Coupling Impedance of Ferrite Devices

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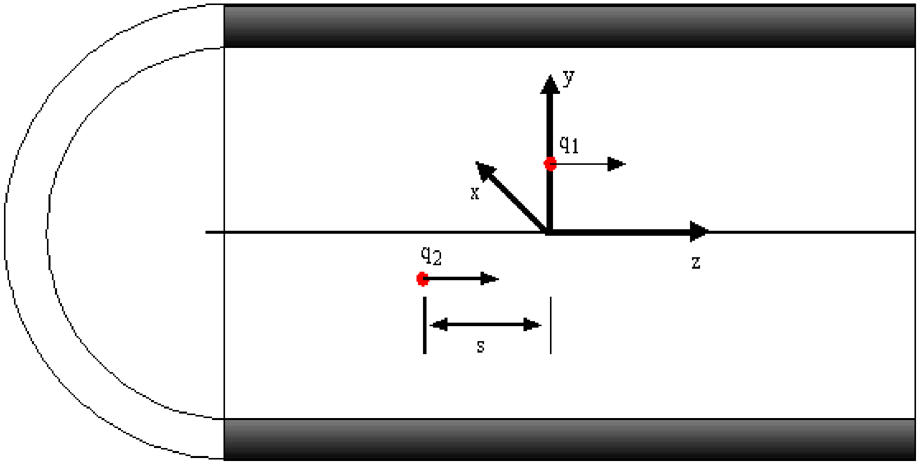
Description of Simulation Approach

Content

- Coupling Impedance Definition and determination
- Finite Integration Technique (FIT)
  →Consistent Discretization
  →Topological Operators
  →Numerical Operators
- Solvers
- Implementation and practical issues
- Boundary Conditions
- Complex frequency dependent material operators
- Present state of work and future plans

Wake Functions

- EM-Fields excited by the leading charge
- Fields depend on surrounding structure
- Measured by the trailing charge



$$\vec{W}(\vec{r}_2, s) := \frac{1}{q_1 q_2} \int_{-\infty}^{\infty} \vec{F}(\vec{r}_2, z_2, \tfrac{z_2+s}{v}) \mathrm{d}z_2$$

Transverse coordinates

$$= \frac{1}{q_1} \int_{-\infty}^{\infty} \left(\vec{E}(\vec{r}_2, z_2, \tfrac{z_2+s}{v}) + \vec{v} \times \vec{B}(\vec{r}_2, z_2, \tfrac{z_2+s}{v}) \right) \mathrm{d}z_2$$

$$\boxed{\Delta\vec{p}(\vec{r}_2, s) = \int_{-\infty}^{\infty} \vec{F} \mathrm{d}t = \frac{q_1 q_2}{v} \vec{W}(\vec{r}_2, s)}$$

Coupling Impedances

$$\underline{Z}_{\|}(\vec{r},\omega) = \frac{1}{v}\int\limits_{-\infty}^{\infty} W_{\|}(\vec{r},s)e^{-i\omega s/v}ds$$

$$\vec{\underline{Z}}_{\perp}(\vec{r},\omega) = \frac{-j}{v}\int\limits_{\infty}^{\infty} \vec{W}_{\perp}(\vec{r},s)e^{-i\omega s/v}ds$$

-j in transverse imp. is convention

- Practice: Wake potentials
- Whole charge distribution as excitation

$$\underline{Z}(\omega) = \frac{\mathcal{F}\{W(s)\}}{\mathcal{F}\{\lambda(s)\}}$$

Time domain calculation requires longer bunch length and wake integration length for lower frequencies! Therefore…

$$\underline{Z}_{\|}(\omega) = -\frac{1}{q^2}\int\limits_{beam} \underline{\vec{E}}\cdot\underline{\vec{J}}_{\|}^{*}\mathrm{d}V$$

$$\underline{Z}_{\perp,x}(\omega) = -\frac{v}{(qd_x)^2\omega}\int\limits_{beam} \underline{\vec{E}}\cdot\underline{\vec{J}}_{\perp}^{*}\mathrm{d}V$$

Details and equivalence of both definitiuons: See e.g. R. Gluckstern, *CERN Accelerator School*, 2000.

Source Terms

Uniform cylindrical disc:

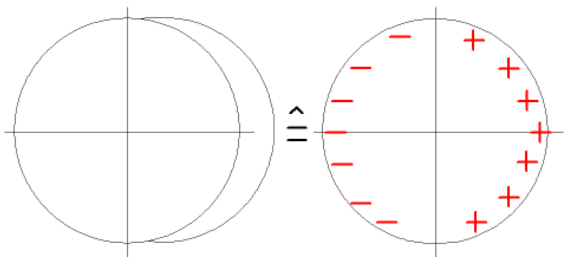
Radius of the beam

Displacement of the beam

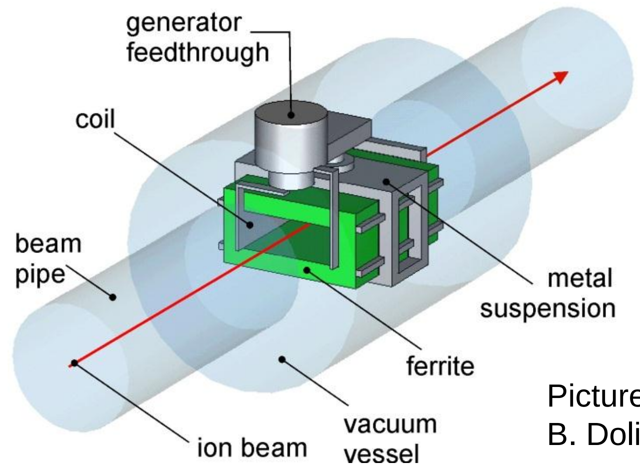
$$\boxed{\sigma(\varrho,\varphi) \approx \frac{q}{\pi a^2}\left(\Theta(a-\varrho) + \delta(a-\varrho)\, d_x \cos\varphi\right)}$$

$$\underline{J}_{s,z}(\varrho,\varphi,z,\omega) = \sigma e^{-i\omega z/v}$$

$$\underline{\varrho}_s(\varrho,\varphi,z,\omega) = \frac{1}{v}\sigma e^{-i\omega z/v}$$



- Rigid beam
- Monopole/Dipole approximated by one/two wires
- Valid for Resistive Wall impedance but not for Space charge impedance



Picture by
B. Doliwa

Coupling Impedance Determination

- From Maxwell's equations we have
- Charge implicitly included by continuity eq.

$$\boxed{\nabla \times \frac{1}{\underline{\mu}} \nabla \times \underline{\vec{E}} - \omega^2 \underline{\varepsilon \vec{E}} = -i\omega \underline{\vec{J}}_{ext}}$$

Coupling impedance determined by integrating E on the beam path

$$\underline{\mu}(\omega) = \mu' - i\mu'' \qquad \underline{\varepsilon}(\omega) = \varepsilon' - i\varepsilon'' + \frac{\kappa}{i\omega}$$

Magnetization / Polarization Losses conductivity

→ Linear and Lossy
→ Hysteresis loop approximated by ellipse in H-B space
→ Excitation of Higher Order Harmonics neglected

Maxwell's equations

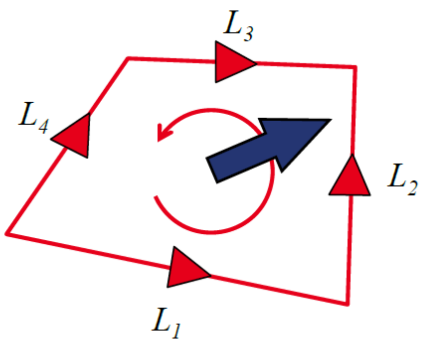
$\mathrm{rot}\,\vec{E}(\vec{r},t) = -\dfrac{\partial \vec{B}(\vec{r},t)}{\partial t}$	$\oint_{\partial A} \vec{E}(\vec{r},t)\cdot d\vec{s} = -\dfrac{d}{dt}\int_A \vec{B}(\vec{r},t)\cdot d\vec{A}$
$\mathrm{rot}\,\vec{H}(\vec{r},t) = \dfrac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}$	$\oint_{\partial A} \vec{H}(\vec{r},t)\cdot d\vec{s} = \int_A \left(\dfrac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}(\vec{r},t)\right)\cdot d\vec{A}$
$\mathrm{div}\,\vec{B}(\vec{r},t) = 0$	$\oint_{\partial V} \vec{B}(\vec{r},t)\cdot d\vec{A} = 0$
$\mathrm{div}\,\vec{D}(\vec{r},t) = \rho(\vec{r},t)$	$\oint_{\partial V} \vec{D}(\vec{r},t)\cdot d\vec{A} = \int_V \rho(\vec{r},t)\,dV$
$\forall \vec{r} \in \Omega \subseteq \mathbf{R}^3$	$\forall A, \forall V \subset \Omega \subseteq \mathbf{R}^3$

Frequency Domain: $\dfrac{\partial}{\partial t} \to i\omega$

FIT is a mimetic discretization based on the INTEGRAL FORMULATION

Picture from T.Weiland, VAdF1

Introduction of the FIT Topology - Faraday's law



$$\widehat{e}_n := \int_{L_n} \vec{E}(\vec{r},t)\cdot d\vec{s}$$

$$\widehat{\widehat{b}}_n := \int_{A_n} \vec{B}(\vec{r},t)\cdot d\vec{A}$$

$$\oint_{\partial A} \vec{E}(\vec{r},t)\cdot d\vec{s} = +\int_{L_1} +\int_{L_2} -\int_{L_3} -\int_{L_4}$$
$$= +\widehat{e}_1 +\widehat{e}_2 -\widehat{e}_3 -\widehat{e}_4$$

$$-\frac{d}{dt}\int_A \vec{B}(\vec{r},t)\cdot d\vec{A} = -\frac{d}{dt}\widehat{\widehat{b}}_n$$

$$+\widehat{e}_1 +\widehat{e}_2 -\widehat{e}_3 -\widehat{e}_4 = -\frac{d}{dt}\widehat{\widehat{b}}_n$$

This is just Kirchhoff's loop – law
→ Exact!!!

Pictures from T.Weiland, VAdF1

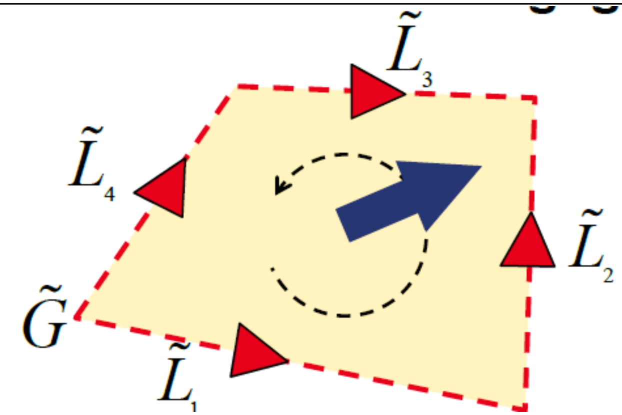
Introduction of the FIT Topology - Ampere's law

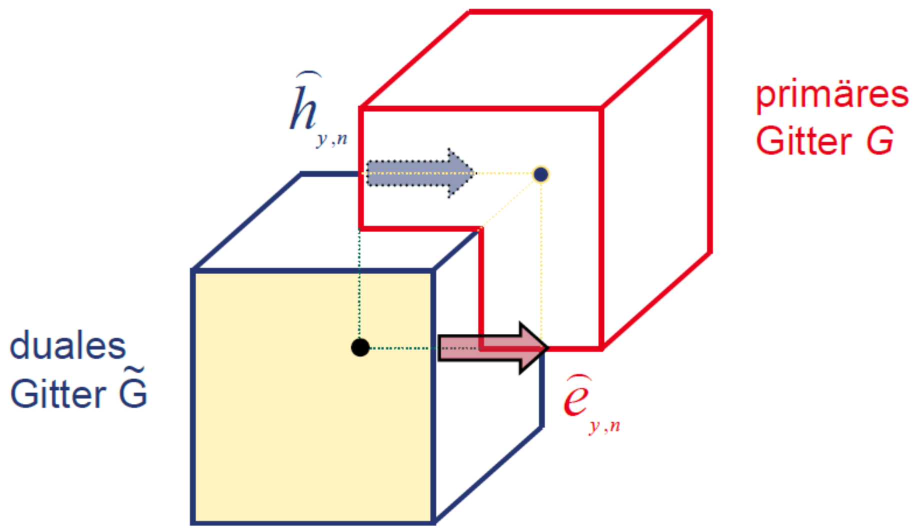
$$+\widehat{h}_1 \; +\widehat{h}_2 \; -\widehat{h}_3 \; -\widehat{h}_4 = \frac{d}{dt}\widehat{\widehat{d}}_n + \widehat{\widehat{j}}_n$$

$$\widehat{h}_n := \int_{\tilde{L}_n} \vec{H}(\vec{r},t)\cdot d\vec{s}$$

$$\widehat{\widehat{d}}_n := \int_{\tilde{A}_n} \vec{D}(\vec{r},t)\cdot d\vec{A}$$

$$\widehat{\widehat{j}}_n := \int_{\tilde{A}_n} \vec{J}(\vec{r},t)\cdot d\vec{A}$$





Staggered grid, originally by Yee, 1966

Pictures from T.Weiland, VAdF1

Introduction of the FIT Topology - Gauss's law and no magnetic charge

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$$+\widehat{\widehat{\mathrm{d}}}_1 + \widehat{\widehat{\mathrm{d}}}_2 + \widehat{\widehat{\mathrm{d}}}_3 - \widehat{\widehat{\mathrm{d}}}_4 - \widehat{\widehat{\mathrm{d}}}_5 - \widehat{\widehat{\mathrm{d}}}_6 = q$$

→dual grid

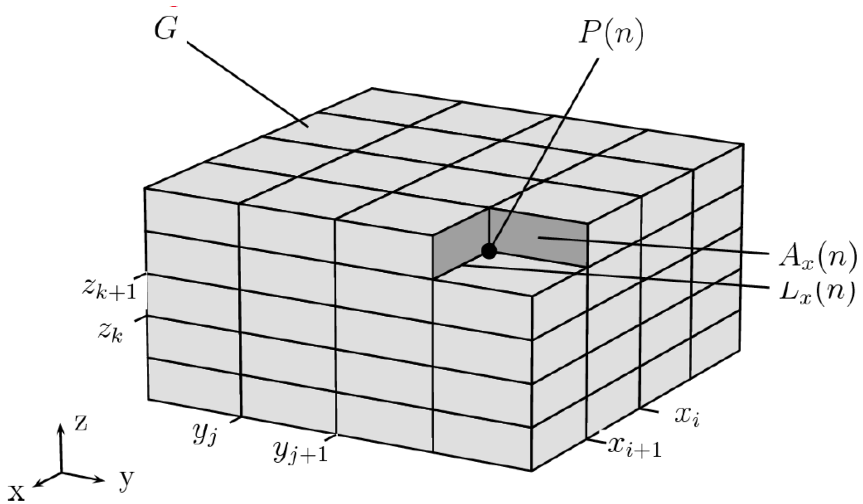
$$+\widehat{\widehat{\mathrm{b}}}_1 + \widehat{\widehat{\mathrm{b}}}_2 + \widehat{\widehat{\mathrm{b}}}_3 - \widehat{\widehat{\mathrm{b}}}_4 - \widehat{\widehat{\mathrm{b}}}_5 - \widehat{\widehat{\mathrm{b}}}_6 = 0$$

→primary grid

The incidence of the signs on the LHS correspond to the topology of the DIV operator!

The incidence of the signs in the discrete Faraday and Ampere law correspond to the topology of the CURL operator!

The Grid

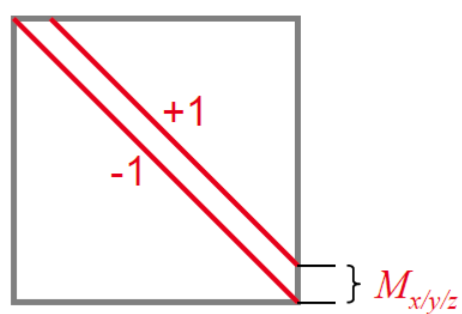


Discrete CURL operator (Matrix)

$$\mathbf{C} = \begin{pmatrix} & -\mathbf{P}_z & \mathbf{P}_y \\ \mathbf{P}_z & & -\mathbf{P}_x \\ -\mathbf{P}_y & \mathbf{P}_x & \end{pmatrix}$$

The same way:
Discrete DIV operator (Matrix)

$$\mathbf{S} = \begin{pmatrix} \mathbf{P}_x & \mathbf{P}_y & \mathbf{P}_z \end{pmatrix}$$



Discrete partial derivative operators

Pictures from T.Weiland, VAdF1

Grid-Maxwell-Equations

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$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d}{dt}\int_{A} \vec{B} \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H} \cdot d\vec{s} = \int_{A} \left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right) \cdot d\vec{A}$$

$$\oint_{\partial V} \vec{D} \cdot d\vec{A} = \int_{V} \rho \, dV$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

FIT

$$\mathbf{C}\widehat{\mathbf{e}} = -\frac{d}{dt}\widehat{\widehat{\mathbf{b}}}$$

$$\widetilde{\mathbf{C}}\widehat{\mathbf{h}} = \frac{d}{dt}\widehat{\widehat{\mathbf{d}}} + \widehat{\widehat{\mathbf{j}}}$$

$$\widetilde{\mathbf{S}}\widehat{\widehat{\mathbf{d}}} = \mathbf{q}$$

$$\mathbf{S}\widehat{\widehat{\mathbf{b}}} = \mathbf{0}$$

The Grid-Equations represent an EVALUATION of Maxwell‘s equations

→ Therefore they are exact

Pictures from T.Weiland, VAdF1

Preservation of topological properties (mimetic discretization)

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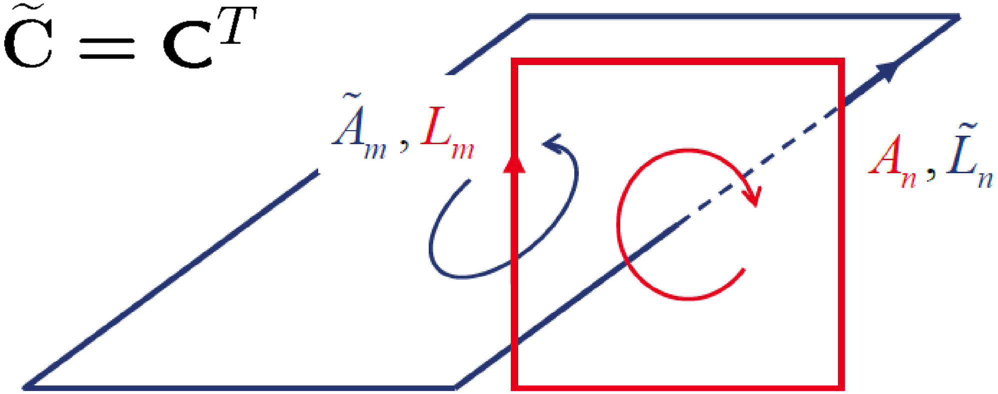
$$\left.\begin{aligned} \mathbf{SC} &= 0 \\ \tilde{\mathbf{S}}\tilde{\mathbf{C}} &= 0 \end{aligned}\right\} \quad \nabla \cdot \nabla \times = 0$$

$$\left.\begin{aligned} \mathbf{CS}^T &= 0 \\ \tilde{\mathbf{C}}\tilde{\mathbf{S}}^T &= 0 \end{aligned}\right\} \quad \nabla \times (\nabla) = 0$$

$$\tilde{\mathbf{C}} = \mathbf{C}^T$$



$$C_{nm} = \tilde{C}_{mn} = +1$$

Pictures from T.Weiland, VAdF1

Numerical operators

- So far no approximations done.
- For the solutions of the Grid equations material relations needed:

$$\underline{\vec{B}} = \underline{\mu}\,\underline{\vec{H}} \qquad\qquad \underline{\vec{D}} + \frac{1}{i\omega}\underline{\vec{J}} = \underline{\varepsilon}\,\underline{\vec{E}}$$

- Problem in the discrete case:

These operators map states from a primary edge to a dual face and vice versa!

Discrete material operators

- So called Hodge operators:

→ Map from k-dimensional to 3-k dimensional Submanifolds of R^3 , cannot be localized

- Practice: Average of the material parameters in the cells connected to one edge
- Results in Diagonal Matrices containing the averages of material parameters weighted with the length of the edge and the area of the face
- Necessitates DUAL-ORTHOGONAL-Grid

Material Matrices

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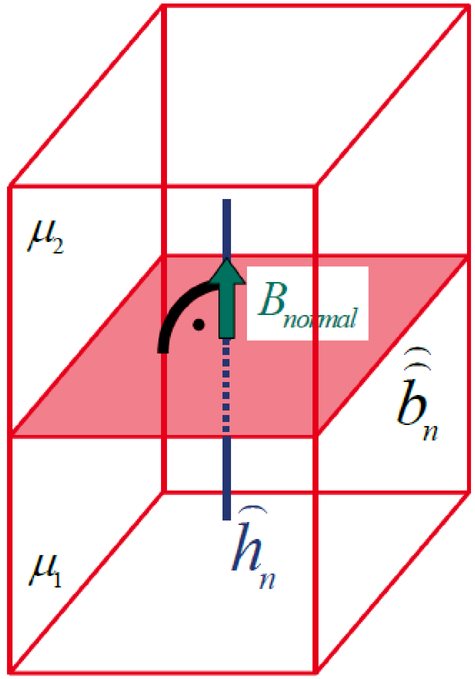


$$\hat{\hat{\mathbf{b}}} = \mathbf{M}_\mu \hat{\mathbf{h}} \qquad \mathbf{M}_\mu : N_A \times \tilde{N}_L \qquad (N_A = \tilde{N}_L)$$

$$\nu = \mu^{-1} \qquad \hat{\mathbf{h}}_i = (\mathrm{M}_\nu)_{ii} \hat{\hat{\mathbf{b}}}_i$$

$$\mathrm{M}_\nu = D_{\tilde{s}} D_\nu {D_A}^{-1}$$

$$(D_\nu)_{ii} = \frac{\hat{\mathbf{h}}_i}{\hat{\hat{\mathbf{b}}}_i} \frac{(D_A)_{ii}}{(D_{\tilde{s}})_{ii}} = \frac{H}{B} + \mathcal{O}(l^{2\ldots3})$$

$$\hat{\hat{b}}_n \approx \frac{\int_{A_n} dA}{\int_{\tilde{L}_n} \mu^{-1} ds} \hat{h}_n$$

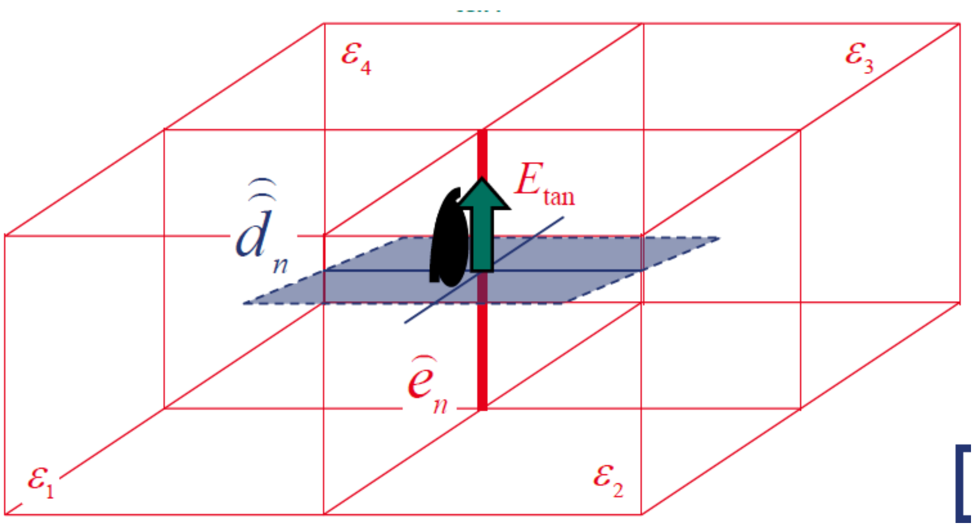
$$(D_\nu)_{ii} = \frac{\nu_i + \nu_{i+Mxyz}}{2}$$

Weighting the average does not improve the order…

Pictures from T.Weiland, VAdF1

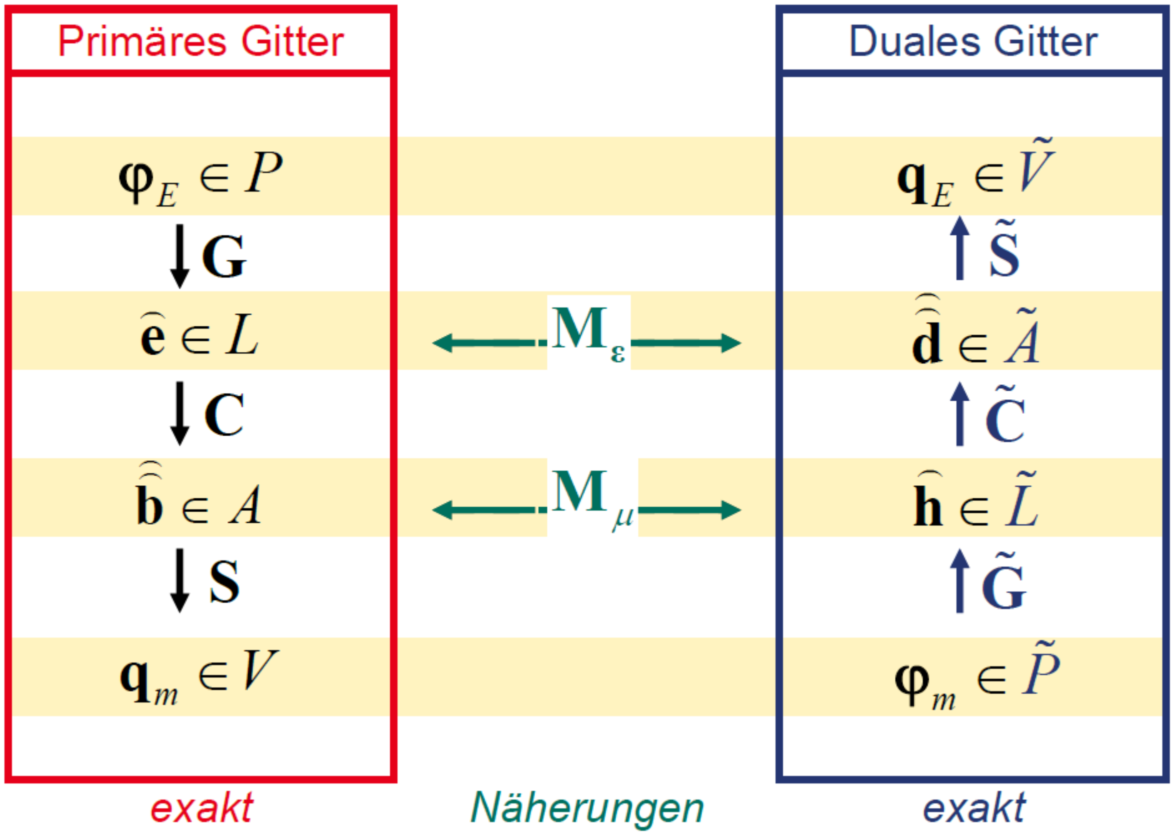
Material Matrices

- Same behaviour for permittivity and conductivity matrix
- 4 cells involved



Pictures from T.Weiland, VAdF1

A complete LINEAR set of equations that allow for solving



Convergence of the method follows from consistency and stability!!!
(Stability analysis in time domain → complicated…)

Pictures from T.Weiland, VAdF1

Back to the original problem…

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$$\boxed{\nabla \times \frac{1}{\underline{\mu}} \nabla \times \underline{\vec{E}} - \omega^2 \underline{\varepsilon \vec{E}} = -i\omega \underline{\vec{J}}_{ext}}$$

$$\Downarrow$$

$$(\widetilde{\mathbf{C}} \mathbf{M}_{\mu^{-1}} \mathbf{C} - \omega^2 \mathbf{M}_\epsilon + i\omega \mathbf{M}_\kappa) \underline{\widehat{\mathbf{e}}} = -i\omega \underline{\widehat{\widehat{\mathbf{j}}}}_{ext}$$

Number of unknowns in $\underline{\widehat{\mathbf{e}}}$: $3N_p$ (complex)

This is in the range of millions!!!

Problem: $\varepsilon \approx 10^{-12}$ and $\kappa \approx 10^6$ (Metal)

$$cond_{||\cdot||}(A) = ||A|| \cdot ||A^{-1}|| \approx |\frac{\lambda_{max}}{\lambda_{min}}|$$

Also nonconductive Ferrite with complex μ causes problems

Solvers for N linear equations

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- The ONLY way: Iterative Solvers (but these do not like matrices with large condition number)
- A “preconditioner“ (an approximate inverse) is needed
- Several ways to obtain preconditioners

  (available as black-box)

  →Now using KSPCG with SOR preconditioner
- In future Algebraic Multigrid (AMG) PC considered

Setup of Linear System

$$(\tilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C} - \omega^2\mathbf{M}_\epsilon + i\omega\mathbf{M}_\kappa)\underline{\widehat{\mathbf{e}}} = -i\omega\underline{\widehat{\widehat{\mathbf{j}}}}_{ext}$$

$$\widehat{\mathbf{e}} = \mathbf{M}_\epsilon^{-1/2}\widehat{\mathbf{e}}' \qquad A_0 = (\mathbf{M}_{\mu^{-1}}^{1/2}\mathbf{C}\mathbf{M}_\epsilon^{-1/2})^H(\mathbf{M}_{\mu^{-1}}^{1/2}\mathbf{C}\mathbf{M}_\epsilon^{-1/2})$$

$$\underbrace{(A_0 - \omega^2 I + i\omega\mathbf{M}_\kappa\mathbf{M}_\epsilon^{-1/2})}_{\text{A}}\underline{\widehat{\mathbf{e}}}' = -i\omega\mathbf{M}_\epsilon^{-1/2}\underline{\widehat{\widehat{\mathbf{j}}}}_{ext}$$

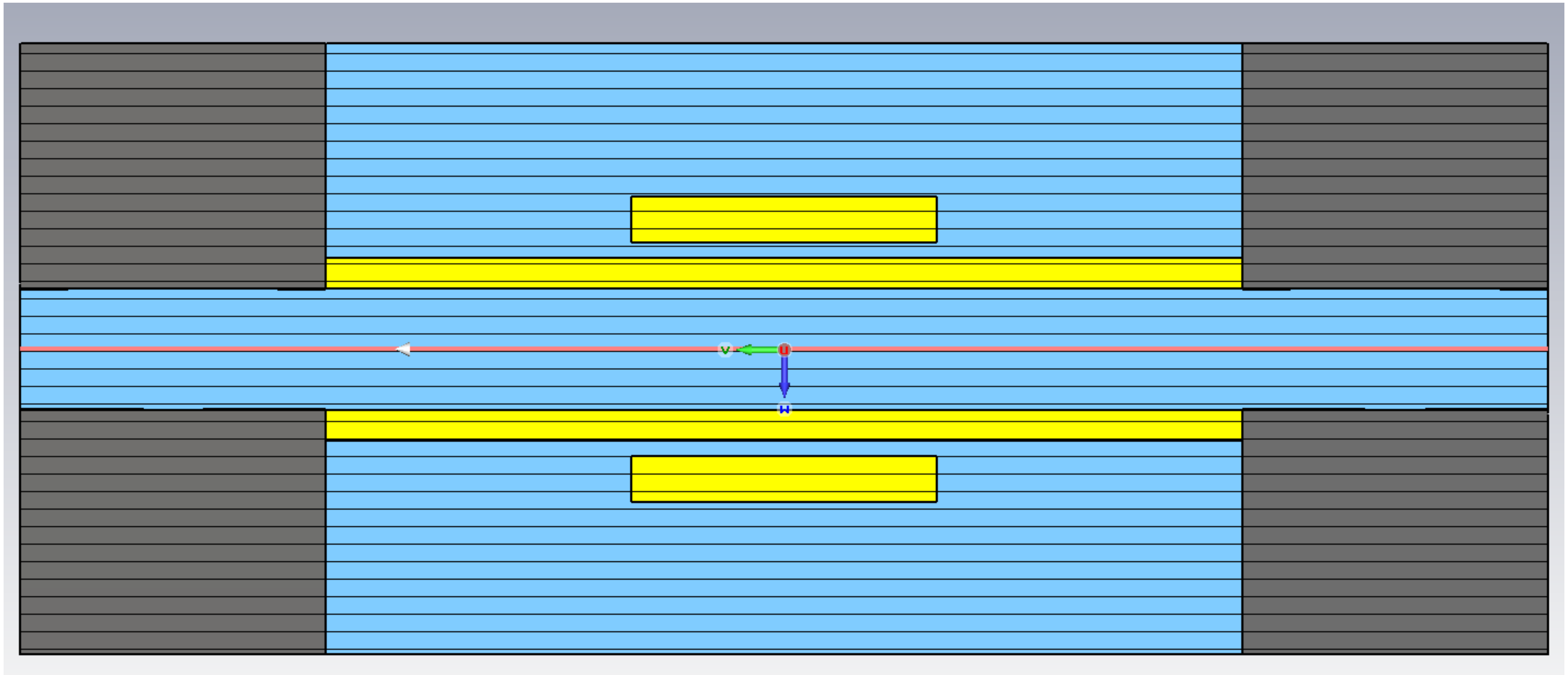
Diagonal dominance needed for SOR

System matrix symmetric but indefinit!

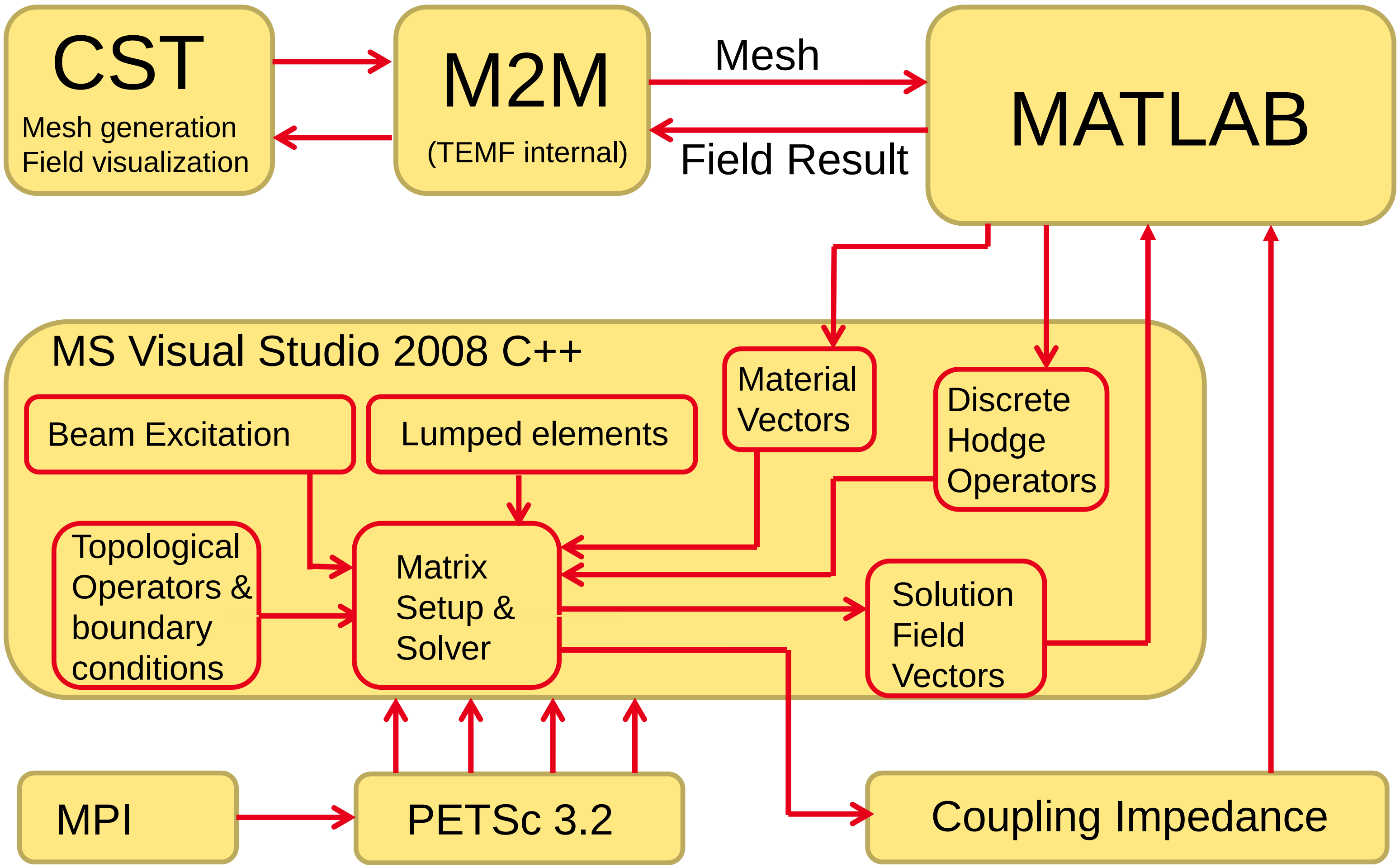
- Minimum frequency required
- Contrast to Doliwa‘s solver for frequencies below first resonance (based on Neumann series expansion of A)

CAD of Model in CST Studio Suite^R

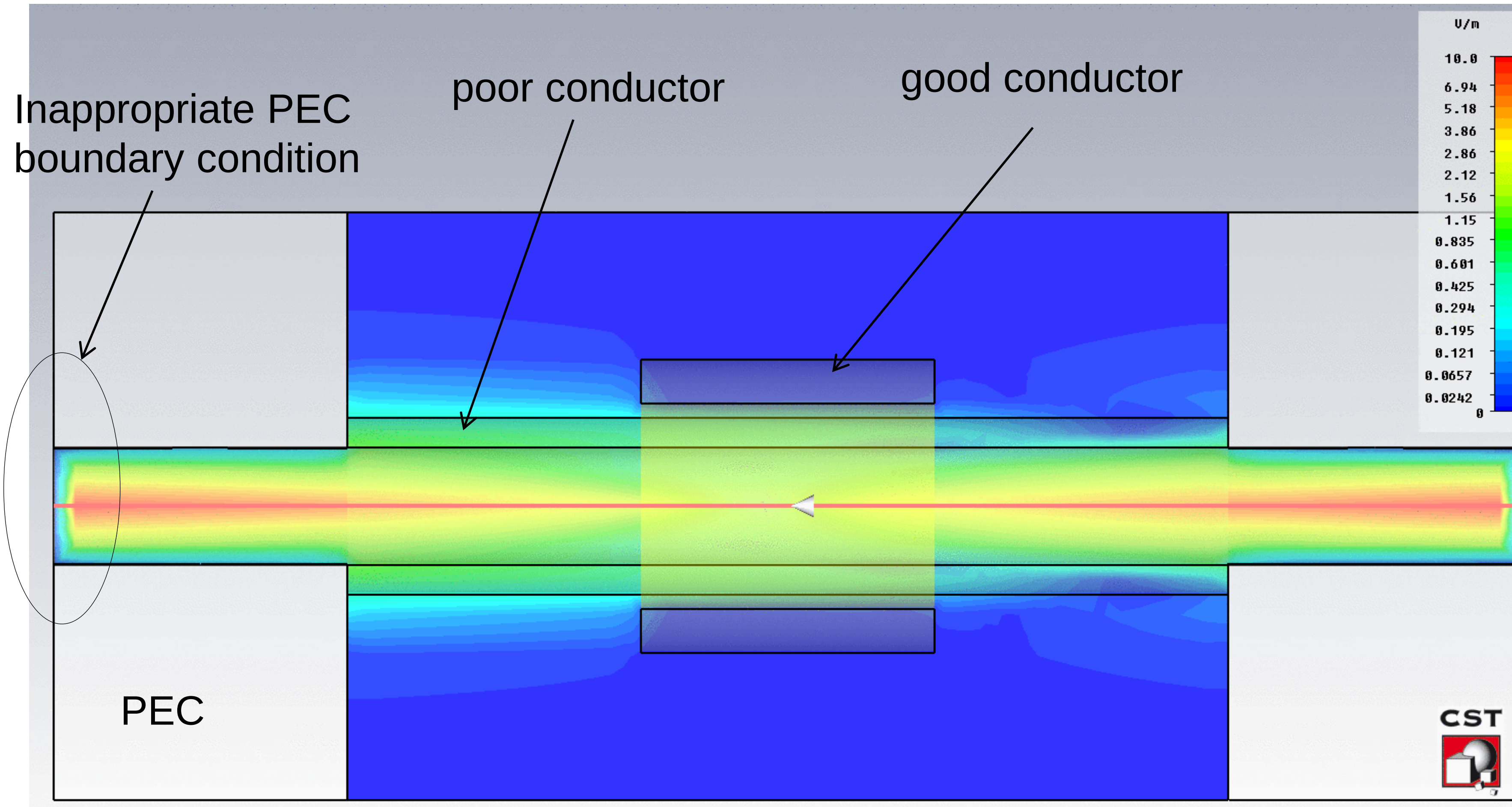
- Vessel for the Device Under Test (DUT)
- PEC boundary conditions, two beampipe stubs



Implementation



First Results for Arbitrary Test Structure



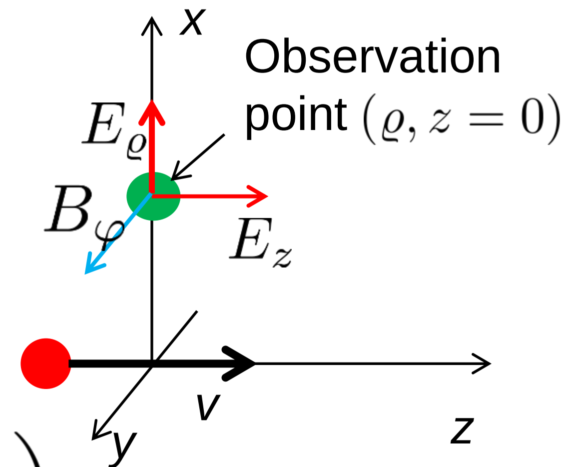
Beampipe-adapted Boundary Conditions

- Included as Neumann BC to preserve symmetry

$$A\underline{\widehat{\mathbf{e}}}' = -i\omega \mathbf{M}_\epsilon^{-1/2}\underline{\widehat{\widehat{\mathbf{j}}}}_{ext} + A\mathbf{M}_\epsilon^{1/2}\underline{\widehat{\mathbf{e}}}_{boundary}$$

- Obtained from relativistic charge in infinite PEC pipe
- Must have proper phase!
- Analytical solution violates discrete charge conservation!!!
- **SC**$(e_n + e_a)$ gives artificial magnetic charges
- Numerical solution pursued

Source Fields

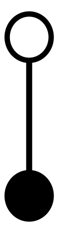


- Coulomb field in moving frame

$$\vec{E}' = \frac{q}{4\pi\varepsilon}\left(\frac{\varrho'}{\sqrt{\varrho'^2+z'^2}^3}\vec{e}_\varrho + \frac{z'}{\sqrt{\varrho'^2+z'^2}^3}\vec{e}_z\right)$$

- Lorentz transformation to lab-frame

$$\vec{E} = \frac{q}{4\pi\varepsilon}\left(\frac{\gamma\varrho}{\sqrt{\varrho^2+(\beta\gamma ct)^2}^3}\vec{e}_\varrho + \frac{-\beta\gamma ct}{\sqrt{\varrho^2+(\beta\gamma ct)^2}^3}\vec{e}_z\right), \quad B_\varphi = \frac{v}{c^2}E_\varrho$$



$$\begin{cases} \underline{E}_z = iq\frac{\mu_0}{2\pi}\frac{\omega}{\beta^2\gamma^2}\mathrm{K}_0\left(\frac{|\omega|}{\beta\gamma c}\varrho\right) \\ \underline{E}_\varrho = q\frac{\mu_0}{2\pi}\frac{|\omega|}{\beta^2\gamma}\mathrm{K}_1\left(\frac{|\omega|}{\beta\gamma c}\varrho\right) \end{cases}$$

Evaluate Erho at the center of primary edges

$$\underline{\widehat{\mathbf{e}}}_{i,x} = e^{\pm i\omega l/2v}\underline{E}_\varrho(x_{i+1/2})\cos\varphi\Delta_{x,i} + O(\Delta_x^2)$$

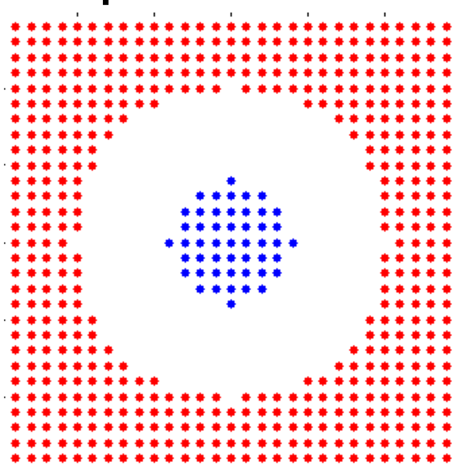
2D Solver for Boundary Conditions

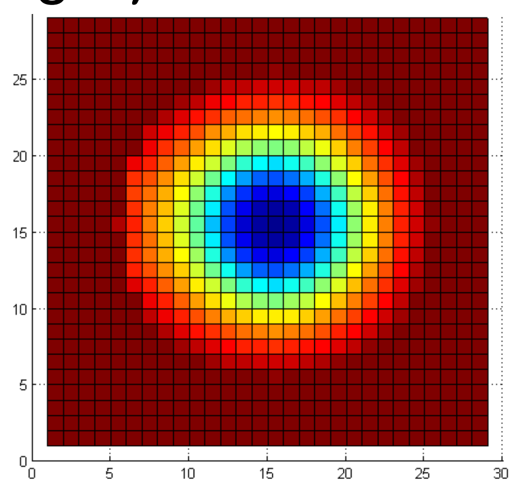
(could also be called 2.5D Solver)

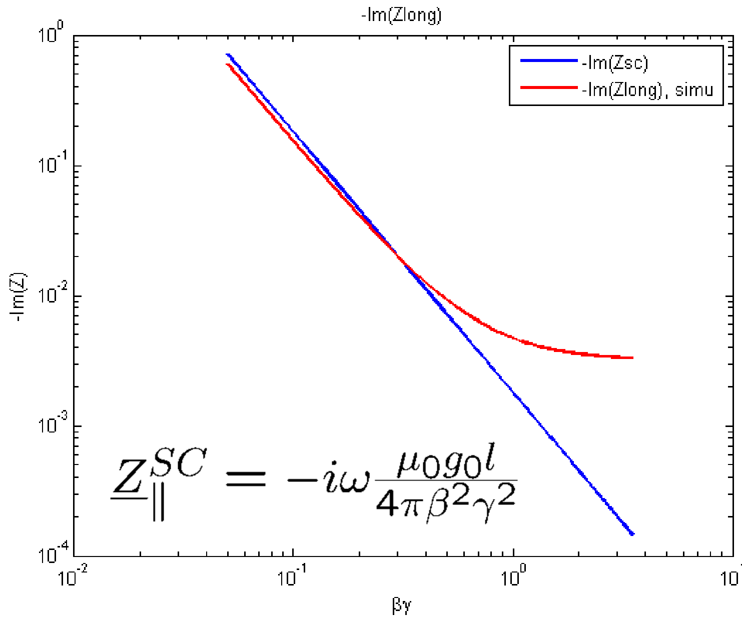
- Only one cell long (nz=2) (Quasi-periodic-BC in z-direction)

$$\partial_z \rightarrow -i\omega/v \qquad P_z \rightarrow diag(-1 + exp(-i\omega \triangle z/v))$$

- Currently under test in MATLAB
- Frequency behaviour as expected but problems at high γ







- This could also be applied for dipolar excitation

Inclusion of complex permeability

- Inversion of material averaging (in MATLAB)
- Difficulties at PEC cells
- Change parameteres for particular material (in C++)
- Reassemble discrete Hodge Operator (within f-loop)

- Lumped impedances can be included in M_κ matrix in the same way (still diagonal if and only if neighboring cells are connected)

Results – functionals on C^{3np}

- Posprocessings to be done for every frequency
- Longitudinal Impedance

$$\underline{Z}_{||}(\widehat{\mathbf{e}}(\omega)) = \frac{1}{q^2}\widehat{\mathbf{e}}^t \widehat{\widehat{\mathbf{j}}}^*_s$$

- Transverse Impedance

$$\underline{Z}_{\perp}(\widehat{\mathbf{e}}(\omega)) = -\frac{v}{(q\cdot 2d_x)^2}\widehat{\mathbf{e}}^t \widehat{\widehat{\mathbf{j}}}^*_s$$

$2d_x$: Distance between excitation paths

- Total Resistive Power Loss

$$P(\omega) = \frac{1}{2}\Re(\widehat{\mathbf{e}}^t \mathbf{M}_\kappa \widehat{\mathbf{e}}^*)$$

Conclusion (Done and To Do)

- Preprocessing in MATLAB, treating of PEC cells ✓
- Construction of CURL-Matrix and Electrical Boundary conditions ✓
- Setup of Hermitian System Matrix ✓
- Preconditioner and solver for up to 10^5 cells ✓
- 2D solver for boundary conditions 1/2 ✓
- Integration of the beam adapted boundary in the main grid ✗
- Reassembling of Hodge operators with frequency dependent material parameters and lumped elements ✗
- Solver optimization for larger systems ✗
- Transverse Impedance ✗

Outlook

- Side effects of Ferrite Insertion
- Kicker device with supply network
- Comparison to lumped element circuit models
- Connection to high frequency (TD) wake-field computations (CST Particle-Studio)
- Verification by dedicated bench measurements
- Final goal:
  Impedance Database for Instabiliy Estimations for SIS100

THE END

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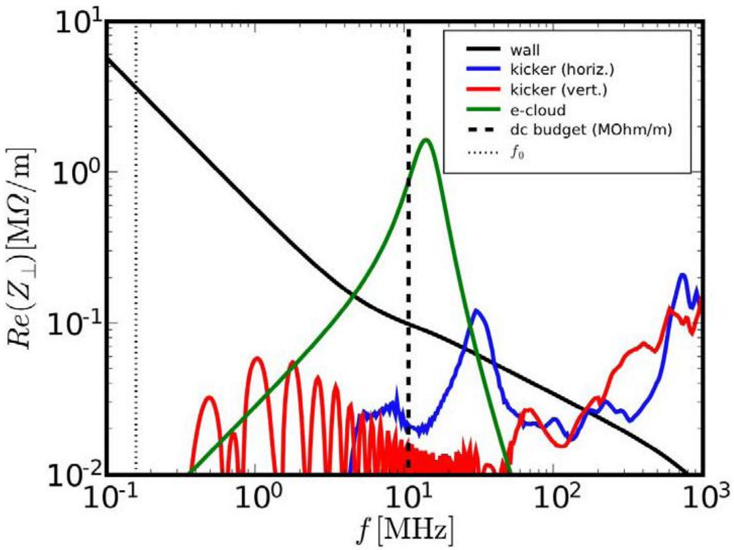
Thank you very much for your kind attention!

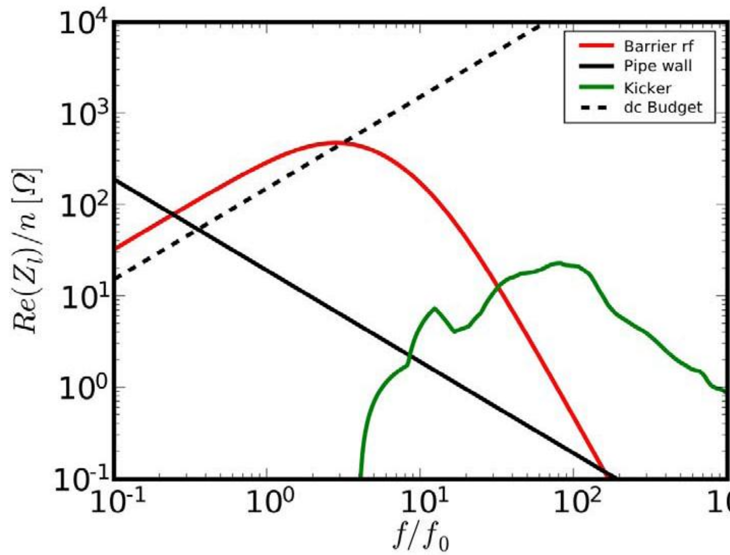
Any questions?

The coupling impedance spectrum in SIS18 and SIS100

Dominating devices   1 MHz   50 MHz   1 GHz   *f*

$\mathrm{Re}\{\underline{Z}_\perp\}$	Beampipe (wall current)	Kicker, loading network (PFN)	Kicker, Ferrite yoke	Waveguide cutoff of beampipe (structural dependence)
$\mathrm{Re}\{\underline{Z}_\parallel\}$	MA Cavities (Broadband)	Ferrite Cavities (Narrowband)	Kicker	





Rough estimate!

$$f_0 = 156\mathrm{kHz}$$

Taken from SIS100 design spec.   Courtesy of O. Boine-Frankenheim

Material of the Kicker

- Material description by complex permeability
  → Linear and lossy
- Nonlinear modeling by weak coupling of higher harmonics
  → Necessitates nonlinear Impedance
  (nonlinear coherent force)

$$\frac{1}{q}\int \underline{\vec{F}} \cdot \mathrm{d}\vec{s} = \underline{Z}(\underline{I}) \cdot \underline{I}$$

- Restriction to linear impedance!
- If possible: Estimation of „Total Harmonic Distortion"