JLU
NEUE WEGE. SEIT 1607.

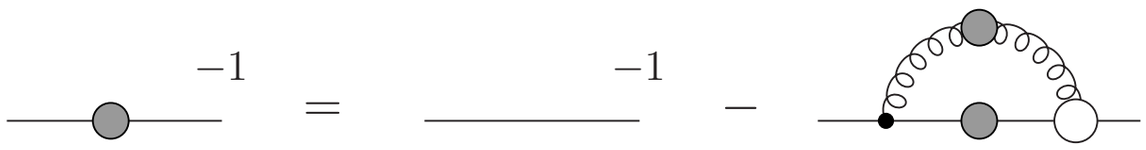
JUSTUS-LIEBIG-
UNIVERSITÄT
GIESSEN

HFHF
Helmholtz Forschungsakademie Hessen für FAIR

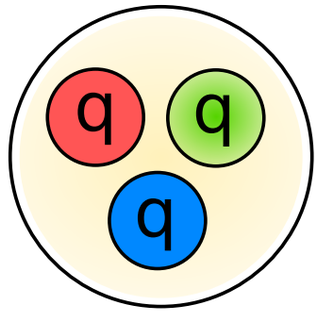
Hadron spectra and properties from functional methods

Overview

1. Mass from nothing

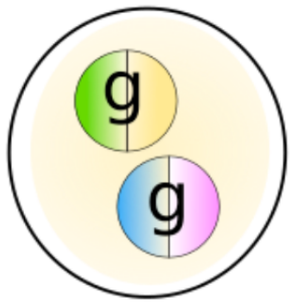


2. Baryons



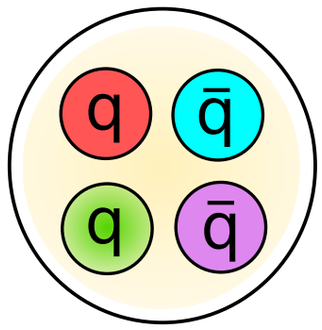
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3. Glueballs



CF, Huber, Sanchis-Alepuz, EPJC 80 11, 1077 (2020), arXiv:2004.00415

4. (Heavy-light) Tetraquarks



Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, arXiv:1905.02615
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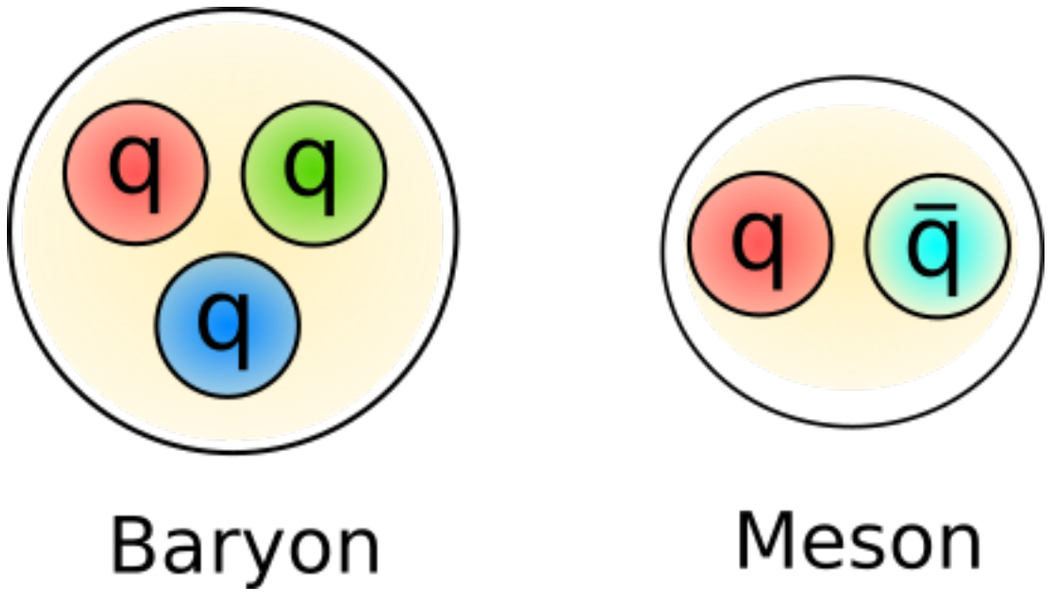
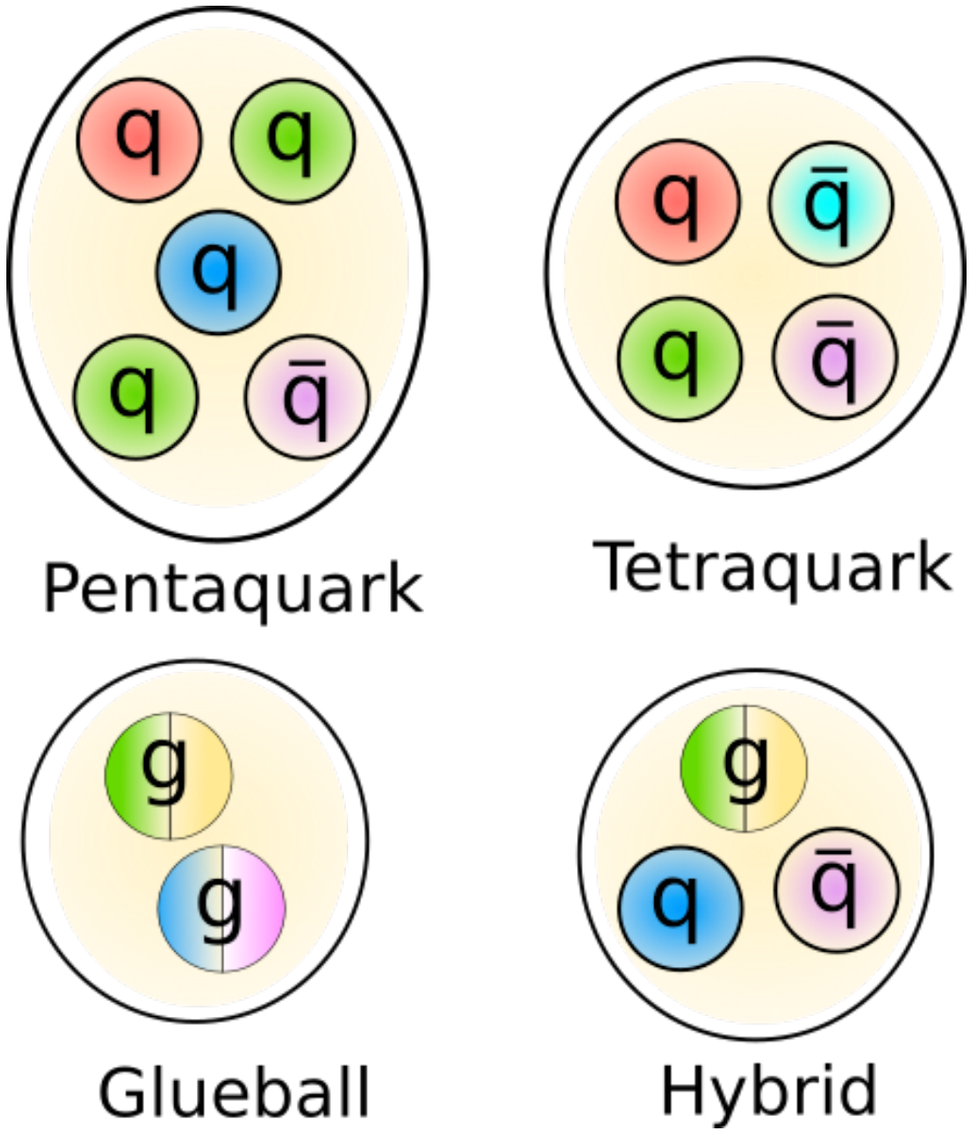
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495

Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4,38, arXiv:2008.10240

Hadrons: baryons, mesons and ... exotics !

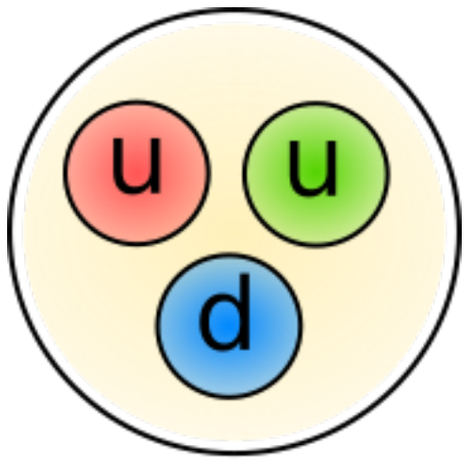




ordinary hadrons

'exotic' hadrons

Properties of QCD: Dynamical mass generation



m_{proton} = 938 MeV

Dynamical quark masses via weak force

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000

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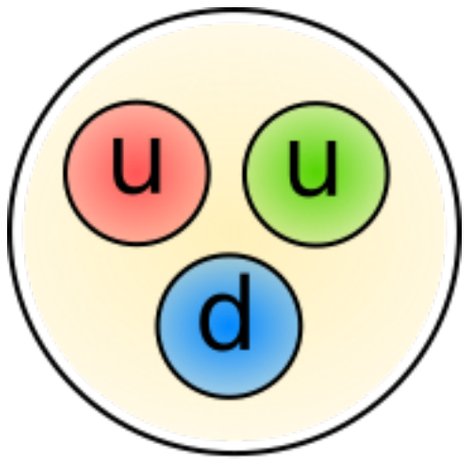

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Dynamical quark masses via weak force and strong force:

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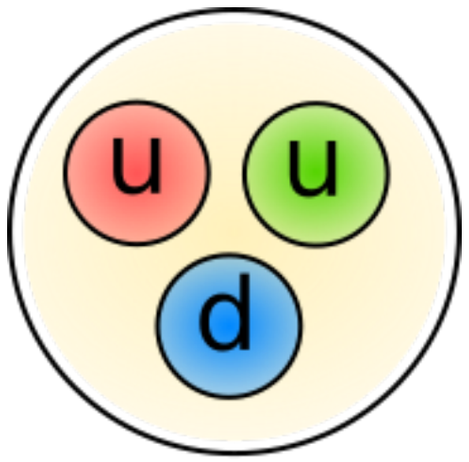

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Yoichiro Nambu,
Nobel prize 2008

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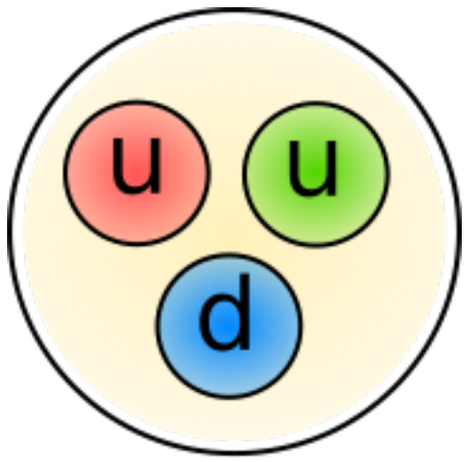

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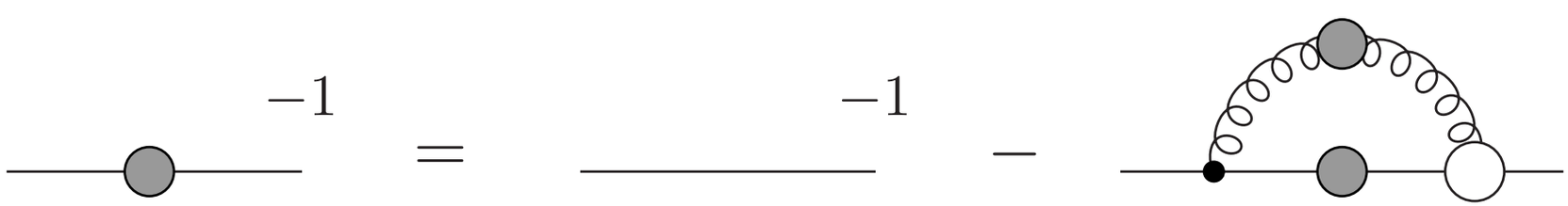

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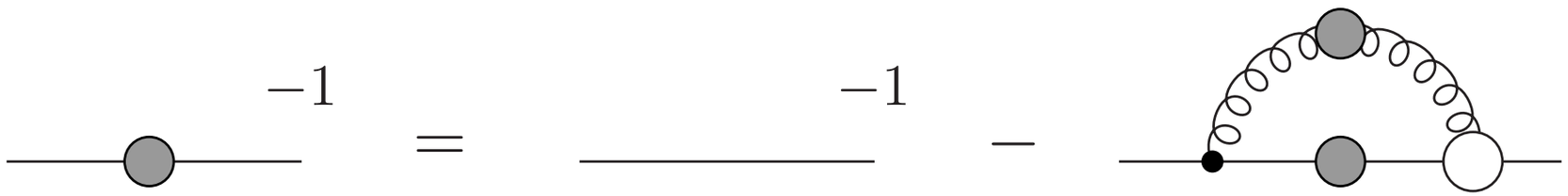

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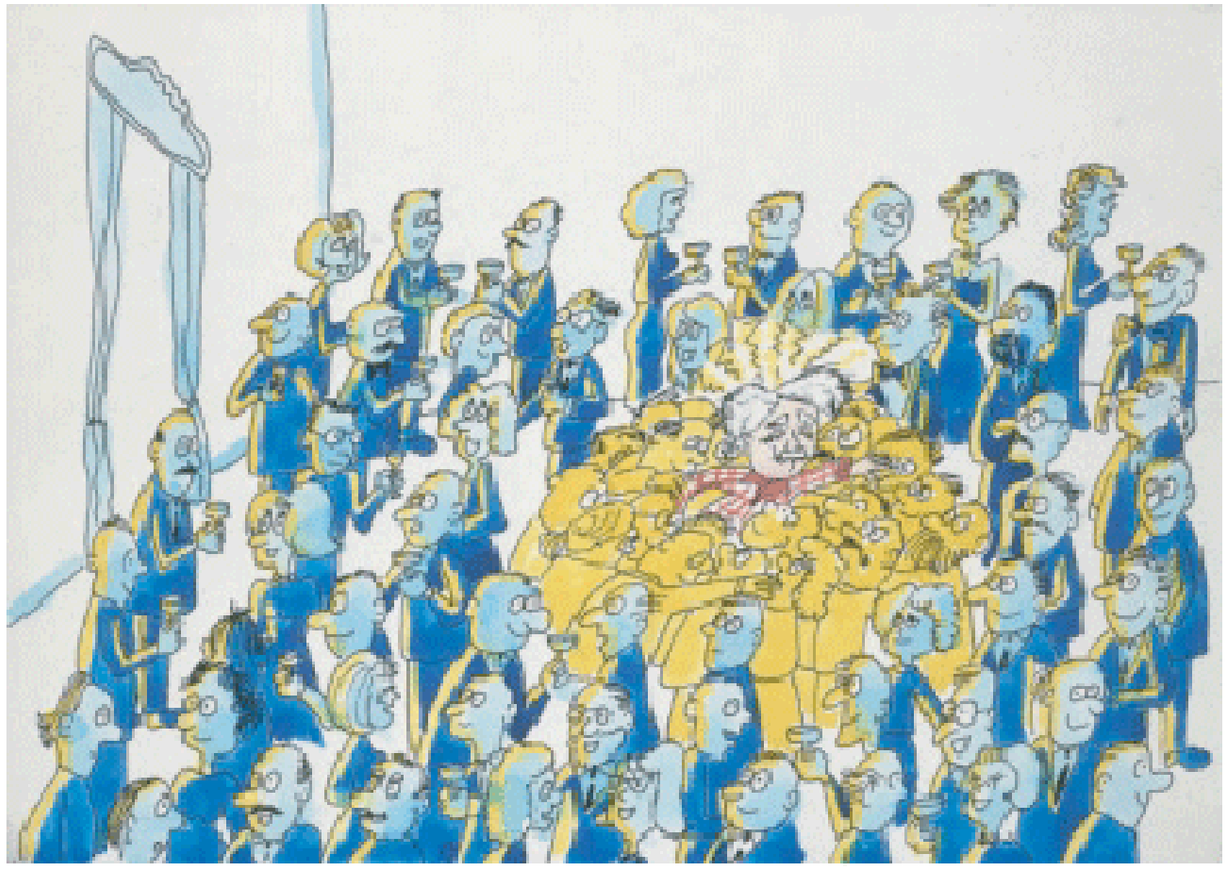
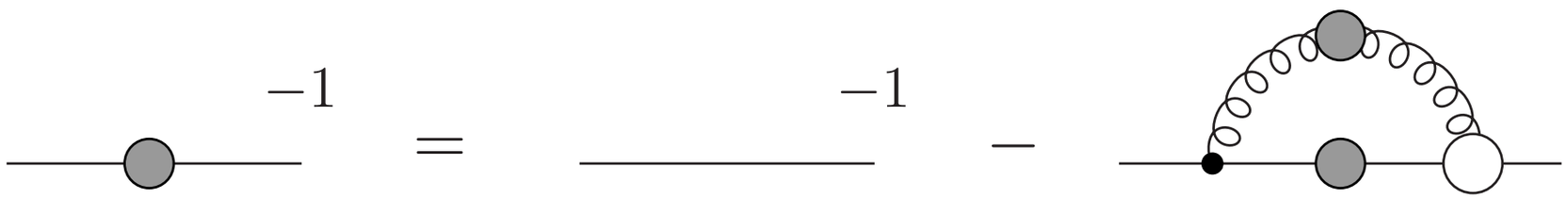
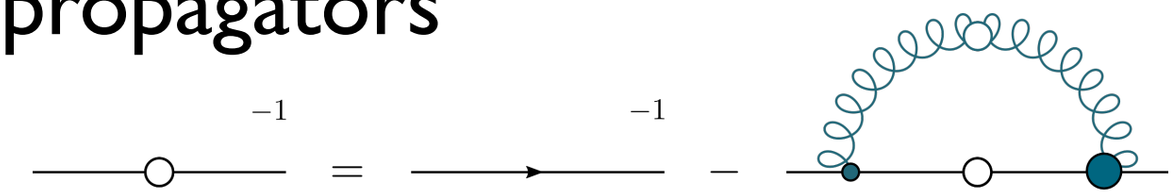




Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, EPJ C77 (2017) no.11, 733

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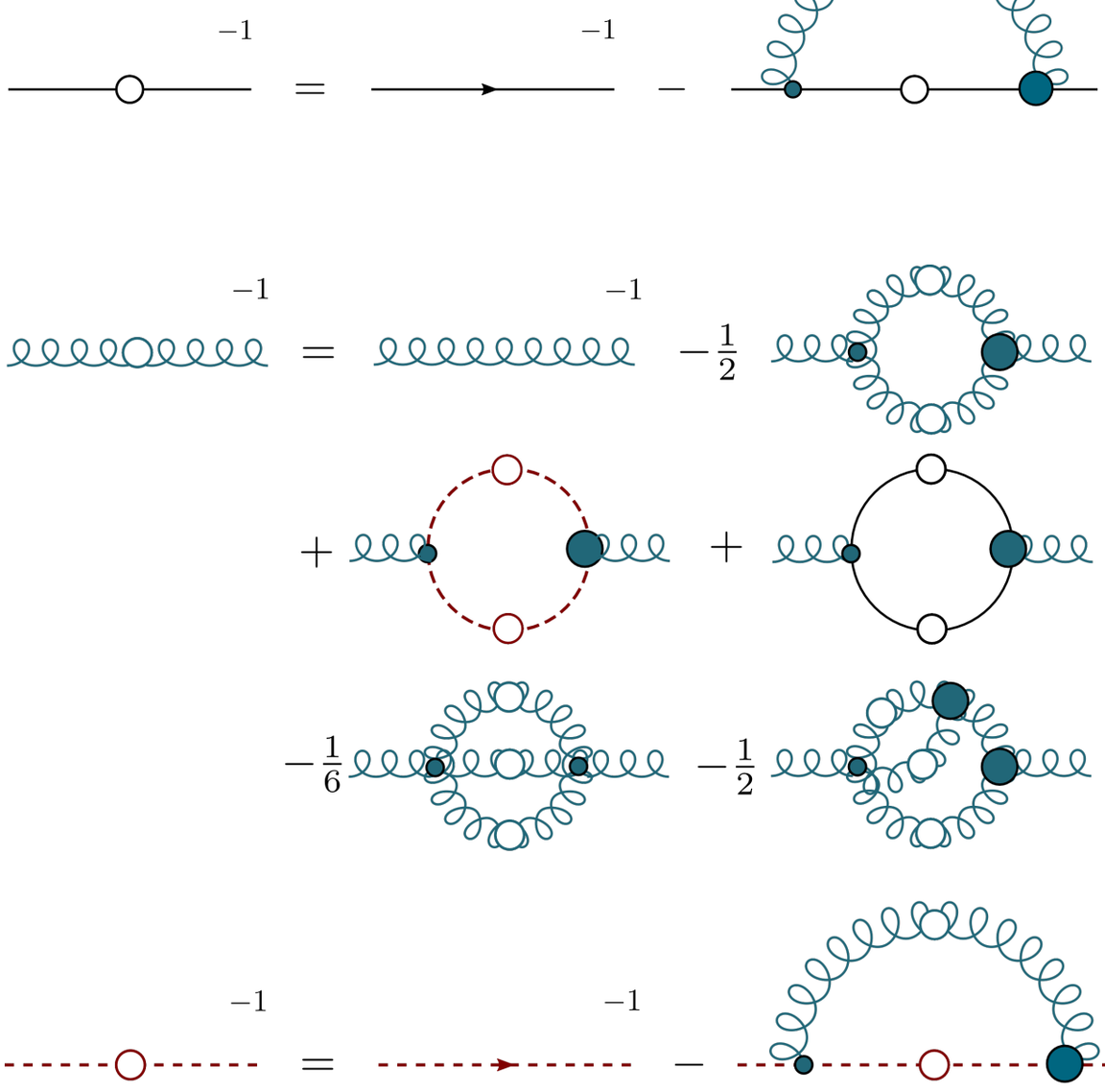

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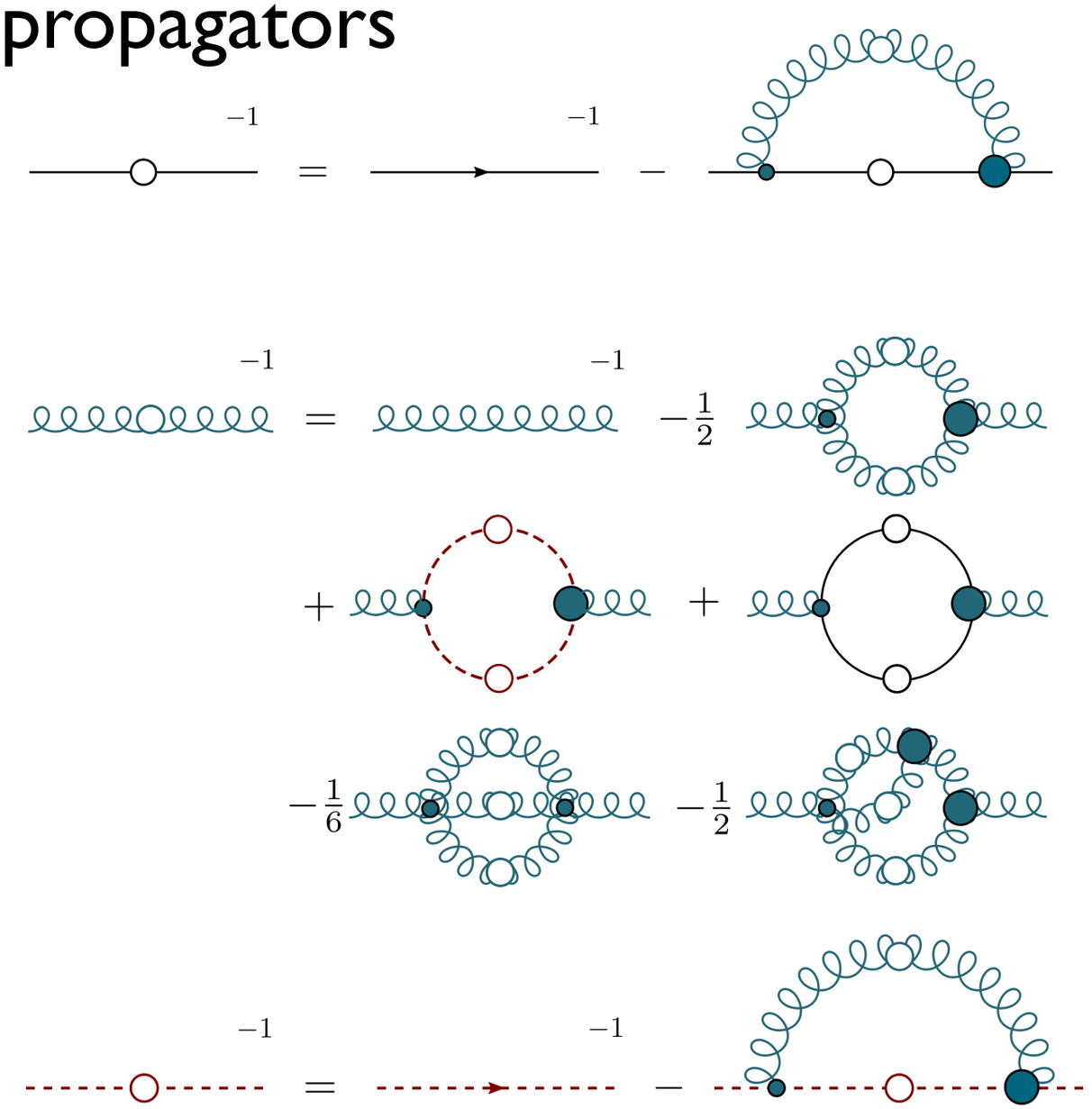
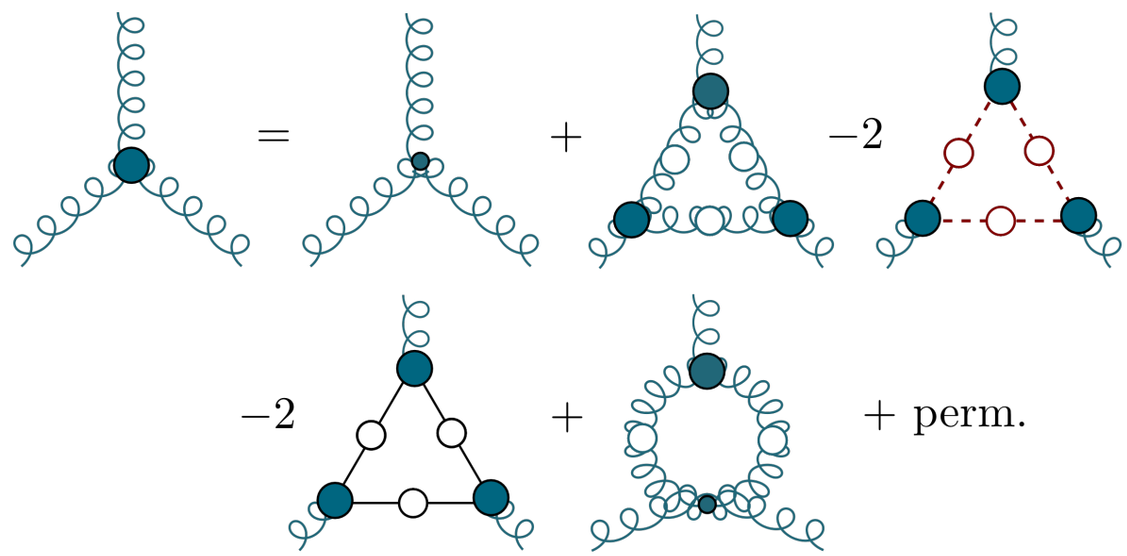


vertices



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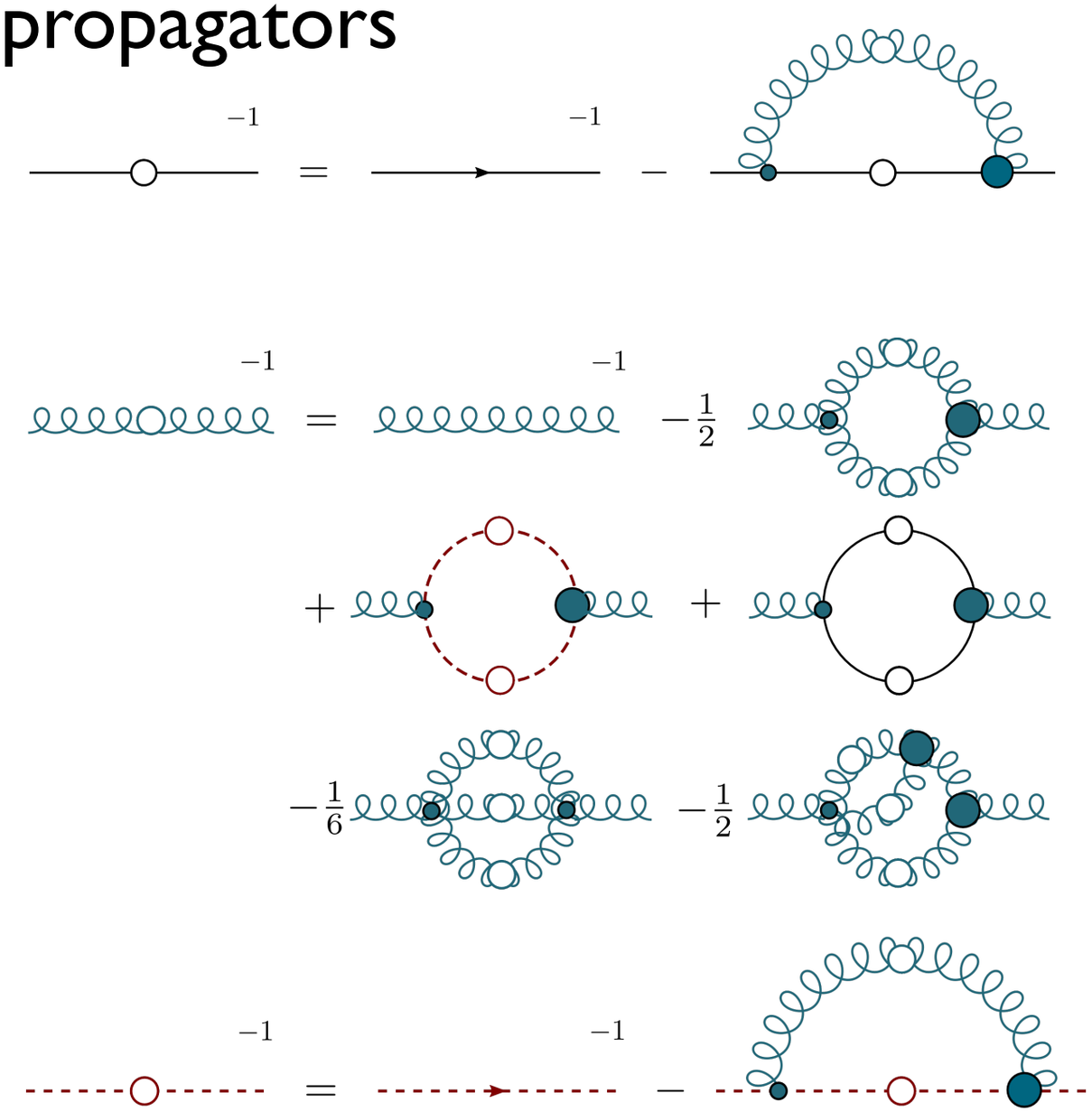

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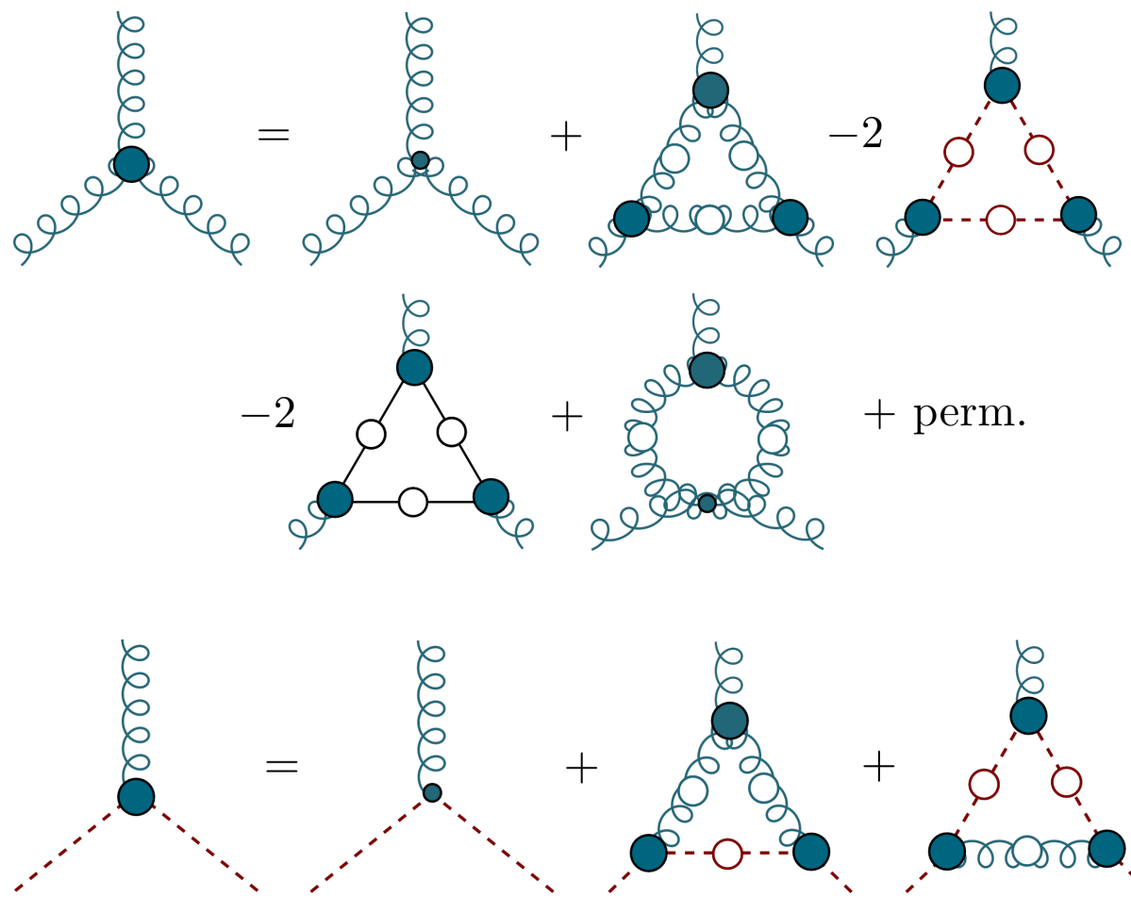

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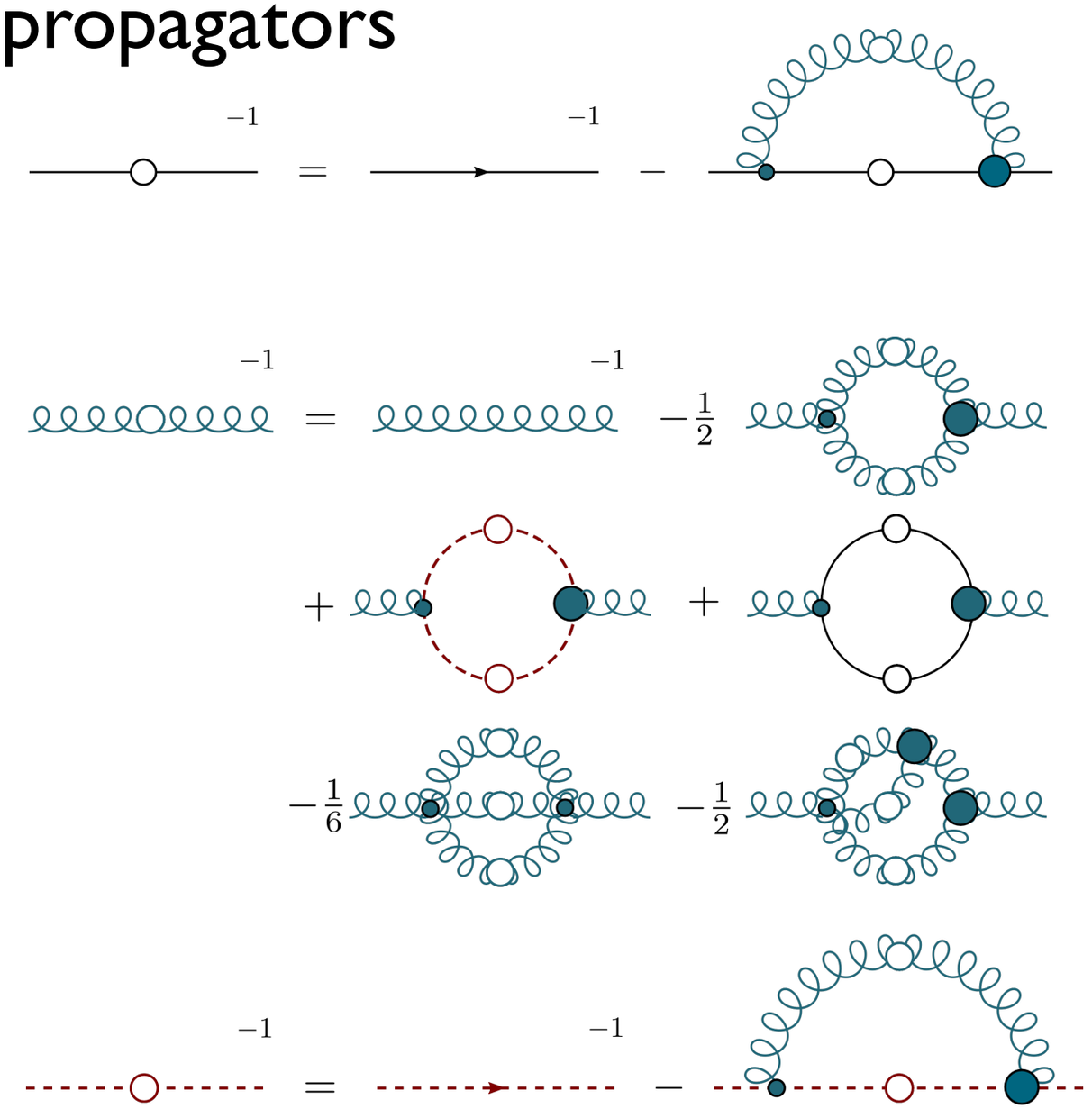

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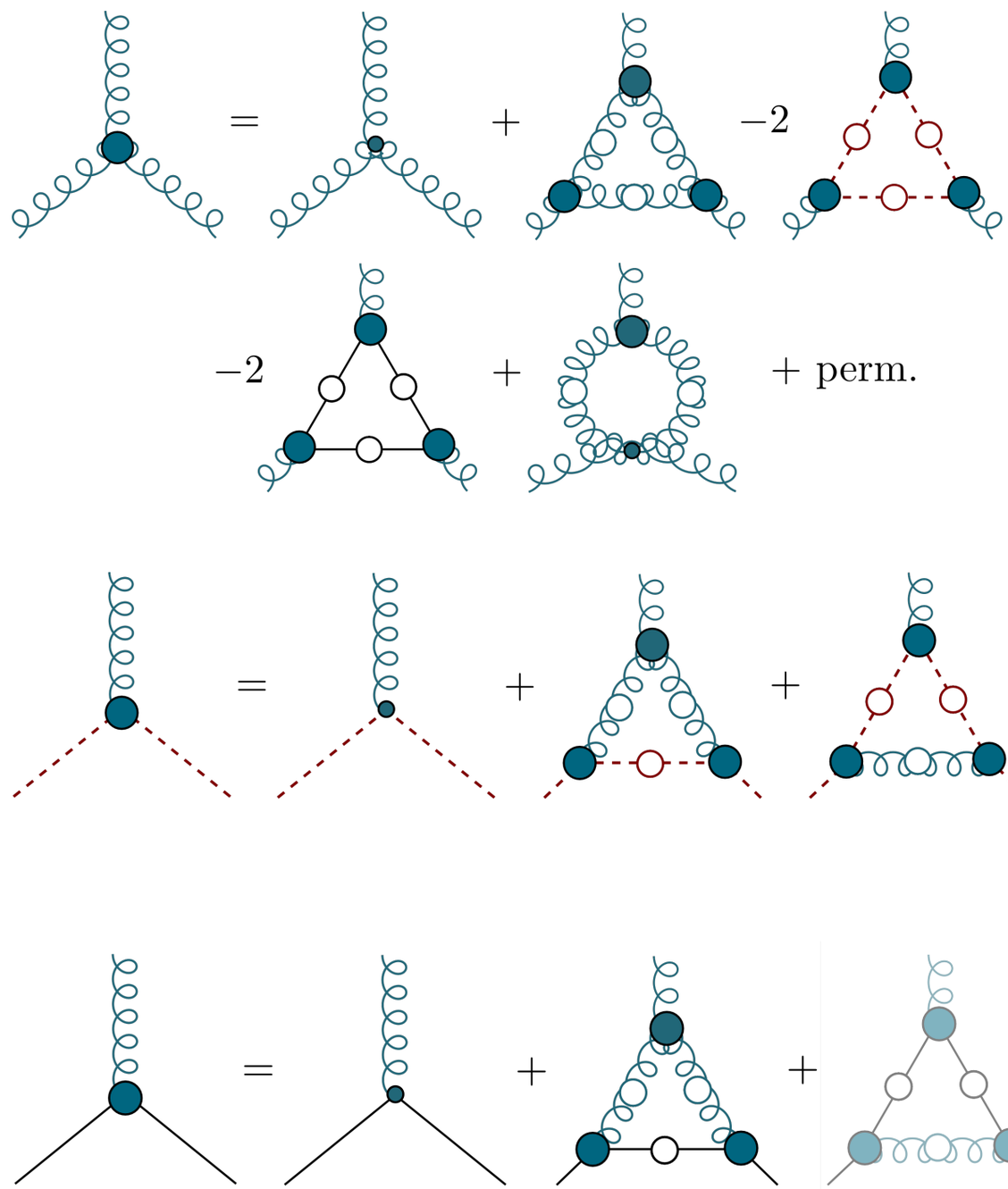

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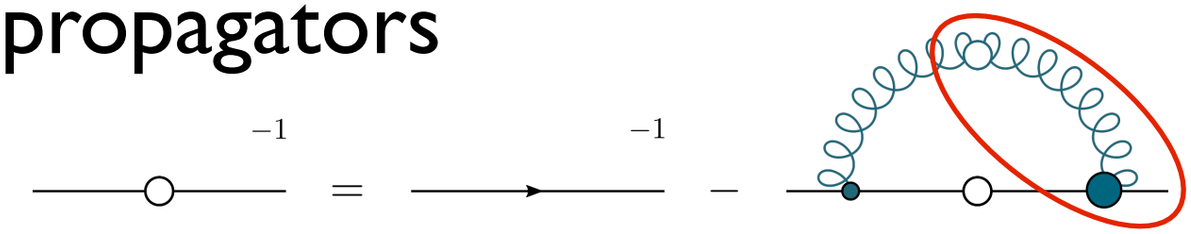
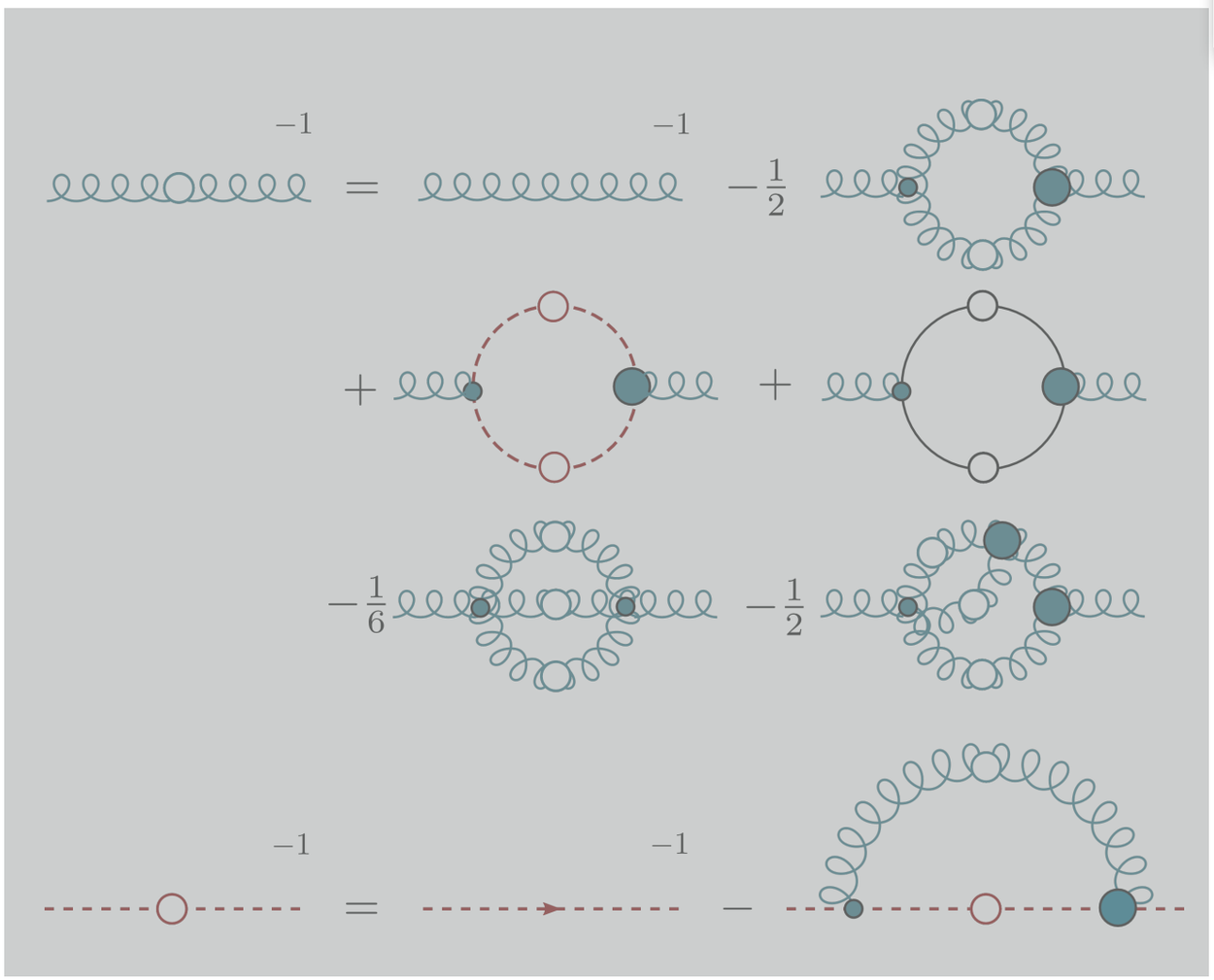
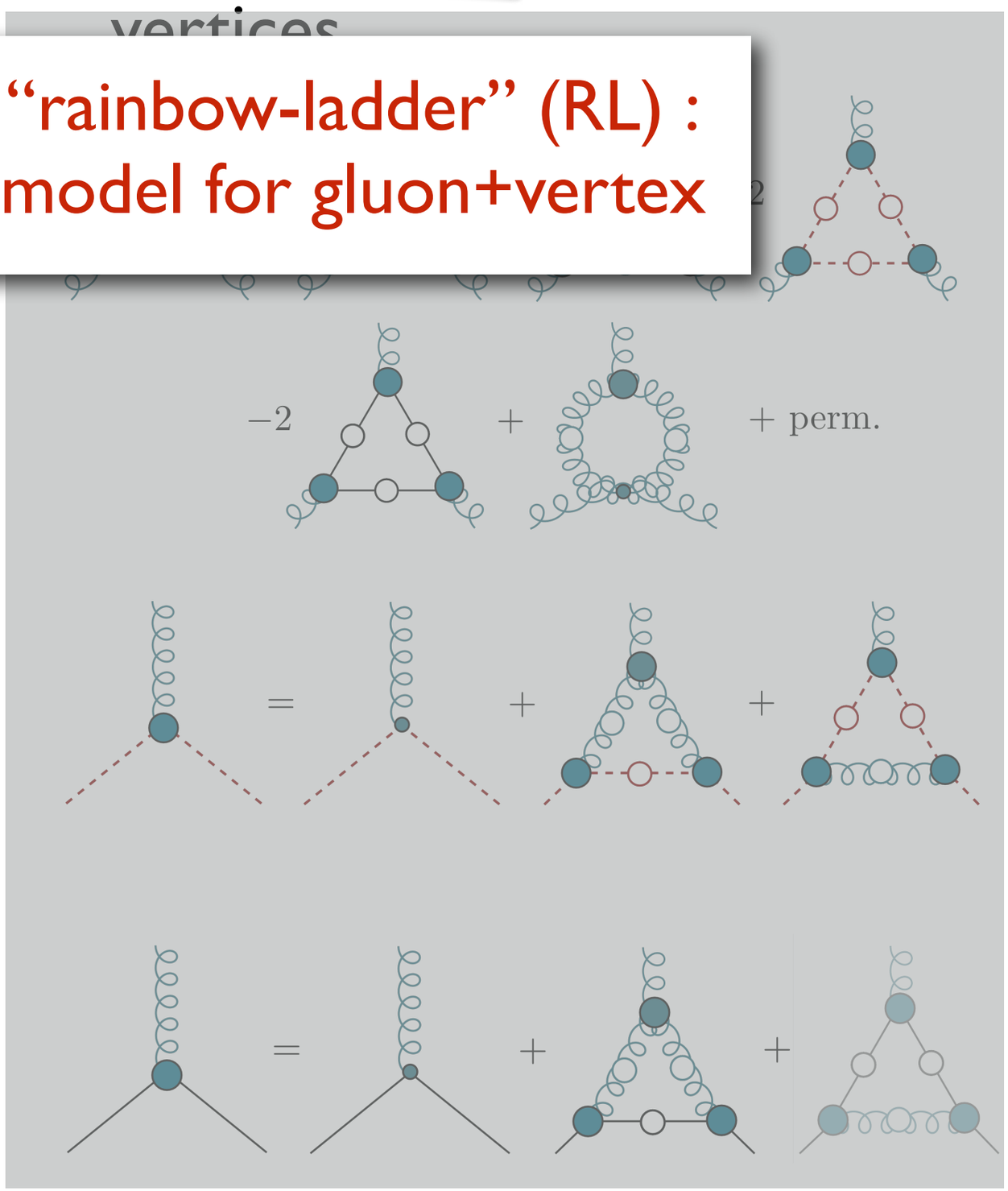




"rainbow-ladder" (RL) : model for gluon+vertex

vertices



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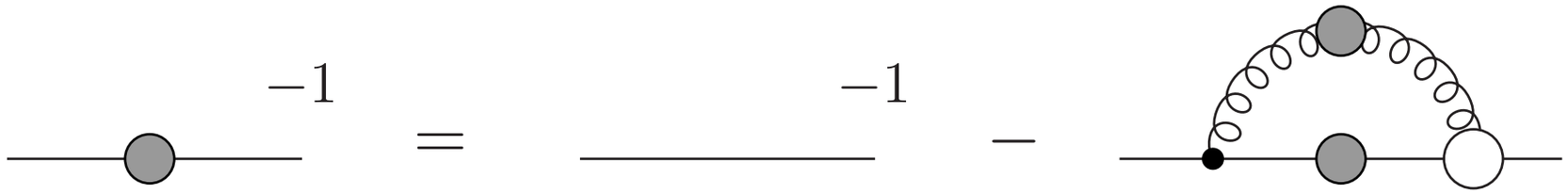
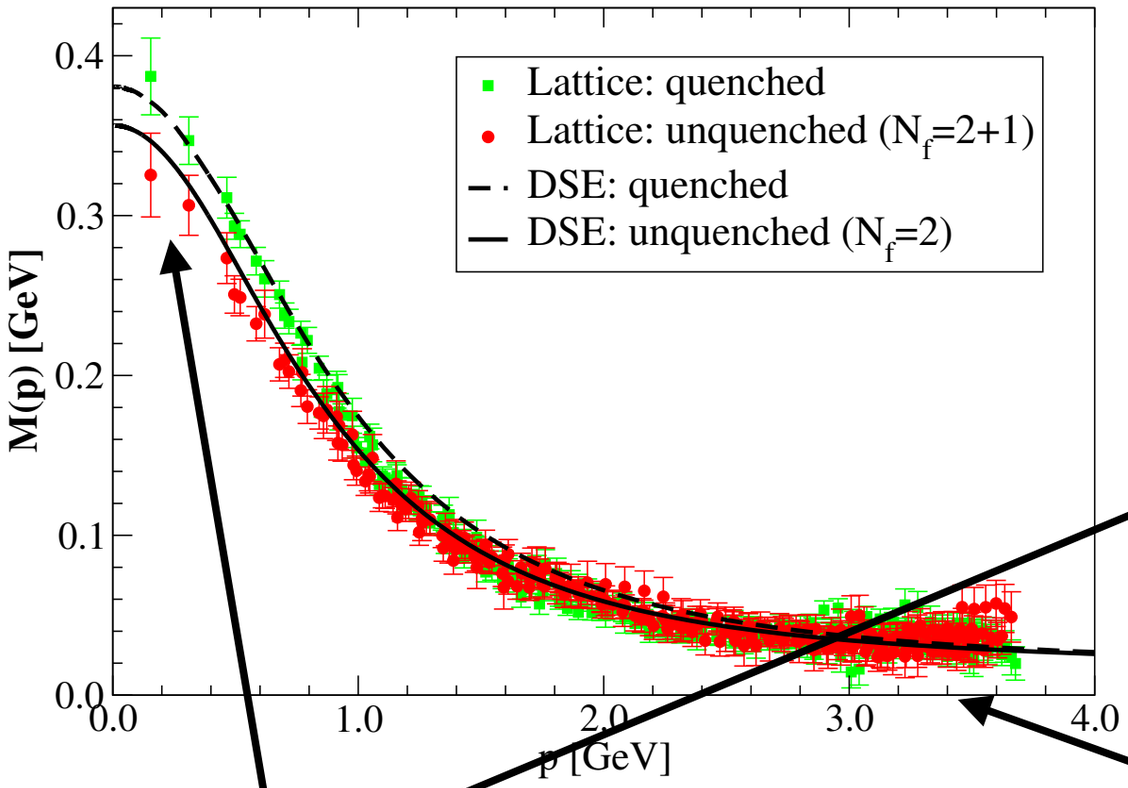
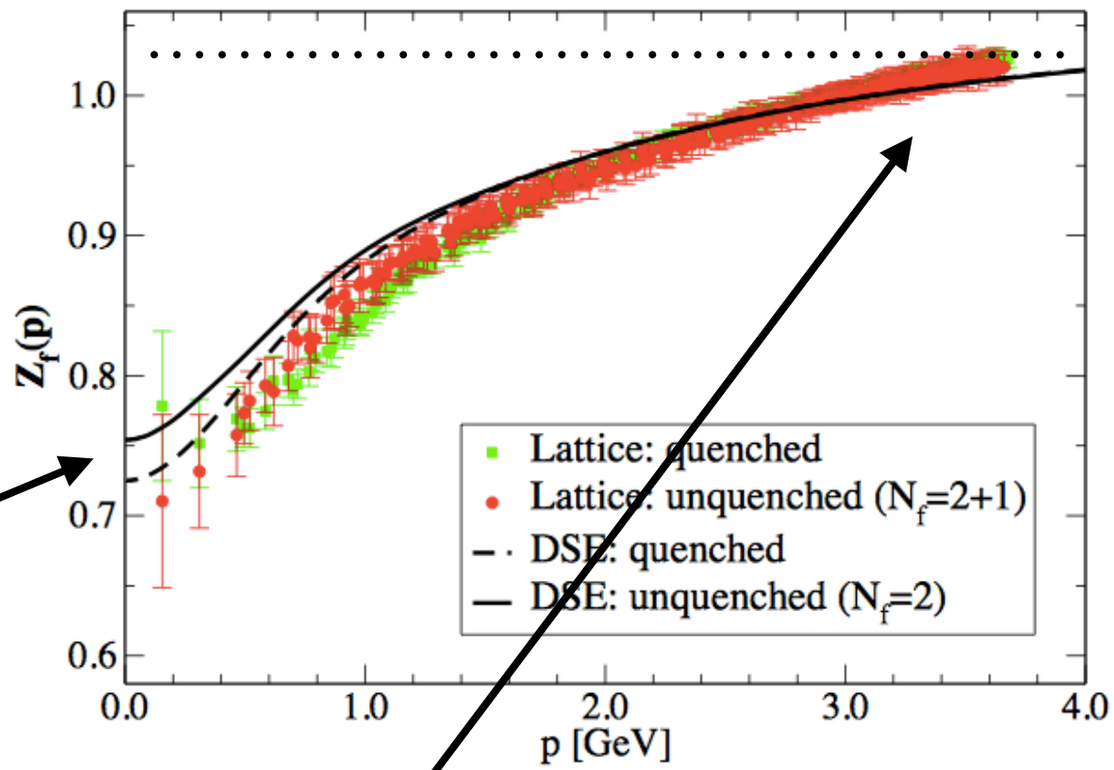
Quarks: mass from interaction



$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

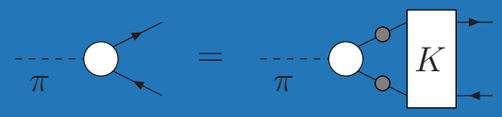


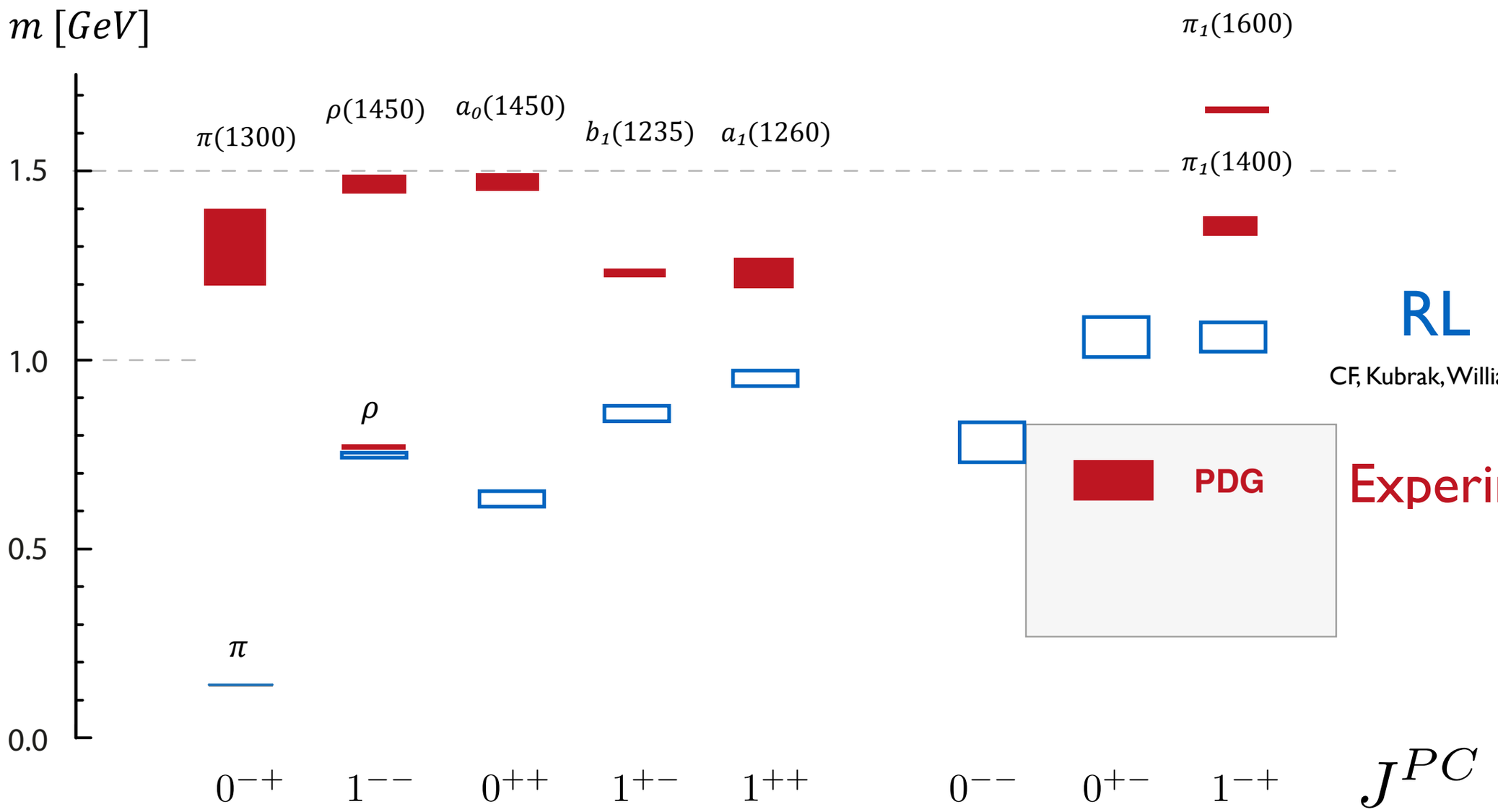


'constituent quark':
large mass; very composite

'current quark':
- small mass; non-composite

Light meson spectrum





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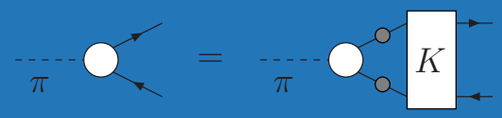

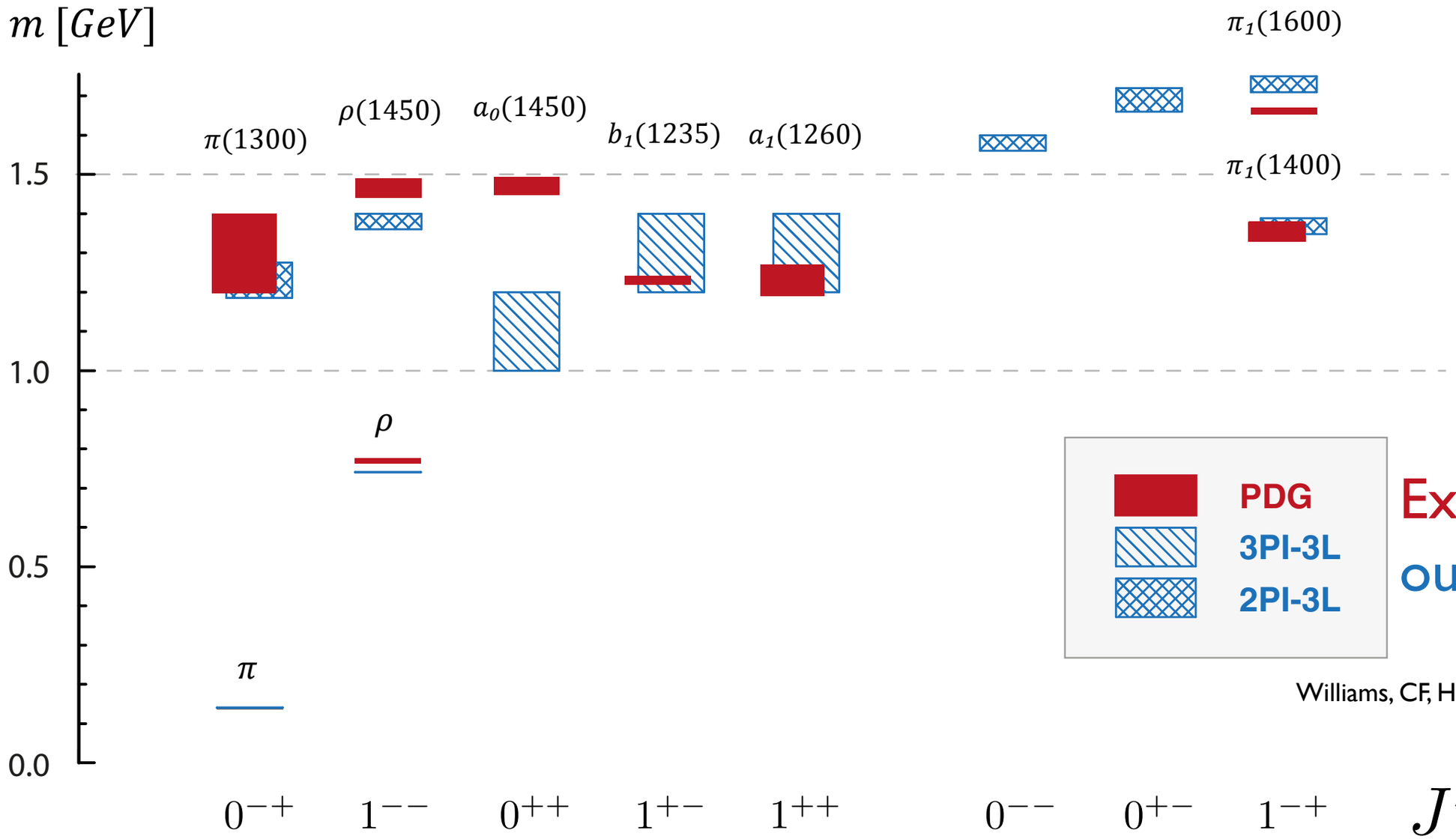

Experiment
our results

Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

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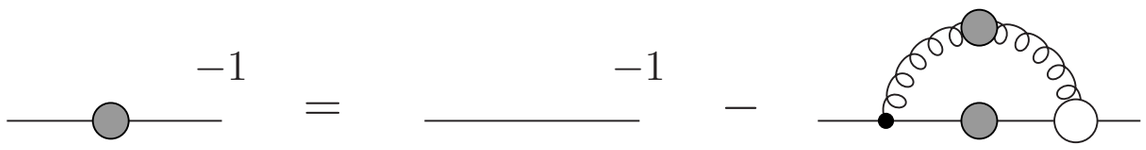

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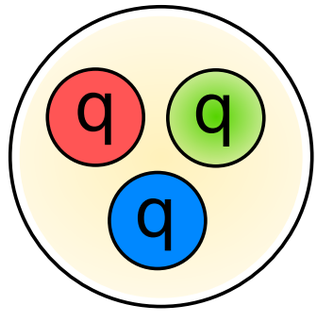

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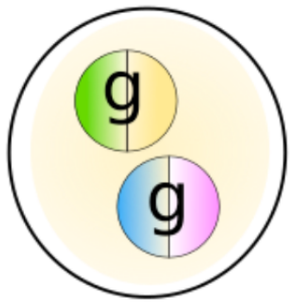

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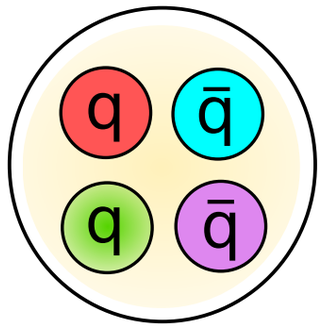

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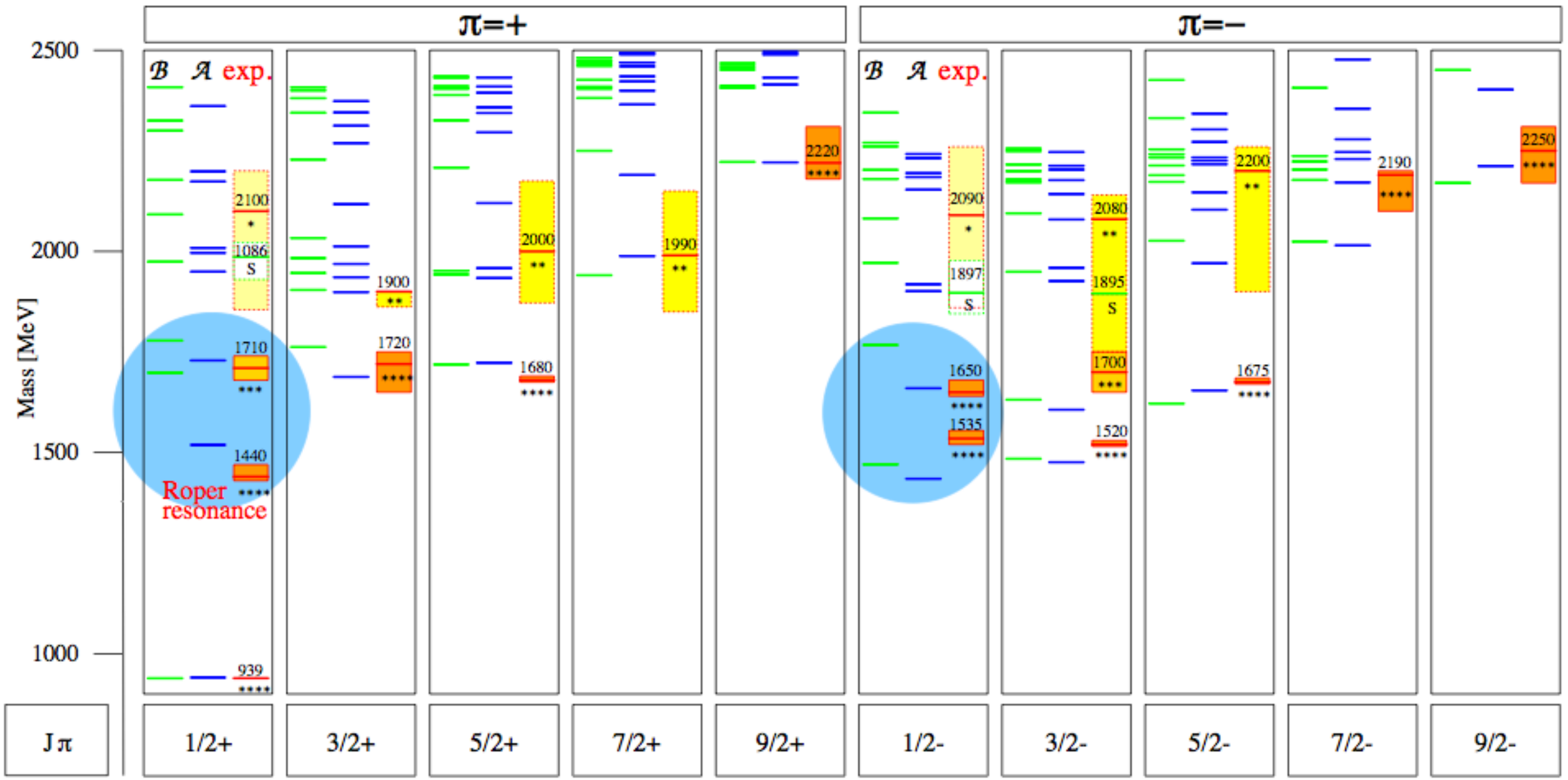
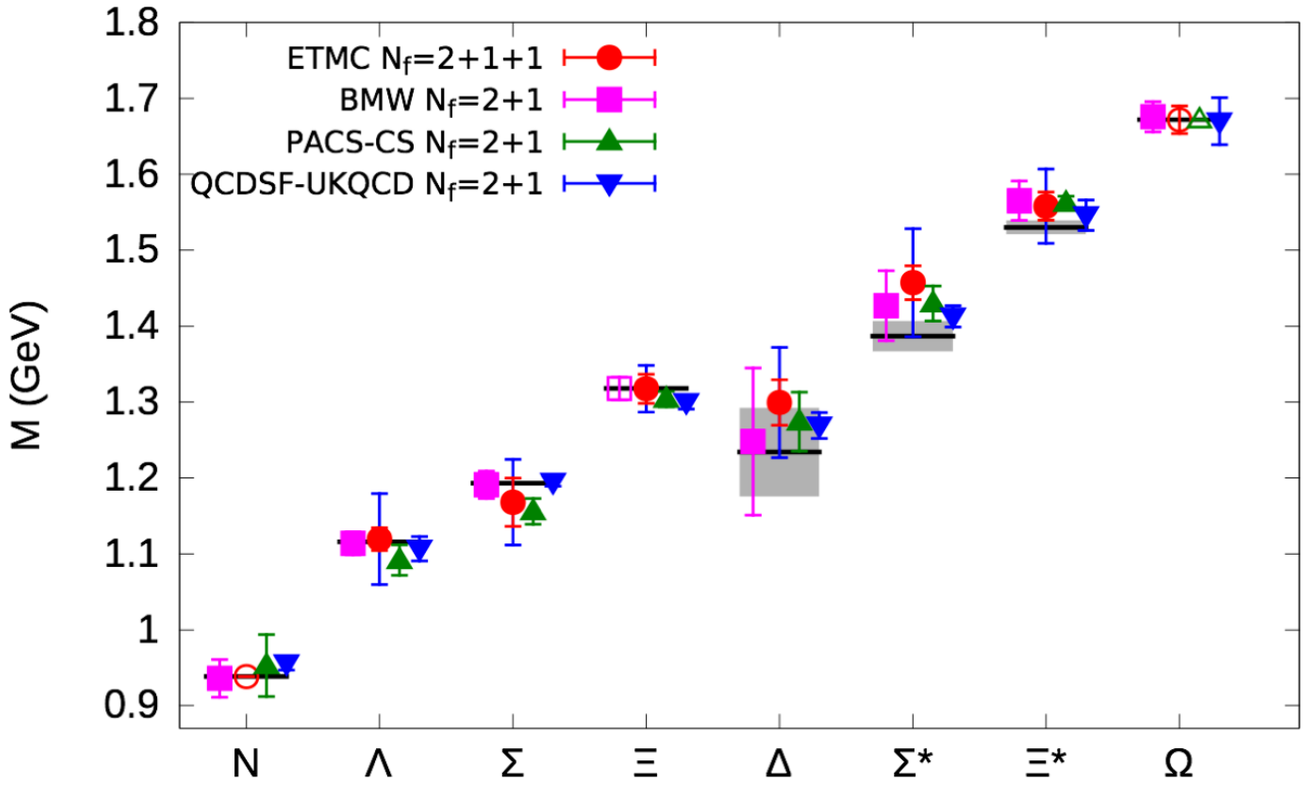
Light baryon spectrum - quark model



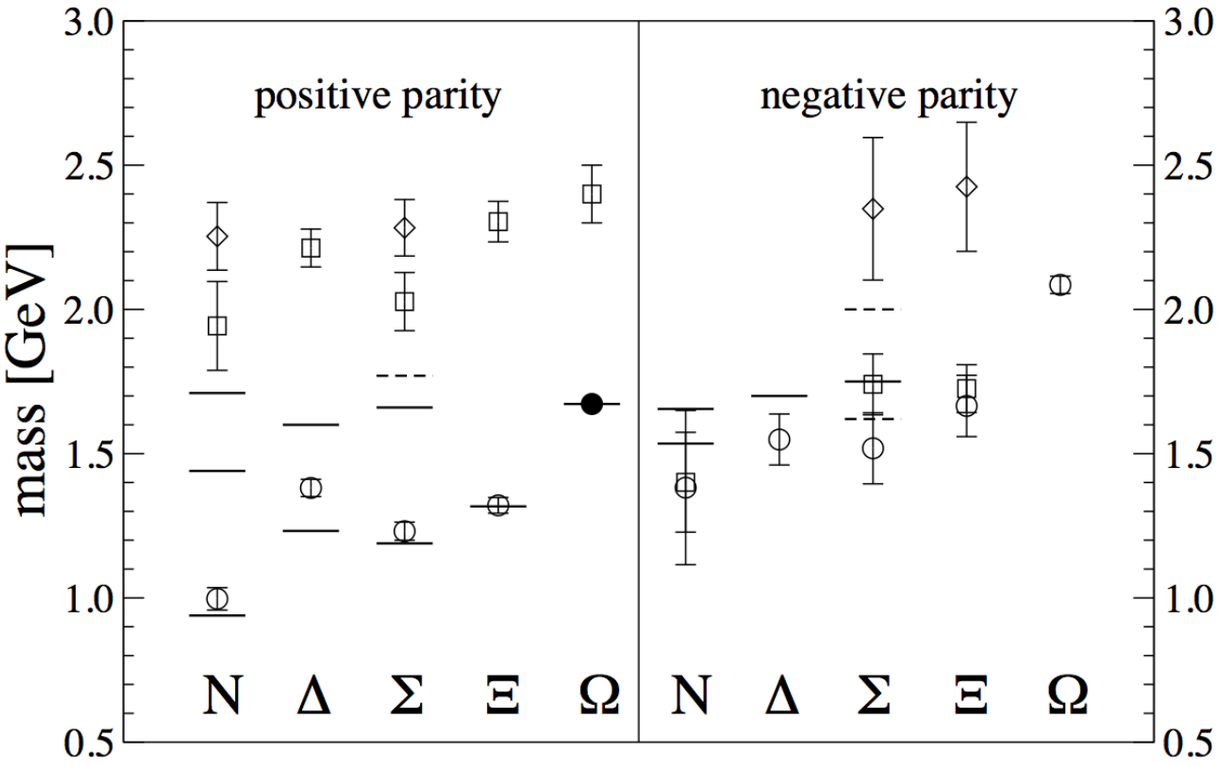
Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances'
- level ordering: $N_{\frac{1}{2}+}$ vs. $N_{\frac{1}{2}-}$

Lattice QCD



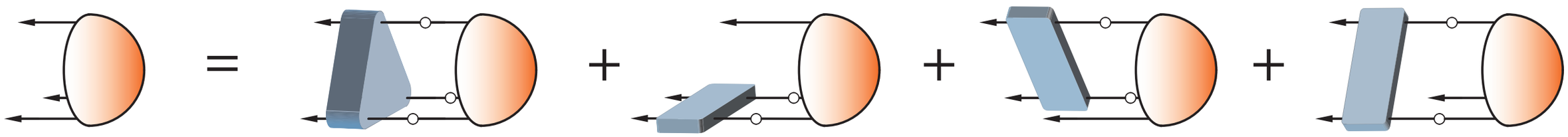
Alexandrou et al. (ETMC), PRD 90 (2014) 7



Engel et al. (BGR), PRD82 (2010) 034505

- baryon ground states well under control
- baryon excited states: very tough problem

Faddeev - equation



- irreducible three-body forces: may be neglected
- two-body interactions:
  - non-perturbative one-gluon exchange (RL)
  - beyond one-gluon exchange

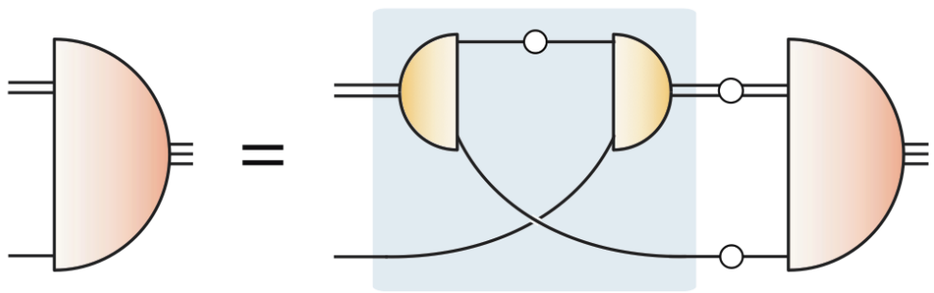
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)

- quark-diquark approximation

Oettel, Hellstern, Alkofer and Reinhardt, PRC 58 (1998) 2459

see also: Bloch, Chang, Chen, Cloet, Thomas, Ramalho, Roberts, Segovia et al.



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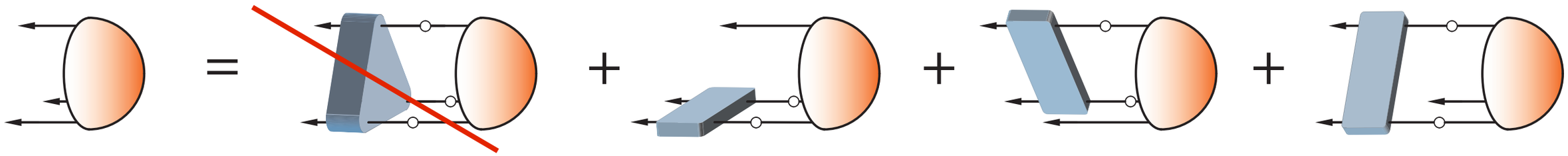

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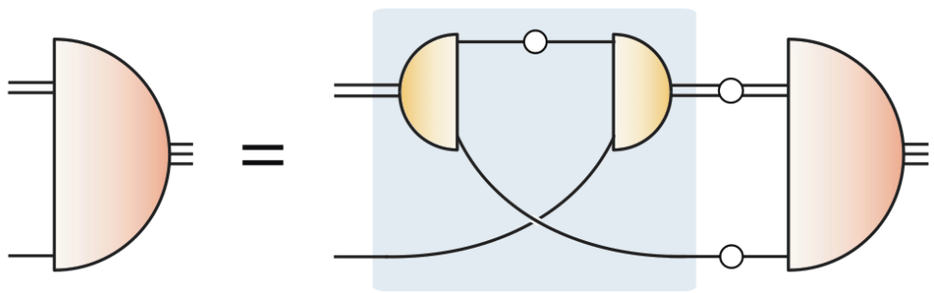
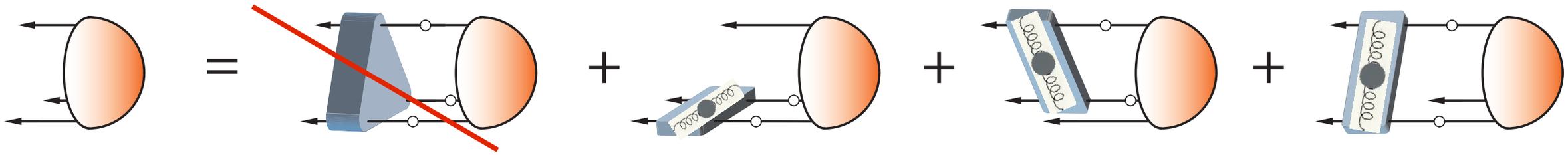


Faddeev - equation



- irreducible three-body forces: may be neglected
- two-body interactions:
  - non-perturbative one-gluon exchange (RL)
  - beyond one-gluon exchange

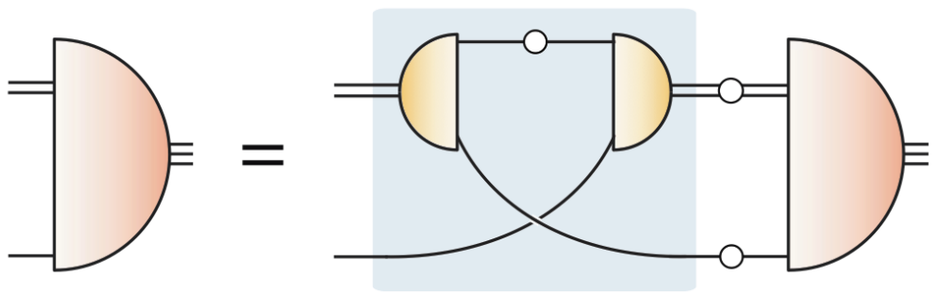
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)

- quark-diquark approximation

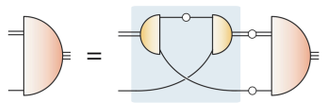
Oettel, Hellstern, Alkofer and Reinhardt, PRC 58 (1998) 2459

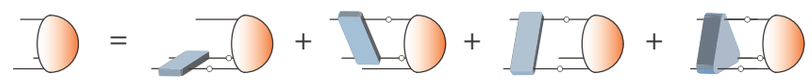
see also: Bloch, Chang, Chen, Cloet, Thomas, Ramalho, Roberts, Segovia et al.



DSE/BSE/Faddeev landscape (2015)

level of complexity

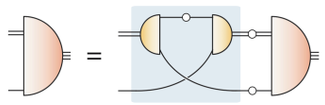


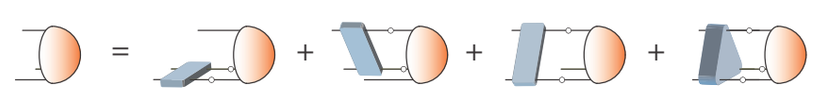


		I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P=+$ N,Δ masses	✓	✓	✓	✓	✓
	$P=+$ N,Δ em. FFs	✓	✓	✓	✓	
	$P=+$ $N\to\Delta\gamma$	✓	✓	✓		
	$P=+$ N^*,Δ^*masses	✓	✓			
	$P=+$ $\gamma N\to N^*/\Delta^*$	✓	✓			
	$P=-$ N^*,Δ^*masses		✓			
	$P=-$ $\gamma N\to N^*/\Delta^*$		✓			
strange	ground states		✓			
	excited states		✓			
	em. FF		✓			
	TFFs		✓			
c/b	ground states					
	excited states					
		Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF

DSE/BSE/Faddeev landscape

level of complexity





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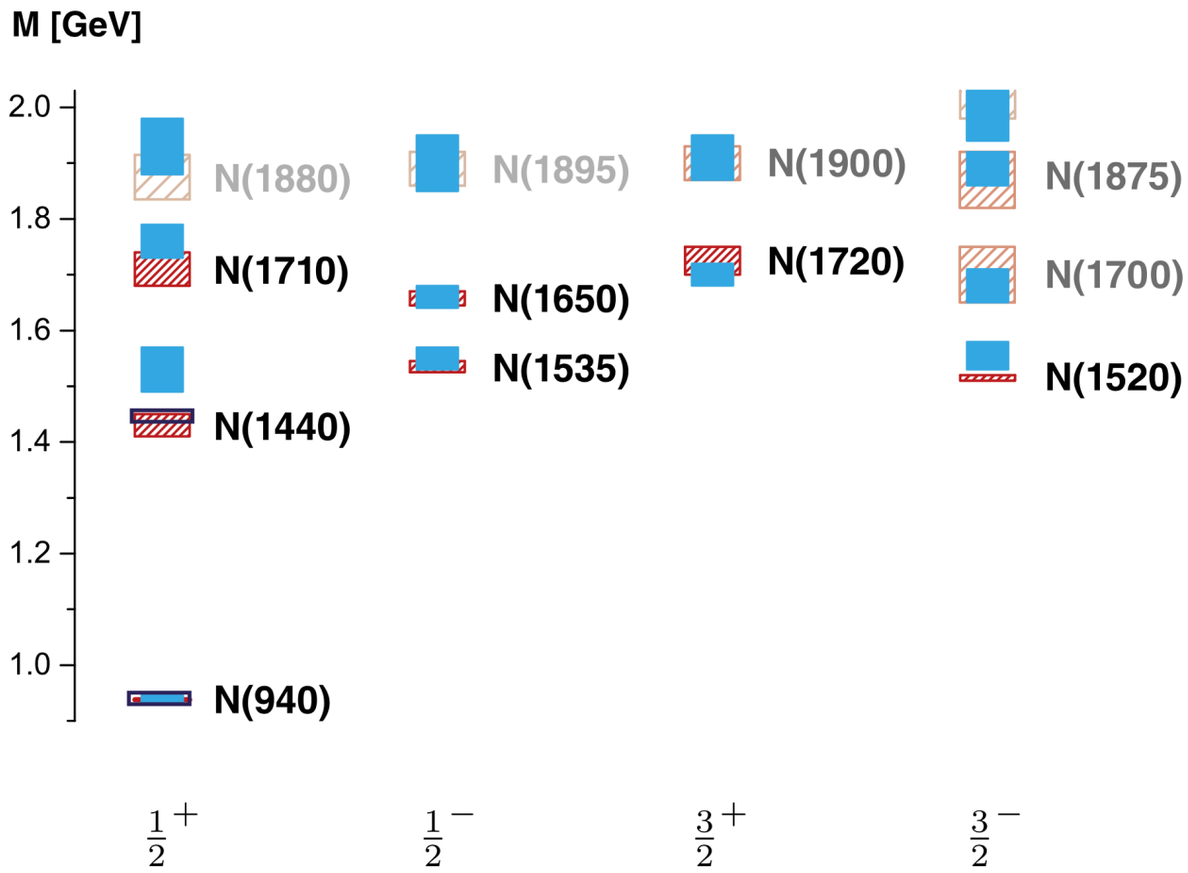
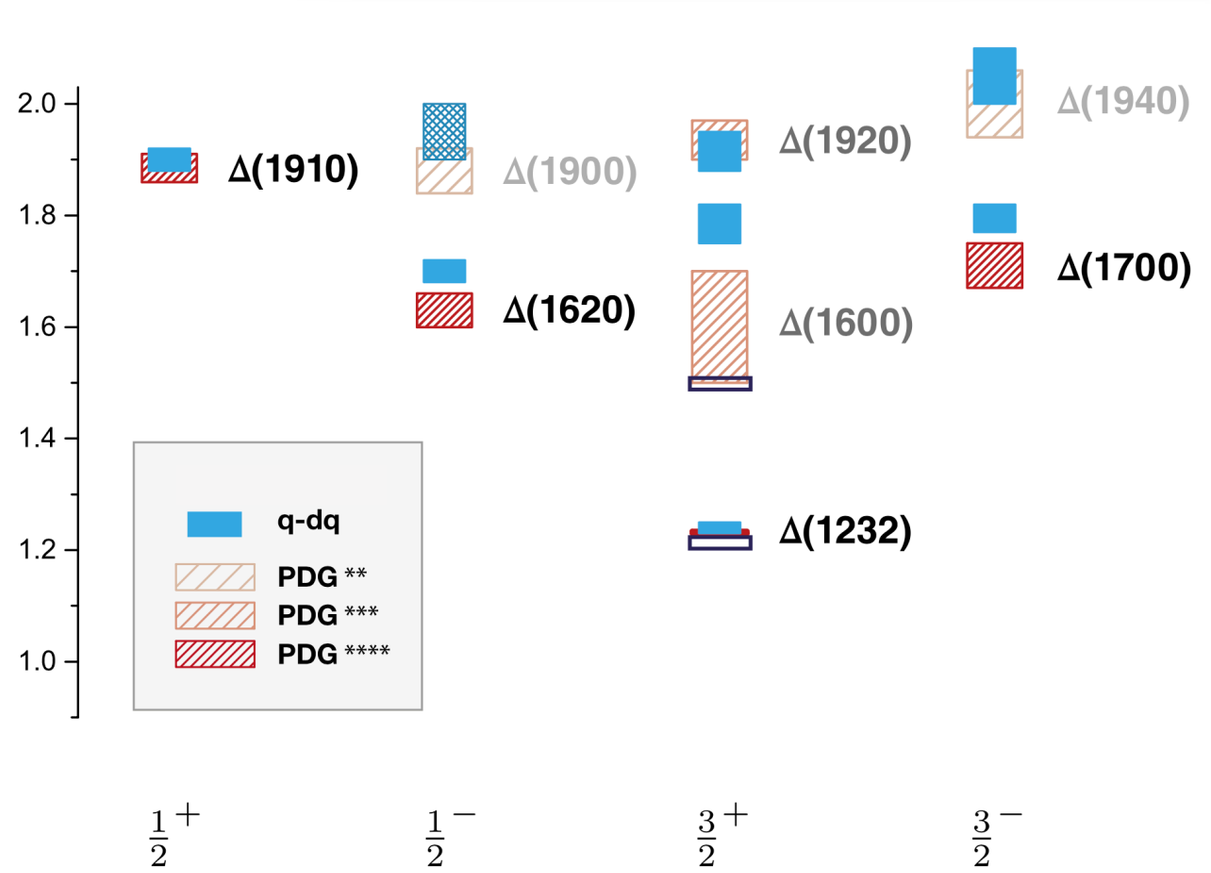
	$P=+$ $N\to\Delta\gamma$	✓	✓	✓	✓	
	$P=+$ N^*,Δ^*masses	✓	✓	✓	✓	
	$P=+$ $\gamma N\to N^*/\Delta^*$	✓	✓			
	$P=-$ N^*,Δ^*masses	✓	✓	✓	✓	
	$P=-$ $\gamma N\to N^*/\Delta^*$					
strange	ground states	✓	✓	✓	✓	
	excited states	✓	✓	✓	✓	
	em. FF			✓	✓	
	TFFs				✓	
c/b	ground states	✓	✓		✓	
	excited states		✓		✓	
		Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF

Light baryon spectrum:

- 3 parameters + $m_{u,d,s}$
  (all fixed in meson sector)





Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)

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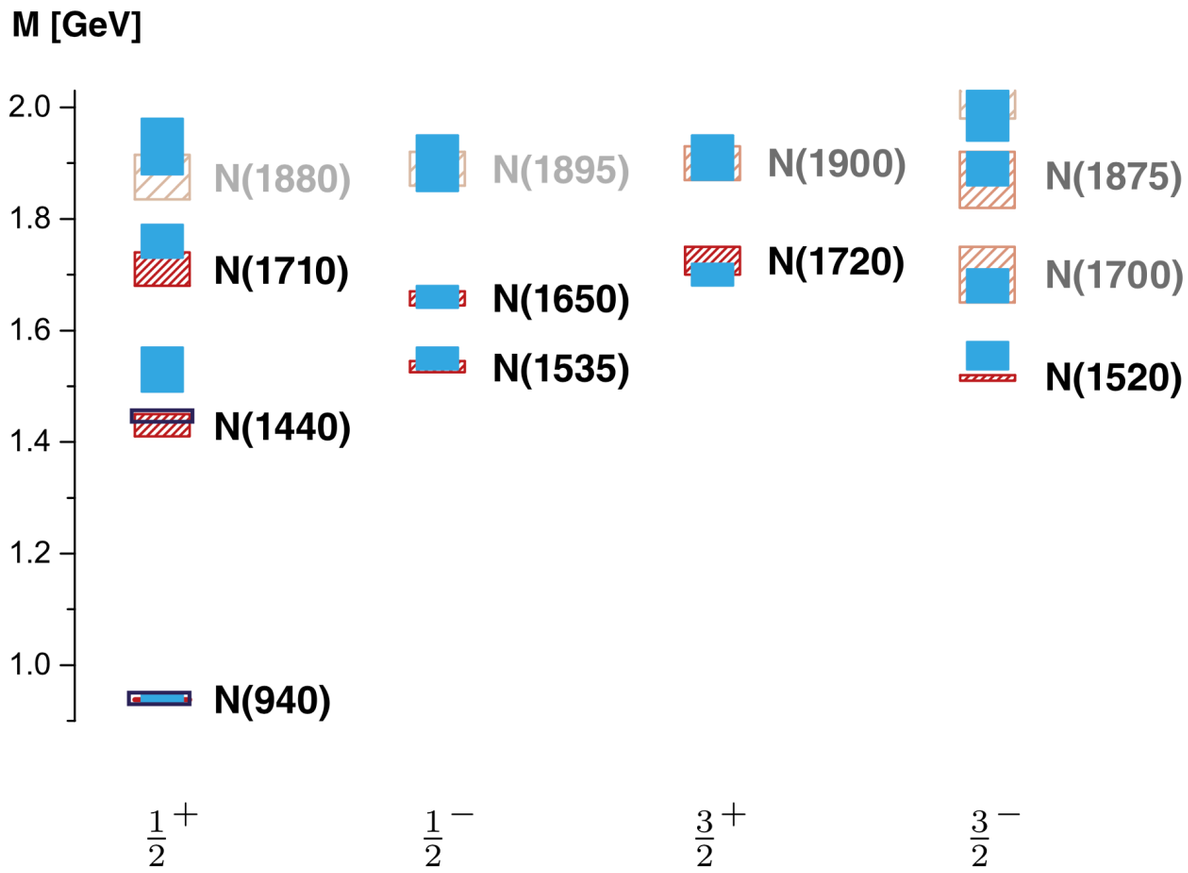

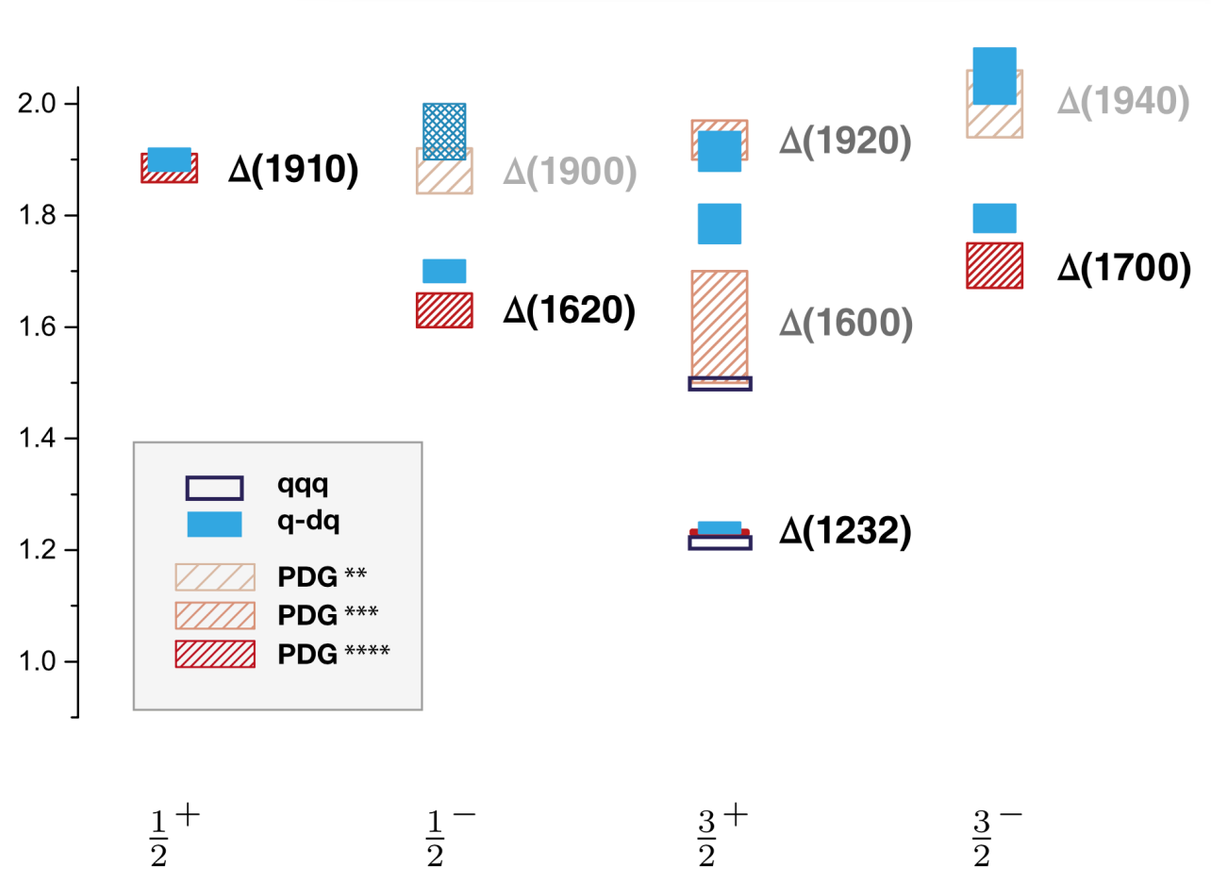

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- three-body agrees with diquark-quark where applicable

Relativistic proton

$$J^P = \left(\frac{1}{2}\right)^+$$

non-relativistic

three quarks with spin 1/2: $S = 1/2 \text{ or } S = 3/2$

parity $P = (-1)^L$: $L = 0 \text{ or } L = 2$

relativistic

$P = (-1)^L$

64 components in wave function: 8 s-wave (L=0)
36 p-wave (L=1)
20 d-wave (L=2)

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

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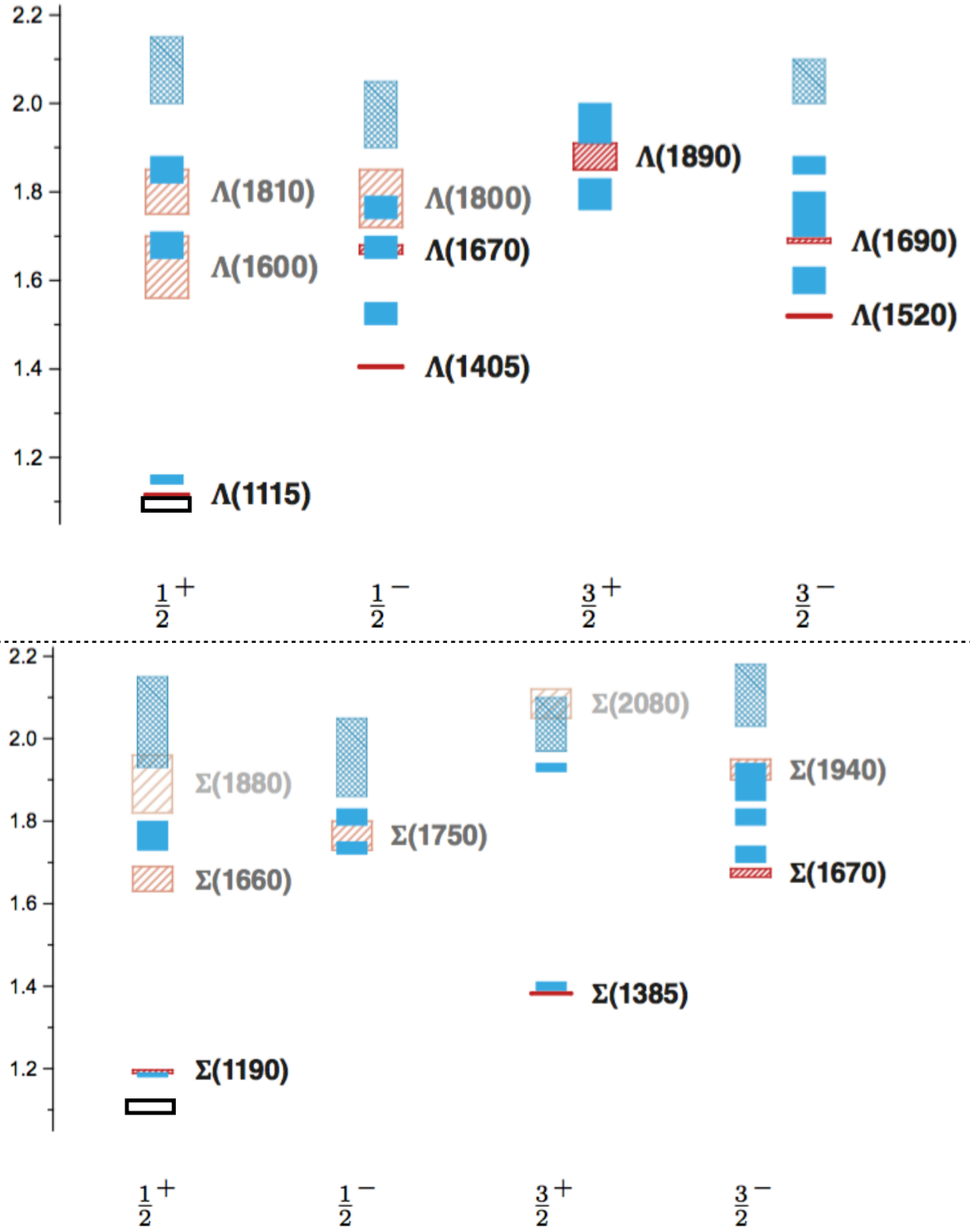
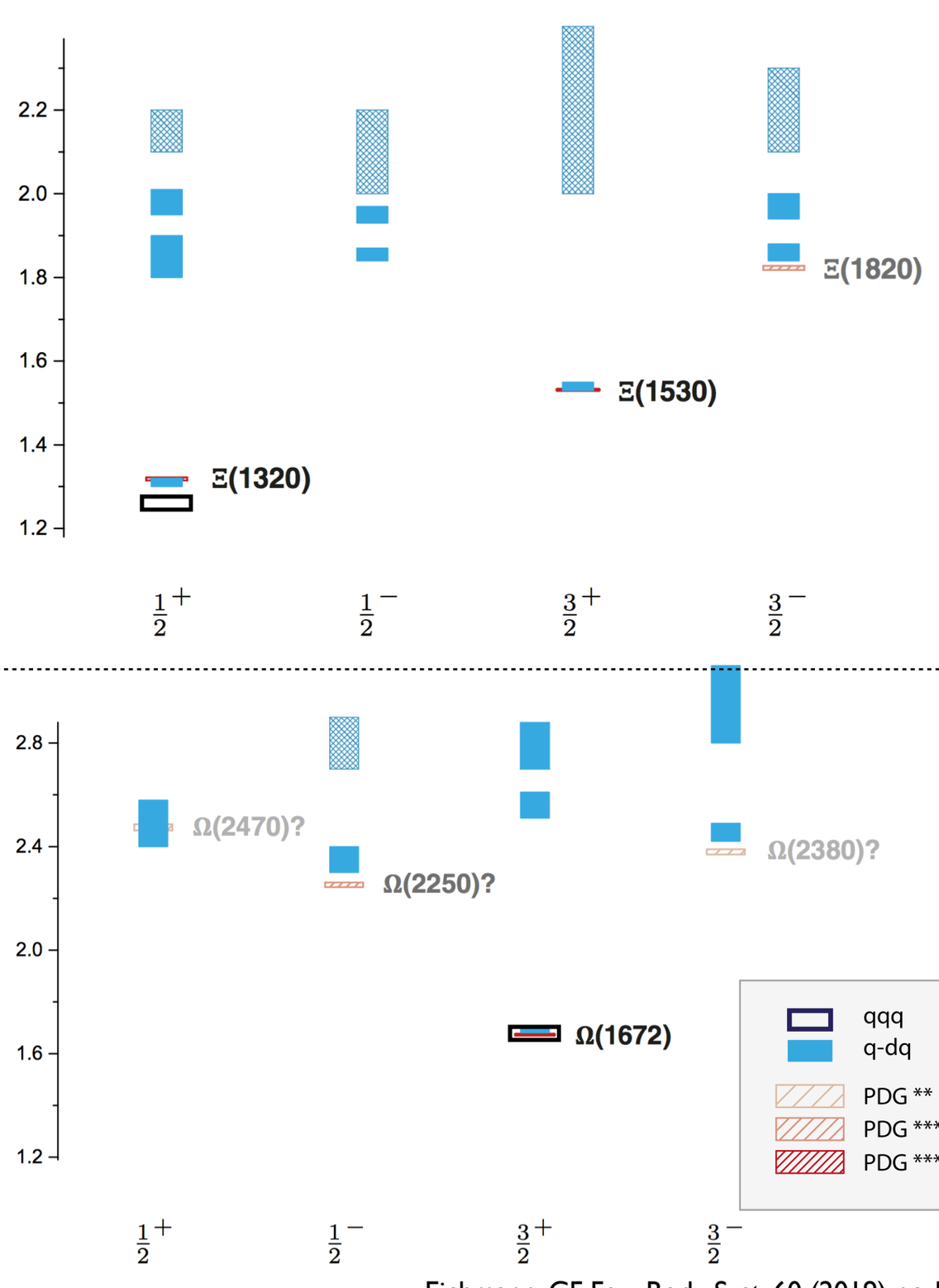
Strange baryon spectrum: DSE-RL (preliminary !)





Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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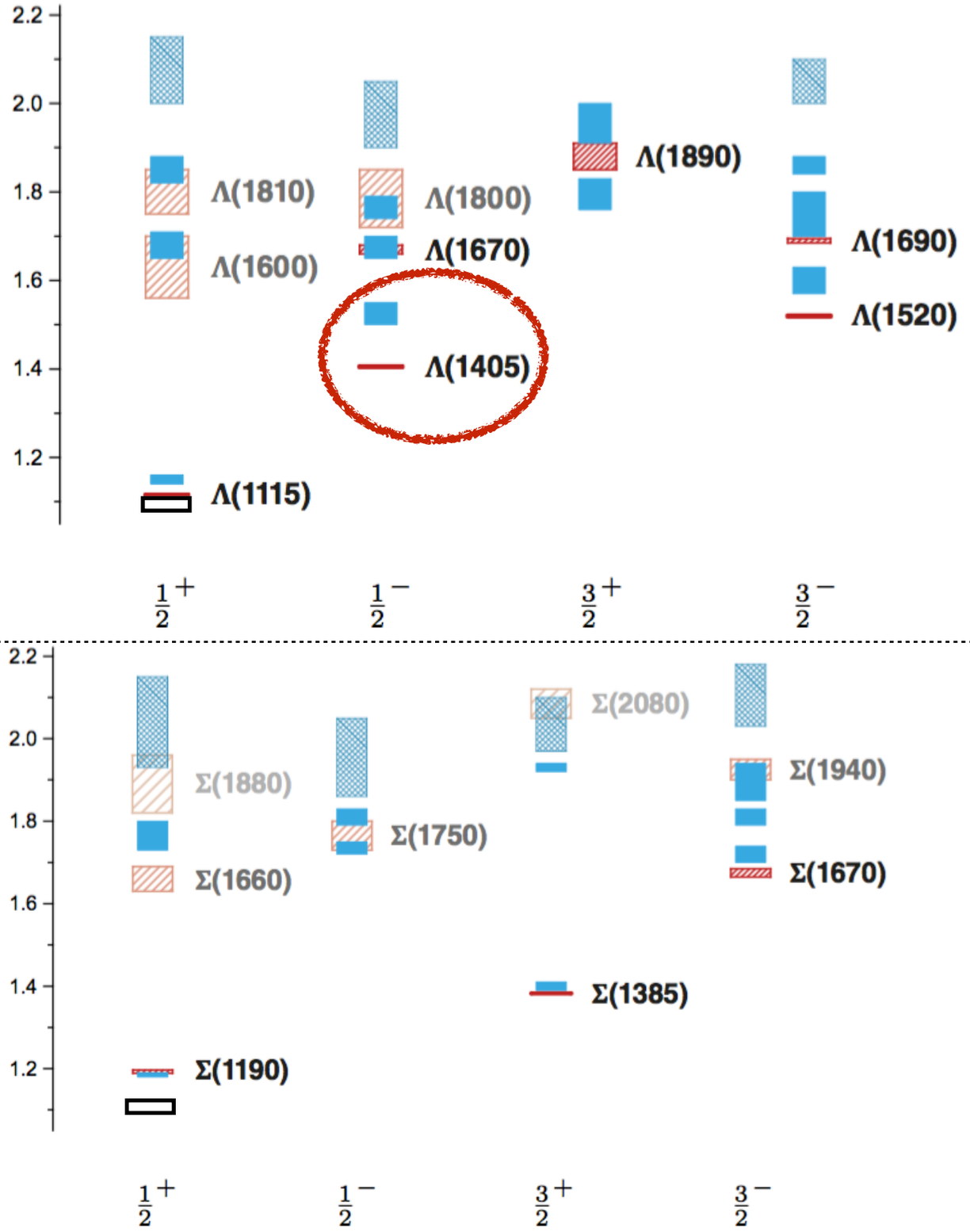

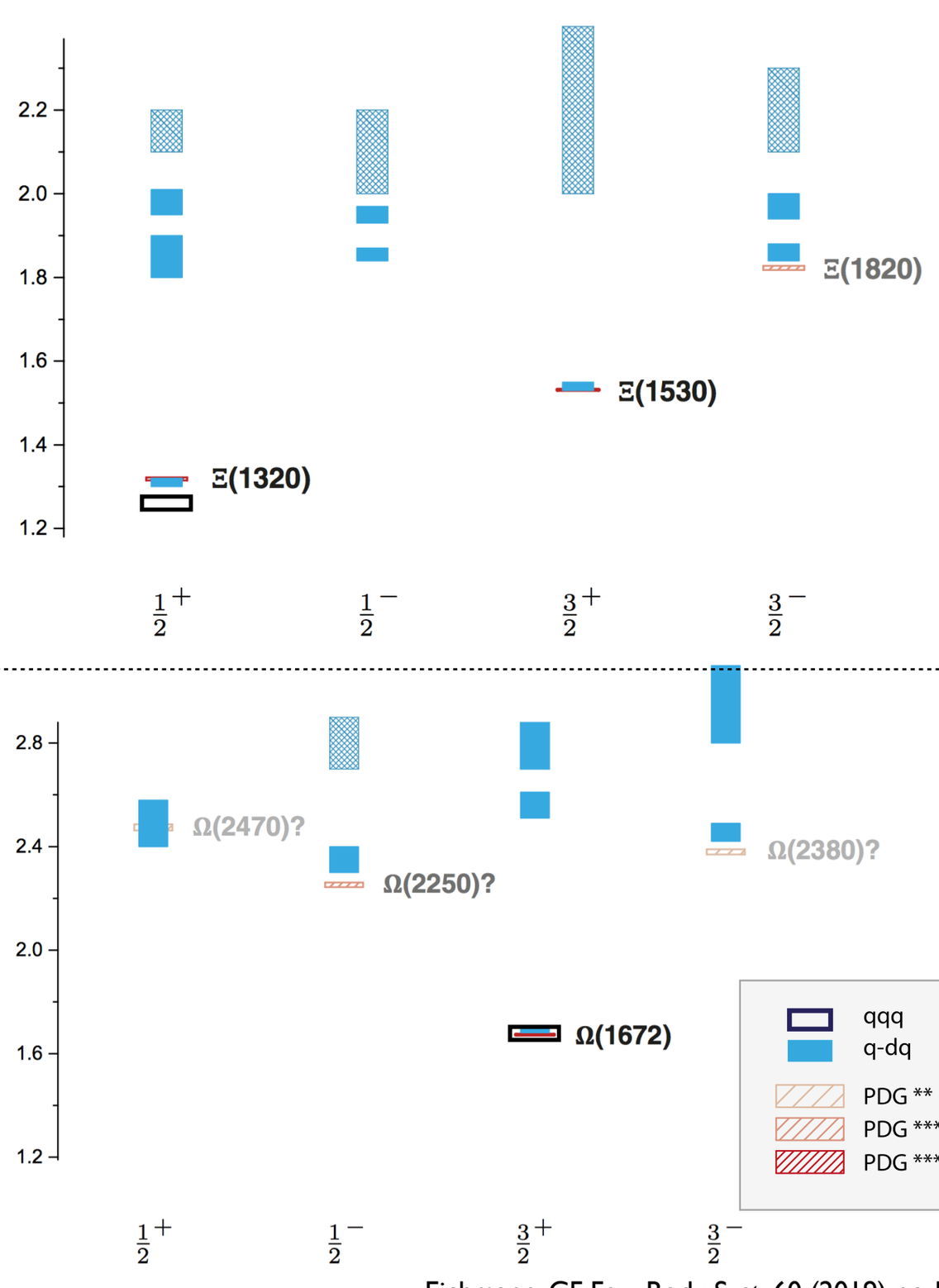

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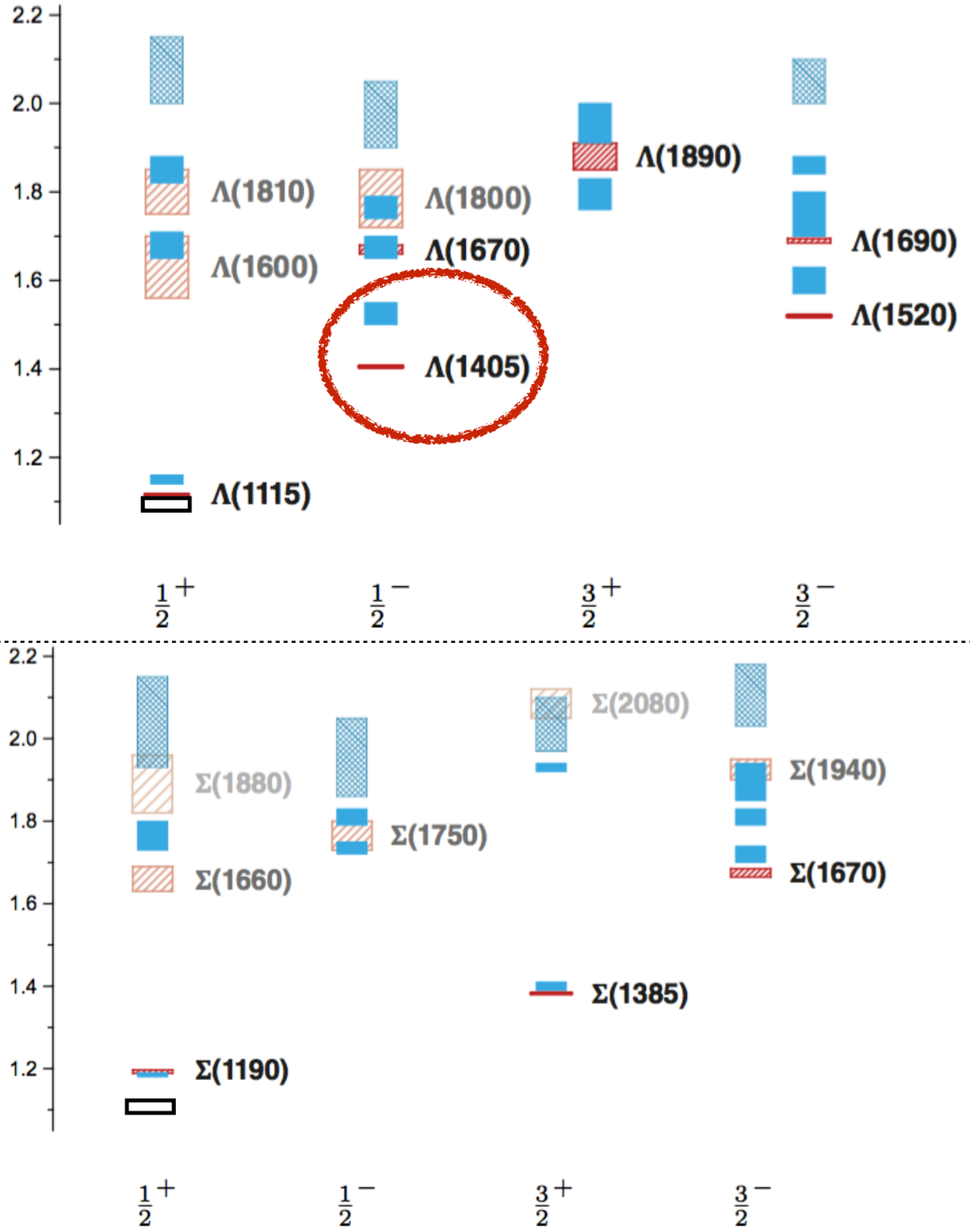

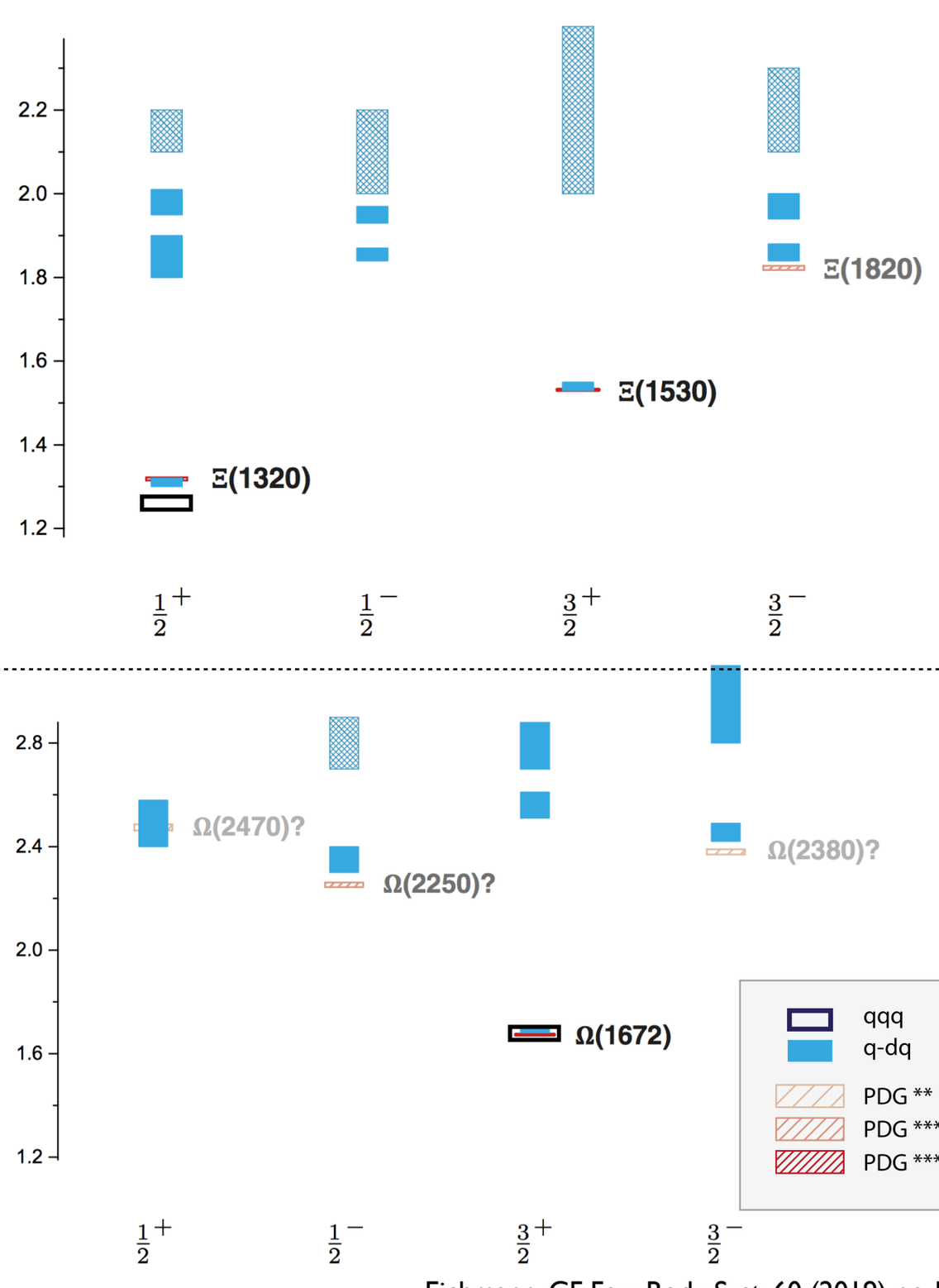

New states: Bonn-Gatchina (talk of M. Matveev at N*2019)

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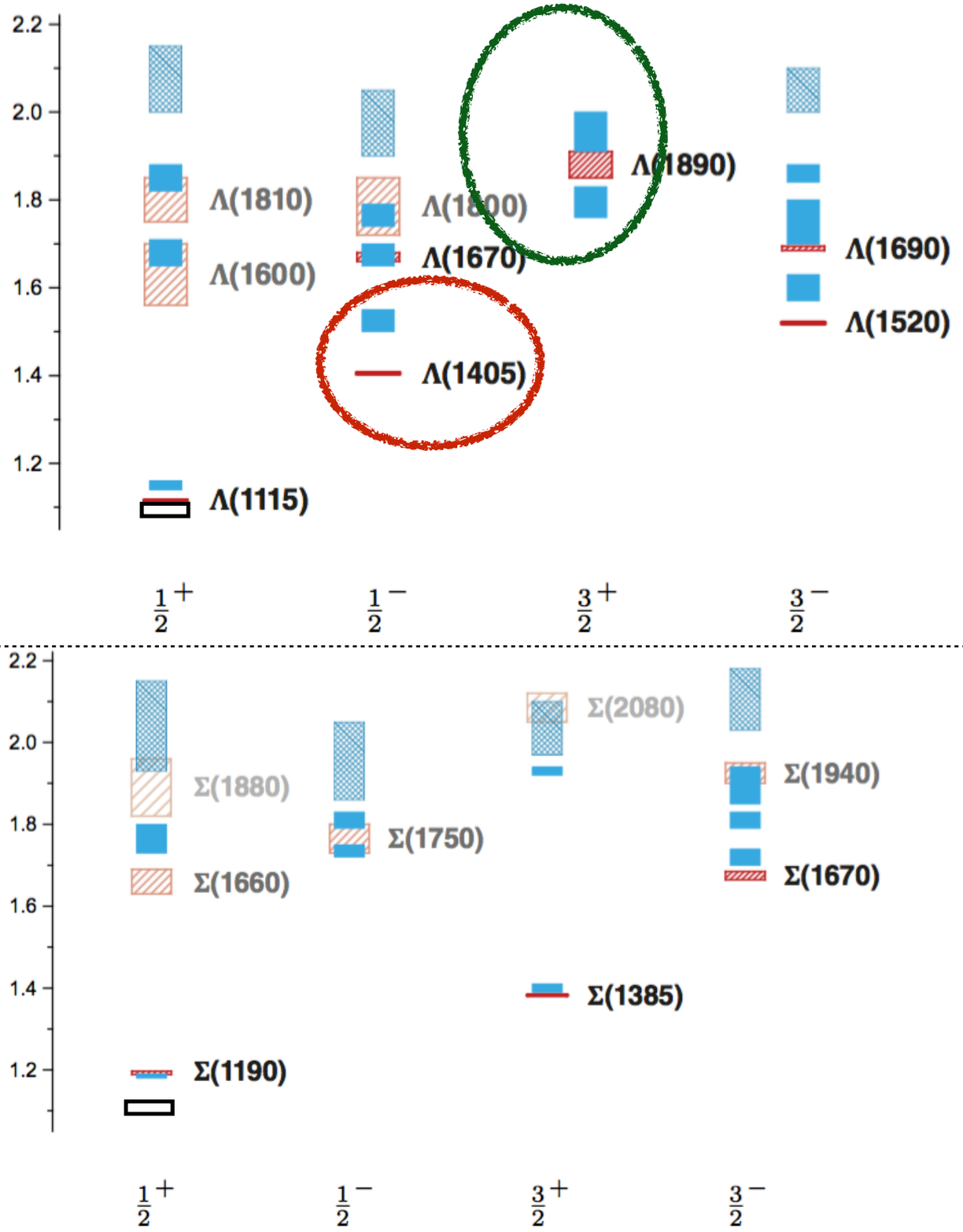

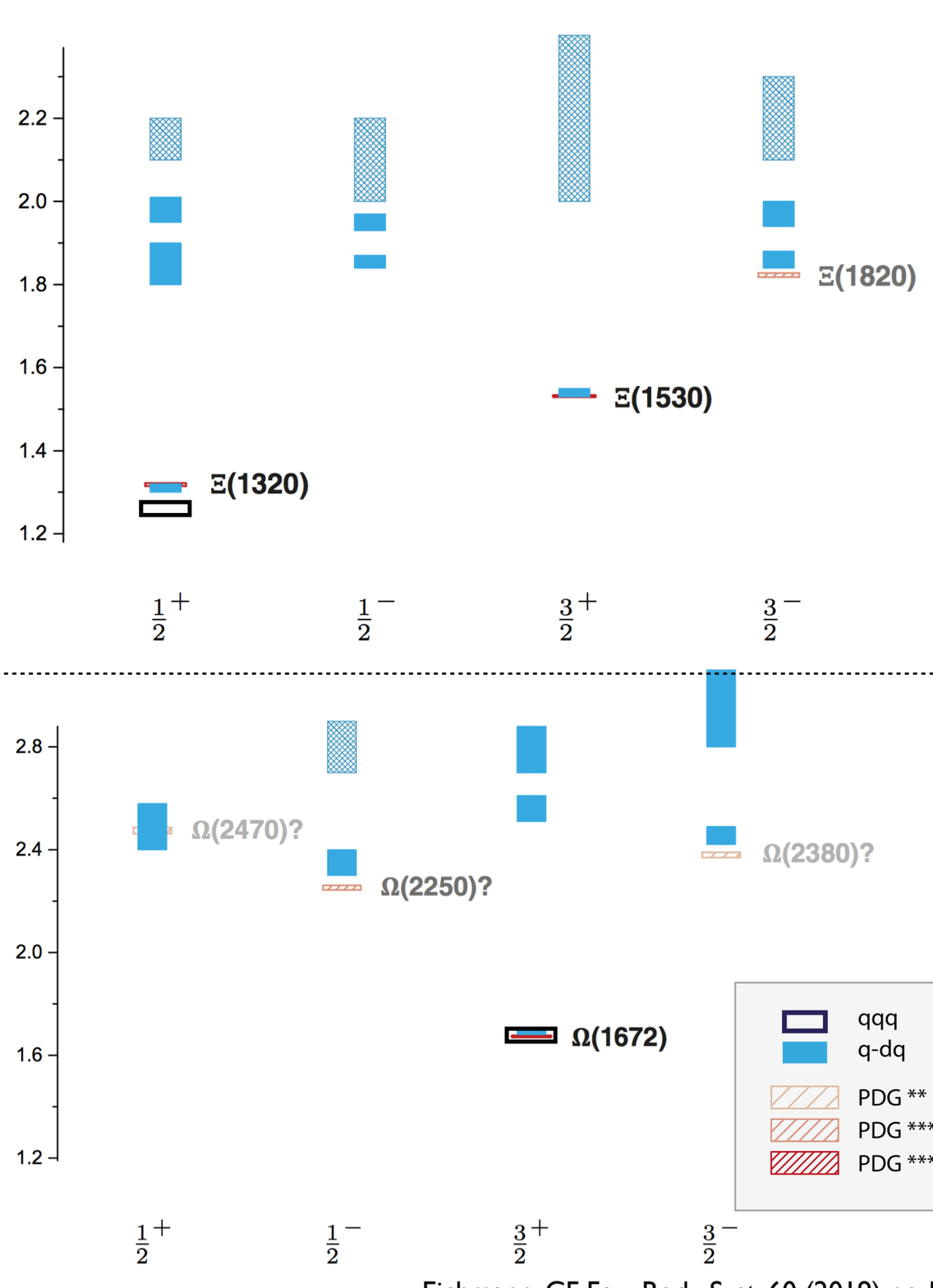

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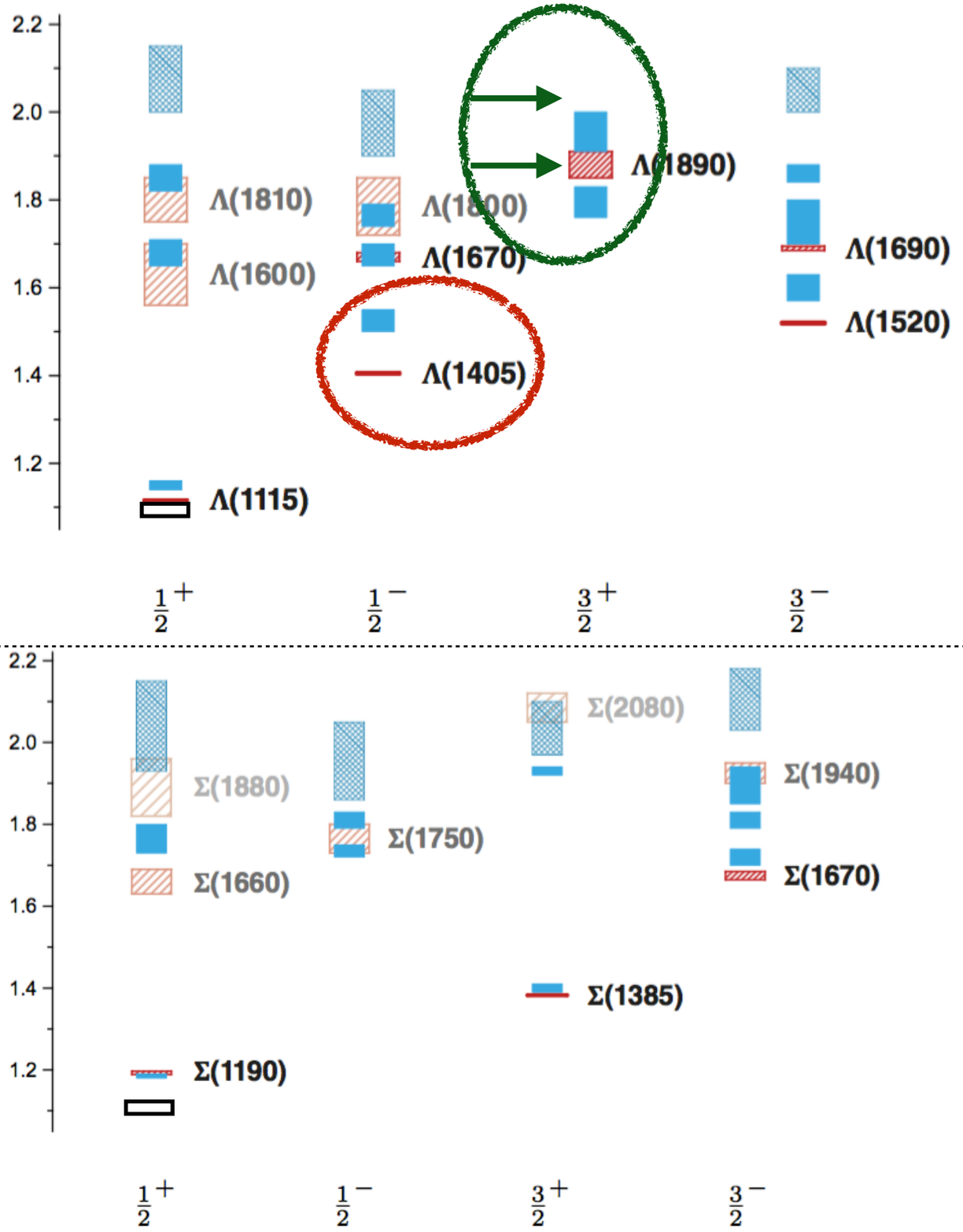

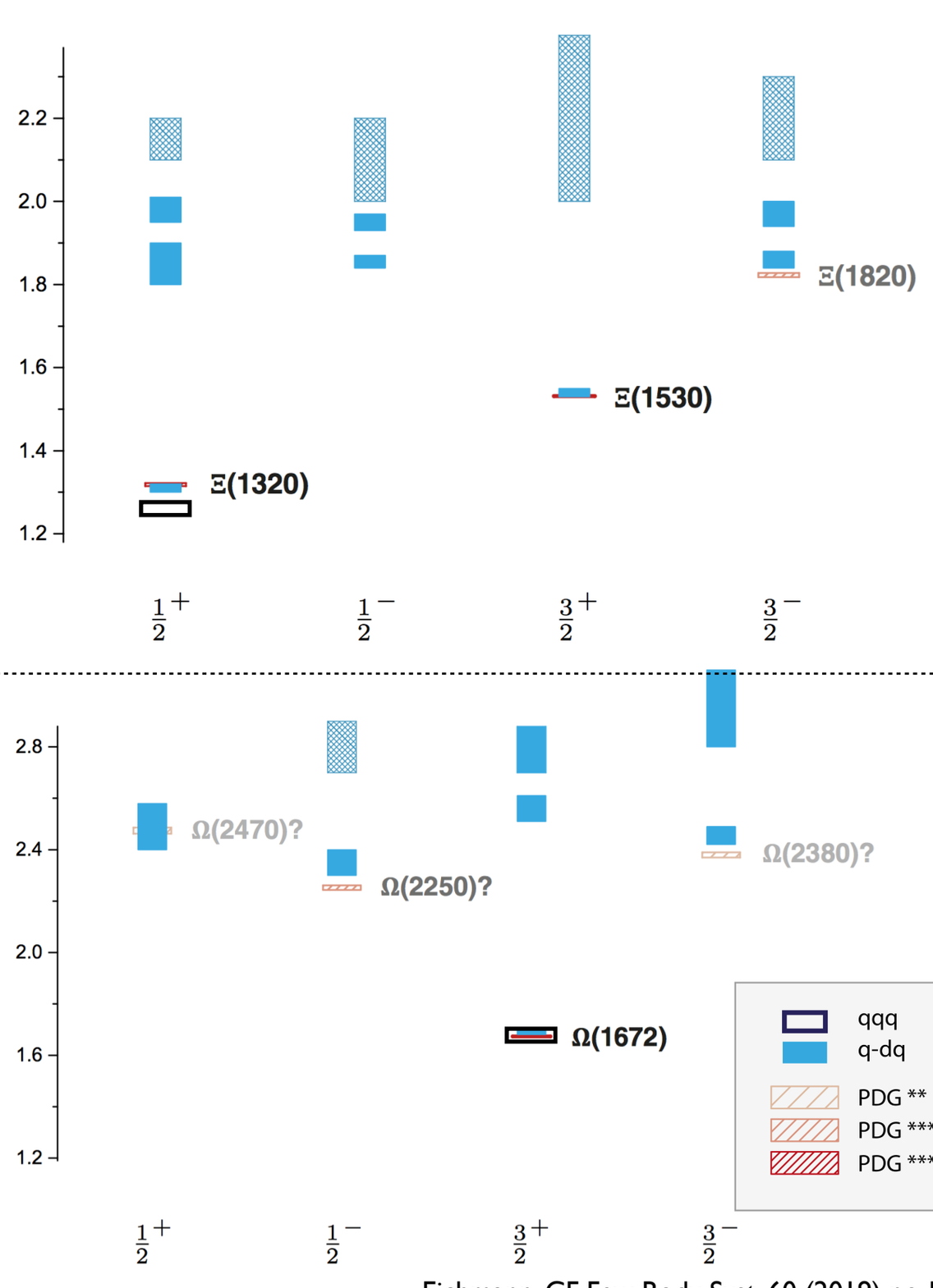

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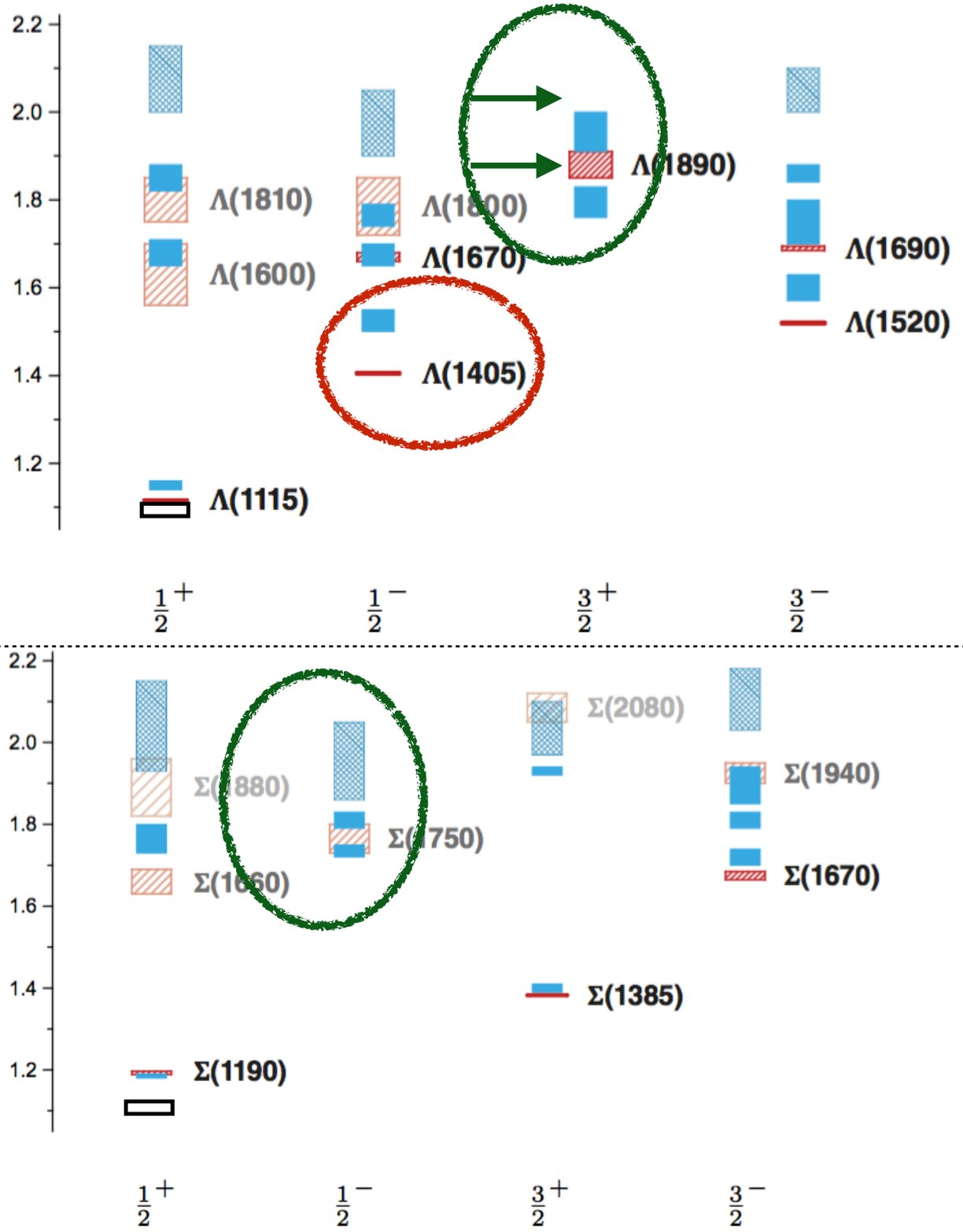

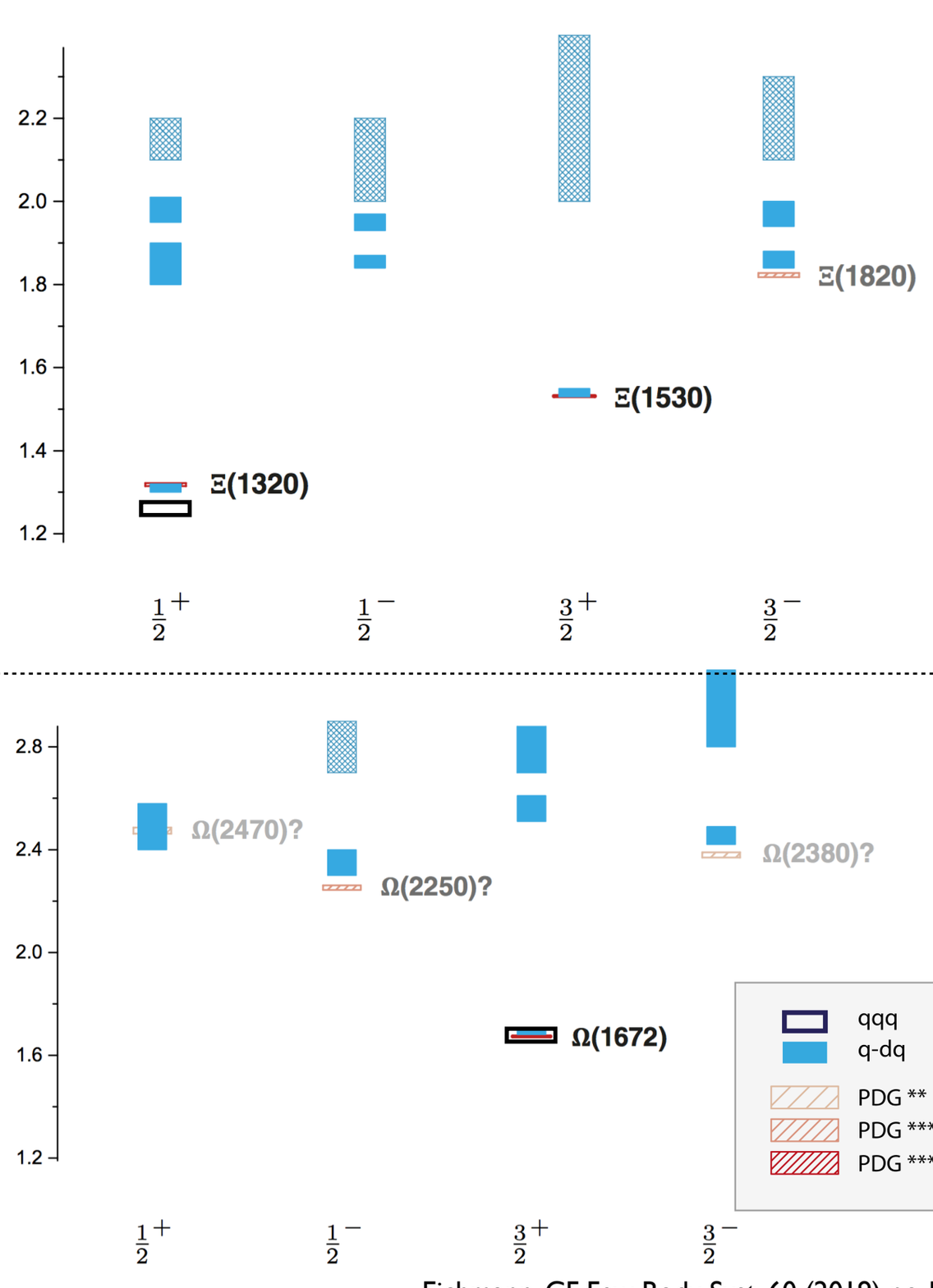

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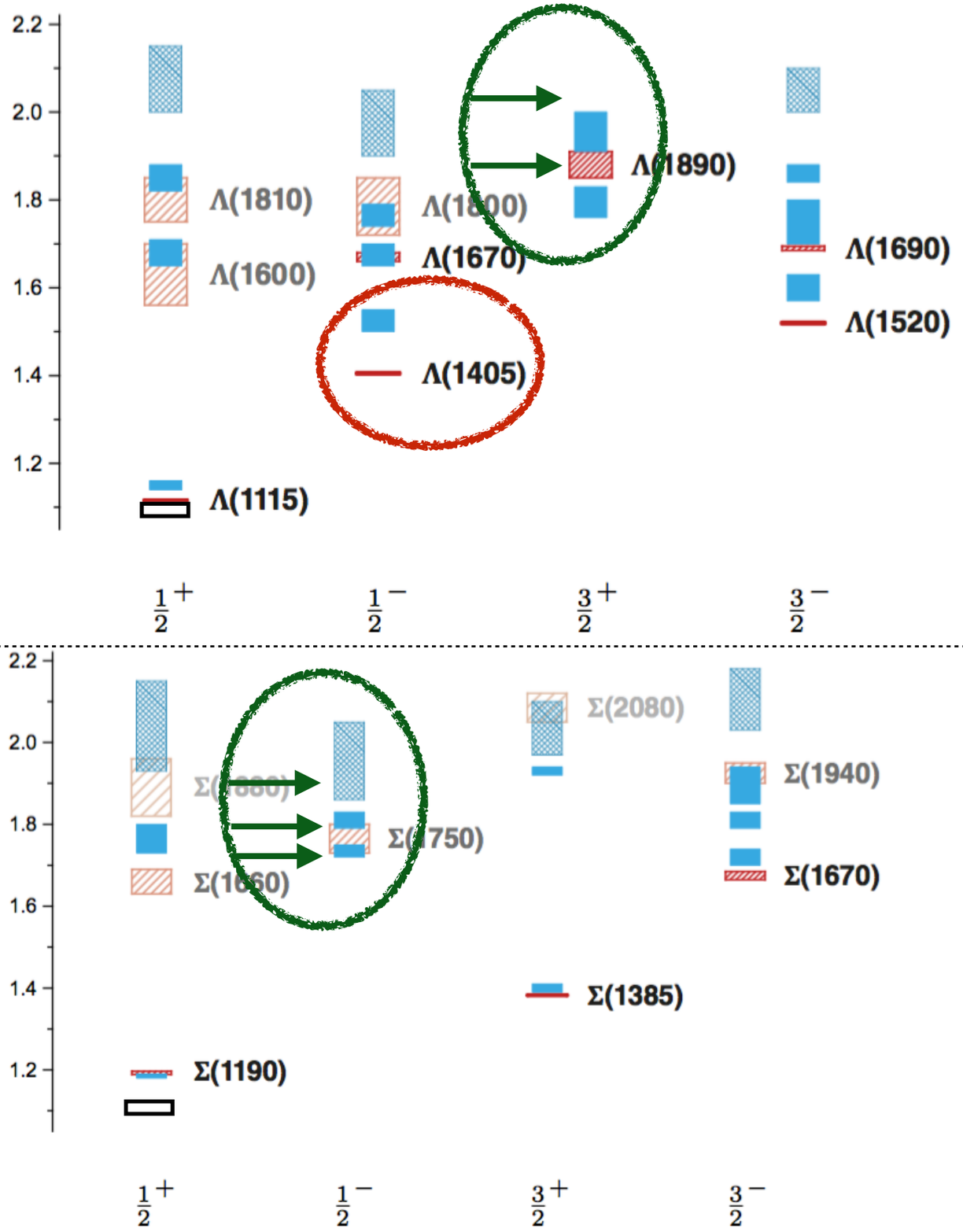

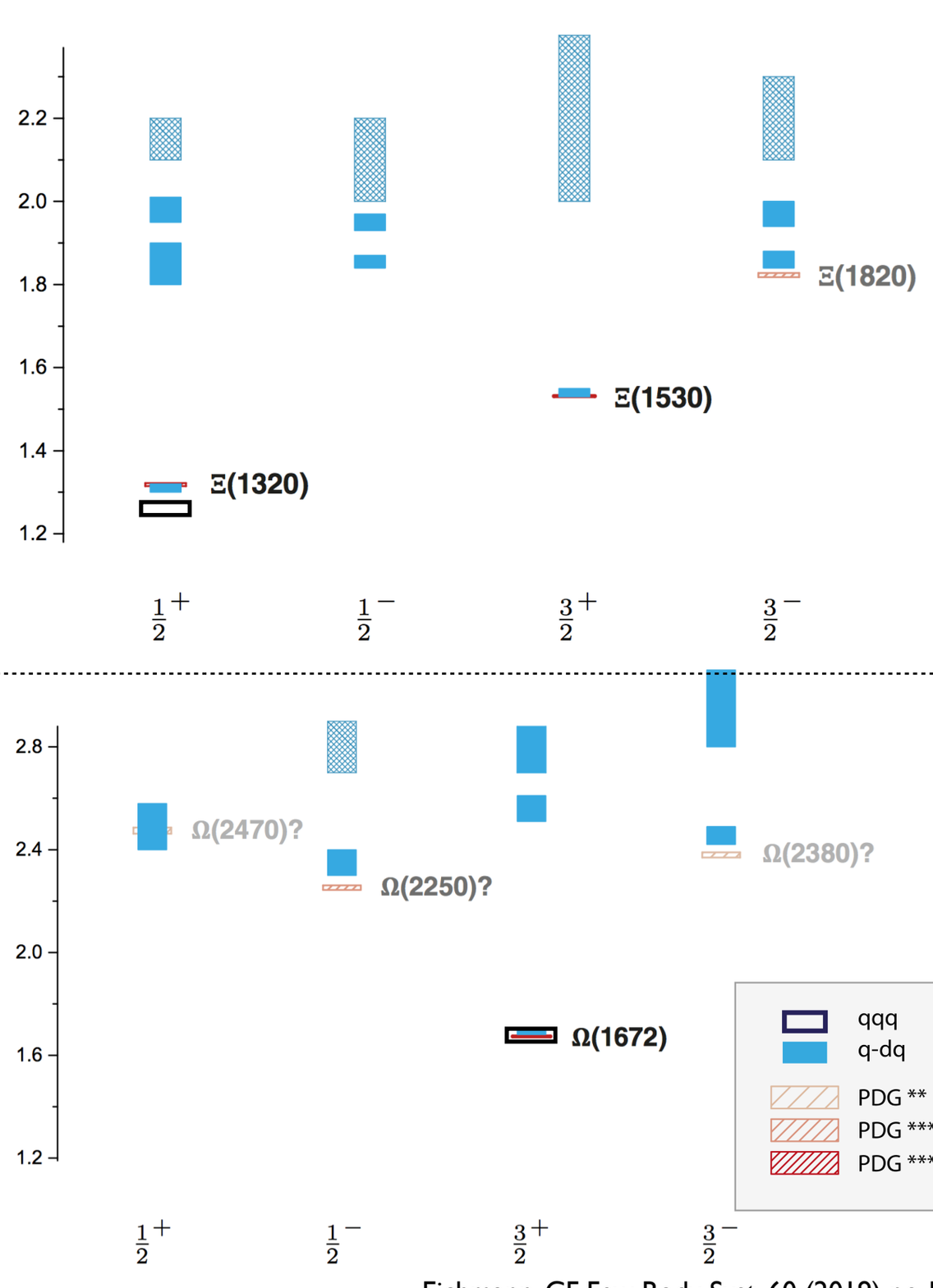

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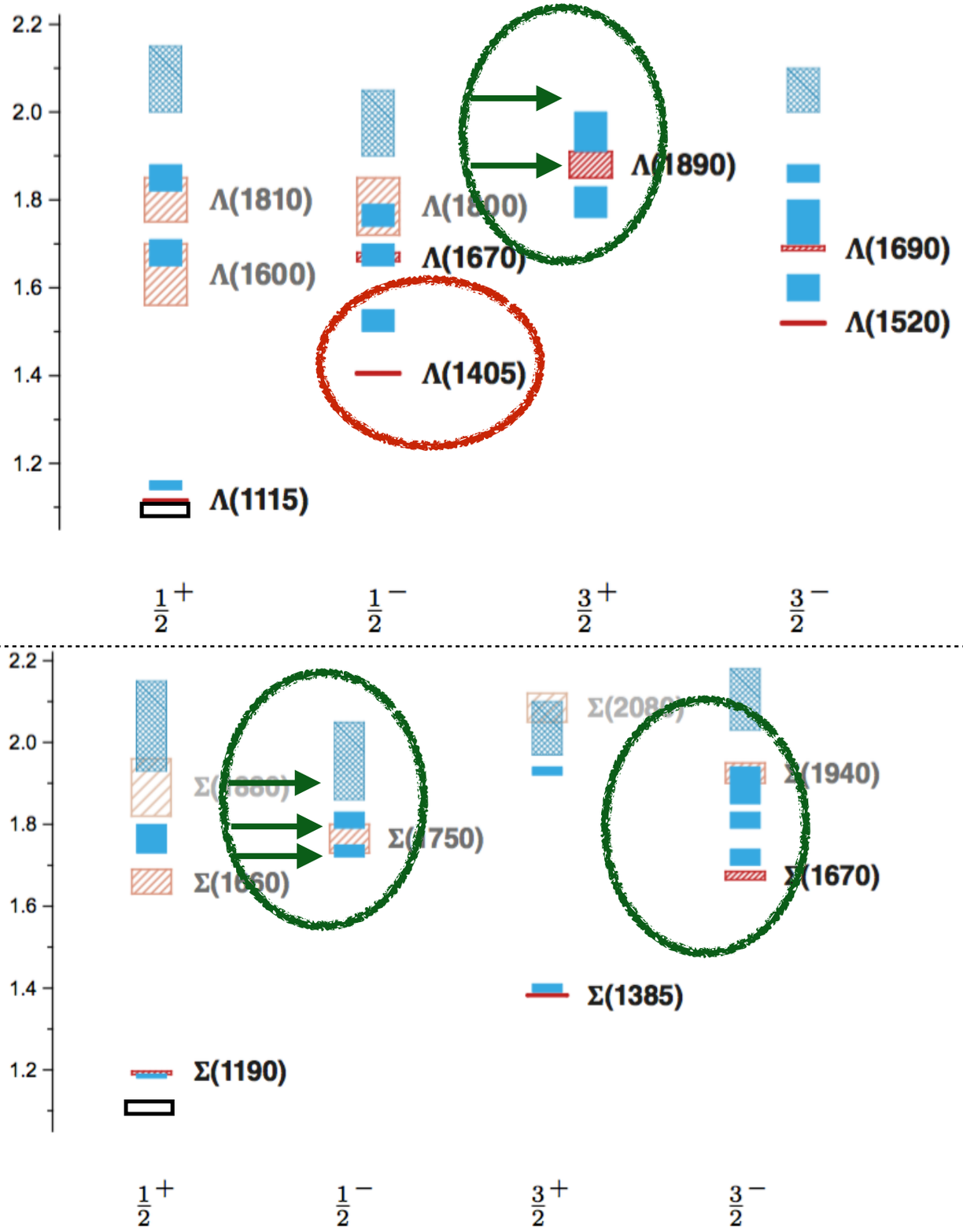

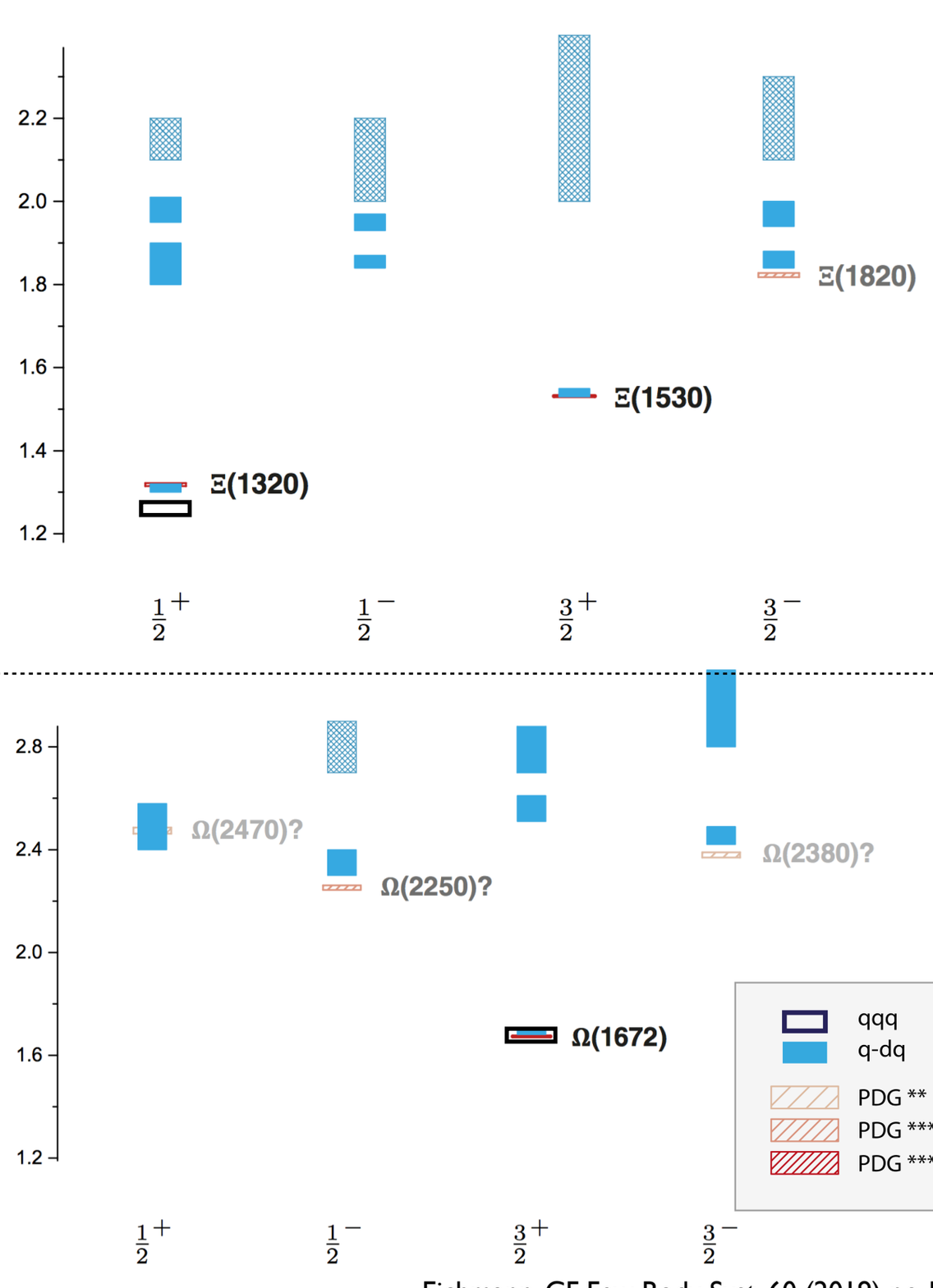

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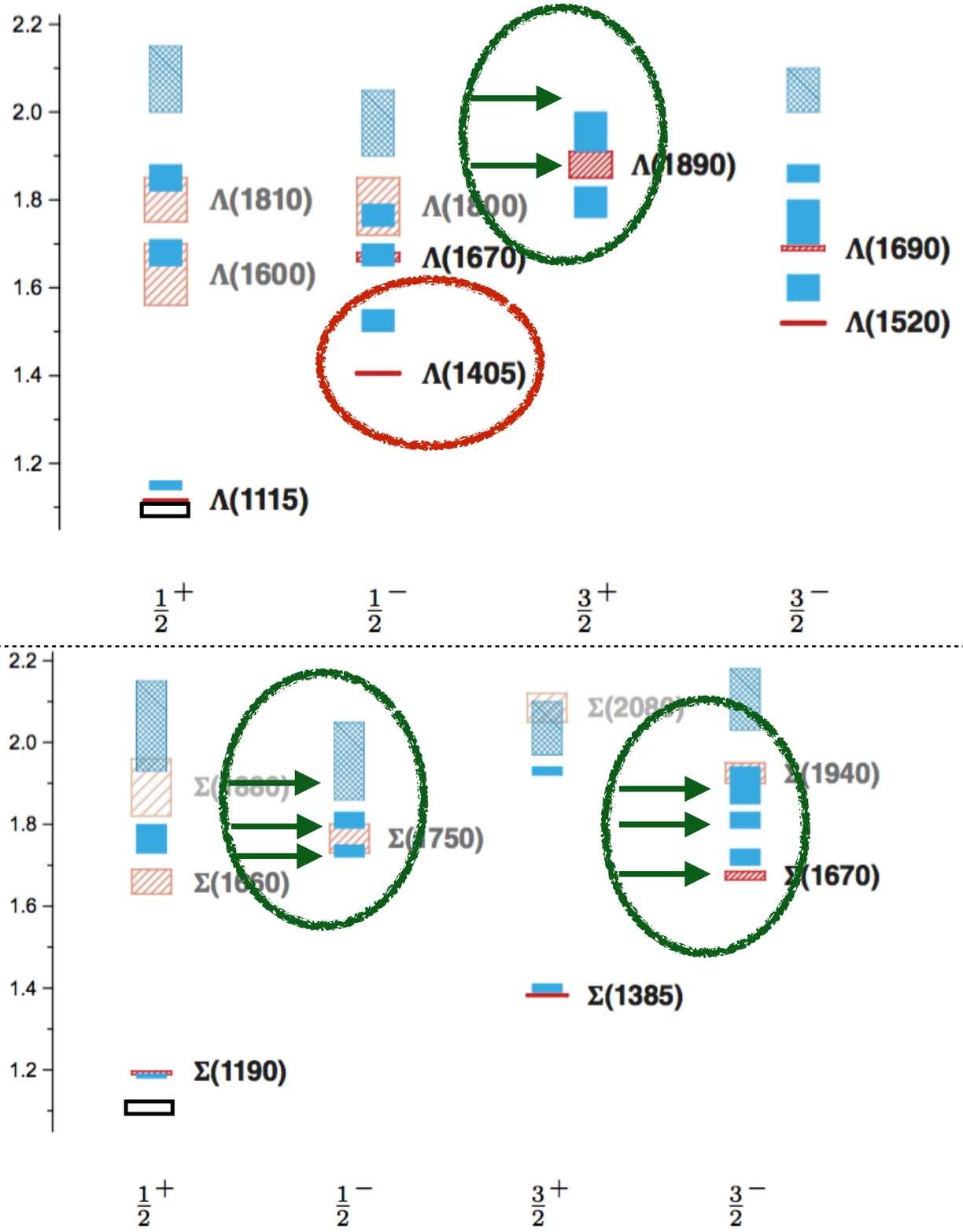

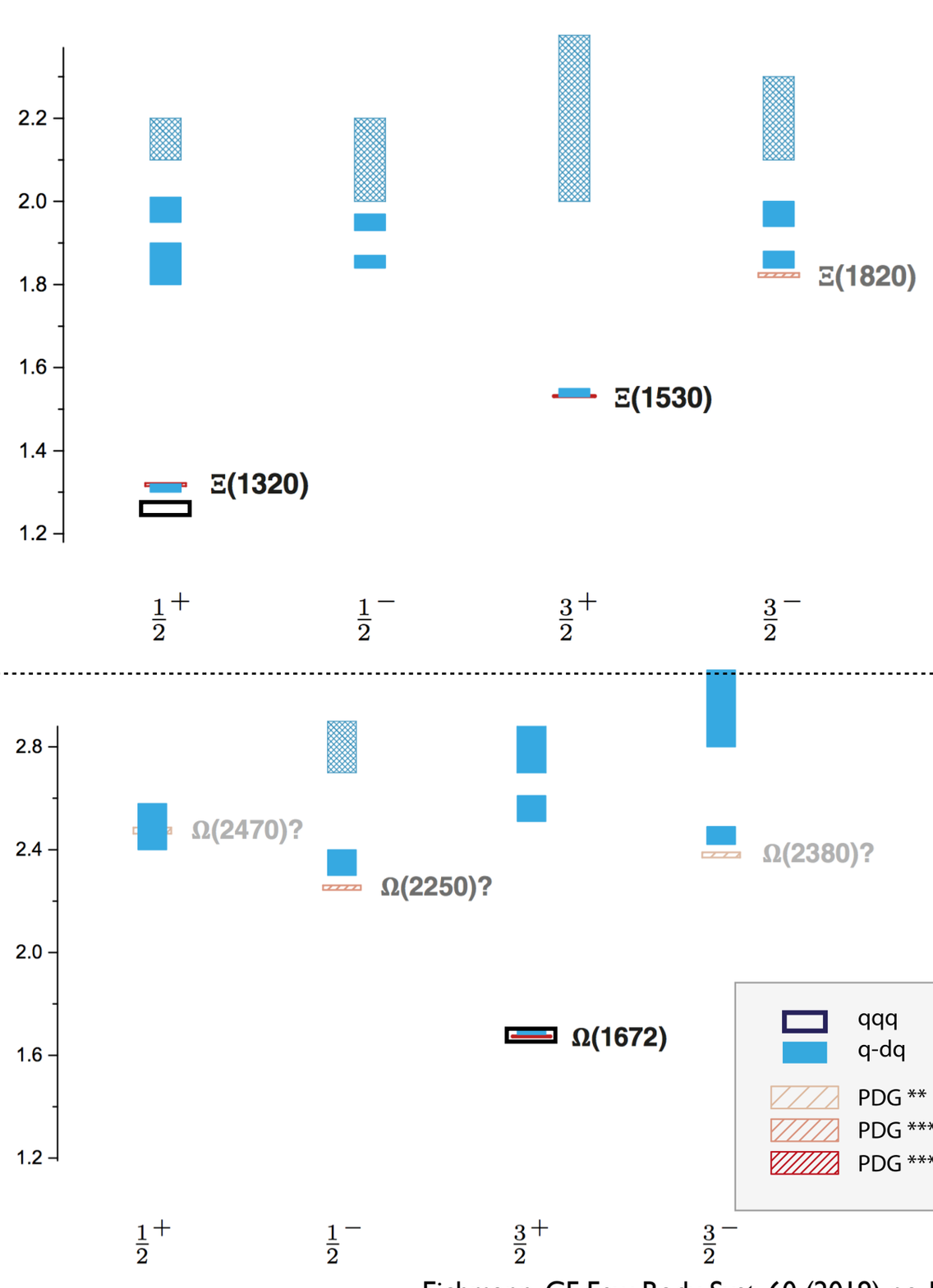

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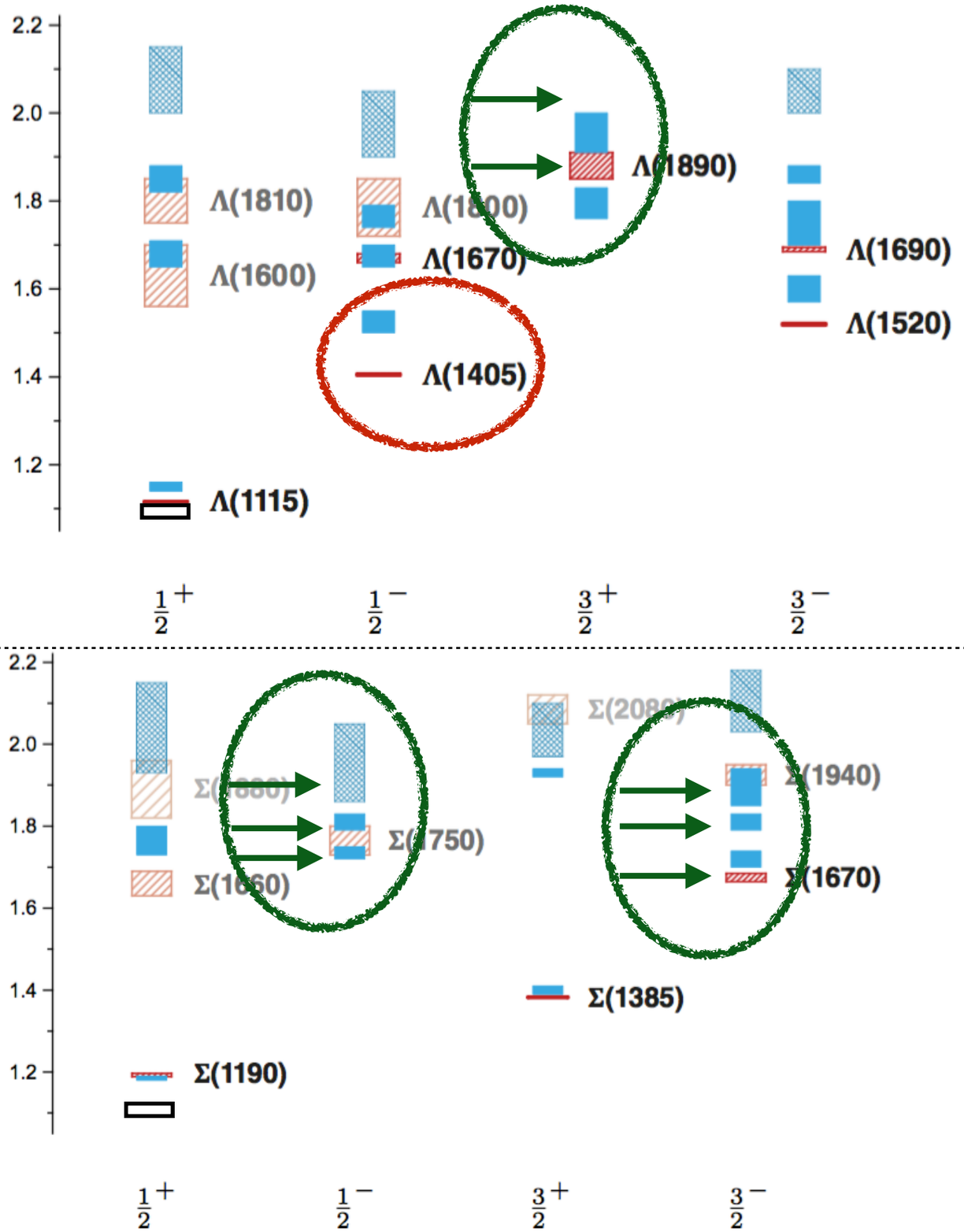

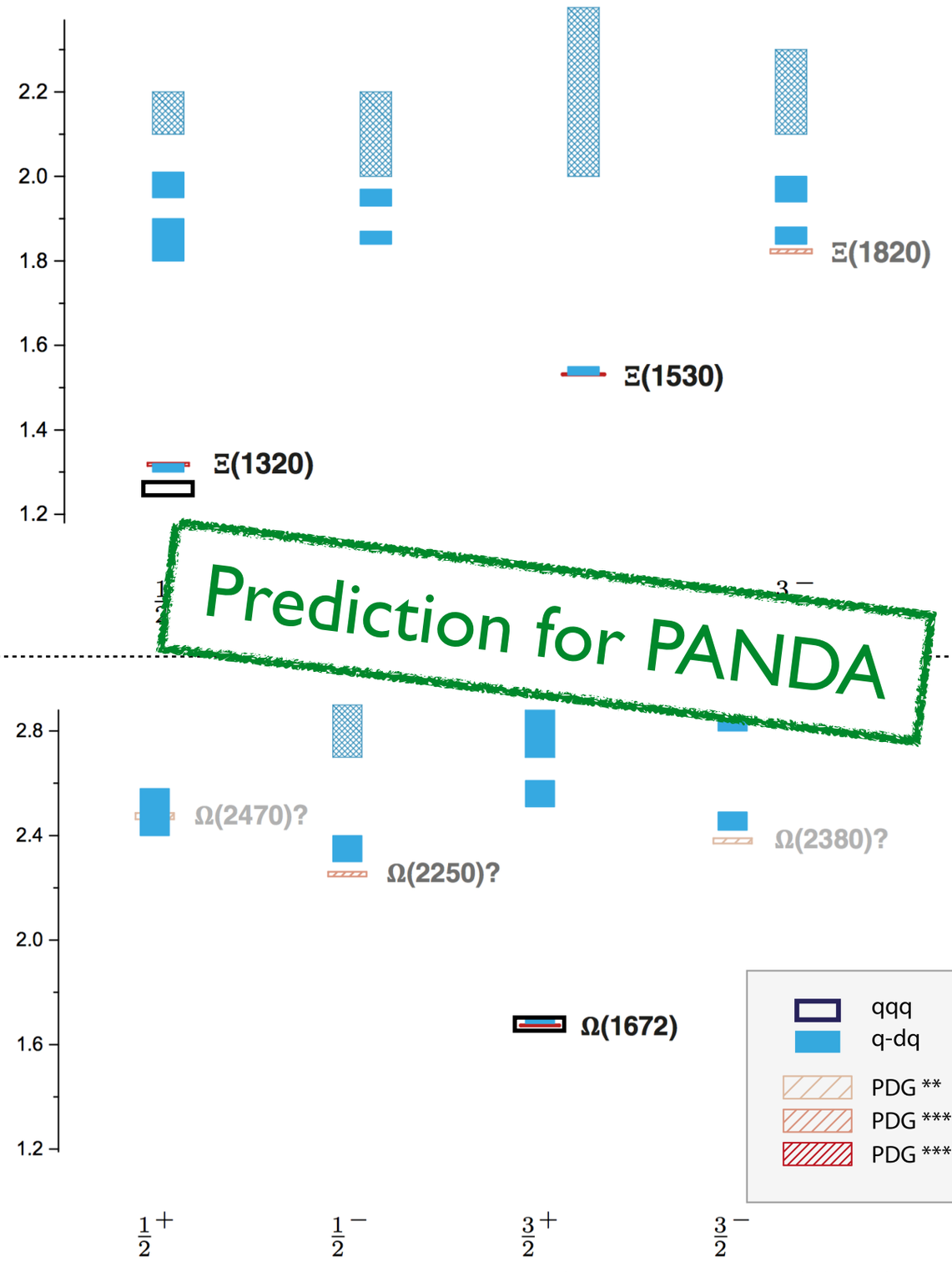

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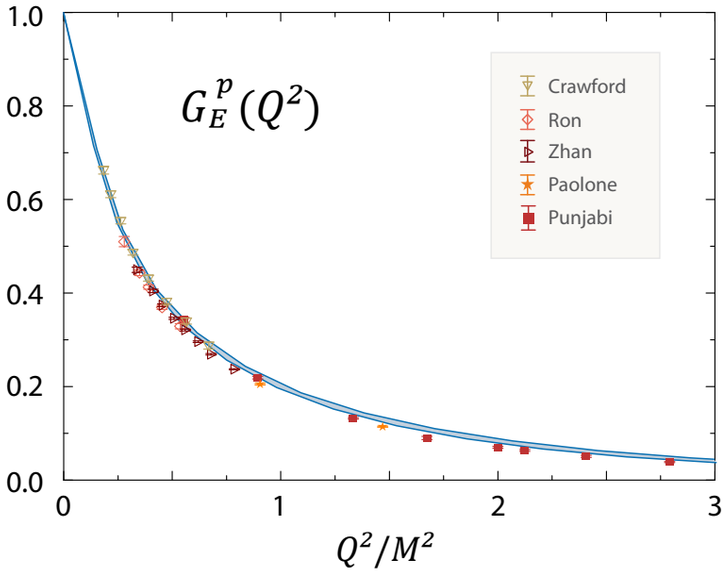
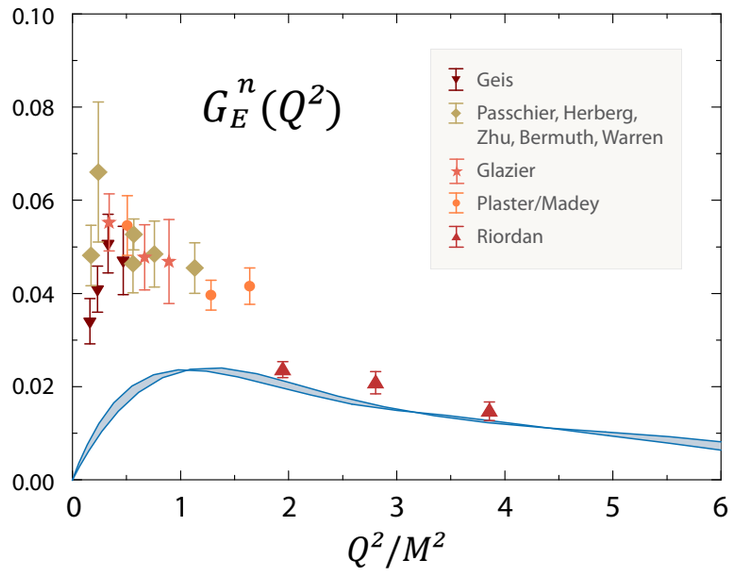
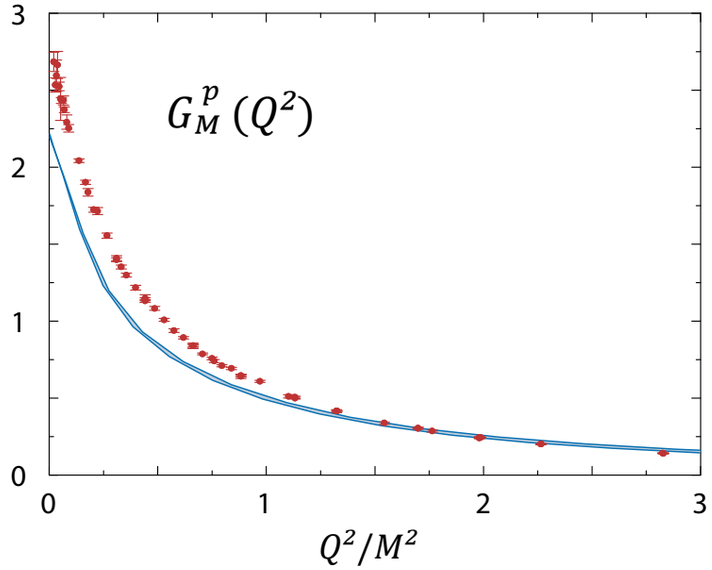
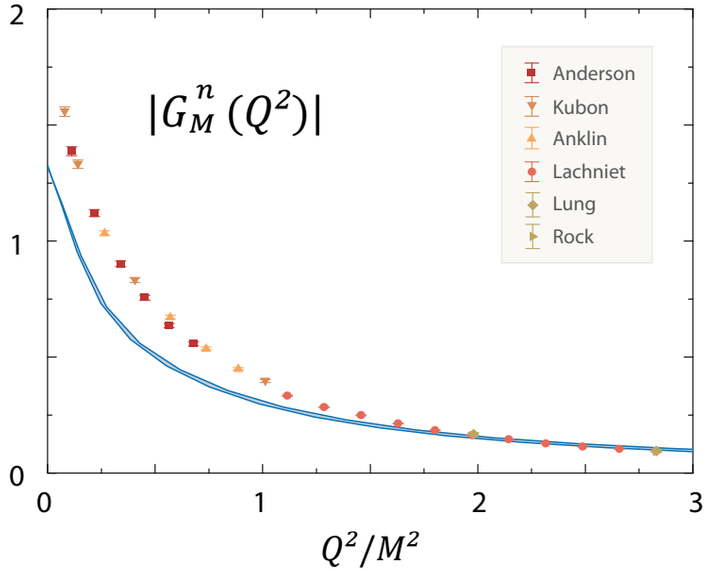
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Nucleon form factors and magnetic moments

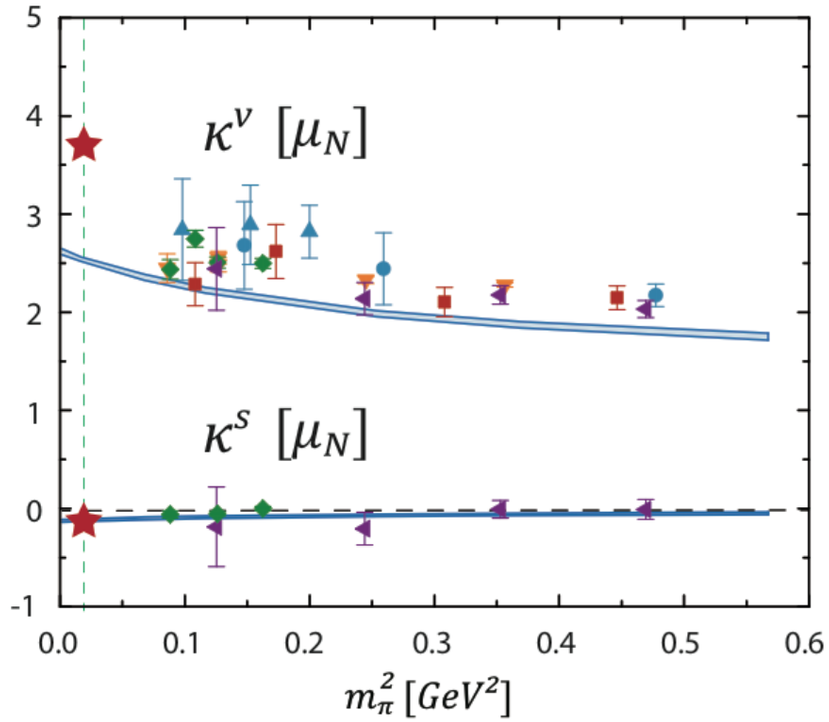








Isovector (p-n), isoscalar (p+n):



- missing pion cloud effects
- similar for axial form factors

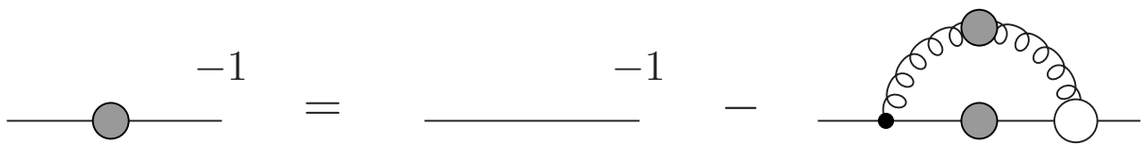
Eichmann, PRD 84 (2011)

Eichmann and CF, EPJ A48 (2012) 9

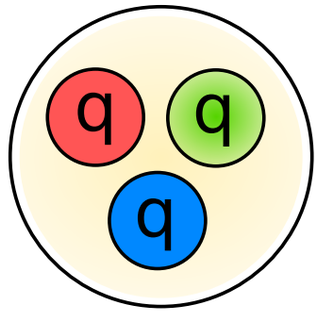
latest results: see Chen, CF, Roberts, Segovia, in preparation

Overview

1. Mass from nothing

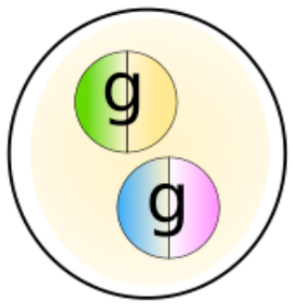


2. Baryons



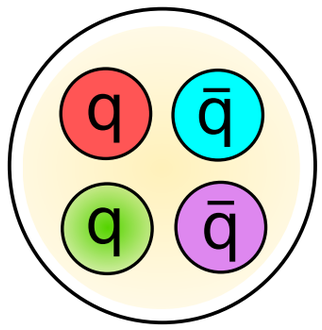
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3. Glueballs



CF, Huber, Sanchis-Alepuz, EPJC 80 11, 1077 (2020), arXiv:2004.00415

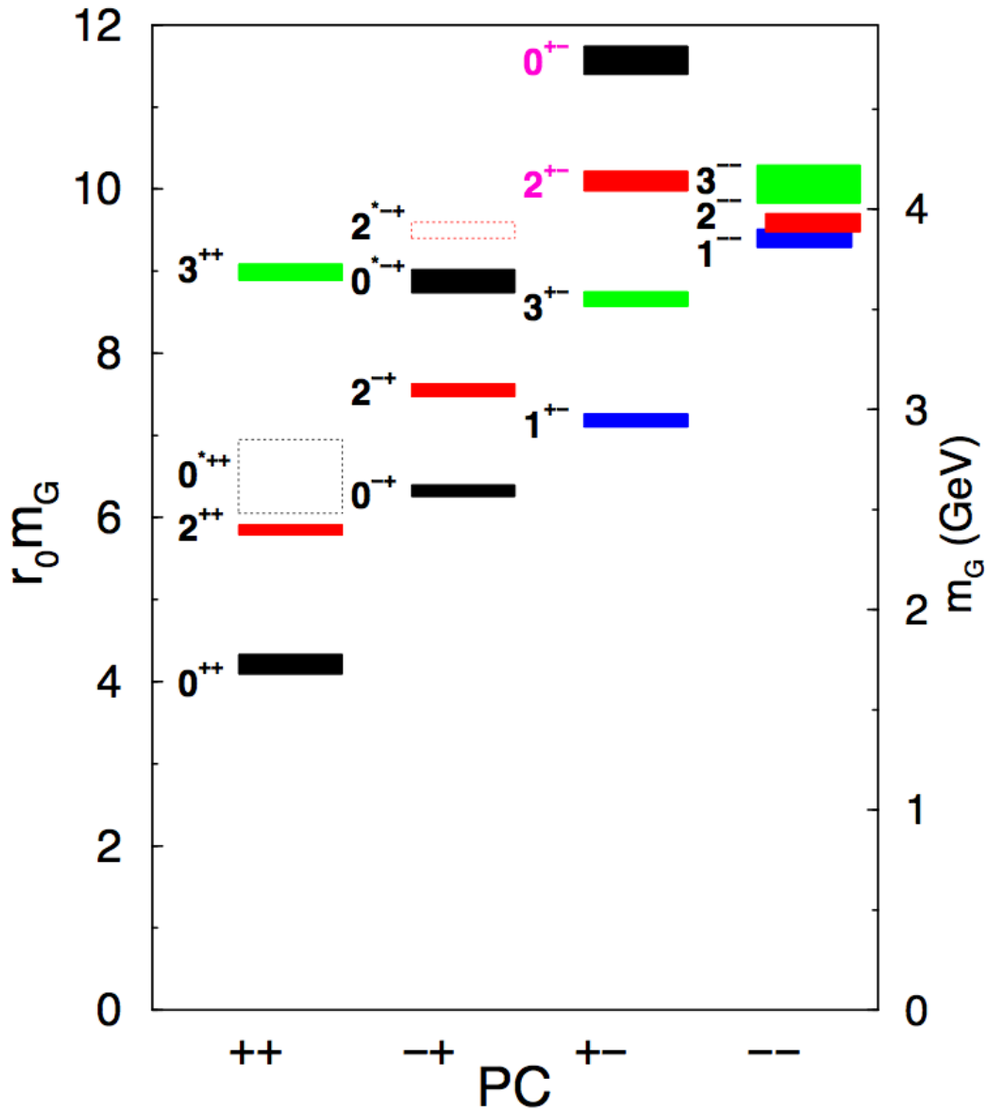
4. (Heavy-light) Tetraquarks



Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, arXiv:1905.02615
Wallbott, Eichmann and CF, PRD102 (2020) no.5, 051501, arXiv:2003.12407
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495

Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4,38, arXiv:2008.10240

Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509
Y.~Chen et al., PRD 73 (2006) 014516

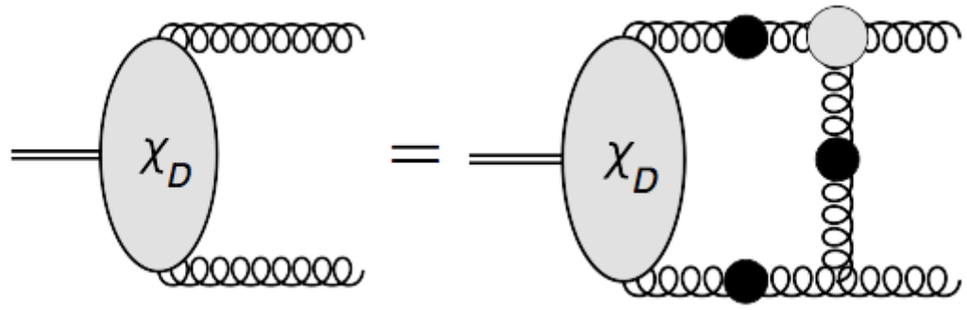
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Unquenched calculations very involved

Gregory et al., JHEP 1210 (2012) 170

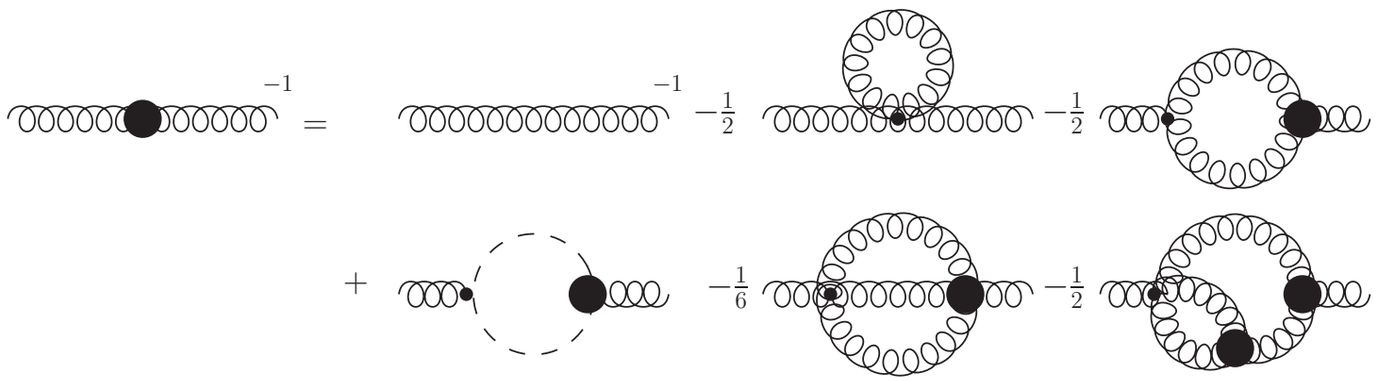
DSE:

- structural information



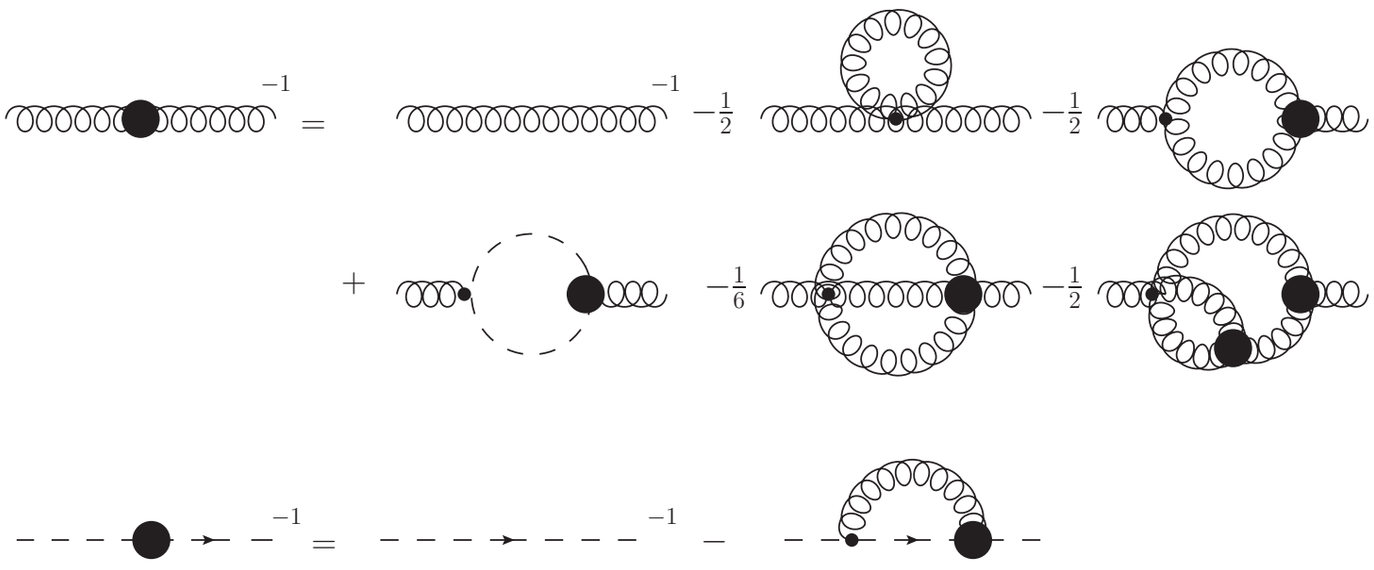
Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Landau gauge - 3PI truncation



Huber, PRD 101 (2020) 114009, arXiv:2003.13703

Landau gauge - 3PI truncation



Huber, PRD 101 (2020) 114009, arXiv:2003.13703

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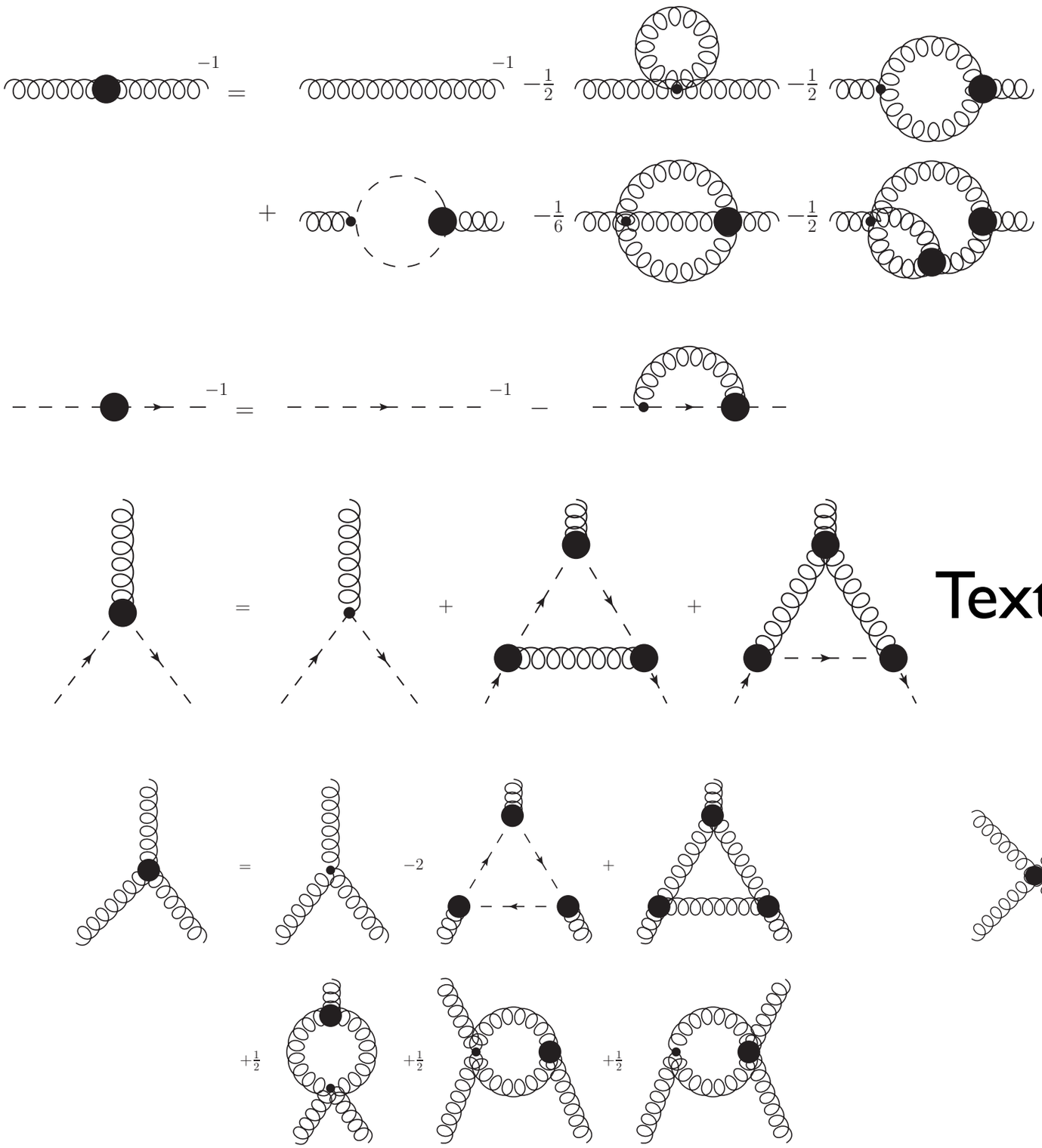
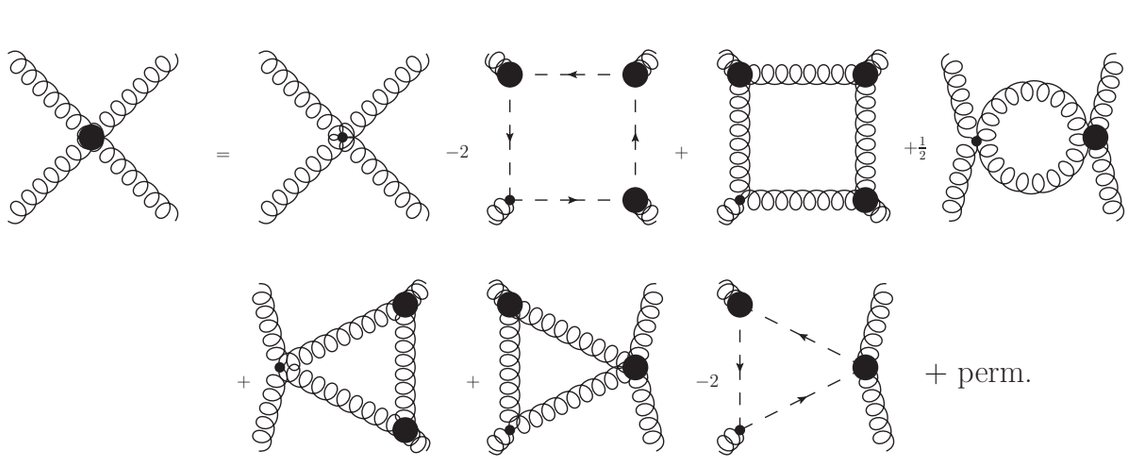
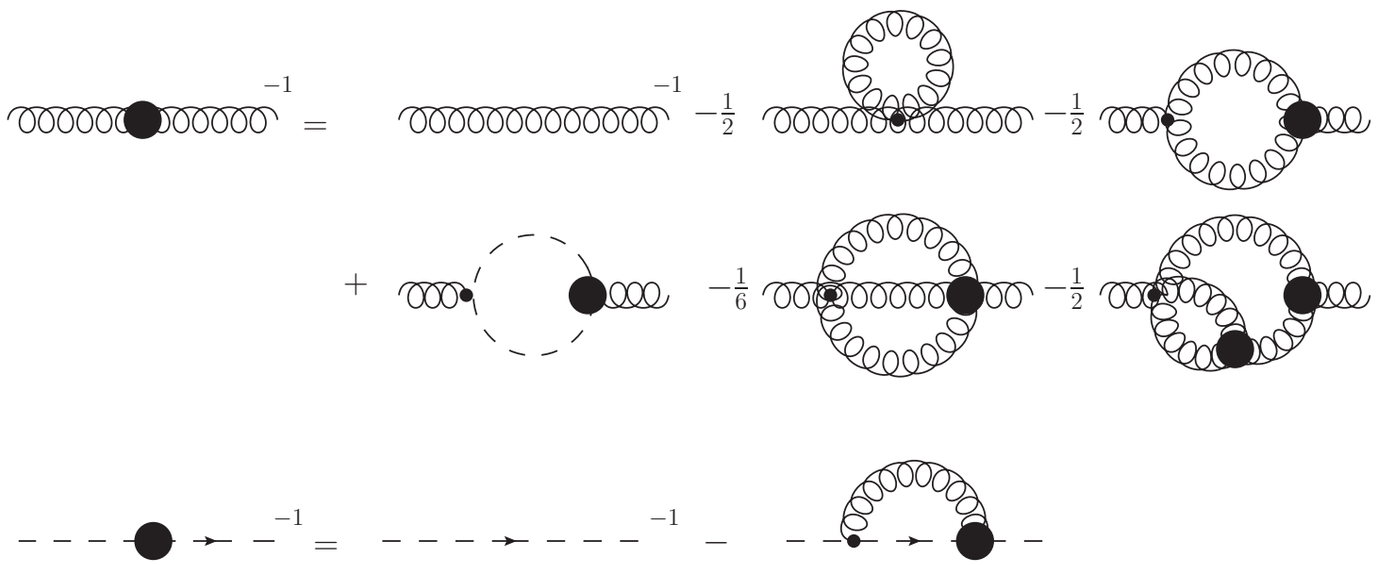


Text



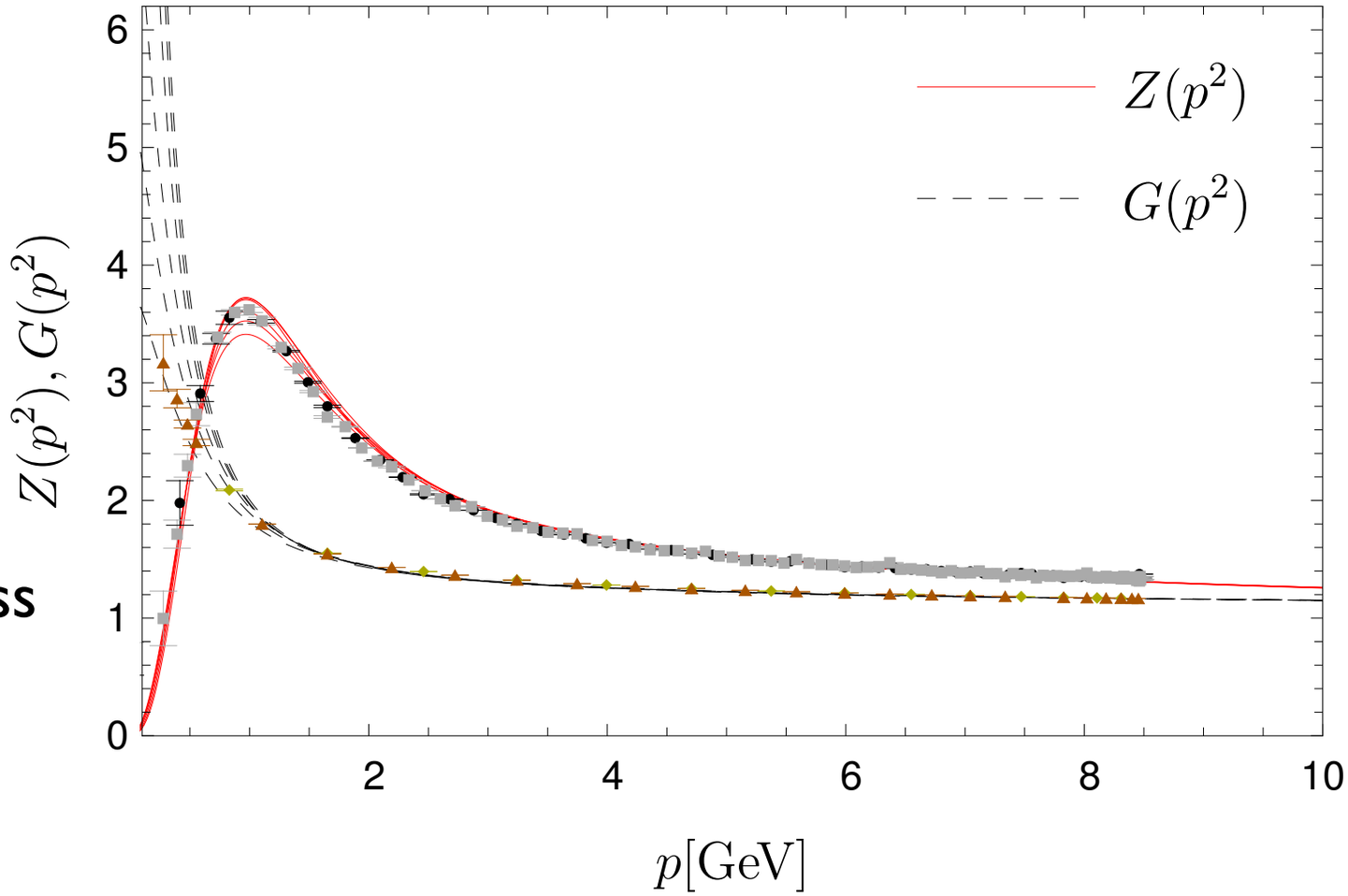
Huber, PRD 101 (2020) 114009, arXiv:2003.13703

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

- spacelike momenta: good agreement with lattice
- fully dressed gluon appears massive
- time-like momenta: work in progress

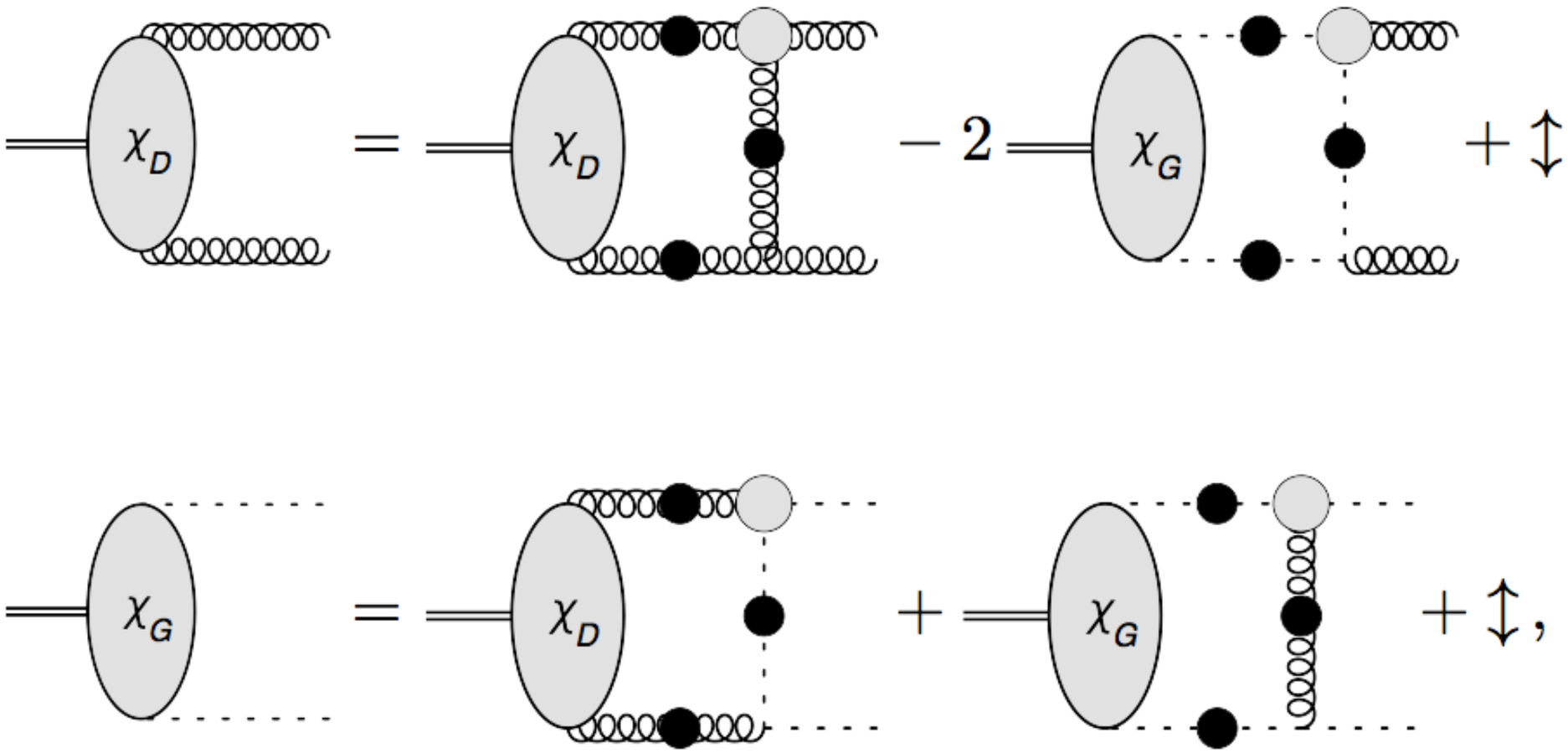


DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

CF, Huber, PRD 102 (2020) 094005, arXiv:2007.11505

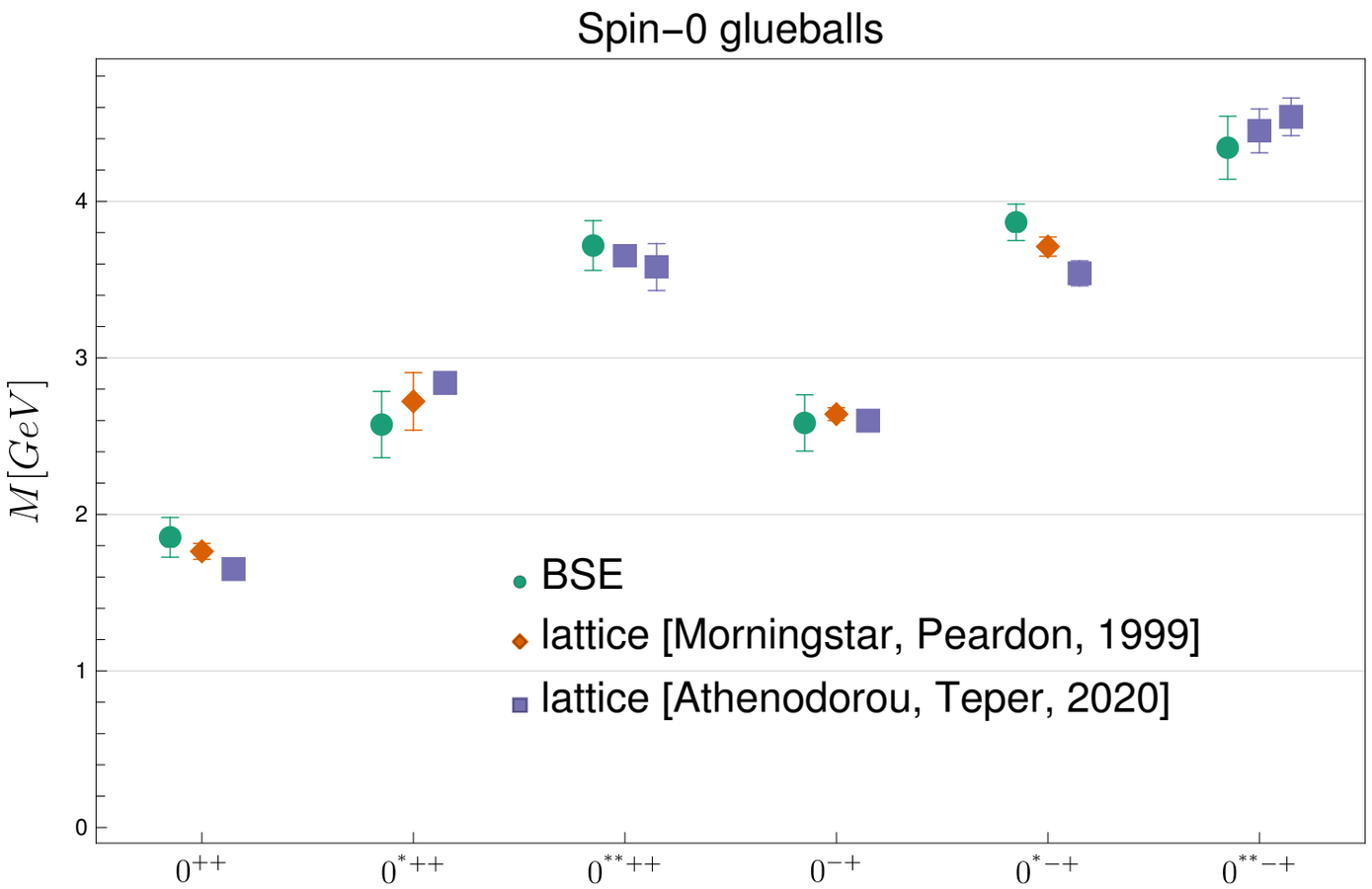
Glueballs from DSE/BSEs



Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

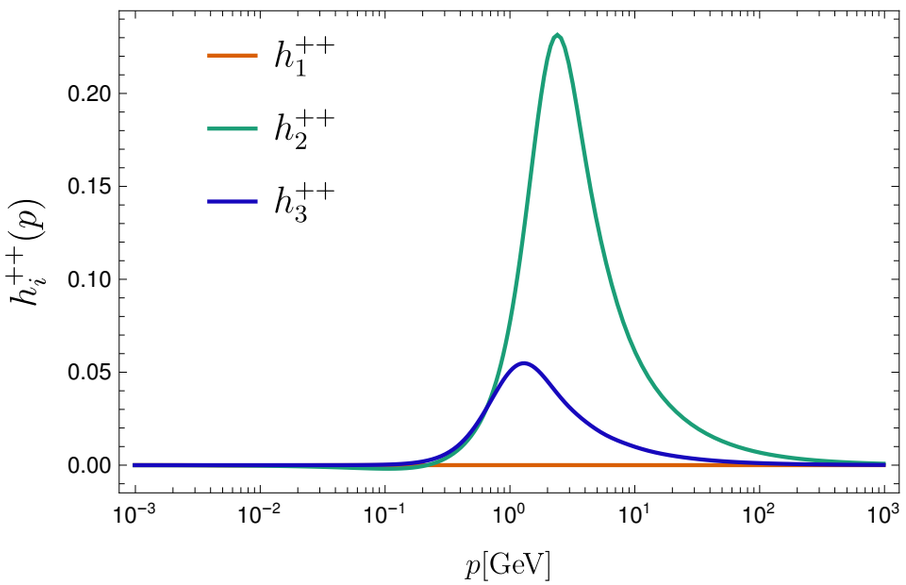
- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

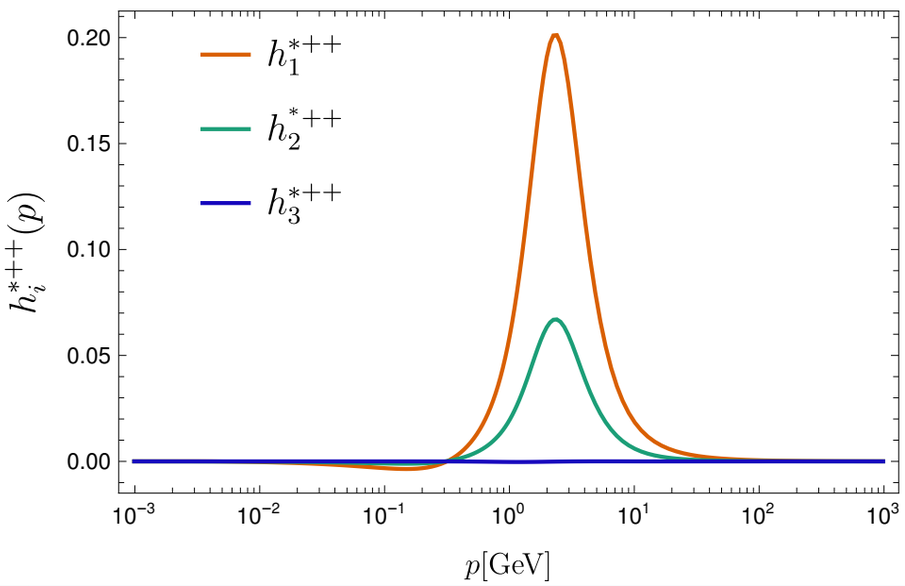
Glueballs: results



- spectrum:
  very good agreement
  lattice vs. DSE/BSE

CF, Huber, Sanchis-Alepuz, accepted by EPJC, arXiv:2004.00415

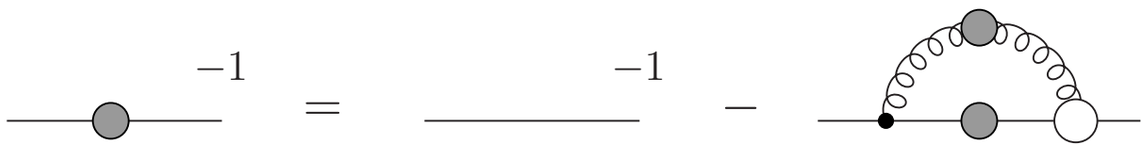




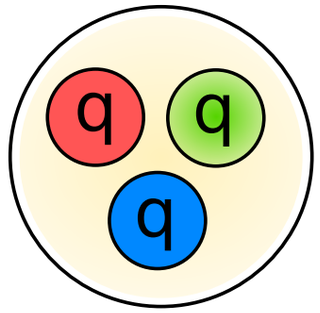
- excited states:
  different internal
  structure

Overview

1. Mass from nothing

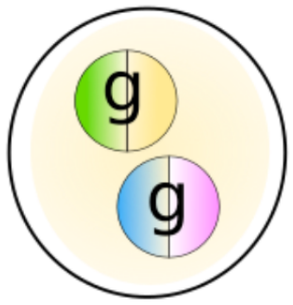


2. Baryons



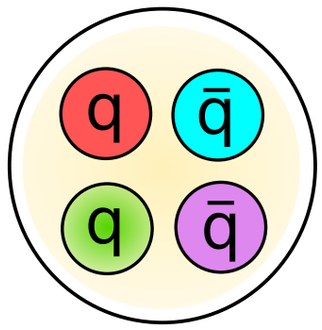
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3. Glueballs



CF, Huber, Sanchis-Alepuz, EPJC 80 11, 1077 (2020), arXiv:2004.00415

4. (Heavy-light) Tetraquarks

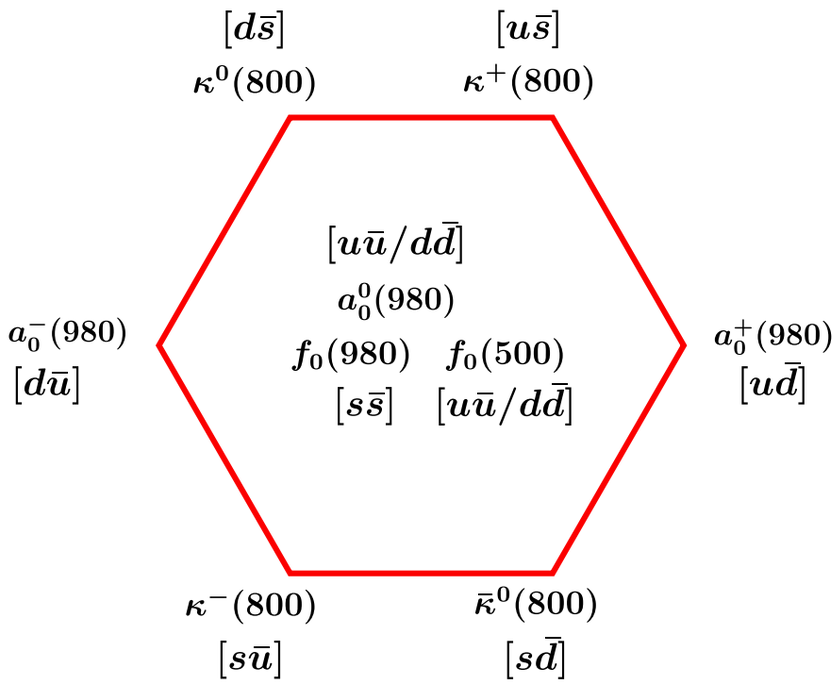


Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, arXiv:1905.02615
Wallbott, Eichmann and CF, PRD102 (2020) no.5, 051501, arXiv:2003.12407
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495

Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4,38, arXiv:2008.10240

Tetraquark candidates with $qq\bar{q}\bar{q}$-content

Light scalar mesons:



wrong level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

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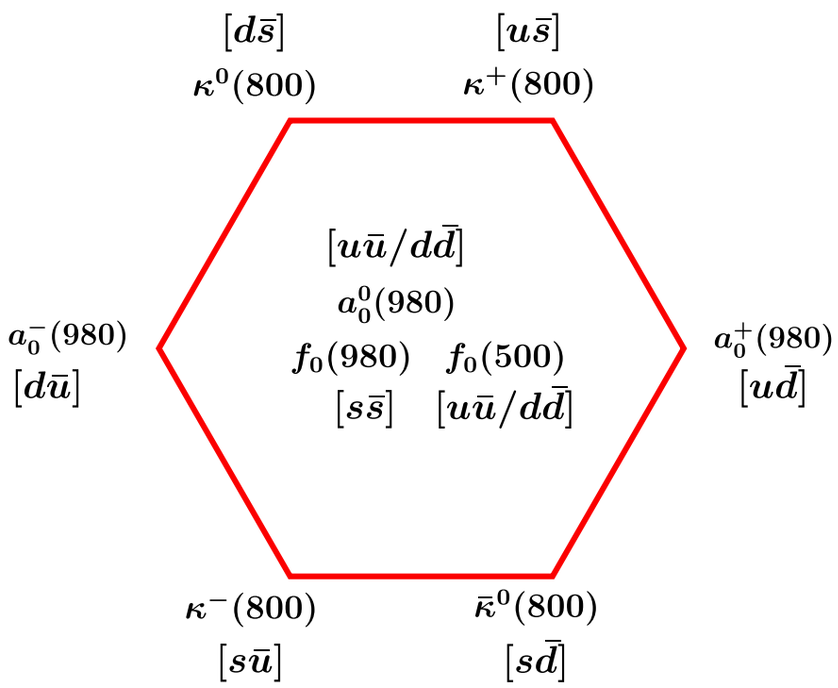

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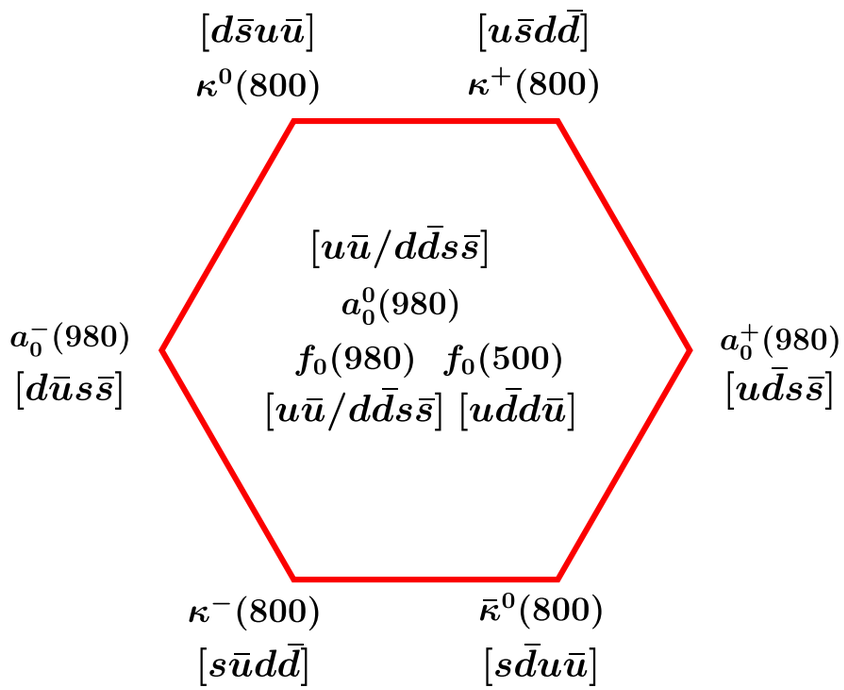

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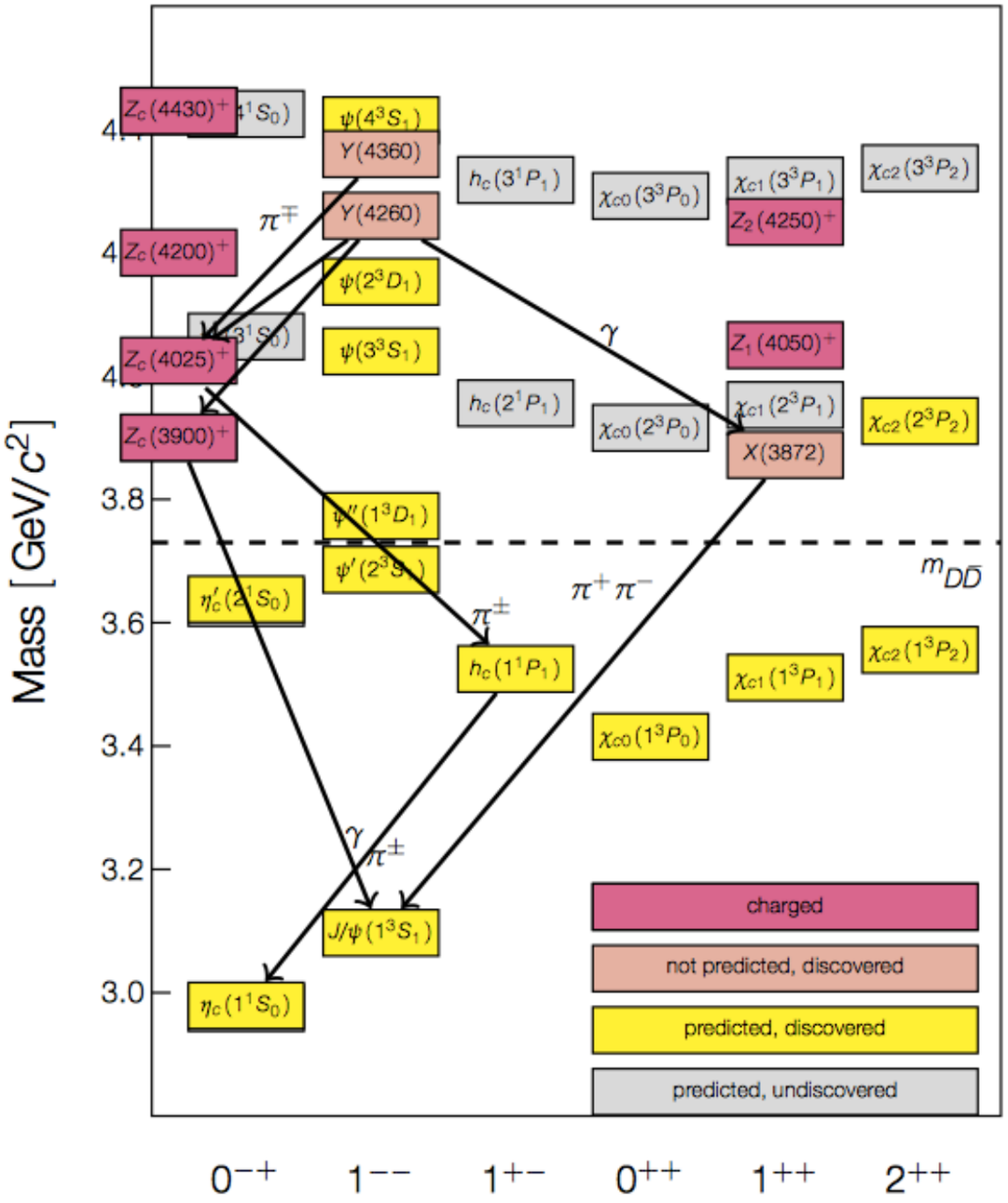
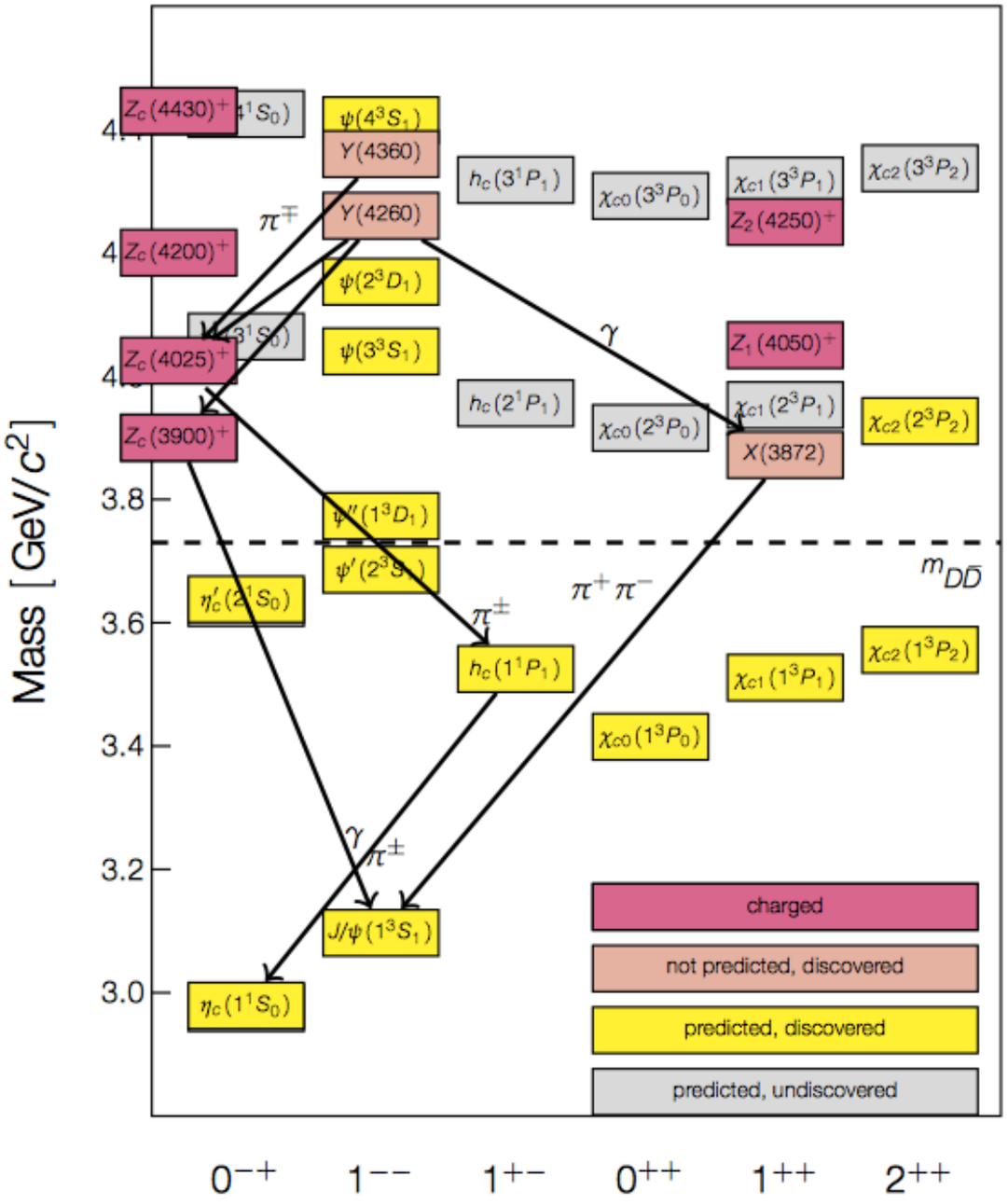
Tetraquark candidates with $cq\bar{q}\bar{c}$-content



Many new unexpected states found: Belle, BABAR, BES, LHCb …

Tetraquark candidates with $cq\bar{q}\bar{c}$-content



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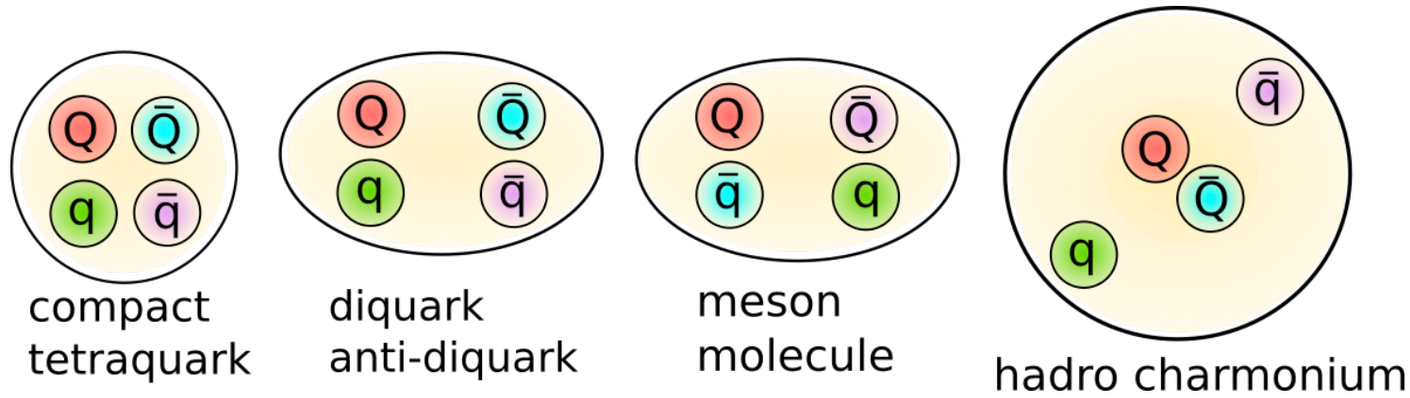
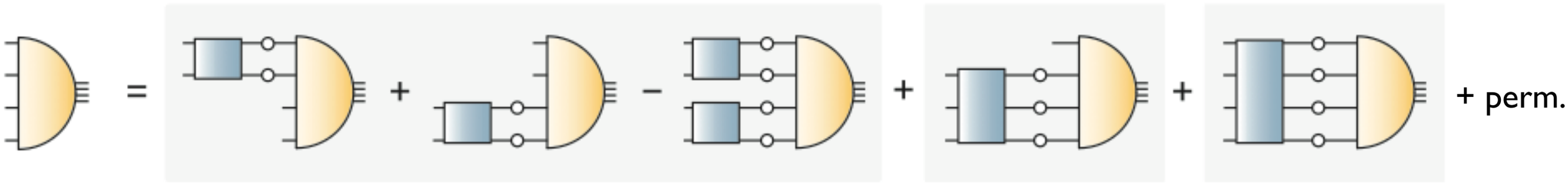
Internal structure ??



Related to details of underlying QCD forces between quarks and gluons

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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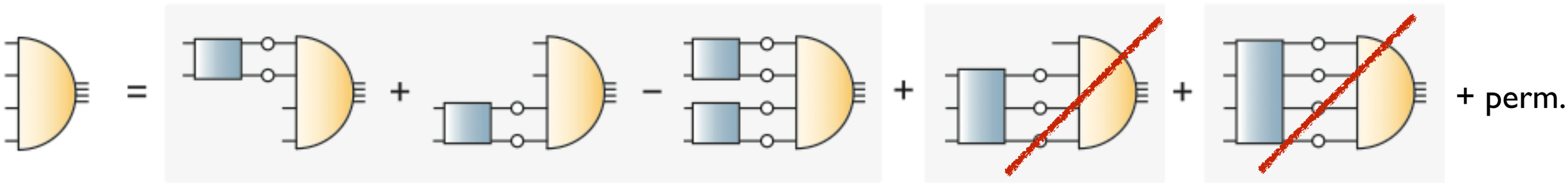

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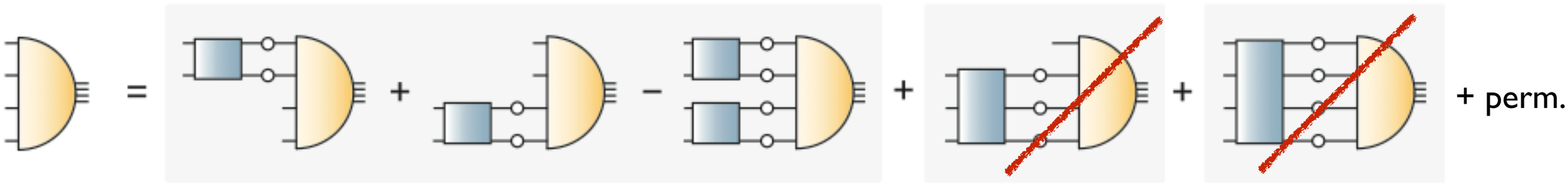

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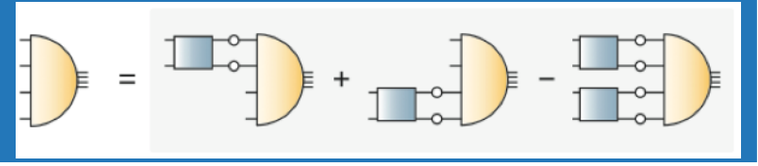
- Two-body interactions: allow for internal clustering
- use RL-approximation

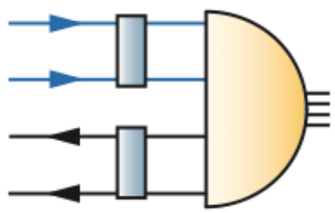
Four-body equation: permutations

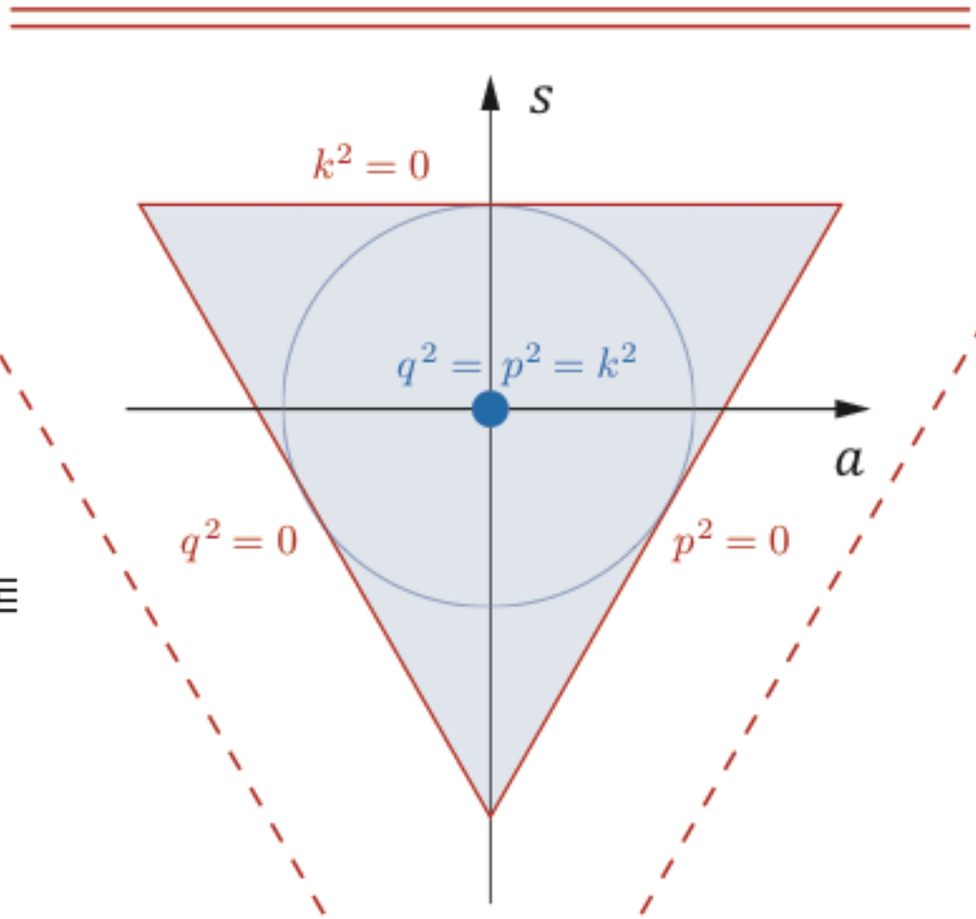


- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$
- Doublet: $s \sim p^2 + q^2 - 2k^2$

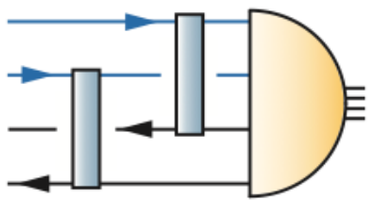
$a \sim q^2 - p^2$

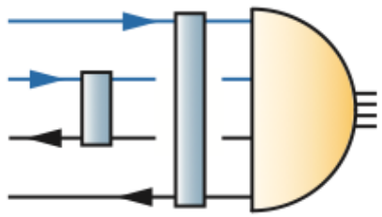
p,q,k: relative momenta





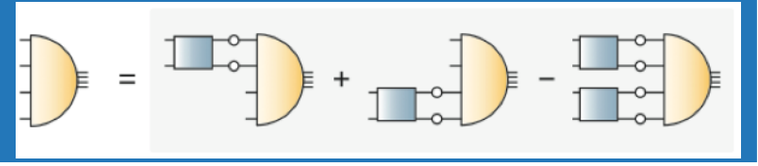
diquark pole





meson poles

Four-body equation: permutations



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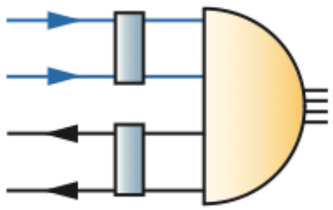

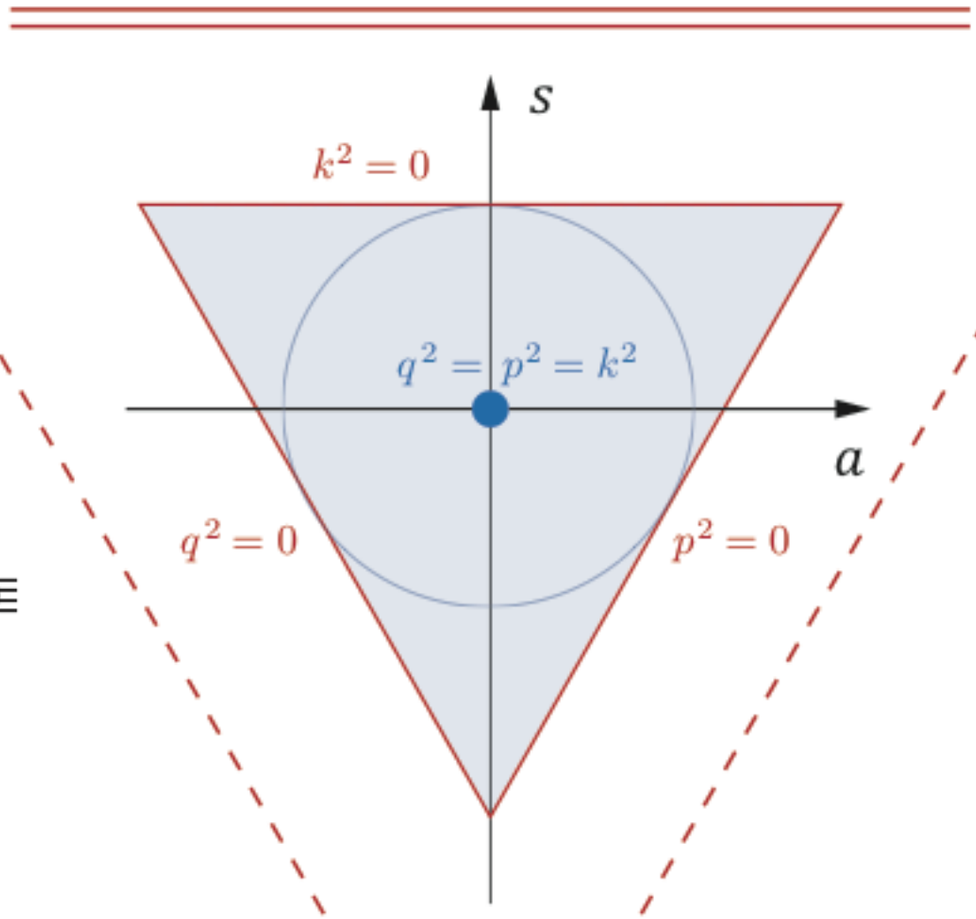

diquark pole

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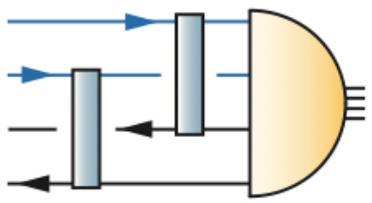
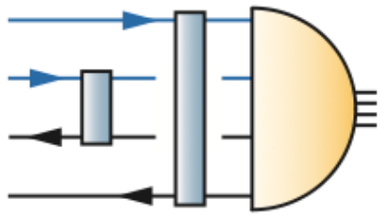
u $\bar{c}$



hadro charmonium



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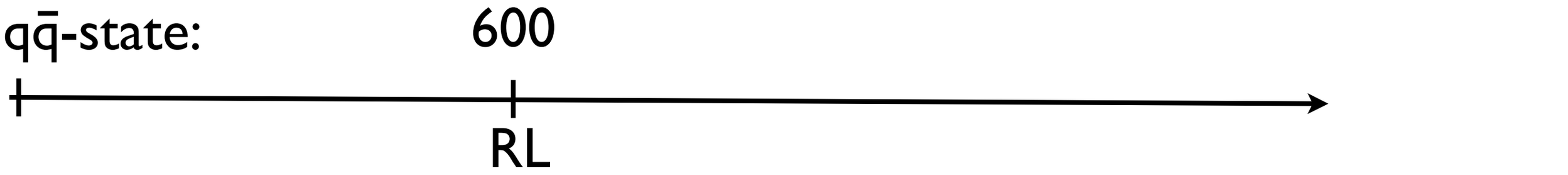
molecule

meson poles

Bound state vs resonance: light scalars



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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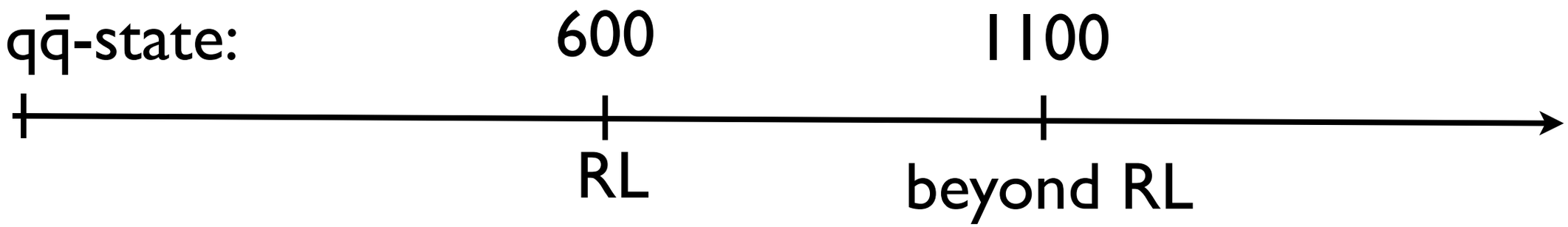

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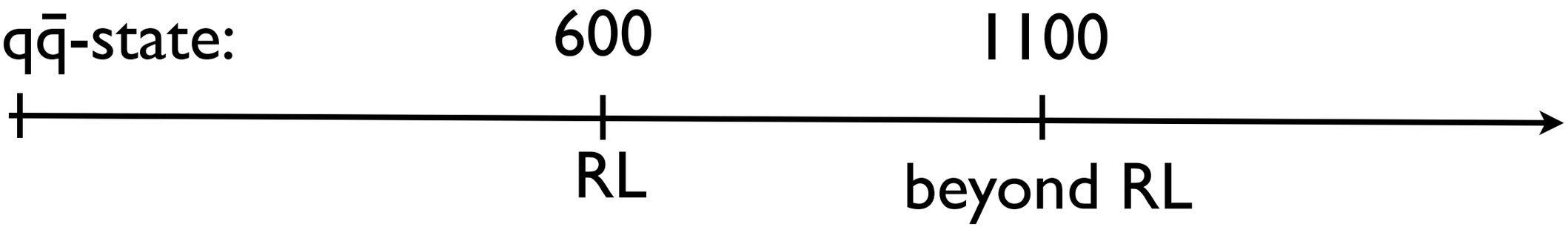


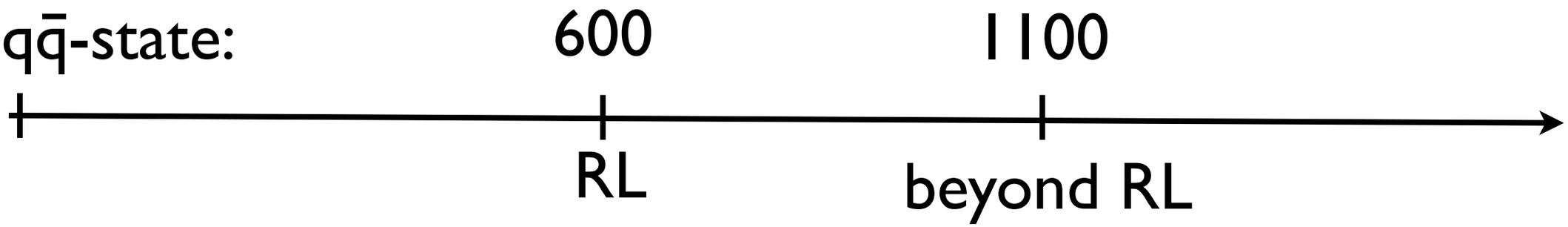
qq̄qq̄ state: $\Gamma(S_0, \cancel{s, a,} \ldots)$

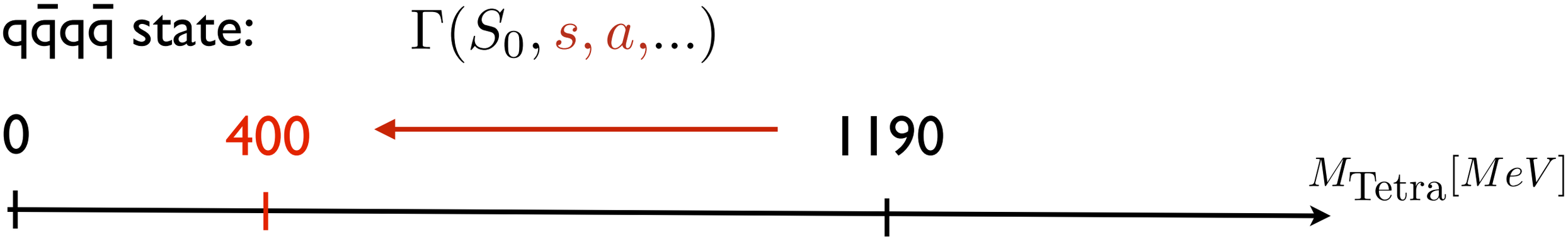
restricted kinematics

Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: light scalars





full kinematics

Two-pion resonance

restricted kinematics

Bound state of
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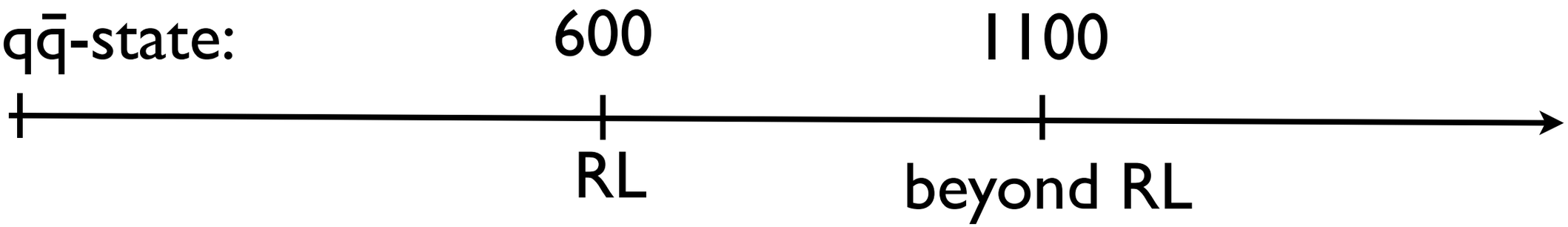
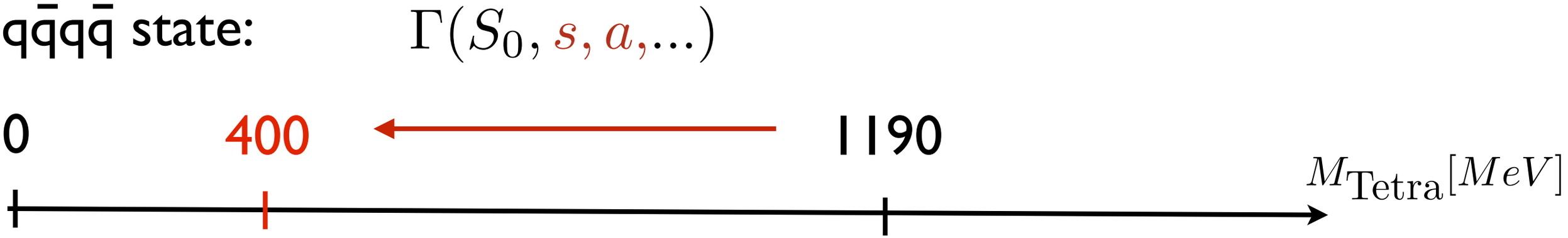
⟶ identify with $f_0(500)$ ('σ-meson')

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: light scalars





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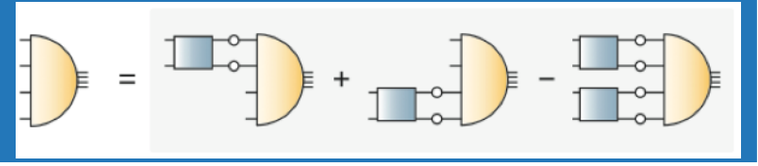
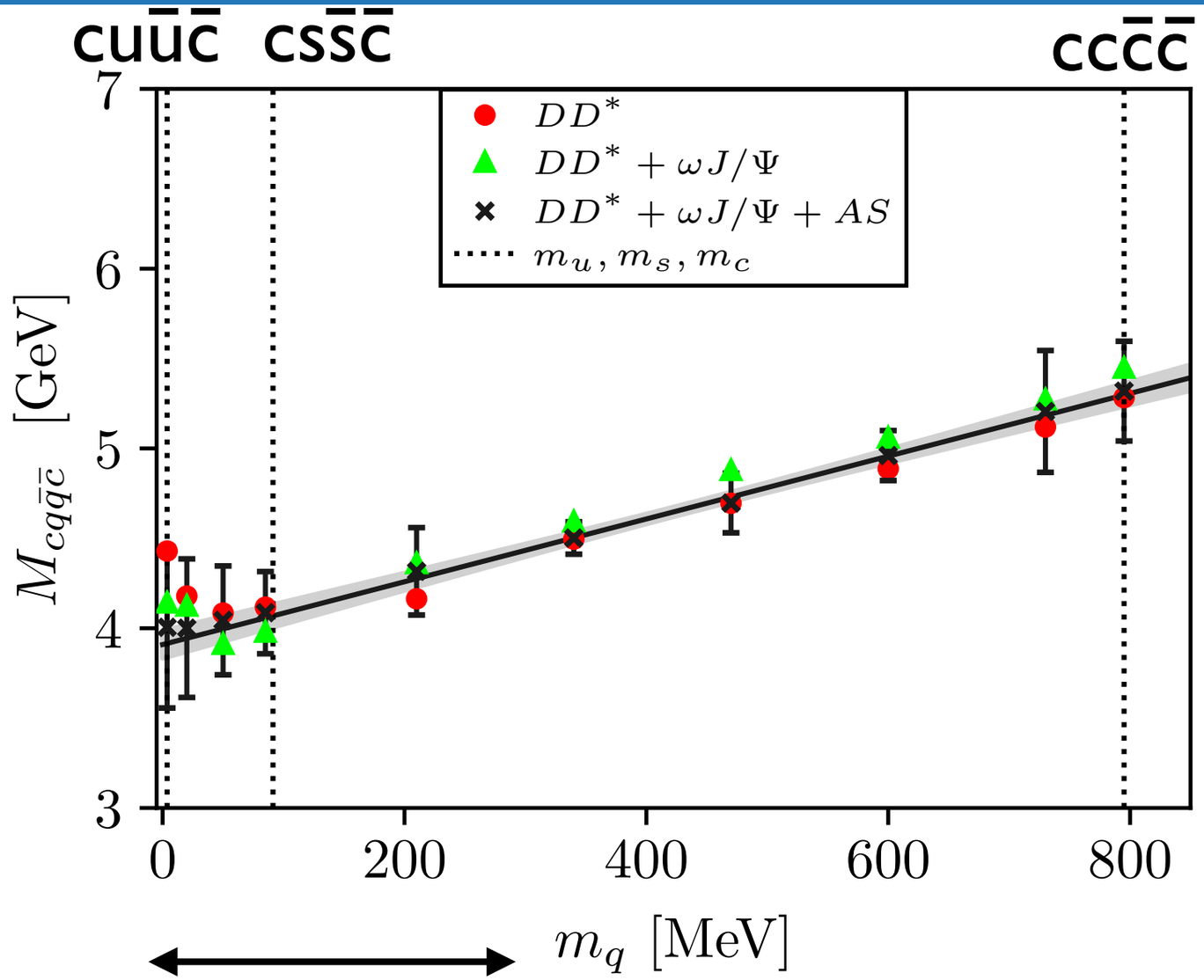
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Mixing with qq̄: small effect

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

$J^{PC} = 1^{++}$





Extrapolation in BSE necessary…

m_c fixed

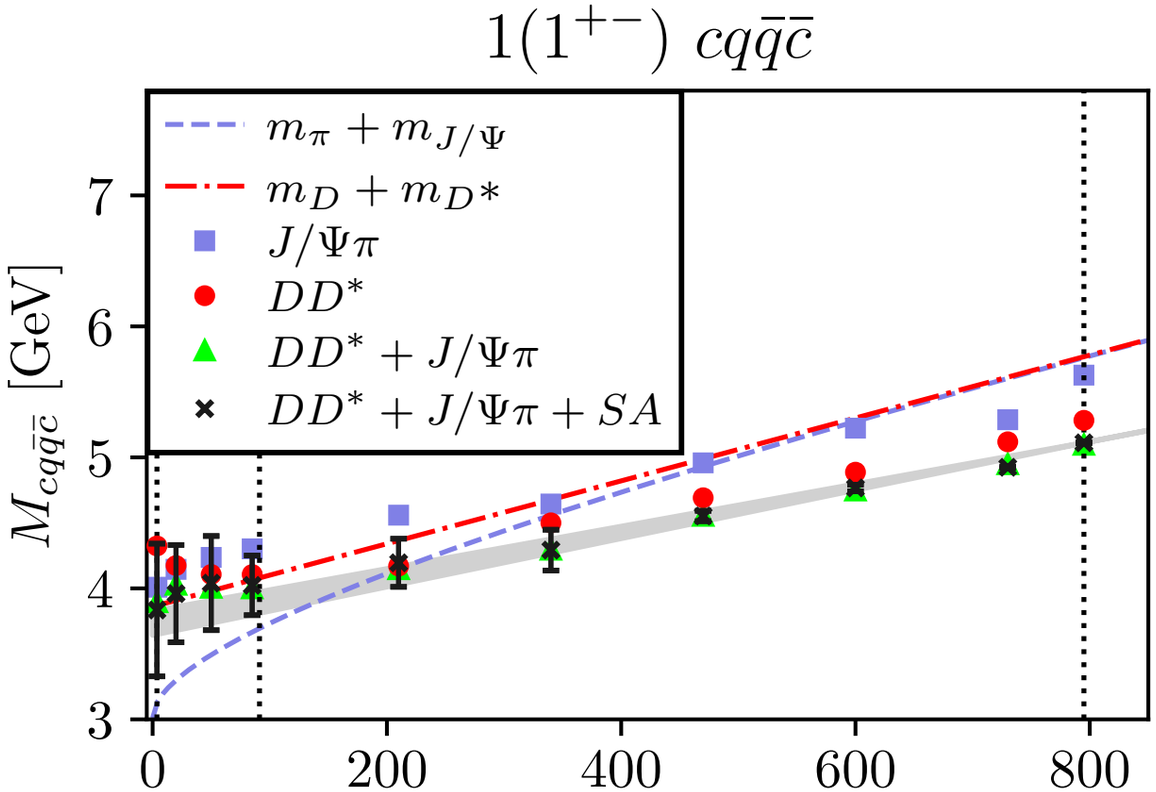
m_q varied

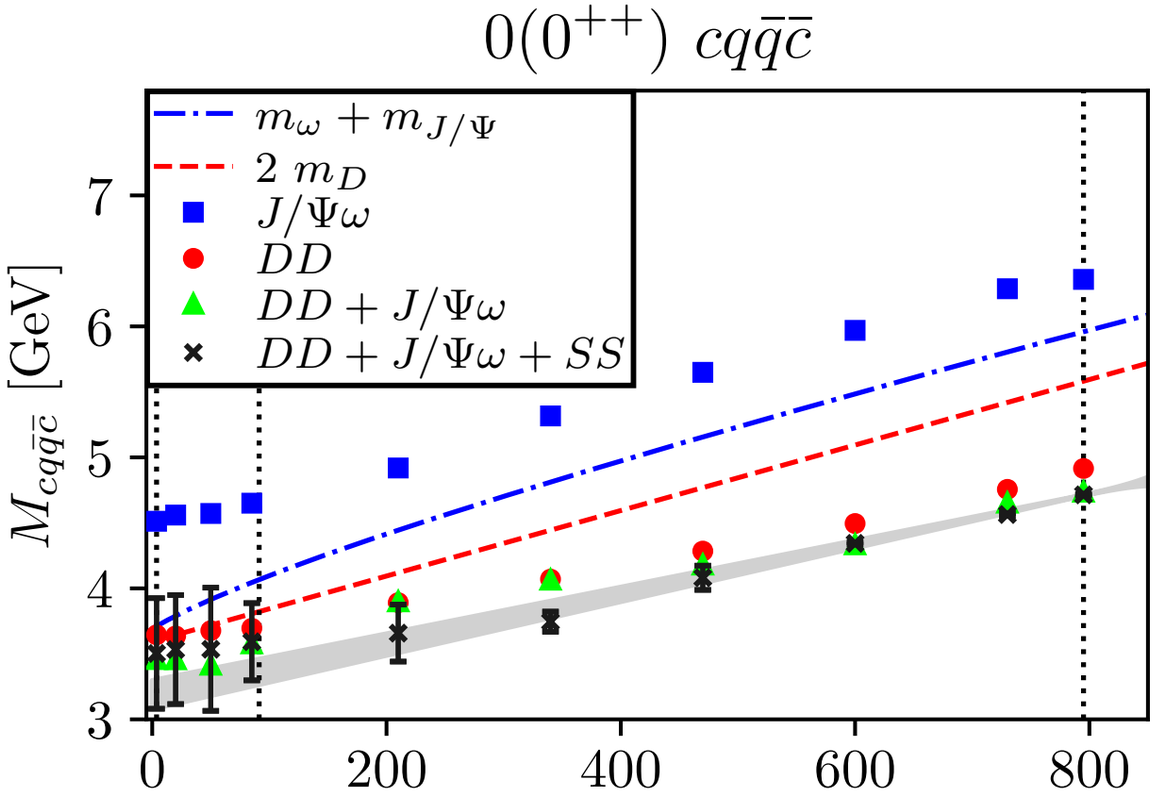
- DD* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74)\,\mathrm{MeV} \longrightarrow X(3872)$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

$J^{PC} = 1^{+-}$ and 0^{++}



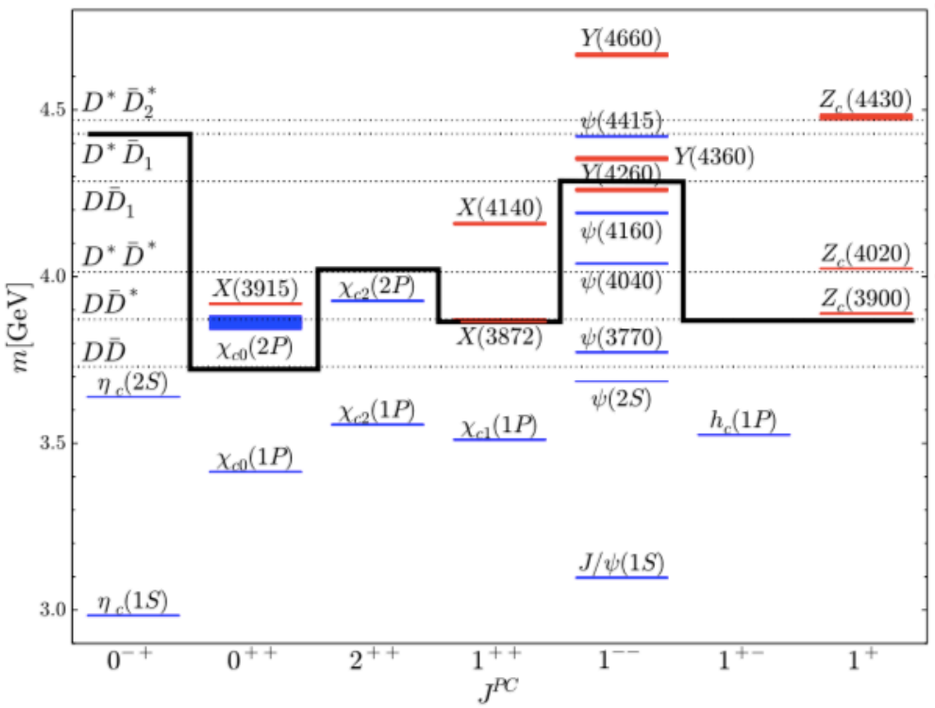


Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

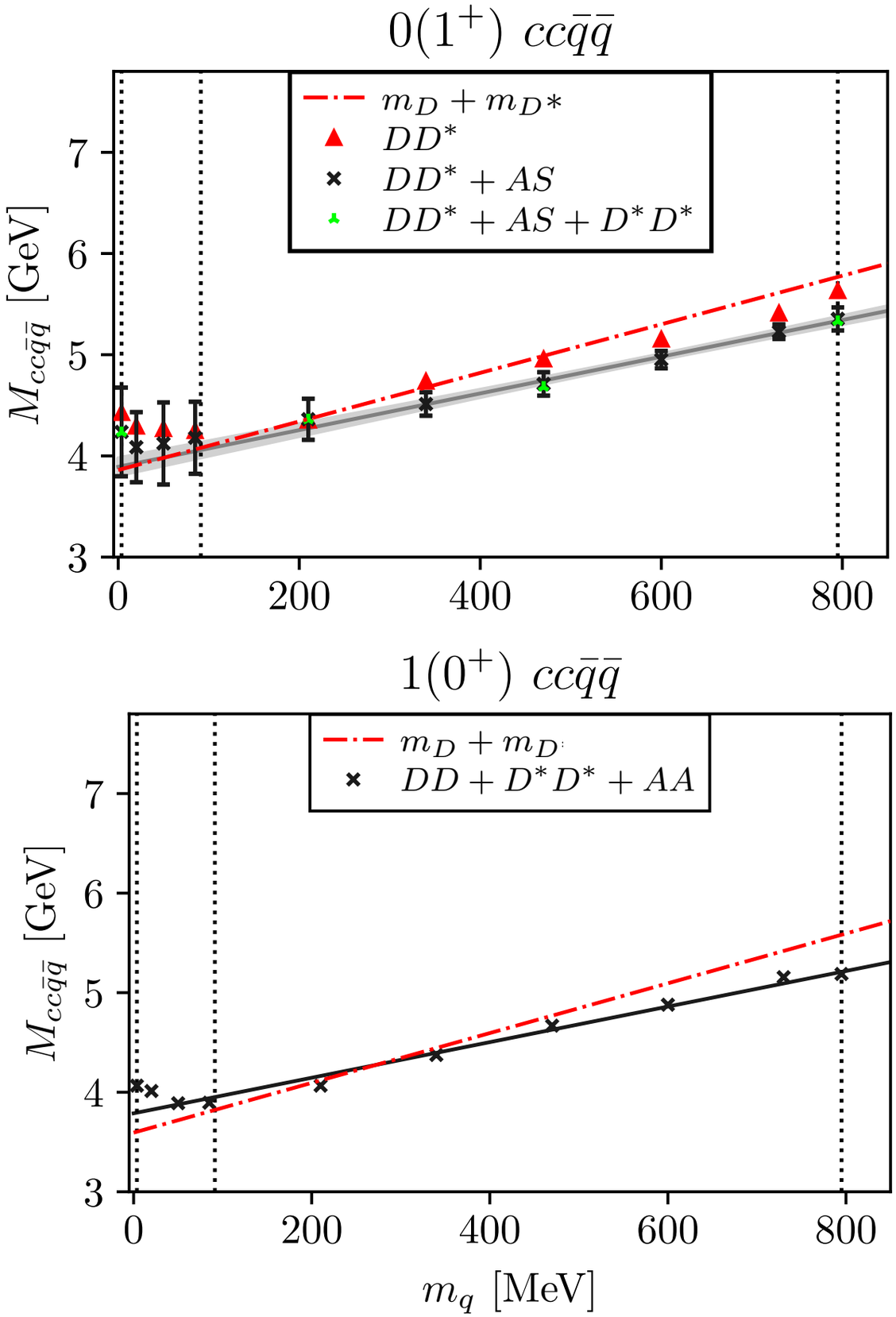
$$M^{cq\bar{q}\bar{c}}_{1^{+-}} = 3741(91) \ \rightarrow \ Z(3900)$$

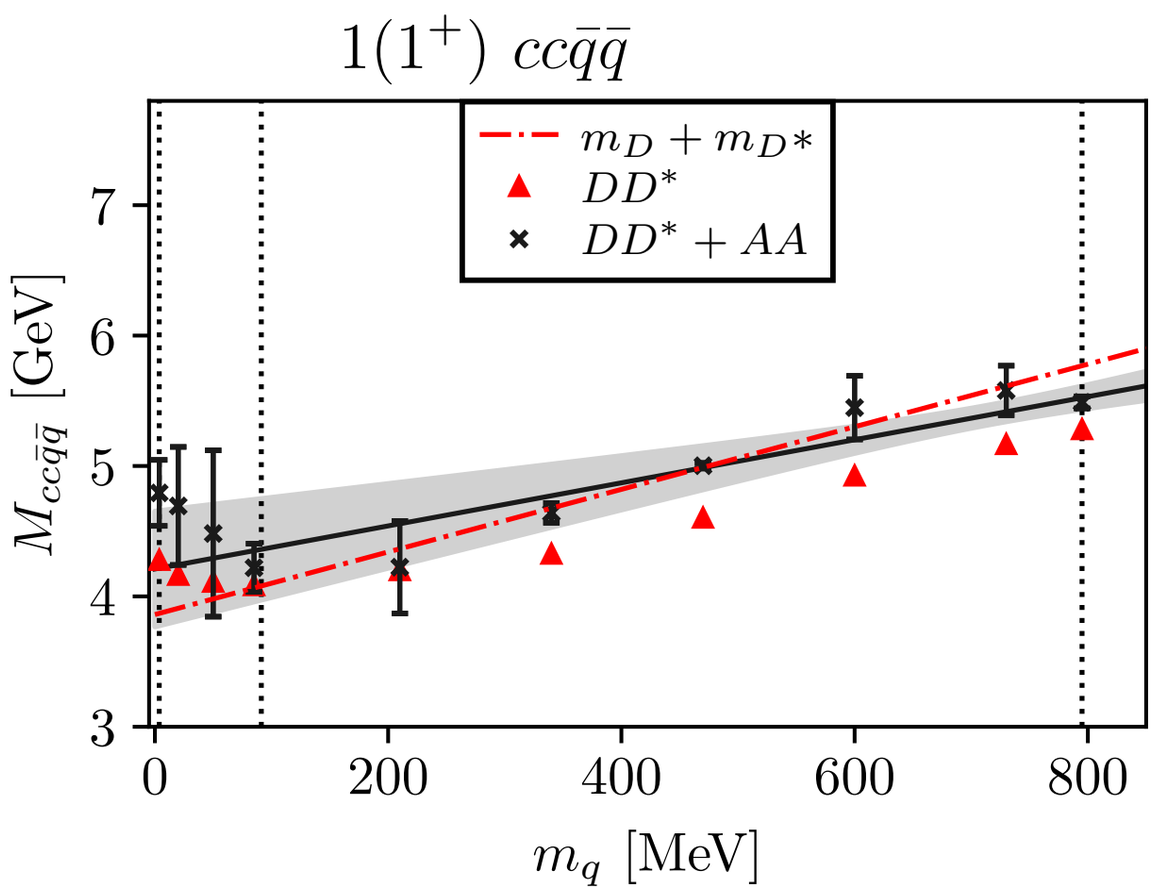
$$M^{cq\bar{q}\bar{c}}_{0^{++}} = 3195(107) \ \rightarrow \ ?$$

mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states





- DD(*) and diquarks important!

Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

Summary

Main goals:

- one framework for all areas of hadron physics:
  - discussed: baryons, mesons, glueballs, tetraquarks
  - not discussed: hybrids, form factors,
    anomalous magnetic moments QCD phase diagram
  - access to DXSB, confinement,…

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016) [1606.09602]
CF, PPNP 105 (2019) [1810.12938]

Main challenge:

- systematic control over error budget:
  intrinsic + cp to other methods like lattice QCD

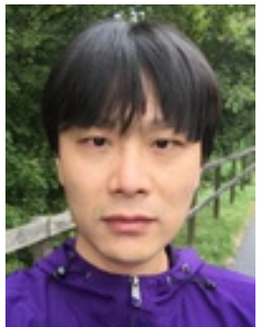
Main results:

- NOT high precision physics
- BUT competitive contributions in many areas

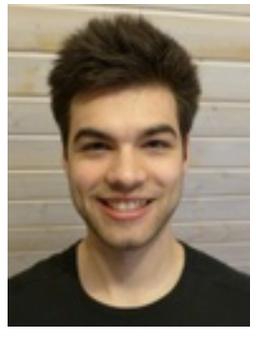
Thanks to…

… the group:

Postdocs:
- Dr. Chen Chen
- PD Dr. Markus Huber




PhD-candidates:
- Julian Bernhardt
- Pascal Gunkel
- Joshua Hoffer
- Philipp Isserstedt
- Nico Santowsky




MSc-students:
- Stephan Hagel
- Felix Keil

… and many external collaborators.

Bundesministerium für Bildung und Forschung

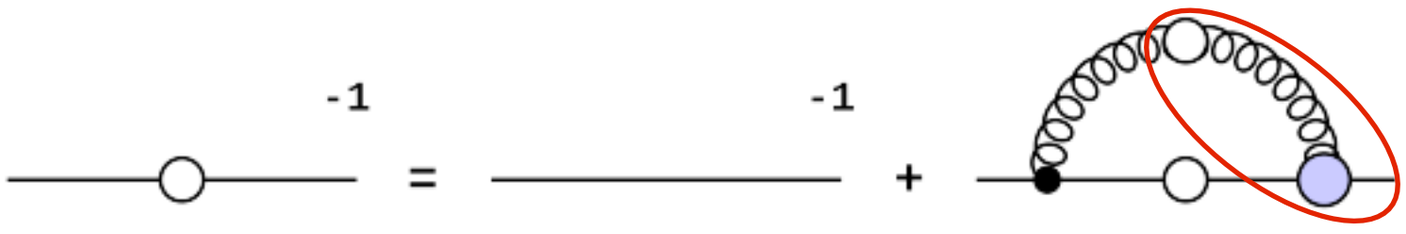
GSI

DFG

HIC for FAIR
Helmholtz International Center

Backup Slides

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

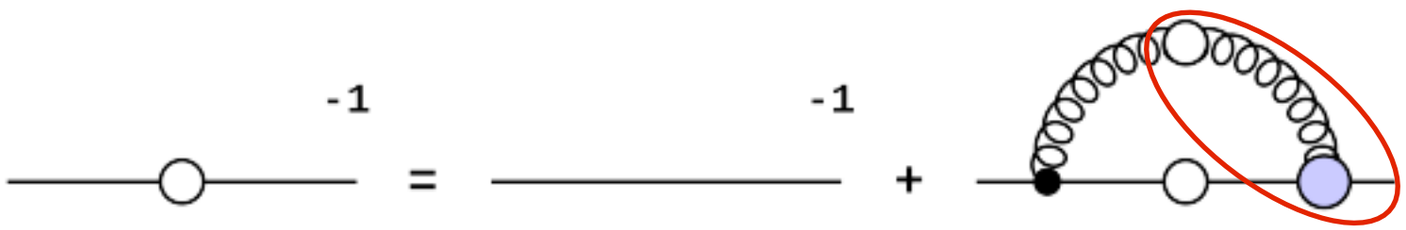
$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

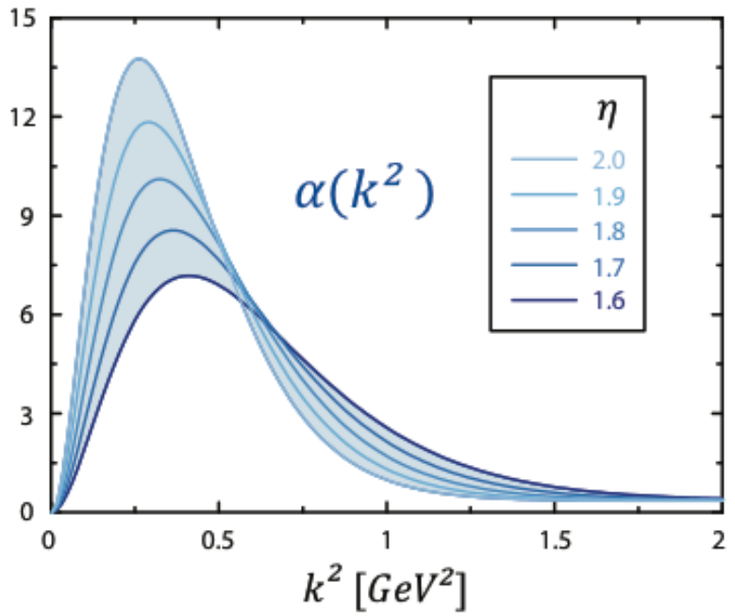
$$\sim \gamma^\mu \tau(k^2)$$ "approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

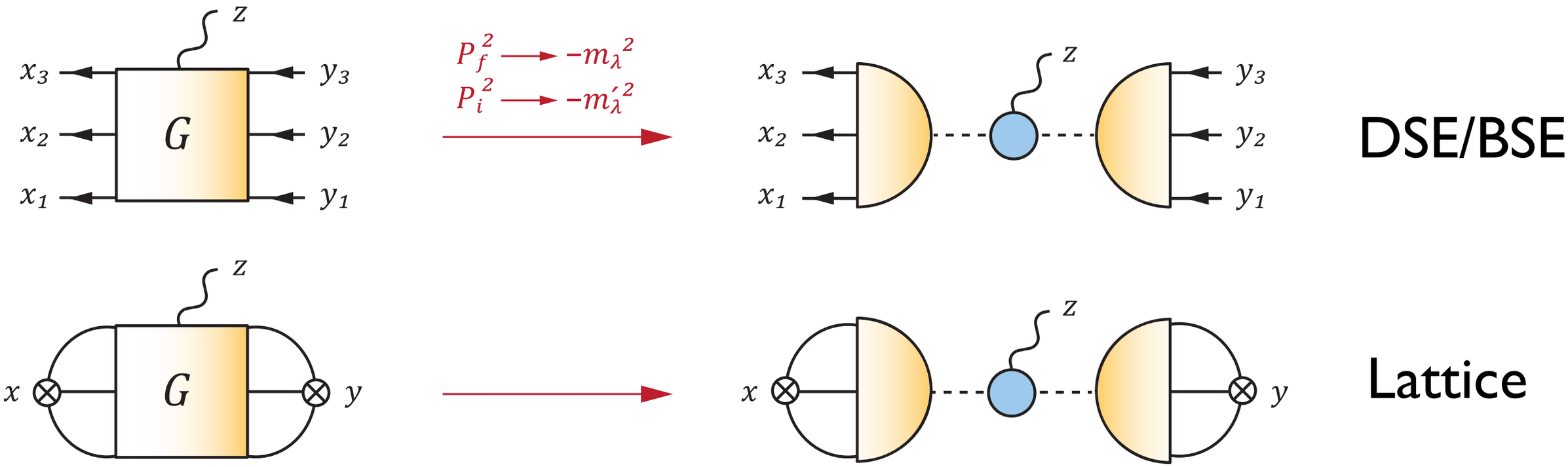
effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

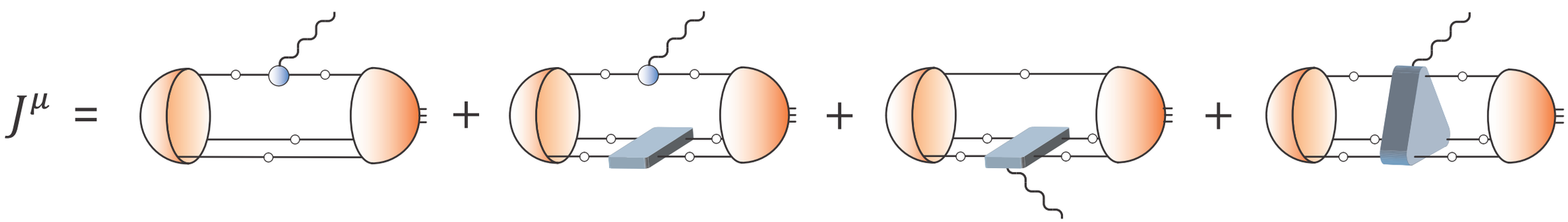
- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Extracting form factors from correlators

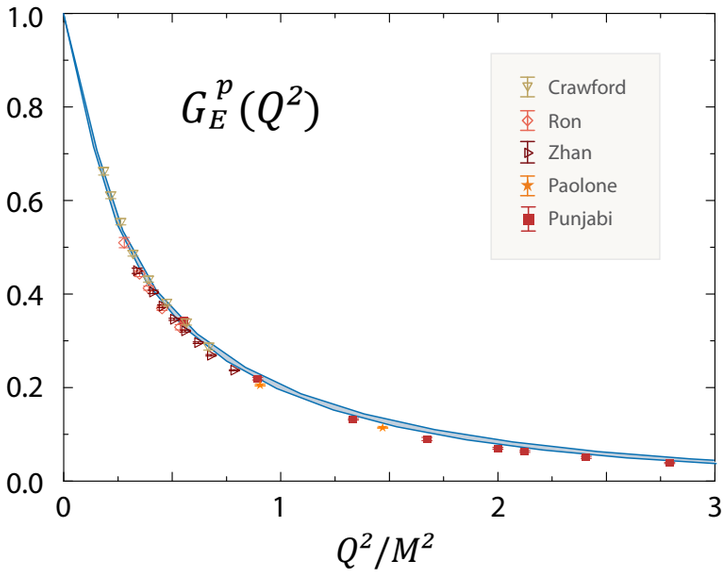


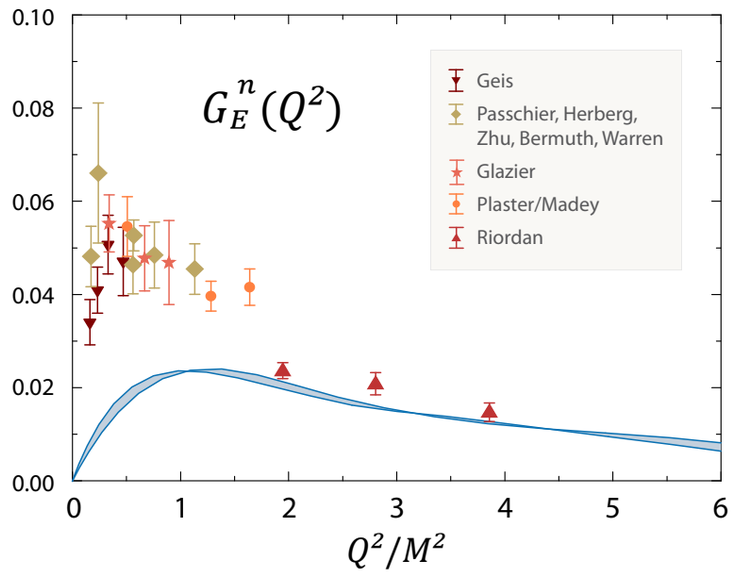
Form factor from BSEs (derived from equation of motion for G and 'gauging')

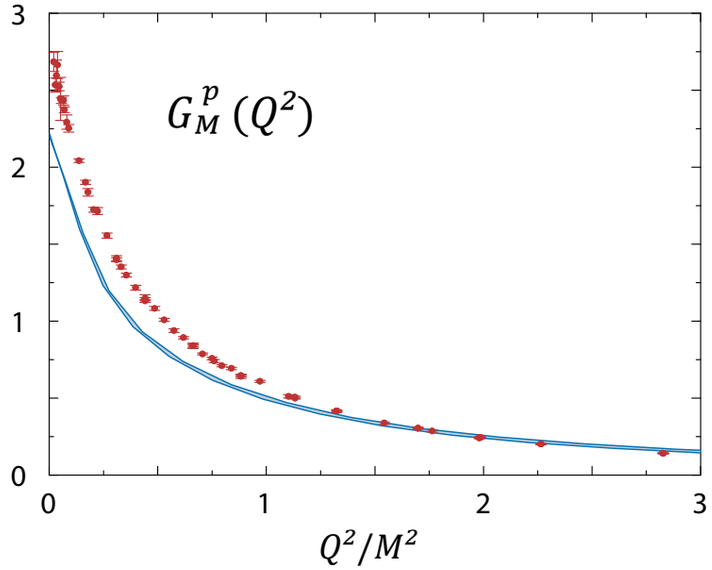


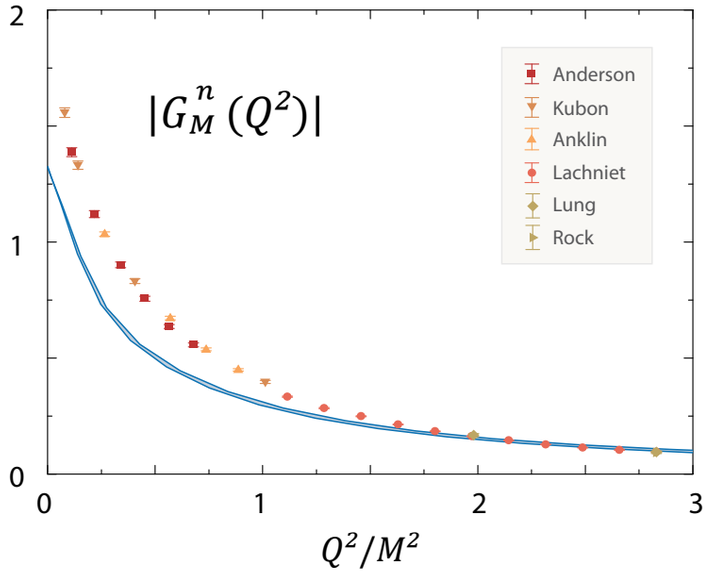
- exact equation for baryon form factors

Nucleon form factors and magnetic moments

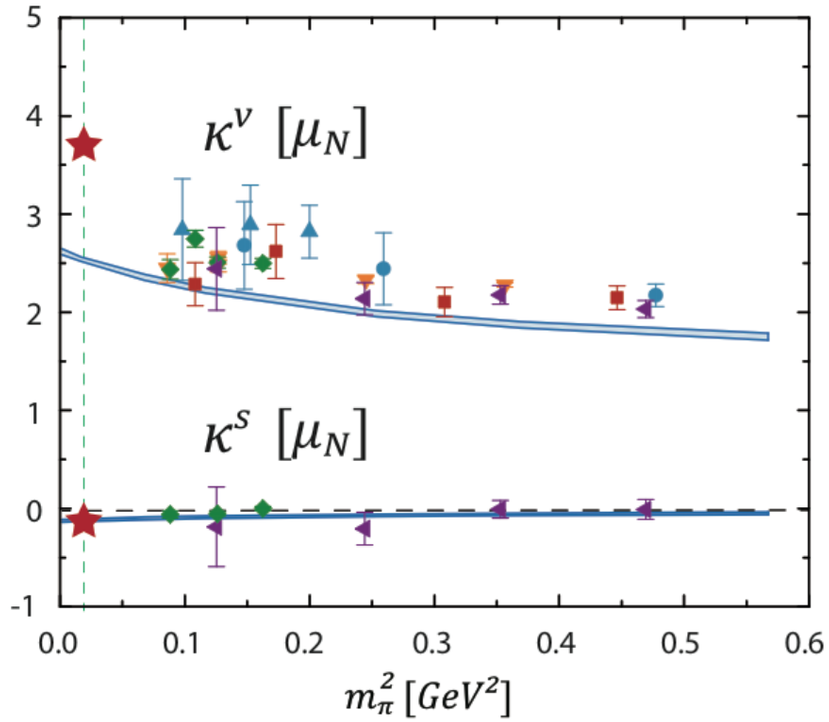








Isovector (p-n), isoscalar (p+n):

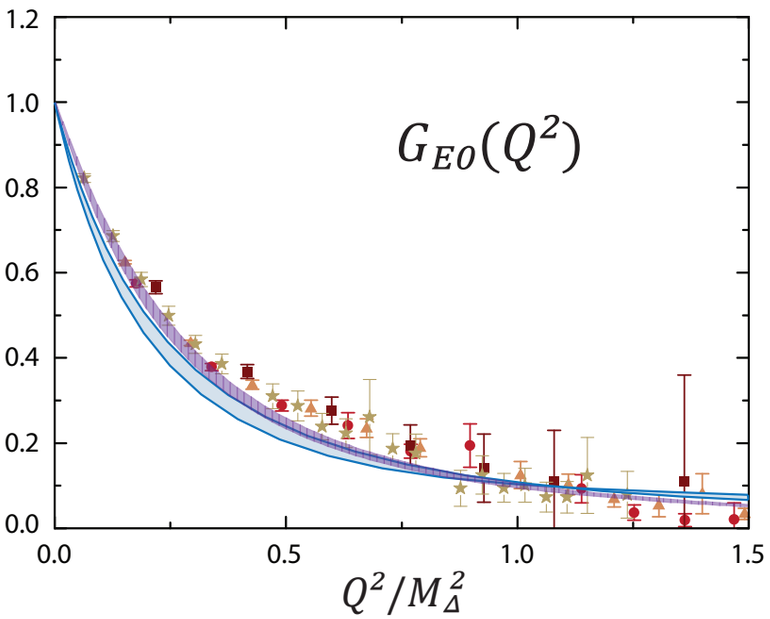


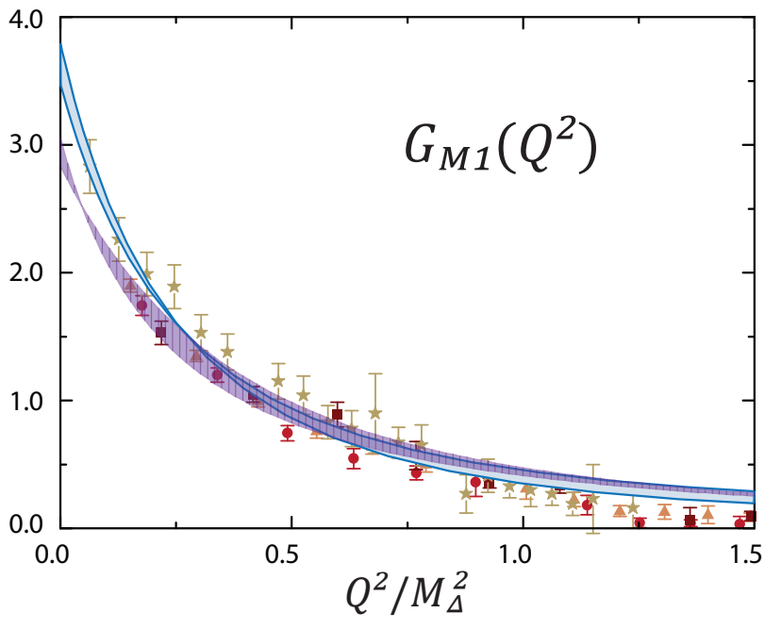
- missing pion cloud effects
- similar for axial form factors

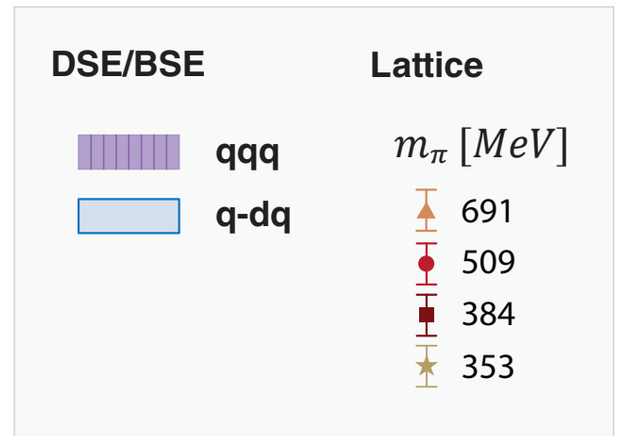
Eichmann, PRD 84 (2011)

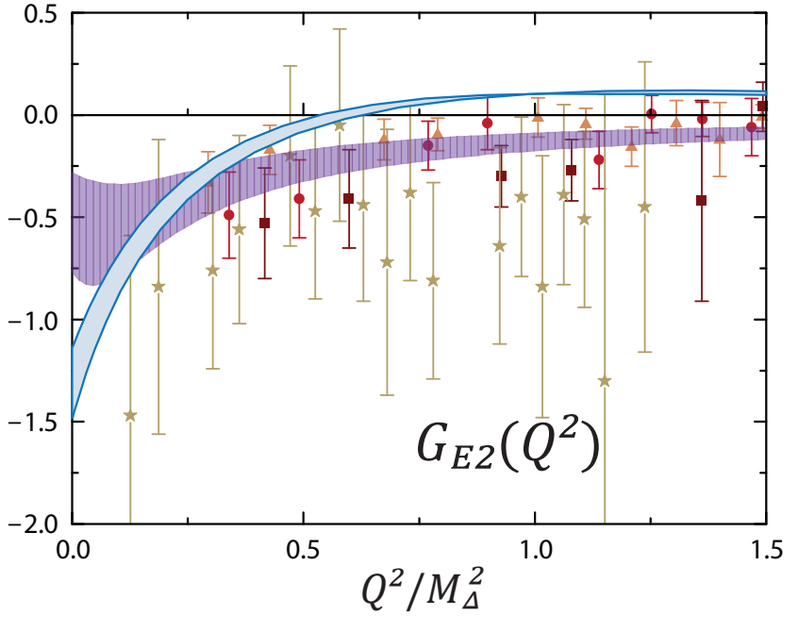
Eichmann and CF, EPJ A48 (2012) 9

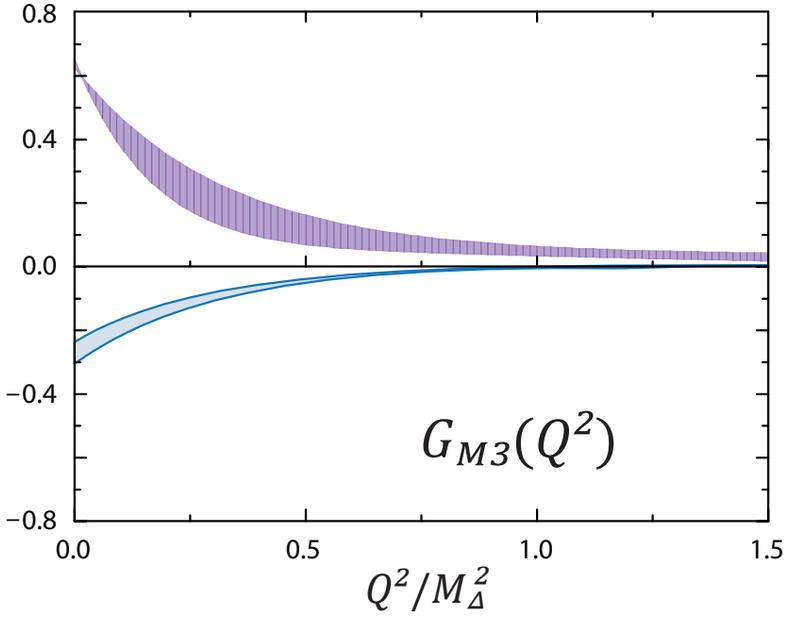
Δ-form factors

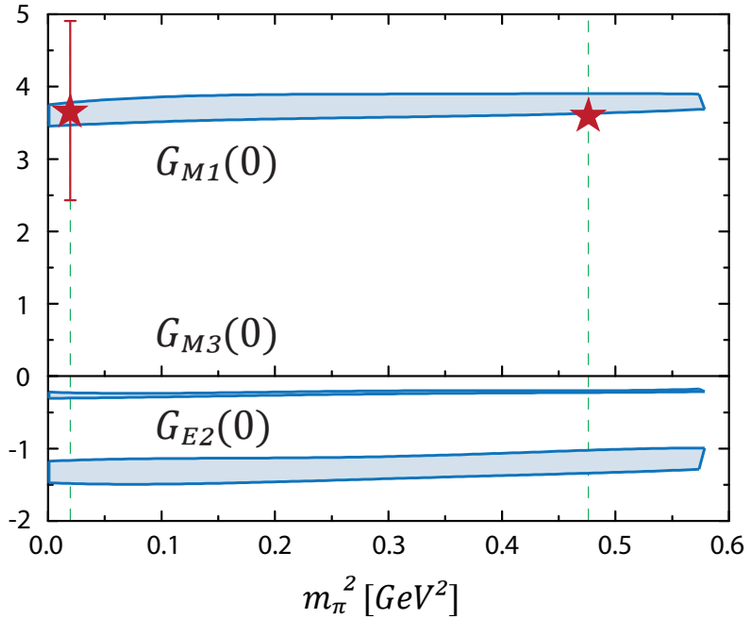








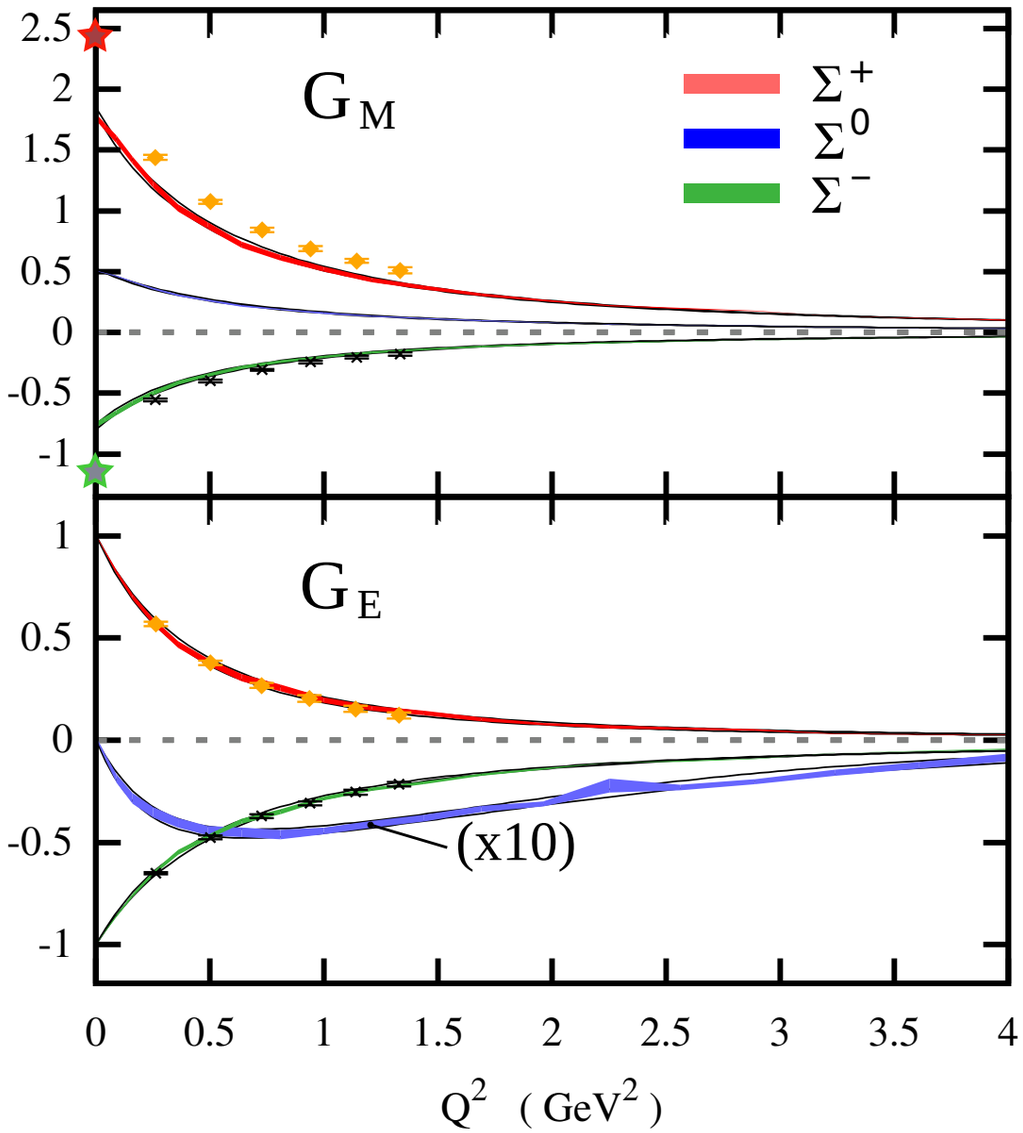


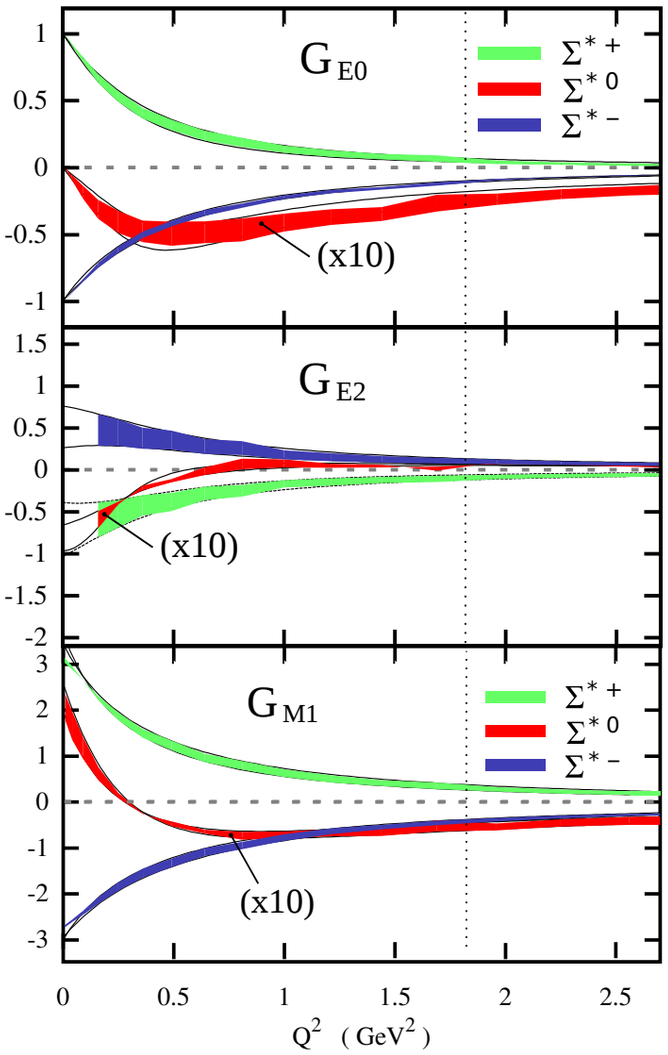


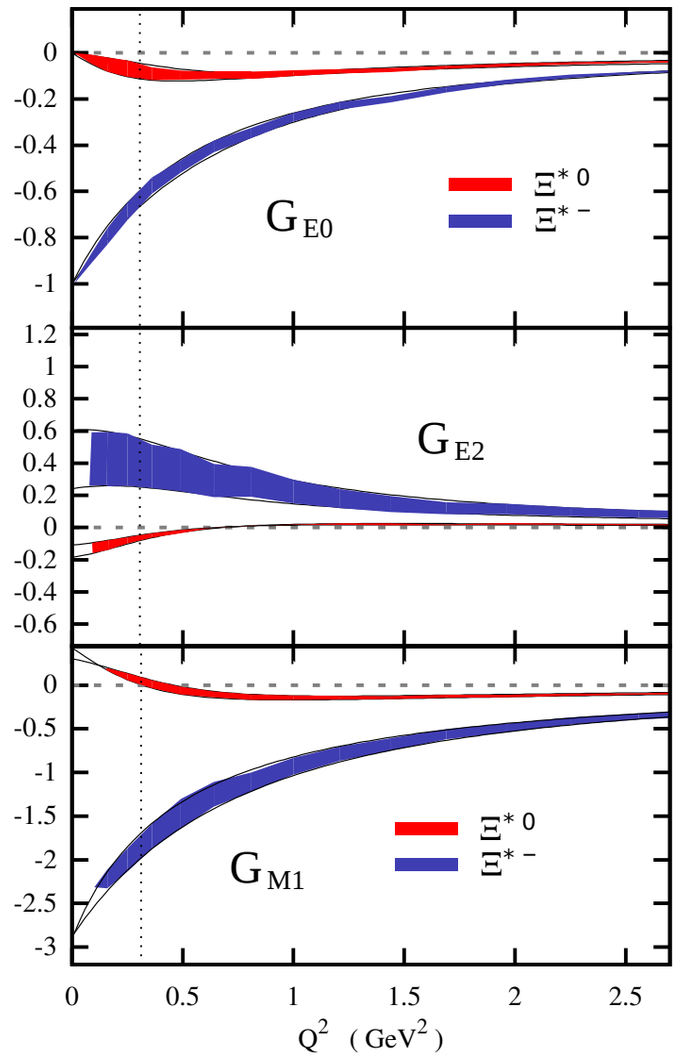
- may serve to distinguish between qqq and q-dq !

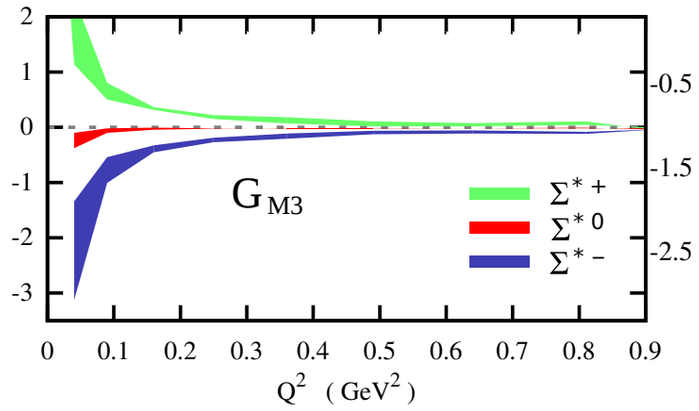
Sanchis-Alepuz, Williams, Alkofer, PRD87 (2013)
Nicmorus, Eichmann, Alkofer, PRD82 (2010)

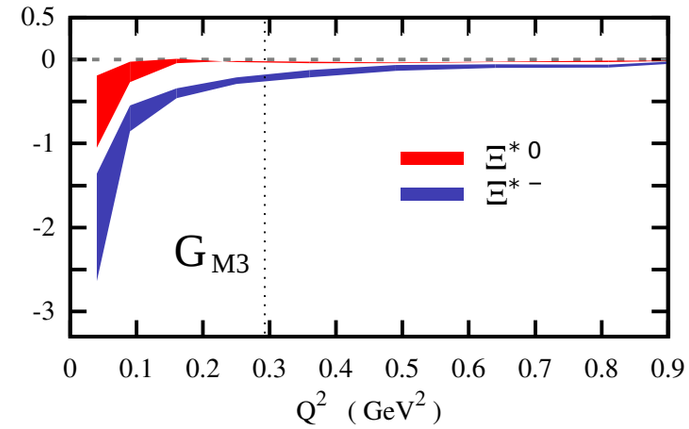
Strange form factors: octet and decuplet









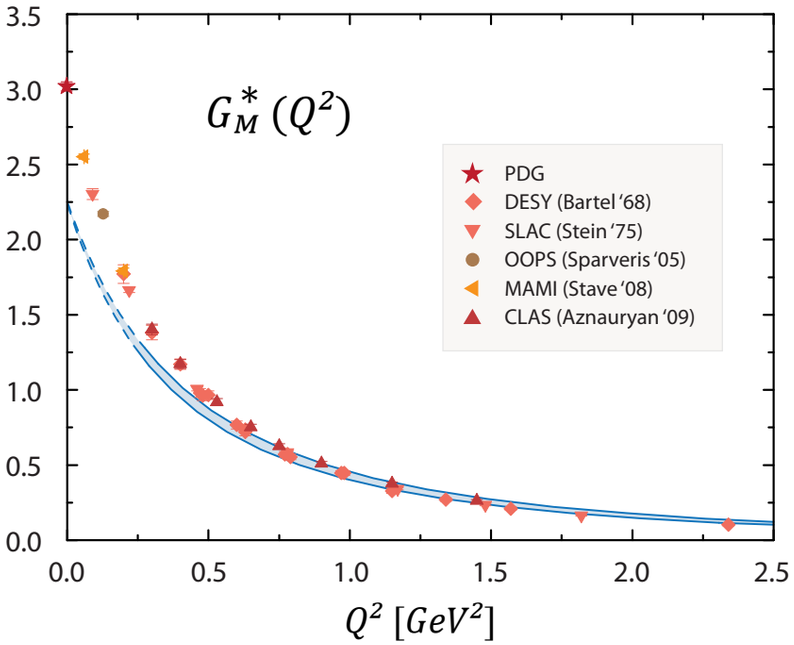


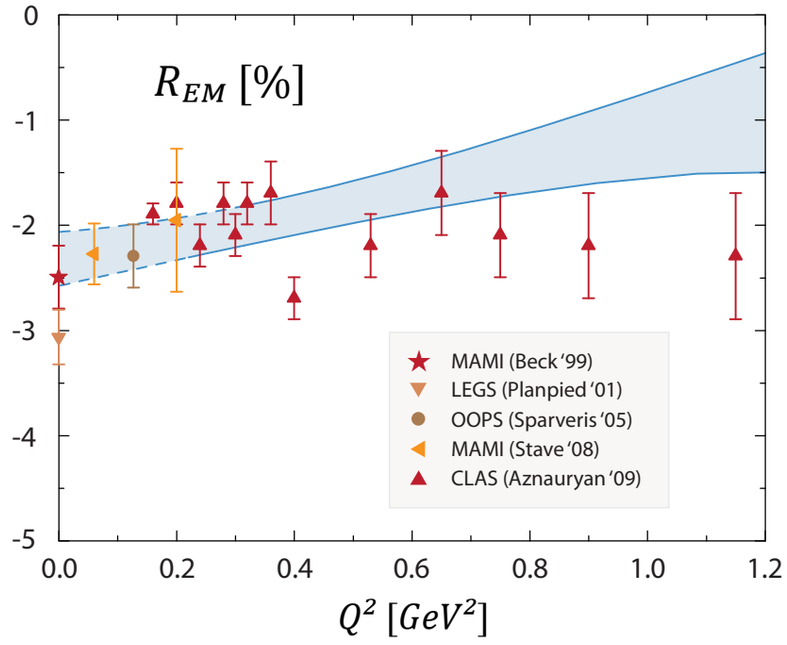
- Decuplet: prediction

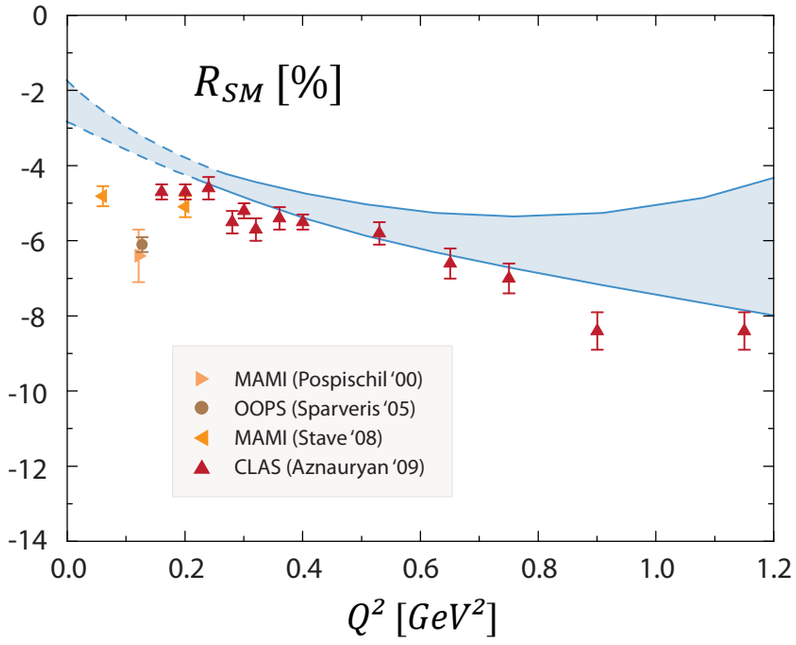
DSE: Sanchis-Alepuz, CF, EPJA 52 (2016)
Lattice: Shanahan et al, PRD 89 (2014), PRD 90 (2014)

Transition form factor: NΔγ

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \qquad R_{SM} = -\frac{|\vec{Q}|}{2M_\Delta}\frac{G_C^*}{G_M^*}$$



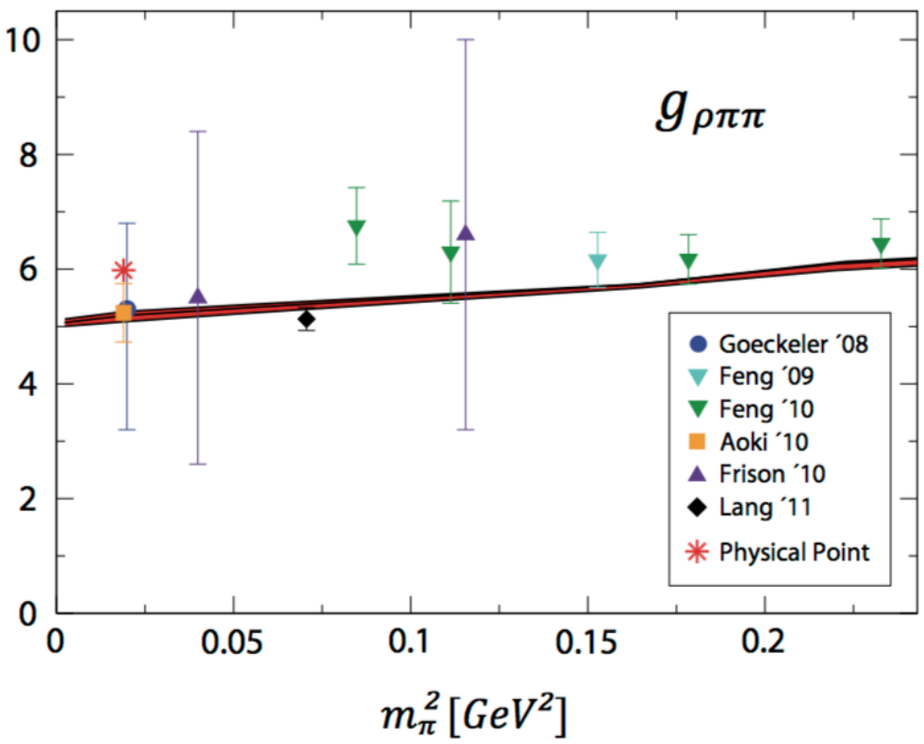


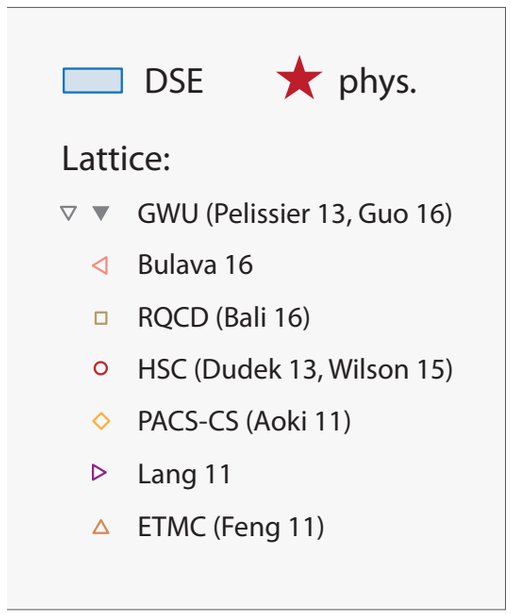


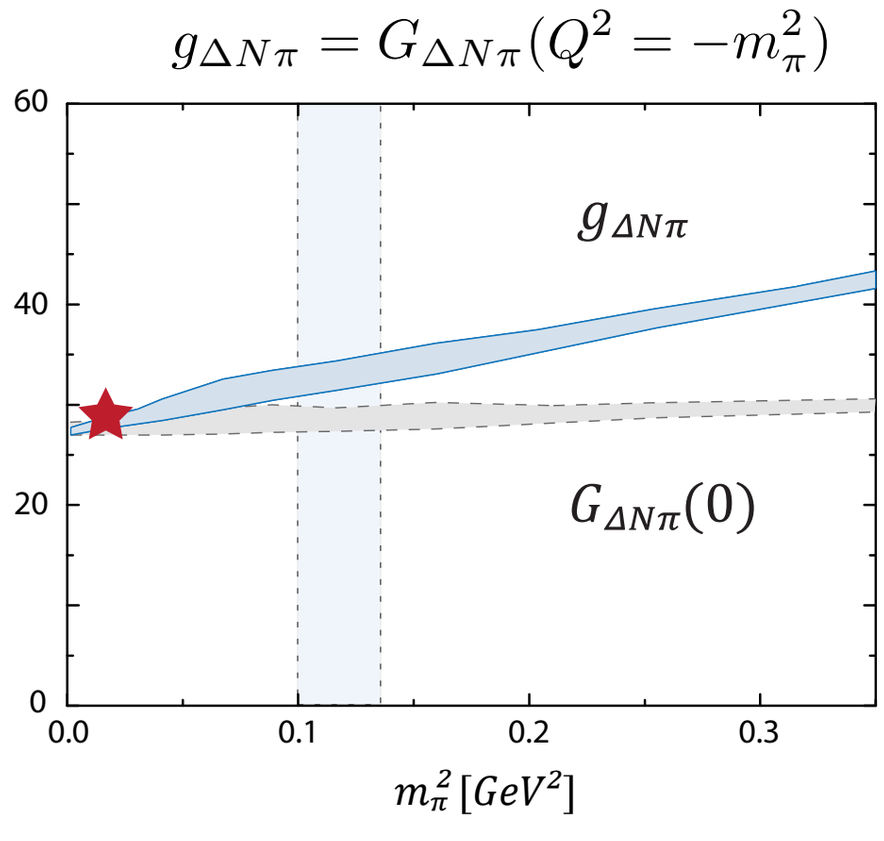
- R_{EM} highly dominated by p-waves !

Eichmann, Nicmorus, PRD 87 (2012)

Decays

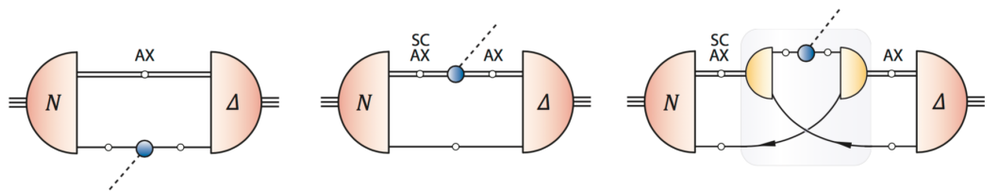




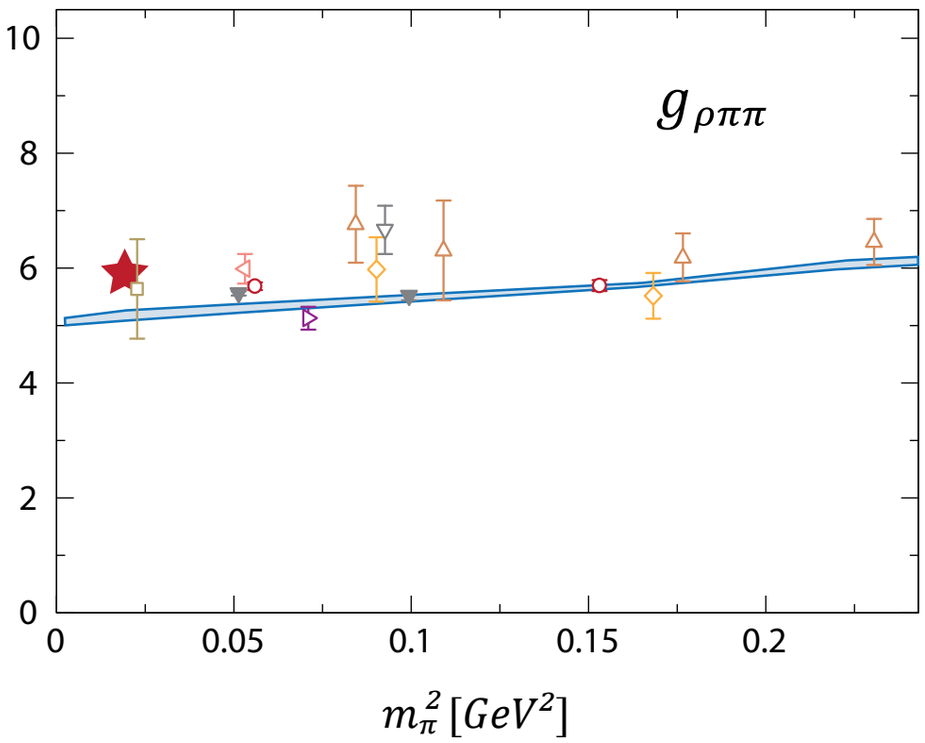


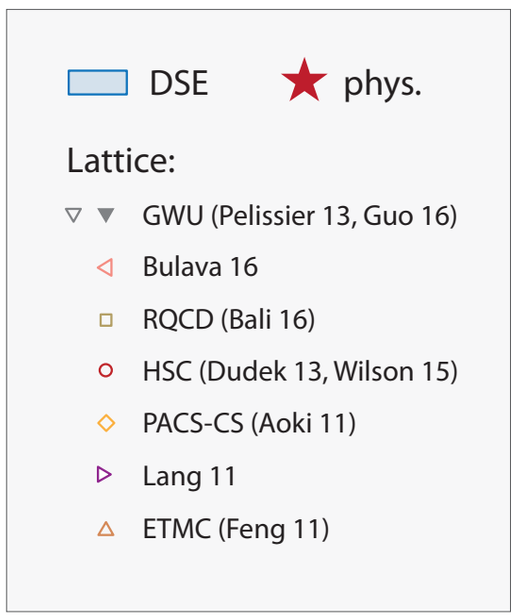
Mader, Eichmann, Blank, Krassnigg PRD84 (2011)

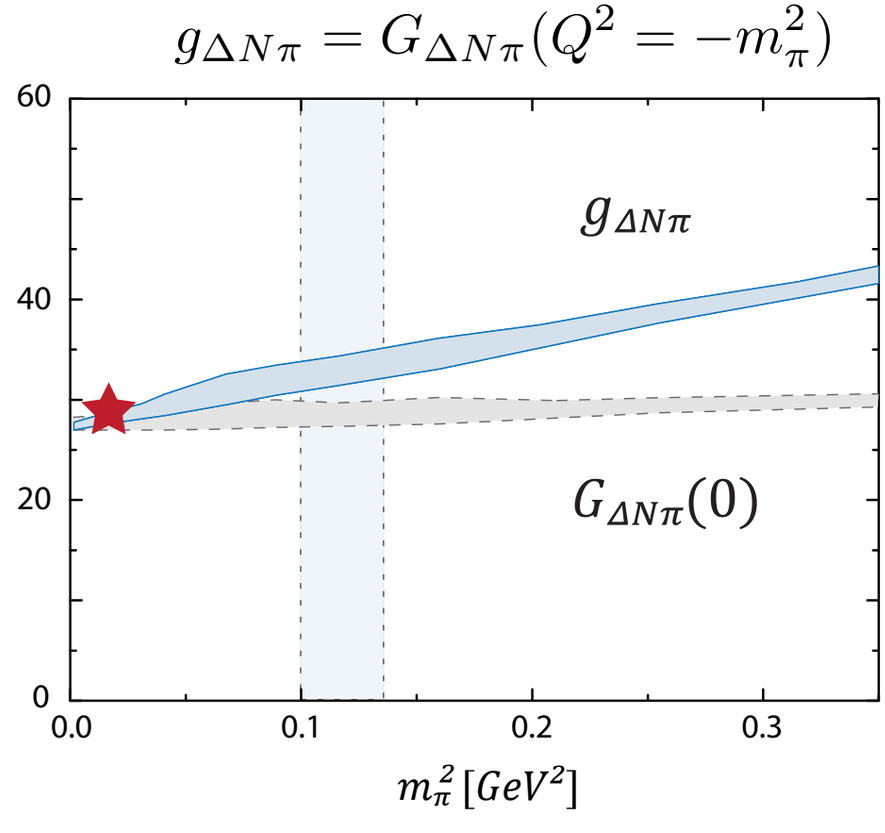
- Decay constants can be calculated in rainbow-ladder
  (although bound states have no width)
- Good agreement with lattice and experiment



Decays







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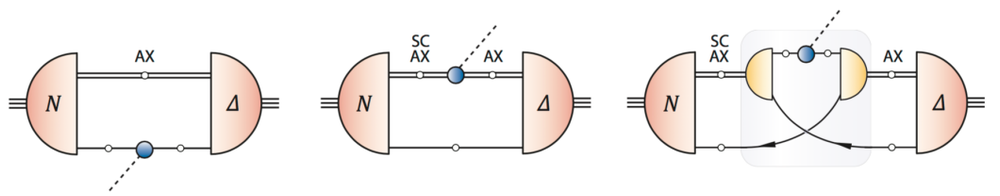
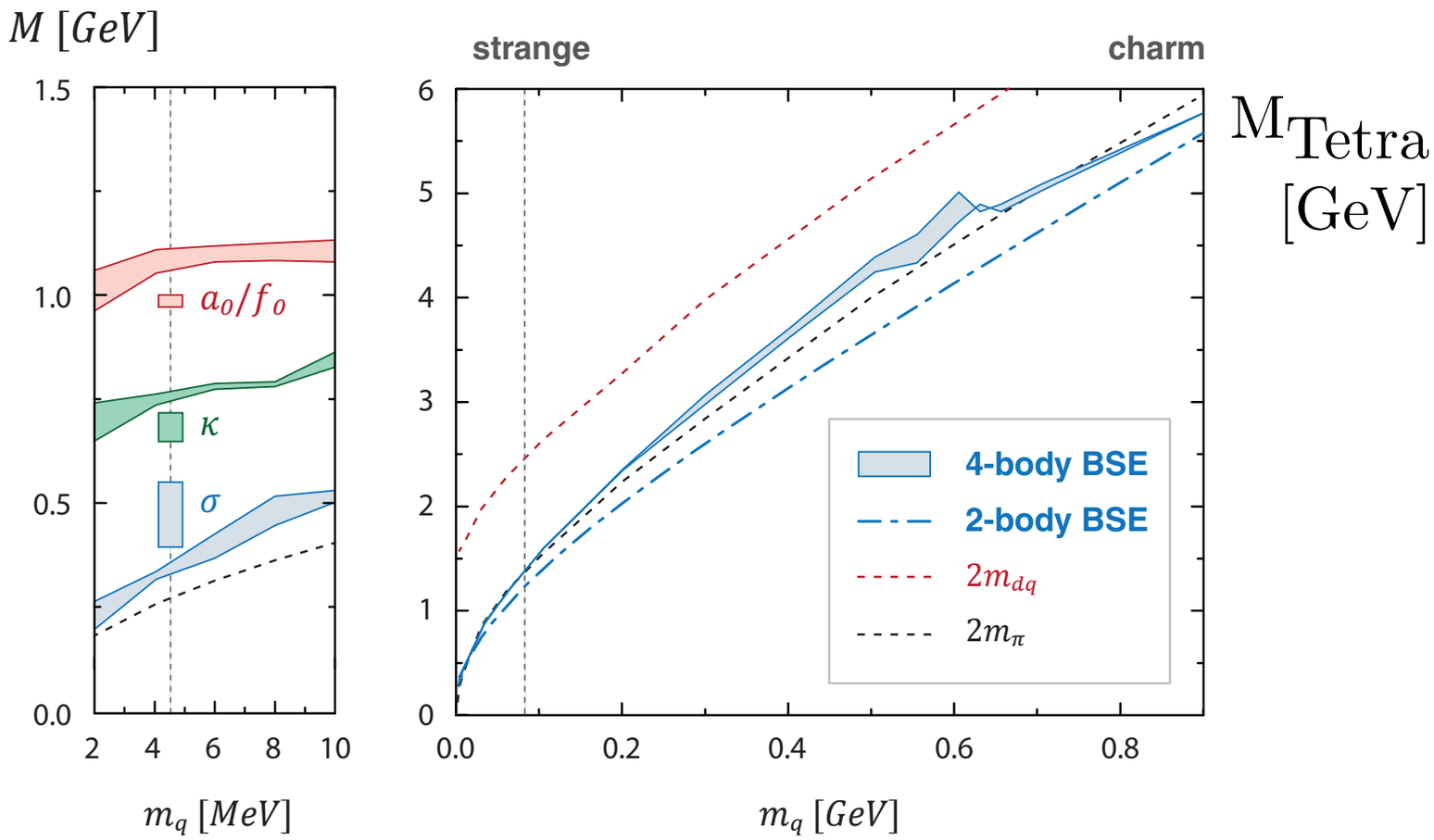


Mass evolution of tetraquark



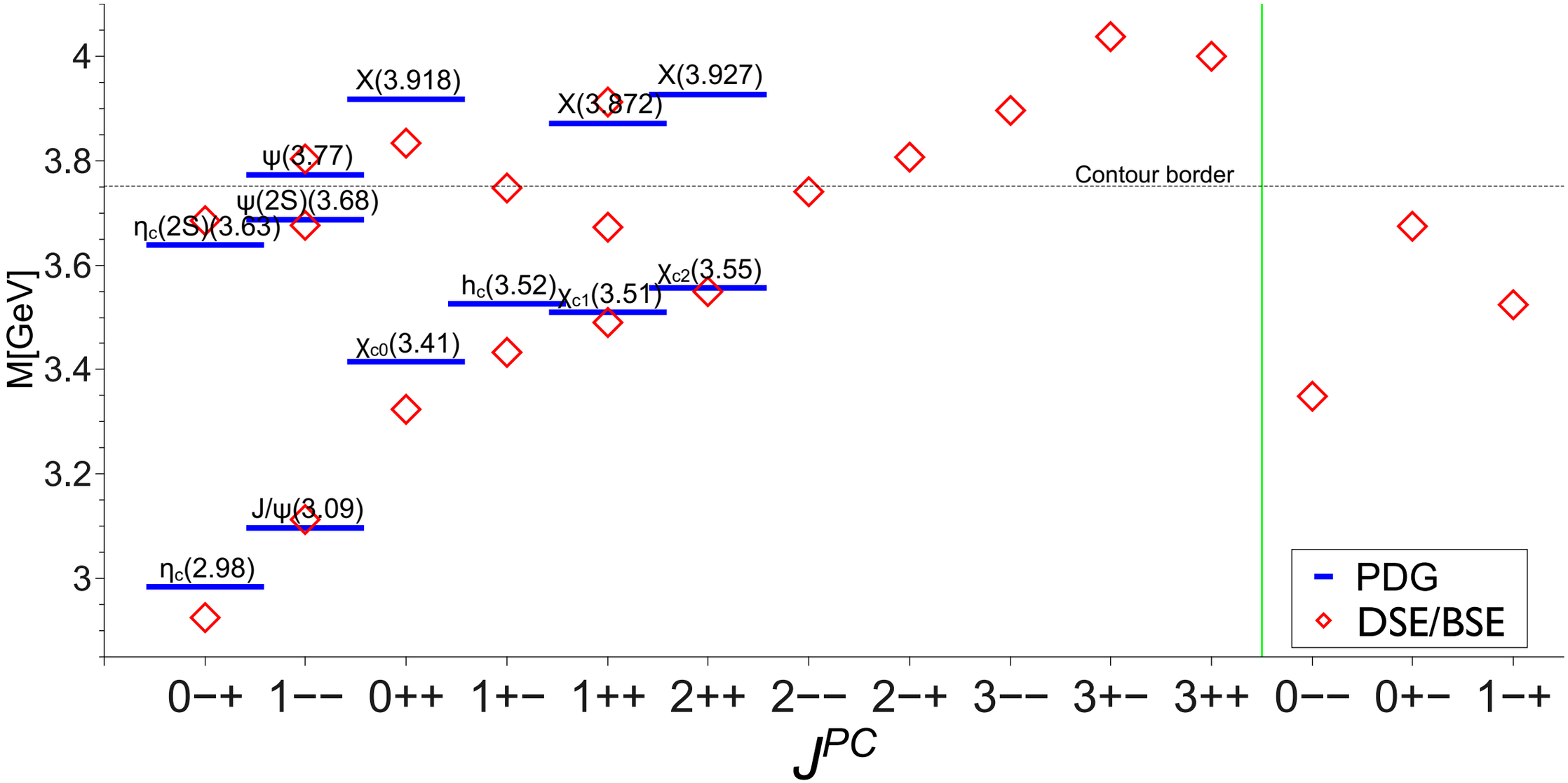
- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\mathrm{MeV}$$
$$m_\kappa \sim 750\,\mathrm{MeV}$$
$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV}$$

$$m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$
$$m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

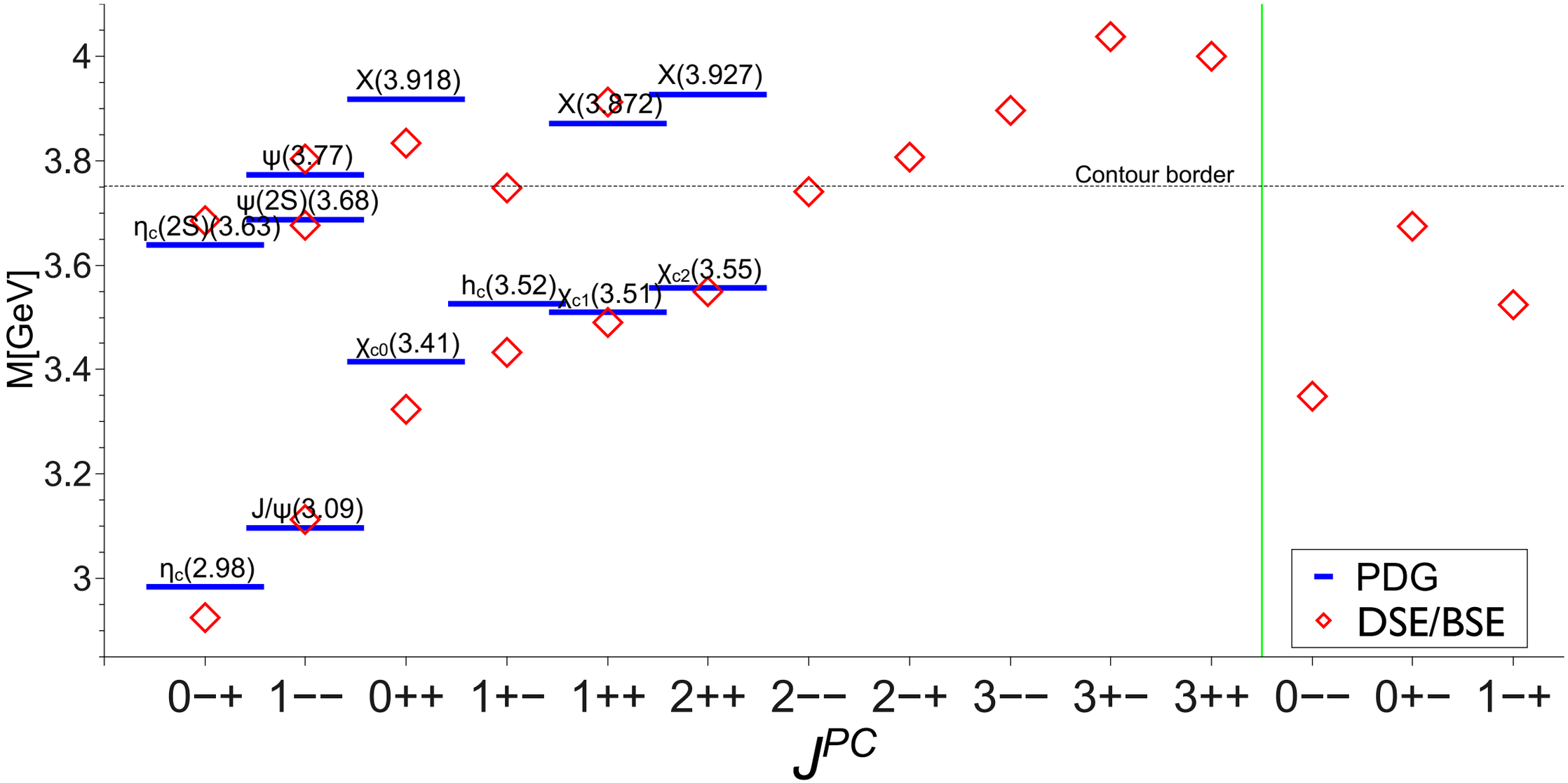
Charmonium spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
- acceptable channels: $0^{-+}, 1^{++}$
- deficiencies in other channels: missing spin-structure

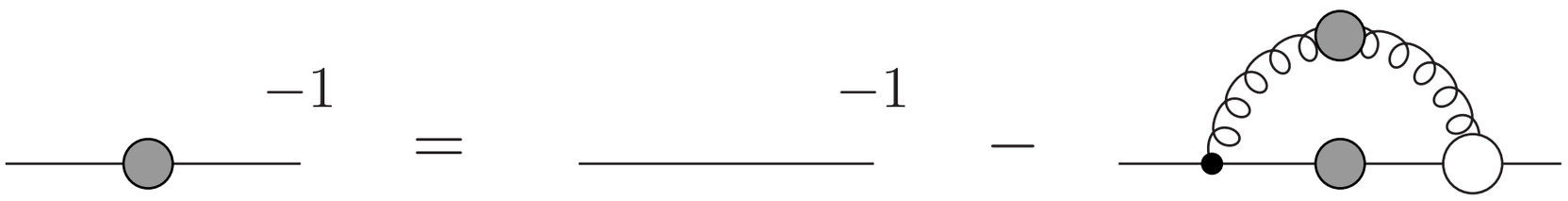
Charmonium spectrum

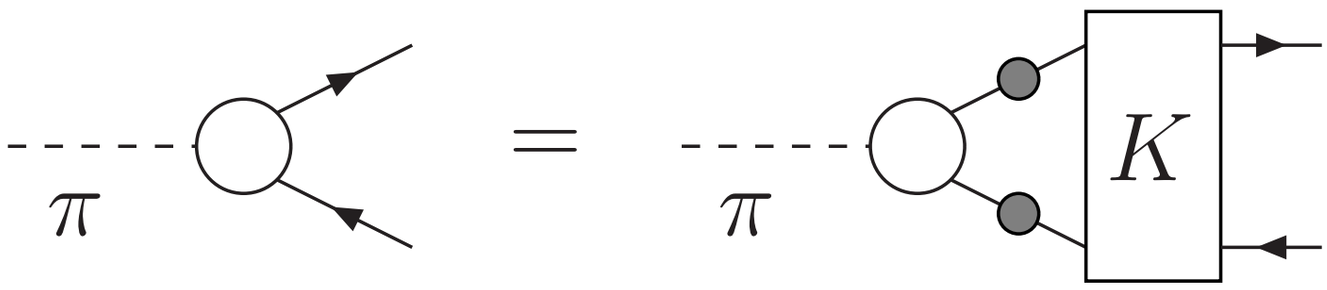


CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
- acceptable channels: $0^{-+}, 1^{++}$
- deficiencies in other channels: missing spin-structure

Theoretical Tools II: DSEs and BSEs





Kernel K uniquely related to quark-DSE via axWTI

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics
- Need approximations for dressed quark-gluon vertex