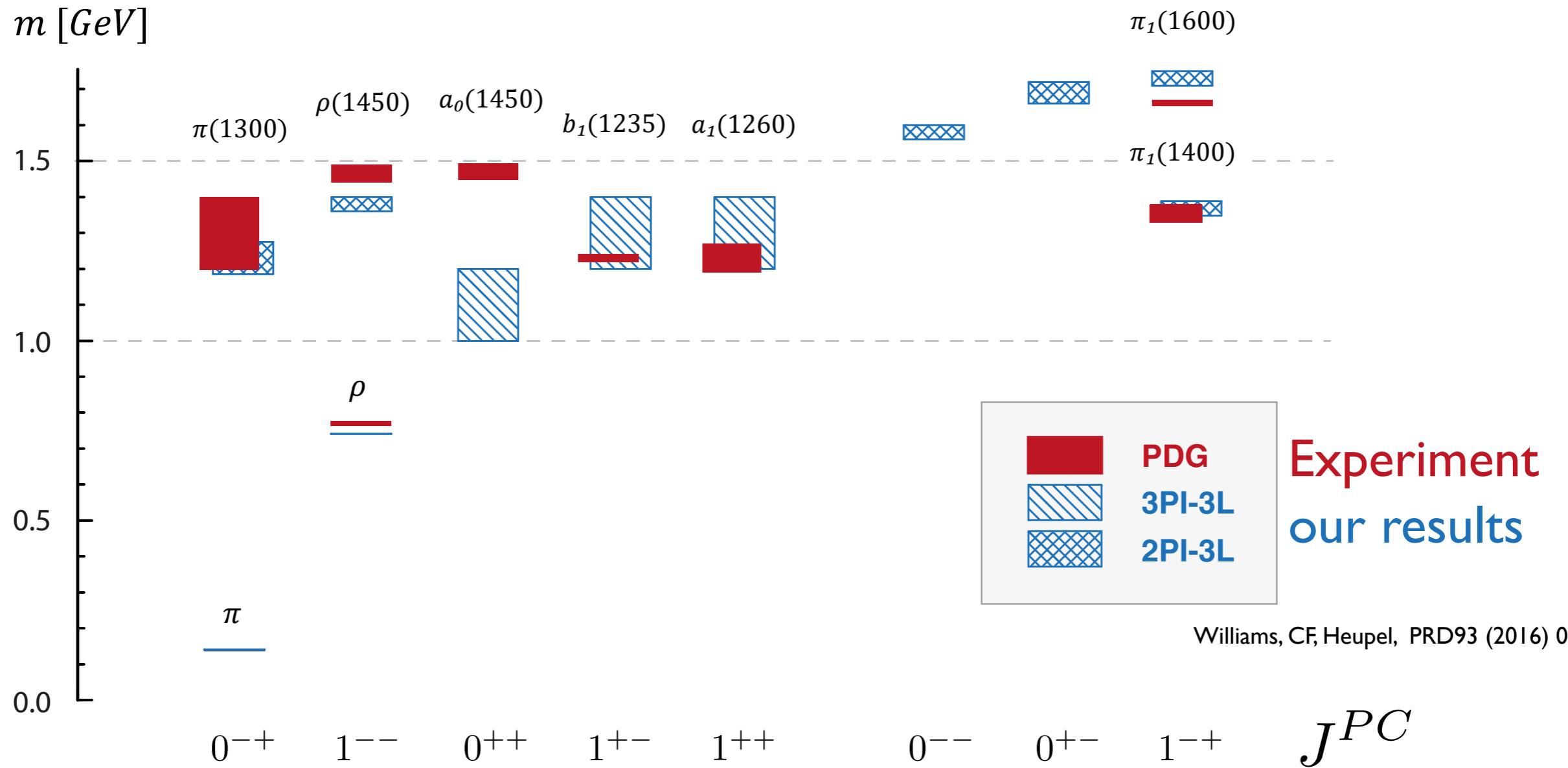
'current quark':
 - small mass; non-composite

Light meson spectrum



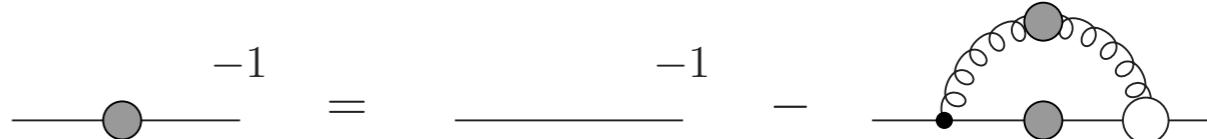
CF, Kubrak, Williams, EPJA (2014) 126

Light meson spectrum

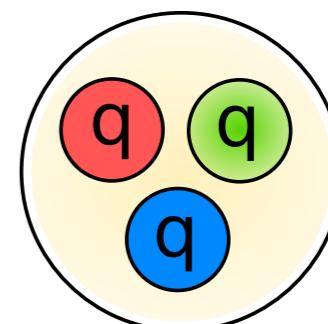


- good agreement with experiment in most channels
- special channels:
 - pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
 - scalar 0^{++} : complicated channel...

I. Mass from nothing

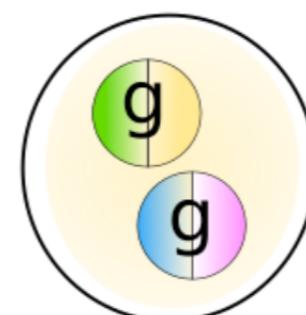


2. Baryons



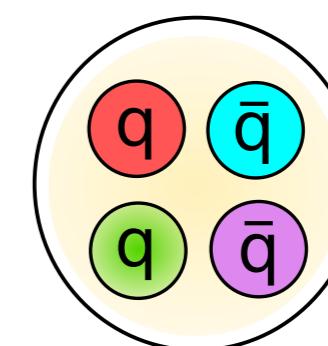
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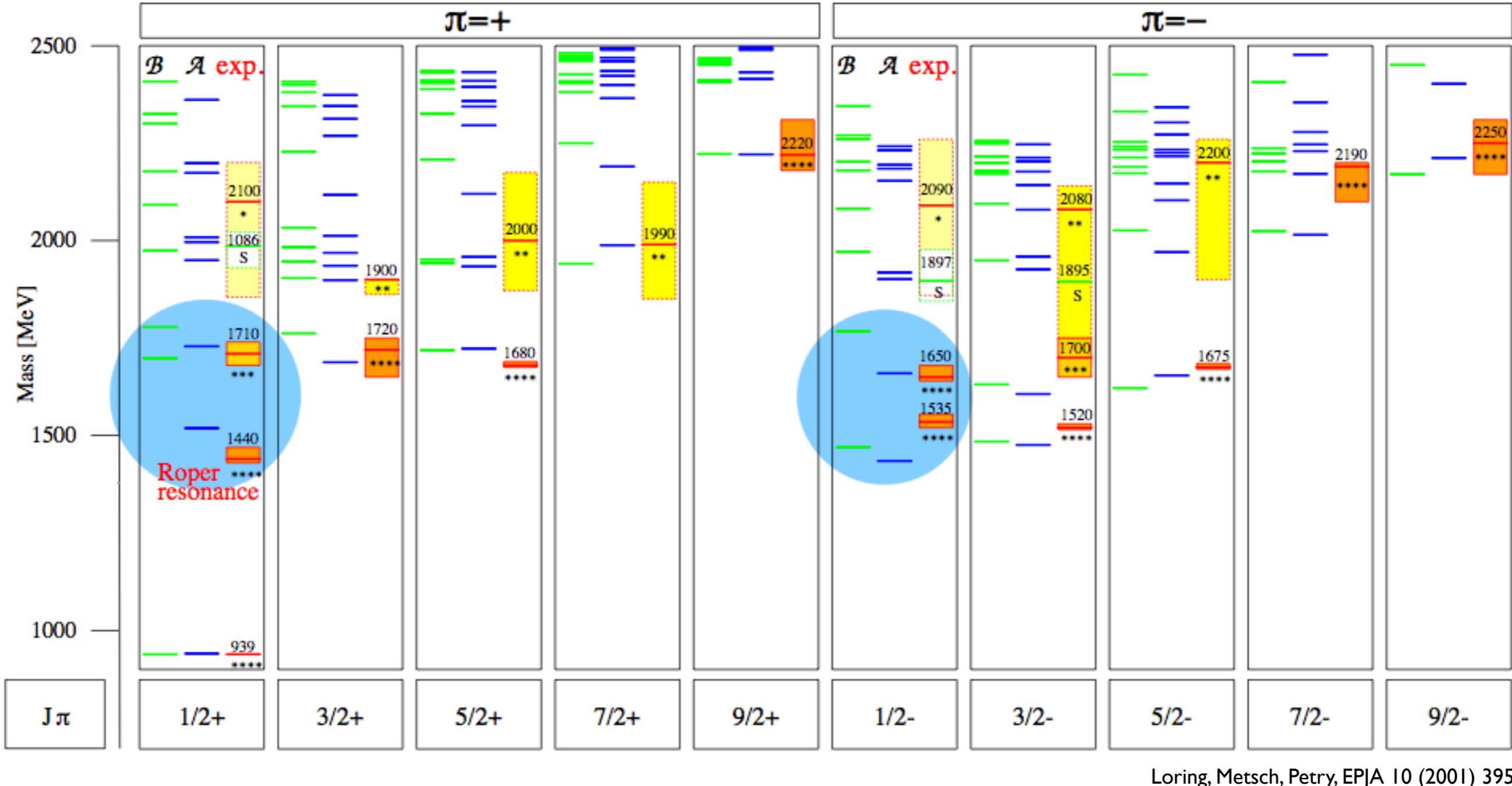
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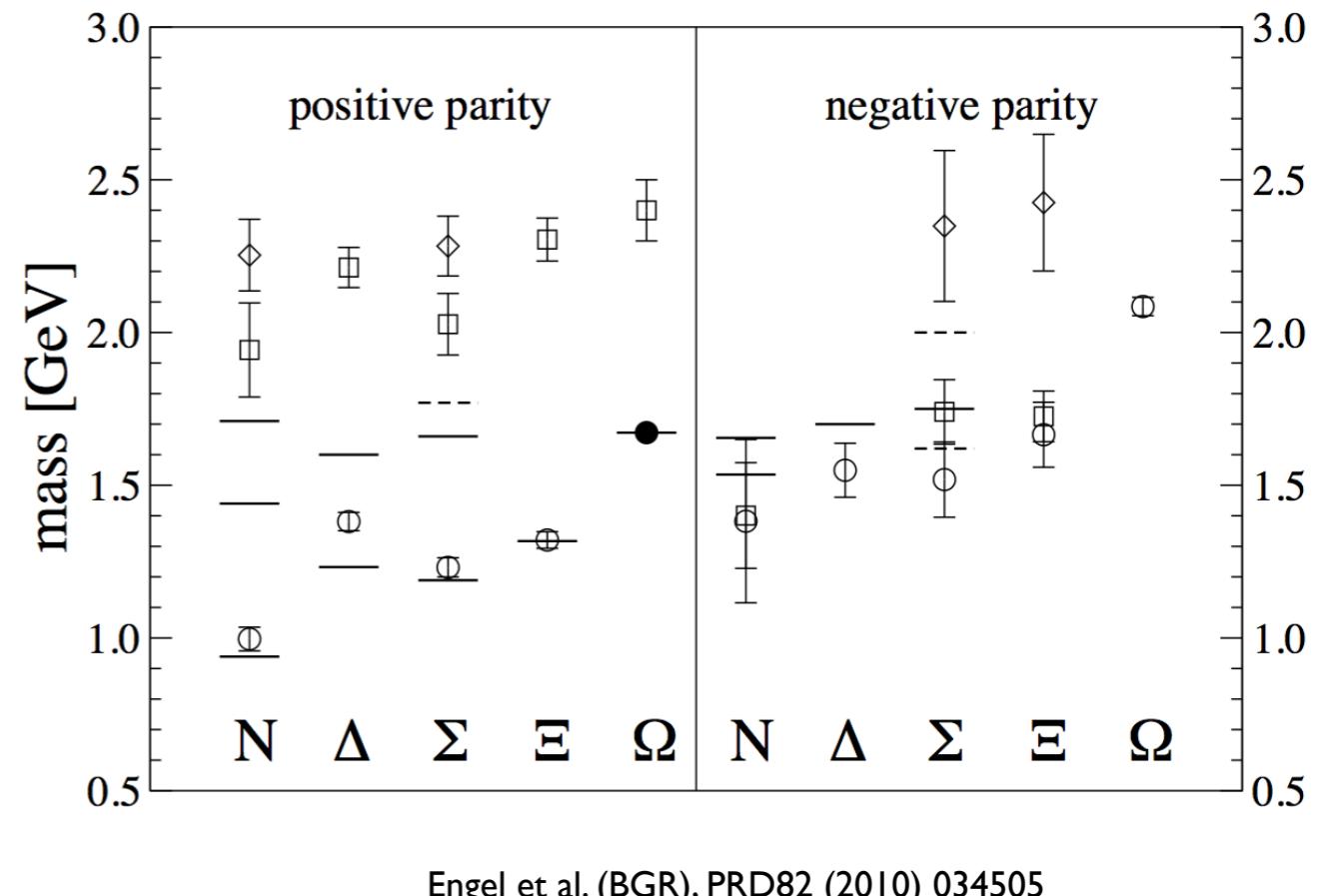
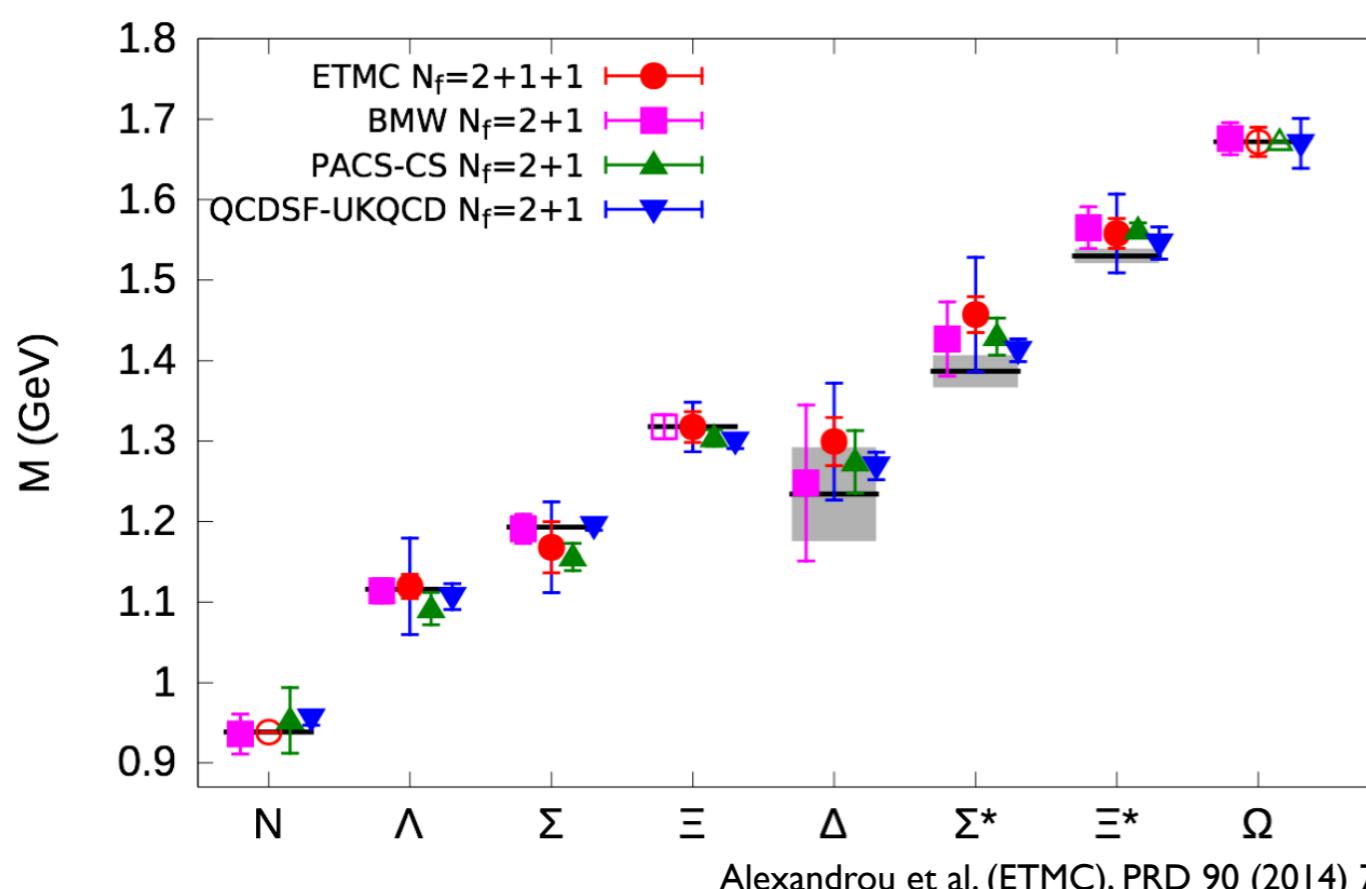
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Light baryon spectrum - quark model



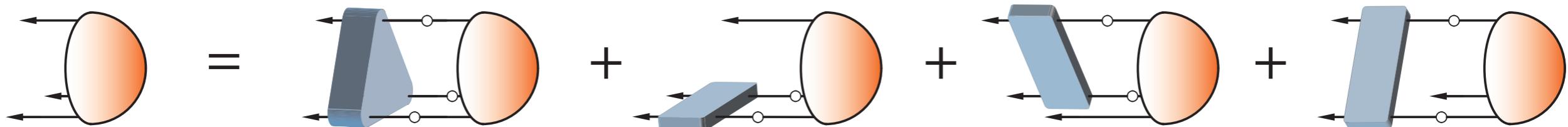
- ‘missing resonances’
- level ordering: $N_{\frac{1}{2}+}$ vs. $N_{\frac{1}{2}-}$

Lattice QCD

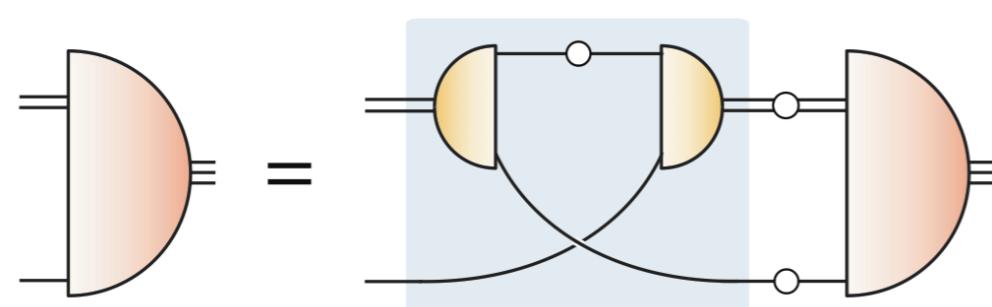


- baryon ground states well under control
- baryon excited states: very tough problem

Faddeev - equation

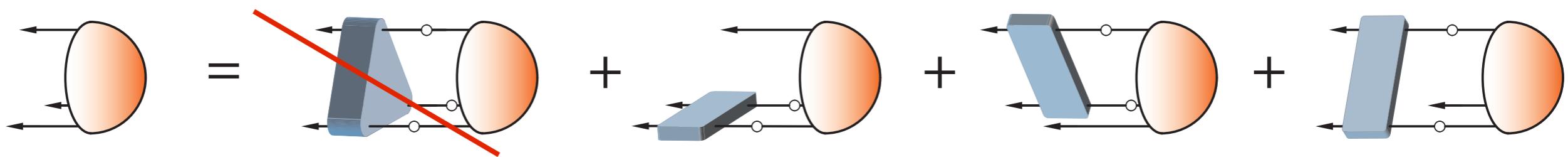


- irreducible three-body forces: may be neglected
- two-body interactions:
 - non-perturbative one-gluon exchange (RL)
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
 - beyond one-gluon exchange
Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)
- quark-diquark approximation
Oettel, Hellstern, Alkofer and Reinhardt, PRC 58 (1998) 2459

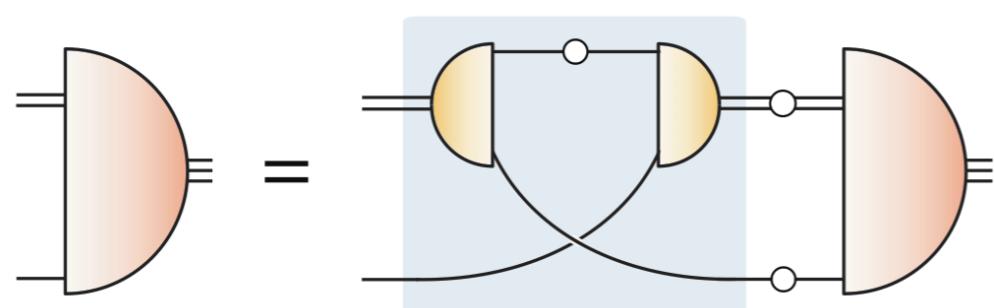


see also: Bloch, Chang, Chen, Cloet, Thomas, Ramalho, Roberts, Segovia et al.

Faddeev - equation

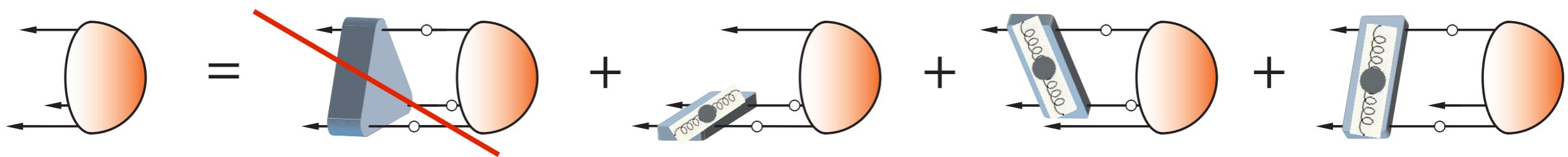


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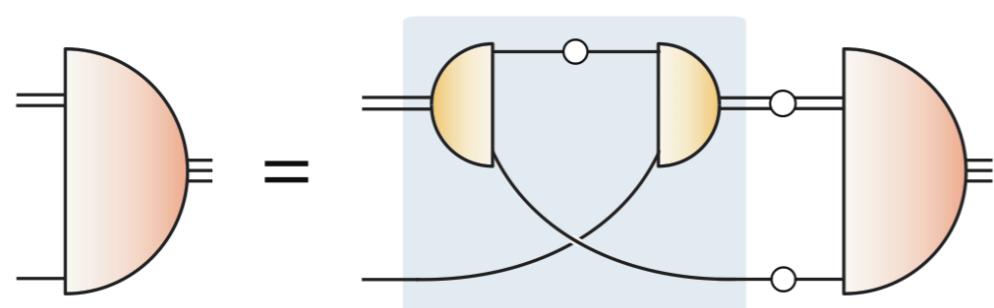


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Faddeev - equation



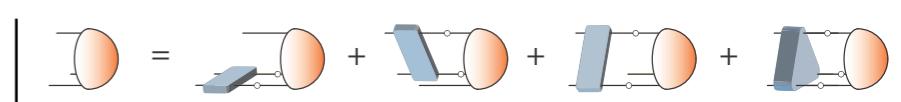
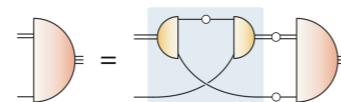
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see also: Bloch, Chang, Chen, Cloet, Thomas, Ramalho, Roberts, Segovia et al.

DSE/BSE/Faddeev landscape (2015)

level of complexity



	I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)	IV) DSE (bRL)
up/down	N, Δ masses	✓	✓	✓
	N, Δ em. FFs	✓	✓	✓
	$N \rightarrow \Delta\gamma$	✓	✓	
$P = +$	N^*, Δ^* masses	✓	✓	
	$\gamma N \rightarrow N^*/\Delta^*$	✓	✓	
$P = -$	N^*, Δ^* masses		✓	
	$\gamma N \rightarrow N^*/\Delta^*$			
strange	ground states			
	excited states	✓		
	em. FF			
	TFFs			
c/b	ground states			
	excited states			

Cloet, Thomas,
Roberts, Segovia,
Chen, et al.

Oettel, Alkofer, Bloch,
Roberts, Segovia, Chen, et al.

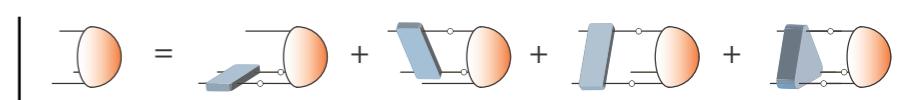
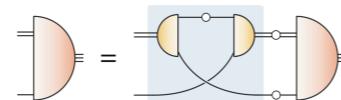
Eichmann, Alkofer,
Kraßnigg, Nicmorus,
Sanchis-Alepuz, CF

Eichmann, Alkofer,
Sanchis-Alepuz, CF,
Qin, Roberts

Sanchis-Alepuz,
Williams, CF

DSE/BSE/Faddeev landscape

level of complexity



	I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)	IV) DSE (bRL)
up/down	N, Δ masses	✓	✓	✓
	N, Δ em. FFs	✓	✓	✓
	$N \rightarrow \Delta\gamma$	✓	✓	✓
$P = +$	N^*, Δ^* masses	✓	✓	✓
	$\gamma N \rightarrow N^*/\Delta^*$	✓	✓	
$P = -$	N^*, Δ^* masses	✓	✓	✓
	$\gamma N \rightarrow N^*/\Delta^*$			
strange	ground states	✓	✓	✓
	excited states	✓	✓	✓
	em. FF		✓	✓
	TFFs			✓
c/b	ground states	✓		✓
	excited states		✓	✓

Cloet, Thomas,
Roberts, Segovia,
Chen, et al.

Oettel, Alkofer, Bloch,
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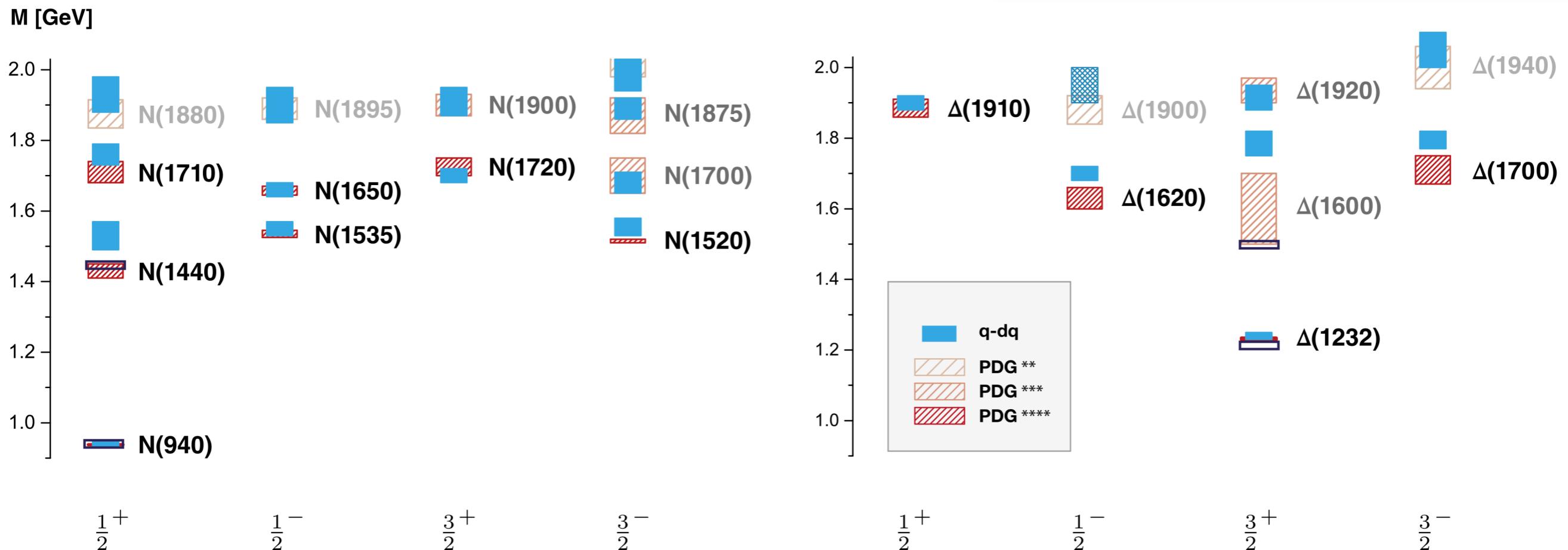
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Sanchis-Alepuz, CF,
Qin, Roberts

Sanchis-Alepuz,
Williams, CF

Light baryon spectrum:

- 3 parameters + $m_{u,d,s}$
(all fixed in meson sector)



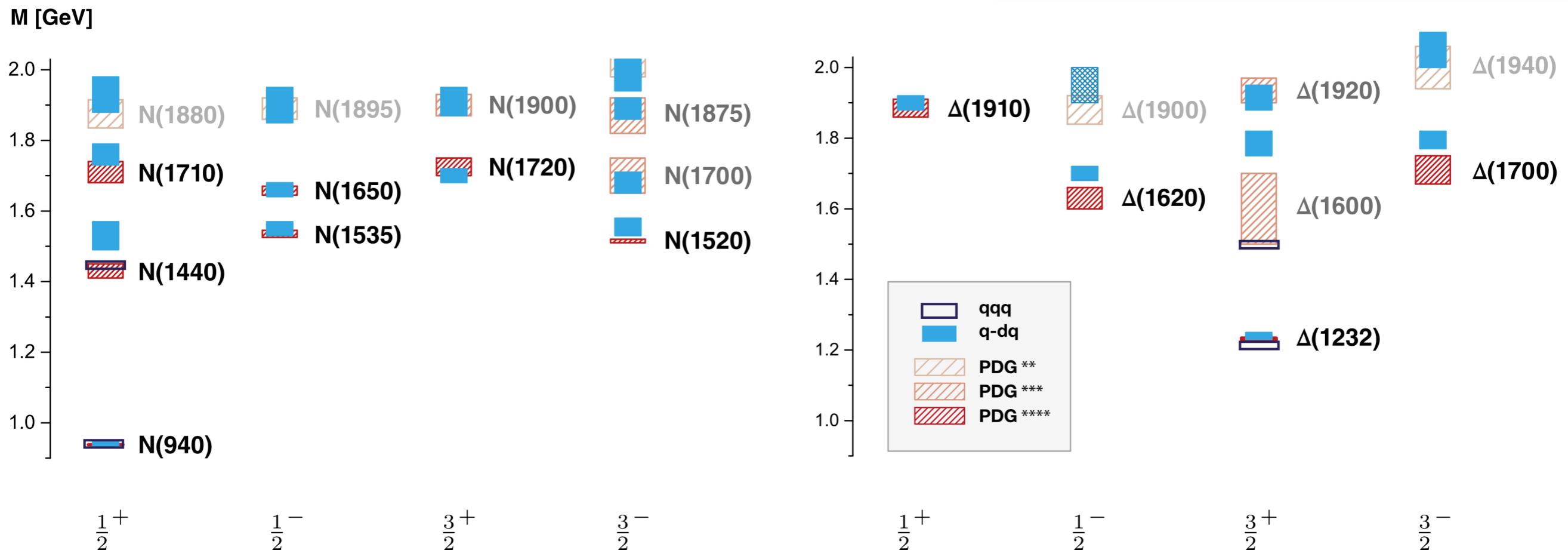
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects...)

!

Light baryon spectrum:

- 3 parameters + $m_{u,d,s}$
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Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects...)
- three-body agrees with diquark-quark where applicable

Relativistic proton

$$J^P = \left(\frac{1}{2}\right)^+$$

non-relativistic

three quarks with spin 1/2:

$S = 1/2$ or $S = 3/2$

parity $P = (-1)^L$:

$L = 0$ or $L = 2$

relativistic

64 components in wave function: 8 s-wave ($L=0$)

36 p-wave ($L=1$)

20 d-wave ($L=2$)

$$P = (-1)^L$$

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	—	—	< 0.5	16

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

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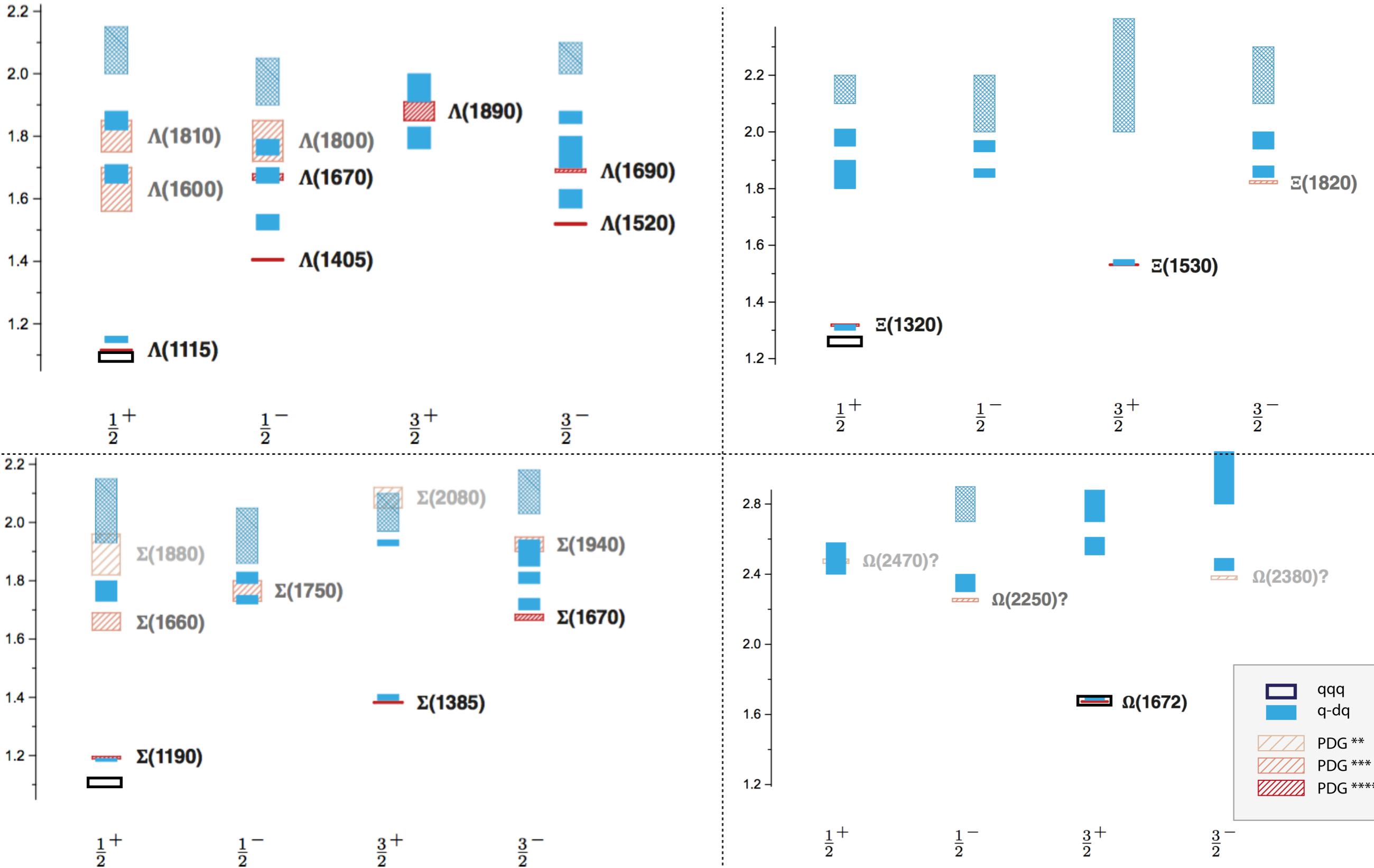
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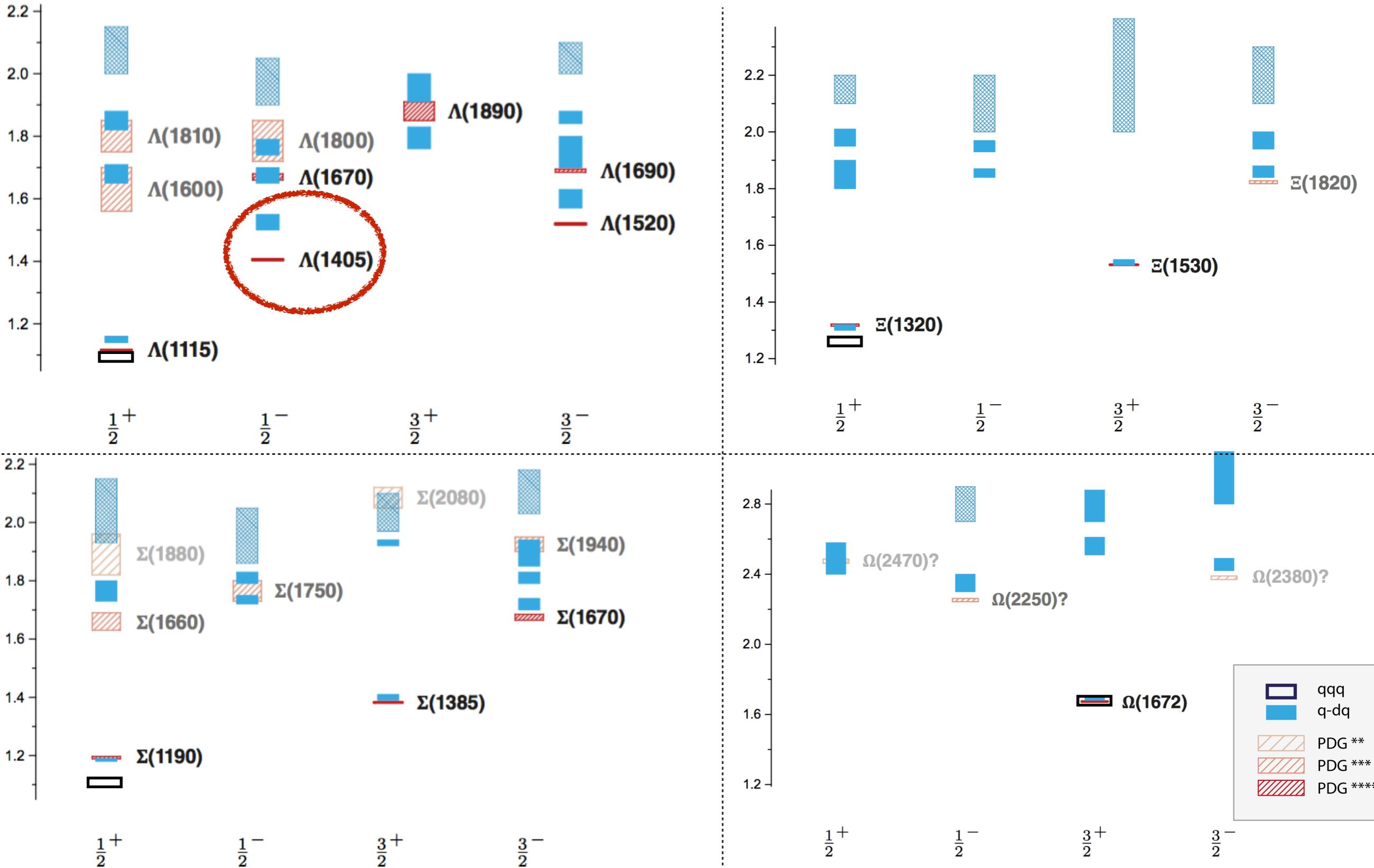
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

Strange baryon spectrum: DSE-RL (preliminary !)



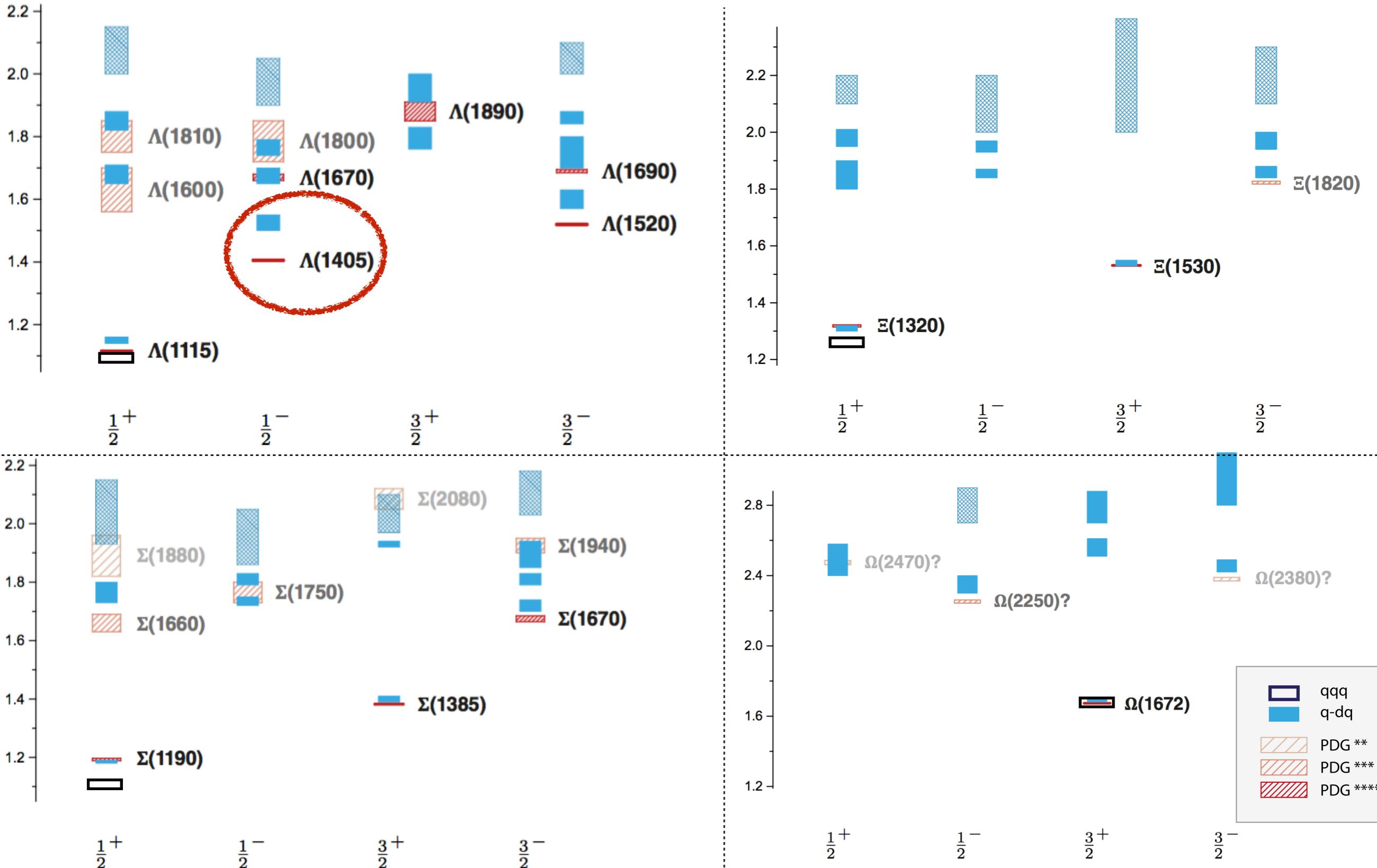
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 CF, Eichmann PoS Hadron 2017 (2018) 007
 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Strange baryon spectrum: DSE-RL (preliminary !)



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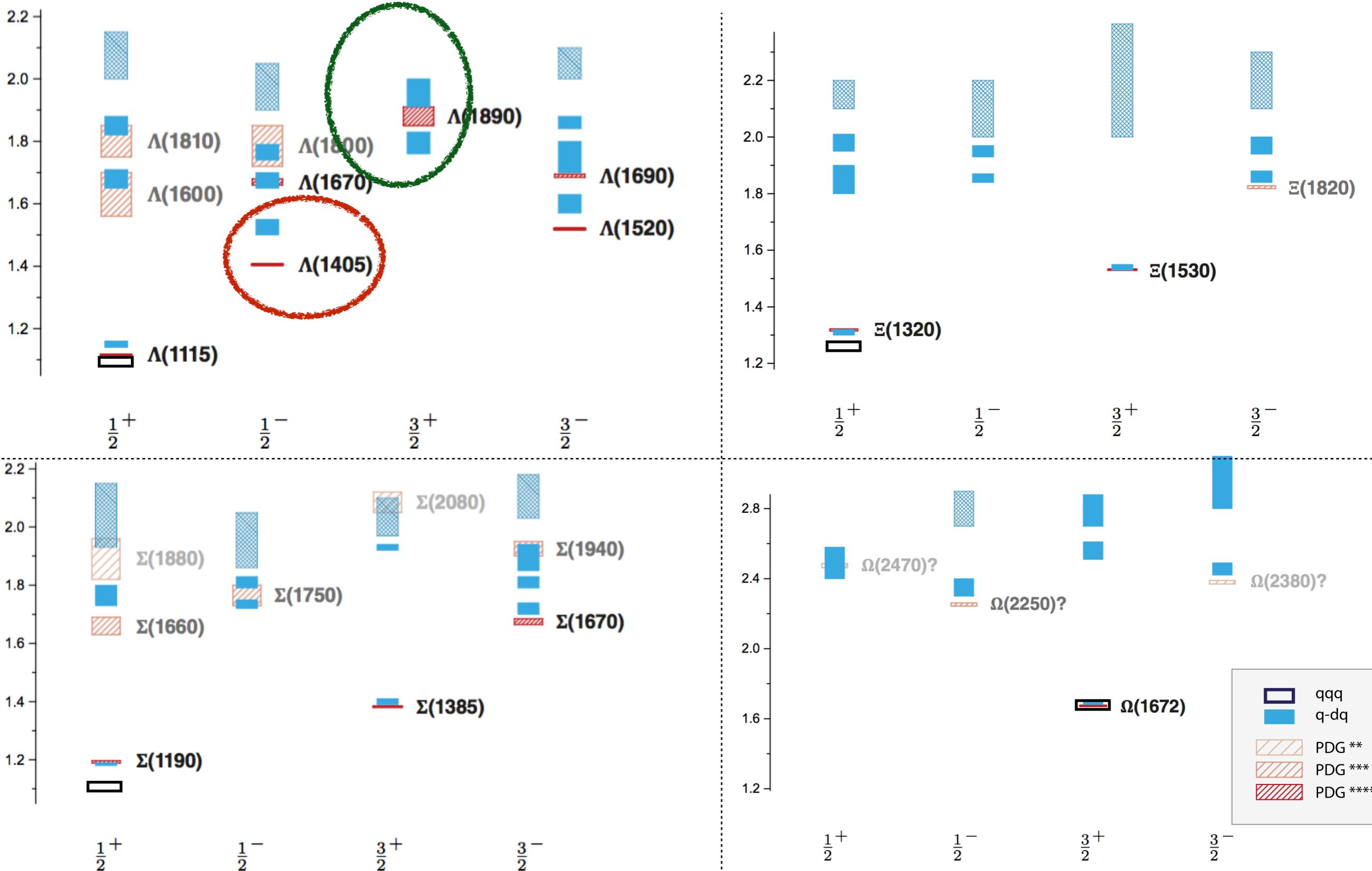
Strange baryon spectrum: DSE-RL (preliminary !)



New states: Bonn-Gatchina (talk of M. Matveev at N*2019)

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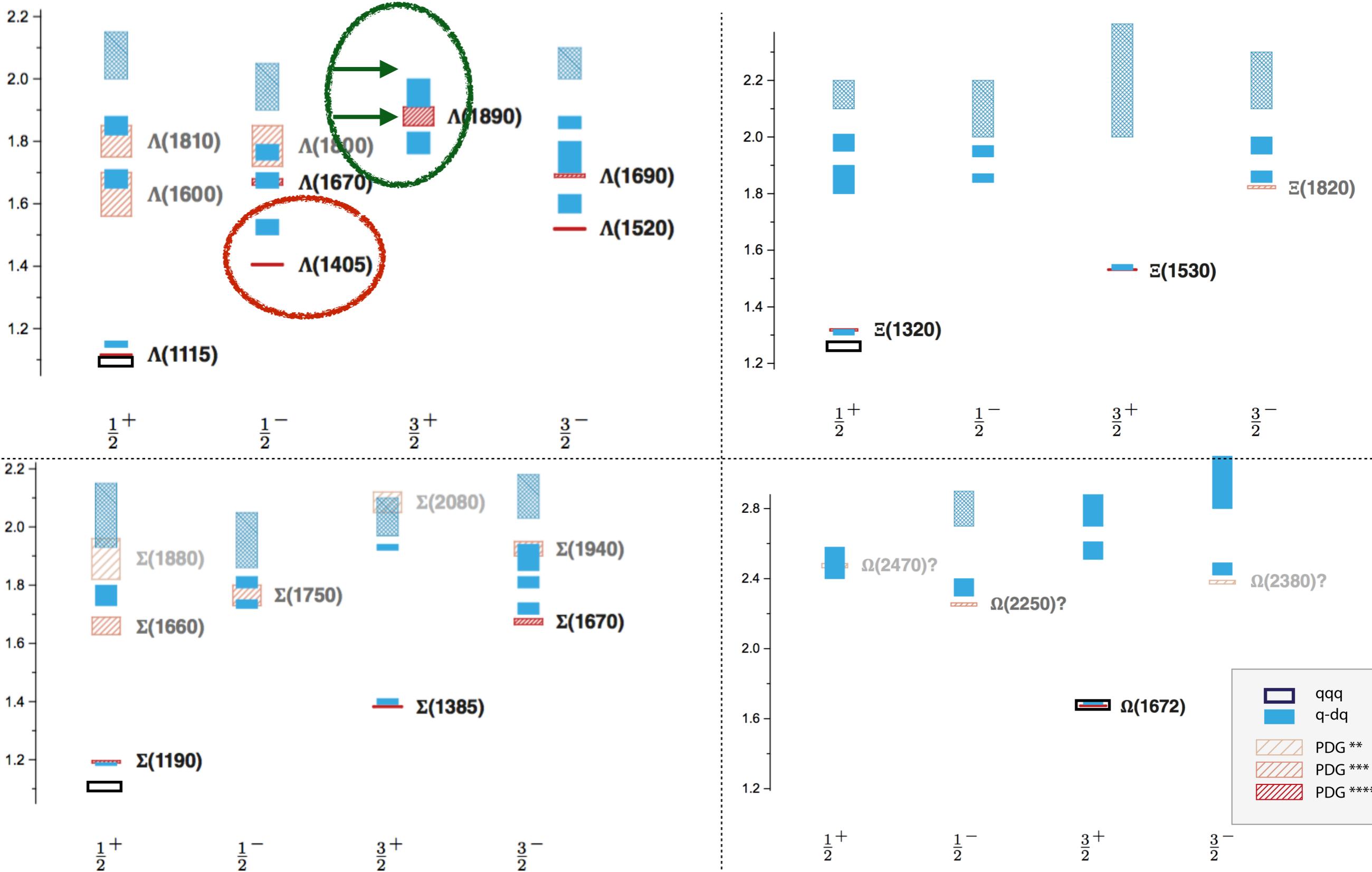
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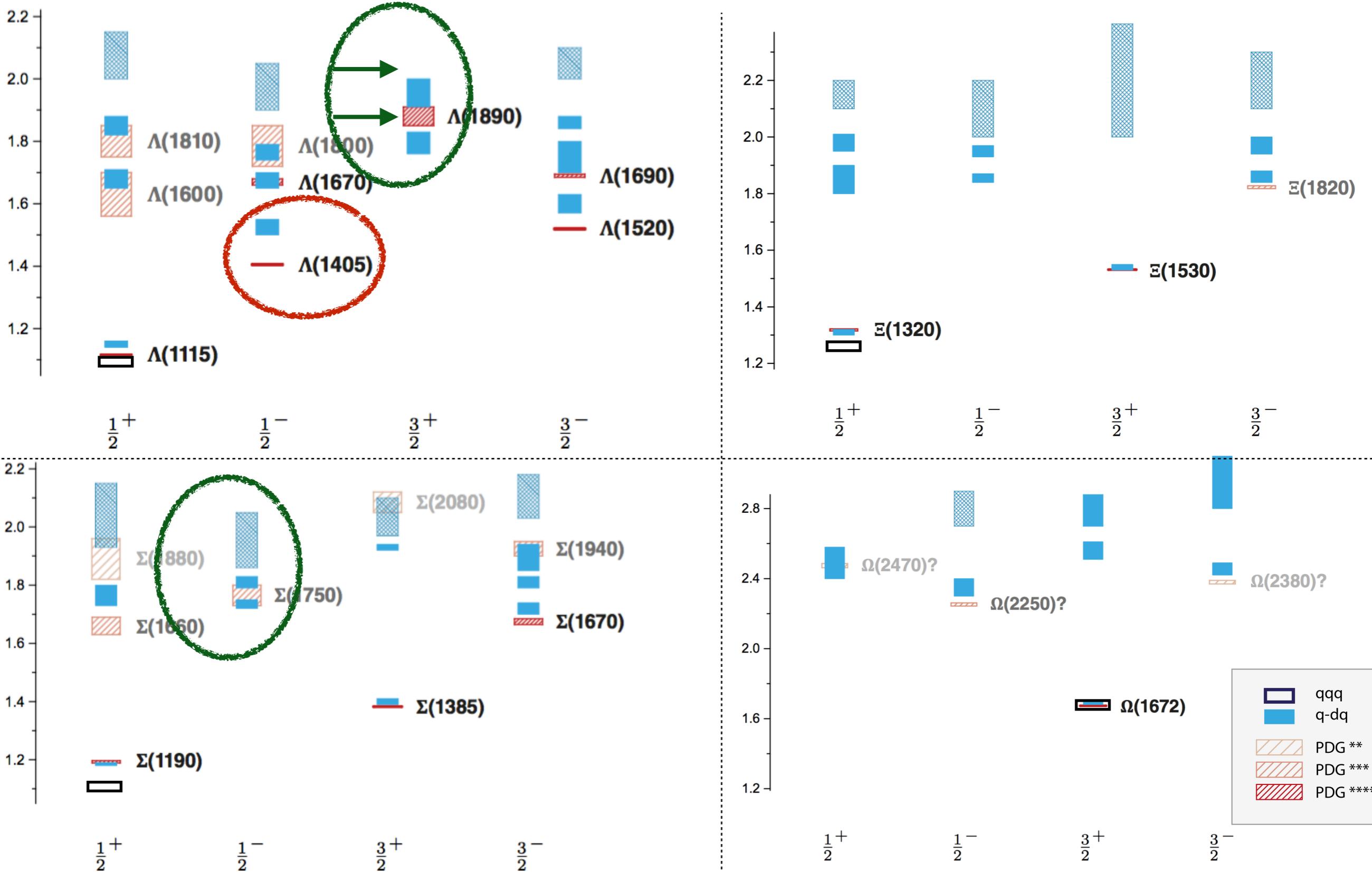
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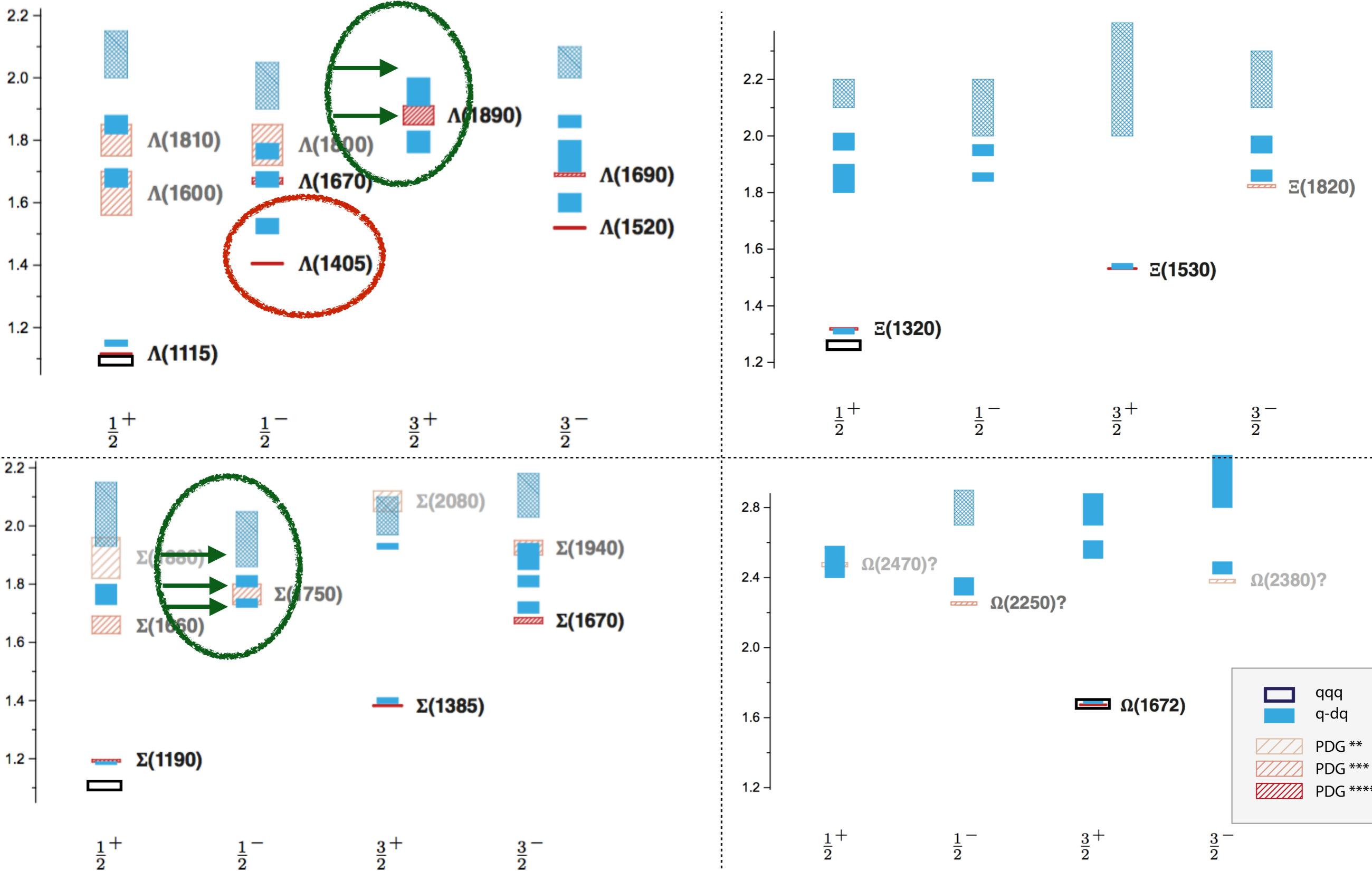
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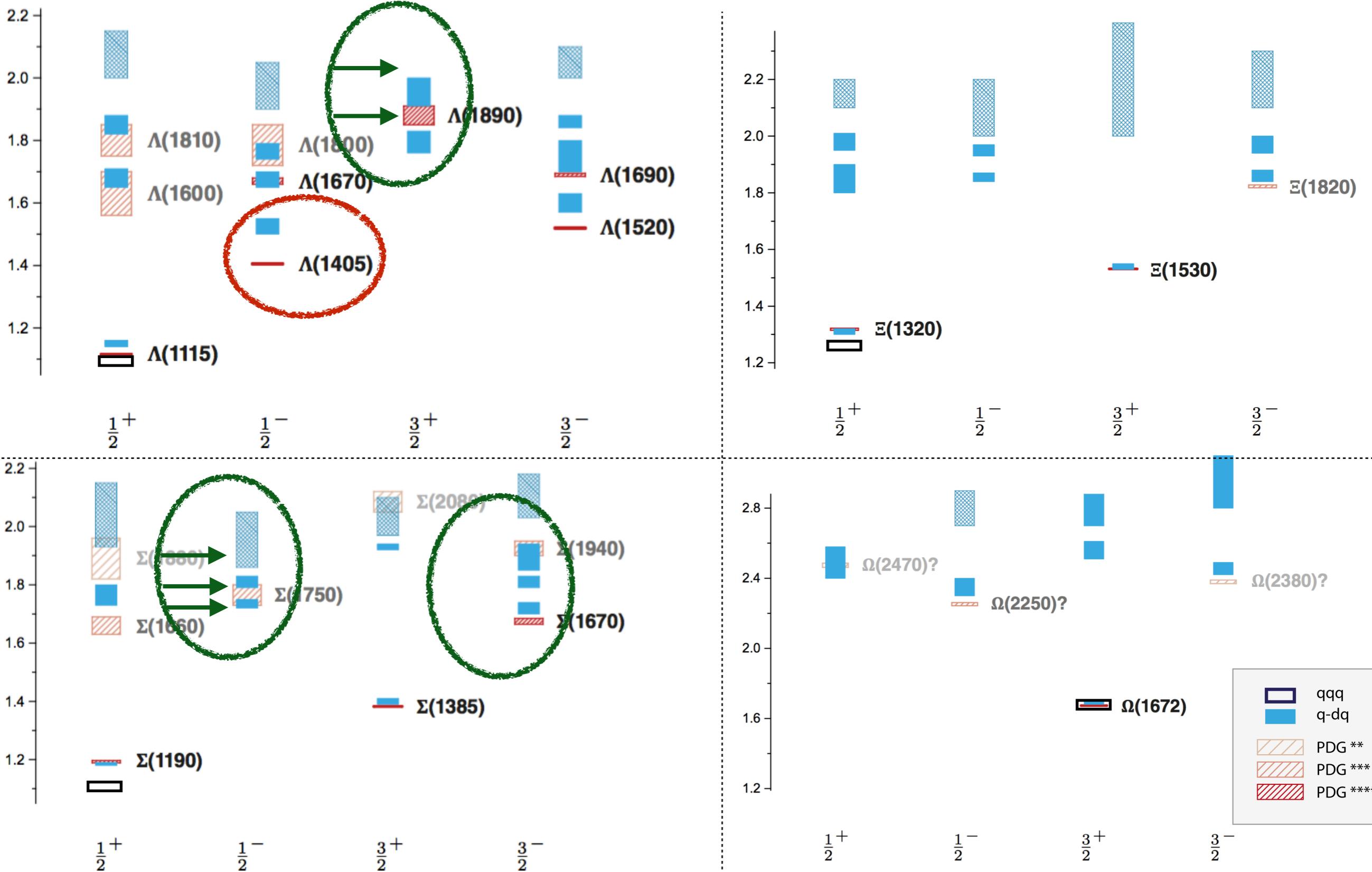
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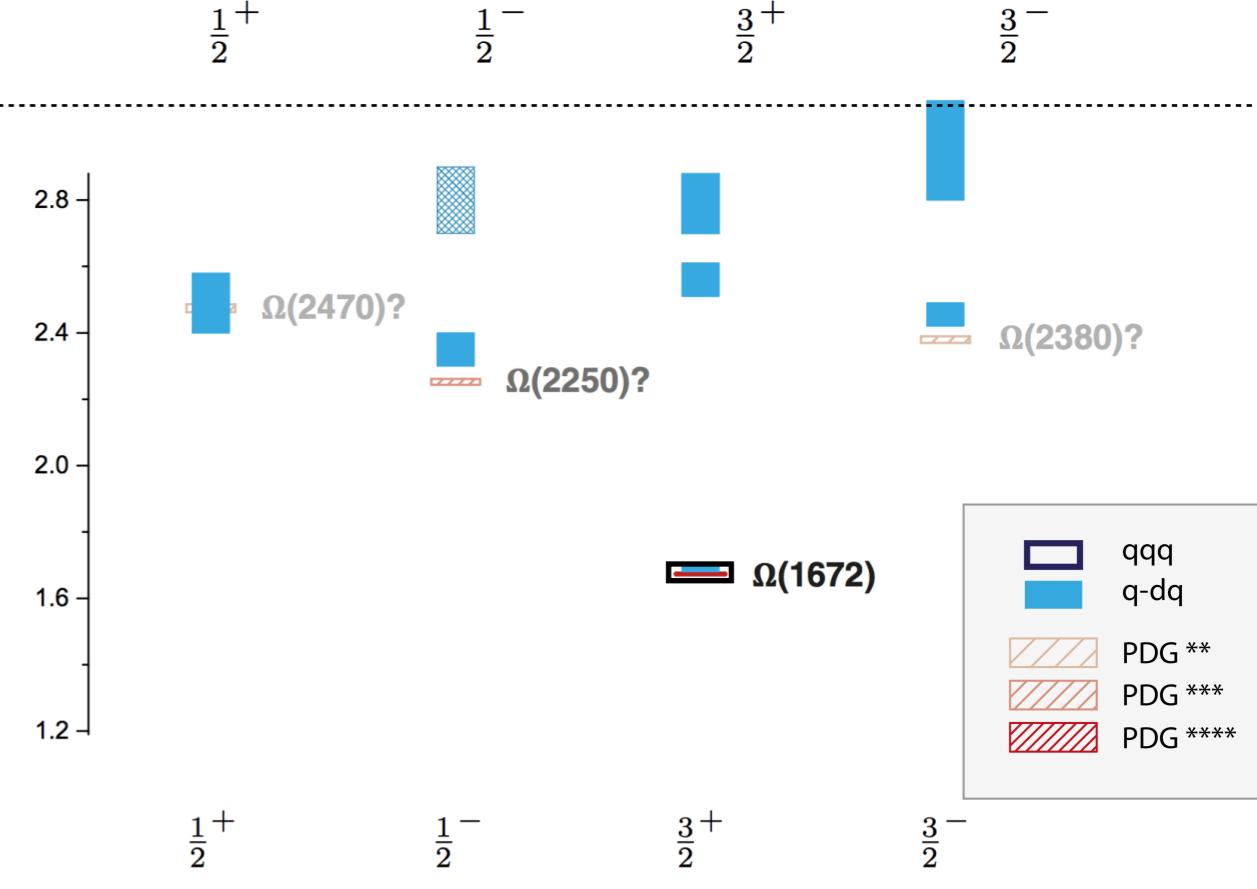
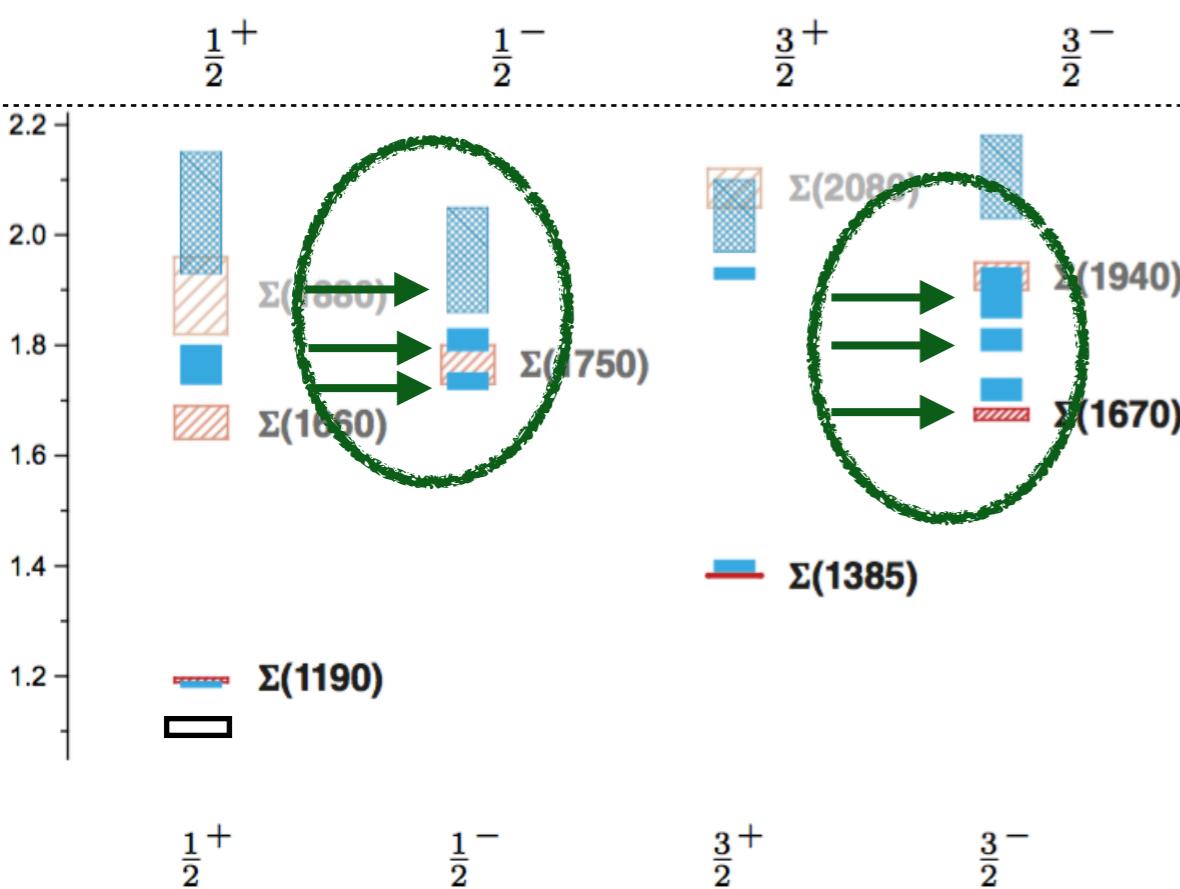
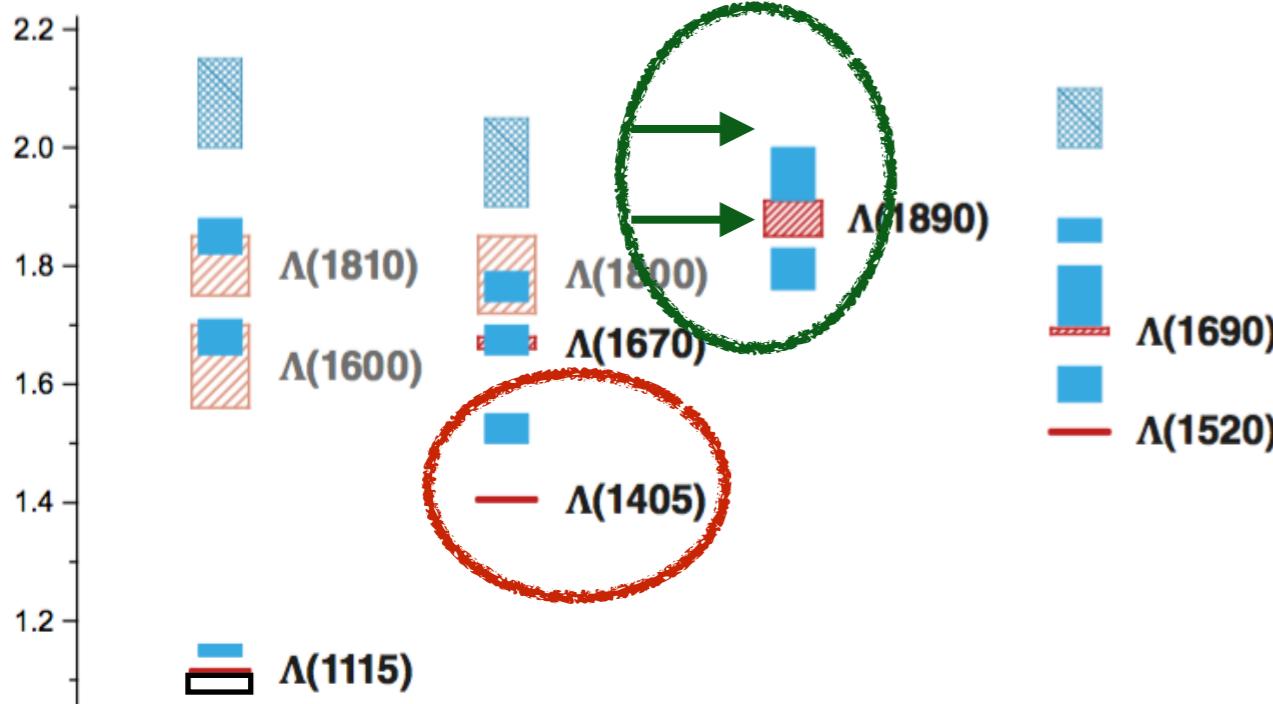
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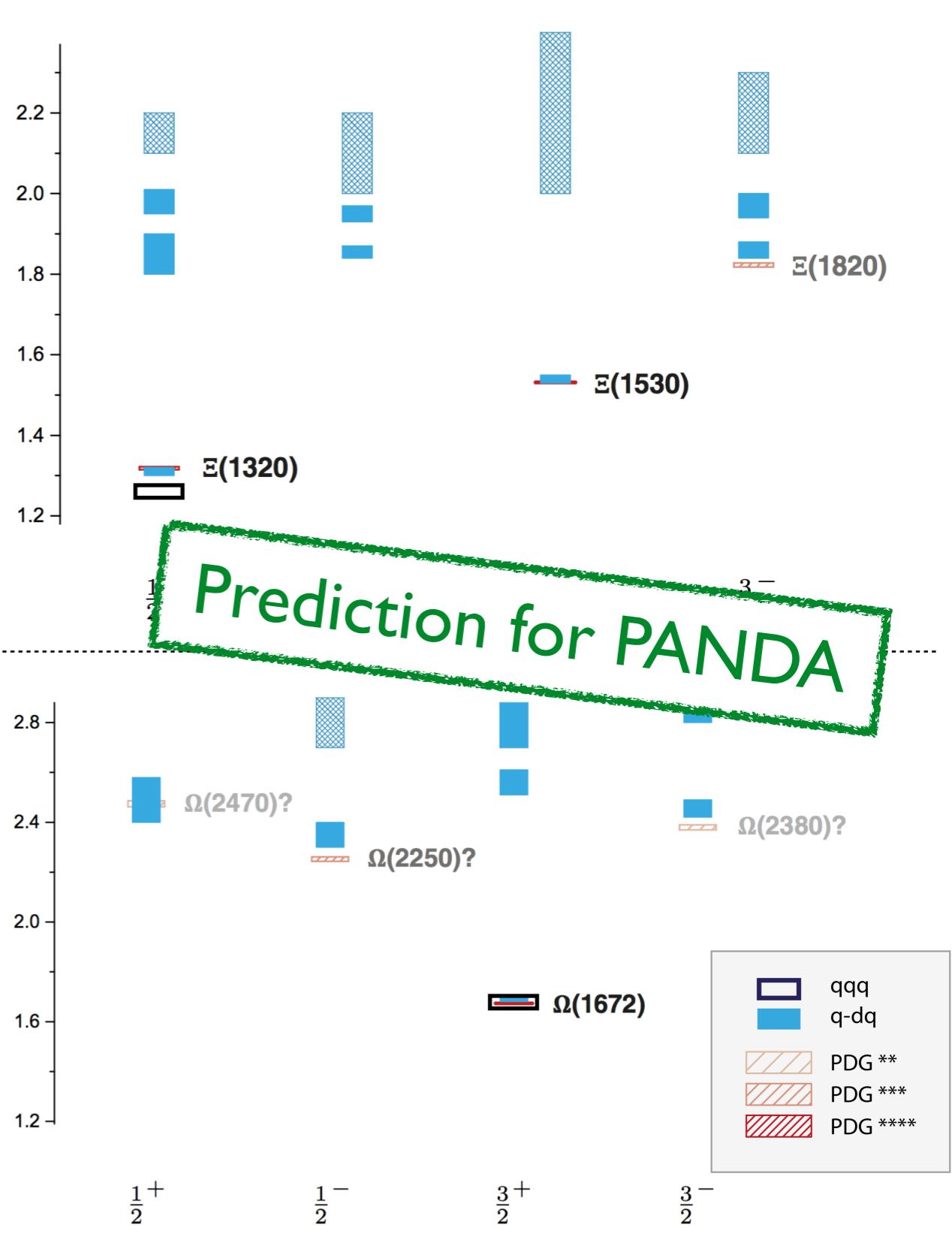
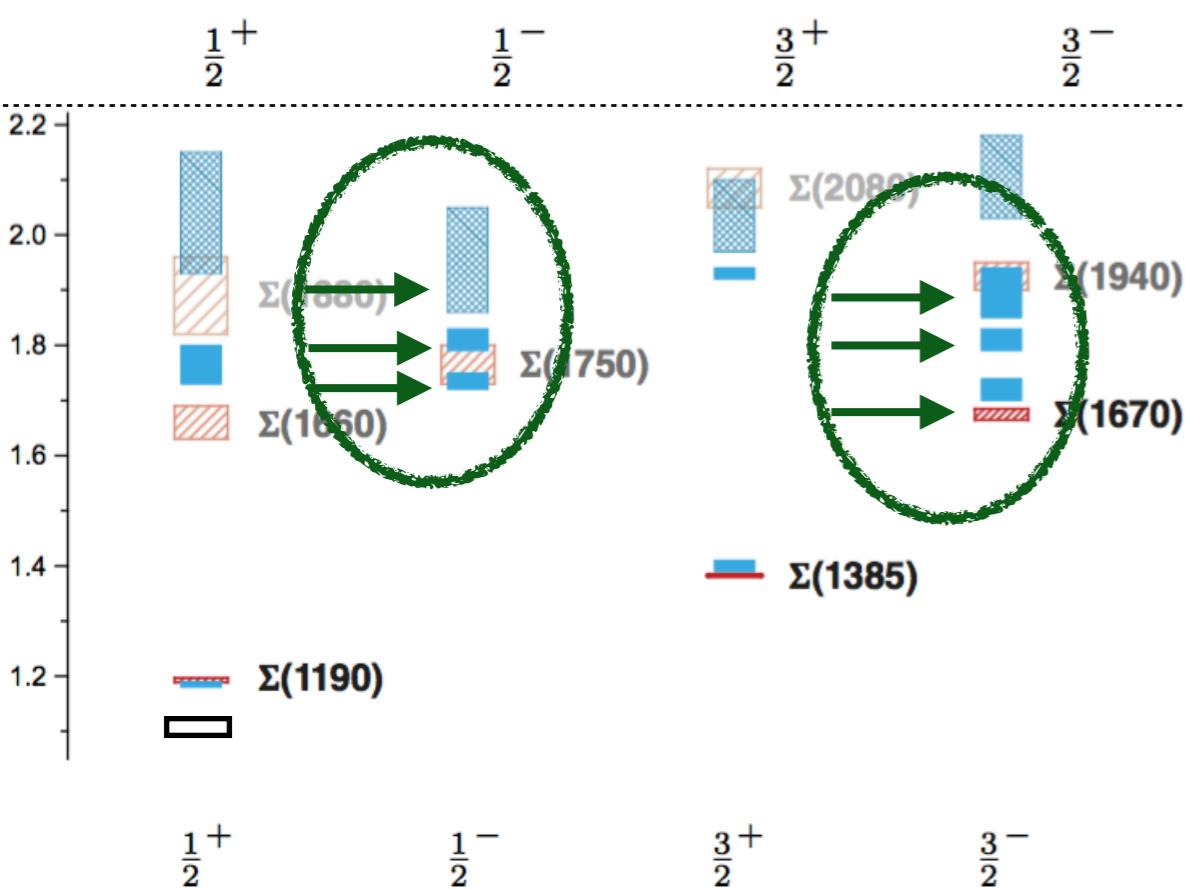
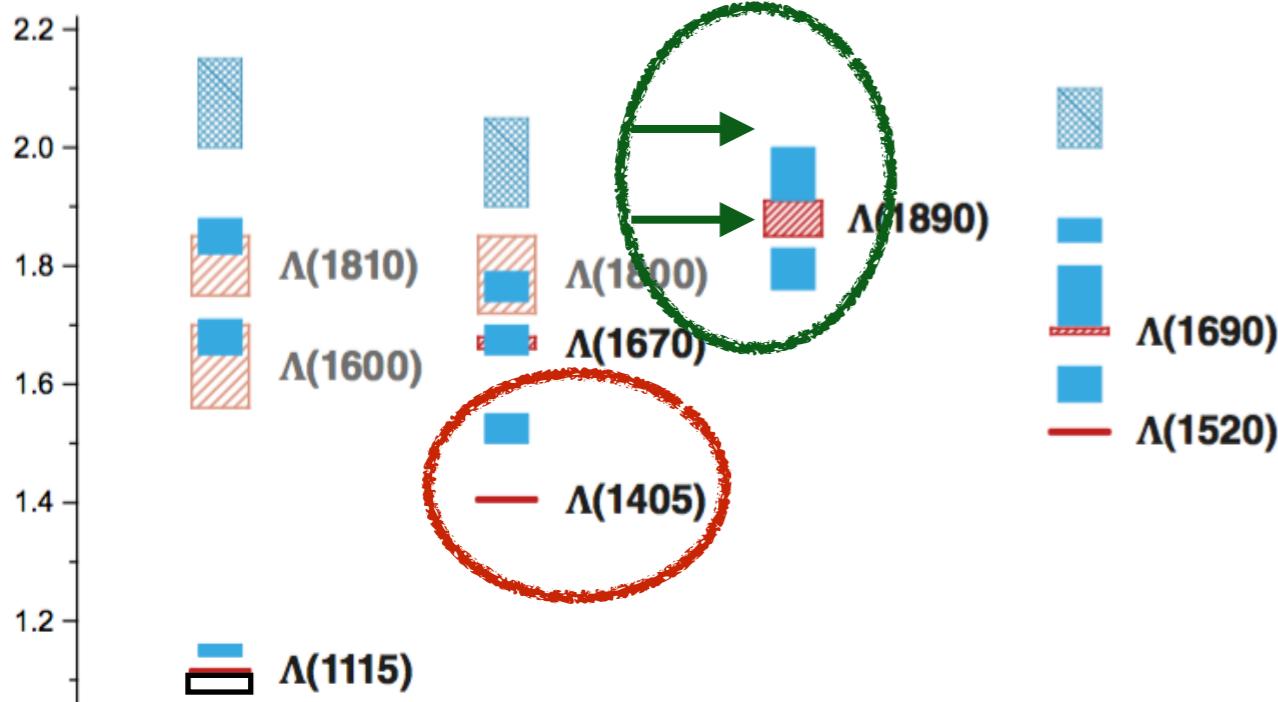
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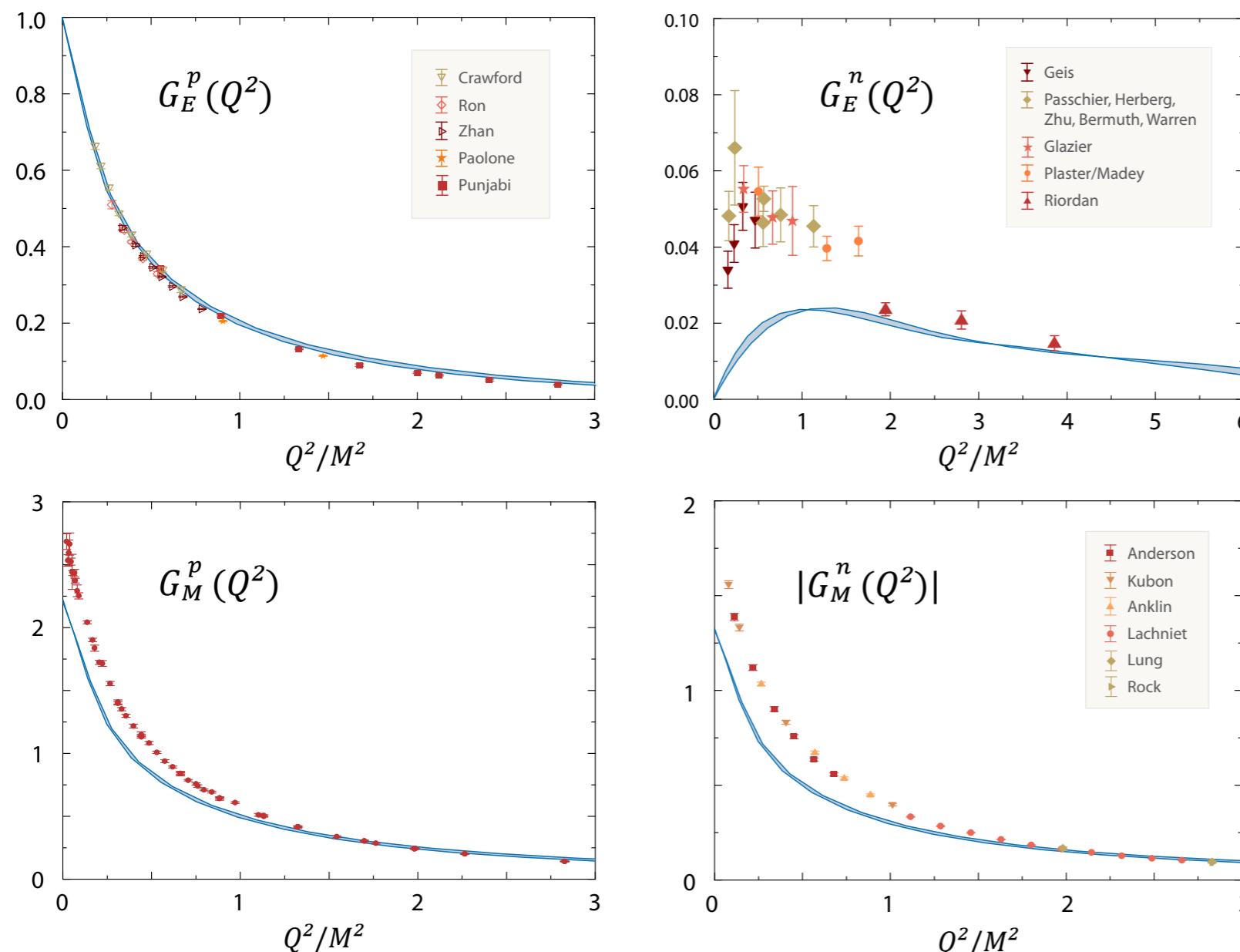
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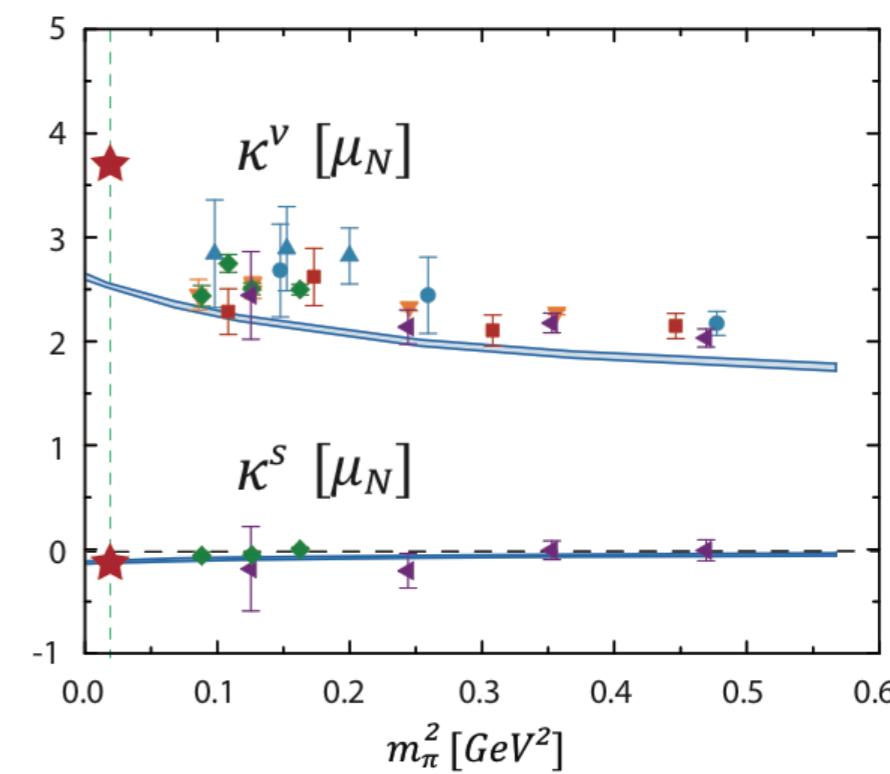
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CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Nucleon form factors and magnetic moments



Isovector (p-n), isoscalar (p+n):



- missing pion cloud effects
- similar for axial form factors

Eichmann, PRD 84 (2011)

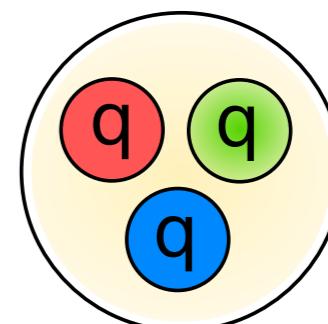
Eichmann and CF, EPJ A48 (2012) 9

latest results: see Chen, CF, Roberts, Segovia, in preparation

I. Mass from nothing

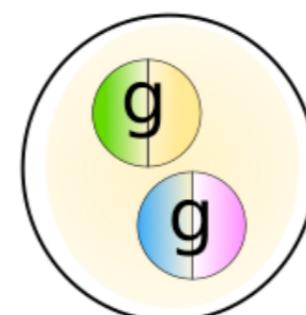


2. Baryons



Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3. Glueballs



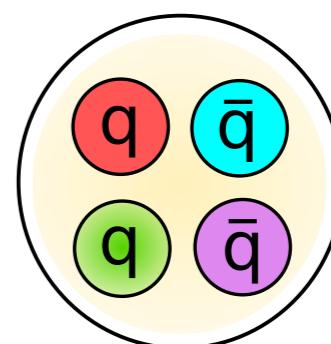
CF, Huber, Sanchis-Alepuz, EPJC 80 11, 1077 (2020), arXiv:2004.00415

4.(Heavy-light) Tetraquarks

Wallbott, Eichmann and CF, PRD 100 (2019) no.1, 014033, arXiv:1905.02615

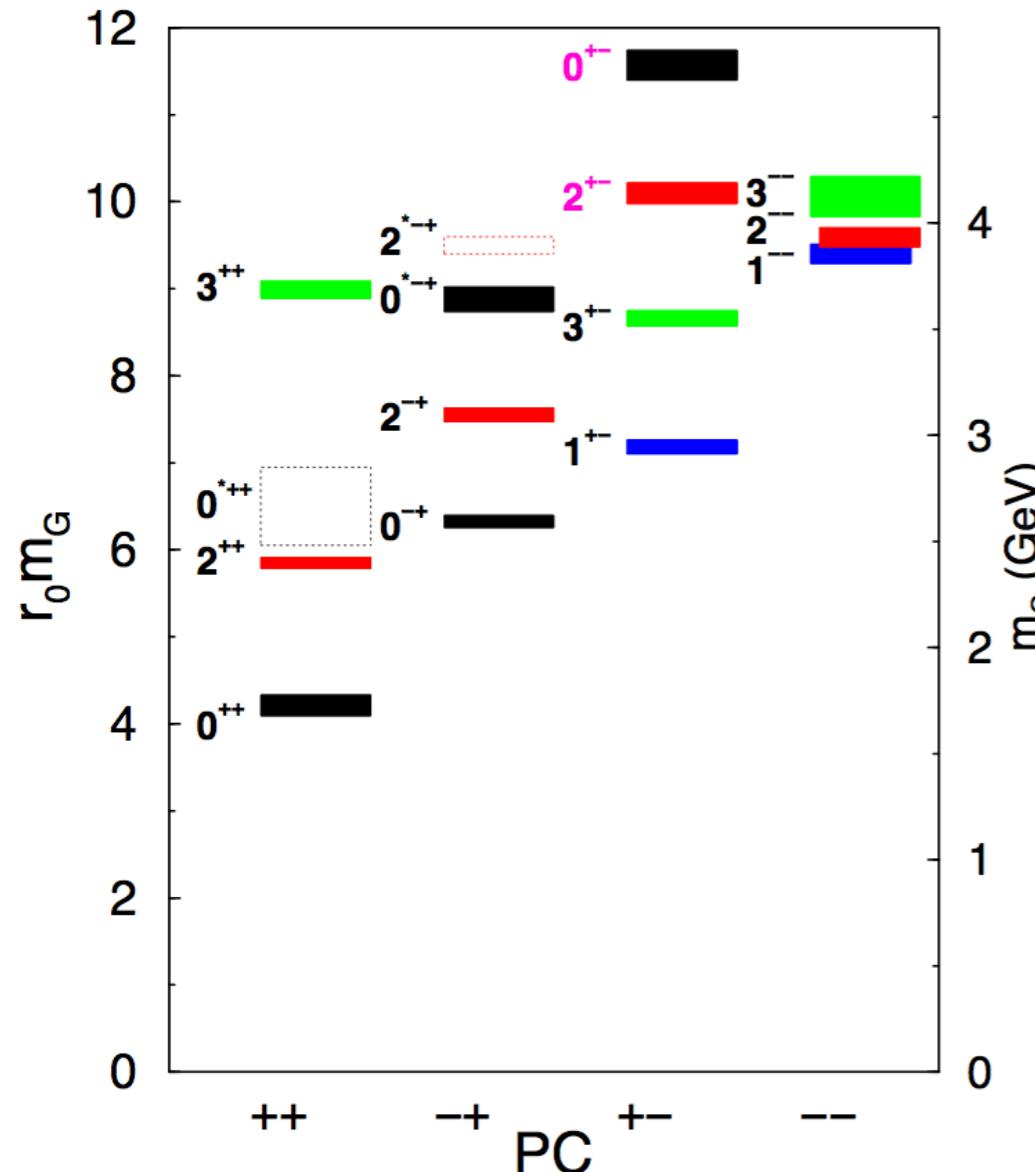
Wallbott, Eichmann and CF, PRD 102 (2020) no.5, 051501, arXiv:2003.12407

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495



Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4,38, arXiv:2008.10240

Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509

Y.-Chen et al., PRD 73 (2006) 014516

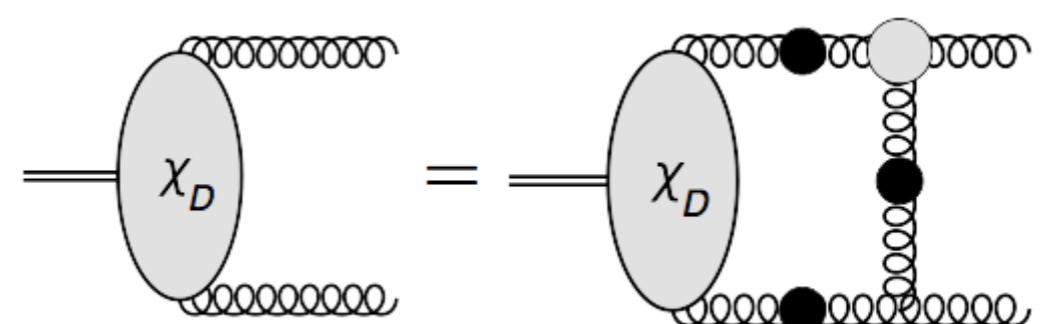
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Unquenched calculations very involved

Gregory et al., JHEP 1210 (2012) 170

DSE:

- structural information



Meyers, Swanson, PRD 87 (2013) 3, 036009

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Landau gauge - 3PI truncation

$$\text{Diagram with a black dot and a label } -1 = \text{Diagram with a black dot and a label } -1 + \frac{1}{2} \text{Diagram with a black dot and a label } -\frac{1}{2} + \text{Diagram with a black dot and a label } -\frac{1}{2} + \text{Diagram with a black dot and a label } -\frac{1}{2} + \text{Diagram with a black dot and a label } -\frac{1}{6}$$

Huber, PRD 101 (2020) 114009, arXiv:2003.13703

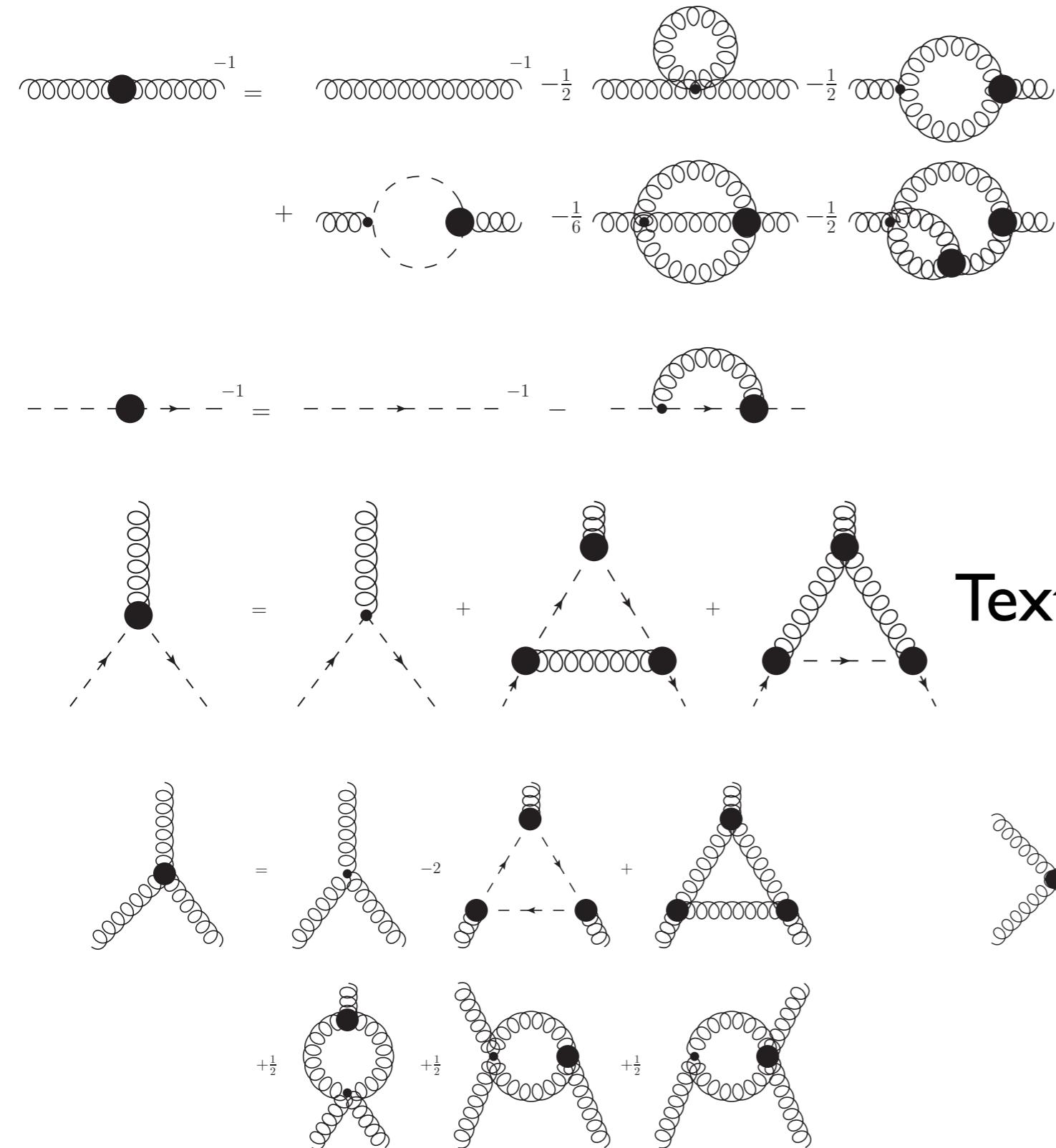
Landau gauge - 3PI truncation

$$\begin{aligned} \text{---} \bullet \text{---}^{-1} &= \text{---} \bullet \text{---}^{-1} \text{---}^{-\frac{1}{2}} \text{---} \bullet \text{---}^{-\frac{1}{2}} \text{---}^{-1} \\ &+ \text{---} \bullet \text{---}^{-\frac{1}{6}} \text{---}^{-\frac{1}{2}} \text{---}^{-\frac{1}{2}} \text{---}^{-1} \\ \text{---} \bullet \text{---}^{-1} &= \text{---} \bullet \text{---}^{-1} \text{---} \end{aligned}$$

The diagram shows the Landau gauge 3PI truncation. It consists of two main parts. The first part is a loop with a dot at the top-left vertex and a black dot at the bottom-right vertex. The second part is a loop with a dot at the top-left vertex and a black dot at the bottom-right vertex, plus a correction term involving a dashed loop with a dot at its center and a black dot at its right vertex.

Huber, PRD 101 (2020) 114009, arXiv:2003.13703

Landau gauge - 3PI truncation



Text

Huber, PRD 101 (2020) 114009, arXiv:2003.13703

Landau gauge gluon propagator

$$\text{Diagrammatic representation of the Landau gauge gluon propagator}$$

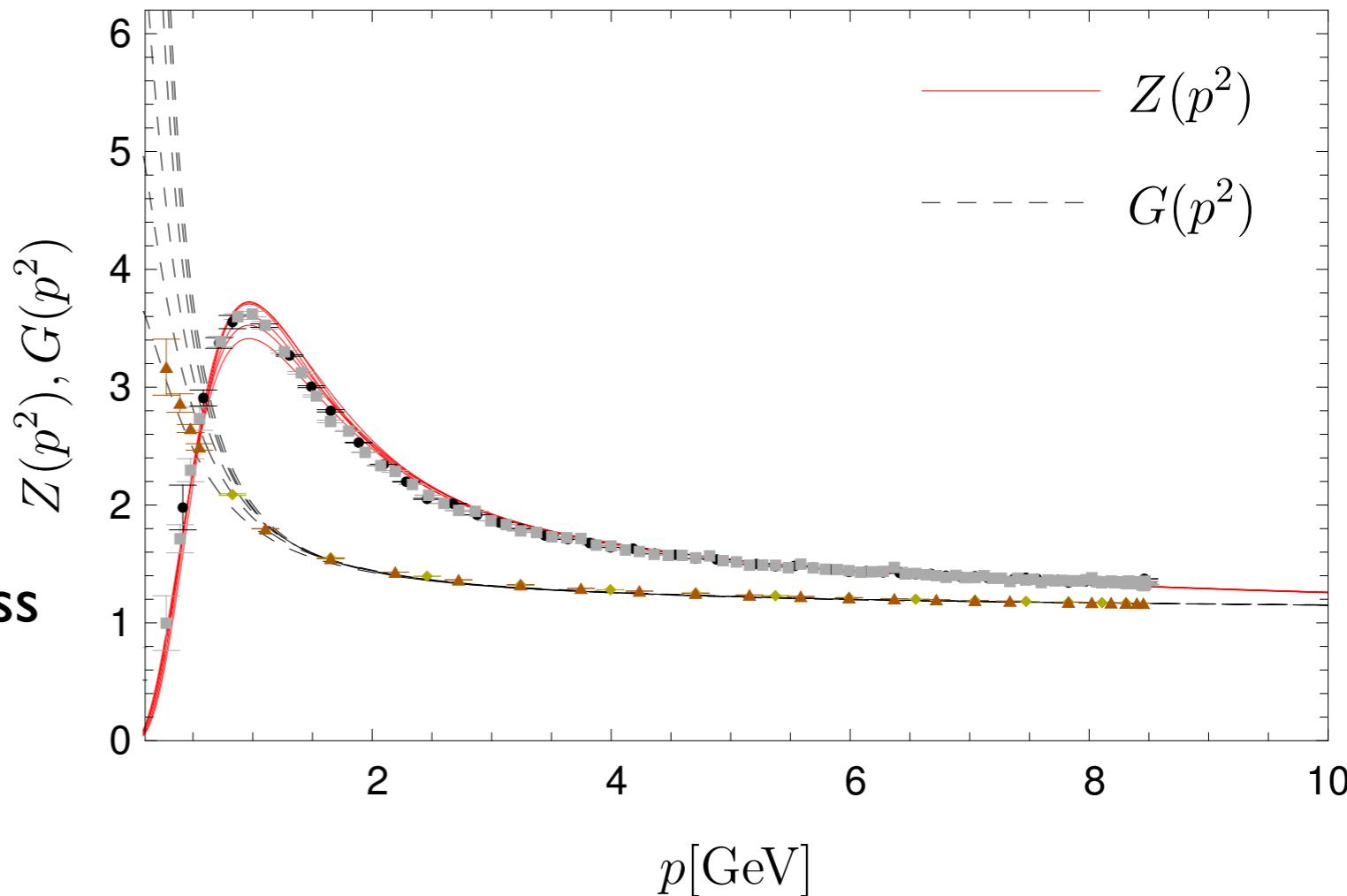
The diagram shows the gluon propagator $\text{---}^{-1}\text{---}$ decomposed into a bare part $\text{---}^{-1}\text{---}$ and a loop correction. The loop correction consists of two terms: one with a single loop and another with a double loop. The coefficients for the single loop term are $-\frac{1}{2}$ and for the double loop term are $-\frac{1}{6}$.

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$

- spacelike momenta:
good agreement with lattice
- fully dressed gluon appears massive
- time-like momenta: work in progress

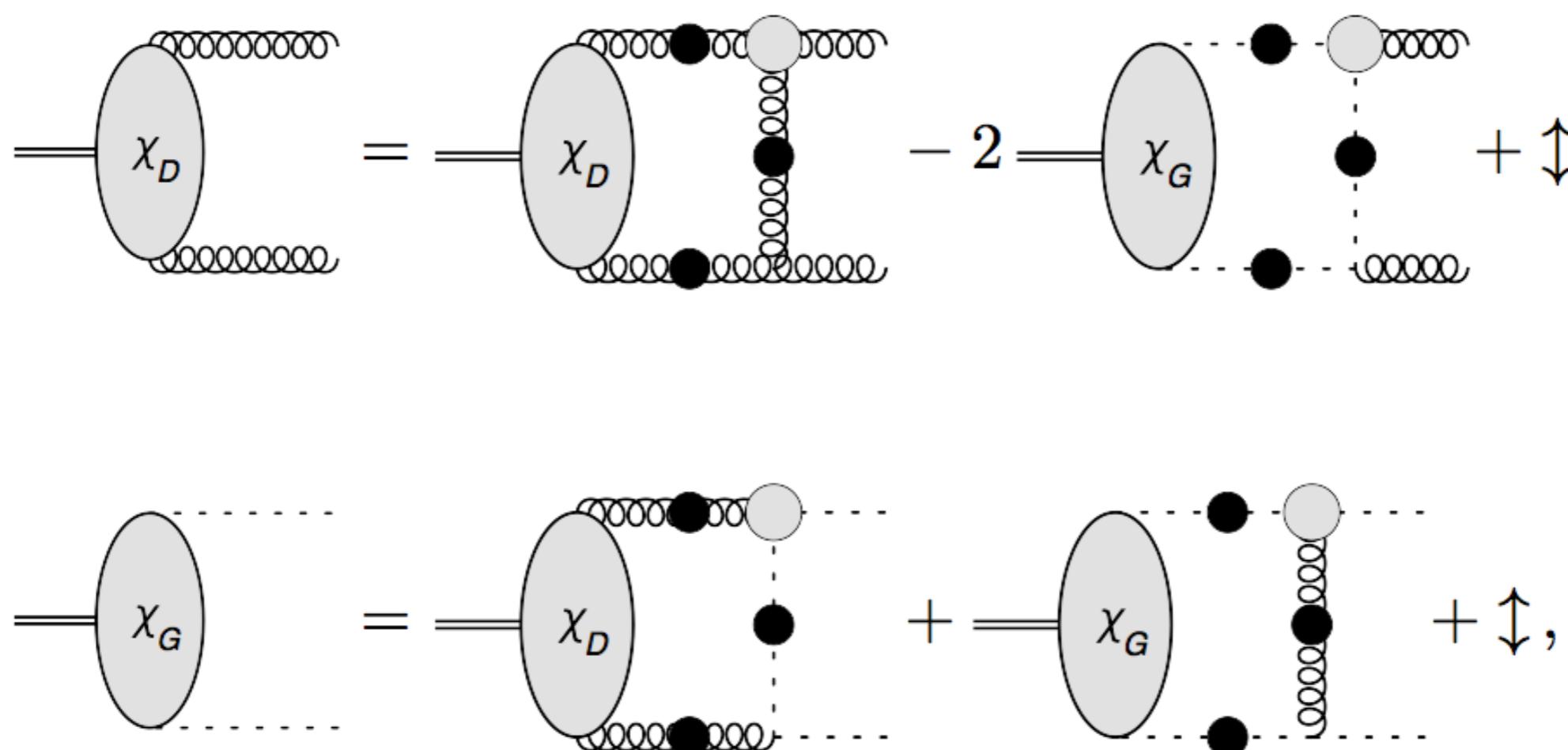
Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawłowski, Annals Phys. 324 (2009) 2408

CF, Huber, PRD 102 (2020) 094005, arXiv:2007.11505



DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

Glueballs from DSE/BSEs



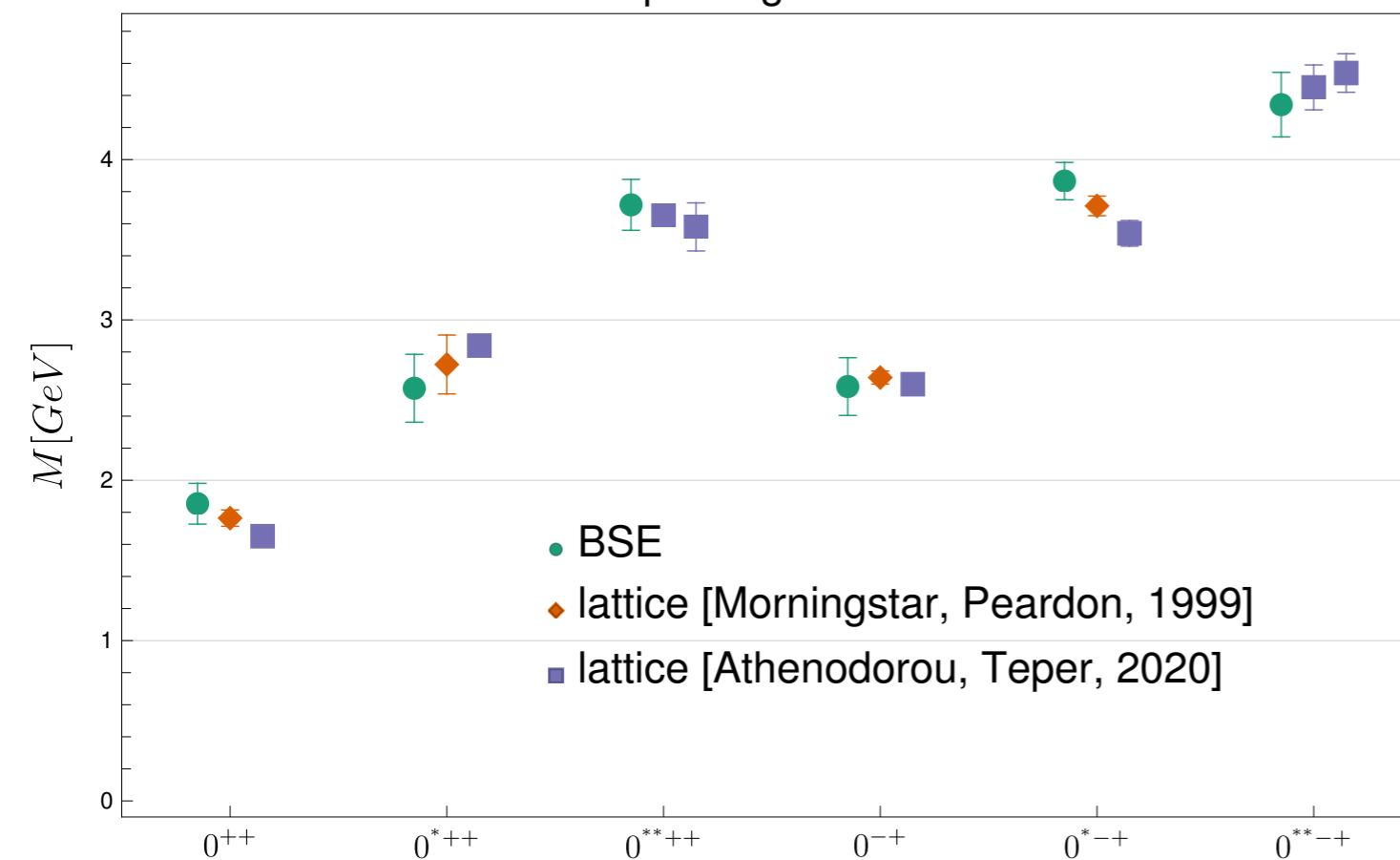
Meyers, Swanson, PRD 87 (2013) 3, 036009

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

Glueballs: results

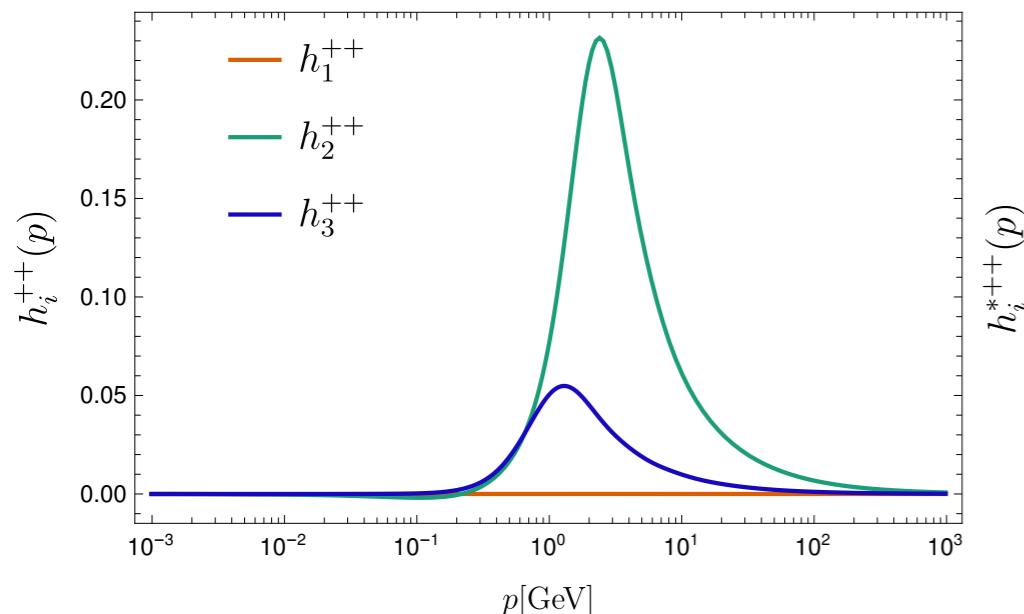
Spin-0 glueballs



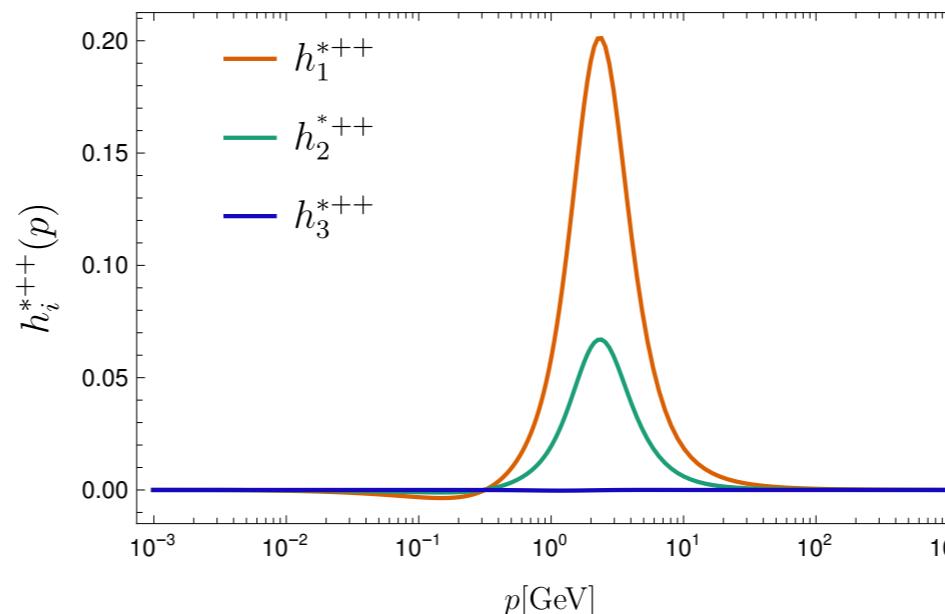
- spectrum:
very good agreement
lattice vs. DSE/BSE

CF, Huber, Sanchis-Alepuz, accepted by EPJC, arXiv:2004.00415

Amplitudes 0^{++}



Amplitudes 0^{*++}

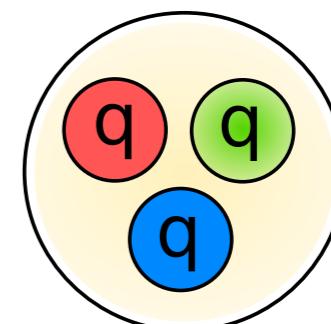


- excited states:
different internal
structure

I. Mass from nothing

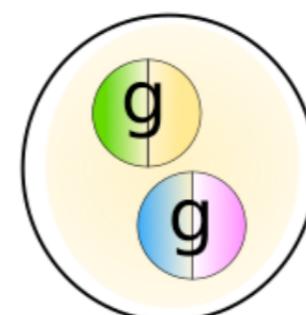


2. Baryons



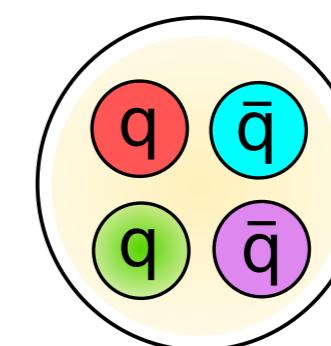
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3. Glueballs



CF, Huber, Sanchis-Alepuz, EPJC 80 11, 1077 (2020), arXiv:2004.00415

4.(Heavy-light) Tetraquarks



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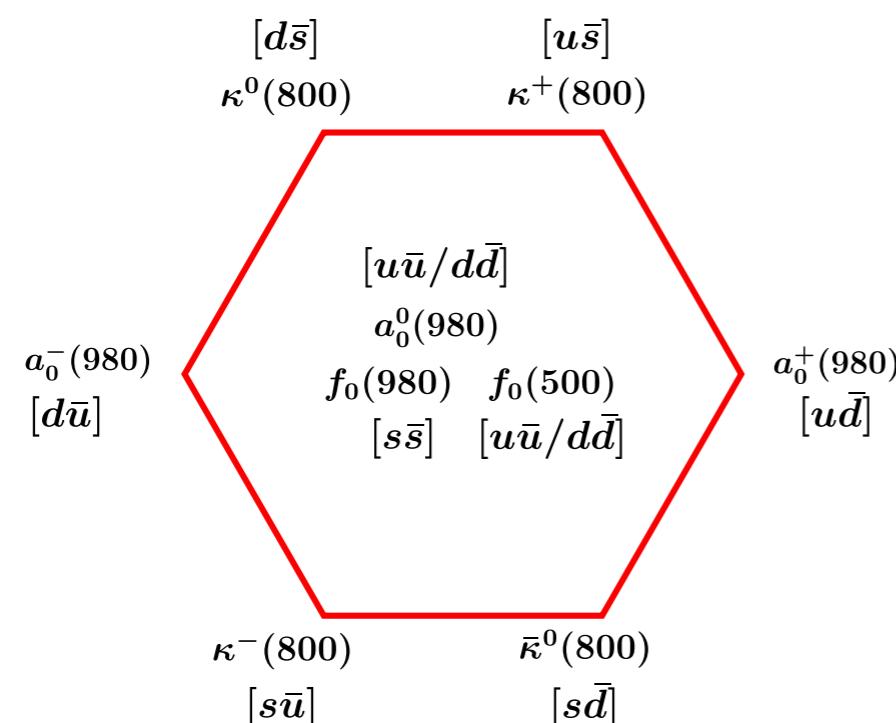
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Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4,38, arXiv:2008.10240

Tetraquark candidates with $qq\bar{q}\bar{q}$ -content

Light scalar mesons:

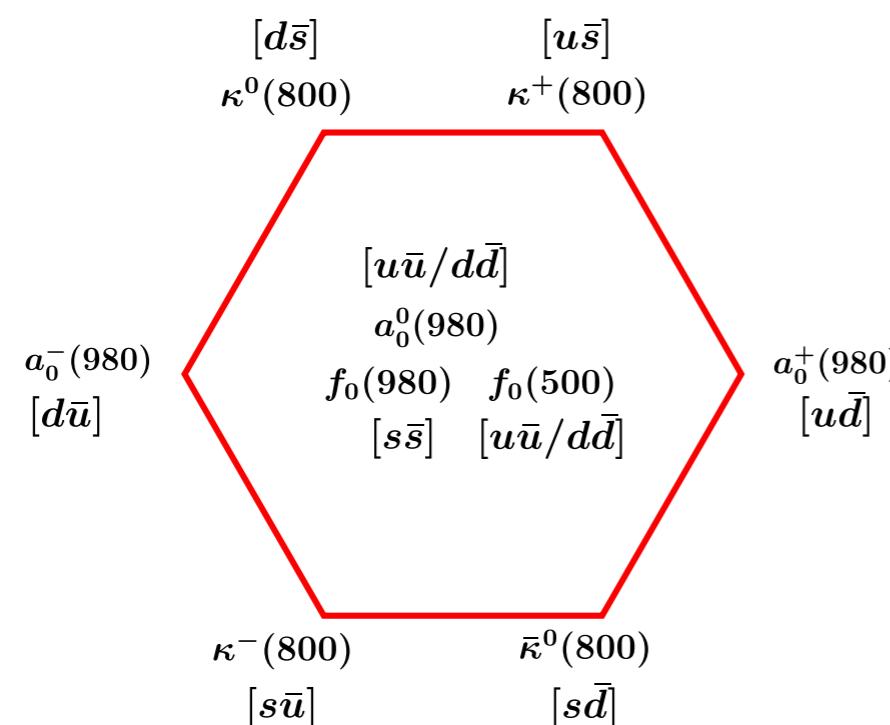


wrong level ordering

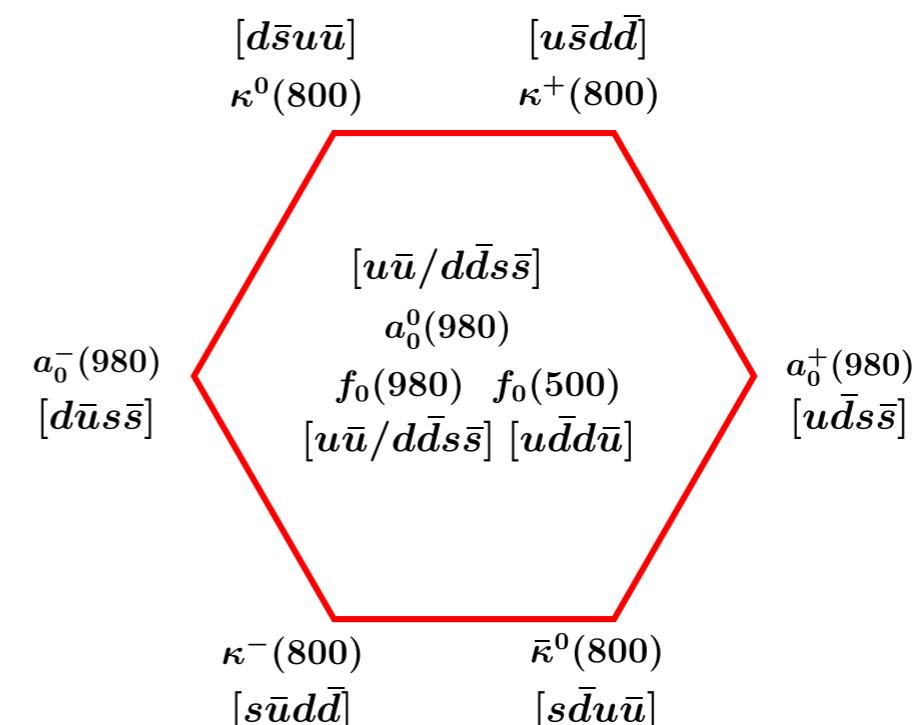
R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

Tetraquark candidates with $qq\bar{q}\bar{q}$ -content

Light scalar mesons:



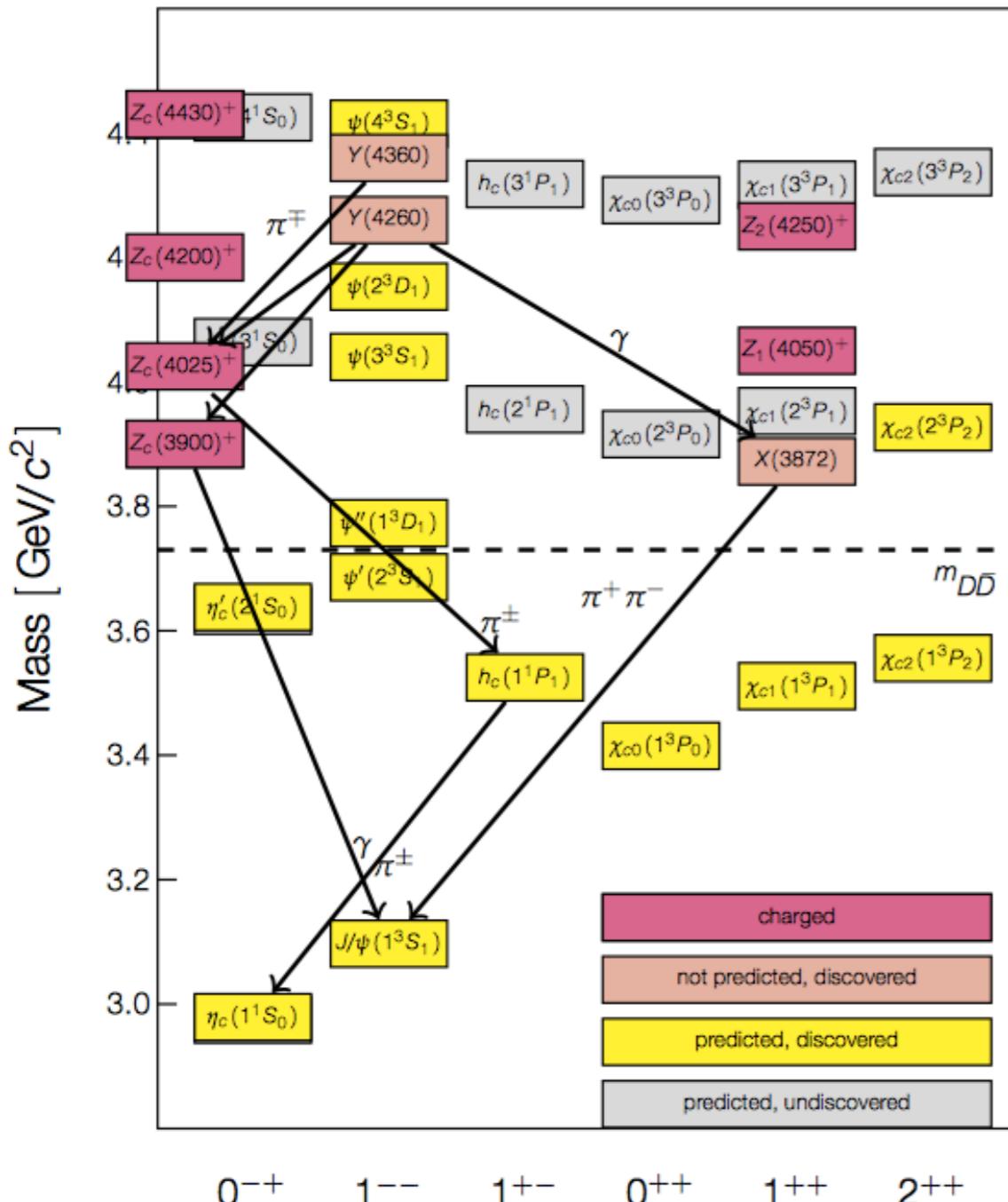
wrong level ordering



correct level ordering

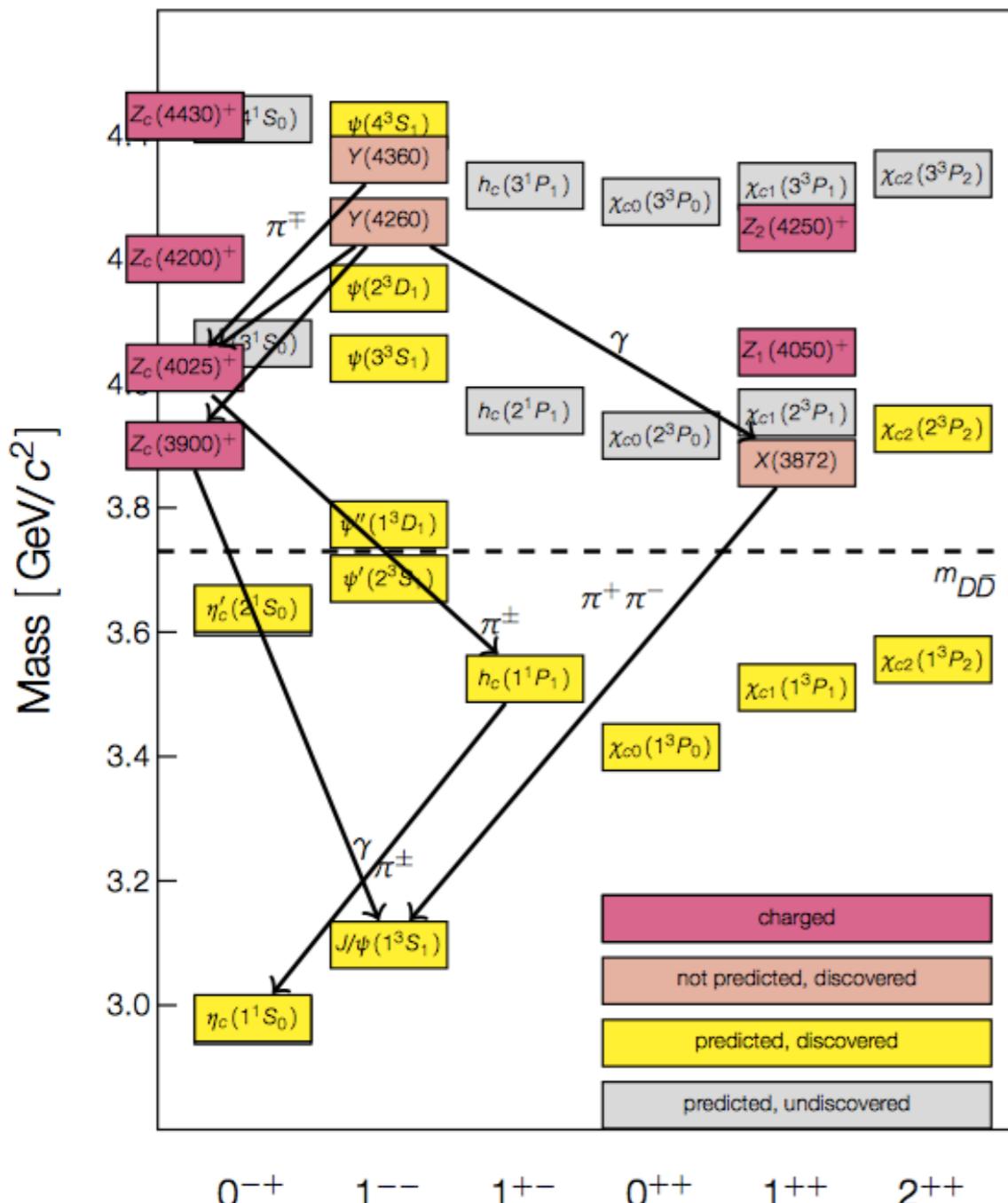
R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

Tetraquark candidates with $c\bar{q}q\bar{c}$ -content



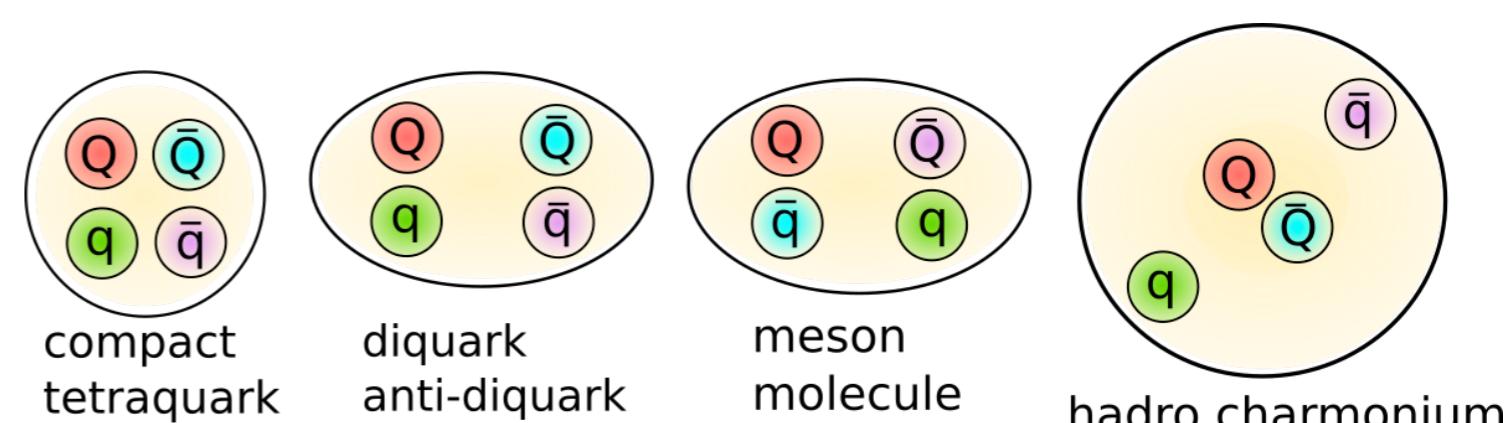
Many new unexpected states found: Belle, BABAR, BES, LHCb ...

Tetraquark candidates with $c\bar{q}\bar{q}c$ -content



Many new unexpected states found: Belle, BABAR, BES, LHCb ...

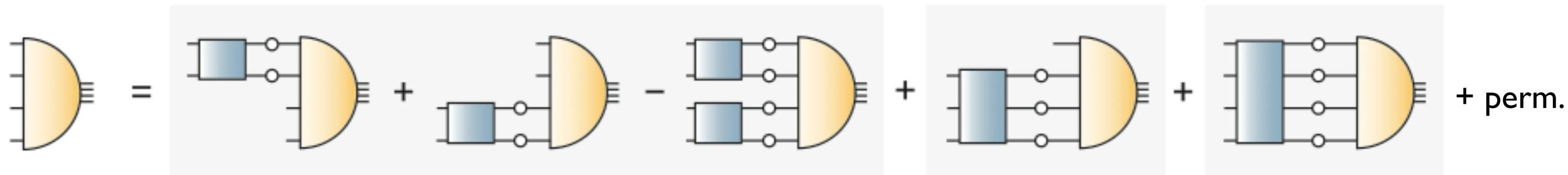
Internal structure ??



Related to details of underlying QCD forces between quarks and gluons

Tetraquarks from the four-body equation

Exact equation:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$
The diagram shows a central yellow circle representing a tetraquark state. It is equated to a sum of five terms. The first term is a single yellow circle. The second term is a yellow circle with a blue square block attached to its top-left edge. The third term is a yellow circle with a blue square block attached to its bottom-left edge. The fourth term is a yellow circle with a blue rectangle block attached to its left edge. The fifth term is a yellow circle with a blue rectangle block attached to its right edge. A plus sign (+) follows each term except the first one. A minus sign (-) follows the third term. A plus sign (+) followed by the text "perm." follows the fifth term.

Two-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Three- and four-body interactions

Tetraquarks from the four-body equation

Exact equation:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$

The diagram shows a central yellow circle representing a tetraquark state. To its left is an equals sign. To the right of the equals sign are five terms separated by plus signs. The first term is a single yellow circle. The second term is a yellow circle with a blue square interaction vertex on its left side. The third term is a blue square interaction vertex with a yellow circle on its right side. The fourth term is a blue square interaction vertex with another blue square interaction vertex on its left side. The fifth term is a blue square interaction vertex with a yellow circle on its right side. A minus sign is placed between the third and fourth terms. A plus sign is placed before the fifth term. A plus sign followed by the word "perm." is placed at the far right. A red diagonal slash is drawn through the fourth and fifth terms.

Two-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

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Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Three- and four-body interactions

Tetraquarks from the four-body equation

Exact equation:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$

The diagram shows a central equals sign followed by a plus sign, a minus sign, another plus sign, and a final plus sign followed by the text '+ perm.'. To the left of the first plus sign is a yellow circle with three external gluon lines. To the right of the minus sign is a blue square with three external gluon lines. The first term is a blue square connected to a yellow circle. The second term is a yellow circle connected to a blue square. The third term is a blue square connected to a blue square. The fourth term is a blue square connected to a yellow circle. The fifth term is a yellow circle connected to a blue square. A red diagonal line through a red circle crosses out the fourth and fifth terms.

Two-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

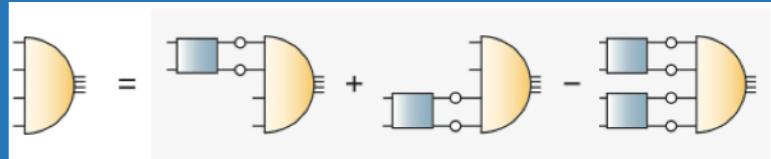
Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Three- and four-body interactions

- Two-body interactions: allow for internal clustering
- use RL-approximation

Four-body equation: permutations

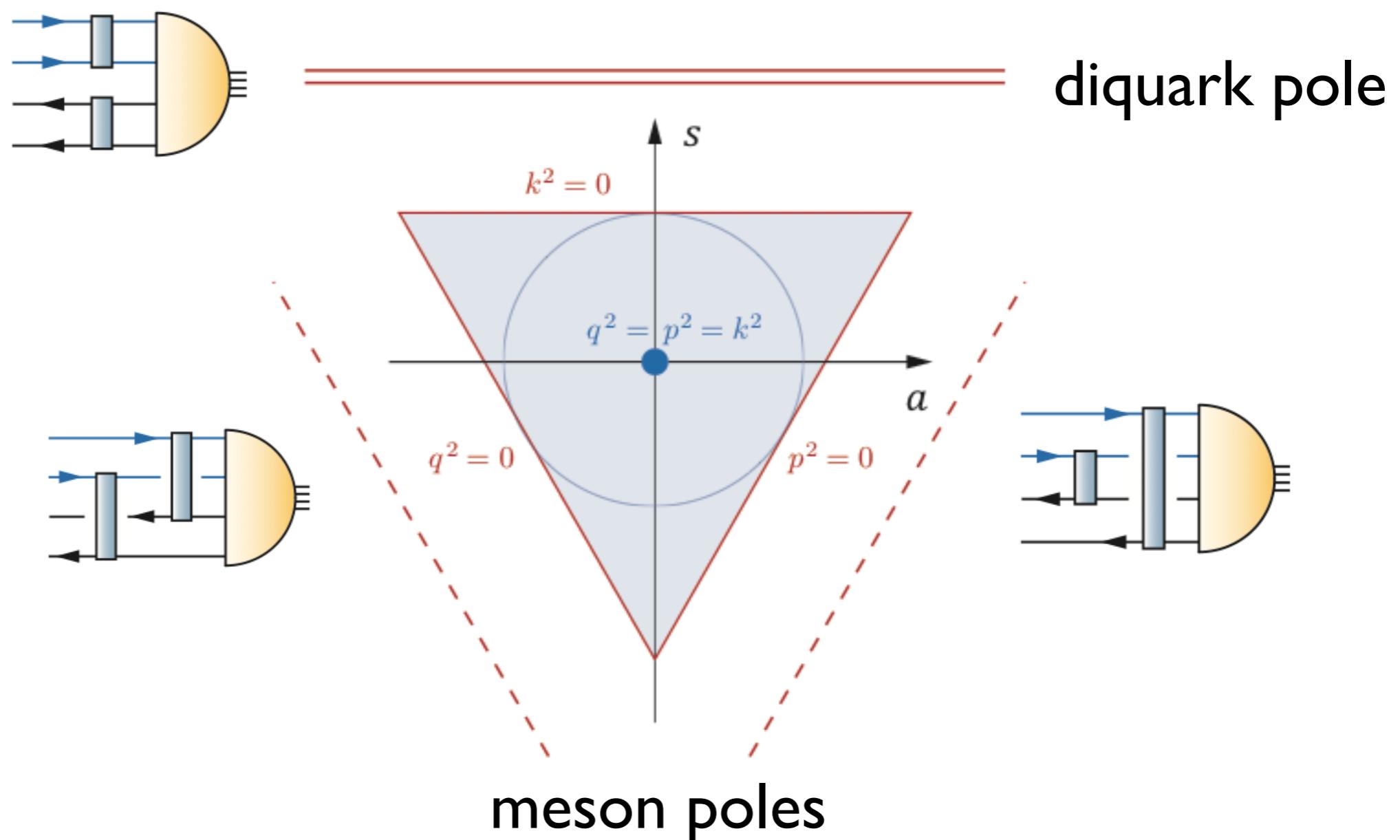


- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$

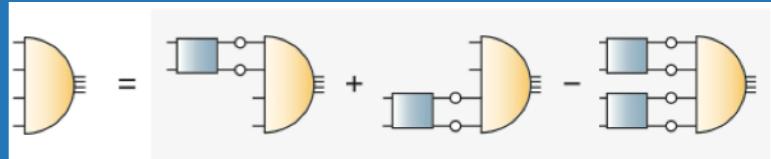
p,q,k: relative momenta

- **Doublet:** $s \sim p^2 + q^2 - 2k^2$

$$a \sim q^2 - p^2$$



Four-body equation: permutations

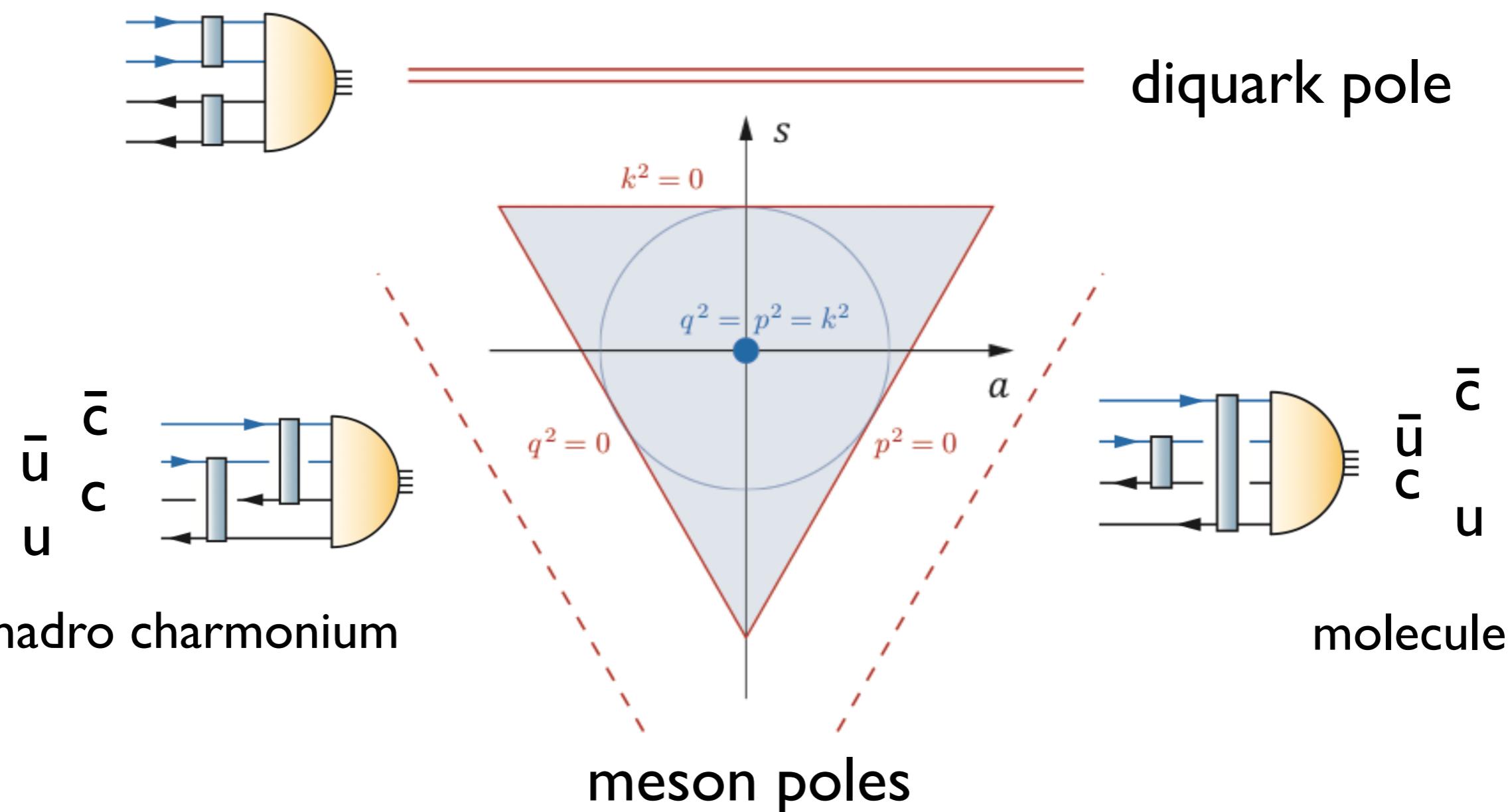


- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$

p, q, k : relative momenta

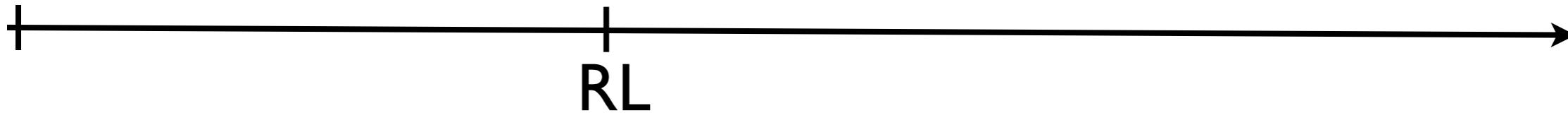
- Doublet: $s \sim p^2 + q^2 - 2k^2$

$$a \sim q^2 - p^2$$



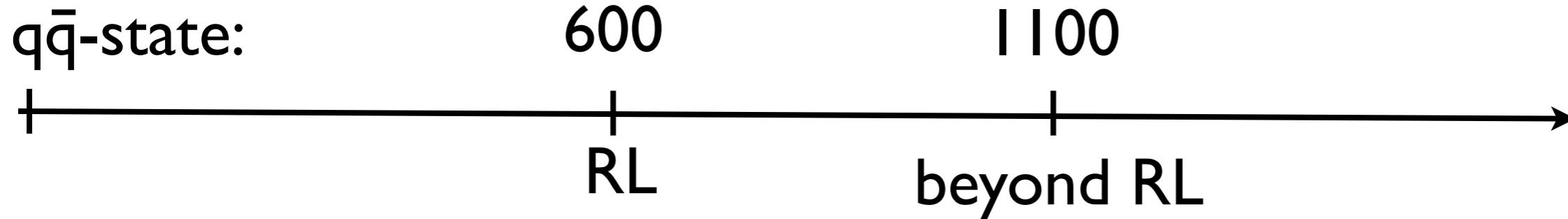
Bound state vs resonance: light scalars

$q\bar{q}$ -state:



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

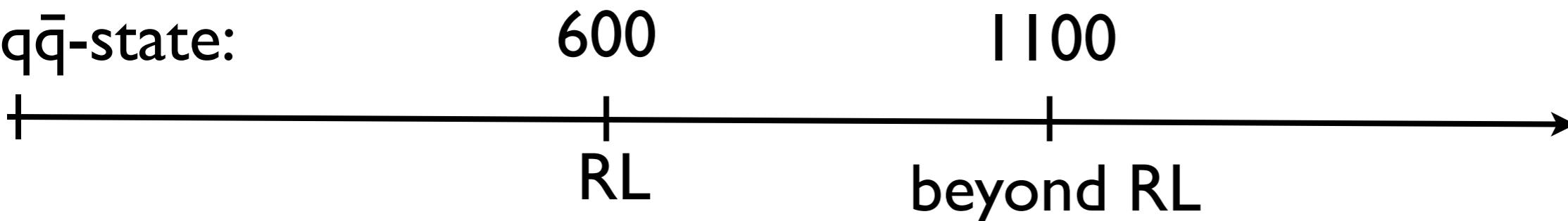
Bound state vs resonance: light scalars



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: light scalars

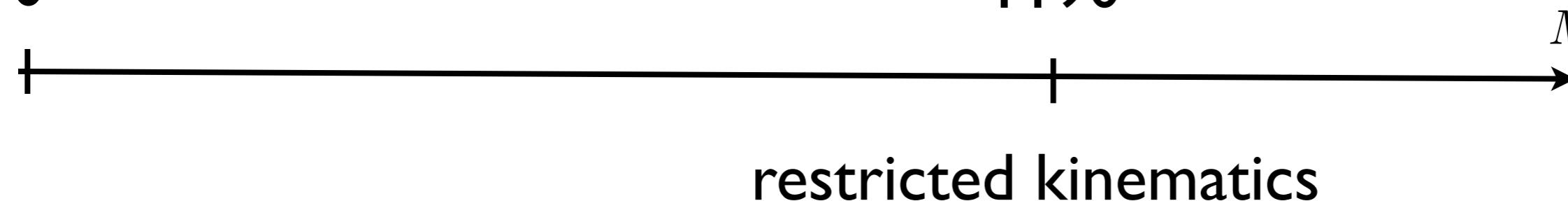
$q\bar{q}$ -state:



$q\bar{q}q\bar{q}$ state:

$$\Gamma(S_0, \cancel{s}, \cancel{a}, \dots)$$

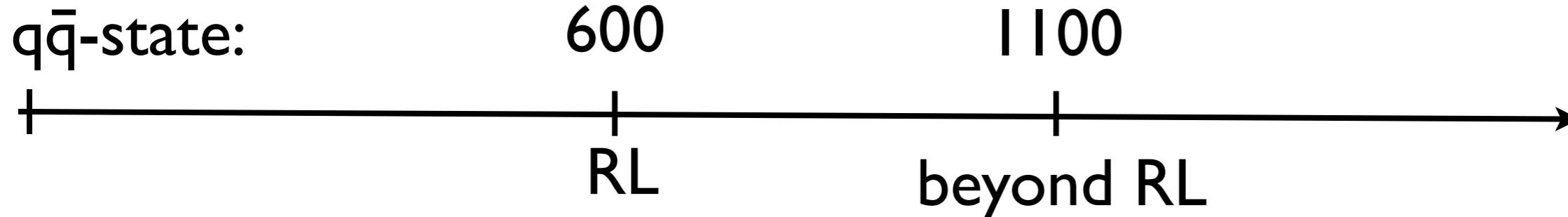
0



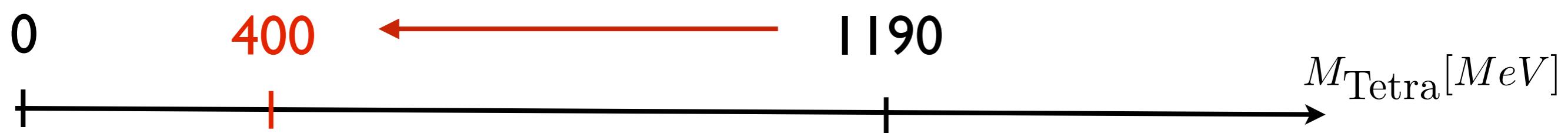
Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: light scalars



q $\bar{q}q\bar{q}$ state: $\Gamma(S_0, s, a, \dots)$



Two-pion resonance

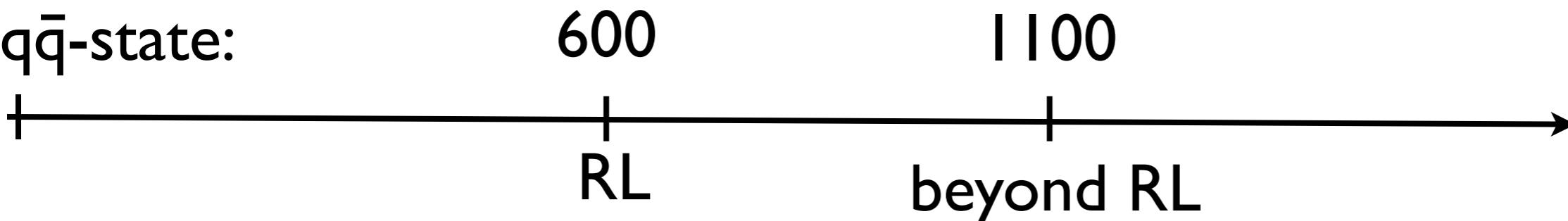
Bound state of
four massive quarks

→ identify with $f_0(500)$ (' σ -meson')

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

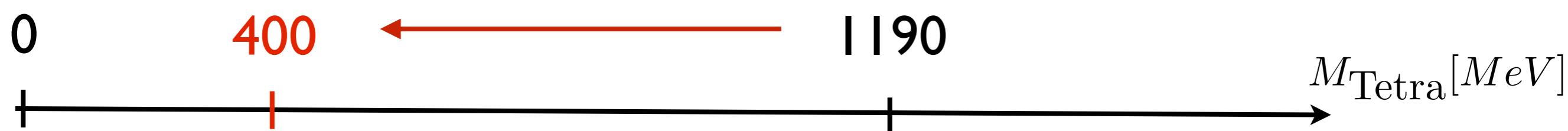
Bound state vs resonance: light scalars

q̄q-state:



$q\bar{q}q\bar{q}$ state:

$$\Gamma(S_0, s, a, \dots)$$



full kinematics

restricted kinematics

Two-pion resonance

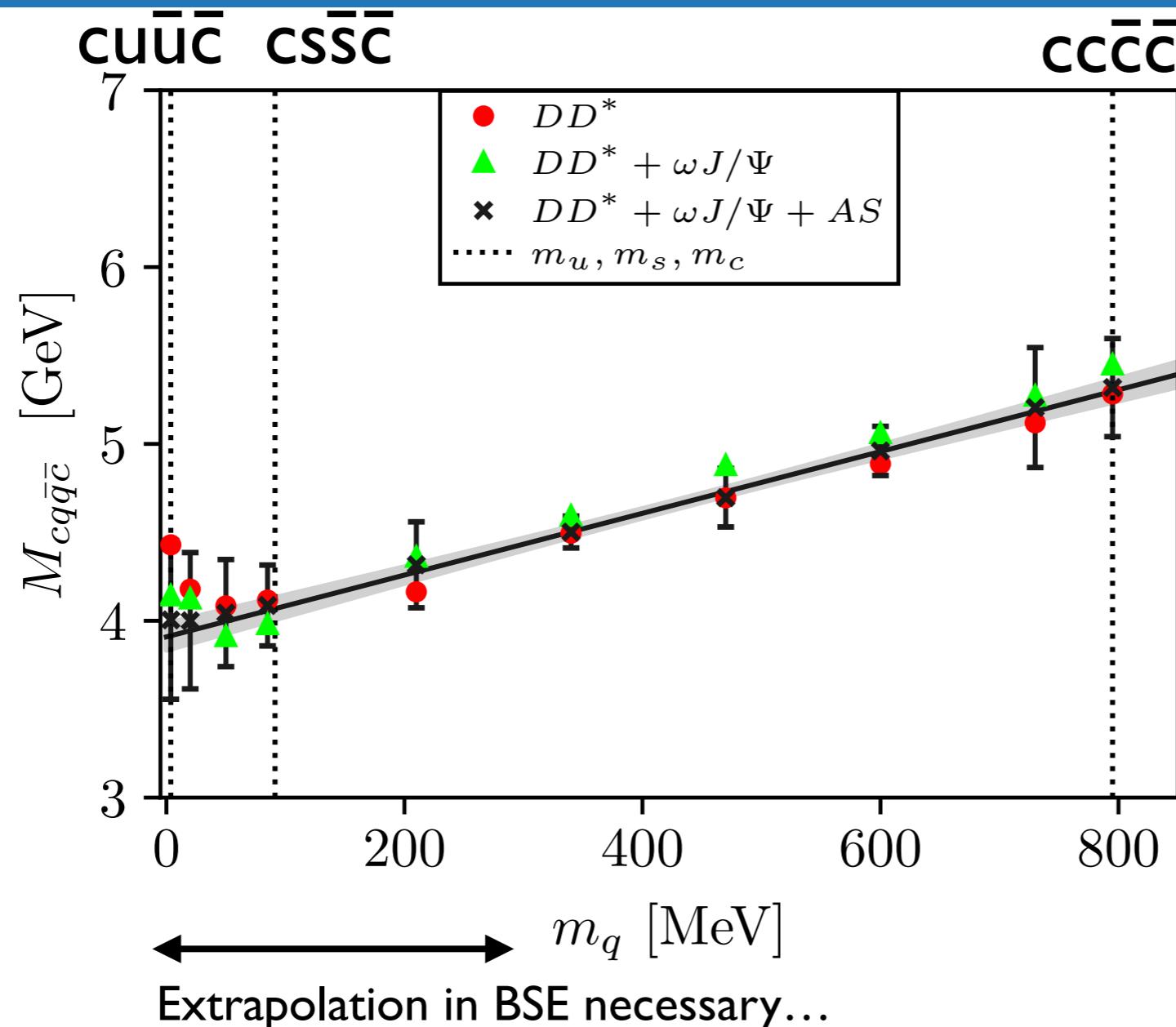
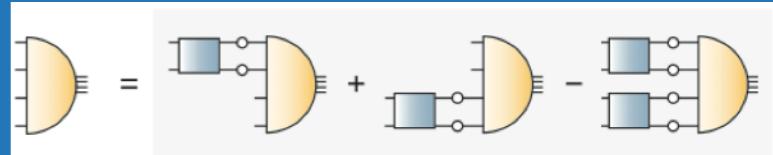
Bound state of four massive quarks

→ identify with $f_0(500)$ (' σ -meson')

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Mixing with $q\bar{q}$: small effect

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.



m_c fixed
 m_q varied

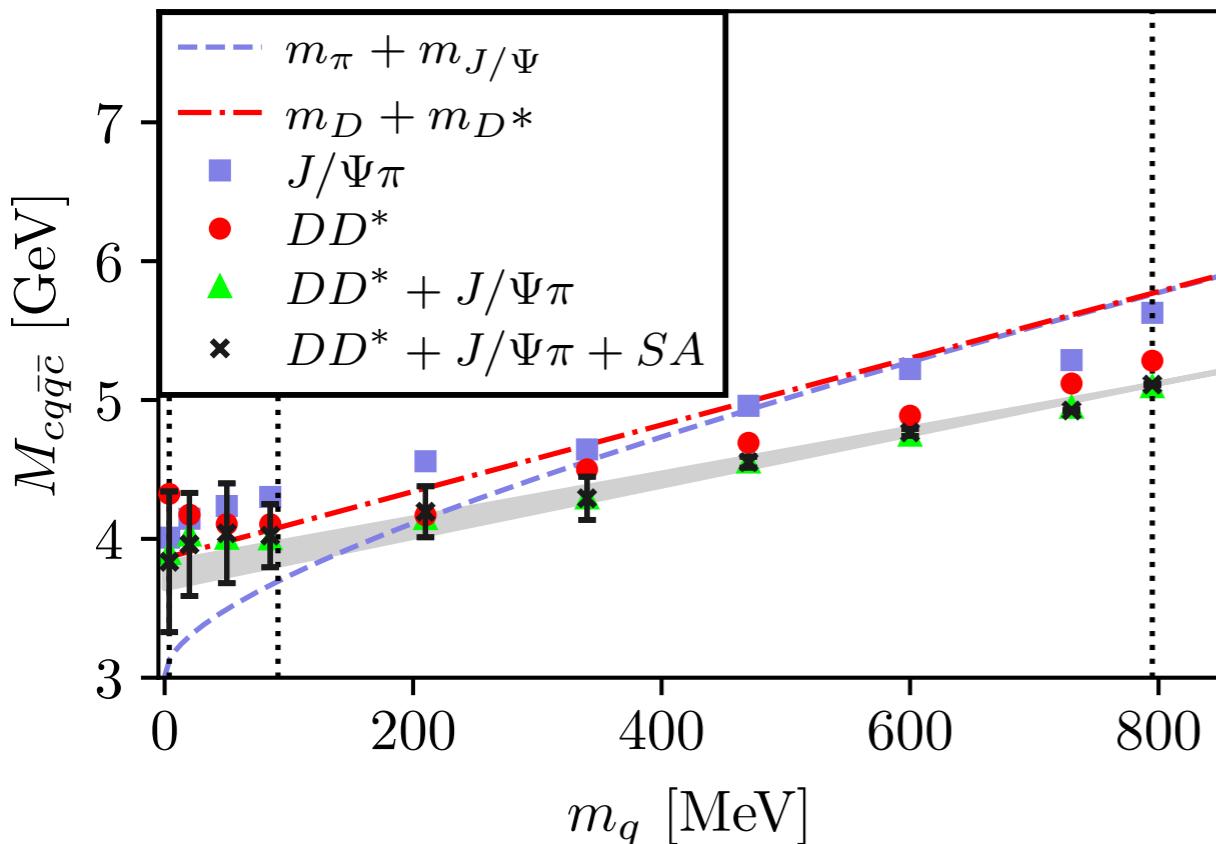
● DD* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74) \text{ MeV} \longrightarrow X(3872)$$

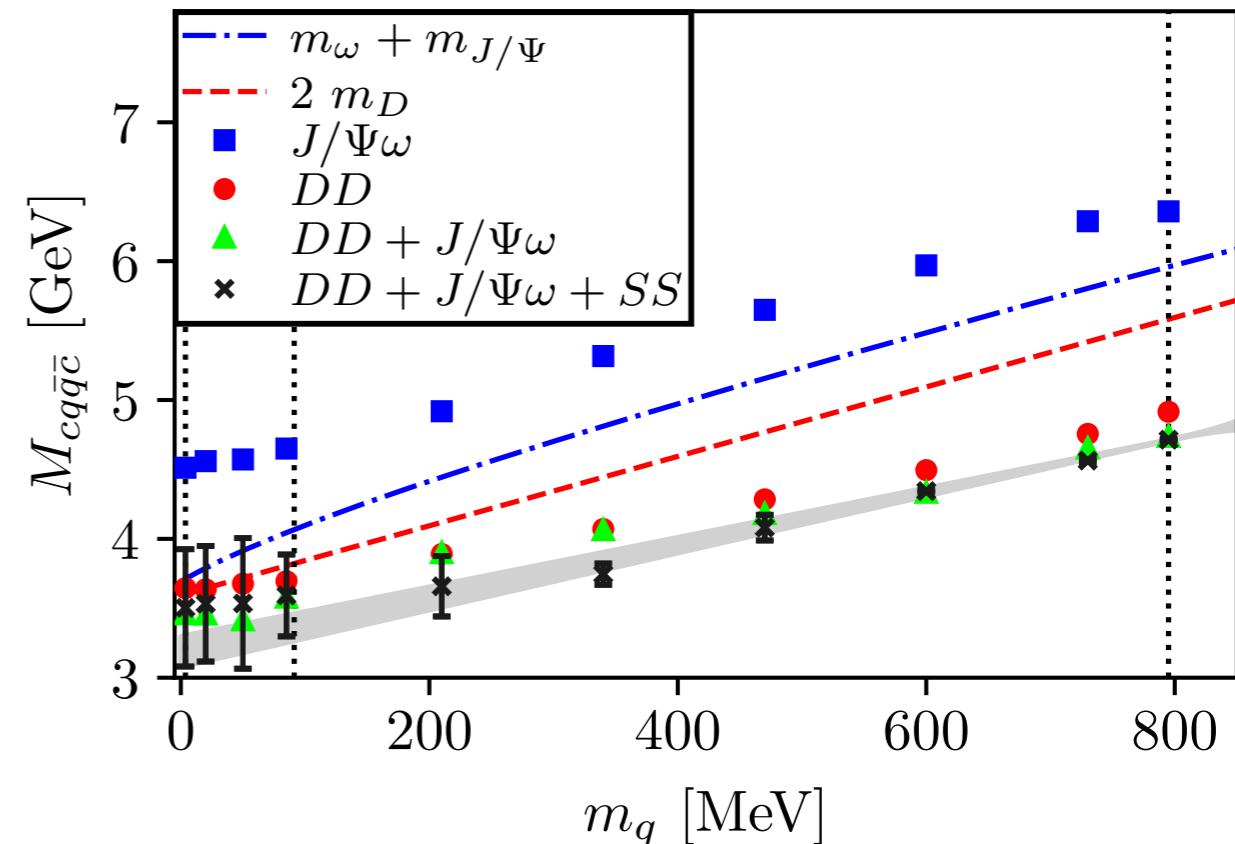
Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

$J^{PC} = 1^{+-}$ and 0^{++}

$1(1^{+-}) \, c q \bar{q} \bar{c}$



$0(0^{++}) \, c q \bar{q} \bar{c}$

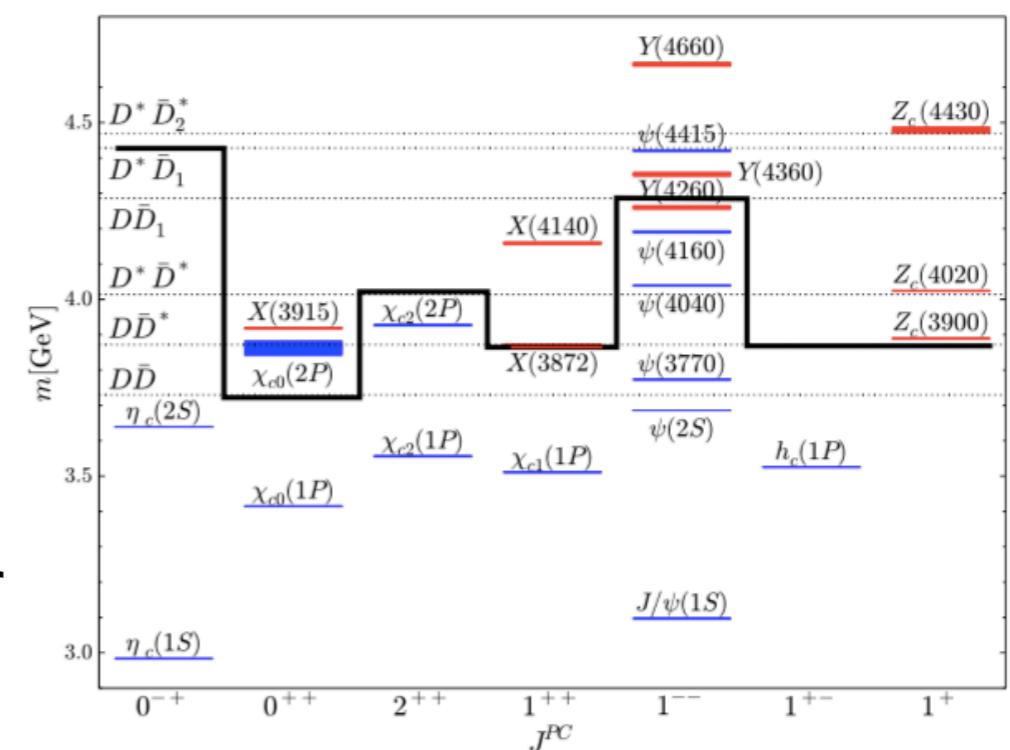


Wallbott, Eichmann and CF, PRD 102 (2020) no.5, 051501, arXiv:2003.12407

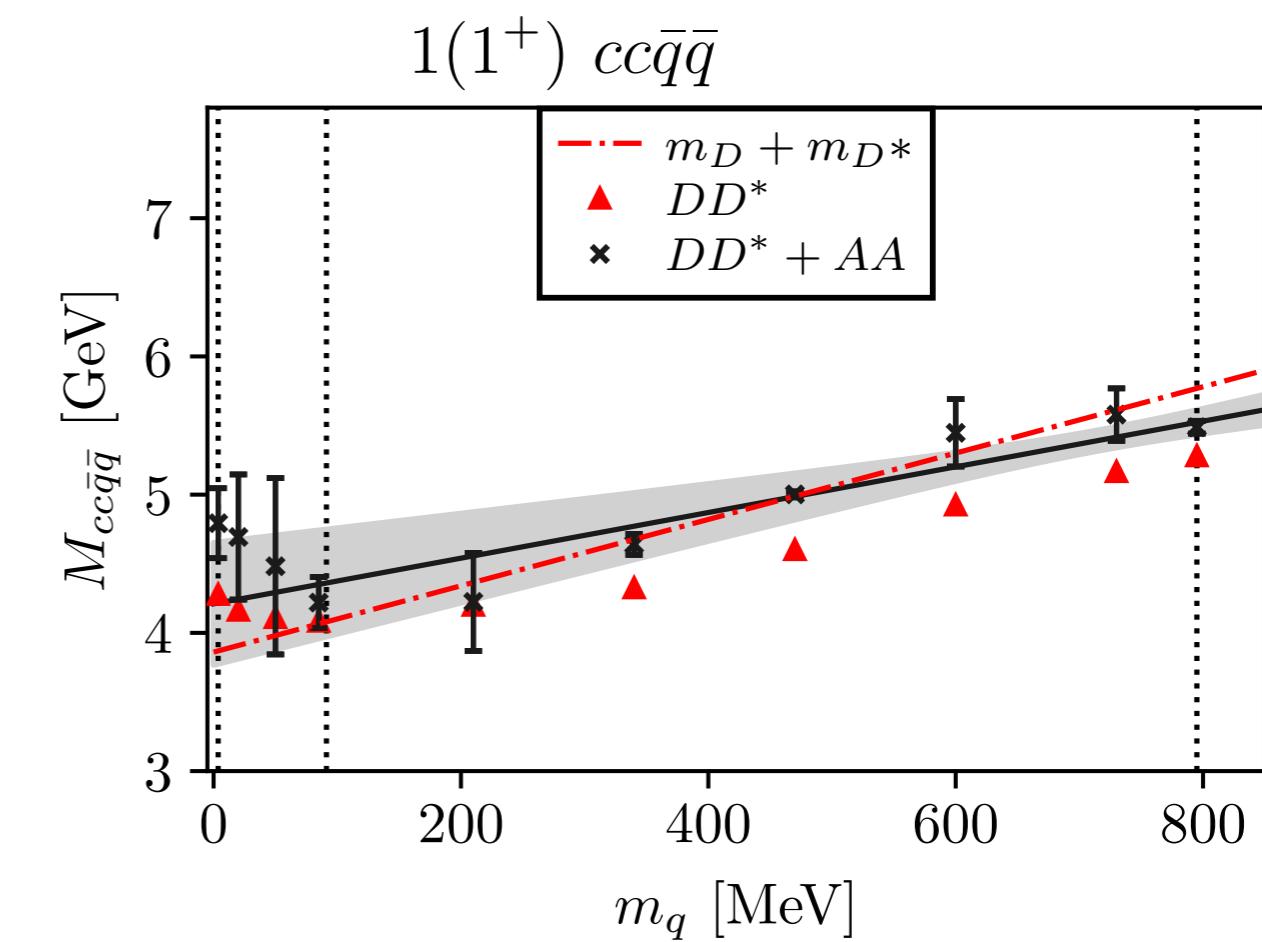
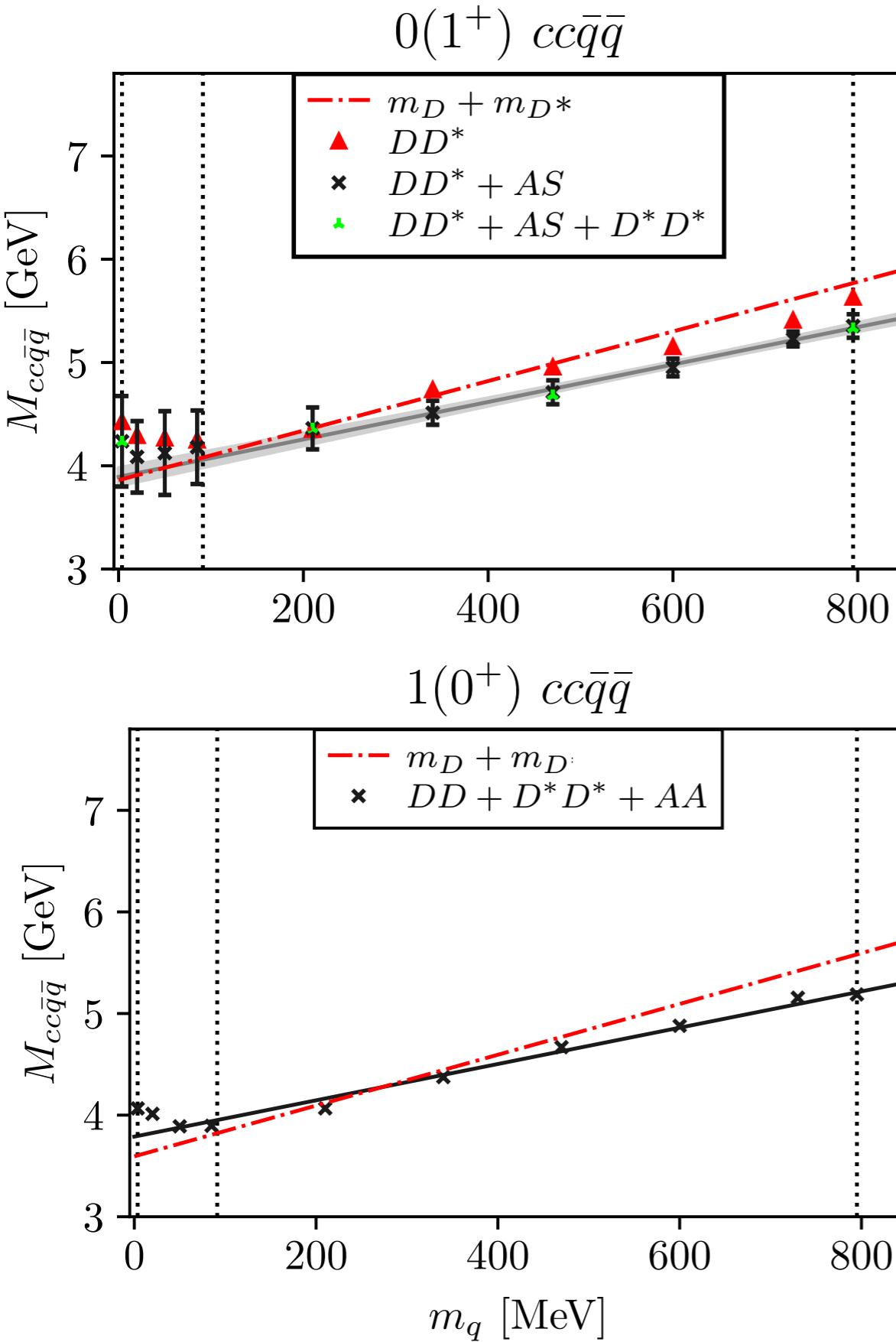
$$M_{1^{+-}}^{cq\bar{q}\bar{c}} = 3741(91) \rightarrow Z(3900)$$

$$M_{0^{++}}^{cq\bar{q}\bar{c}} = 3195(107) \rightarrow ?$$

mass pattern matches molecule picture of
Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states



● DD(*) and diquarks important!

Summary

Main goals:

- one framework for all areas of hadron physics:
 - * discussed: baryons, mesons, glueballs, tetraquarks
 - * not discussed: hybrids, form factors, anomalous magnetic moments QCD phase diagram
 - * access to DXSB, confinement,...

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016) [1606.09602]

CF, PPNP 105 (2019) [1810.12938]

Main challenge:

- systematic control over error budget:
intrinsic + cp to other methods like lattice QCD

Main results:

- NOT high precision physics
- BUT competitive contributions in many areas

Thanks to...

... the group:

Postdocs:

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PD Dr. Markus Huber



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Pascal Gunkel
Joshua Hoffer
Philipp Isserstedt
Nico Santowsky



MSc-students:

Stephan Hagel
Felix Keil

... and many external collaborators.



Backup Slides

Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

$$\Gamma^\mu(p, k) = \sum_{i=1,12} \tau_i(p, k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2) \quad \text{“approximation” !}$$

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

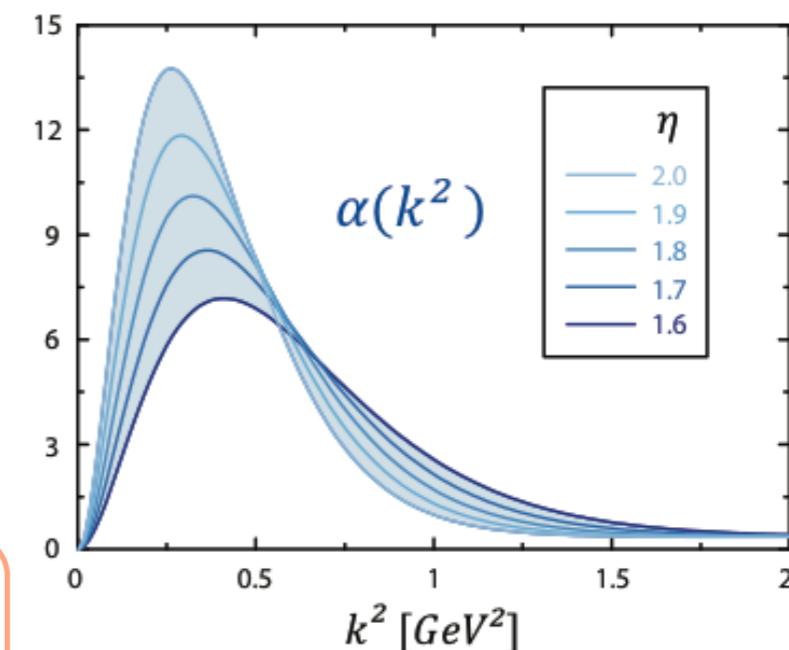
Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

effective coupling

$$\alpha(k^2) = \pi \eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

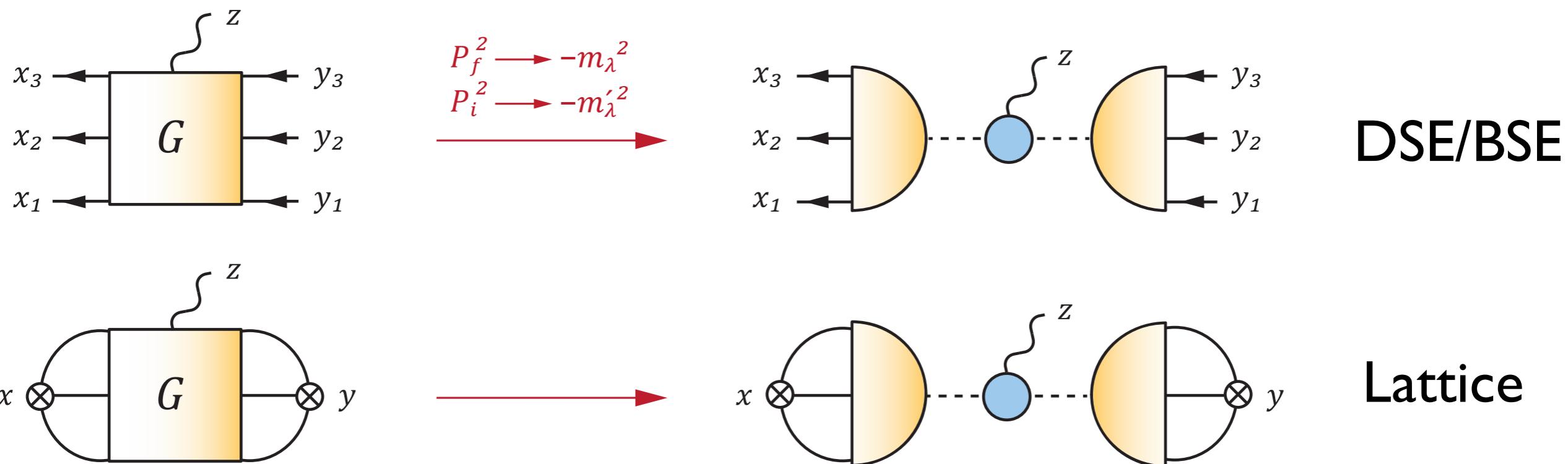


Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

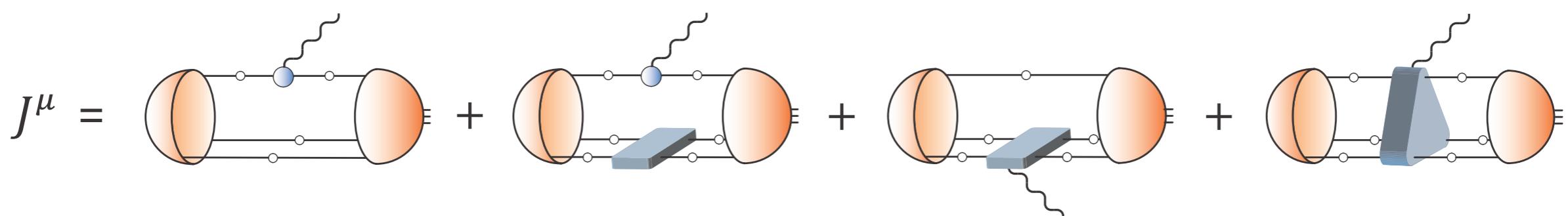
- scale Λ from f_π , masses $m_u = m_d, m_s$ from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF Heupel, PRD93 (2016) 034026, and refs. therein

Extracting form factors from correlators

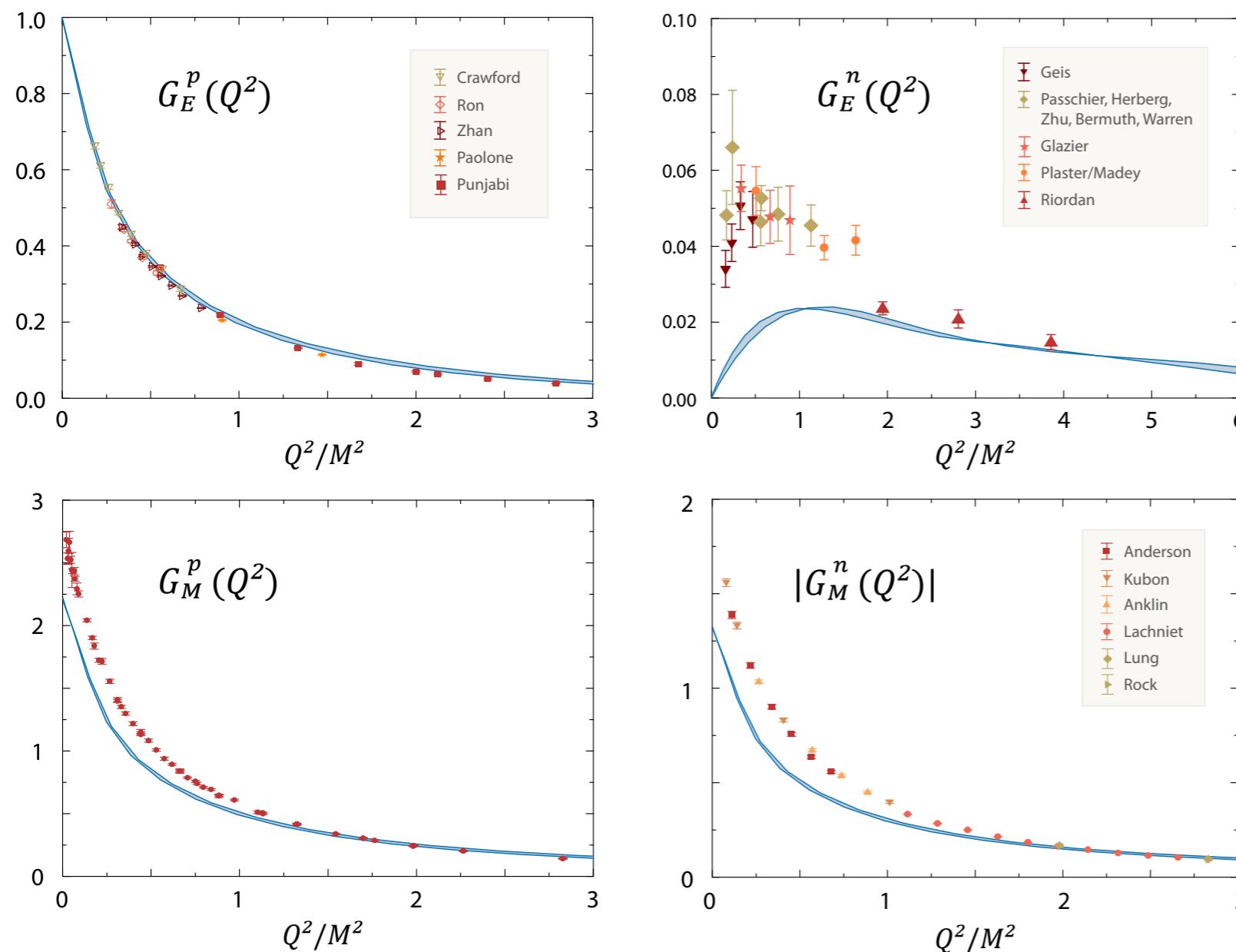


Form factor from BSEs (derived from equation of motion for G and ‘gauging’)

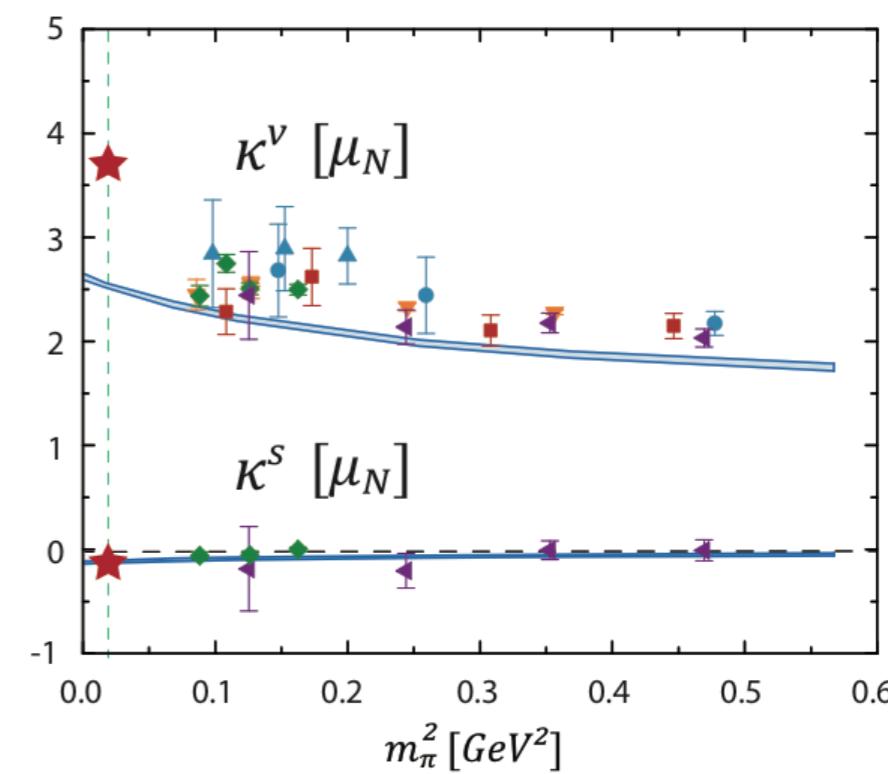


- exact equation for baryon form factors

Nucleon form factors and magnetic moments



Isovector (p-n), isoscalar (p+n):

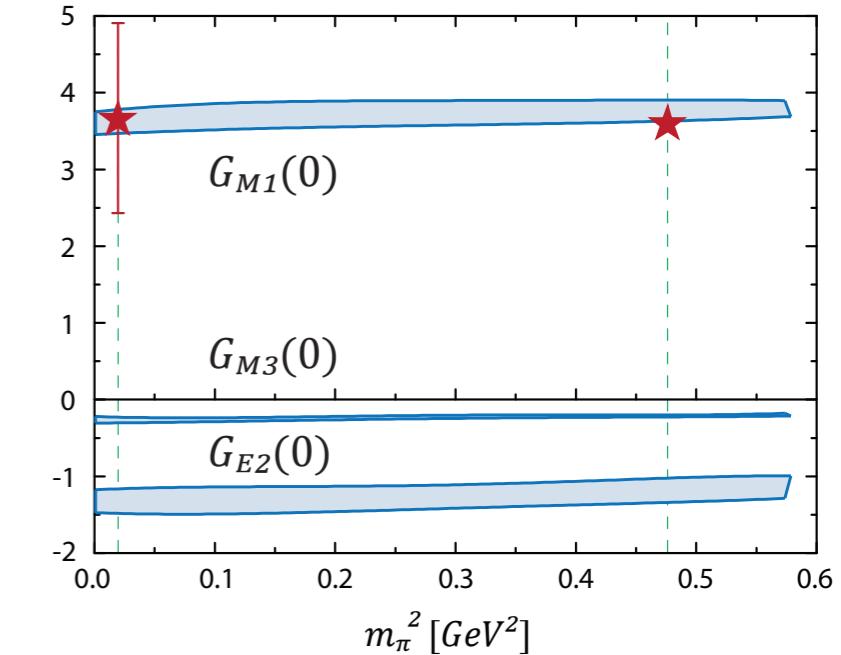
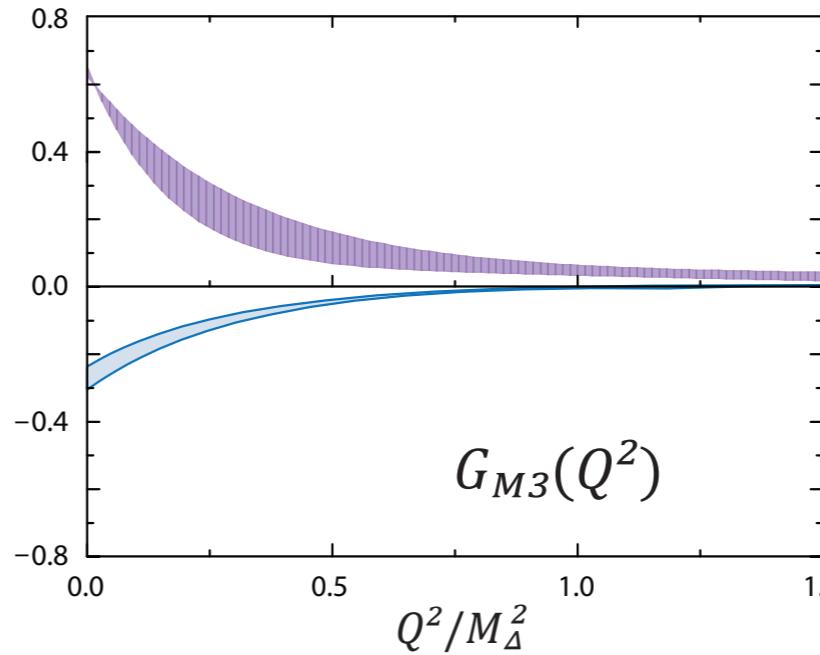
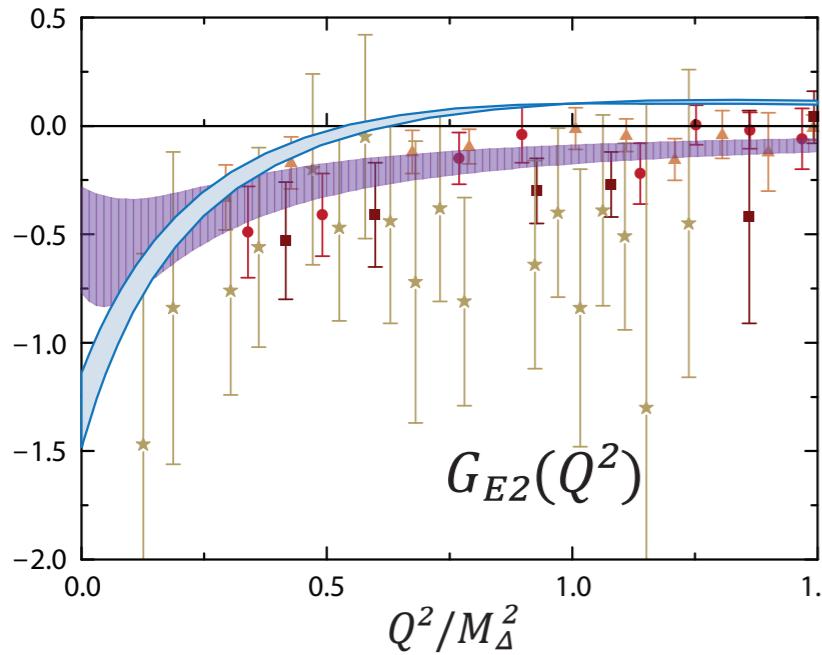
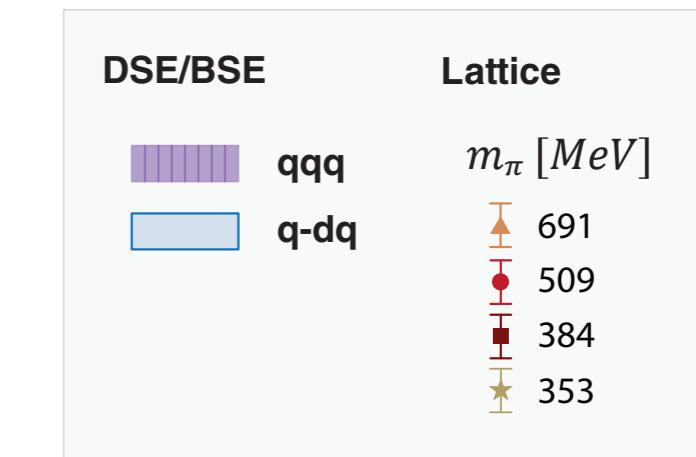
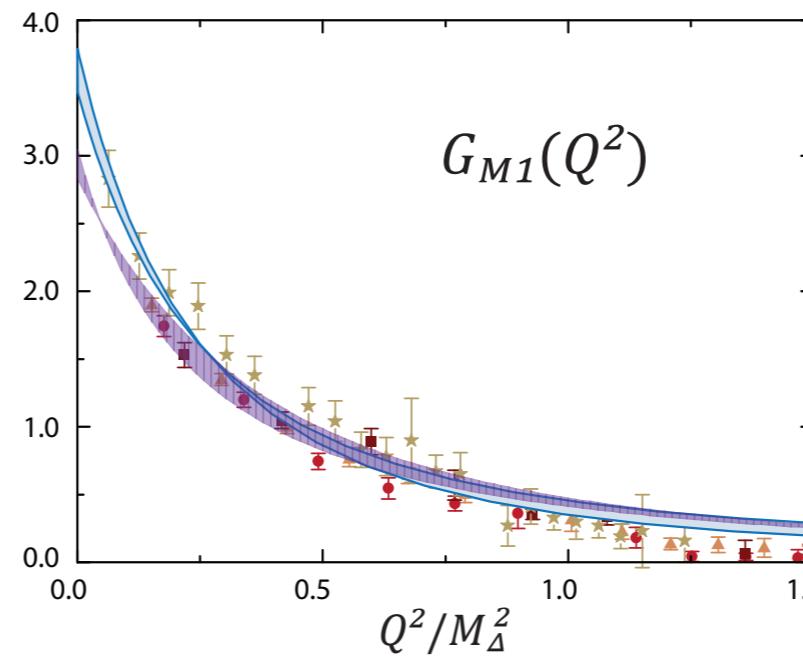
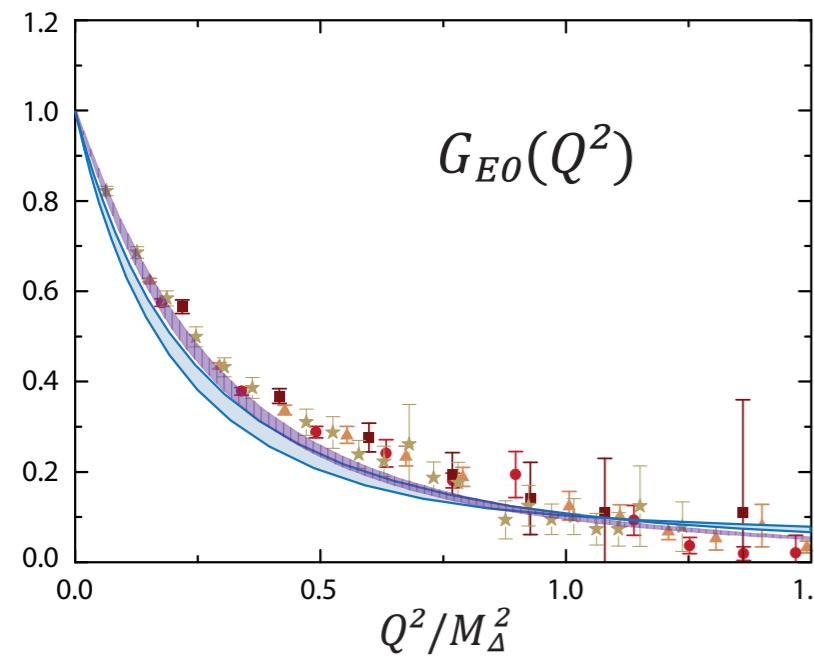


- missing pion cloud effects
- similar for axial form factors

Eichmann, PRD 84 (2011)

Eichmann and CF, EPJ A48 (2012) 9

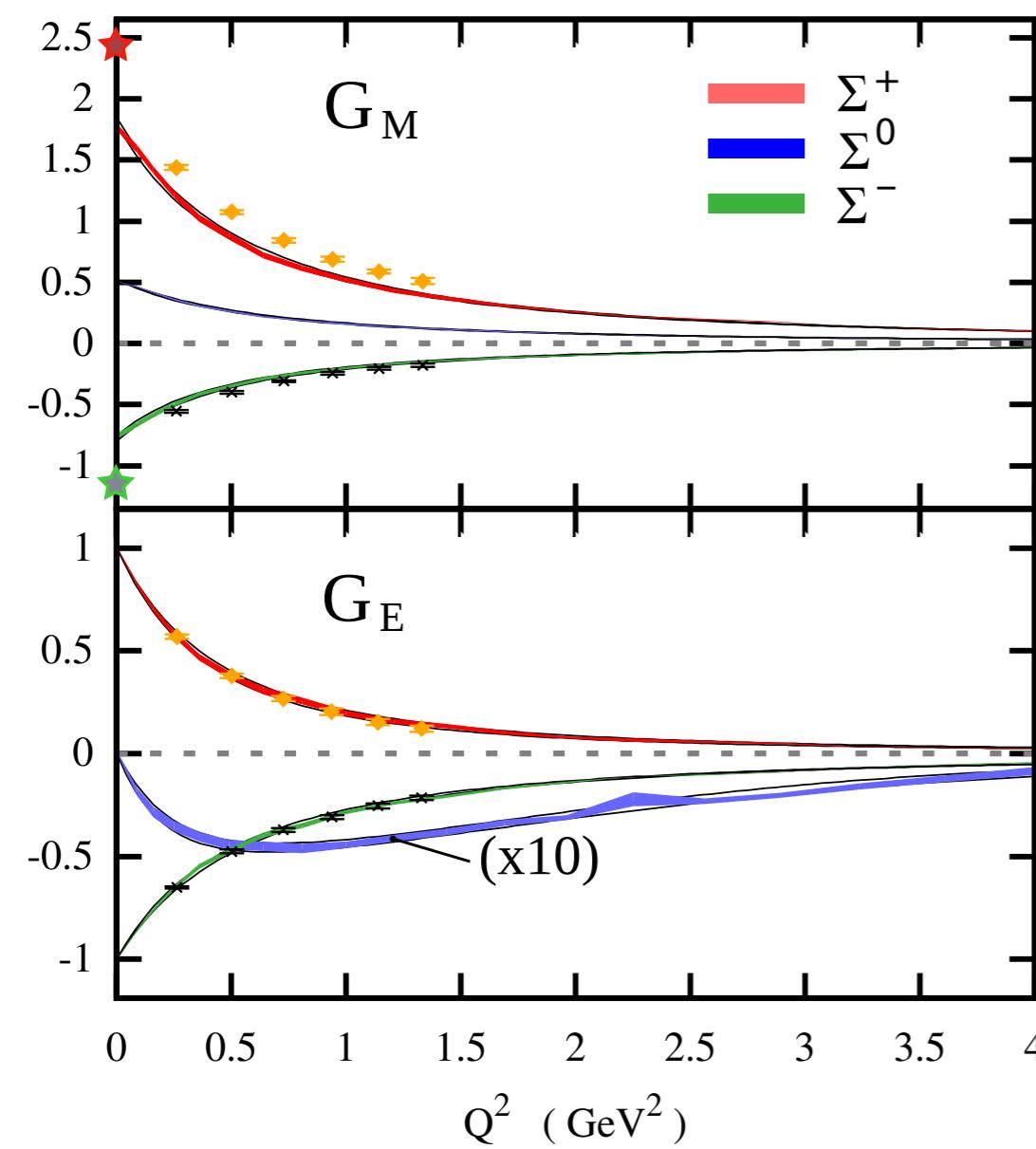
Δ -form factors



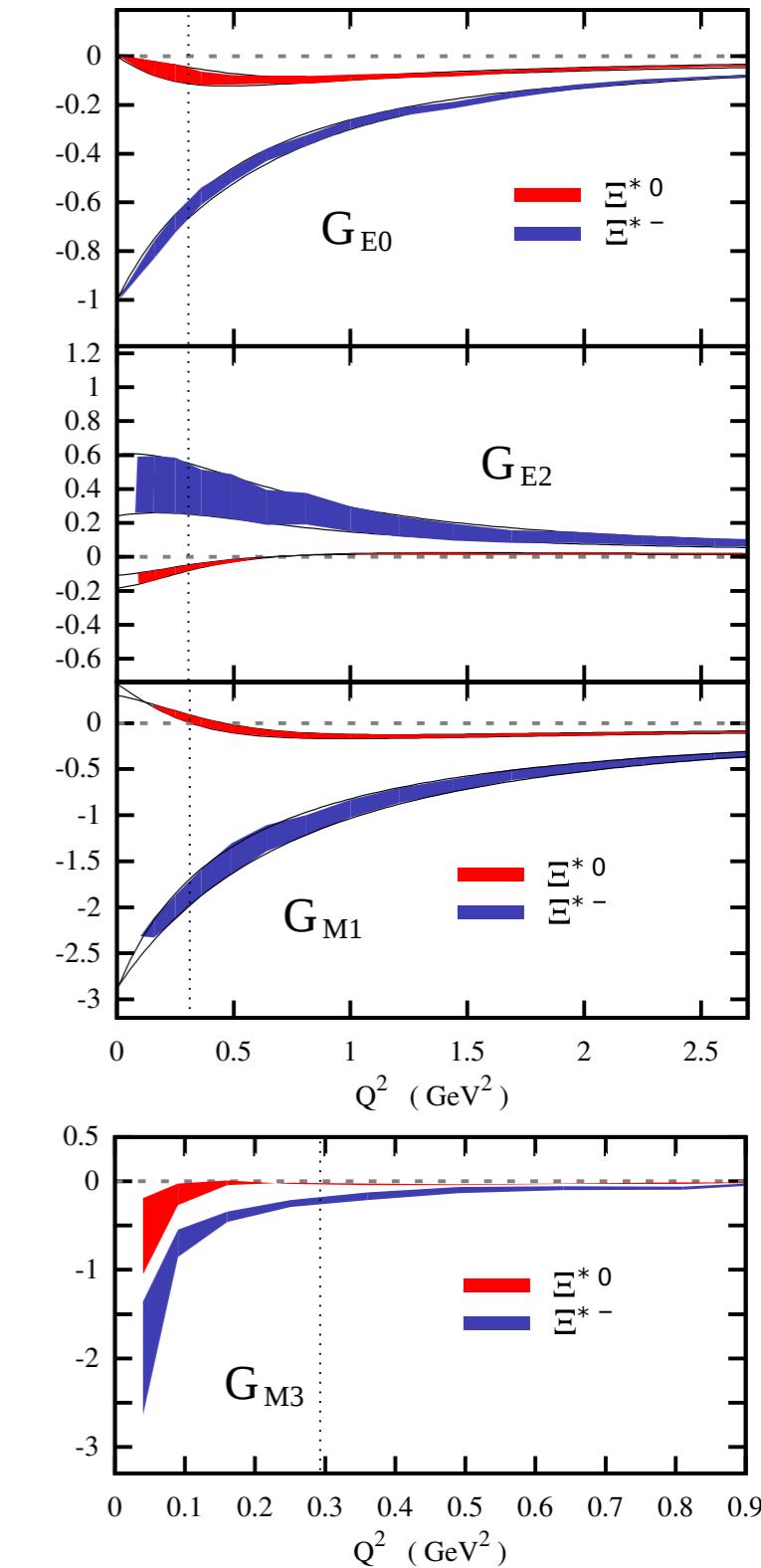
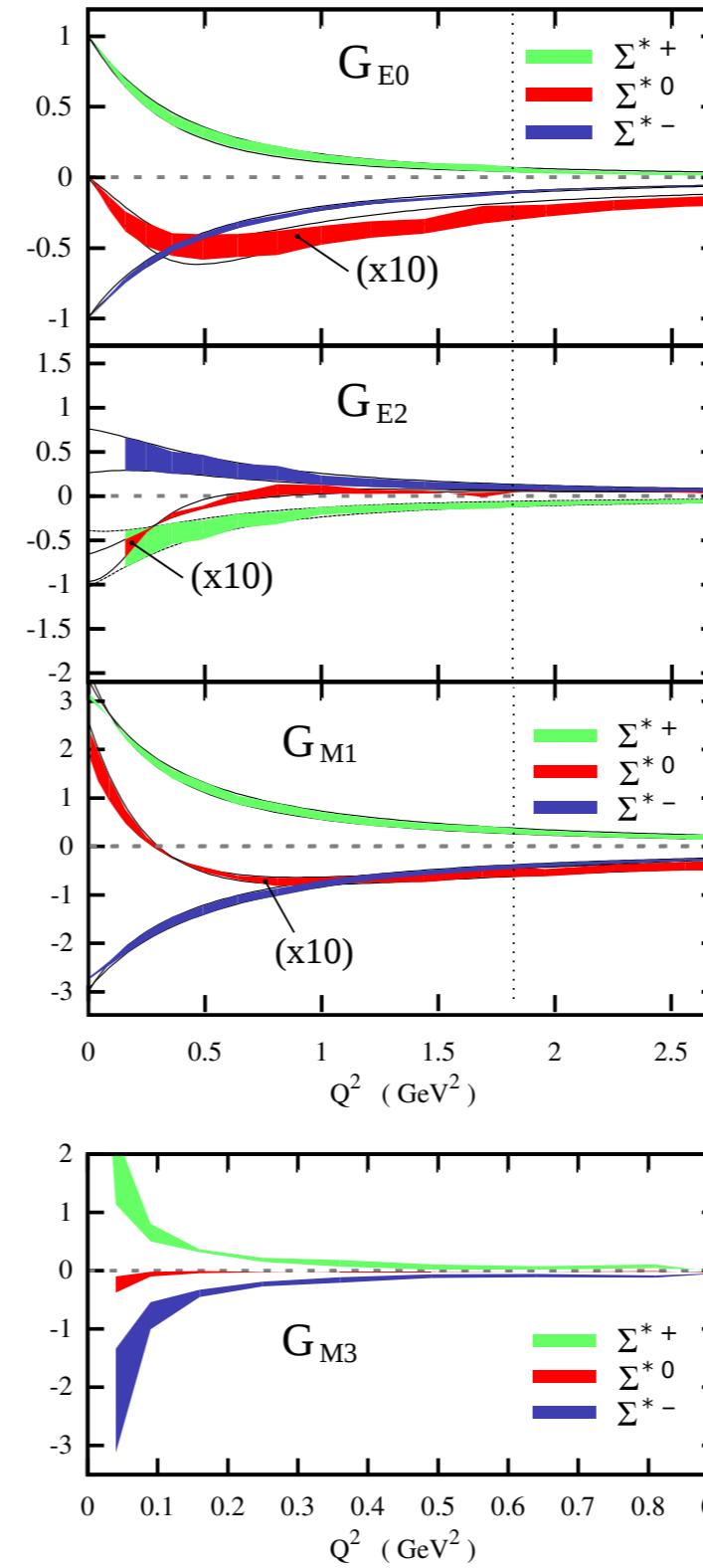
● may serve to distinguish between qqq and q-dq !

Sanchis-Alepuz, Williams, Alkofer, PRD87 (2013)
Nicmorus, Eichmann, Alkofer, PRD82 (2010)

Strange form factors: octet and decuplet



● Decuplet: prediction

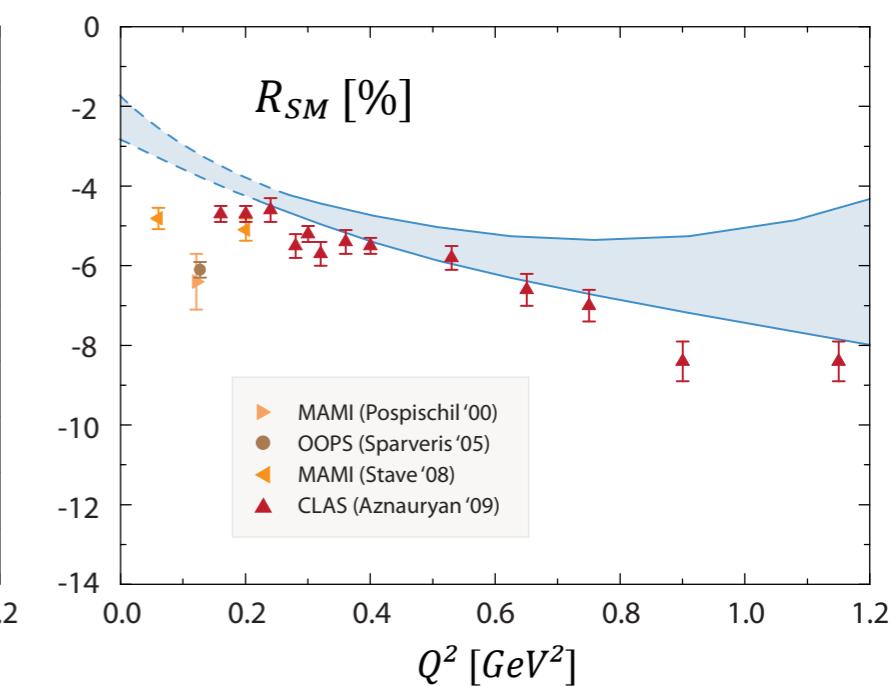
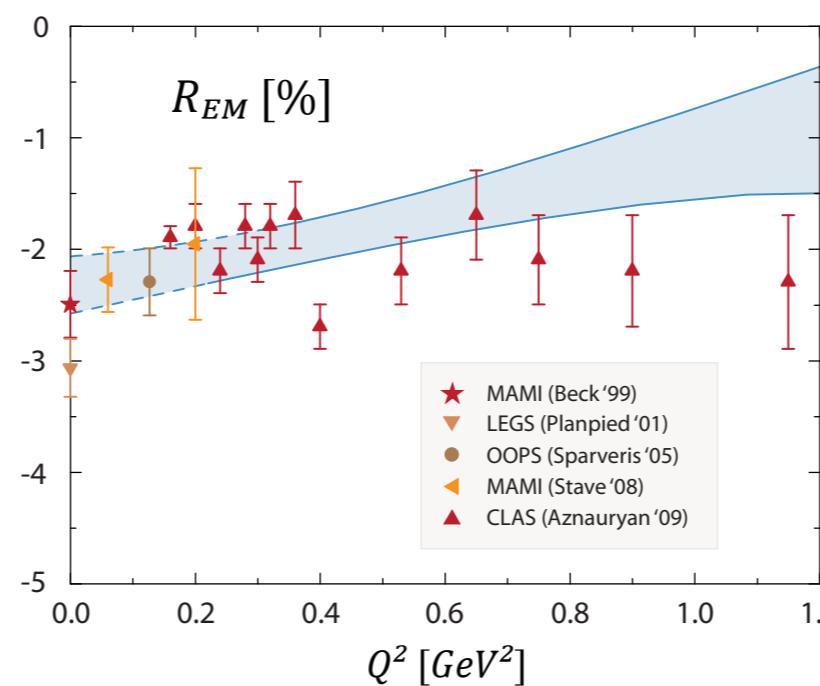
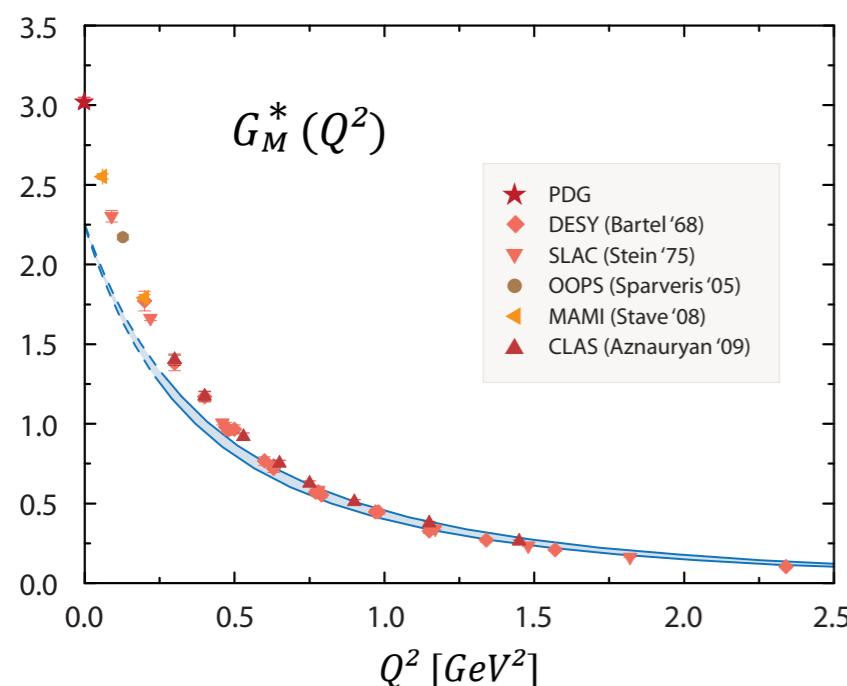


DSE: Sanchis-Alepuz, CF, EPJA 52 (2016)
Lattice: Shanahan et al, PRD 89 (2014), PRD 90 (2014)

Transition form factor: $N\Delta\gamma$

$$R_{EM} = -\frac{G_E^*}{G_M^*},$$

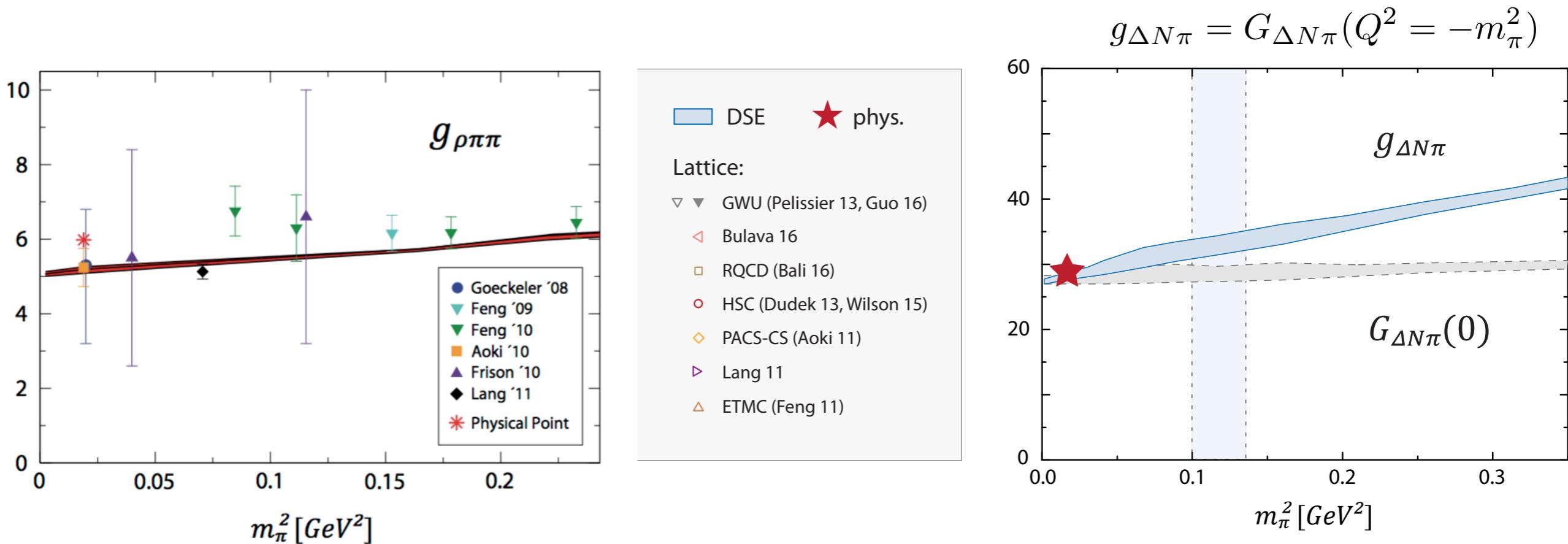
$$R_{SM} = -\frac{|\vec{Q}|}{2M_\Delta} \frac{G_C^*}{G_M^*}$$



- R_{EM} highly dominated by p-waves !

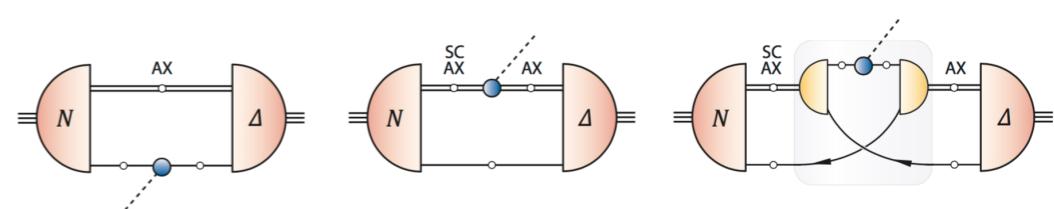
Eichmann, Nicmorus, PRD 87 (2012)

Decays

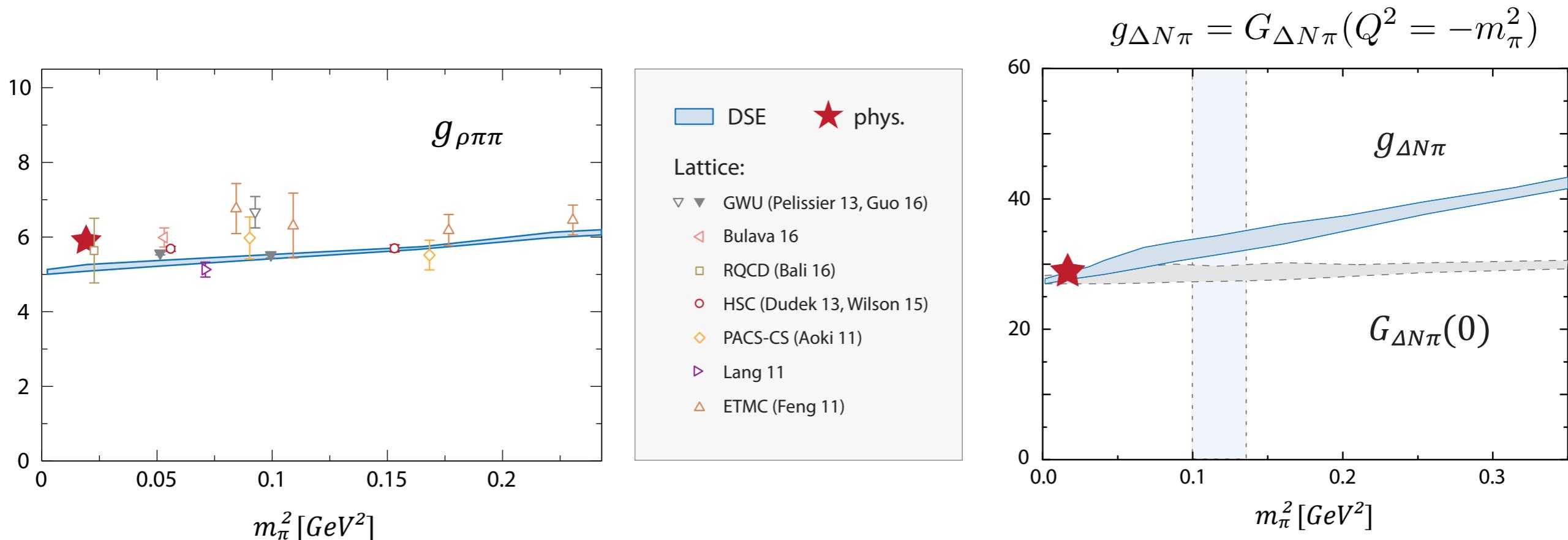


Mader, Eichmann, Blank, Krassnigg PRD84 (2011)

- Decay constants can be calculated in rainbow-ladder (although bound states have no width)
- Good agreement with lattice and experiment

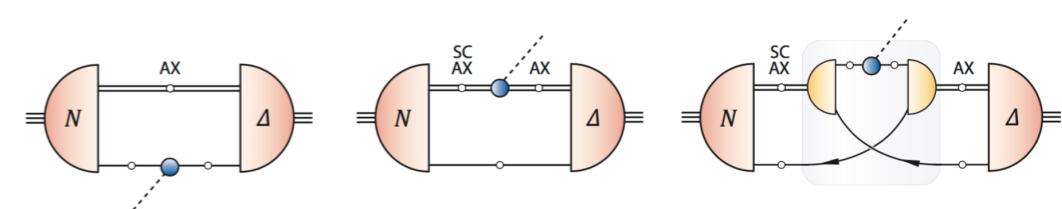


Decays

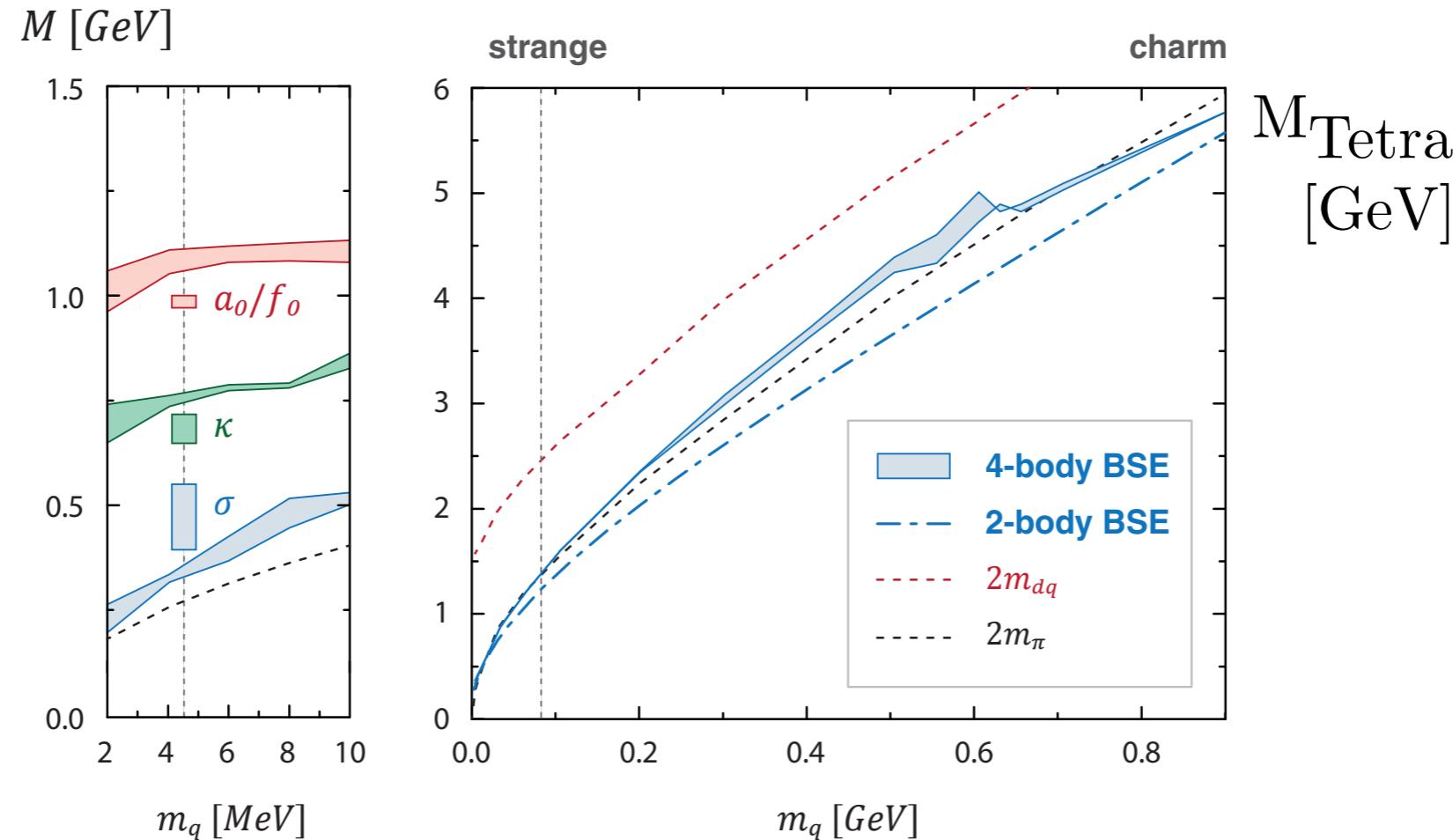


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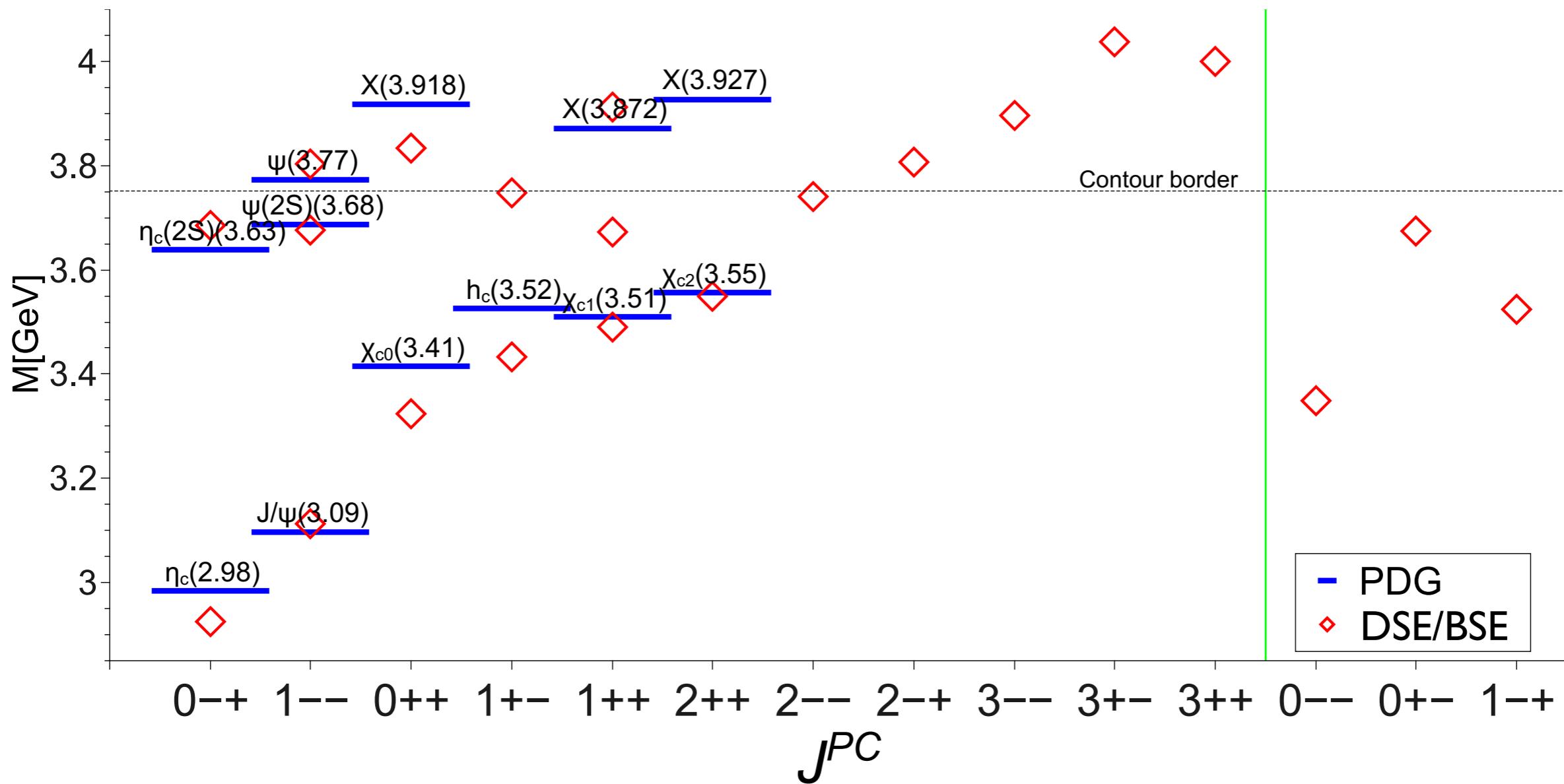
Mass evolution of tetraquark



- Resonance becomes bound state for large m_q
- Dynamical decision: **meson clusters, not diquarks**
- Results:
 - $m_\sigma \sim 350 \text{ MeV}$
 - $m_\kappa \sim 750 \text{ MeV}$
 - $m_{a_0, f_0} \sim 1080 \text{ MeV}$
 - $m_{ss\bar{s}\bar{s}} \sim 1.5 \text{ GeV}$
 - $m_{cc\bar{c}\bar{c}} \sim 5.7 \text{ GeV}$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

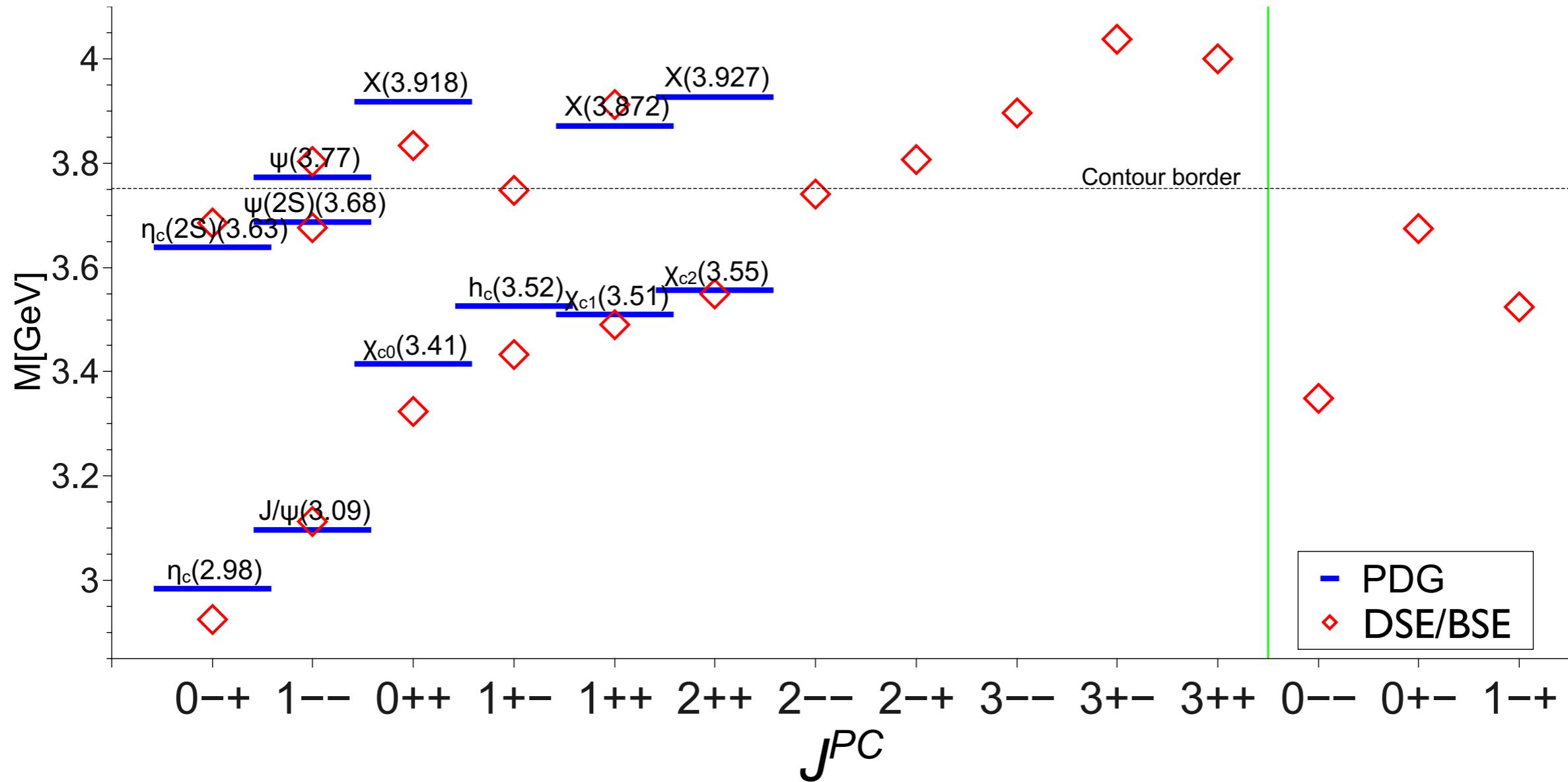
Charmonium spectrum



- good channels: $1--, 2^{++}, 3--, \dots$
- acceptable channels: $0-+, 1^{++}$
- deficiencies in other channels: missing spin-structure

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

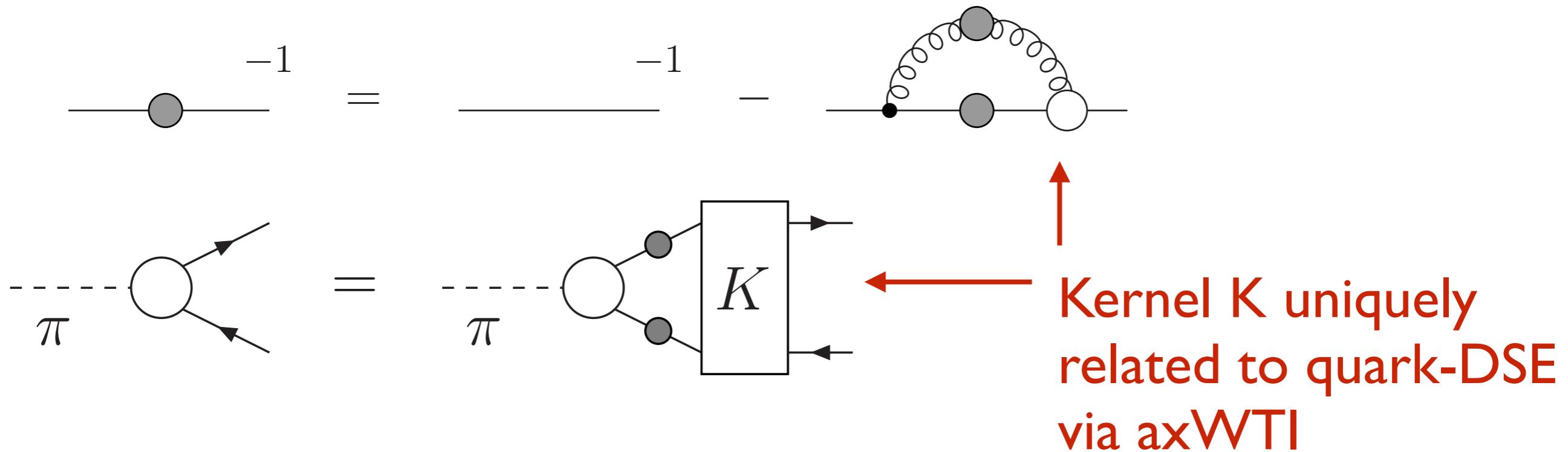
Charmonium spectrum



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- acceptable channels: $0^{-+}, 1^{+-}$
- deficiencies in other channels: missing spin-structure

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

Theoretical Tools II: DSEs and BSEs



→ Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics
- Need approximations for dressed quark-gluon vertex