Collisional neutrino flavor instabilities
in
spherically symmetric supernova models

ZX, M.-R. Wu, G. Martínez-Pinedo, T. Fischer, M. George, C.-Y. Lin, L. Johns, arXiv: 2210.08254

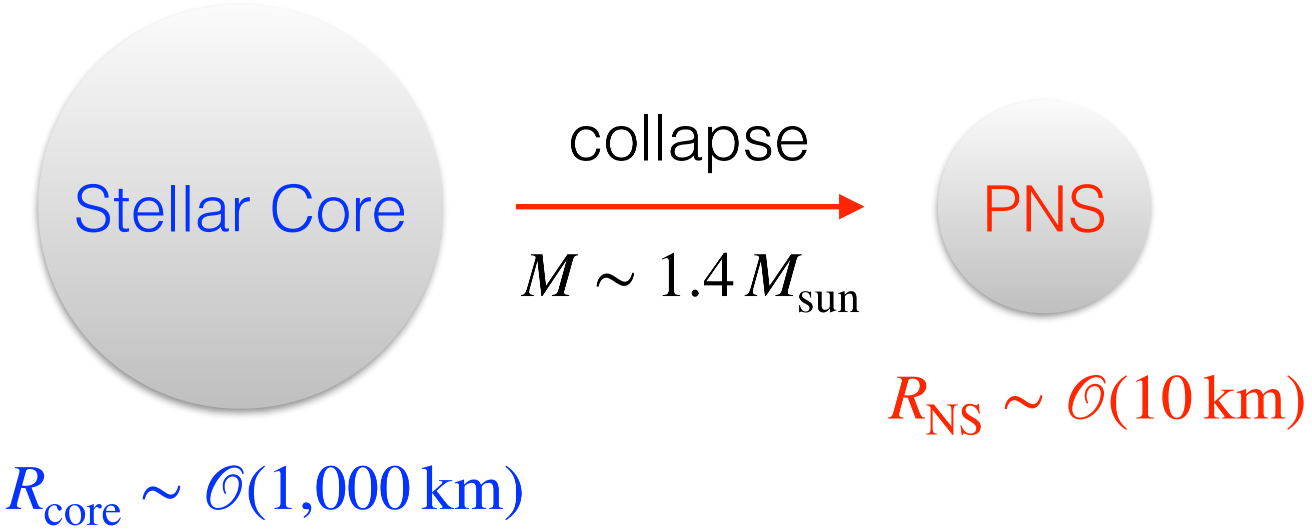
Zewei Xiong
GSI Helmholtzzentrum für Schwerionenforschung GmbH

EMMI+IReNA Workshop "Remnants of neutron-star mergers –
Connecting hydrodynamics models to nuclear, neutrino, and kilonova physics"
Oct. 19, 2022

European Research Council
Established by the European Commission

GSI Helmholtzzentrum für Schwerionenforschung

Neutrinos in core-collapse supernovae



Stellar Core

collapse

$M \sim 1.4\, M_{\rm sun}$

PNS

$R_{\rm NS} \sim \mathcal{O}(10\,{\rm km})$

$R_{\rm core} \sim \mathcal{O}(1{,}000\,{\rm km})$

- Profuse source of neutrinos: $\nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$
- Dynamics: revive the stalled shock, neutrino-driven wind
- Nucleosynthesis: weak interactions change electron fraction Y_e

Neutrino oscillations

- Equation for neutrino flavor evolution:

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = -i[\mathbf{H}_{\mathrm{vac}} + \mathbf{H}_{\mathrm{mat}} + \mathbf{H}_{\nu\nu}, \varrho]$$

vacuum mixing

coherent forward scatterings

with neutrino flavor density matrix: $\varrho = \begin{pmatrix} n_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho^*_{e\mu} & n_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho^*_{e\tau} & \varrho^*_{\mu\tau} & n_{\nu_\tau} \end{pmatrix}$

- Neutrino oscillations:
  - Vacuum mixing
  - Mikheyev–Smirnov-Wolfenstein (MSW) matter effect
  - Collective neutrino oscillations and flavor instabilities:

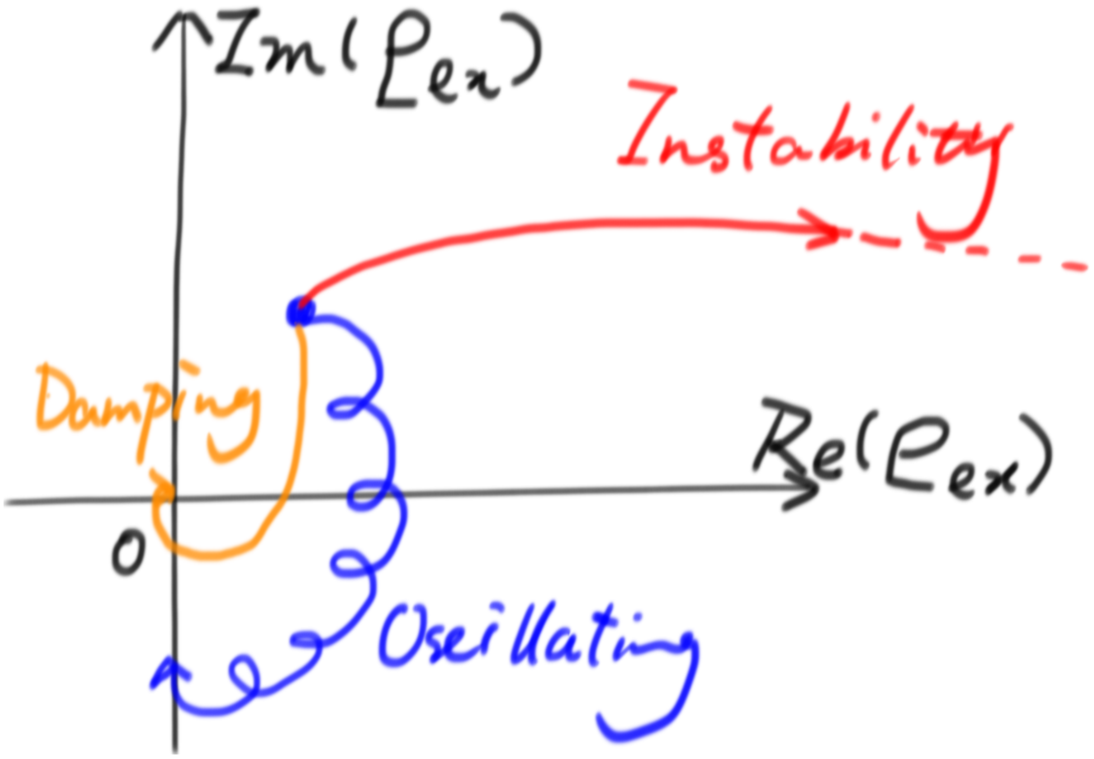
$$\mathbf{H}_{\nu\nu} = \sqrt{2}G_F \int d\mathbf{p}'(1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')[\varrho(\mathbf{p}') - \bar{\varrho}^*(\mathbf{p}')]$$

Neutrino flavor instability

- What is flavor instability?
  special flavor mode enhancing the small flavor mixing resulted from neutrino propagation

$$\varrho = \begin{pmatrix} n_{\nu_e} & \varrho_{e\mu} \\ \varrho_{e\mu}^{*} & n_{\nu_\mu} \end{pmatrix}$$

$$\begin{tikzcd}
\varrho_{e\mu} \ll n_{\nu_e}, n_{\nu_\mu} \arrow[d, "\text{non-linear mechanism}"', "\varrho_{e\mu} \propto e^{\kappa t}"] \\
\varrho_{e\mu} \sim n_{\nu_e}, n_{\nu_\mu}
\end{tikzcd}$$



- Different types of flavor instabilities depending on how those oscillation terms interplay:
  - Slow flavor instability: different vacuum oscillation frequencies for different energies
  - Fast flavor instability: angular spectral crossing

Neutrino collisional processes

- Equation for neutrino flavor evolution with incoherent collisions:

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = -i[\mathbf{H}_{\mathrm{vac}} + \mathbf{H}_{\mathrm{mat}} + \mathbf{H}_{\nu\nu}, \varrho] + \mathbf{C}(\varrho)$$

EA	Emission and absorption	$\nu_e + n \leftrightarrows p + e^-$
		$\bar{\nu}_e + p \leftrightarrows n + e^+$
		$\bar{\nu}_e + p + e^- \leftrightarrows n$
NNS	Neutrino-nucleon scattering	$\nu + N \leftrightarrows \nu + N$
		$\bar{\nu} + N \leftrightarrows \bar{\nu} + N$
NES	Neutrino-electron scattering	$\nu + e^{\pm} \leftrightarrows \nu + e^{\pm}$
NPR	Neutrino pair reactions	$\nu + \bar{\nu} \leftrightarrows e^- + e^+$
		$\nu + \bar{\nu} + N + N \leftrightarrows \nu + \bar{\nu}$

- EA in classical neutrino transport: $\mathrm{d}f_{\nu_e}/\mathrm{d}t = j_e(1 - f_{\nu_e}) - \chi_e f_{\nu_e}$ with emissivity j_e and opacity χ_e.

- EA taking the account of coherent flavor mixing in quantum kinetic equation (QKE) [A. Vlasenko, G. Fuller, V. Cirigliano, 2014; D.N. Blaschke, V. Cirigliano, 2016; ....]:

$$\mathbf{C}_{\mathrm{EA}} \sim \begin{pmatrix} j_e(1-\varrho_{ee}) - \chi_e \varrho_{ee} & -(j_e + \chi_e)\varrho_{e\mu}/2 \\ -(j_e + \chi_e)\varrho_{e\mu}^{*}/2 & 0 \end{pmatrix}.$$

Collisional flavor instability in homogeneous model

coherent forward scatterings

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = -i[\mathbf{H}_{\nu\nu}, \varrho] = -i\sqrt{2}G_F[(\varrho - \bar{\varrho}^*), \varrho]$$

$$\mathrm{d}\varrho_{e\mu}/\mathrm{d}t = i\sqrt{2}G_F(\bar{n}_\nu \varrho_{e\mu} - n_\nu \bar{\varrho}^*_{e\mu})$$

$$\mathrm{d}\bar{\varrho}^*_{e\mu}/\mathrm{d}t = i\sqrt{2}G_F(\bar{n}_\nu \varrho_{e\mu} - n_\nu \bar{\varrho}^*_{e\mu})$$

incoherent collisions

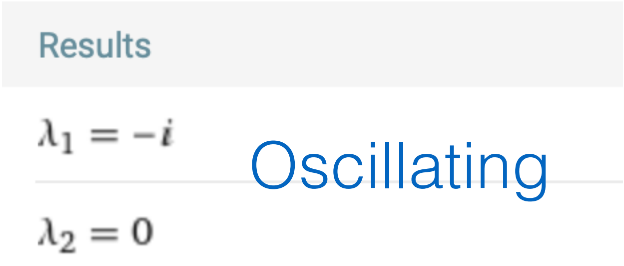
$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = \mathrm{C}_{\mathrm{EA}} \sim \begin{pmatrix} j_e(1-\varrho_{ee}) - \chi_e\varrho_{ee} & -(j_e+\chi_e)\varrho_{e\mu}/2 \\ -(j_e+\chi_e)\varrho^*_{e\mu}/2 & 0 \end{pmatrix}$$

$$\mathrm{d}\varrho_{e\mu}/\mathrm{d}t = -C\varrho_{e\mu}$$

$$\mathrm{d}\bar{\varrho}^*_{e\mu}/\mathrm{d}t = \qquad -\bar{C}\bar{\varrho}^*_{e\mu}$$

$$\begin{tikzcd}
\varrho_{e\mu} \ll n_{\nu_e}, n_{\nu_\mu} \arrow[d, "\varrho_{e\mu} \propto e^{\kappa t}"] \\
\varrho_{e\mu} \sim n_{\nu_e}, n_{\nu_\mu}
\end{tikzcd}$$

eigenvalues	$\begin{pmatrix} 9i & -10i \\ 9i & -10i \end{pmatrix}$



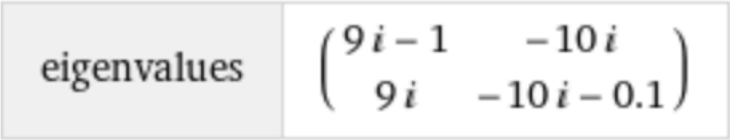
Results

$\lambda_1 = -i$

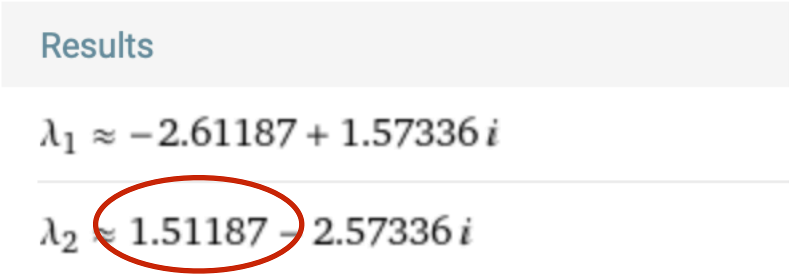
$\lambda_2 = 0$

Oscillating

(snapshots from www.wolframalpha.com)



eigenvalues	$\begin{pmatrix} 9i-1 & -10i \\ 9i & -10i-0.1 \end{pmatrix}$



Results

$\lambda_1 \approx -2.61187 + 1.57336\,i$

$\lambda_2 \approx 1.51187 - 2.57336\,i$

Collisional instability!

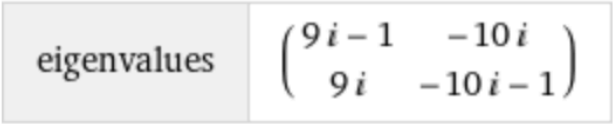
eigenvalues	$\begin{pmatrix} -1 & 0 \\ 0 & -0.1 \end{pmatrix}$

Results

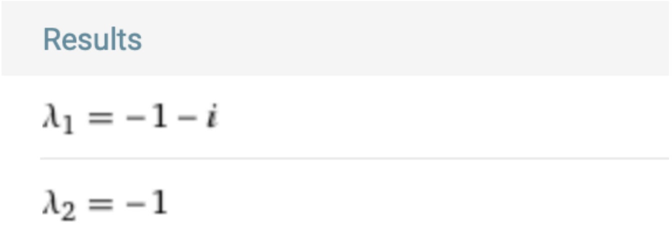
$\lambda_1 = -1$

$\lambda_2 = -\frac{1}{10}$

Damping



eigenvalues	$\begin{pmatrix} 9i-1 & -10i \\ 9i & -10i-1 \end{pmatrix}$



Results

$\lambda_1 = -1 - i$

$\lambda_2 = -1$

Collisional flavor instability in homogeneous model

- Collisional flavor instability needs asymmetric collisional rates

- Growth rate [L. Johns, 2021]:

$$\mathrm{Re}(\kappa) \approx (|C+\bar{C}|/2)\left(\alpha_C/\alpha_n - 1\right)$$

independent of the strength of $H_{\nu\nu}$ when $H_{\nu\nu} \gg C$

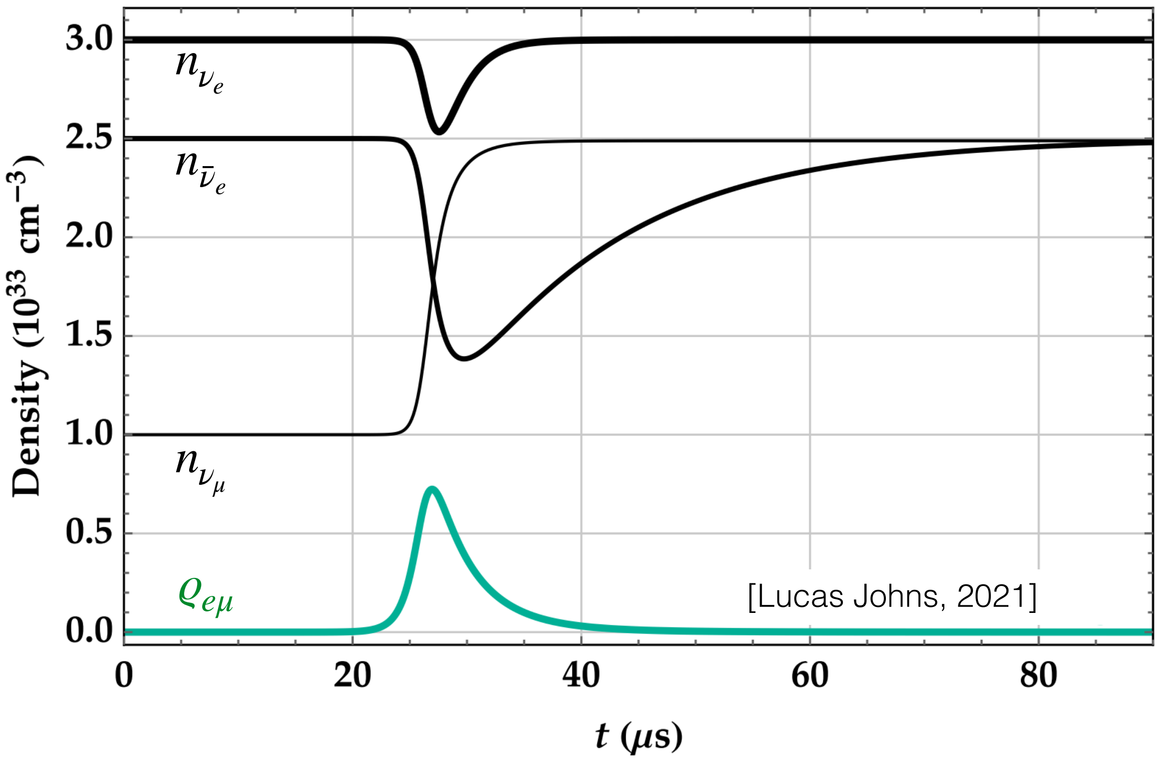
with $C = (j_e + \chi_e)/2$, $\bar{C} = (\bar{j}_e + \bar{\chi}_e)/2$, and asymmetry factors

$$\alpha_n = \left| \frac{n_{\nu_e} - n_{\nu_\mu} - n_{\bar{\nu}_e} + n_{\bar{\nu}_\mu}}{n_{\nu_e} - n_{\nu_\mu} + n_{\bar{\nu}_e} - n_{\bar{\nu}_\mu}} \right|, \ \alpha_C = \left| \frac{C - \bar{C}}{C + \bar{C}} \right|.$$

- Collisional flavor instability develops when $\alpha_n \lesssim \alpha_C$.

- Provide a different condition from fast flavor instability:
  - Y_e is small near PNS
  - Heavy-lepton neutrinos are decoupling and number densities are diluted

Isotropic collisional instability



[Lucas Johns, 2021]

Models with advection

$$(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho = -i[a_{\nu\nu}\mathbf{H}_{\nu\nu}, \varrho] + \mathbf{C}(\varrho)$$

- Advection in spherical symmetric geometry with multiple energies and angles

- Iso-energetic NNS:

$$\mathbf{C}_{\mathrm{NNS}} = \int dv_r' \, R_{\mathrm{NNS}}(E, v_r, v_r')[\varrho(E, v_r') - \varrho(E, v_r)]$$

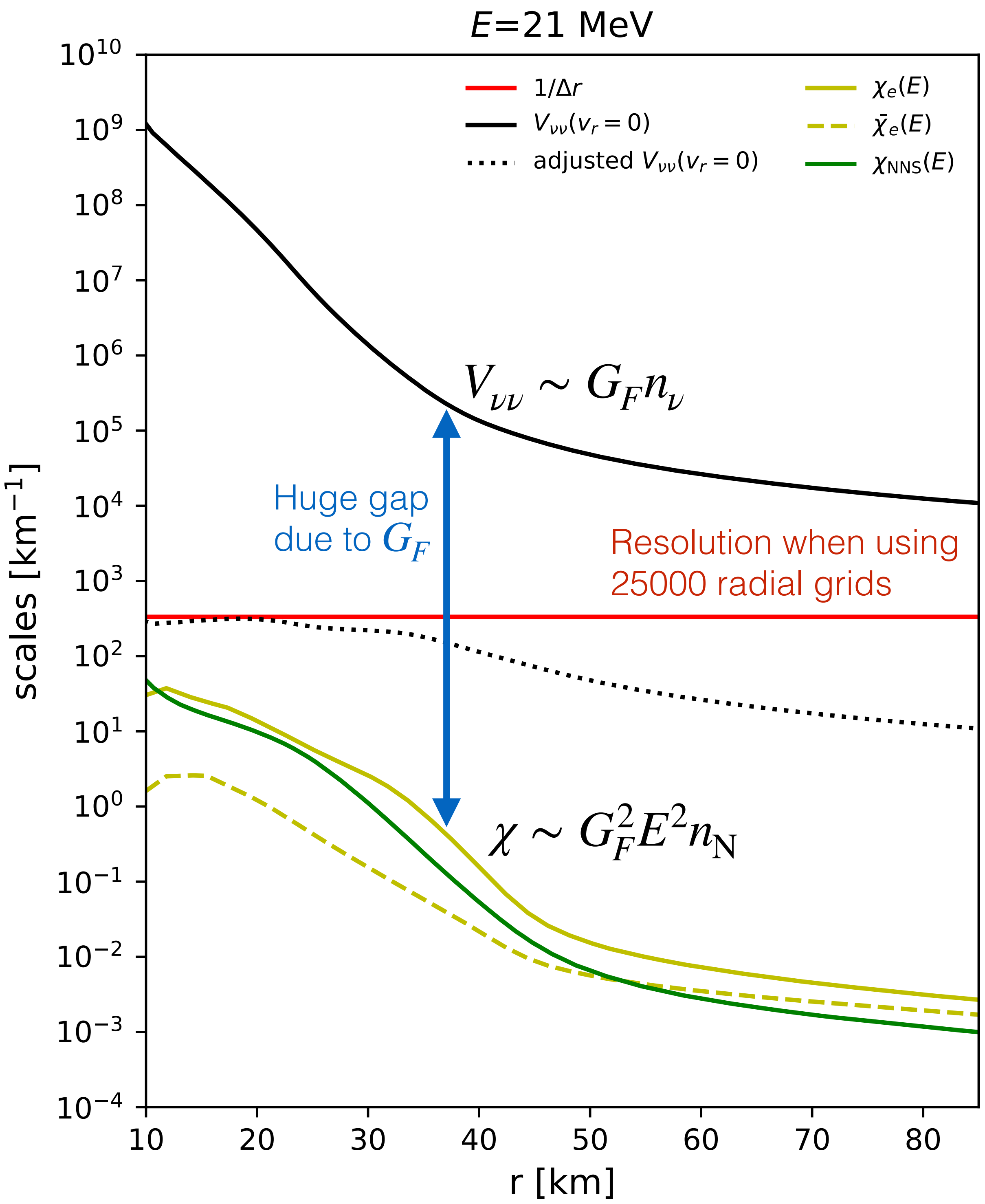
with the opacity

$$\chi_{\mathrm{NNS}}(E) = \int dv_r' R_{\mathrm{NNS}}(E, v_r, v_r').$$

- **Trilemma** among
  1. self-consistency
  2. advection
  3. exact rates

- Strategical attenuation with the factor:

$$a_{\nu\nu}(r) = \frac{a_1}{1 + e^{(a_2 - r)/a_3}}$$



$V_{\nu\nu} \sim G_F n_\nu$

Huge gap due to G_F

Resolution when using 25000 radial grids

$\chi \sim G_F^2 E^2 n_{\mathrm{N}}$

Models with advection

- Use background matter profiles from spherically symmetric CCSN simulations ($\texttt{AGILE}-\texttt{BOLTZTRAN}$) with an $18M_\odot$ progenitor at four post-bouncing time points: $t_{\text{pb}} \approx$ 150 ms, 250ms, 500ms, 1000ms

- Solve the neutrino flavor evolution equation

$$(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho(E, v_r) = -i[a_{\nu\nu}\mathbf{H}_{\nu\nu}, \varrho(E, v_r)] + \mathbf{C}_{\text{EA}} + \mathbf{C}_{\text{NNS}}$$

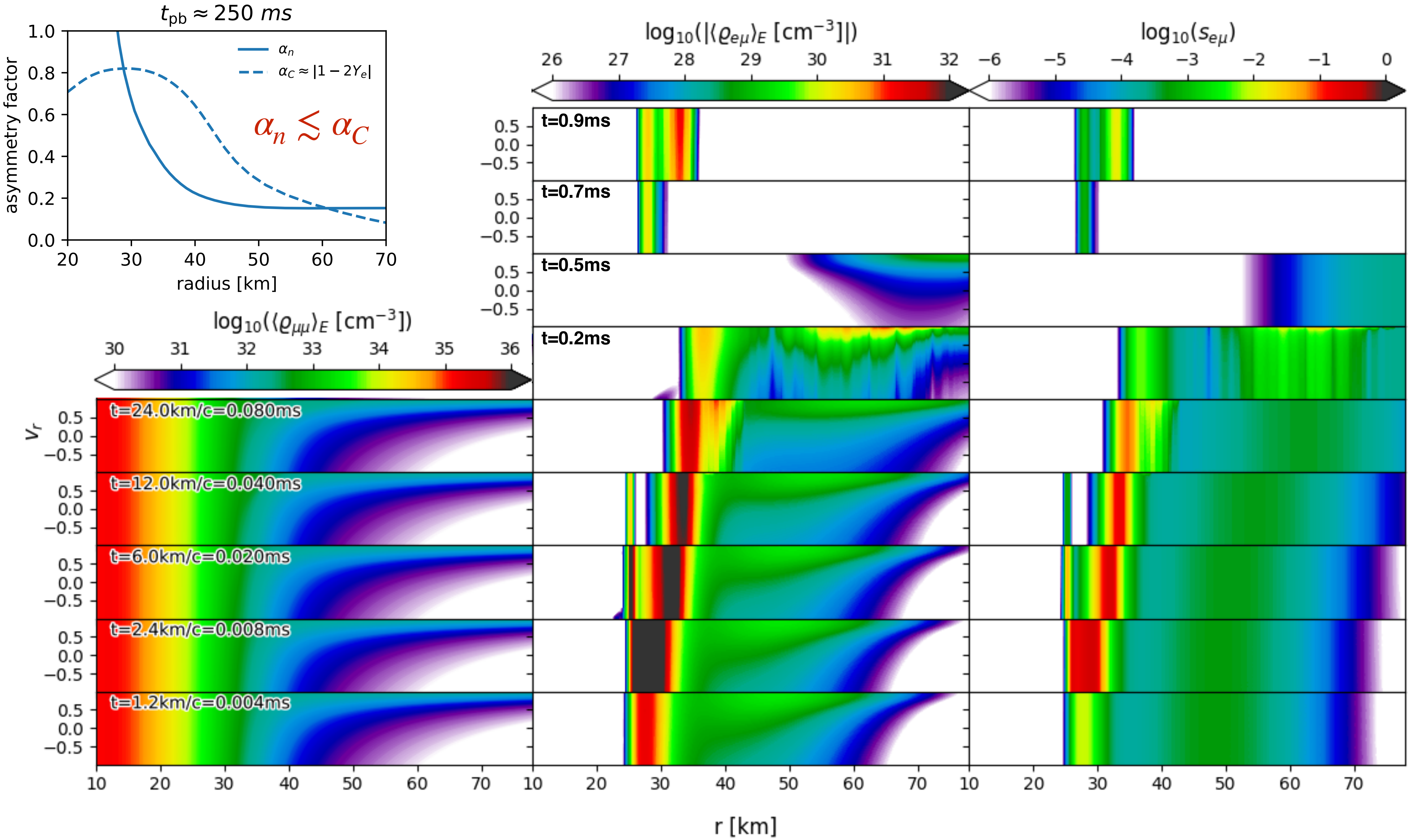
  - in $\texttt{COSE}\nu$ for two flavors ν_e and ν_μ up to ~1 ms
  - in the absence of fast flavor instability
  - Resolutions: $N_r = 25000$, $N_{v_r} = 50$, $N_E = 20$
  - NES, NPR is more computationally expensive because of $R_{\text{NES/NPR}}(E, E', v_r, v_r')$

- Boundary conditions:
  - Inner boundary: neutrinos in thermal equilibrium with matter between 10 and 16 km to mimic NPR
  - Outer boundary: freely stream out at 85 km

- Initial perturbation (flavor mixing seed): radial-dependent perturbation in Gaussian function

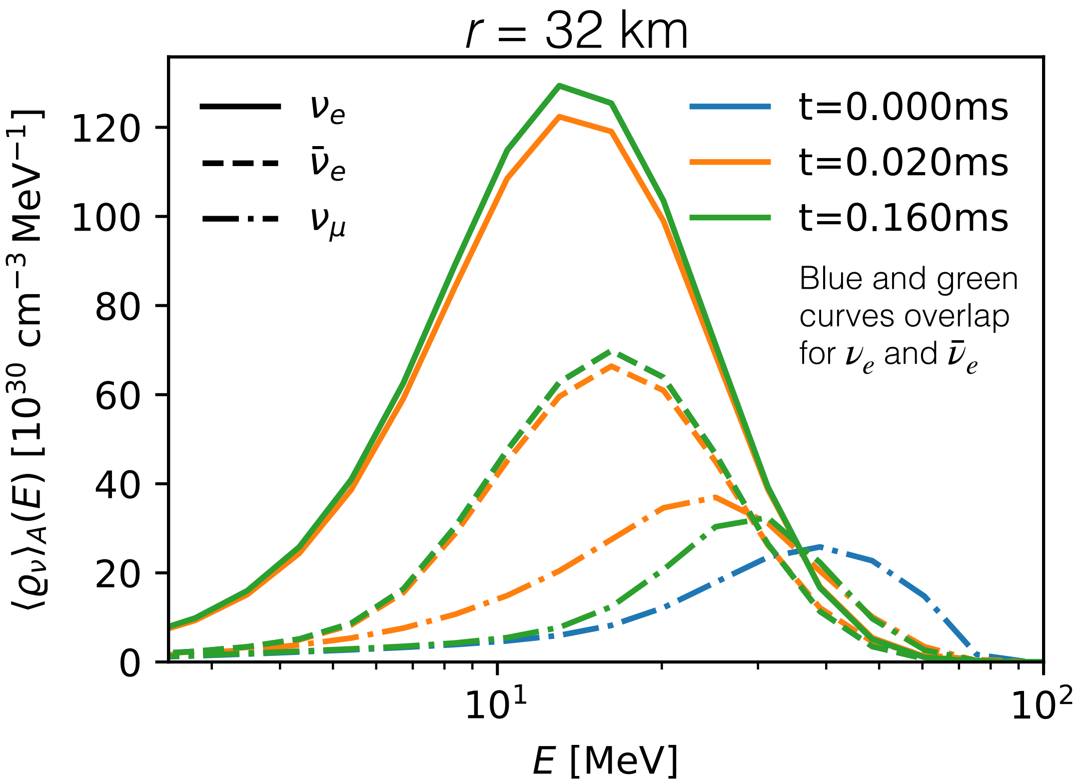
Evolution of collisional flavor instability

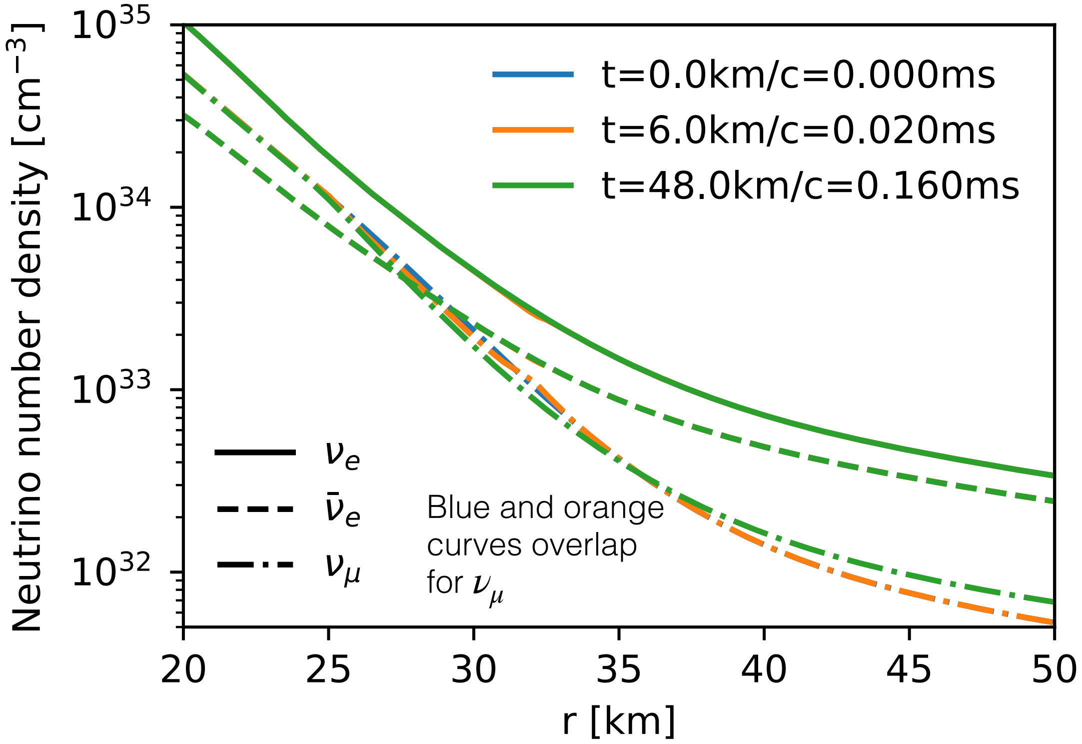
Definitions for analysis: $\langle \varrho \rangle_E(v_r) = \int dE\, \varrho(E, v_r)$, $\langle \varrho \rangle_A(E) = \int dv_r\, \varrho(E, v_r)$, $s_{e\mu} = \dfrac{|\langle \varrho_{e\mu} \rangle_E|}{|\langle \varrho_{ee} \rangle_E - \langle \varrho_{\mu\mu} \rangle_E|}$

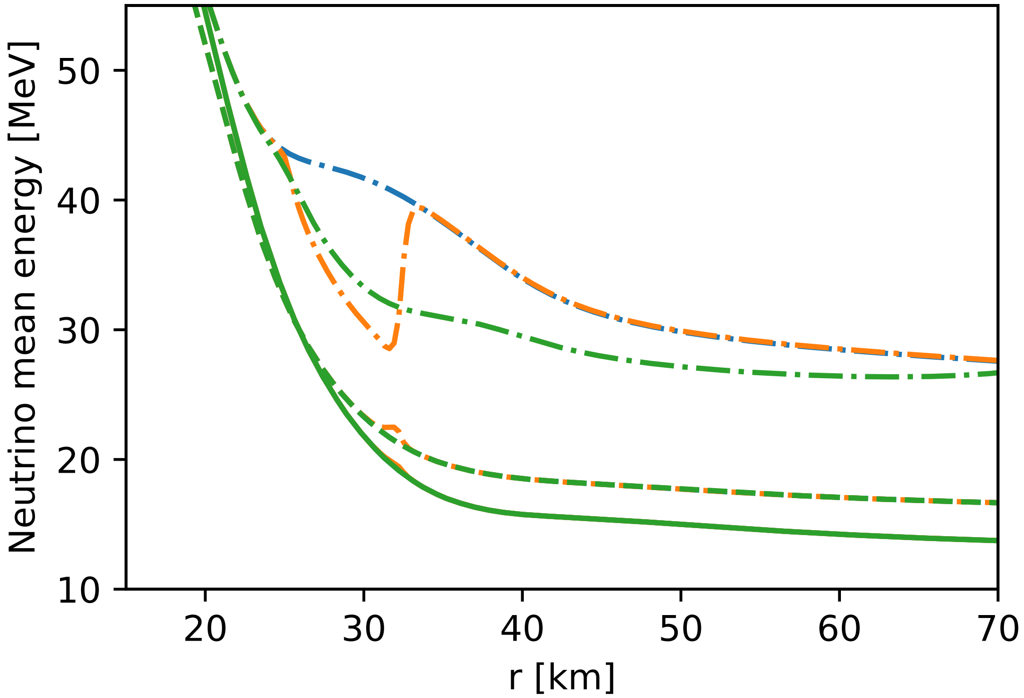


Evolution of collisional flavor instability

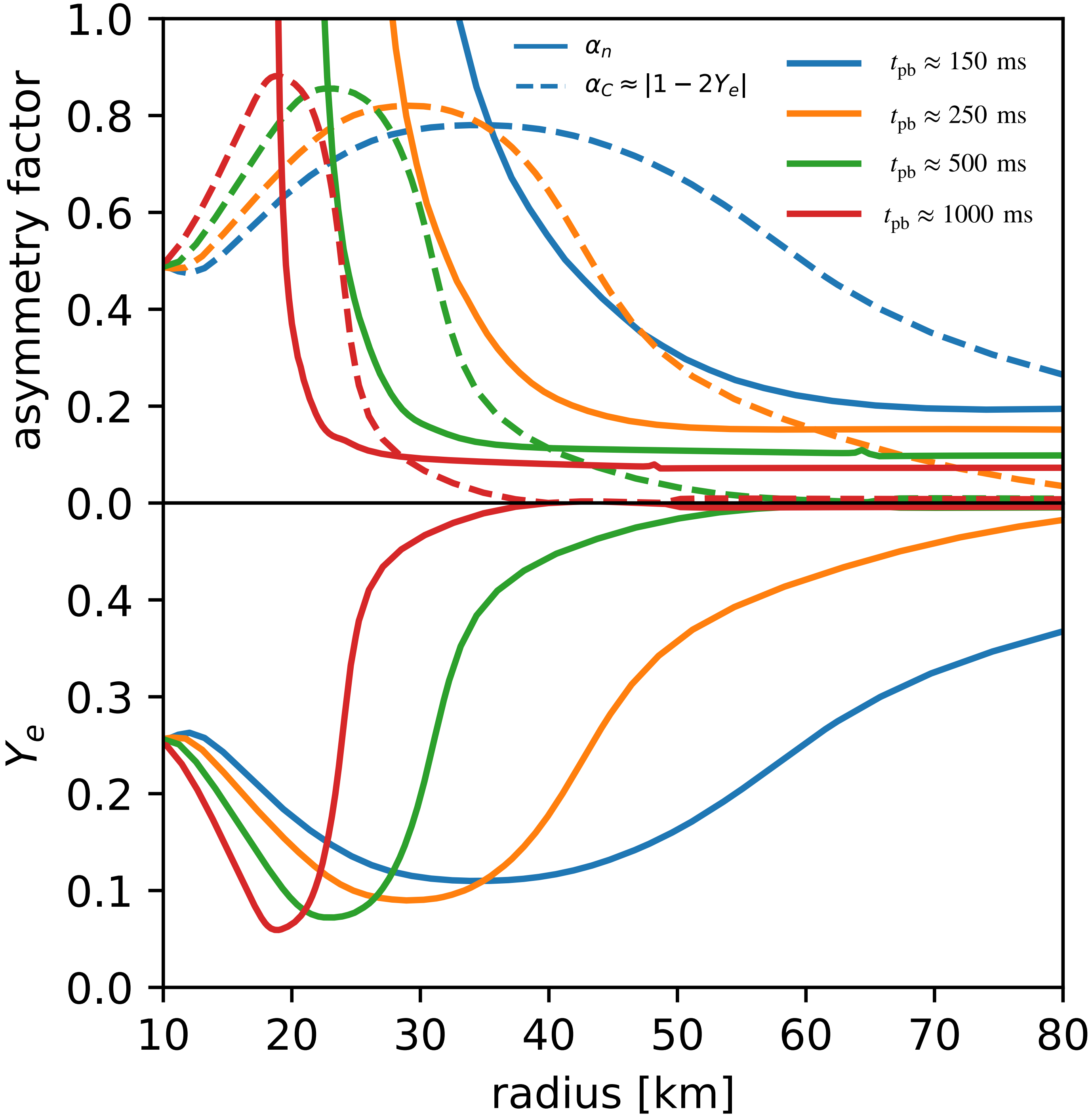
- distributions of ν_e and $\bar{\nu}_e$ are affected at the onset of the flavor conversion, but quickly restored by large EA rates
- In contrast to the homogeneous model, spectrum of heavy-lepton (anti)neutrinos does not converge to that of electron flavor
- leave imprints in the spectra of heavy-lepton (anti)neutrinos at the free-streaming regime







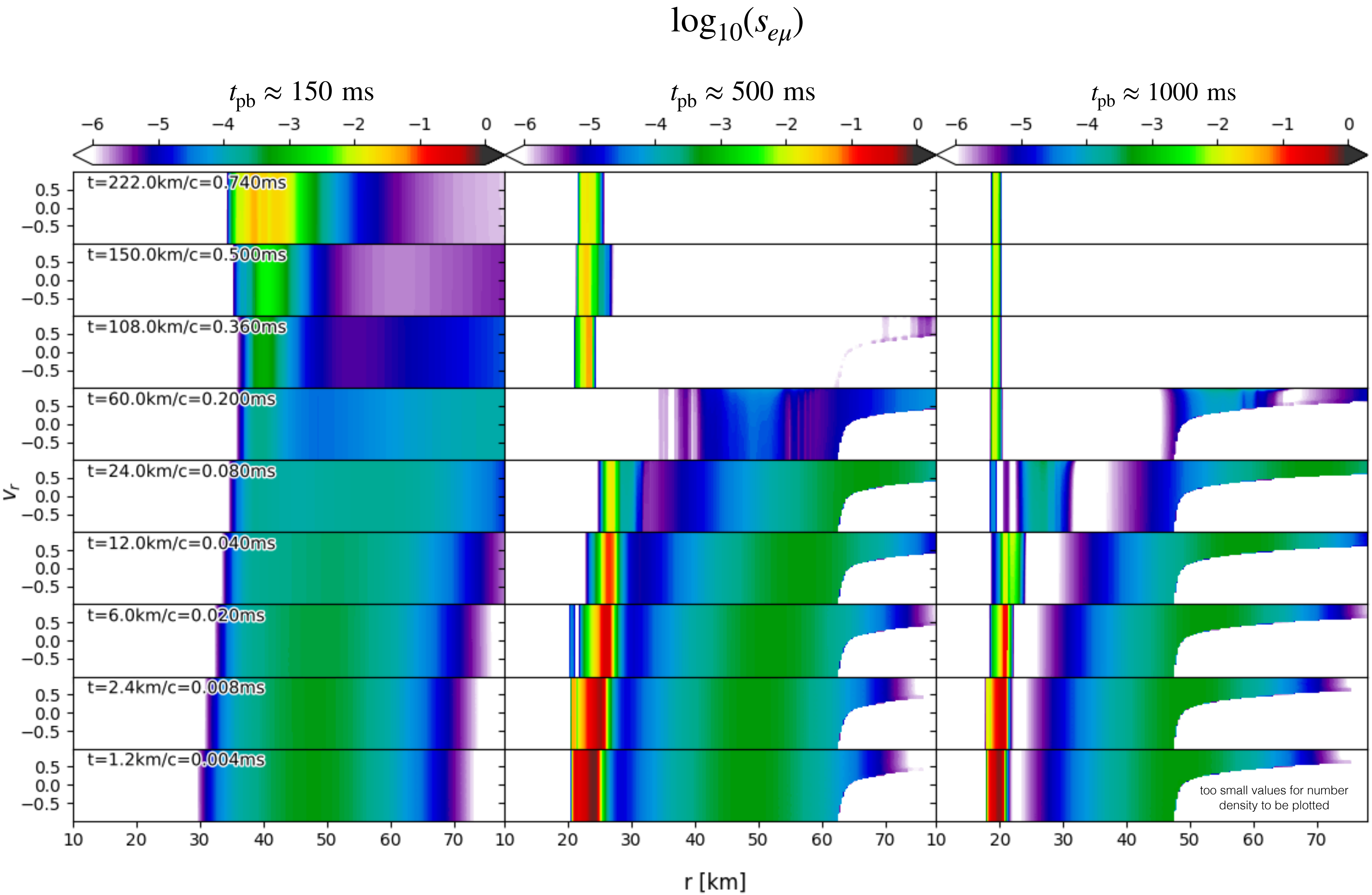
Conditions at different supernova snapshots



- α_C increases due to the deleptonization

- α_n decreases after the neutrino burst.

Evolution of collisional flavor instability



Summary and outlook

- We implement:
  - a multi-energy and multi-angle collective neutrino oscillation simulator
  - to study collisional flavor instability in a spherically symmetric geometry
  - including advection on a global scale and realistic collisional rates
- We find:
  - collisional instability leads to conversions near decoupling region
  - the strength of the instability increases with respect to the post-bouncing time
  - more likely to manifest in the CCSN dynamics and the emitted neutrino signals
  - may moderately affect neutrino (induced) nucleosynthetic processes that are sensitive to the spectra of heavy-lepton neutrino flavors
- Outlook:
  - include NES
  - artificial attenuation?
  - dynamic evolution and matter feedback?
  - fast flavor conversion?