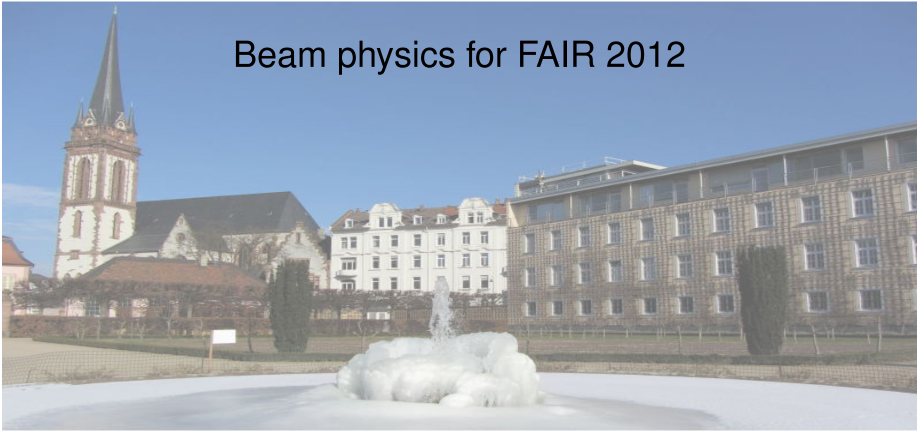
Simulation of ferrite-loaded cavity resonators

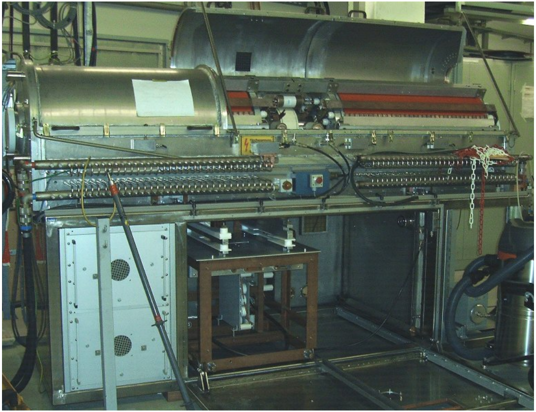
Klaus Klopfer, Wolfgang Ackermann, Thomas Weiland
Institut für Theorie Elektromagnetischer Felder,
TU Darmstadt



Contents

TECHNISCHE UNIVERSITÄT DARMSTADT

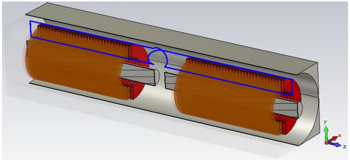
GSI SIS18 Ferrite Cavity

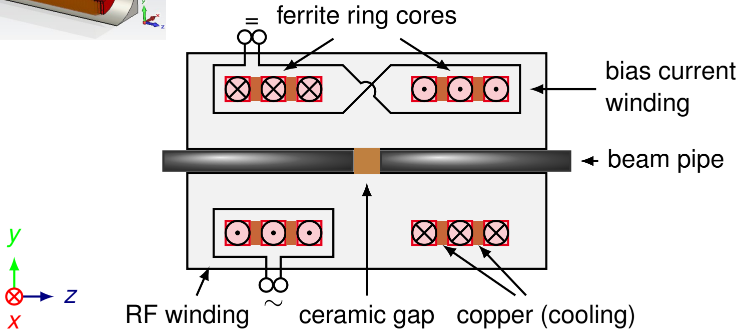


Main benefits of ferrite cavities:

- reduction of wavelength $\Rightarrow$ more compact cavity
- modification of resonance frequency in a wide range SIS 18 cavity: $\sim 0.6 - 5$ MHz

GSI SIS18 Ferrite Cavity
Main components





Fundamental relations

TECHNISCHE UNIVERSITÄT DARMSTADT

Assumptions: $|\vec{H}_d| \ll |\vec{H}_{\text{bias}}|$, effect of hysteresis negligible

$$\vec{B}(t) = \mu_0 \mu_{\text{bias}} \vec{H}_{\text{bias}} + \mu_0 \overset{\leftrightarrow}{\mu}_d \vec{H}_d(t) = \mu_0 \mu_{\text{bias}} \vec{H}_{\text{bias}} + \mu_0 \overset{\leftrightarrow}{\mu}_d \operatorname{Re}\left(\vec{H}_d \cdot e^{-\mathrm{i}\omega t}\right)$$

- Maxwell's equations:

$$\nabla \times \vec{H}_d(t) = \frac{\partial}{\partial t} \vec{D}(t)$$
$$\nabla \times \vec{E}\,(t) = -\mu_0 \overset{\leftrightarrow}{\mu}_d \frac{\partial}{\partial t} \vec{H}_d(t)$$

- Eigensolutions determined by:

$$\boxed{\epsilon^{-1} \nabla \times \left(\mu_0 \overset{\leftrightarrow}{\mu}_d^{-1} \nabla \times \vec{E}\right) = \omega^2 \vec{E}}$$

Boundary condition: $\vec{n} \times \vec{E} = 0$ on cavity boundary

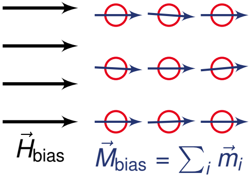
Permeability tensor

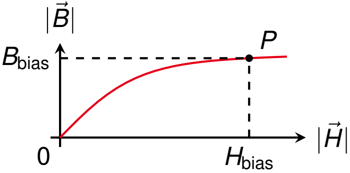
TECHNISCHE UNIVERSITÄT DARMSTADT

[D. Polder, Phil. Mag., 40 (1949)]

Aim: Determination of permeability tensor $\overleftrightarrow{\mu}_d$

$$\vec{B}_{\text{bias}} = \mu_0 \left(\underbrace{\vec{H}_{\text{bias}} + \vec{M}_{\text{bias}}}_{=: \mu_{\text{bias}} \vec{H}_{\text{bias}}} \right) = \mu_0 \mu_{\text{bias}} \vec{H}_{\text{bias}}$$





Permeability tensor

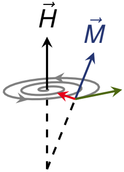
[D. Polder, Phil. Mag., 40 (1949)]

Aim: Determination of permeability tensor $\overset{\leftrightarrow}{\mu}_d$

$$\vec{B}(t) = \mu_0\left(\vec{H}(t) + \vec{M}(t)\right) = \mu_0\left(\underbrace{\vec{H}_{\text{bias}} + \vec{M}_{\text{bias}}}_{=:\mu_{\text{bias}}\vec{H}_{\text{bias}}} + \underbrace{\vec{H}_d(t) + \vec{M}_d(t)}_{=:\overset{\leftrightarrow}{\mu}_d \vec{H}_d(t)}\right) = \mu_0\mu_{\text{bias}}\vec{H}_{\text{bias}} + \mu_0\overset{\leftrightarrow}{\mu}_d\vec{H}_d(t)$$

Equation of motion for magnetization:

$$\frac{d\vec{M}}{dt} = \gamma\left(\vec{M}\times\vec{H}\right) + \frac{\alpha}{M_0}\vec{M}\times\frac{d\vec{M}}{dt}$$



$$\Rightarrow \overset{\leftrightarrow}{\mu}_d\left(\vec{H}_{\text{bias}},\omega\right) = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad (\text{for } \vec{H} = H_{\text{bias}}\cdot\vec{e}_z)$$

$\mu_{1,2} = \mu_{1,2}\left(\vec{H}_{\text{bias}},\omega\right)$; $\text{Im}(\mu_{1,2}) \neq 0$ (including magnetic losses)

Calculation of eigenvectors of biased ferrite cavities: Finite Integration Technique

TECHNISCHE UNIVERSITÄT DARMSTADT

$$\epsilon^{-1}\nabla \times \left(\mu_0 \overset{\leftrightarrow}{\mu}_d^{-1} \nabla \times \vec{E}\right) = \omega^2 \vec{E}$$

Discretization by Finite Integration Technique (FIT):

$$M_\epsilon^{-1}\widetilde{\mathbf{C}} M_{d,\mu}^{-1}\mathbf{C}\widehat{\mathbf{e}} = \omega^2 \widehat{\mathbf{e}}$$

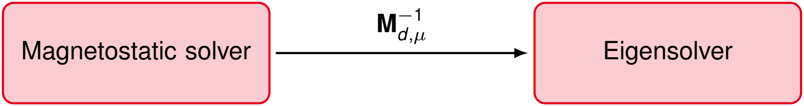
- permeability tensor $M_{d,\mu}$:
  - non-diagonal
  - dependend on $\vec{H}_{\text{bias}}$ and ω
  - if magnetic losses included:
- requirements on eigensolver:

  $\Rightarrow$ non-linear

  $\Rightarrow$ non-Hermitian

Calculation of eigenvectors of biased ferrite cavities: Concept

TECHNISCHE UNIVERSITÄT DARMSTADT



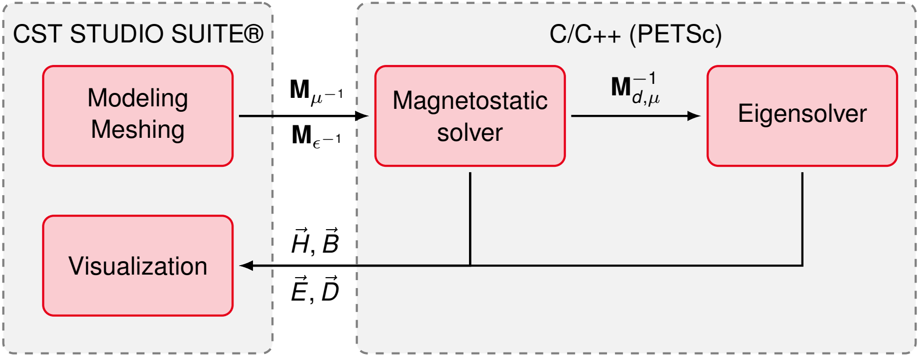
- calculation of bias magnetic field
- determination of permeability matrix $\mathbf{M}_{d,\mu}^{-1}$

- calculation of eigenfrequencies including $\vec{E}$- and $\vec{B}$-field

General requirements:

- support of non-linear material
- support of lossy material
- parallel computation with distributed memory (scalability)

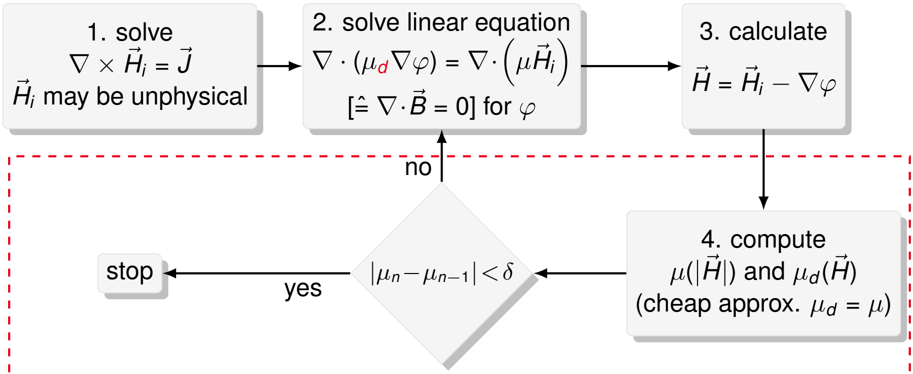
Implementation (Finite Integration Technique)



Implementation (FIT)
Magnetostatic solver: H_i-algorithm

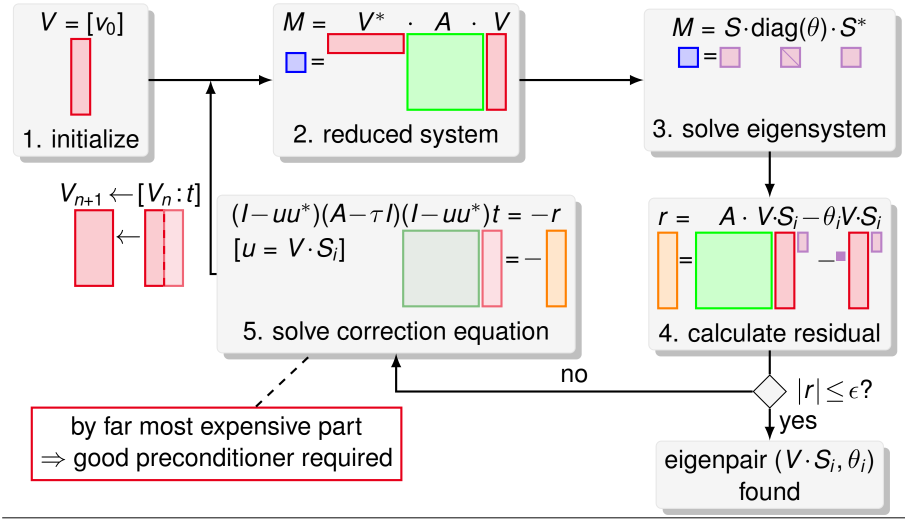
TECHNISCHE UNIVERSITÄT DARMSTADT

Decomposition of $\vec{H}$-field: $\vec{H} := \vec{H}_i + \vec{H}_h$ with $\nabla \times \vec{H}_i = \vec{J}$ and $\vec{H}_h = -\nabla\varphi$



Non-linear material: $\vec{B} = \mu(|\vec{H}|)\vec{H} \Rightarrow$ non-linear equation $\Rightarrow$ Newton's method

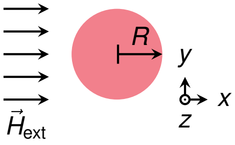
Implementation (FIT)
Eigensolver: Jacobi-Davidson (with target value)
Basic idea

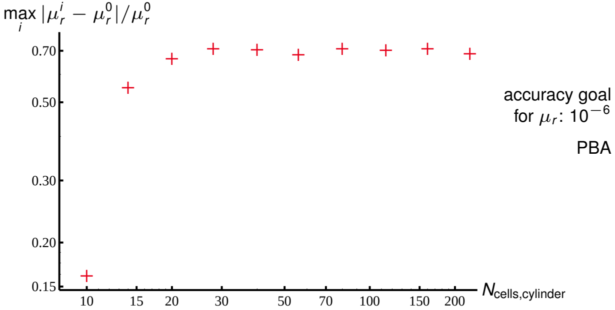


Verification of the magnetostatic solver

TECHNISCHE UNIVERSITÄT DARMSTADT

- test model: iron cylinder in external homogenous magnetic field $\vec{H}_{\text{ext}} = H_0 \cdot \vec{e}_x$ (c.f. [Wagner])
- analytical solution:
  homogenous field inside cylinder $\vec{H}_{\text{in}} \propto \vec{H}_{\text{ext}}$
  e.g. $H_0 = 10^6 \ \frac{\text{A}}{\text{m}} \Rightarrow \mu_r^0 = 12.99$ (interpolation method "straight BH-lines")



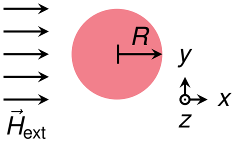


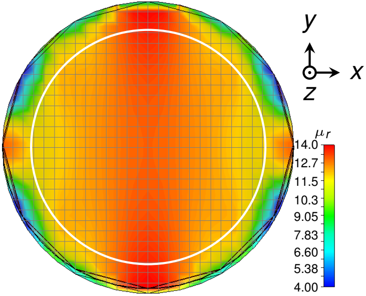
$N_{\text{cells,cylinder}}$: number of mesh cells for diameter of cylinder

Verification of the magnetostatic solver

TECHNISCHE UNIVERSITÄT DARMSTADT

- test model: iron cylinder in external homogenous magnetic field $\vec{H}_{\text{ext}} = H_0 \cdot \vec{e}_x$ (c.f. [Wagner])
- analytical solution:
  homogenous field inside cylinder $\vec{H}_{\text{in}} \propto \vec{H}_{\text{ext}}$
  e.g. $H_0 = 10^6\ \frac{\text{A}}{\text{m}} \Rightarrow \mu_r^0 = 12.99$ (interpolation method "straight *BH*-lines")





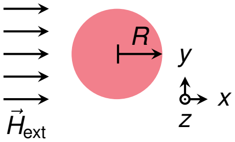
accuracy goal for μ_r: 10^{-6}

PBA

$N_{\text{cells,cylinder}} = 28$

Verification of the magnetostatic solver

- test model: iron cylinder in external homogenous magnetic field $\vec{H}_{\text{ext}} = H_0 \cdot \vec{e}_x$ (c.f. [Wagner])
- analytical solution:
  homogenous field inside cylinder $\vec{H}_{\text{in}} \propto \vec{H}_{\text{ext}}$
  e.g. $H_0 = 10^6 \ \frac{\text{A}}{\text{m}} \Rightarrow \mu_r^0 = 12.99$ (interpolation method "straight BH-lines")

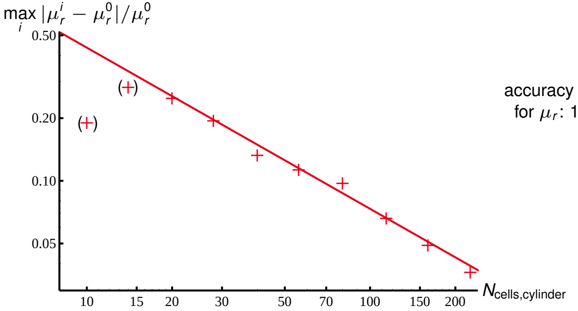


for cells inside cylinder with

$\sqrt{x_i^2 + y_i^2} < 0.8\ R$

$N_{\text{cells,cylinder}}$: number of mesh cells

for diameter of cylinder



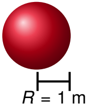
accuracy goal for μ_r: 10^{-6}

PBA

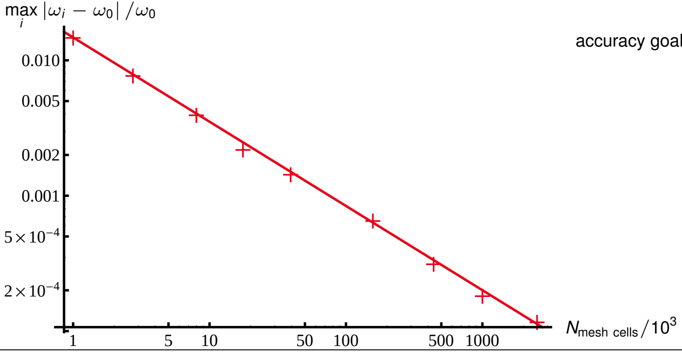
Verification of the Jacobi-Davidson eigensolver
Linear eigenvalue problem

TECHNISCHE UNIVERSITÄT DARMSTADT

test model: Eigenvalues ω of vacuum sphere (radius 1 m)

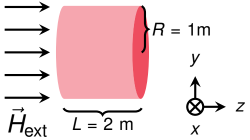


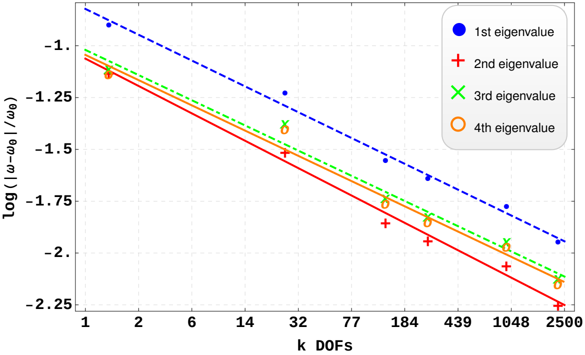
analytical solution:
2nd smallest eigenvalue: $\omega_0 = 2\pi \cdot 184.6624\ldots$ MHz with degree of degeneracy 5



Verification of the Jacobi-Davidson eigensolver
Non-linear eigenvalue problem

- test model: $(|\vec{H}_{\mathrm{ext}}| = 2750\,\frac{\mathrm{A}}{\mathrm{m}};\ \mu_r = 7)$
  lossless, ferrite-filled cylindrical cavity resonator
  longitudinally biased by homogeneous magnetic field
- semi-analytical solution available [Chinn, Epp and Wilkins]





accuracy goal: 10^{-6}

staircase

Conclusion and outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

- Final goal: calculation of eigenvectors for biased ferrite cavities
  1. Magnetostatic solver (non-linear material):
     $\Rightarrow$ permeability tensor $\overleftrightarrow{\mu}_d$
  2. Eigensolver (Jacobi-Davidson):
     $\Rightarrow$ non-linear complex eigenvalue problem
- Status (✔) and future plans (✘):

	sequential	parallel
Magnetostatic solver (linear)	✔	✔
Magnetostatic solver (non-linear)	✔	✘
Eigensolver (real, linear and non-linear)	✔	✔
Eigensolver (complex)	✘	✘
combined solver	✘	✘

Recommended literature

TECHNISCHE UNIVERSITÄT DARMSTADT

- **GSI SIS 18 ferrite cavity:**

H. Klingbeil.
Ferrite cavities.
CERN Yellow Report CERN-2011-007, pp. 299-317, 2012.
`arXiv:1201.1154v1`.

- **Finite Integration Technique**

T. Weiland.
Verfahren und Anwendungen der Feldsimulation I.
Lecture notes, Technische Universität Darmstadt, 2012.