Proyecto trasgo Project

4D tracking

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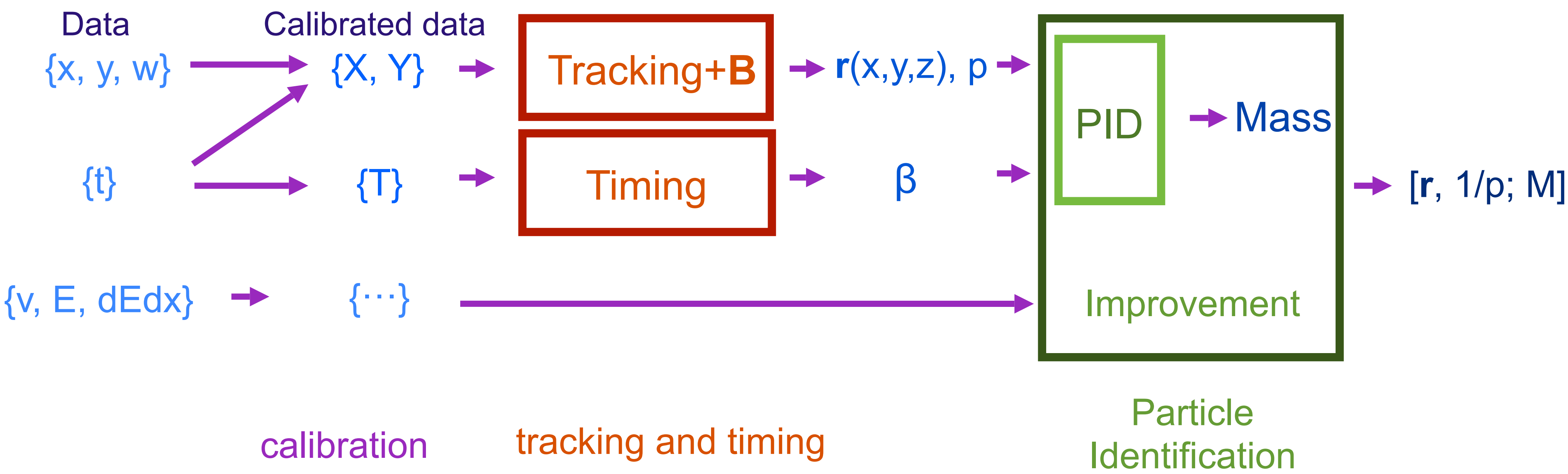
OUTLOOK

- The time as a tracking variable
- 4D tracking
    - The *timtrack* as a 4D tracking example
    - 4D tracking at CBM
- Beyond 4D tracking
- Summary and Reflections

The time as a tracking variable

The time as a tracking variable



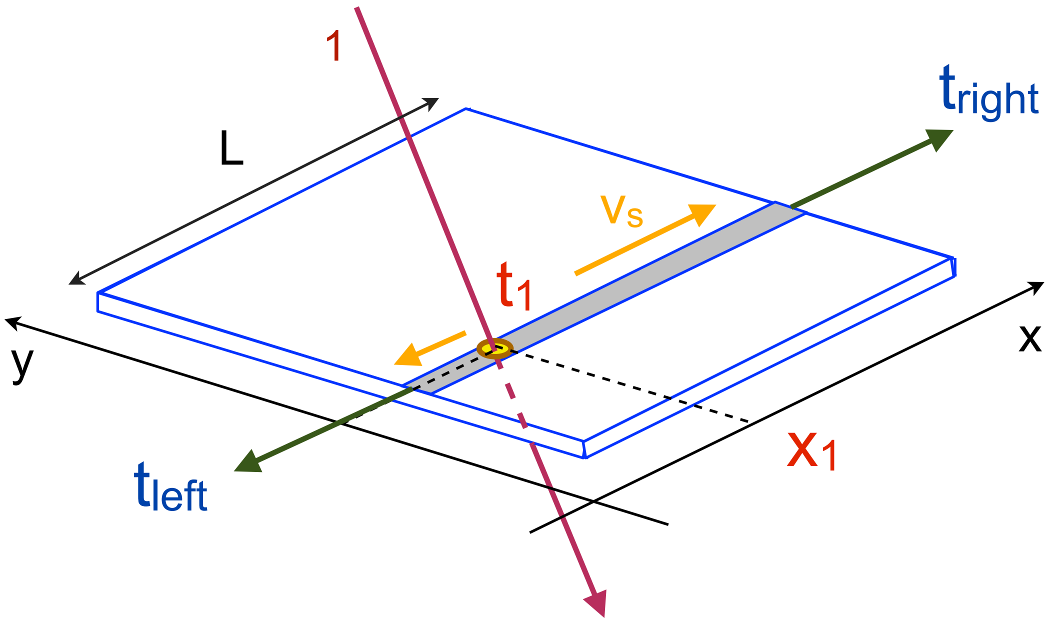
A typical tracking flow

- External t=0 time needed

- This scenario offers some problems. Let see some examples:

The time as a tracking variable

Example 1: Strip detectors



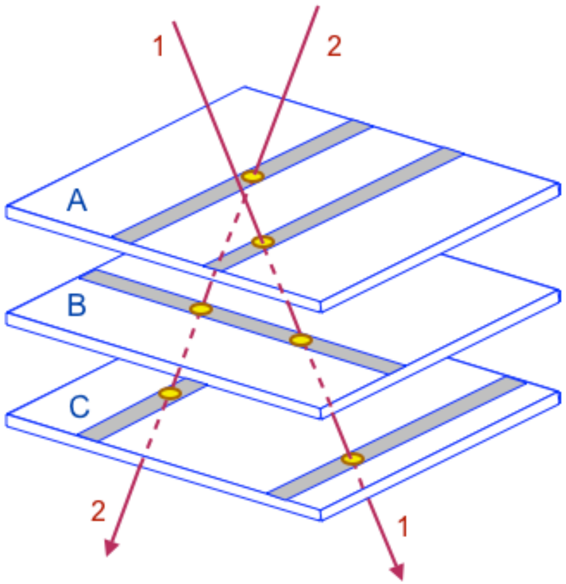
$$t_{left}(x_1, t_1; v_s) = t_1 + x_1/v_s$$
$$t_{right}(x_1, t_1; v_s) = t_1 + (L - x_1)/v_s$$

Usually we reduce both times to get the arrival time and the transverse coordinate:

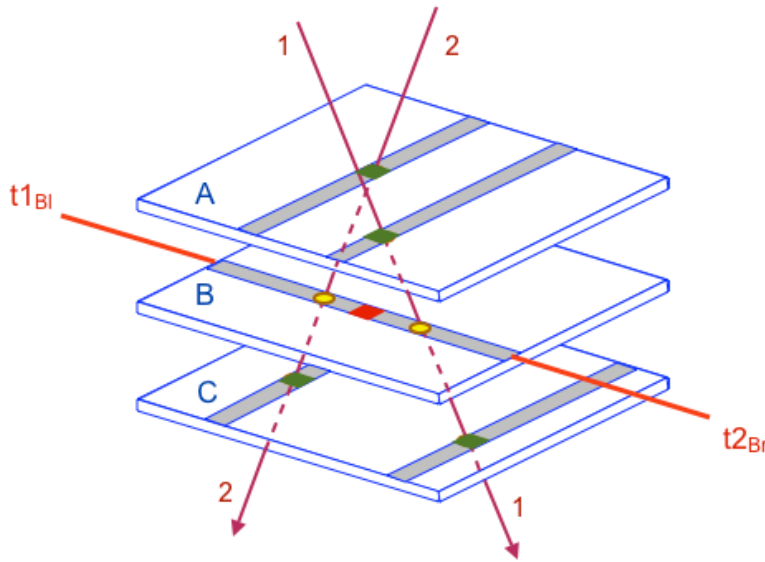
$$t_1 = 1/2\,(t_{left} + t_{right}) - L/v_s$$
$$x_1 = 1/2\,(t_{left} - t_{right})\,v_s + L/2$$

The time as a tracking variable

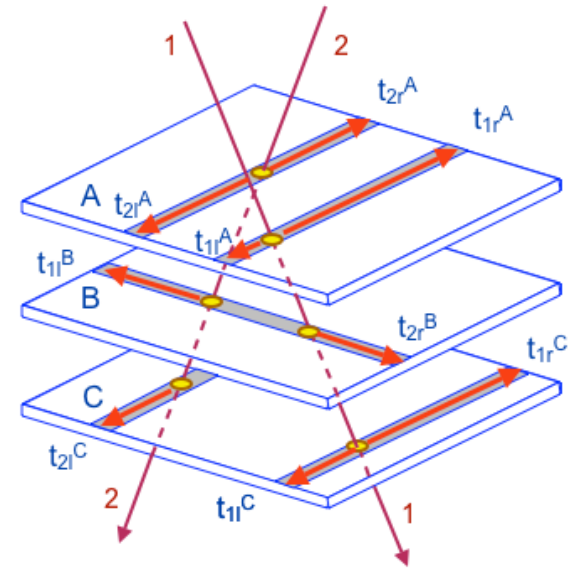
Example 1: Strip detectors



2 particles hit in the same strip of the second plane



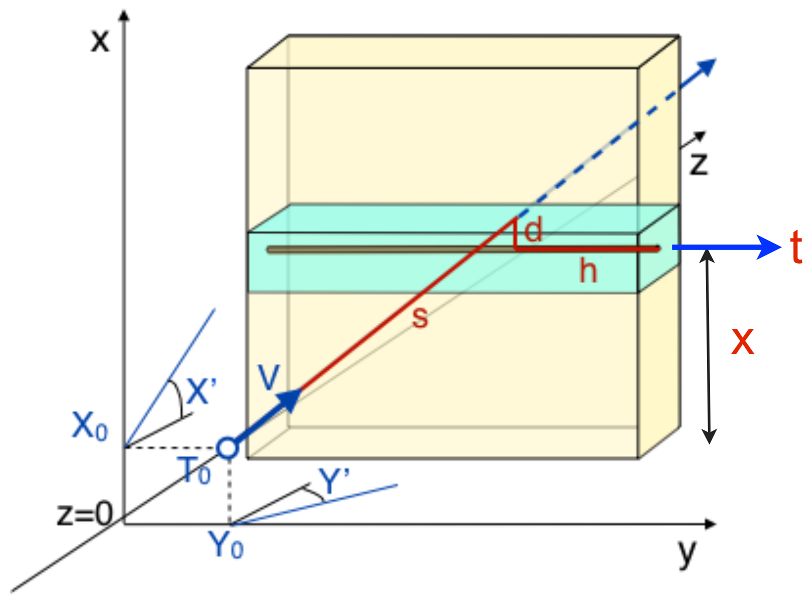
Using time reduction the second plane information is lost



Using time measurements directly in the tracking algorithm does not waste useful information

The time as a tracking variable

Example 2: Drift chambers

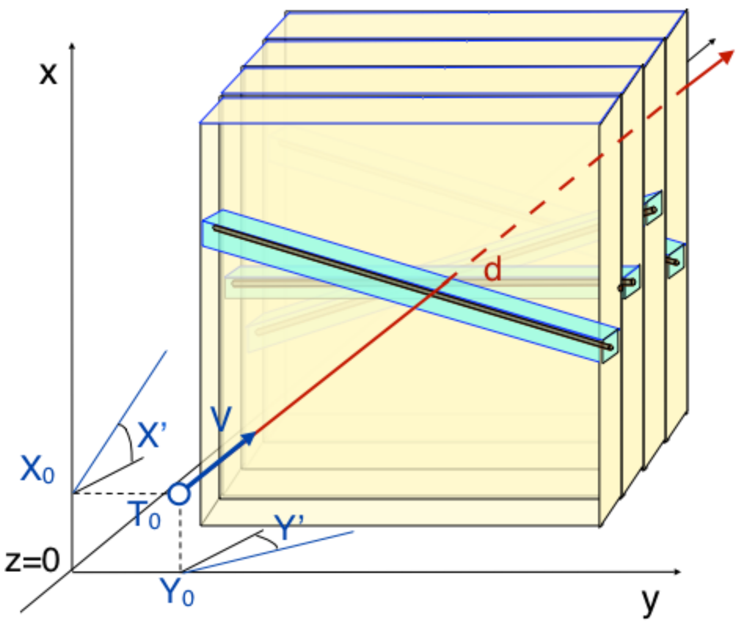


In a drift chamber, every channel gives 2 data:

1 coordinate (cell position)
1 time (t_0 + flight + drift + wire times) → 1 coordinate

The time as a tracking variable

Example 2: Drift chambers



In a drift chamber, every channel gives 2 data:

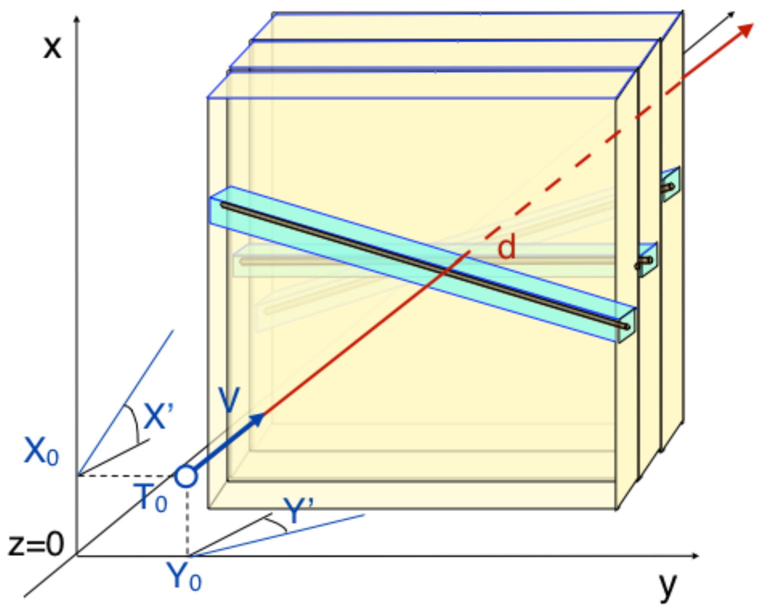
1 coordinate (cell position)
1 time (t_0 + flight + drift + wire times) → 1 coordinate

At least 4 cells (= 4 coordinates) are necessary for defining a 4-parameters track with NDF=0, (assuming an external reference time t_0 and the velocity are well known)

+ slope correction and others
+ ghosts

The time as a tracking variable

Example 2: Drift chambers with time as a separate variable



Using coordinates and time separately:

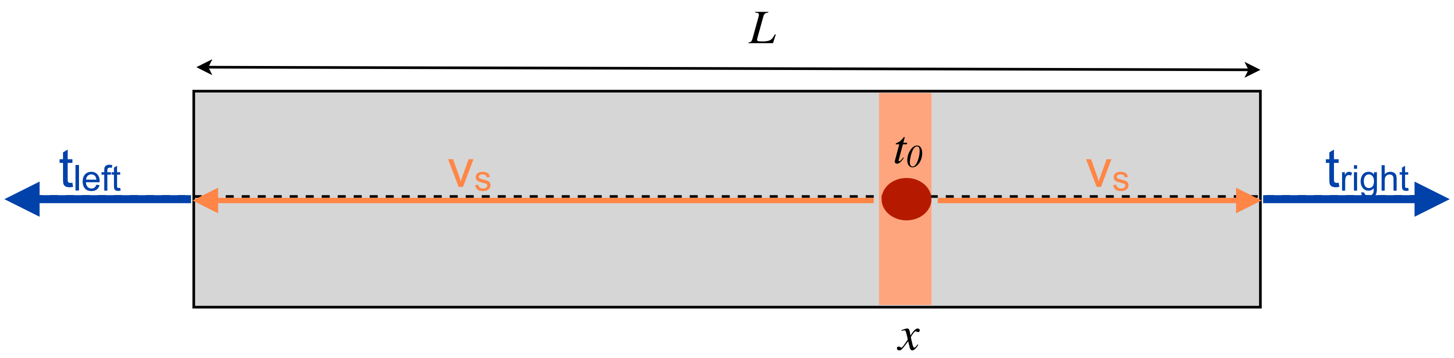
1 coordinate (cell positions)
1 times (t_0 + flights + drifts + wire times) → 2 data

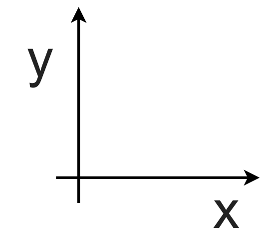
3 cells (= 6 parameter) are enough for defining a 6-parameter track (4 parameters + reference t_0 and velocity v) with NDF=0.

- Of course, the accuracy is very bad but constraints about t_0 and v can be introduced later.
- Separating coordinates and times gives more flexibility in hybrid set-up's, for introducing weigths....

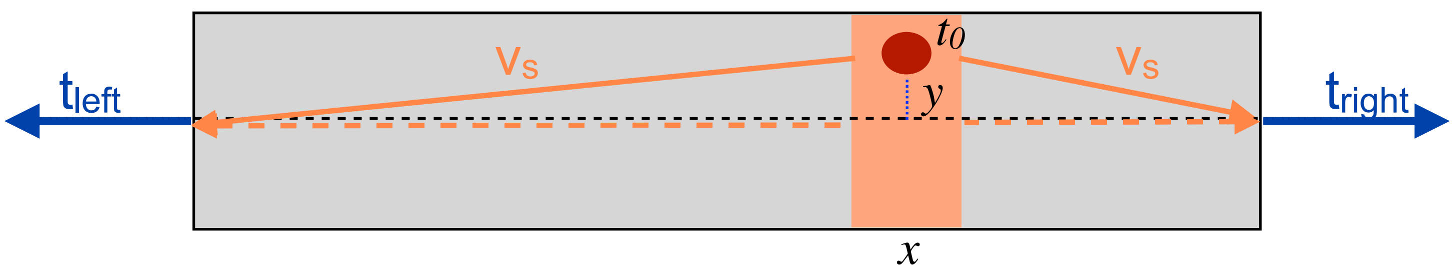
The time as a tracking variable

Example 3: Strip detectors





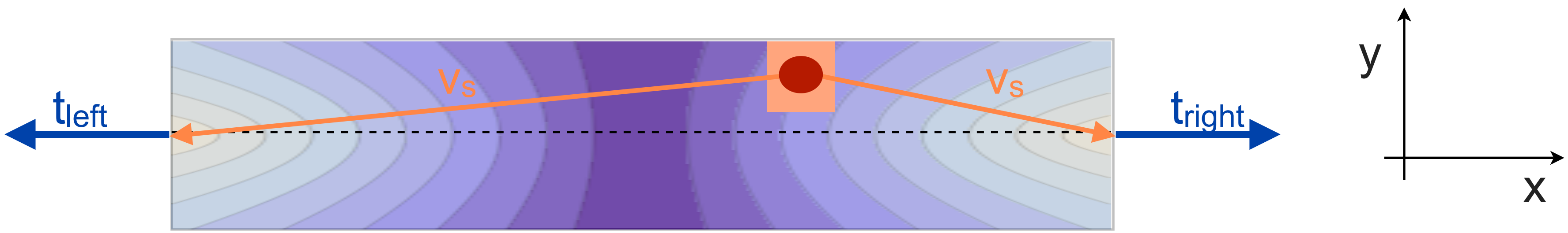
$$x = \frac{1}{2}[L + v_s(t_{left} - t_{right})]$$



t_{left} and t_{right} are a function of all t_0, x and y !

The time as a tracking variable

Example 3: Strip detectors



$$t_{left}(x,y) = t_0 + \sqrt{\frac{x^2+y^2}{v_s}} \qquad t_{right}(x,y) = t_0 + \sqrt{\frac{(L-x)^2+y^2}{v_s}}$$

Using the correct dependence of t_{left} and t_{right} on t_0, x and y avoids timing and position errors

The time as a tracking variable

Summary:

Using time as an independent variable has many advantages:

- More efficient
- More precise
- More powerful

Disadvantages:

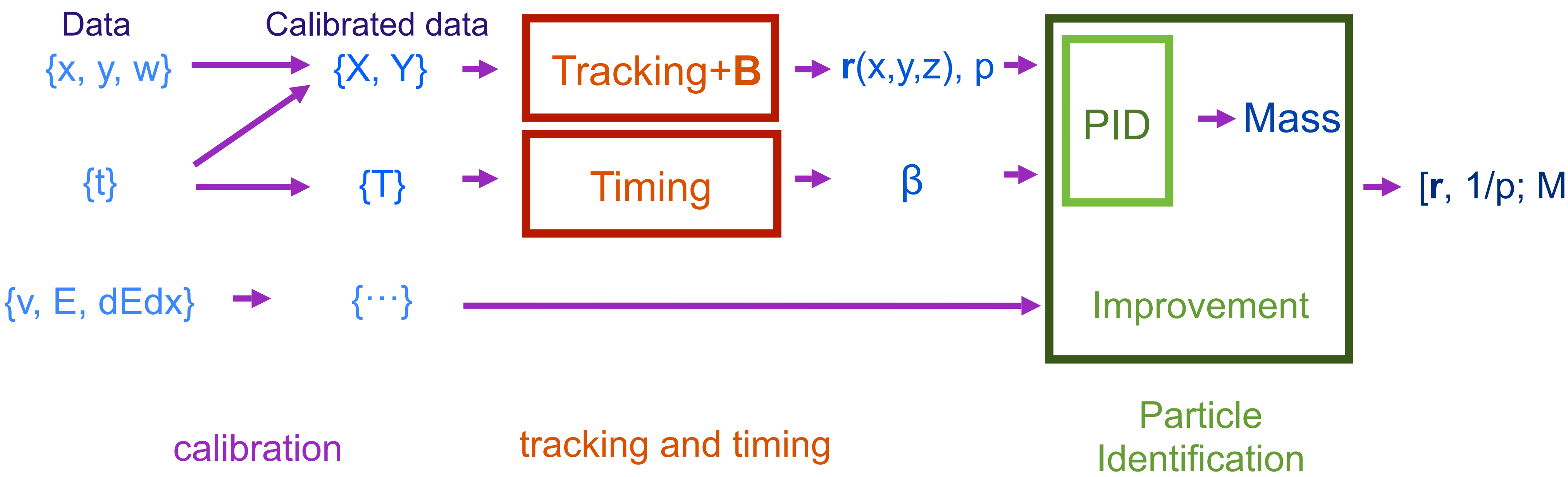
Working in bigger dimensions may be more difficult:

- Some danger of settle points in the hyperspace
- The curse of dimensionality: higher the space dimension, bigger the difficulty of arriving to the good solution region.

4D tracking: 3D + time

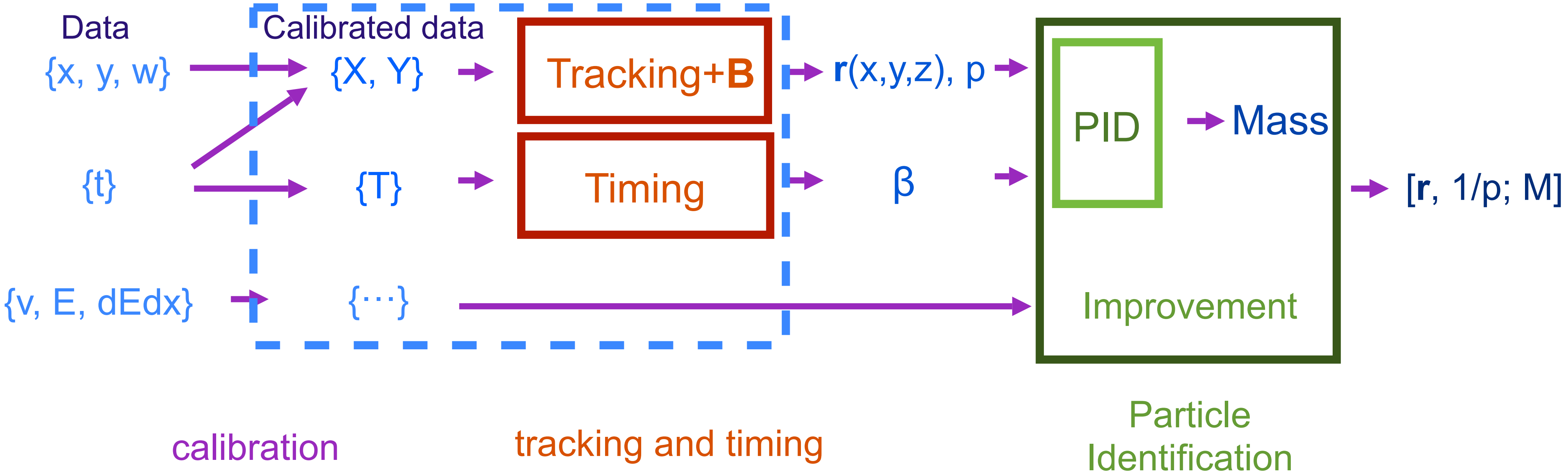
- The *timtrack* approach
- 4D Kalman Filter

timtrack as a 4D tracking example



A typical tracking flow

timtrack as a 4D tracking example



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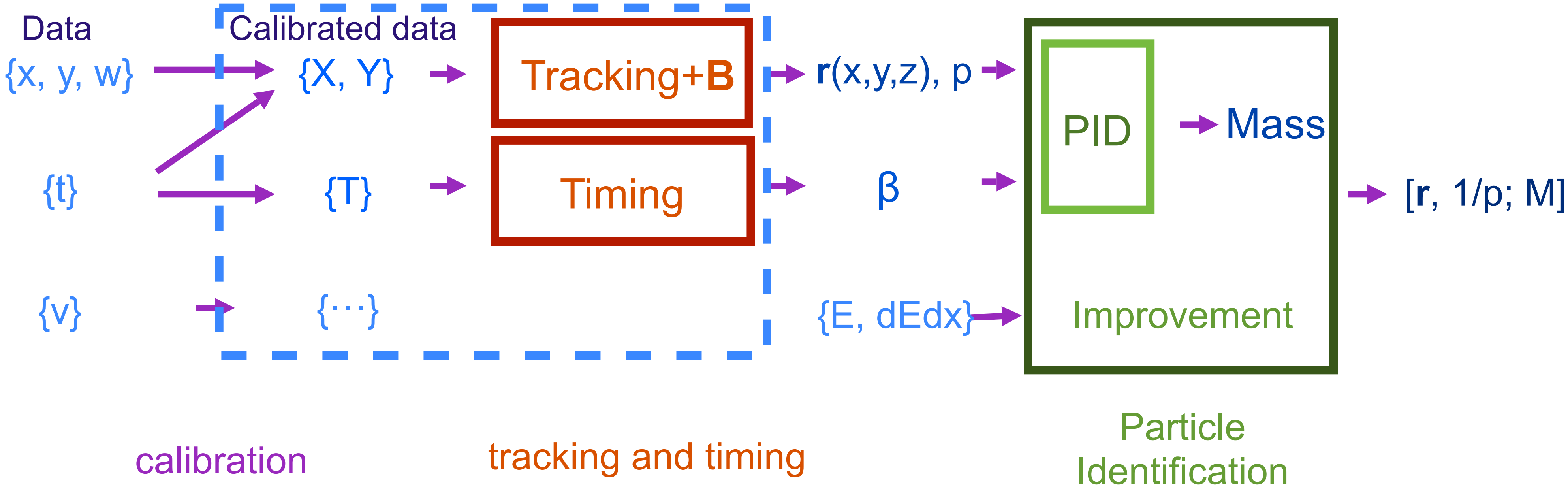
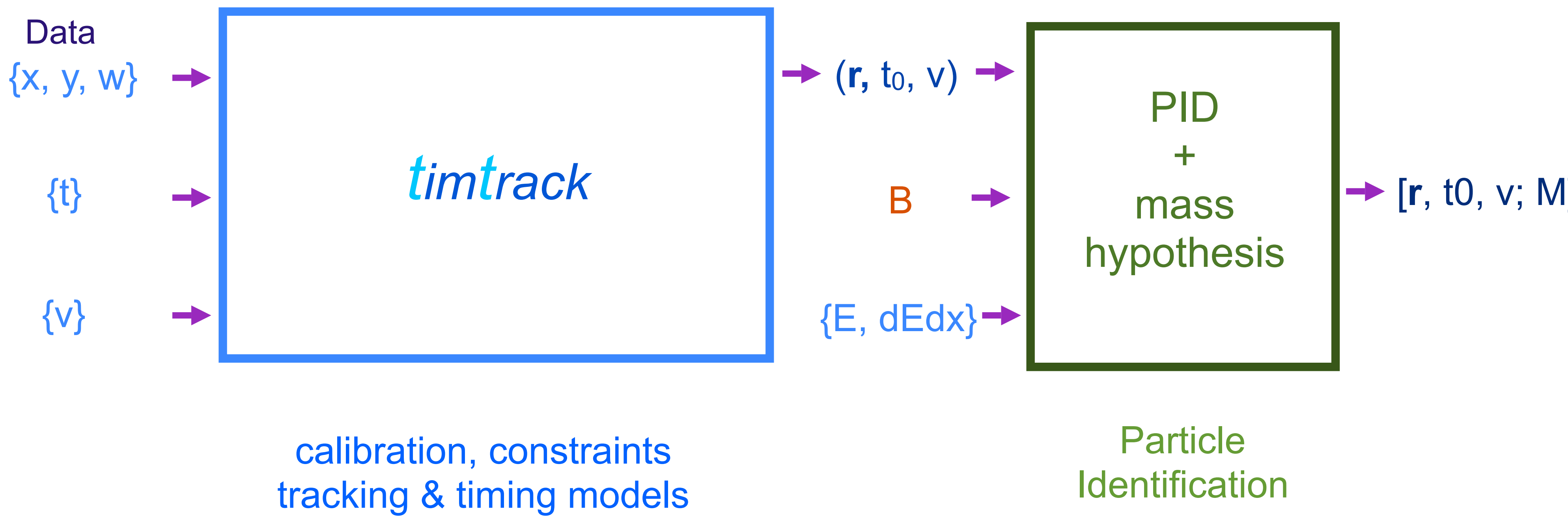


timtrack as a 4D tracking example



External t_0 and velocity v are let as free parameters

timtrack as a 4D tracking example

Timtrack basics:

- TimTrack is a Least Squares fitting Method using a general Matrix Formalism

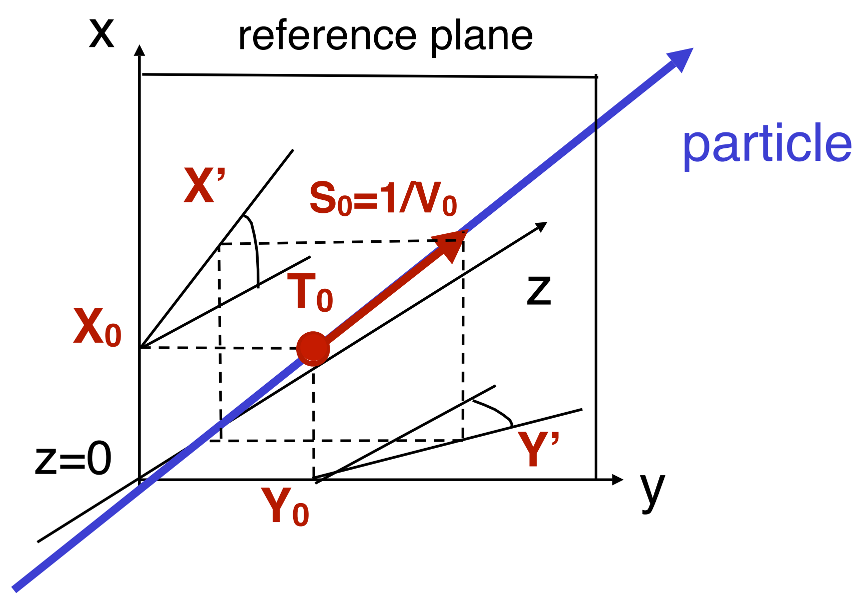
- TimTracks work with the primary data, without full reduction.

  Calibration parameters, or even alignment parameters, could be let free in the fit. Then, it provides a common formalism useful for tracking, calibration and alignment.

- TimTrack works always with the six parameters that define completely the movement of a particle:
  - (X,Y): 2 coordinates at a given plane
  - (X',Y'): 2 projected slopes
  - T: The time at the given plane
  - S (=1/V) or slowness: The inverse of the velocity at the given plane

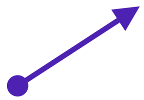
timtrack as a 4D tracking example

The saeta



- We call the set of these six parameters a SAETA*:

$$\mathbf{s} = (X_0, X', Y_0, Y', T_0, S)$$



- The SAETA can be extended introducing a mass hypothesis:

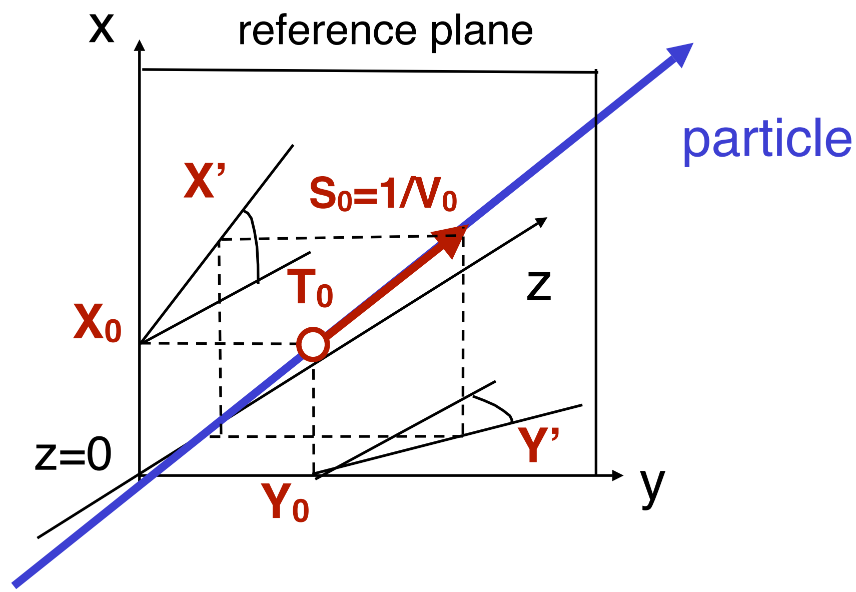
$$\mathbf{s}_m = (\mathbf{s};m)$$

- An extended SAETA contains everything necessary to describe all the possible effects acting on a particle: magnetic field, energy loss, multiple scattering...

*) SAETA: SmAllest sET of pArameters (≡ Sagitta = Arrow, in Spanish)

timtrack as a 4D tracking example

The saeta

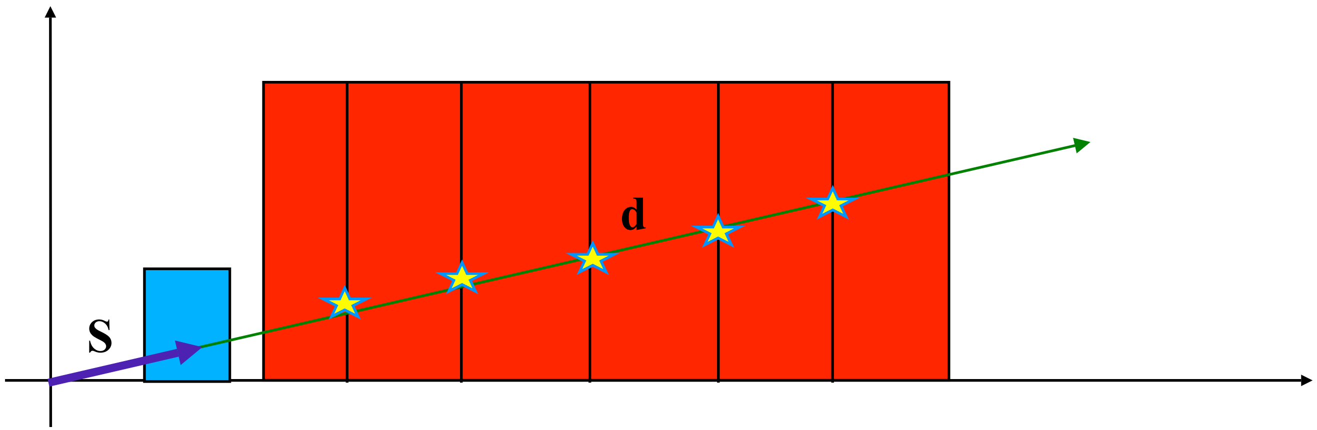


Why slowness S instead velocity, momentum or energy ?

- S has ~gaussian uncertainties
- S is a "natural" parameter: ToF detectors provide them directly
- S allows to introduce a very useful universal constraint: $S \geq 1/c$

timtrack as a 4D tracking example

Timtrack is a Least Squares Method:



Let suppose that we have:

- a set of n_m data: $\mathbf{d}$
- a set of n_s parameters: $\mathbf{s}$
- a model relating both sets: $\mathbf{d}=\mathbf{m}(\mathbf{s})$

The Least Squares Method (LSM) consists in finding the set of parameters $\mathbf{s}$ minimizing the function S:

$$S = \sum_i \left(\frac{d_i - m_i(\mathbf{s})}{\sigma_i} \right)^2$$

timtrack as a 4D tracking example

Timtrack: summary of steps for a lineal model: $\mathbf{m(s)}$

1. $$\boxed{S = [\mathbf{d} - \mathbf{m}(\mathbf{s})]' \cdot W \cdot [\mathbf{d} - \mathbf{m}(\mathbf{s})]}$$

2 $$\mathbf{m}(\mathbf{s}) = G \cdot \mathbf{s} + \mathbf{g}(\mathbf{s}) \qquad \mathbf{g}(\mathbf{s}) = \mathbf{m}(\mathbf{s}) - G \cdot \mathbf{s} = \mathbf{g}_0 \quad \text{(constant)}$$

3. $$K = G' \cdot W \cdot G$$
$$\mathbf{a} = G' \cdot W \cdot (\mathbf{d} - \mathbf{g}_0)$$
$$s_0 = (\mathbf{d} - \mathbf{g}_0)' \cdot W \cdot (\mathbf{d} - \mathbf{g}_0)$$
$$\longrightarrow \boxed{S = \mathbf{s}' \cdot K \cdot \mathbf{s} - 2\mathbf{s}' \cdot \mathbf{a} + s_0}$$

4. (Minimum condition)
$$\frac{\partial S}{\partial \mathbf{s}} = K \cdot \mathbf{s}_m - \mathbf{a} = \mathbf{0} \longrightarrow K \cdot \mathbf{s}_m = \mathbf{a} \quad \text{(Normal equations)}$$

$$\boxed{\mathbf{s}_m = K^{-1} \cdot \mathbf{a}} \quad \textit{(Solution)}$$

5. $$\mathcal{E} = \left(\frac{1}{2}\frac{\partial^2 S}{\partial \mathbf{s}^2}\right)^{-1} = K^{-1} \quad \text{(Variance-covariance matrix)}$$

timtrack as a 4D tracking example

Corrections for non-linear models - 1

When the model $\mathbf{m}(\mathbf{s})$ to be fitted is *non-linear*, the S function should be minimized using an iterative process:

- We look for an initial value of the set of parameters $\mathbf{s}_0$, that is near to the minimum of S.

- Then, we follow the same steps used with the linear model with the following differences:

Step 2.
We calculate $G(\mathbf{s})$ and then, we build: $\boxed{\mathbf{g}(\mathbf{s}) = \mathbf{m}(\mathbf{s}) - G(\mathbf{s}) \cdot \mathbf{s}}$

Step 3.
We calculate the matrix K and the vector $\mathbf{a}$ at $\mathbf{s} = \mathbf{s}_0$:

$$\boxed{\begin{aligned} K_0 &= G'(\mathbf{s}_0) \cdot W \cdot G(\mathbf{s}_0) \\ \mathbf{a}_0 &= G'(\mathbf{s}_0) \cdot W \cdot (\mathbf{d} - \mathbf{g}(\mathbf{s}_0)) \end{aligned}}$$

timtrack as a 4D tracking example

Corrections for non-linear models - 2

Step 4.

We estimate a new set of parameters, $\mathbf{s}_1$, using the equation:

$$\mathbf{s}_1 = K_0^{-1} \cdot \mathbf{a}_0$$

We repeat this step iteratively until the convergence is reached:

$$\boxed{\mathbf{s}_i = K_{i-1}^{-1} \cdot \mathbf{a}_{i-1}}$$

Corrections for non-linear model and with constraints in the parameters - 1

Let consider the case where the set parameters $\mathbf{s}$ do fulfill:

$$\boxed{\mathbf{f}(\mathbf{s}) = 0 \qquad (n_c \text{ equations})}$$

- Now, we minimize iteratively the Lagrange function defined as:

$$\boxed{L(\mathbf{s}) = (\mathbf{d} - \mathbf{m}(\mathbf{s}))' \cdot W \cdot (\mathbf{d} - \mathbf{m}(\mathbf{s})) + 2 \cdot \lambda' \cdot \mathbf{f}(\mathbf{s})}$$

where λ is a vector of n_c elements: the Lagrange multipliers.

- We introduce the jacobian matrix of the constraint functions:

$$\boxed{R(\mathbf{s}) = \partial_{\mathbf{s}} \mathbf{f}(\mathbf{s})}$$

timtrack as a 4D tracking example

Corrections for non-linear model and with constraints in the parameters - 2

- Starting from a point, close enough to the minimum:

$$\mathbf{s} = \mathbf{s}_0,$$
$$R_0 = R(\mathbf{s}_0)$$
$$\mathbf{f}_0 = \mathbf{f}(\mathbf{s}_0)$$

The matrix equation:

$$\boxed{\begin{pmatrix} \delta\mathbf{s}_1 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} K_0 & R_0' \\ R_0 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \mathbf{a}_0(\mathbf{s}_0) - K_0 \cdot \mathbf{s}_0 \\ -\mathbf{f}_0 \end{pmatrix}}$$

provides the next-step $\mathbf{s}$ and λ vectors:

$$\boxed{\begin{array}{l} \mathbf{s}_1 = \mathbf{s}_0 + \delta\mathbf{s}_1 \\ \lambda_1 \end{array}}$$

- The iteration is repeated until convergence and $\lambda_i = 0$.

timtrack as a 4D tracking example

Corrections for non-linear model and with constraints in the parameters - 3

Uncertainty analysis:

- We build:

$$\boxed{\begin{pmatrix} K & R' \\ R & 0 \end{pmatrix}^{-1} = \begin{pmatrix} H & Q' \\ Q & Z \end{pmatrix}}$$

-At the minimum, the *Error Matrix* is:

$$\boxed{\mathcal{E} = K^{-1}\left(I - R'Q\right)}$$

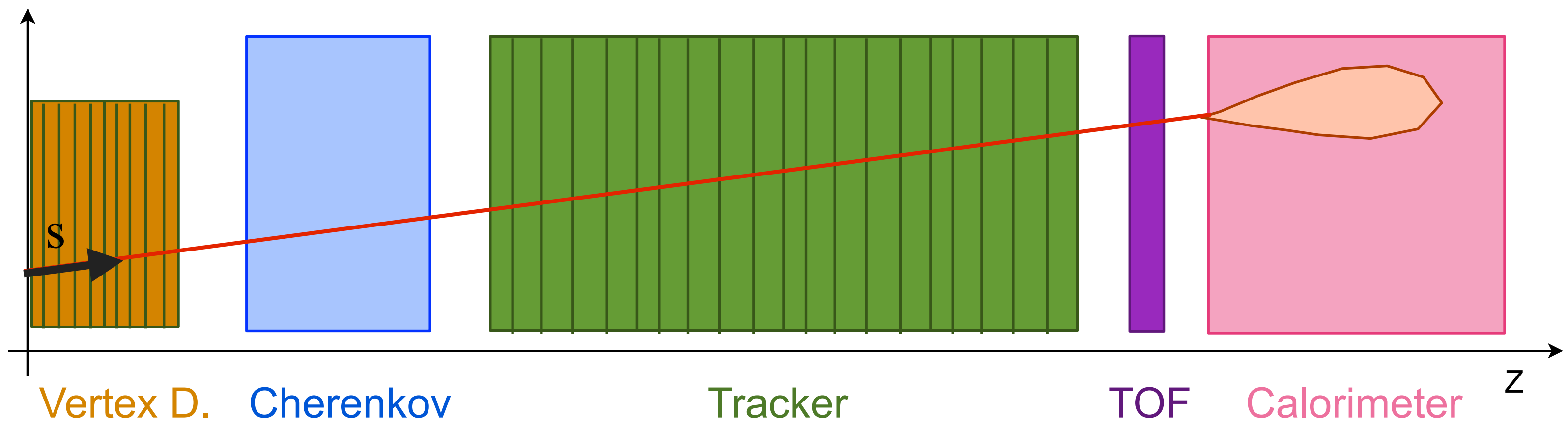
K has dimensions: $n_s \times n_s$
R' has dimensions: $n_s \times n_c$
Q' has dimensions: $n_c \times n_s$

-The number of degrees of freedom of the fit is: $\boxed{n_f = n_m - n_s + n_c}$

timtrack as a 4D tracking example

Hybrid set-ups:

If we have a set of different detectors (different models):



models:	$\mathbf{m}_1$	$\mathbf{m}_2$	$\mathbf{m}_3$	$\mathbf{m}_4$	$\mathbf{m}_5$
data (f.ex):	(x,y)	(S=1/V)	(x,y,w)	(t)	(E)
K matrices:	K_1	K_2	K_3	K_4	K_5
a vectors:	$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$	$\mathbf{a}_4$	$\mathbf{a}_5$

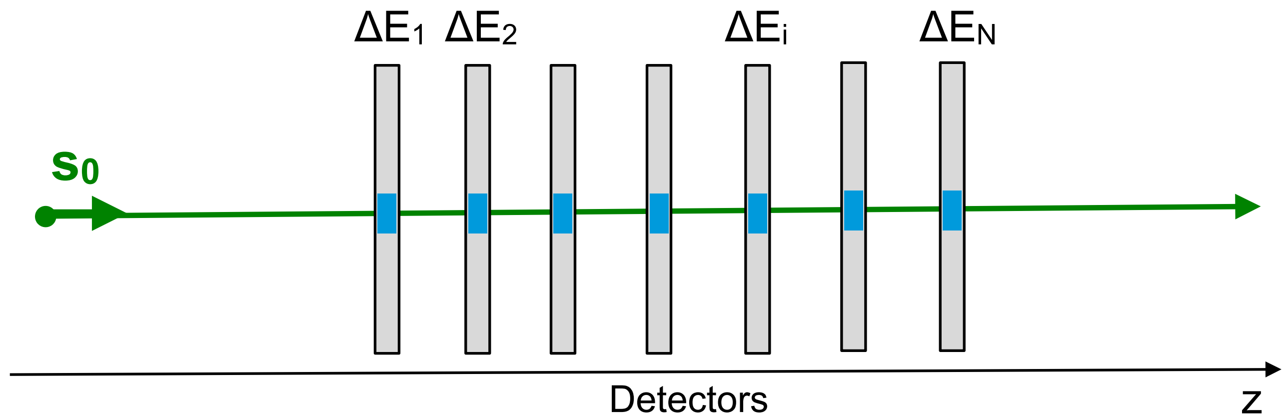
and, finally:

$$K = \sum K_i$$

$$\mathbf{a} = \sum \mathbf{a}_i$$

$$\rightarrow \boxed{\mathbf{s} = K^{-1}\mathbf{a}}$$

Other physical corrections: Energy loss (Bethe Bloch) - 1



Energy losses are related to the velocity (β) of a particle through the Bethe-Bloch formula:

$$-\frac{dE}{dx} \simeq k \cdot \frac{1}{\beta^2}\left(\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right)$$

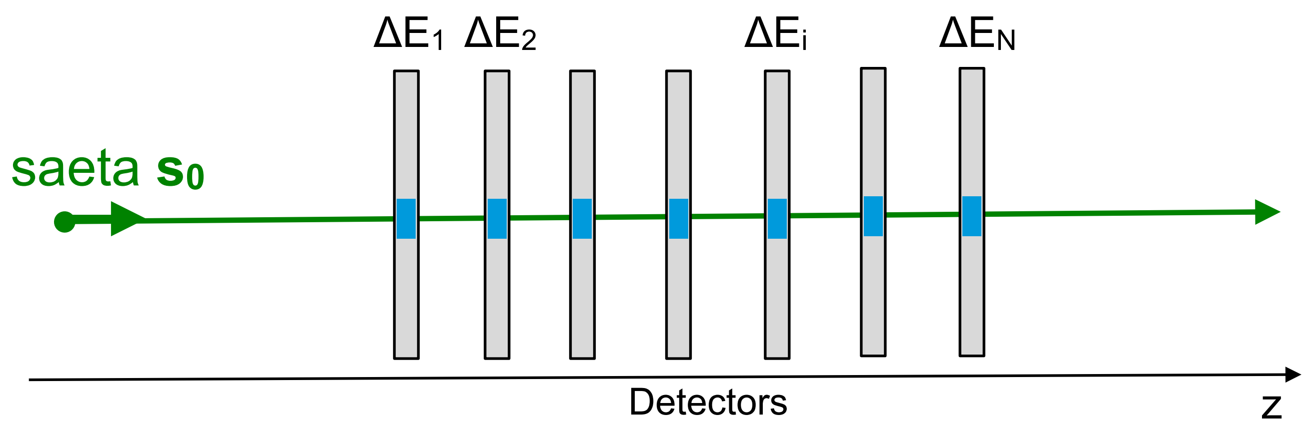
that can be written in a simplified form as:

$$\boxed{-\frac{dE}{dx} \simeq k \cdot \frac{1}{\beta^2}\left(\frac{1}{2}\ln\frac{1}{I_c^2}\frac{\beta^2}{(1-\beta^2)} - 1\right)}$$

with:

$$I_c = \frac{I}{2m_e c^2}$$

Other physical corrections: Energy loss (Bethe Bloch) - 2



The Bethe- Bloch formula can be written as funcion of the slowness S (=1/V) as:

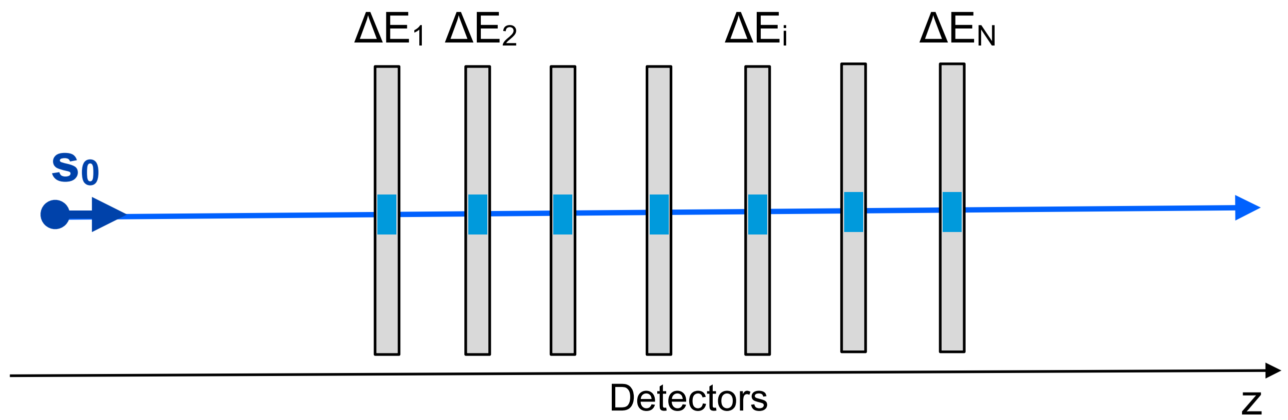
$$-\frac{dE}{dx} \simeq k \cdot S^2 \left(\frac{1}{2} \ln \frac{2 m_e c^2 T_{max}}{(S^2-1)^2 I^2} - \frac{1}{S^2} - \frac{\delta(S)}{2} \right)$$

that can be approximated and simplified as:

$$\boxed{-\frac{dE}{dx} \simeq k \left(S^2 \ln \frac{1}{(S^2-1) I_c} - 1 \right)}$$

timtrack as a 4D tracking example

Other physical corrections: Energy loss (Bethe Bloch) - 3



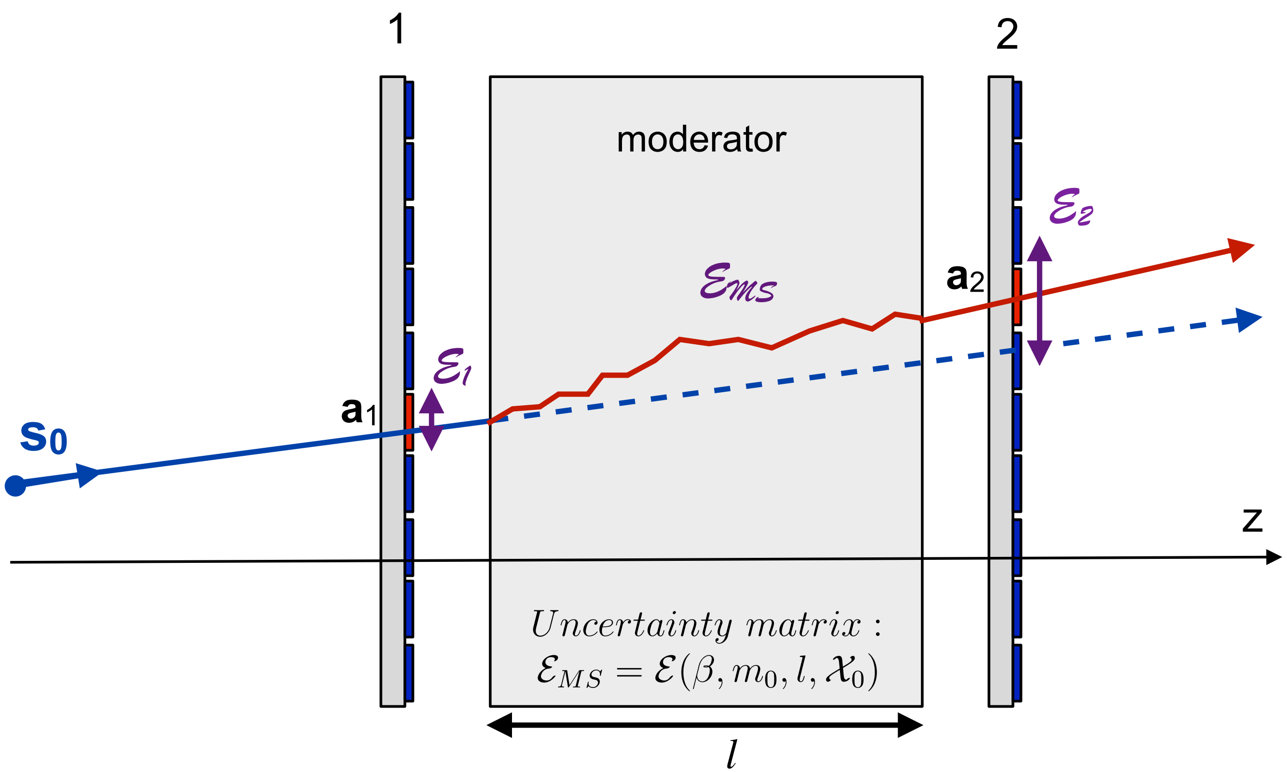
Observe that the equation:

$$\boxed{-\frac{dE}{dx} \simeq k\left(S^2 \ln \frac{1}{(S^2-1)I_c} - 1\right)}$$

relates the energy loss measurements to the unknown parameter S and, as consequence, those measurements (either separately for each detector or promediated to all the detectors) can be included as a new model in the formalism.

timtrack as a 4D tracking example

Other physical corrections: Multiple scattering



K matrices: K_1 $K_{MS} = \mathcal{E}_{MS}^{-1}$ $K_2 = \mathcal{E}_2^{-1}$

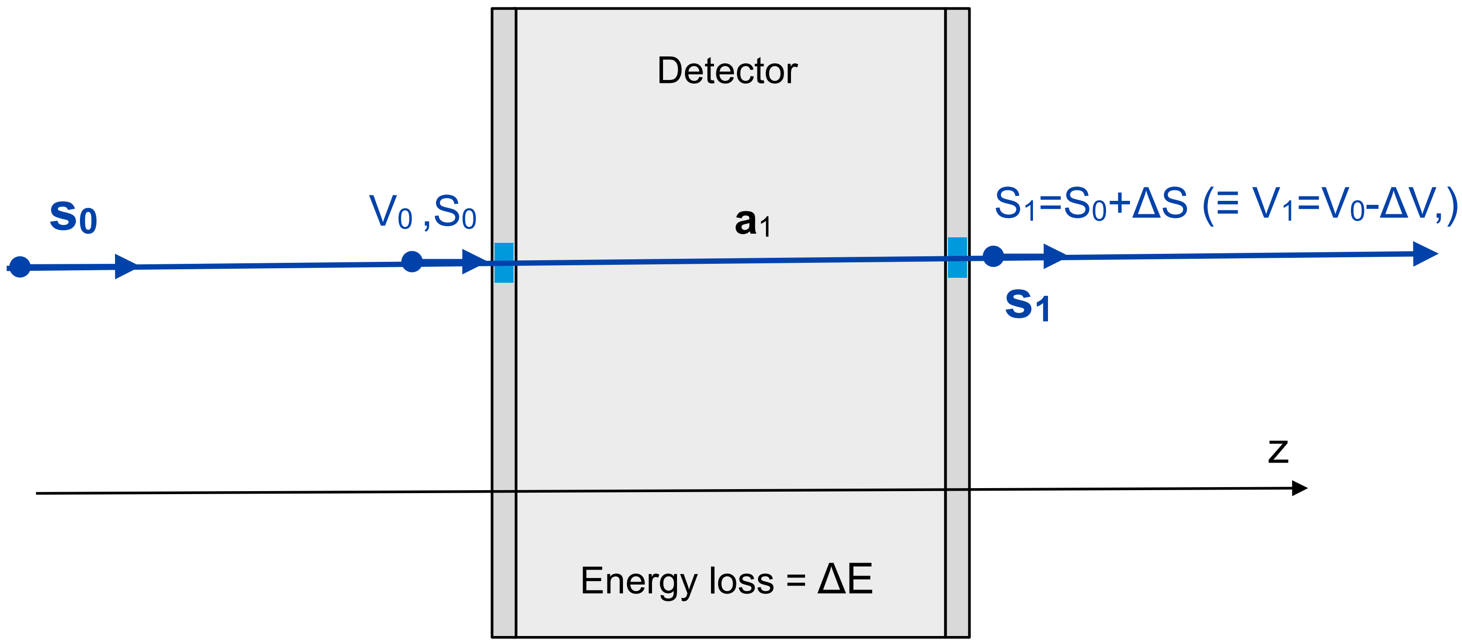
Following the TT formalism, we expect:

- $\mathbf{a}_1 = K_1\, \mathbf{s}_0$
- $\mathbf{a}_2 = K_{MS,2}\, \mathbf{s}_0$ with $K_{MS,2} = (\mathcal{E}_{MS} + \mathcal{E}_2)^{-1}$

$\Rightarrow$ $\boxed{\mathbf{s} = K_1^{-1}\, \mathbf{a}_1 + K_{MS,2}^{-1}\, \mathbf{a}_2}$

timtrack as a 4D tracking example

Other physical corrections: Relationship between energy loss, slowness and mass



The slowness S (=1/V) increases with the loss of energy, ΔE, according to:

$$\boxed{\Delta S = -\frac{(S^2-1)^{\frac{3}{2}}}{m_0}\,\Delta E} \qquad \left(\ \Delta\beta = \frac{(1-\beta^2)^{\frac{3}{2}}}{\beta\ m_0}\,\Delta E\ \right)$$

If ΔE has been well measured, the increasing in the slowness S can be introduced as a constraint to improve the fit.

TimTrack as a calibration tool -1:

The model of the data, $\mathbf{m}$, can be expanded as a function of both, the set of parameters **s** and the set of some alignment and calibration constants **α**:

$$\mathbf{m} = \mathbf{m}(\mathbf{s};\boldsymbol{\alpha})$$

and

$$\mathbf{m}(\mathbf{s};\boldsymbol{\alpha}) = \left(\frac{\partial \mathbf{m}}{\partial \mathbf{s}}\right)\mathbf{s} + \left(\frac{\partial \mathbf{m}}{\partial \boldsymbol{\alpha}}\right)\boldsymbol{\alpha} + \mathbf{g}(\mathbf{s};\boldsymbol{\alpha})$$

$$\mathbf{m}(\mathbf{s},\boldsymbol{\alpha}) = \left(\frac{\partial \mathbf{m}}{\partial \mathbf{s}} \;\; \frac{\partial \mathbf{m}}{\partial \boldsymbol{\alpha}}\right)\begin{pmatrix} \mathbf{s} \\ \boldsymbol{\alpha} \end{pmatrix} + \mathbf{g}(\mathbf{s};\boldsymbol{\alpha})$$

$$\mathbf{m}(\mathbf{s};\boldsymbol{\alpha}) = G_A \cdot \mathbf{s}_\alpha + \mathbf{g}(\mathbf{s};\boldsymbol{\alpha})$$

where: $G_A = \begin{pmatrix} \dfrac{\partial \mathbf{m}}{\partial \mathbf{s}} & \dfrac{\partial \mathbf{m}}{\partial \boldsymbol{\alpha}} \end{pmatrix}$ and $\mathbf{s}_\alpha = \begin{pmatrix} \mathbf{s} \\ \boldsymbol{\alpha} \end{pmatrix}$

timtrack as a 4D tracking example

TimTrack as a calibration tool -2:

The matrix G_A takes the form:

$$G_A = \begin{pmatrix} \frac{\partial m_1}{\partial s_1} & \cdots & \frac{\partial m_1}{\partial s_i} & \cdots & \frac{\partial m_1}{\partial s_n} & \frac{\partial m_1}{\partial \alpha_1} & \cdots & \frac{\partial m_1}{\partial \alpha_j} & \cdots & \frac{\partial m_1}{\partial \alpha_m} \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ \frac{\partial m_d}{\partial s_1} & \cdots & \frac{\partial m_d}{\partial s_i} & \cdots & \frac{\partial m_d}{\partial s_n} & \frac{\partial m_d}{\partial \alpha_1} & \cdots & \frac{\partial m_d}{\partial \alpha_j} & \cdots & \frac{\partial m_d}{\partial \alpha_m} \end{pmatrix}$$

timtrack as a 4D tracking example

TimTrack as a calibration tool -3:

Particles with well known parameters may be used to determine the unknown calibration constants.

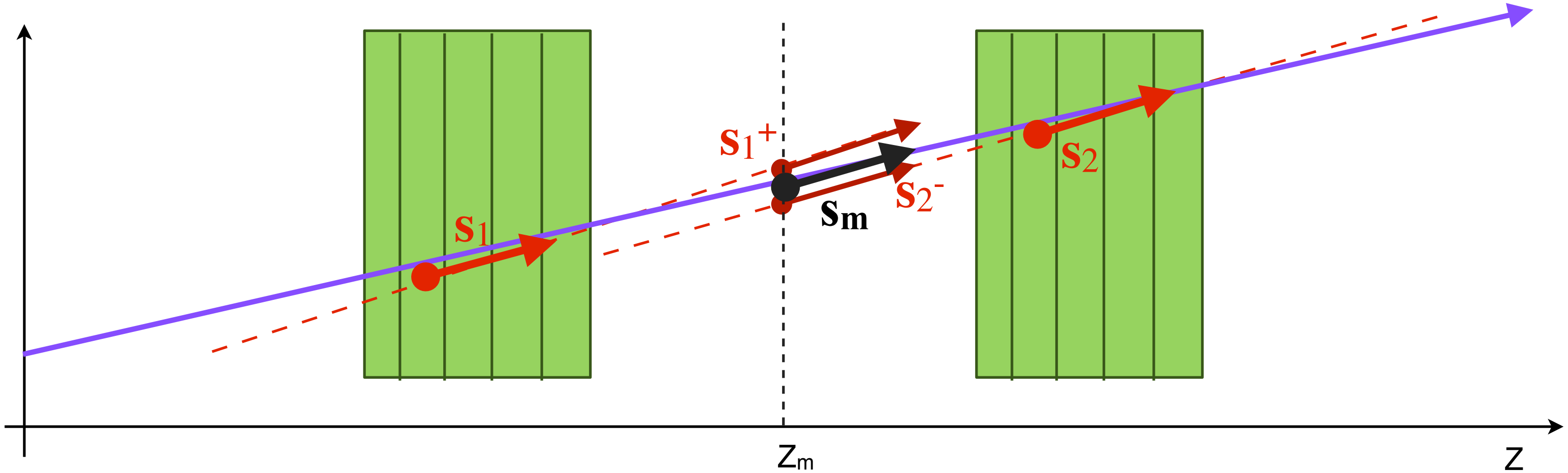
$$G_A = \begin{pmatrix} \frac{\partial m_1}{\partial s_1} & \cdots & \frac{\partial m_1}{\partial s_i} & \cdots & \frac{\partial m_1}{\partial s_n} & \frac{\partial m_1}{\partial \alpha_1} & \cdots & \frac{\partial m_1}{\partial \alpha_j} & \cdots & \frac{\partial m_1}{\partial \alpha_m} \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ \frac{\partial m_d}{\partial s_1} & \cdots & \frac{\partial m_d}{\partial s_i} & \cdots & \frac{\partial m_d}{\partial s_n} & \frac{\partial m_d}{\partial \alpha_1} & \cdots & \frac{\partial m_d}{\partial \alpha_j} & \cdots & \frac{\partial m_d}{\partial \alpha_m} \end{pmatrix}$$

The columns corresponding both to the known parameters and to the known calibration constants can be eliminated from G_A before calculating the K matrix and finding the solution:

$$\mathbf{s}_\alpha = \begin{pmatrix} \mathbf{s} \\ \boldsymbol{\alpha} \end{pmatrix}$$

timtrack as a 4D tracking example

Algebra of saetas



- Compatibility (Mahalanobis distance)

$$(\mathbf{s}_1, \mathcal{E}_1) \rightarrow (\mathbf{s}_1^+ = F\cdot\mathbf{s}_1,\ \mathcal{E}_1^+ = F'\cdot\mathcal{E}_1\cdot F)$$
$$(\mathbf{s}_2, \mathcal{E}_2) \rightarrow (\mathbf{s}_2^- = B\cdot\mathbf{s}_2,\ \mathcal{E}_2^- = B'\cdot\mathcal{E}_2\cdot B)$$

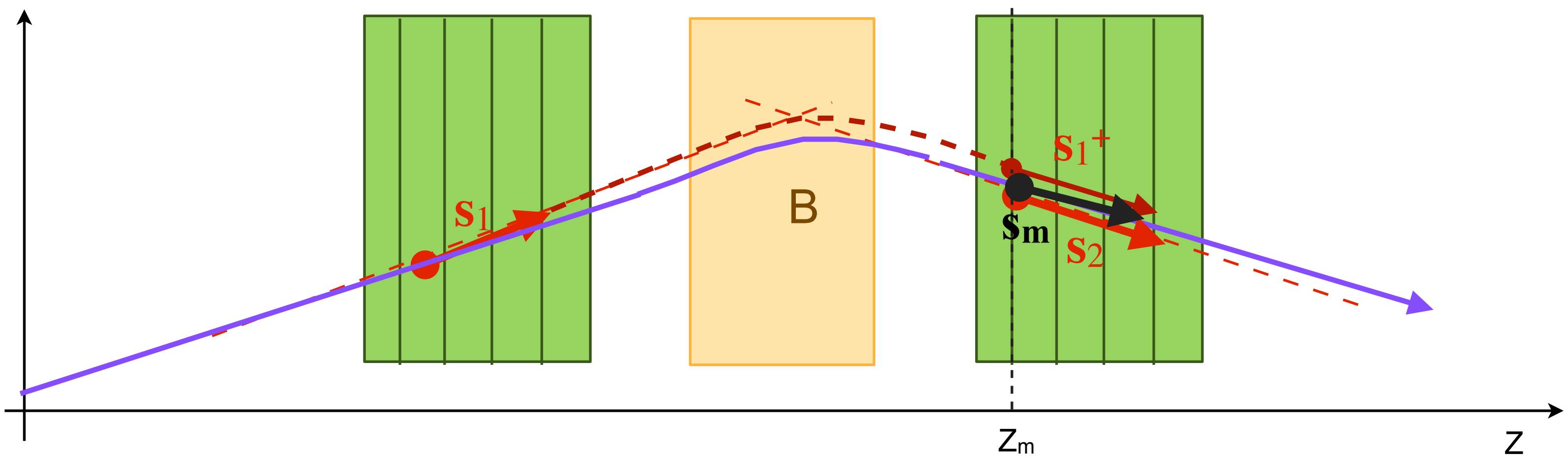
$$d_M(\mathbf{s}_1^+,\mathbf{s}_2^-) = \sqrt{(\mathbf{s}_1^+ - \mathbf{s}_2^-)'\cdot(\mathcal{E}_1^+ + \mathcal{E}_2^-)\cdot(\mathbf{s}_1^+ - \mathbf{s}_2^-)}$$

- Reduction of saetas:

$$\mathbf{s_m} = [(\mathcal{E}_1^+)^{-1} + (\mathcal{E}_2^-)^{-1}]^{-1}\cdot[(\mathcal{E}_1^+)^{-1}\cdot\mathbf{s}_1^+ + (\mathcal{E}_2^-)^{-1}\cdot\mathbf{s}_2^-]$$

timtrack as a 4D tracking example

Algebra of saetas



- Compatibility (Mahalanobis distance)

$$(\mathbf{s}_1, \mathcal{E}_1) \rightarrow (\mathbf{s}_1^+ = F\cdot\mathbf{s}_1,\ \mathcal{E}_1^+ = F'\cdot\mathcal{E}_1\cdot F)$$
$$(\mathbf{s}_2, \mathcal{E}_2) \rightarrow (\mathbf{s}_2^- = B\cdot\mathbf{s}_2,\ \mathcal{E}_2^- = B'\cdot\mathcal{E}_2\cdot B)$$

$$\longrightarrow \quad d_M(\mathbf{s}_1^+,\mathbf{s}_2^-) = \sqrt{(\mathbf{s}_1^+-\mathbf{s}_2^-)'\cdot(\mathcal{E}_1^+ + \mathcal{E}_2^-)\cdot(\mathbf{s}_1^+-\mathbf{s}_2^-)}$$

- Reduction of saetas:

$$\mathbf{s}_m = [(\mathcal{E}_1^+)^{-1} + (\mathcal{E}_2^-)^{-1}]^{-1}\cdot[(\mathcal{E}_1^+)^{-1}\cdot\mathbf{s}_1^+ + (\mathcal{E}_2^-)^{-1}\cdot\mathbf{s}_2^-]$$

timtrack as a 4D tracking example

Tim Track flow summary

Parameter space	Linear TT	Measurement space
		$\mathbf{d}$, $W_d = V_d^{-1}$
	G: Measurement Matrix $\mathbf{m}(\mathbf{s}) = G \cdot \mathbf{s}_p + \mathbf{g}_0$	$\mathbf{d}_c = \mathbf{d} - \mathbf{g}_0$
	$\mathbf{a} = G' \cdot W_d \cdot \mathbf{d}_c$ $K = G' \cdot W_d \cdot G$	
$\mathbf{s} = K^{-1} \cdot \mathbf{a}$		

Parameter space	Non linear TT	Measurement space
$\mathbf{s}_p$		$\mathbf{d}$, $W_d = V_d^{-1}$
	G: Measurement Matrix $\mathbf{m}(\mathbf{s}_p) = G \cdot \mathbf{s}_p + \mathbf{g}(\mathbf{s}_p)$	$\mathbf{d}_c = \mathbf{d} - \mathbf{g}(\mathbf{s}_p)$
	$\mathbf{a}_p = G(\mathbf{s}_p)' \cdot W_d \cdot \mathbf{d}_c$ $K_p = G(\mathbf{s}_p)' \cdot W_d \cdot G(\mathbf{s}_p)$	
$\mathbf{s}_{p+1} = K_p^{-1} \cdot \mathbf{a}_p$		

timtrack as a 4D tracking example

Summary:

TimTrack provides a framework for the reconstruction of particles in hybrid environments, taking into account all possible physical effects.

- It works always in the highest dimension parameter space of the particle
- It allows to include any constraint affecting the movement of the particle
- It works with particle parameters and calibration or alignment constants at the same level

Disadvantages:

- Still not tested in many environments
  - It has been tested with the HADES MDrift Chambers + RPC wall with comparable results with the existing ones (although using part of the existing tools : drif velocity parametrization...)
- Recursive and transporting approach not yet tested (≅ Kalman Filter)
- Working in higher dimension space may include new convergence problems

timtrack as a 4D tracking example

One comment:

Using time as a free parameter needs, together with the alignment, a new very important task in any spectrometer or detector setup:

SYNCHRONIZATION

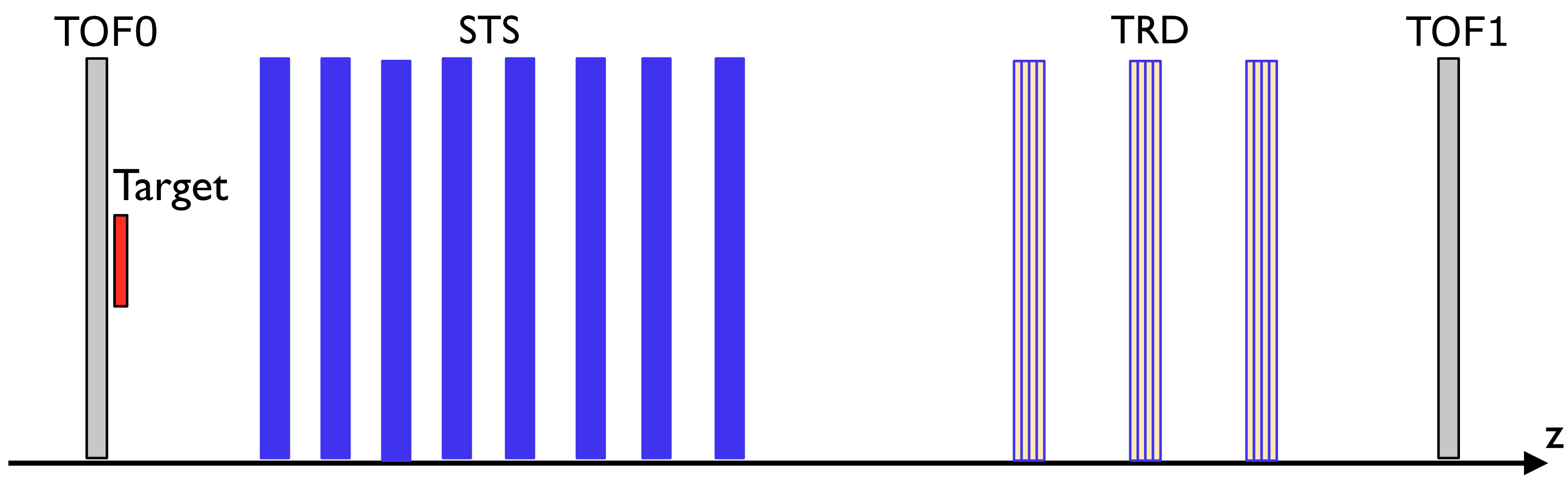
4D Tracking at CBM

4D Tracking at CBM

timtrack at CBM

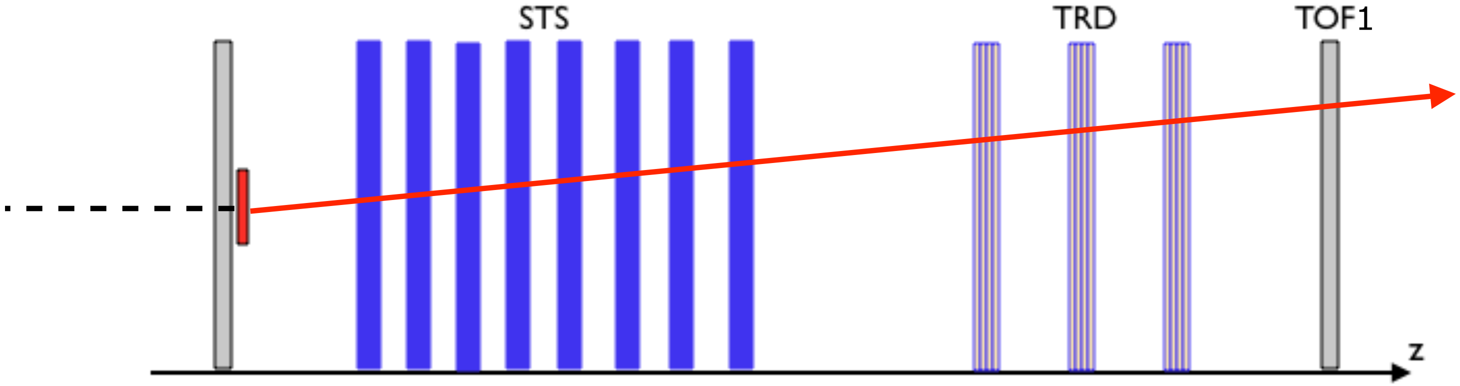
Example: D_0 decay analysis at CBM (GSI)

Layout



z(cm)	0	30 35 40 50 60 75 95 100	600 700 800	1000
σ_x	1cm	~17μm	0.3mm	1cm
σ_y	1cm	~17μm	2.5mm	1cm
σ_t	80ps	$3 \cdot 10^6$ps	2000ps	80ps

Example: D_0 decay analysis at CBM (GSI)



Single Track

-Track fitting model:

$$x_i = X_0 + X' \cdot z_i$$
$$y_i = Y_0 + Y' \cdot z_i$$
$$t_i = T_0 + S \cdot \sqrt{1 + X'^2 + Y'^2} \cdot z_i$$

$$S = \frac{1}{V}$$

SAETA: **s**=(X_0, X', Y_0, Y', T_0, S=1/V) (6 parameters)

Single Track: "reduced" Error Matrix

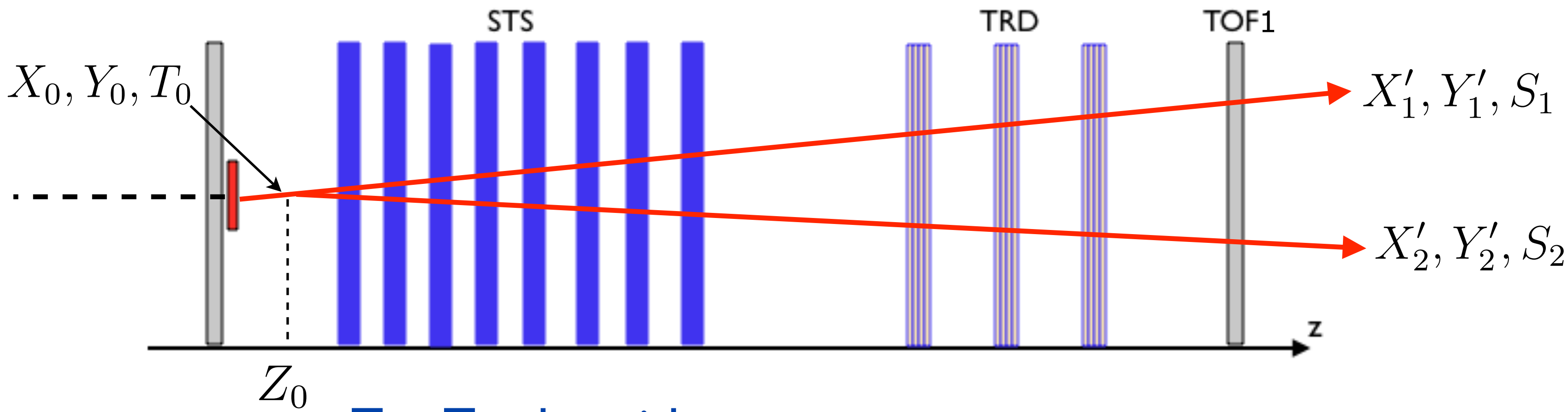
σ's are in the diagonal

X0	X'	Y0	Y'	T0	S=1/V
8.0311e-003	-5.8000e-001	0	0	0	0
	3.3457e-006	0	0	0	0
		8.0558e-003	-5.8000e-001	0	0
			3.4401e-006	0	0
				8.6771e+001	-4.1000e-001
					5.2507e-002

corr coefs.

(Only TOF1)

Example: D_0 decay analysis at CBM (GSI)



Two Tracks with common vertex:

-Track fitting model:

$$x_i^1 = X_0 + X'_1 \cdot z_i$$
$$y_i^1 = Y_0 + Y'_1 \cdot z_i$$
$$t_i^1 = T_0 + S_1 \cdot \sqrt{1 + X_1'^2 + Y_1'^2} \cdot z_i$$
$$x_i^2 = X_0 + X'_2 \cdot z_i$$
$$y_i^2 = Y_0 + Y'_2 \cdot z_i$$
$$t_i^2 = T_0 + S_2 \cdot \sqrt{1 + X_2'^2 + Y_2'^2} \cdot z_i$$

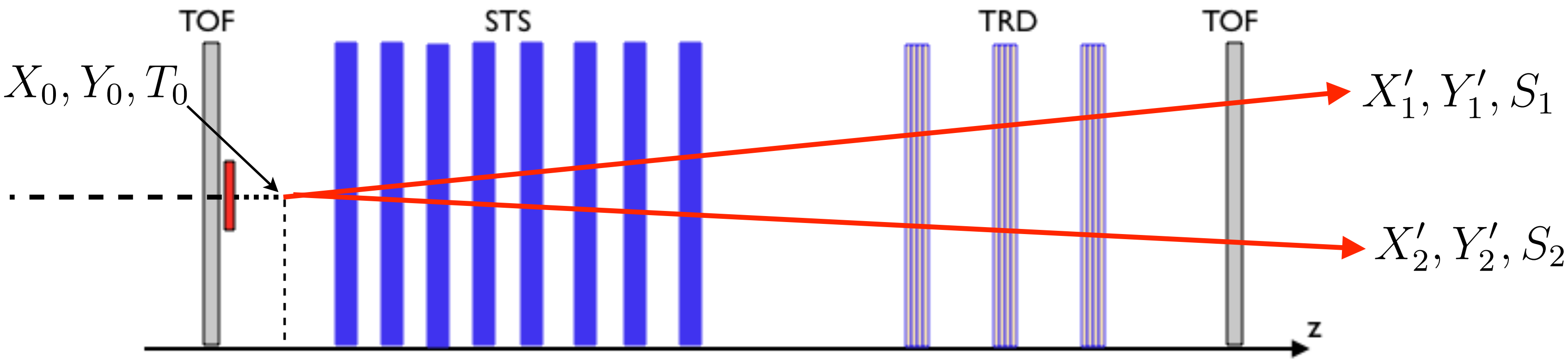
multi-SAETA: **s** = (X_0, X_1', X_2', Y_0, Y_1', Y_2', S_1, S_2, T_0, Z_0) (10 parameters)

TwoTracks with common vertex: "Reduced" error matrix

X_0	X'_1	X'_2	Y_0	Y'_1	Y'_2	S_1	S_2	T_0	Z_0
5.6793e-003	-4.5000e-001	-4.5000e-001	0	0	0	0	0	0	1.0000e-002
	3.0595e-006	2.0000e-001	1.0000e-002	-3.0000e-002	3.0000e-002	0	0	0	-8.0000e-002
		3.0595e-006	-1.0000e-002	3.0000e-002	-3.0000e-002	0	0	0	8.0000e-002
			5.7078e-003	-4.4000e-001	-3.9000e-001	0	0	0	-6.0000e-002
				3.4300e-006	1.0000e-002	0	0	0	4.1000e-001
					3.4300e-006	0	0	0	-4.1000e-001
						7.0578e-002	9.0000e-002	-3.0000e-001	0
							6.8720e-002	-3.0000e-001	0
								6.1356e+001	0
									1.1150e-002

Numerical estimation for D0's with K=2GeV at polar angle 0 and decay with azimuthal angle= 45°

Example: D_0 decay analysis at CBM (GSI)



TwoTracks with common vertex and mass constraint

-Track fitting model = Two Tracks with common vertex
- Mass constraint (**f**(**s**)=0):

$$m(D_0) - \sqrt{(m_K^2 + m_\pi^2 + m_K \cdot m_{pi} \cdot \gamma_K \cdot \gamma_{pi}(1 + \beta_K \cdot \beta_\pi \cdot cos(\alpha))} = 0$$

multi-SAETA: **s** = (X_0, X_1', X_2', Y_0, Y_1', Y_2', S_1, S_2, T_0, Z_0; m1,m2) (10 parameters)

TwoTracks with D0 mass constraint: “Reduced” error matrix

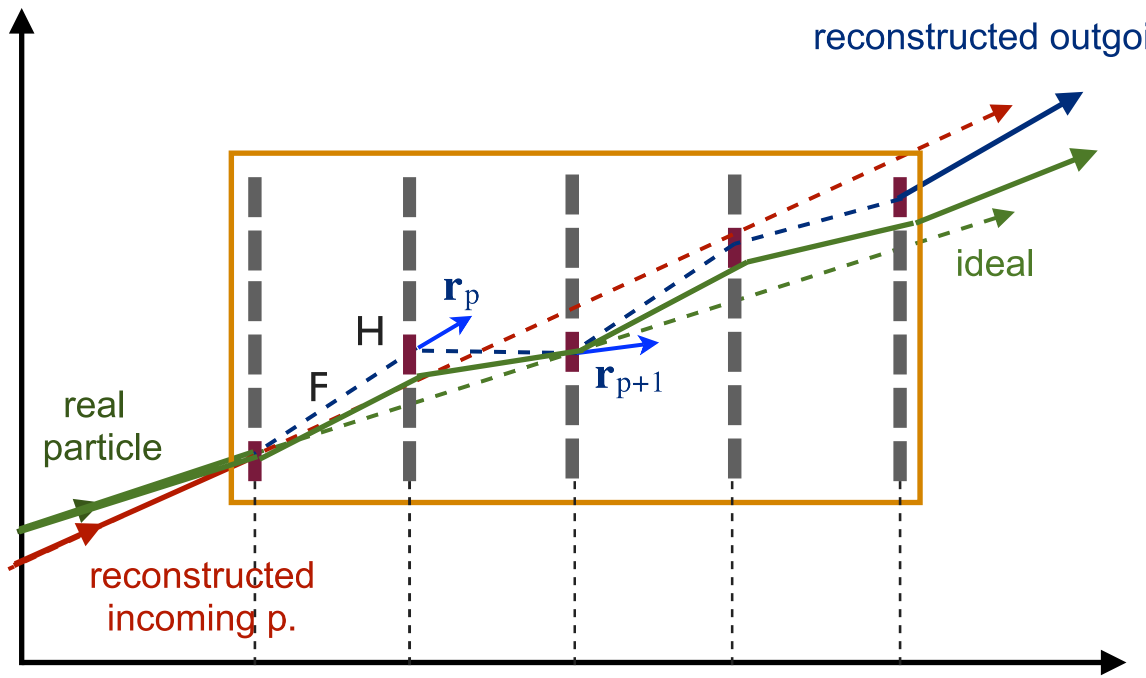
X_0	X'_1	X'_2	Y_0	Y'_1	Y'_2	S_1	S_2	T_0	Z_0
5.6793e-003	-4.5000e-001	-4.5000e-001	0	0	0	0	0	0	1.0000e-002
	3.0595e-006	2.0000e-001	1.0000e-002	-3.0000e-002	3.0000e-002	0	0	0	-8.0000e-002
		3.0595e-006	-1.0000e-002	3.0000e-002	-3.0000e-002	0	0	0	8.0000e-002
			5.7078e-003	-4.4000e-001	-3.9000e-001	0	0	0	-6.0000e-002
				3.4300e-006	1.0000e-002	0	0	0	4.1000e-001
					3.4300e-006	0	0	0	-4.1000e-001
						7.0578e-002	9.0000e-002	-3.0000e-001	0
							6.8720e-002	-3.0000e-001	0
								6.1356e+001	0
									1.1150e-002

4D Kalman Filter at CBM

4D Kalman Filter

4D Kalman Filter at CBM

Kalman Filter is a very well known and broadly used track finding and track reconstruction method



4D Kalman Filter at CBM

Kalman Filter algorithm flow

Kalman Filter		
Parameter space		Measurement space
$\mathbf{r}_p$, $\mathcal{E}_p$		
F: Transport		
$\mathbf{r} = F \cdot \mathbf{r}_p$ $\mathcal{E}_s = F \cdot \mathcal{E}_p \cdot F'$		$\mathbf{d}_p$, V_p
	H: Measurement $\mathbf{m}(\mathbf{r}) = H \cdot \mathbf{r} + \boldsymbol{\eta}$ $\rightarrow$	$\mathbf{d}_s = H \cdot \mathbf{r}$ $V_s = H' \cdot \mathcal{E}_s \cdot H$
		$\boldsymbol{\delta}\mathbf{d} = \mathbf{d}_p - \mathbf{d}_s$ $V_c = V_p + V_s$
$\delta\mathbf{r} = K \cdot \delta\mathbf{d}$ $\delta\mathcal{E}_s = K \cdot H \cdot \mathcal{E}_s$	$K = \mathcal{E}_s \cdot H' \cdot V_c^{-1}$ $\leftarrow$	
$\mathbf{r}_{p+1} = \mathbf{r} + \delta\mathbf{r}$ $\mathcal{E}_{p+1} = \mathcal{E}_s - \delta\mathcal{E}_s$		

4D Kalman Filter at CBM

We are implementing the ideas developed for the TimTrak in the CBM Kalman Filter environment

Status:

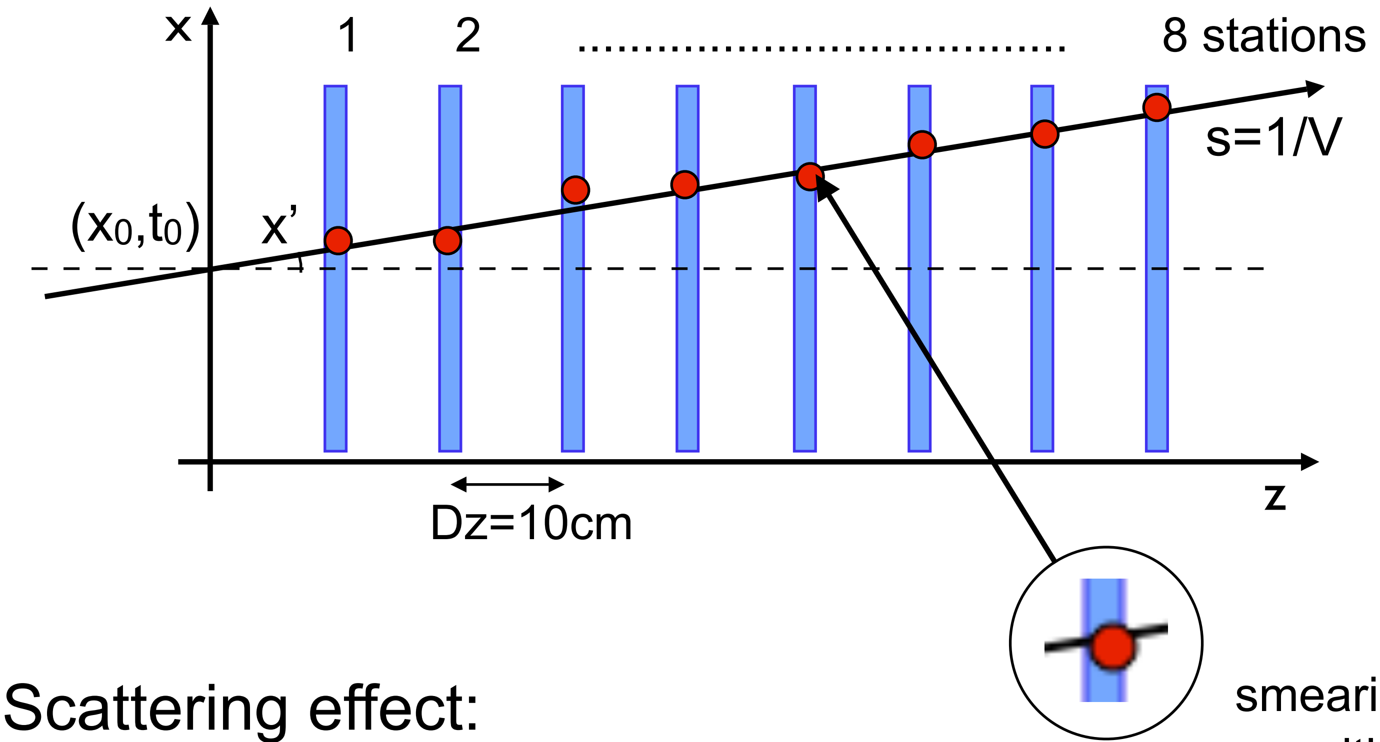
- Simple straight lines 4D examples are working in:

  - Mathematica: good environment for calculating measurement matrices
  - Matlab: good environment for developing
  - C++ CbmRoot

- Analisis of performances has been started
- No significative results yet

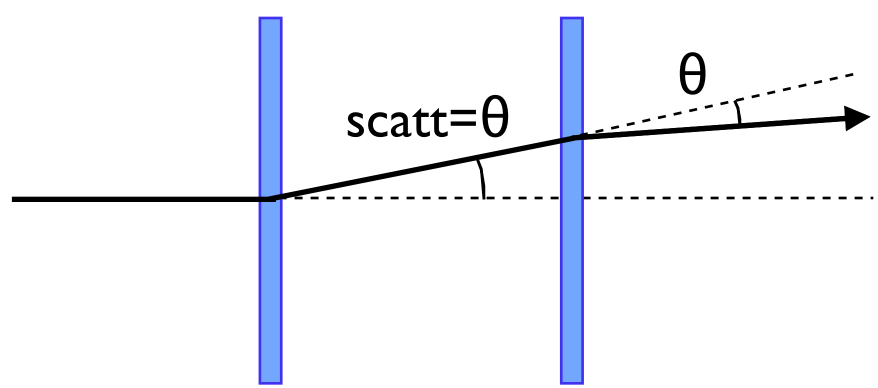
2D+T Kalman Filter at CBM

Detector layout example:



smearing in
- position: σ_x
- time: σ_t

M.Scattering effect:



4D tracking

2D+T Kalman Filter at CBM

Work done:

- A program for a straigth line fitting in 2-dimensions (x,z) has been adapted to include time and slowness s (s= 1/velocity): 3-dimensions (x,t,z).
- We have generated pure samples of 10000 tracks each, both in 2-dimension and 3-dimension modes, for several combinations of the initial parameters:

Fixed parameters:

$x_0 = 0$: x coordinate at z=0
$t_0 = 0$: time at z=0
$\beta = 1$: velocity of the particle
$\sigma_x = 0.1$ mm: gaussian smearing in the position readout at all the planes

Variable parameters:

$x' = 0, 0.1, 0.5$: initial slopes
σ_t = 1ns, 2ns

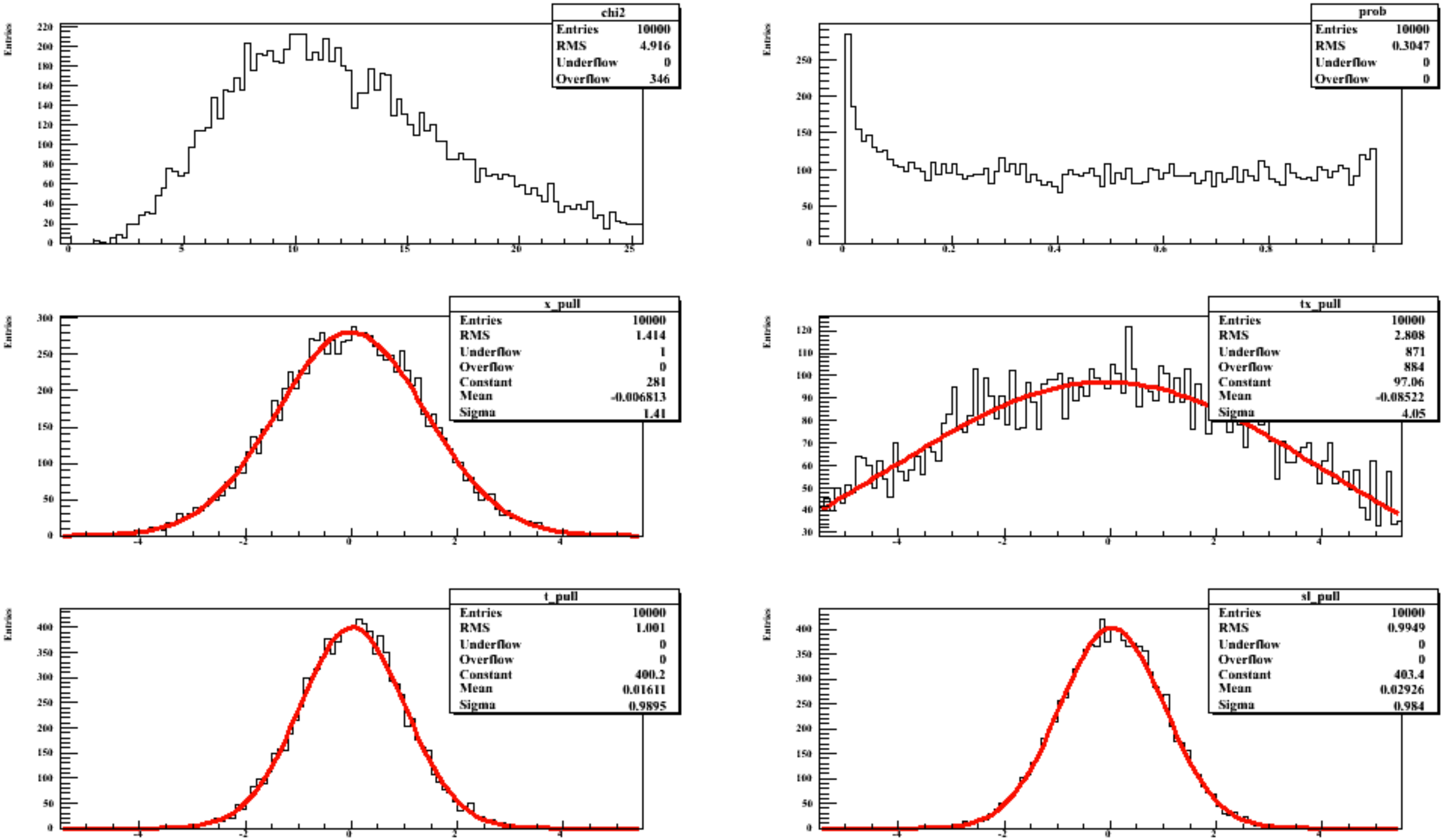
For every combination of parameters, tracks have been propagated with several multiscattering effects given by a gaussian distribution with the width:

scatt = $\tan\theta$ (being θ, the scattering angle) = 0.0, 0.01, 0.05, 0.1 (4 values)

2D+T Kalman Filter at CBM

8 STS stations, DZ = 10cm / σ_x = 0.1 mm , β= 1						
x' = 0.0	2D: x-x' fit		3D: x-x'-t_0-s fit / σ_t = 2 ns			
scatt	Pull(x)	Pull(x')	Pull(x)	Pull(x')	Pull(t)	Pull(s)
0.0	0,99	0,99	1,00	1,00	0,99	0,98
0.01	0,98	0,98	0,99	1,00	0,99	0,98
0.05	0,98	0,99	0,99	1,01	0,99	0,98
0.1	0,99	1,04	0,98	1,05	0,99	0,98
scatt	Res(x)	Res(x')	Res(x)	Res(x')	Res(t)	Res(s)
0.0	0,007	0,010	0,007	0,010	1,270	0,030
0.01	0,010	0,011	0,010	0,011	1,280	0,030
0.05	0,010	0,051	0,010	0,051	1,280	0,030
0.1	0,010	0,107	0,010	0,110	1,280	0,030

2D+T Kalman Filter at CBM



Summary of results for: scatt=0, σ_x = 0.1mm, σ_t = 1ns, x'=0.5 and β=1

Beyond 4D tracking

Beyond 4D

The amplitude (or the charge) of the read-out pulses may also de related with the tracking parameter. As consequence they may be used also as a tracking variables.

Other effects (slewing corrections, walk corrections) are also related either with the time or the position

Why not introduce them also all this effects in the tracking model (without reduction)?

Let see some examples:

Beyond 4D

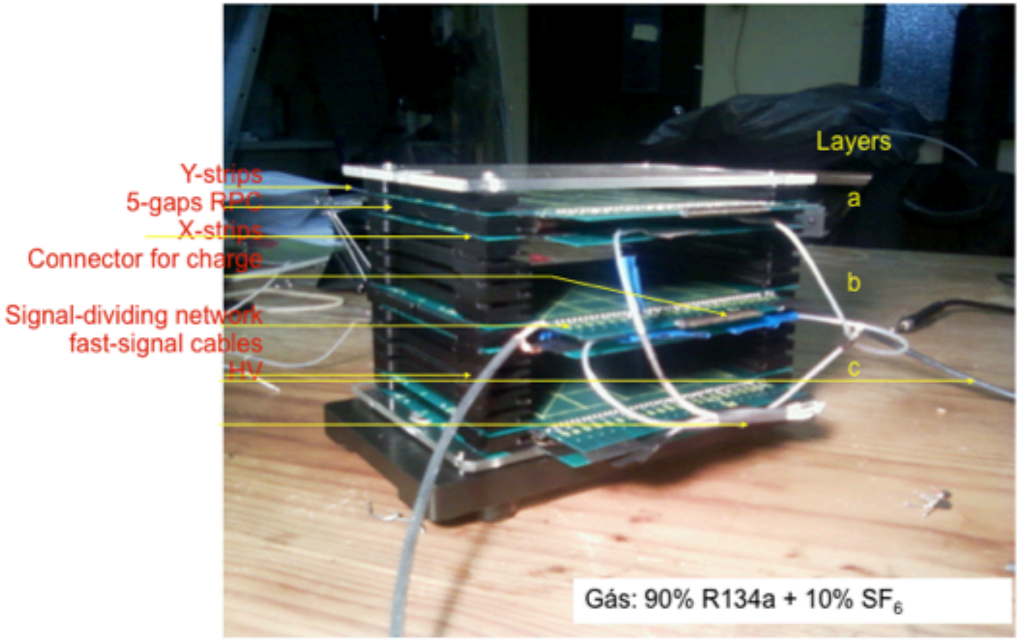
Example 1:

TOFtracker: combination of time-of-flight and high-accuracy bidimensional tracking in a single gaseous detector

A.Blanco[1], P.Fonte[1,2], L.Lopes[1], P. Martins[1], M. Palka[3], J. Michel[3], M. Kajetanowicz[4], G. Korcyl[5], M.Traxler[6], R. Ferreira Marques[1,7]

RPC2012
INFN Istituto Nazionale di Fisica Nucleare
XI Workshop on Resistive Plate Chambers and Related Detectors

- strip electrodes, analog charge readout and charge interpolation for position
- separate the high-frequency parts of the signal for timing and the low-frequency ones for position

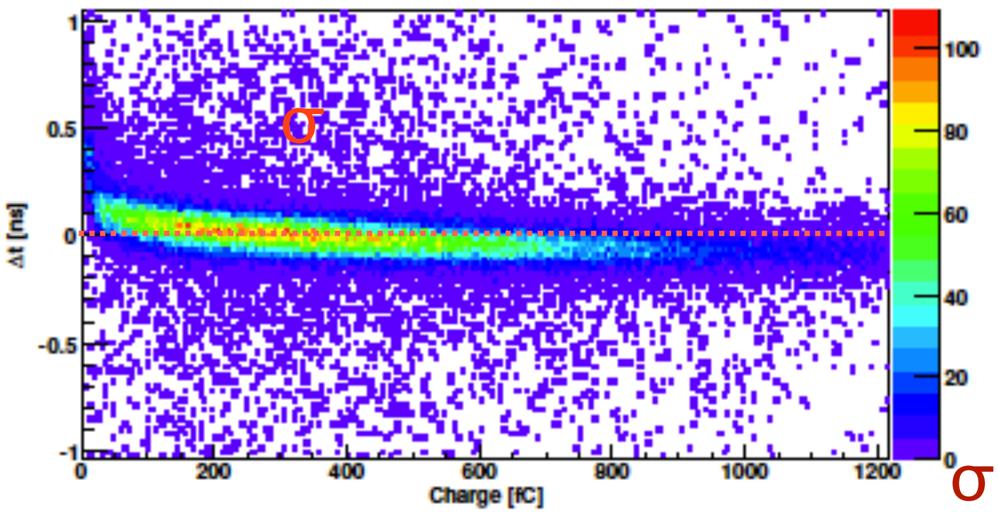


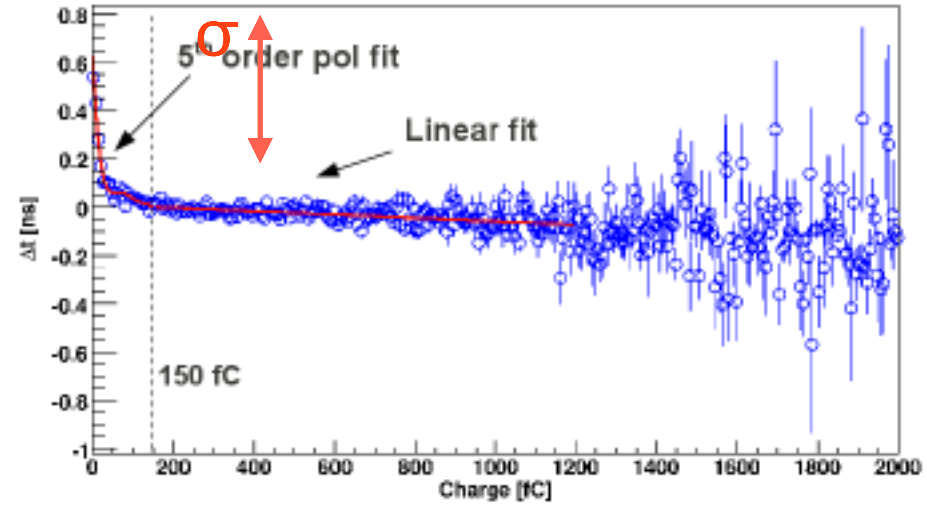
Position resolution below 50 μm can be achieved in multigap RPCs by analog charge readout (high accuracy digital readout is limited to single-gap) of 4 mm pitch strips.

Beyond 4D

Example 2:

Implementation of slewing or walk corrections in the tracking model





Slewing correction in HADES tRPC TOF wall

$$\Delta t = t_{particle} - t_{RPC} = \varepsilon \approx f(Q)$$

The slewing correction function f(Q) can be introduced as a new equation in the fit

$$t_{particle} = t_{RPC} + f(Q) \quad (\pm\,\sigma)$$

Beyond 4D

Example 3:

Working out the position resolution on large scintillating detectors, through the "light attenuation method"

A. Huerta, R. Guerrero, Q. Curiel, J. Huelgas S., P. Rodríguez, F. Favela, D. Marín, M.E. Ortiz, L. Barrón, and E. Chávez
Instituto de Física, Universidad Nacional Autónoma de México,
México D.F. 04510.

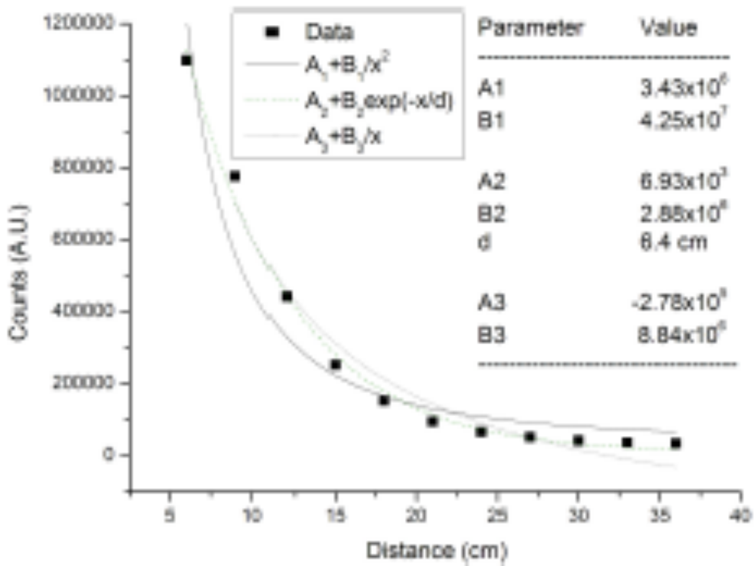


FIGURE 5. For the large plate configuration, this plot shows the integral of each spectrum as a function of the distance between the ^{60}Co source to the photomultiplier. The two fits correspond to a pure exponential and an inverse square law.

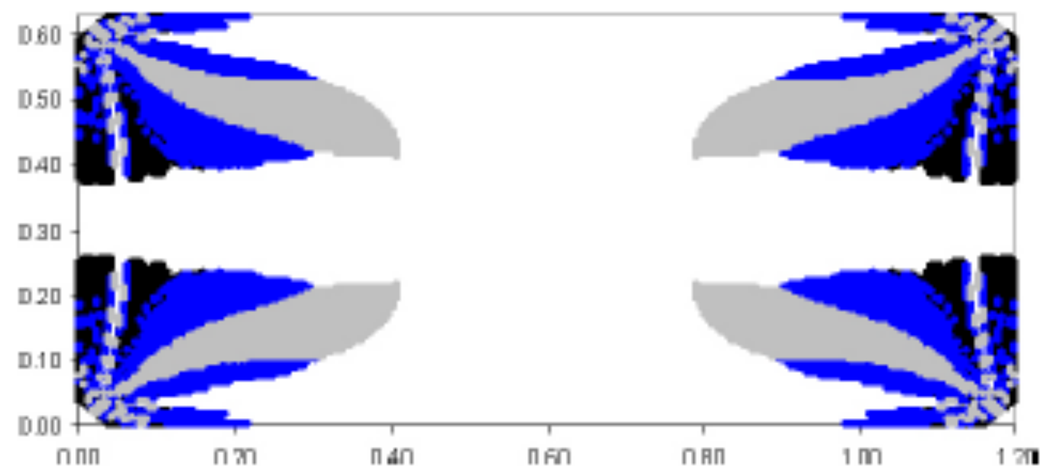
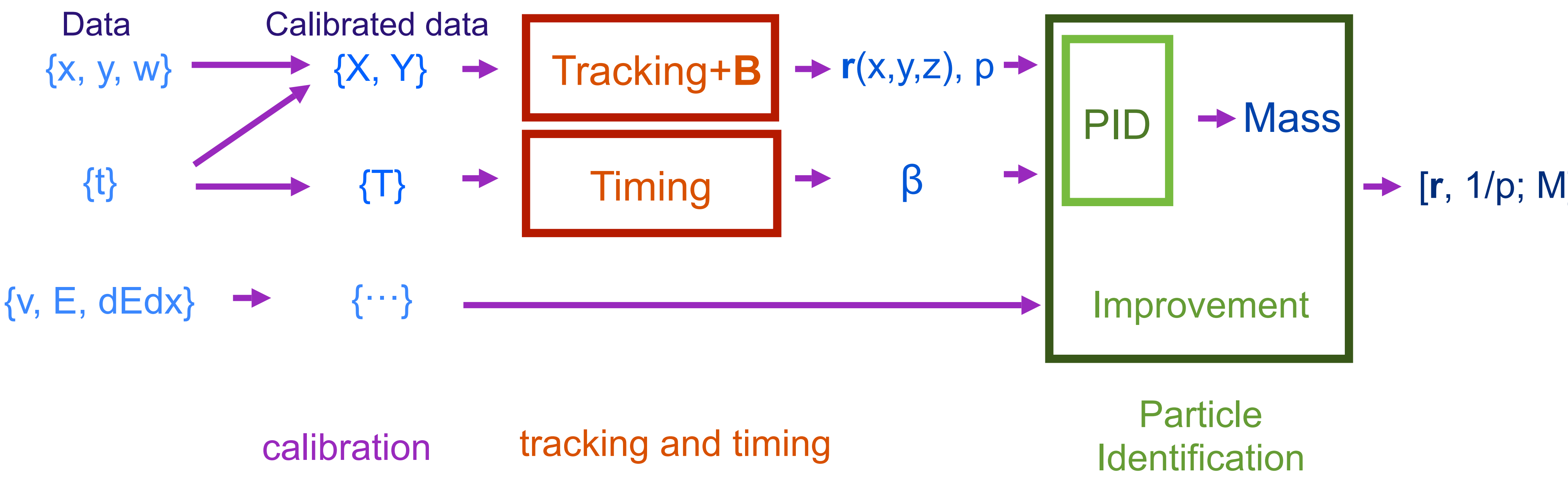
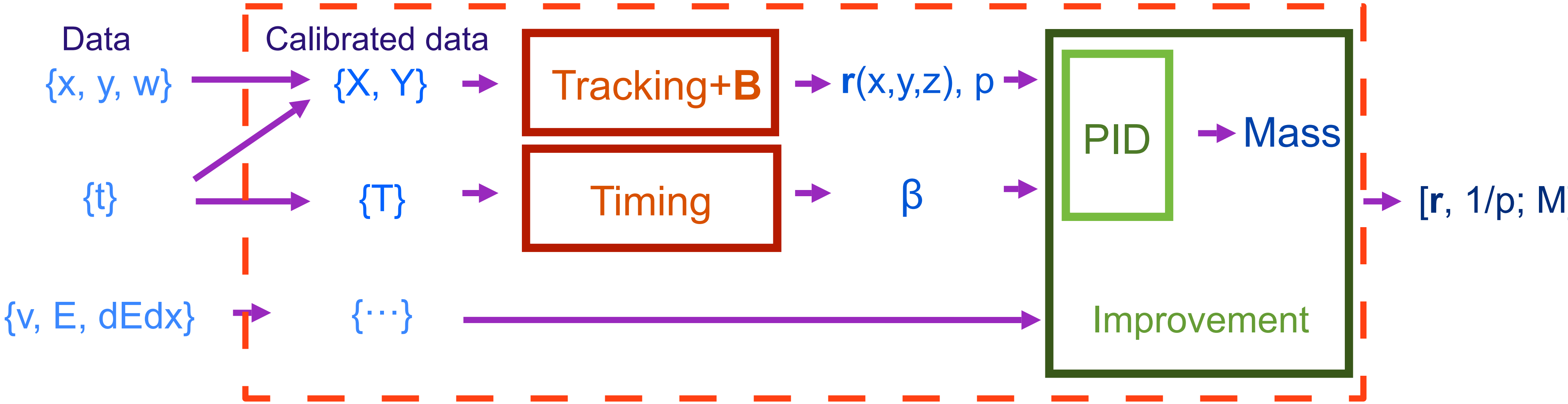


FIGURE 6. Calculation. For the large plate configuration hypothetically viewed by 4 photomultipliers placed in the corners cut at 45 deg. A 1% pulse high resolution is assumed. Position information with a sensitivity better than 1 cm is obtained throughout most of the active area, near the photomultipliers, it degrades to better than 2, 3 and 5 cm as we move closer to the Photomultipliers and the edge of the detector.

Beyond 4D


Data
Calibrated data
{x, y, w}
{X, Y}
Tracking+B
r(x,y,z), p
PID
Mass
{t}
{T}
Timing
β
[r, 1/p; M]
{v, E, dEdx}
{···}
Improvement
calibration
tracking and timing
Particle Identification

Beyond 4D



tracking models, constraints...
Alignment + Synchronization
slewing corrections, walk corrections..

MIDAS

Data
{x, y, w} →

{t} →

{v, E, dEdx} →

MIDAS
Multisampling IDentification of pArticleS

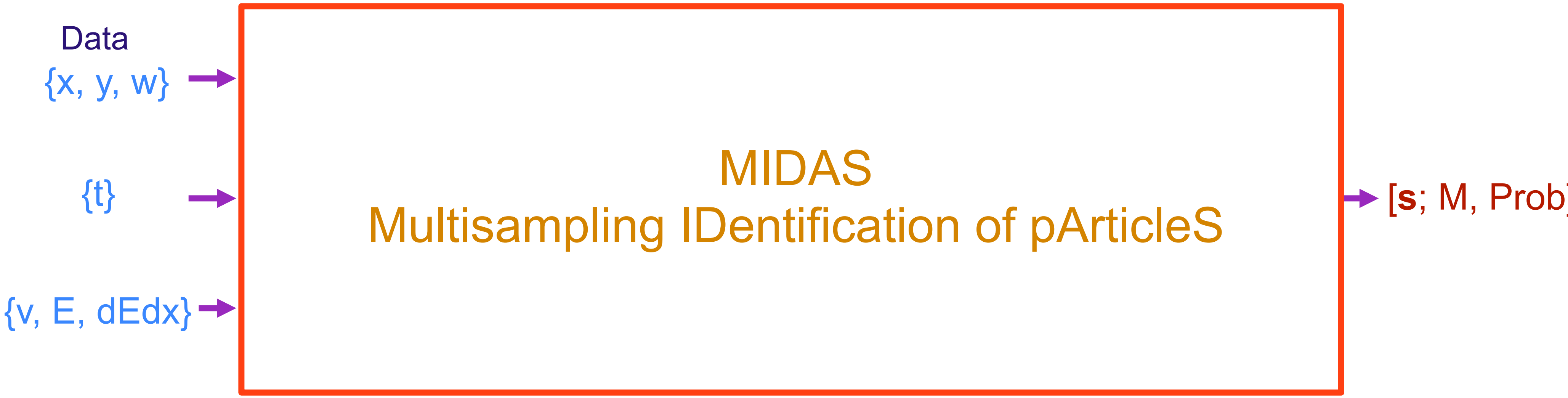
→ [**s**; M, Prob]

tracking models, constraints...
Alignment + Synchronization
slewing corrections, walk corrections..
(everything is inside)

Basic idea:

- Working in a higher dimension space may make more difficult to find the solution to the tracking tasks. As consecuence any piece of useful information shouldn't thrown out.
- Pulses amplitudes or charges, when related with the particle parameters, may be also used as tracking variables.

MIDAS



Perhaps using all the information is a utopia but we are developing very general tools to do it. Bad information could be always thrown out later

- Basic philosophy:
  - Try to make redundant systems
  - Try to increase the NDF as much as possible
  - Try to avoid mathematical reduction of data
  - Try to put as much as possible ingredients in the tracking model

Recollections and Reflections

- The time may become a very important tracking and reconstruction parameter.
- Working separately with time and coordinates may offer important advantages mainly in hybrid environments
- It's true that time resolution is very poor compared with position resolutions. But some detectors may offer many timing data (Ex. straw tubes). Why not try to improve their timing resolution as much as possible?
- Using all the measurements separately at the tracking level could be very usefull for
    - reducing the background
    - increase the redundancy of data
    - improve the estimation of parameters
    - finding hidden correlations and relationships between particle parameters, calibration constants, alignment or synchronization parameters and others
- A lot of matrix algebra tools and multivariate analysis tools are available
- Using time separately allows to run detectors independently of external triggers or external conditions
- Every detector can run at its maximum allowed data acquisition rate: later all the information can be merged together
- Excess of information made advisable to make the reduction of information as close as possible to the detectors: FPGAs, GPUs...

aknowledgments

- G. Kornakov
  (+ J. Markert + V. Pechenov)
- I. Kisel
- I. Kulakov & M. Zyzak
- FIAS

Thanks!

The end