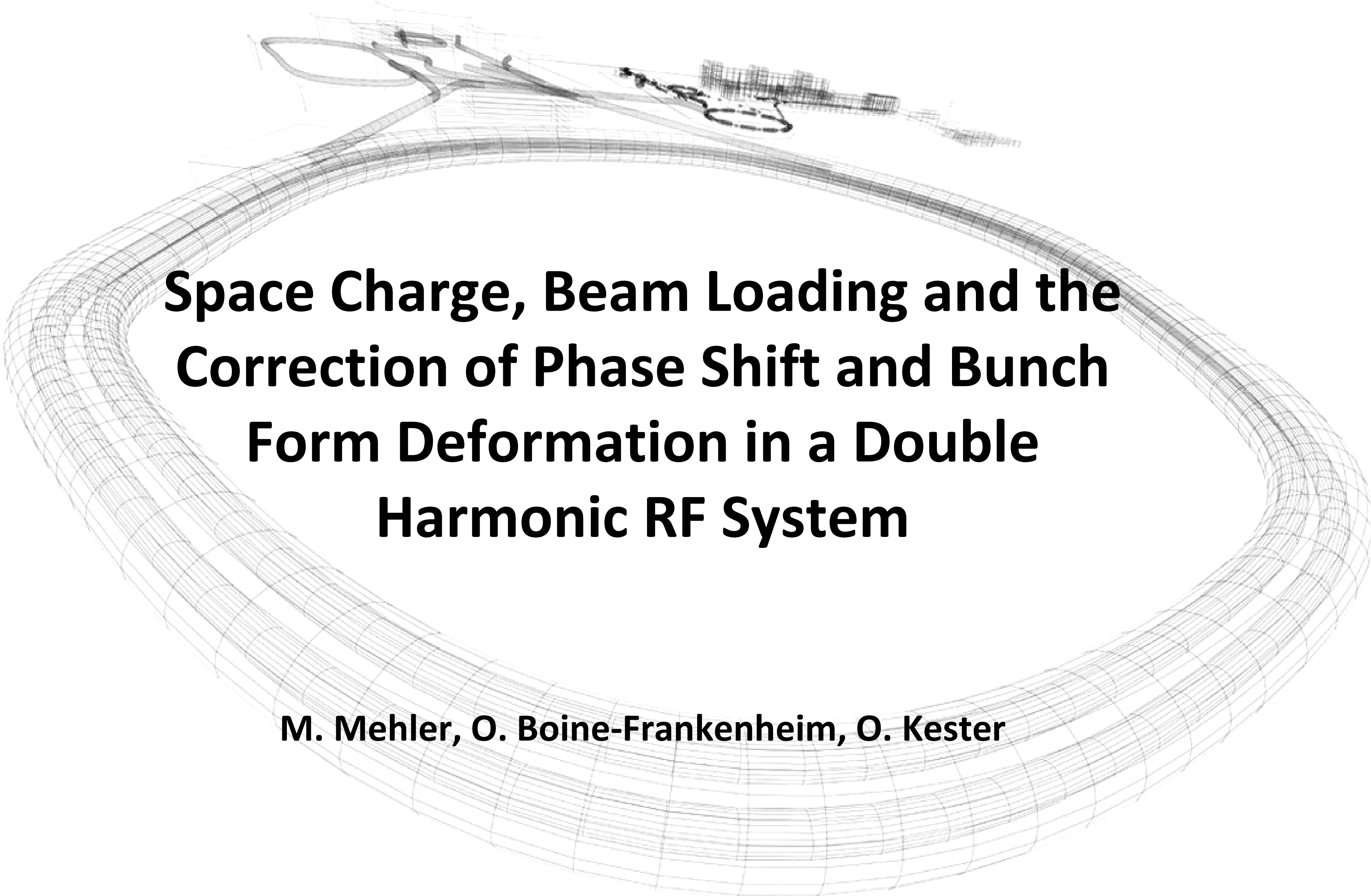


Space Charge, Beam Loading and the Correction of Phase Shift and Bunch Form Deformation in a Double Harmonic RF System

M. Mehler, O. Boine-Frankenheim, O. Kester

Contents

Parameters of the synchrotron SIS-100:

Circumference: 1086 m

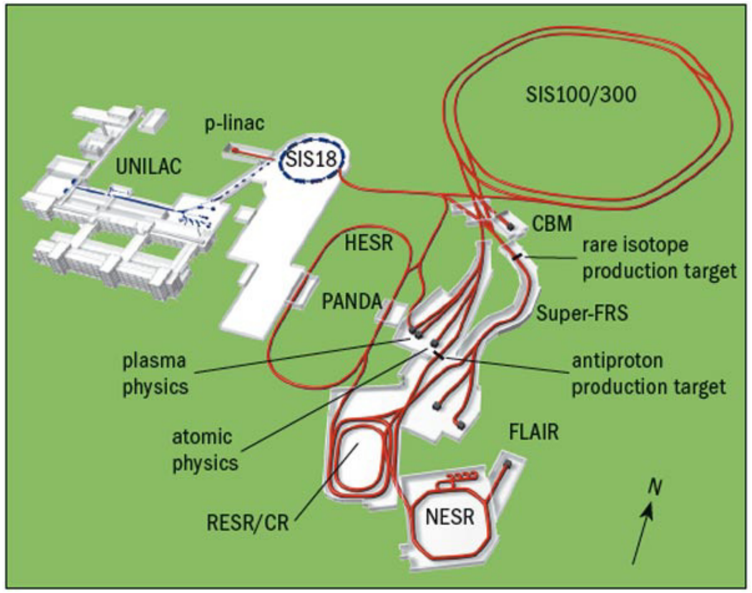
Heavy ions

Max. intensity: $5.0 \cdot 10^{11}$

Energy E_{kin}: (0.2 - 1.5) GeV/u

Operating RF Frequency f_{RF}: (1.5 - 2.6) MHz

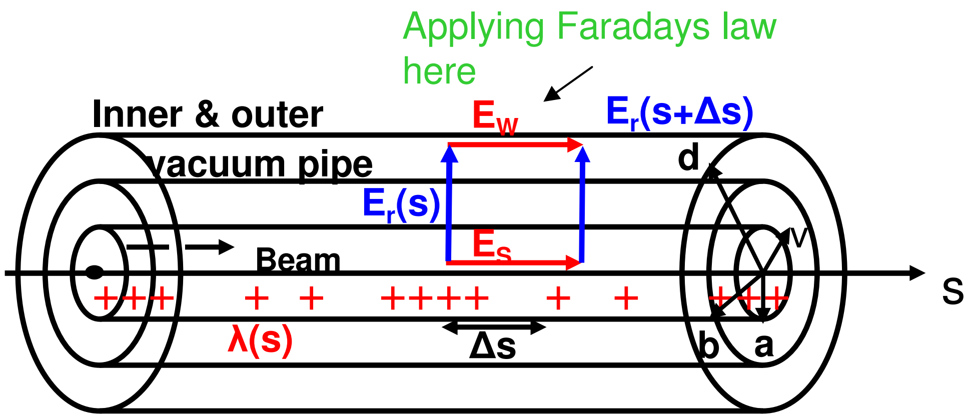
Momentum spread δ: $5.0 \cdot 10^{-4}$



Source: Web

- $\hat{U}_{RF}$ = 178 kV for given δ and h = 10 so that bucket area of 8 buckets is equal to that of the coasting beam
- Ion beam with the current $I_B = e\lambda(s)\beta c$ circulating in a beam pipe

Space charge



- For small perturbations of λ(s):
  Using Gauß' and Ampère's law for the quasi-static case

$$E_r = \frac{e\lambda(s)}{2\pi\varepsilon_0}\frac{1}{r} \qquad B_\phi = \frac{\mu_r\mu_0 e\lambda(s)\beta c}{2\pi}\frac{1}{r} \qquad r \geq a$$

$$E_r = \frac{e\lambda(s)}{2\pi\varepsilon_0}\frac{r}{a^2} \qquad B_\phi = \frac{\mu_0 e\lambda(s)\beta c}{2\pi}\frac{r}{a^2} \qquad r \leq a$$

Faradays' law: $\oint_{Line} \vec{E}d\vec{l} = -\frac{\partial}{\partial t}\int_{Surface} \vec{B}d\vec{\sigma} = -\frac{\partial}{\partial t}\Delta s\int_0^b B_\phi dr$

With $-\frac{\partial\lambda}{\partial t} = -\frac{\partial\lambda}{\partial s}\frac{ds}{dt} = -\beta c\frac{\partial\lambda}{\partial s}$ and $g_0 = 1 + 2\ln\frac{b}{a}$ as geometry factor for a circular vacuum pipe.

$$E_S - E_W = -\frac{eg_0}{4\pi\varepsilon_0\gamma^2}\left[1 + \frac{2\gamma^2}{g_0\varepsilon_r}\ln\frac{d}{b}\right]\frac{\partial\lambda(s)}{\partial s} + 2e\beta^2c^2\frac{L}{2\pi R}\frac{\partial\lambda(s)}{\partial s}$$

R: Mean radius of synchrotron

Passive space charge compensation

- The total voltage per turn is:

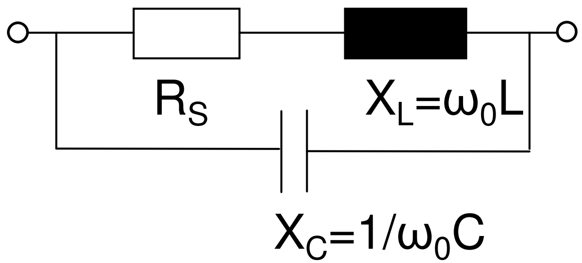
$$U_S - U_W = -e\beta c R \frac{\partial \lambda(s)}{\partial s}\left[\frac{g_0 Z_0}{2\beta\gamma^2} - \omega_0 L\right]$$

- *For compensation the impedance term in the brackets must be zero!*

- For a given length l the dimensions of the insert can be evaluated by the following formula:

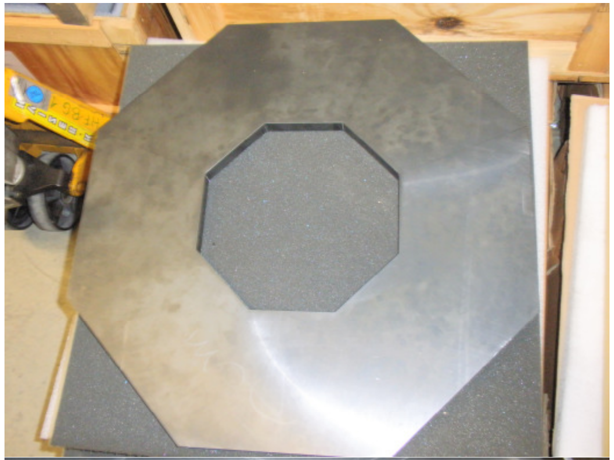
$$L = \mu_r L_0 = \left(\mu_r' - i\mu_r''\right)\frac{\mu_0 l}{2\pi}\ln\frac{d}{b}$$

Serial equivalent circuit of the inductive impedance



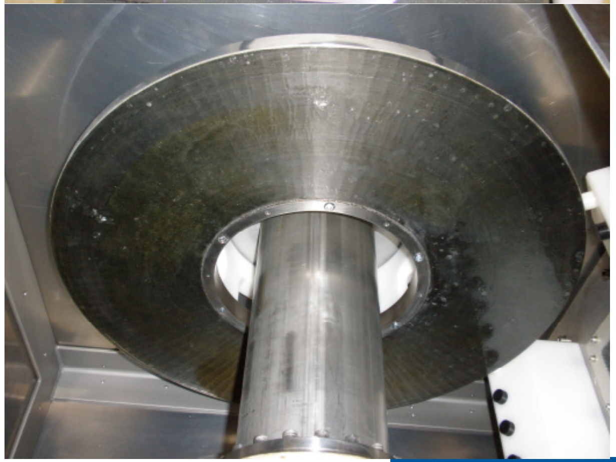
Material

Ferromagnetic and magnetic alloy material with a frequency dependent complex permeability was measured: d: 660 mm, b: 290 mm, thickness: 25 mm



MN8CX soft magnetic MnZn ferrite:

Broadband ferrite with high permeability.



FINEMET® FT3-M magnetic alloy:

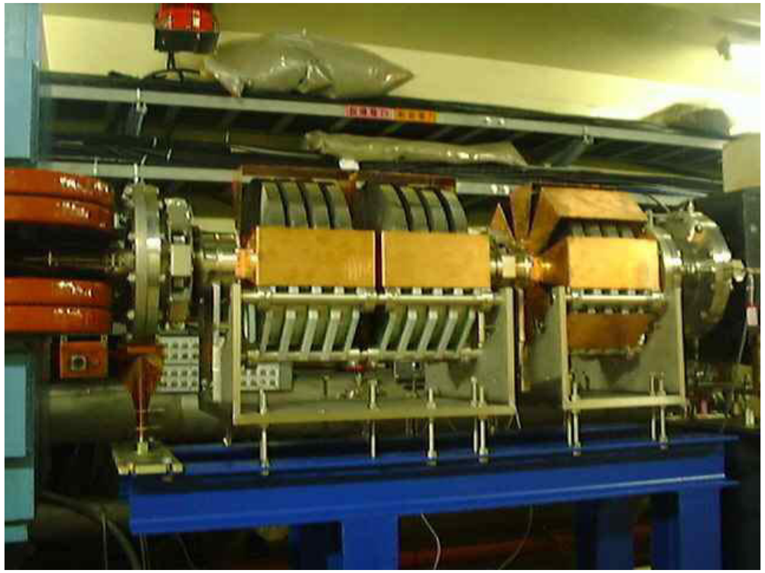
Real part permeability μ' seven times higher than for MnZn ferrite.

Therefore material of choice.

Impedances

Koba et al., Rev. Sci. Instrum., Vol. 70, No. 7 (1999)

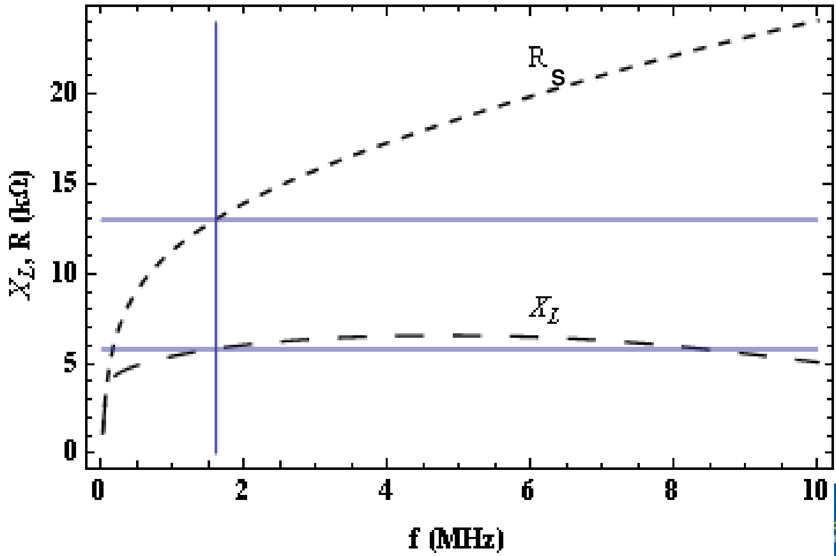
- Impedance tuner installed in the KEK PS.
- 12 Finemet ring cores.



- SIS-100: 78 ring cores necessary for 2 m insert length.
- Complex impedance: $Z_{||}(f) = R_S(f) + iX_L(f)$

$f_{RF} = f = 10 * f_0 = 1.57$ MHz
$X_L = 6.4$ kΩ
$R_S = 13.0$ kΩ

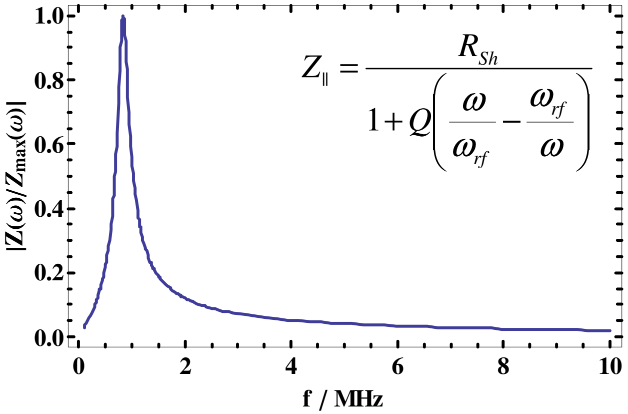
Values of X_L and R_S @ f_{RF}

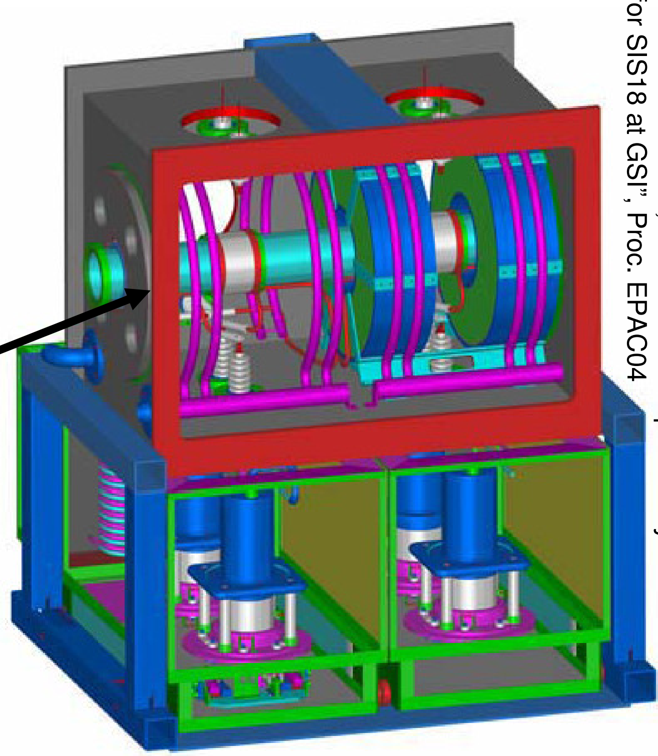


Cavity impedance

Example of an existing cavity with amplifier:

- Determined in addition to the ring cores and the gap by the RF power station
- E.g.: Bunch Compressor cavity SIS18
- Ring cores 2 kΩ shunt impedance per half cavity
- About 1 kΩ per half amplifier
- Makes about 2 kΩ for the total cavity
- Quality factor Q = 4.2 @ 839 kHz





P. Hülsmann et.al., "The bunch compressor system for SIS18 at GSI", Proc. EPAC04

Ch.Dimopoulou, "On behalf of FAIR Technical Division, GSI Accelerator Division & collaborators…", RuPAC08

Potential-well distortion

- Potential-well distortion: Alteration of bunch distribution.
- Influence of time-independent perturbation of wake potential to the ion bunch.
- Static perturbation changes the shape of the bunch.
- Self-consistent solution for a Gaussian ion density distribution is the Haissinski equation:

$$\lambda(\tau) = \lambda(0)\exp\left[-\left(\frac{\omega_{S0}\beta^2 E_0}{\eta\sigma_E}\right)^2 \frac{\tau^2}{2} + \frac{e^2\beta^2 E_0}{2\pi\eta T_0\sigma_E^2}\int_0^{\tau} d\tau'' \int_{\tau''}^{\infty} d\tau' \rho(\tau') \int_{-\infty}^{\infty} d\omega Z_0^{\parallel}(\omega) e^{-i\omega(\tau'-\tau'')}\right]$$

- Expression in brackets describes the motion of a particle in a potential-well.

K. Y. Ng, “Physics of Intensity Dependent Beam Instabilities”, (World Scientific, Singapore, 2006)

Potential-well distortion

- Purely inductive longitudinal impedance, $Z_0^{\parallel}(\omega) = X_L$

→ Distribution will stay left-right symmetric
→ Bunch lengthening or shortening

$$\lambda(\tau) e^{\alpha_L \lambda(\tau)} = k e^{-\tau^2/(2\sigma_\tau^2)}$$

$$\alpha_L = e^2 \beta^2 E_0 L / (\eta T_0 \sigma_E^2)$$

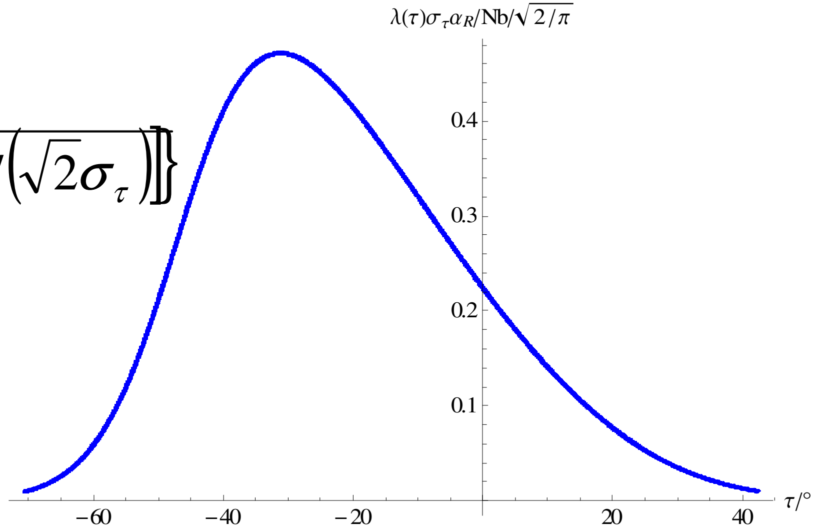
- Purely resistive longitudinal impedance, $Z_0^{\parallel}(\omega) = R_S$

→ Parasitic loss of the beam particle, which is largest at the peak
→ Peak moves forward above transition and backward below transition

$$\lambda(\tau) = \frac{\sqrt{2/\pi}\, e^{-\tau^2/(2\sigma_\tau^2)}}{\alpha_R \sigma_\tau \left\{\coth(\alpha_R N/2) - erf\left[\tau/\left(\sqrt{2}\sigma_\tau\right)\right]\right\}}$$

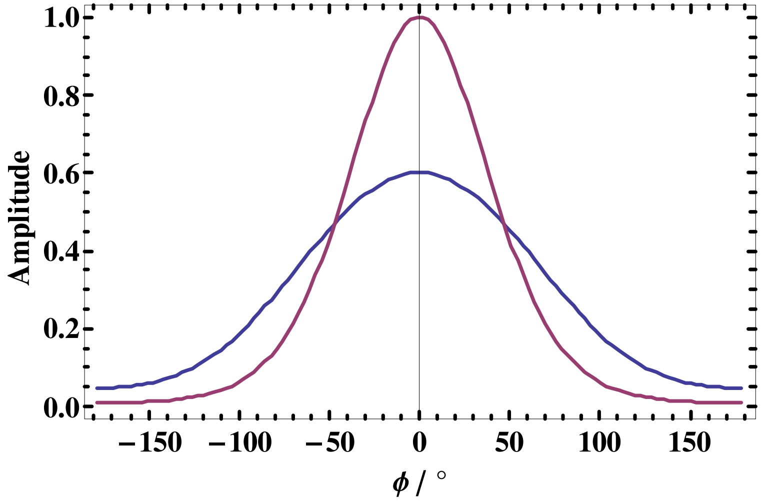
$$\sigma_\tau = \frac{|\eta| \sigma_E}{\beta^2 \omega_{S0} E_0}, \quad \alpha_R = \frac{e^2 \beta^2 E_0 R_S}{\eta T_0 \sigma_E^2}$$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$



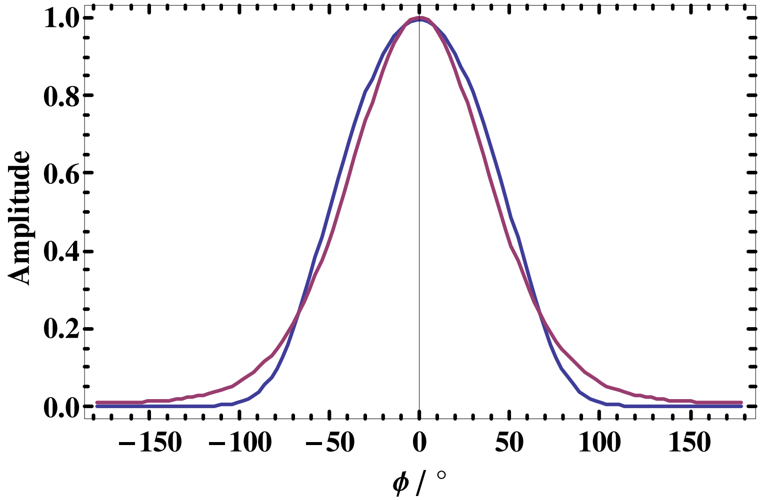
K. Y. Ng, "Physics of Intensity Dependent Beam Instabilities", (World Scientific, Singapore, 2006)

Single harmonic RF system



Without sc compensating insert:

Lower peak current and bunch-lengthening by sc in a single harmonic system. $Z_0^{\parallel}(\omega) = X_C$



With sc compensating insert (ideal imaginary one): $Z_0^{\parallel}(\omega) = X_L$

Comparing the blue curve in the top figure with the one in the bottom figure by imaginary part of insert sc impedance is fully compensated.

Line charge density distribution with low sc impedance

Line charge density distribution with high sc impedance

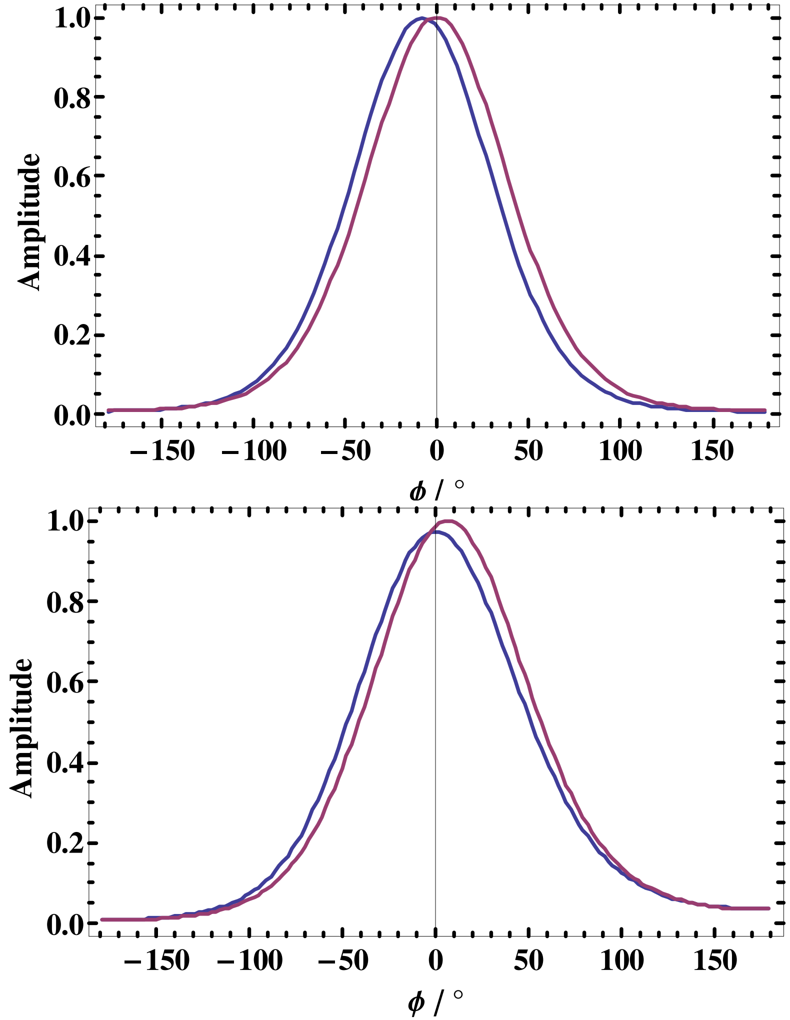
Single harmonic RF system

<u>Effect of resistive part of insert impedance in addition to the imaginary part:</u>

- Peak moves backwards compared to the sc free distribution in red
- Phase shift (~10° for 10^{10} U^{28+}, $\Sigma \approx 0.03$)

  sc parameter Σ defined as: $\Sigma = \frac{1}{\frac{V_{RF}}{V_{SC}} - 1}$

- This can be corrected by giving the synchronous phase Φ_S of the RF voltage a shift of -0.12 rad.
- Asymmetric bunch form not correctable; it increases because of accelerated bucket form.
- With a space charge parameter of about Σ = 0.08 the bunch form starts to become asymmetric.

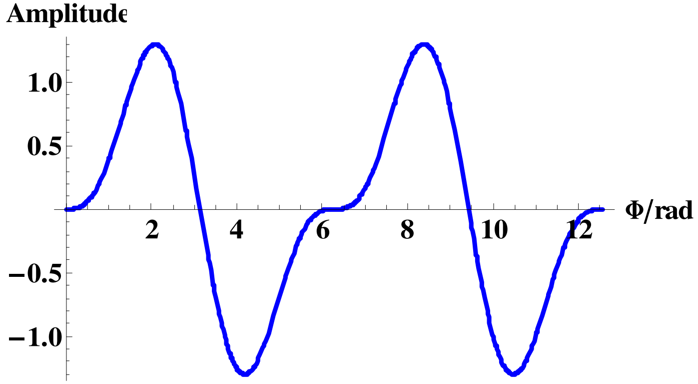


Line charge density distribution with low sc impedance

Line charge density distribution with high sc impedance

Double harmonic RF system

- Voltage of a double harmonic system $V(\Phi) = \sin\Phi - \sin\Phi_S - \alpha\left\{\sin\left[\Phi_{S2} + \frac{h_2}{h_1}(\Phi - \Phi_S)\right] - \sin\Phi_{S2}\right\}$

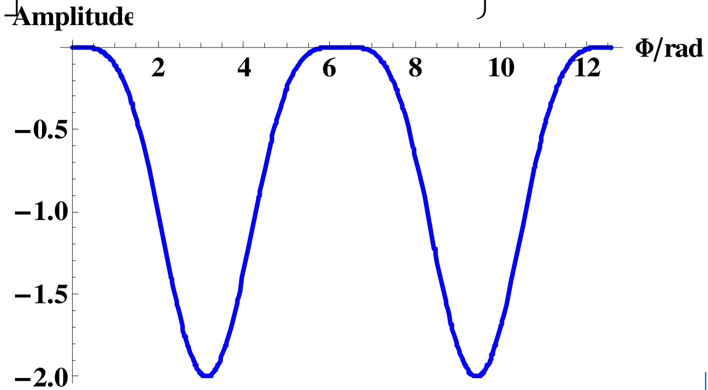


- Synchronous phases of the 2 harmonics:
  $\Phi_S = \Phi_{S2} = 0$, $\alpha = V_2/V_1 = 0.5$, $h_2/h_1 = 2$

- An area with no RF voltage appears comparable to a small barrier bucket region

- The RF potential then is

$$Y(\Phi) = \cos\Phi - \cos\Phi_S + (\Phi - \Phi_S)\sin\Phi_S - \alpha\frac{h_1}{h_2}\left\{\cos\left[\Phi_{S2} + \frac{h_2}{h_1}(\Phi - \Phi_S)\right] - \cos\Phi_{S2} + (\Phi - \Phi_S)\sin\Phi_{S2}\right\}$$



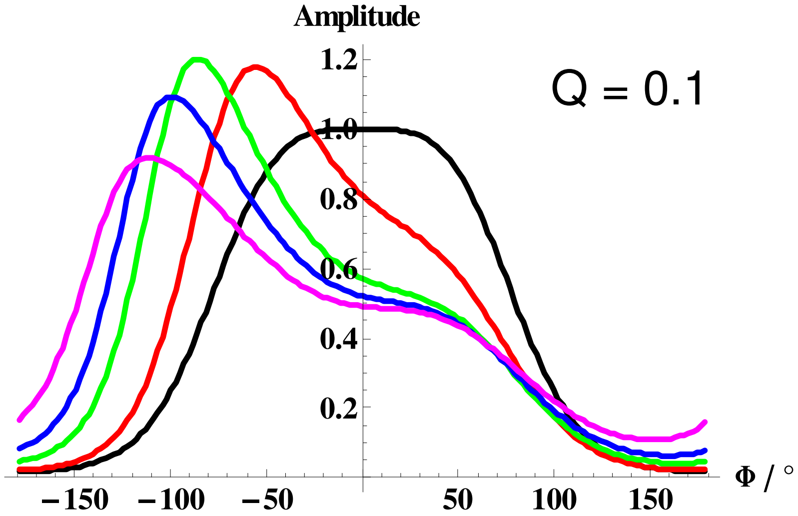
- In the potential free area $\lambda(\tau) = \dfrac{\lambda_0}{1 + \alpha_R N_b \lambda_0 \tau}$

  is the beam profile

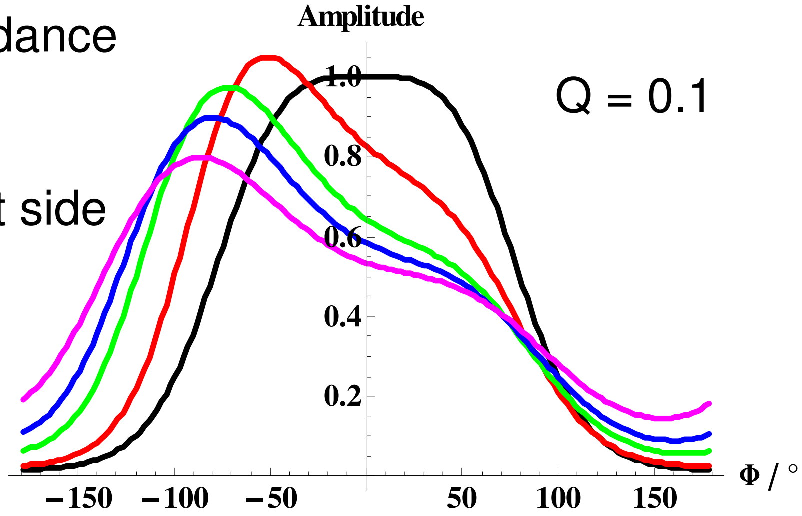
K. Y. Ng, "Physics of Intensity Dependent Beam Instabilities", (World Scientific, Singapore, 2006)

Double harmonic RF system

- Only potential-well distortion
- Interaction with the resistive part of the longitudinal impedance
- Haissinski equation only contains resistive part
- This leads to asymmetry in the beam profile



- Potential-well distortion and space charge impedance
- Shorter bunch length because of capacitive space charge impedance below transition
- Lower peak amplitude but amplitude on the right side of the beam distribution higher



Undisturbed double harmonic beam distribution
Σ = 0.1: double harmonic beam distribution
Σ = 0.3: double harmonic beam distribution
Σ = 0.5: double harmonic beam distribution
Σ = 1.0: double harmonic beam distribution

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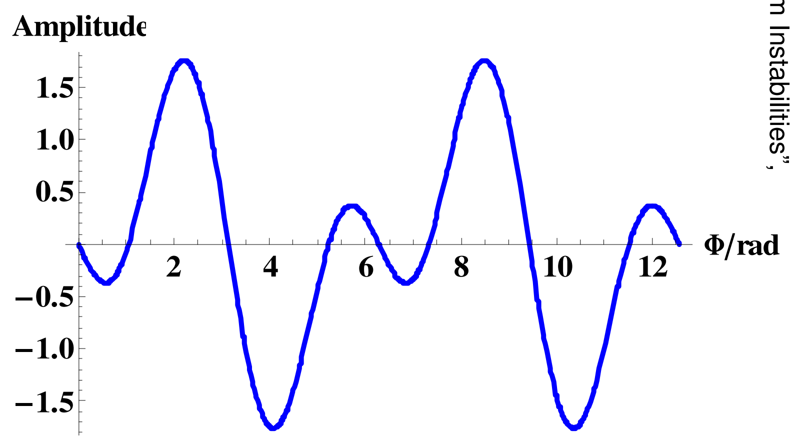
Double harmonic RF system

- Such asymmetric beam profiles were also observed in a barrier bucket system of the Fermilab Recycler Ring.
- The asymmetric beam profile could be compensated by adding a small voltage into the RF free region.

$$V_b = \frac{\eta \sigma_E^2 T_0}{2\beta^2 E_0 T_2} \alpha_R N_b = \frac{e N_b R_S}{T_2}$$

- Restores the energy dissipated by the beam to the resistive part of the circumference

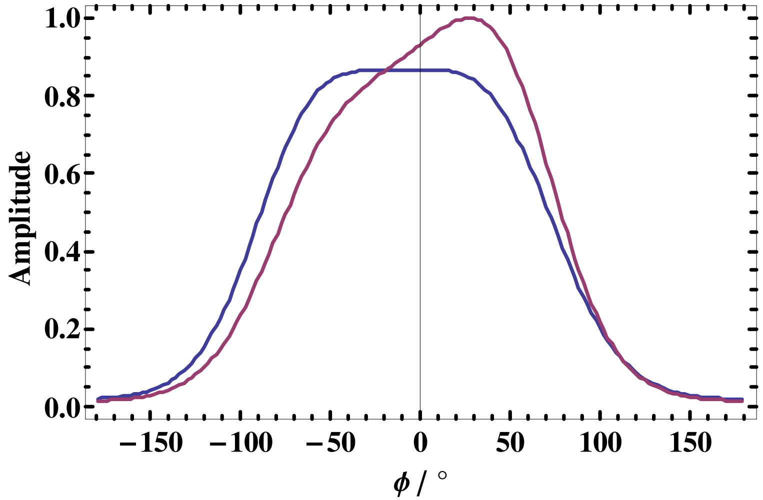
- Voltage of a double harmonic system with α = 1.0 is doing this by itself.
- Adding synchrotron frequency spread into this region



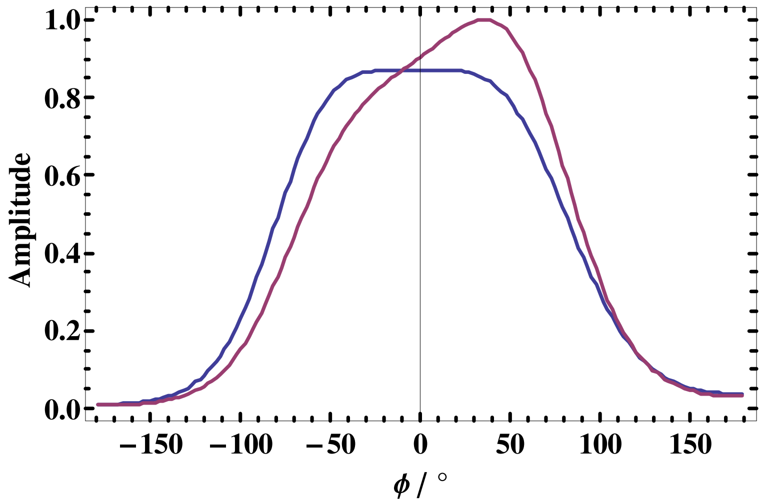
K. Y. Ng, "Physics of Intensity Dependent Beam Instabilities", (World Scientific, Singapore, 2006)

Double harmonic RF system

- α = 0.5, only potential-well distortion
- Correction of asymmetric beam profile through phase between the two harmonics $\Delta\Phi_S$
- Phase shift can be seen after this correction
- By correction beam distribution without sc is shifted to the right (red line)



- α = 0.5, only potential-well distortion
- Correction of phase shift through tuning of synchronous phase Φ_S of the main harmonic
- Because of the correction the distribution with low sc in red now is asymmetric in the opposite direction
- The distribution with sc is flat



Line charge density distribution with low sc impedance

Line charge density distribution with high sc impedance

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Double harmonic RF system

- In the following a numerical program was used to get the static beam profile distribution by solving the Haissinski equation for different sc parameters Σ starting from 0.05 up to 1.35 in steps of 0.05

Steps in short:

- The start values of $Y_{BL}(\Phi)$, $Y_{RF}(\Phi)$ for given impedances and voltages are determined by using FFT
- These values are included into the Haissinski equation giving the line charge density $\lambda(\Phi)$
- Inverse FFT of the FFT of $\lambda(\Phi)$ multiplied with impedance $Z_0^{||}(\Phi)$ gives the new $Y_{BL}(\Phi)$
- This value goes back into the Haissinski equation and so on till convergence is reached

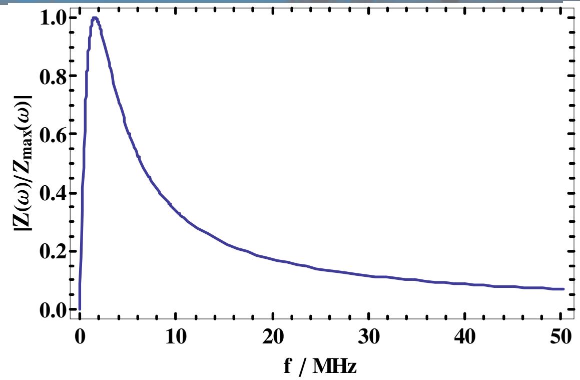
Double harmonic RF system

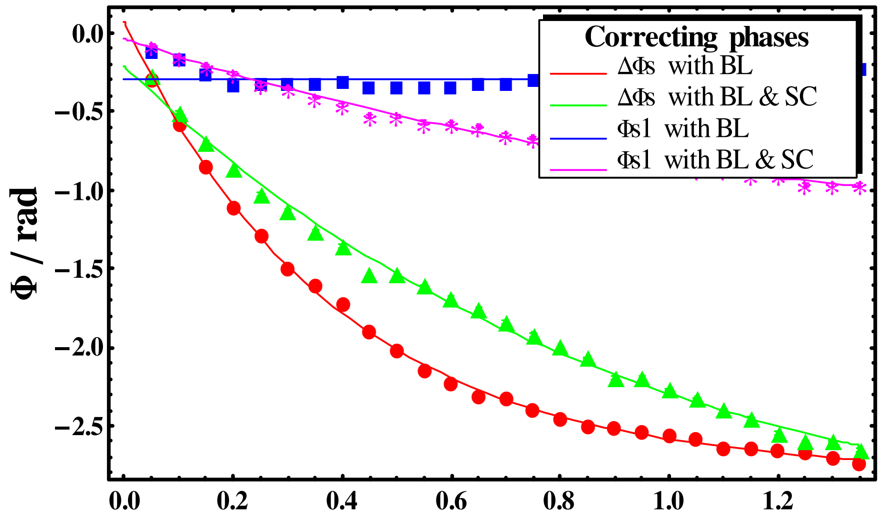
Characteristics of the following 4 figures:

- The correction phases $\Delta\Phi_S = \Phi_S - \Phi_{S2}$ and Φ_S for different impedances over the space charge parameter Σ were collected
- $Z_{\parallel} = \dfrac{R_{Sh}}{1+Q\left(\dfrac{\omega}{\omega_{rf}} - \dfrac{\omega_{rf}}{\omega}\right)}$ has been used for R_{Sh} = 16kΩ, Q = 0.45 and Q = 0.1 @ f_{RF} = 1.57 MHz.
- The impedance table evaluated by the measured Finemet ring core values was used (frequency response on transparency 6.
- An impedance table with a constant resistive value over the frequency points was used (R_{Sh} = 16 kΩ).
- Values with and without taking space charge into account.
- For the fit curves the function $\left.\begin{matrix}\Delta\Phi_S \\ \Phi_S\end{matrix}\right\} = -a + be^{-k\Sigma}$ has been used.

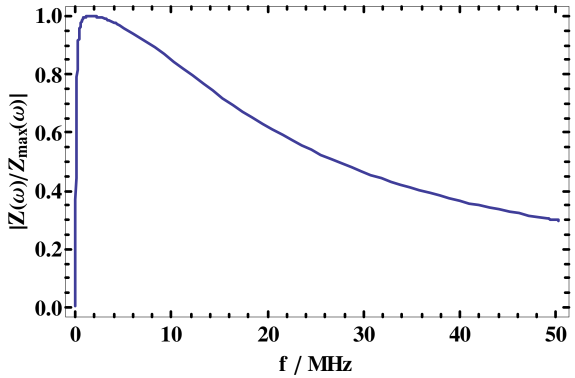
Double harmonic RF system

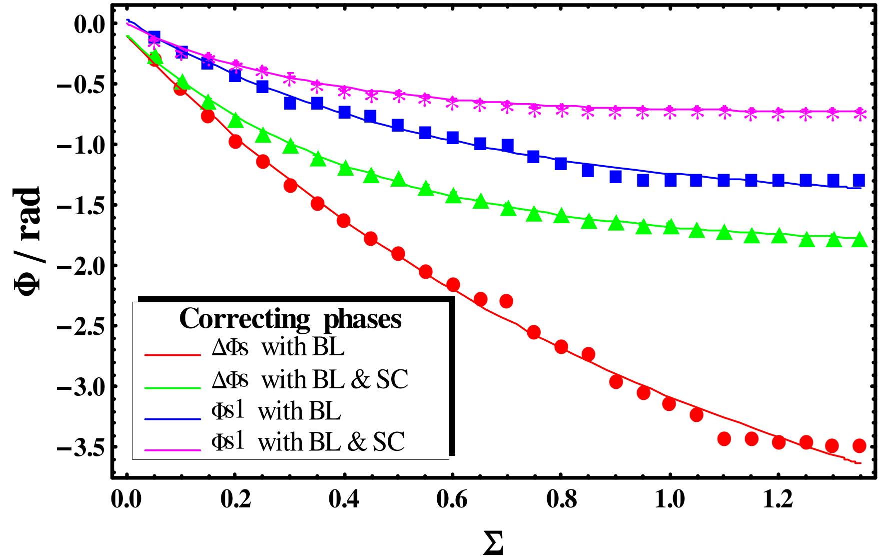
Q = 0.45





Q = 0.10



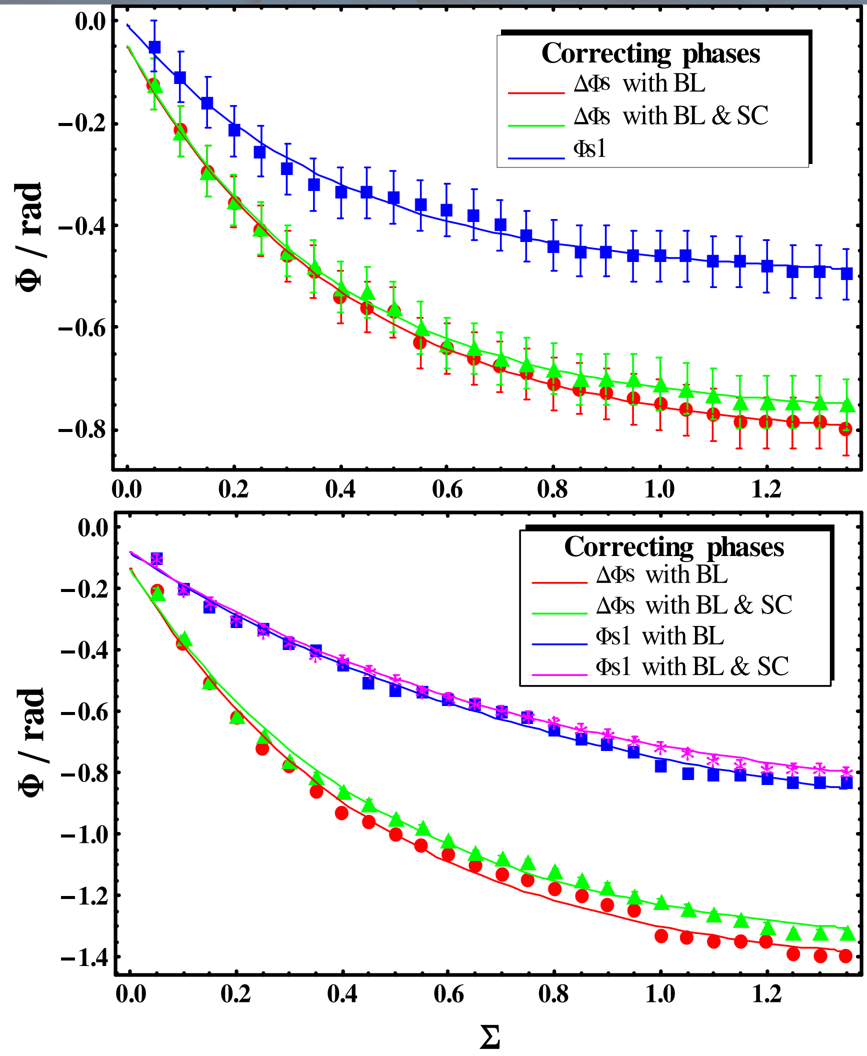


Double harmonic RF system

Ferrite insert impedance table (Finemet FT3-M)

- The correction phases are smaller with sc
- The correction phases are smaller with less higher harmonics

Table with constant impedance with value @ f_{RF} = 1.57 MHz.



Outlook and conclusion

- The difference of the behavior between a double harmonic system with α = 0.5 and α = 1.0 has to be better understood.
- The dependency of the correction phases over the quality factor has to be better understood
- Space charge can be compensated by an insert in SIS-18 and SIS-100.
- The impedances of broadband cavities and inserts are comparable
- For cavities beam loading is relevant
- For inserts without amplifier potential-well distortion is relevant
- In single harmonic RF systems only phase shifts are correctable
- In double harmonic RF systems asymmetries in bunch form and phase shifts are correctable
- Space charge is helpful for these corrections

Conclusion

Thank you for your attention!