New developments on the physics of neutrino fast flavor conversion in dense astrophysical media

Ian Padilla-Gay

Niels Bohr Institute, Copenhagen University

GSI Workshop, 18 October 2022

ian.padilla@nbi.ku.dk

Presentation outline

- Introduction
- Neutrino fast flavor conversion: the pendulum equivalence
- Numerical simulation of fast flavor conversion in mergers
- Conclusions

Neutrinos in dense astrophysical environments

Copiously produced ~10^{58} neutrinos (MeV)

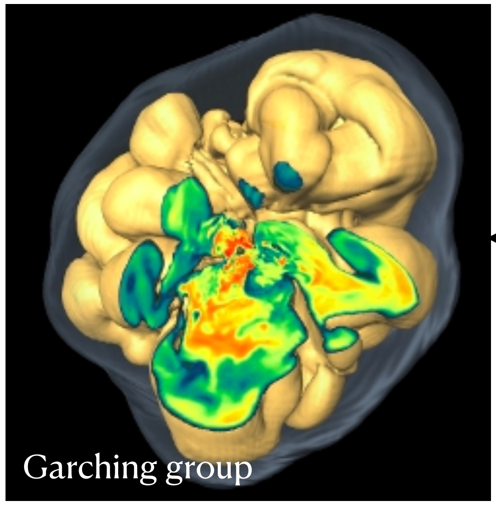
In core-collapse supernovae, neutrinos:

- Carry 99% of the gravitational energy
- Can revive the stalled shock

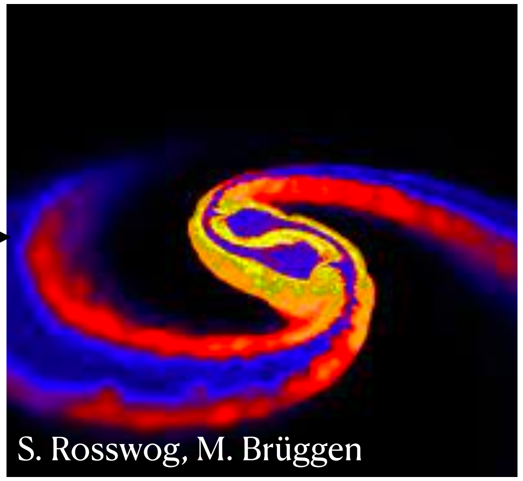
In compact binary mergers, neutrinos:

- Cool the disk
- Dominate ejecta in polar region

- Affect nucleosynthesis



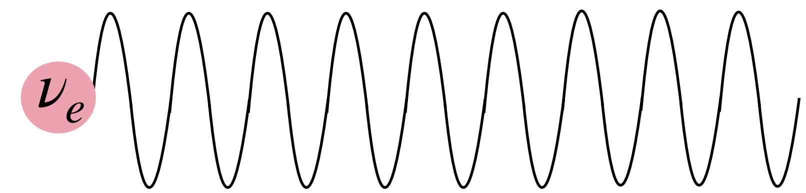
Flavor dependent processes



Neutrino flavor conversion

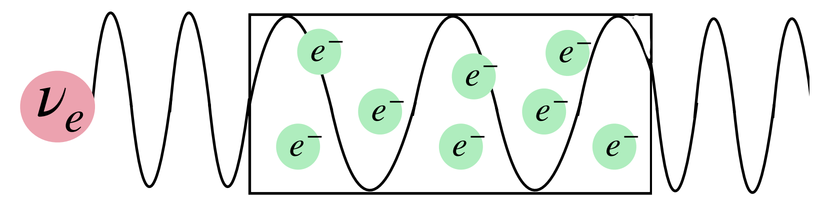
$\omega \sim \mathcal{O}(10)\ \mathrm{km}^{-1}$

- **Vacuum oscillations -** driven by Δm^2



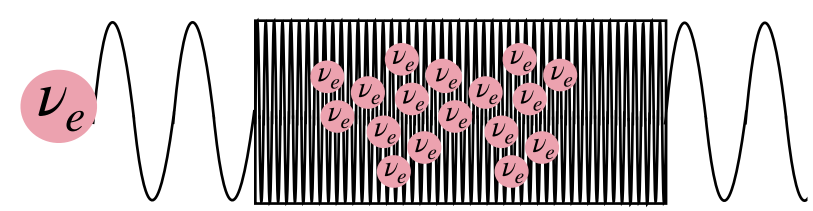
$\lambda \sim \mathcal{O}(10^4)\ \mathrm{km}^{-1}$

- **MSW effect -** coherent forward scattering with electrons

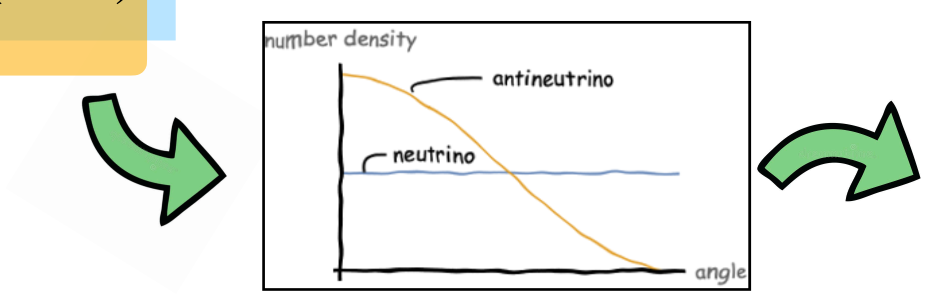


$\mu \sim \mathcal{O}(10^5)\ \mathrm{km}^{-1}$

- **Neutrino-neutrino interaction —** coherent forward scattering with background neutrinos → *Fast pairwise neutrino flavor conversion*



Key input: Electron-lepton-number (ELN) distribution



Determines the flavor dynamics

Neutrino-neutrino interaction

- Neutrinos also constitute a background for other neutrinos
- Neutrino-neutrino interaction induces *fast pairwise conversions*

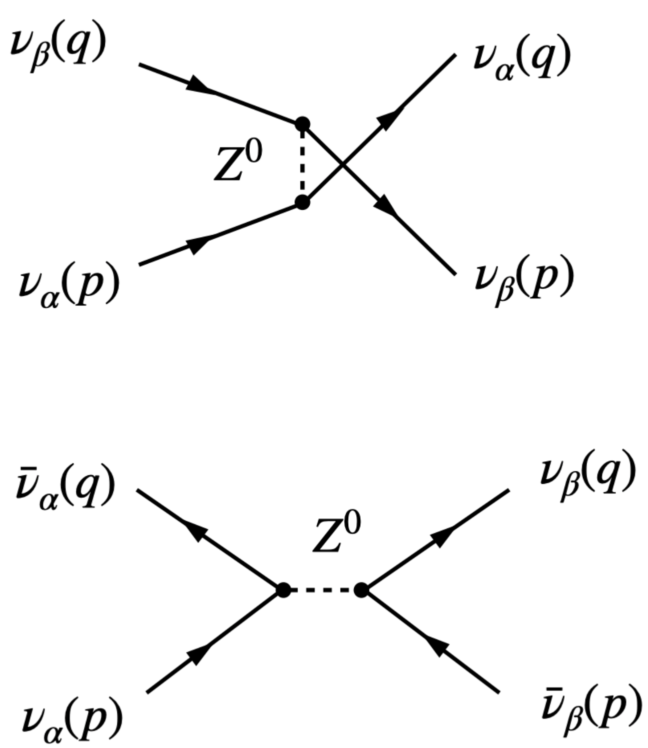
Momentum of test neutrino

$$H_{\nu\nu}(\vec{p}) = \sqrt{2}G_F n_\nu \int d\vec{q}\left[\rho(\vec{q}) - \bar{\rho}(\vec{q})\right]\left(1 - \frac{\vec{p}\cdot\vec{q}}{|\vec{p}||\vec{q}|}\right)$$

Neutrino interaction strength

Velocity of background neutrino

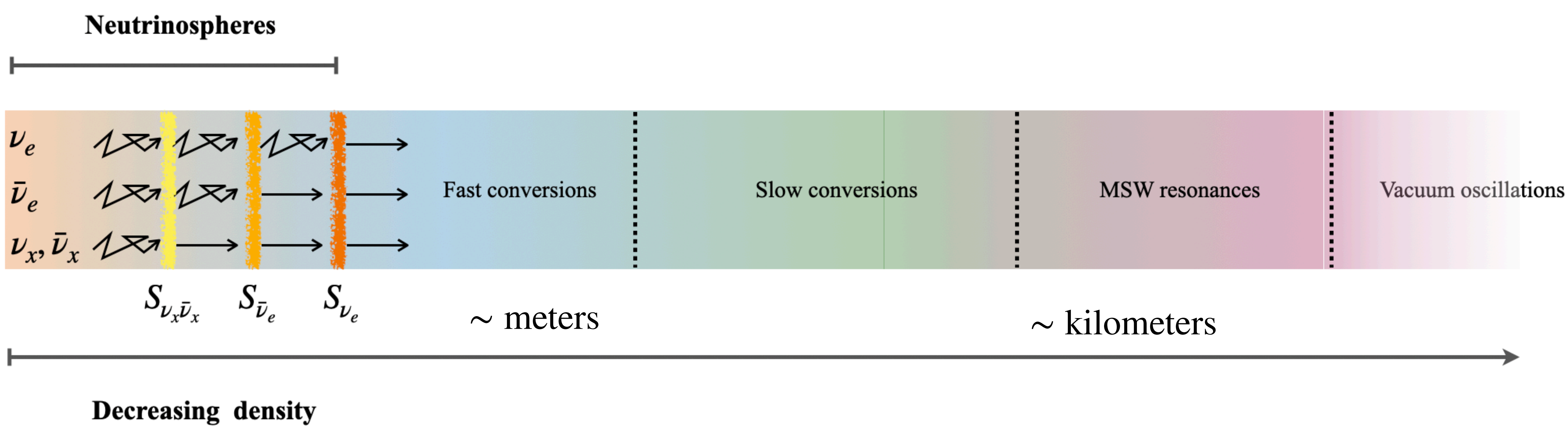
Density matrices for ν and $\bar{\nu}$



- *Non-linear* behavior of flavor evolution and *collective* conversion

Regions of flavor conversion

- Neutrinos with different E_ν have different interaction rates → flavor-dependent decoupling regions



- Different flavor conversion regimes

Collective neutrino conversion

- Neutrinos with different momenta evolve collectively
- The equations of motion describing neutrino flavor conversion are:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_x\right)\rho(\vec{x},\vec{p},t) = -i\left[H(\vec{x},\vec{p},t),\rho(\vec{x},\vec{p},t)\right] + \mathscr{C}\left(\rho(\vec{x},\vec{p},t),\bar{\rho}(\vec{x},\vec{p},t)\right)$$

$$\left(\frac{\partial}{\partial t} + \underbrace{\vec{v}\cdot\vec{\nabla}_x}_{\text{advection}}\right)\bar{\rho}(\vec{x},\vec{p},t) = -i\underbrace{\left[\bar{H}(\vec{x},\vec{p},t),\bar{\rho}(\vec{x},\vec{p},t)\right]}_{\text{refraction}} + \underbrace{\bar{\mathscr{C}}\left(\rho(\vec{x},\vec{p},t),\bar{\rho}(\vec{x},\vec{p},t)\right)}_{\text{collisions}}$$

Spatial inhomogeneities

Coherent forward scattering

Non-forward scattering
(neglected here)

Collective neutrino conversion

- Neutrinos with different momenta evolve collectively
- The equations of motion describing neutrino flavor conversion are:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_x\right)\rho(\vec{x}, \vec{p}, t) = -i\left[H(\vec{x}, \vec{p}, t), \rho(\vec{x}, \vec{p}, t)\right] + \mathscr{C}\left(\rho(\vec{x}, \vec{p}, t), \bar{\rho}(\vec{x}, \vec{p}, t)\right)$$

$$\left(\frac{\partial}{\partial t} + \underbrace{\vec{v} \cdot \vec{\nabla}_x}_{\text{advection}}\right)\bar{\rho}(\vec{x}, \vec{p}, t) = -i\underbrace{\left[\bar{H}(\vec{x}, \vec{p}, t), \bar{\rho}(\vec{x}, \vec{p}, t)\right]}_{\text{refraction}} + \underbrace{\bar{\mathscr{C}}\left(\rho(\vec{x}, \vec{p}, t), \bar{\rho}(\vec{x}, \vec{p}, t)\right)}_{\text{collisions}}$$

Spatial inhomogeneities

Coherent forward scattering

Non-forward scattering
(neglected here)

→ Collisional damping
(**arXiv:2209.11235**)

What are we missing?

- Full solution of quantum neutrino transport
- Implementation of neutrino conversion in hydrodynamical simulations
- Self-consistent treatment of neutrino conversion within nucleosynthesis networks

🤔

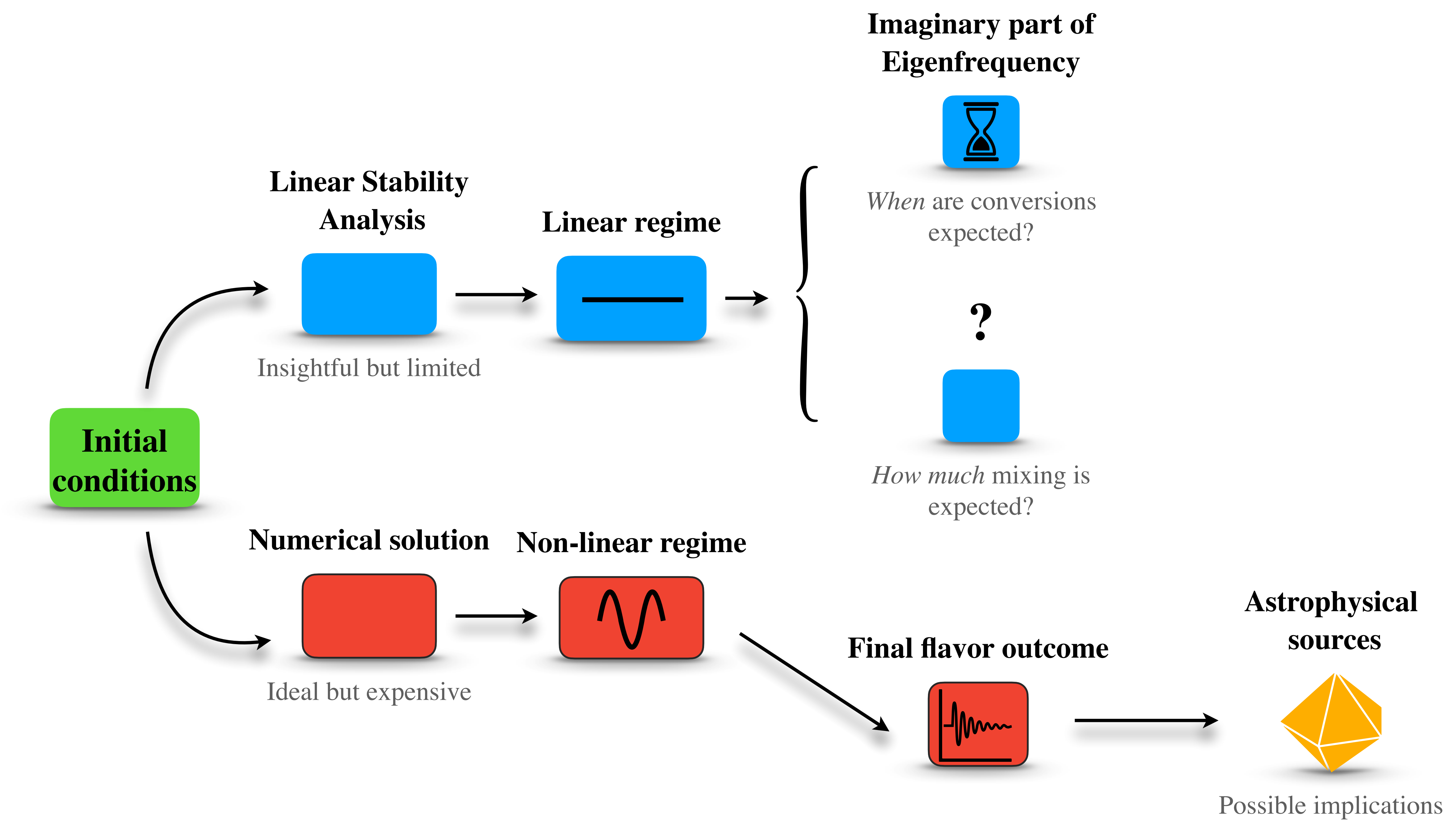
Ian Padilla-Gay, Irene Tamborra, Georg G. Raffelt, *Neutrino Flavor Pendulum Reloaded: The Case of Fast Pairwise Conversion*, Phys. Rev. Lett. 128 121102, arXiv:2109.14627

Ian Padilla-Gay, Shashank Shalgar, Irene Tamborra, *Multi-Dimensional Solution of Fast Neutrino Conversions in Binary Neutron Star Merger Remnants*, JCAP01(2021)017, arXiv:2009.01843

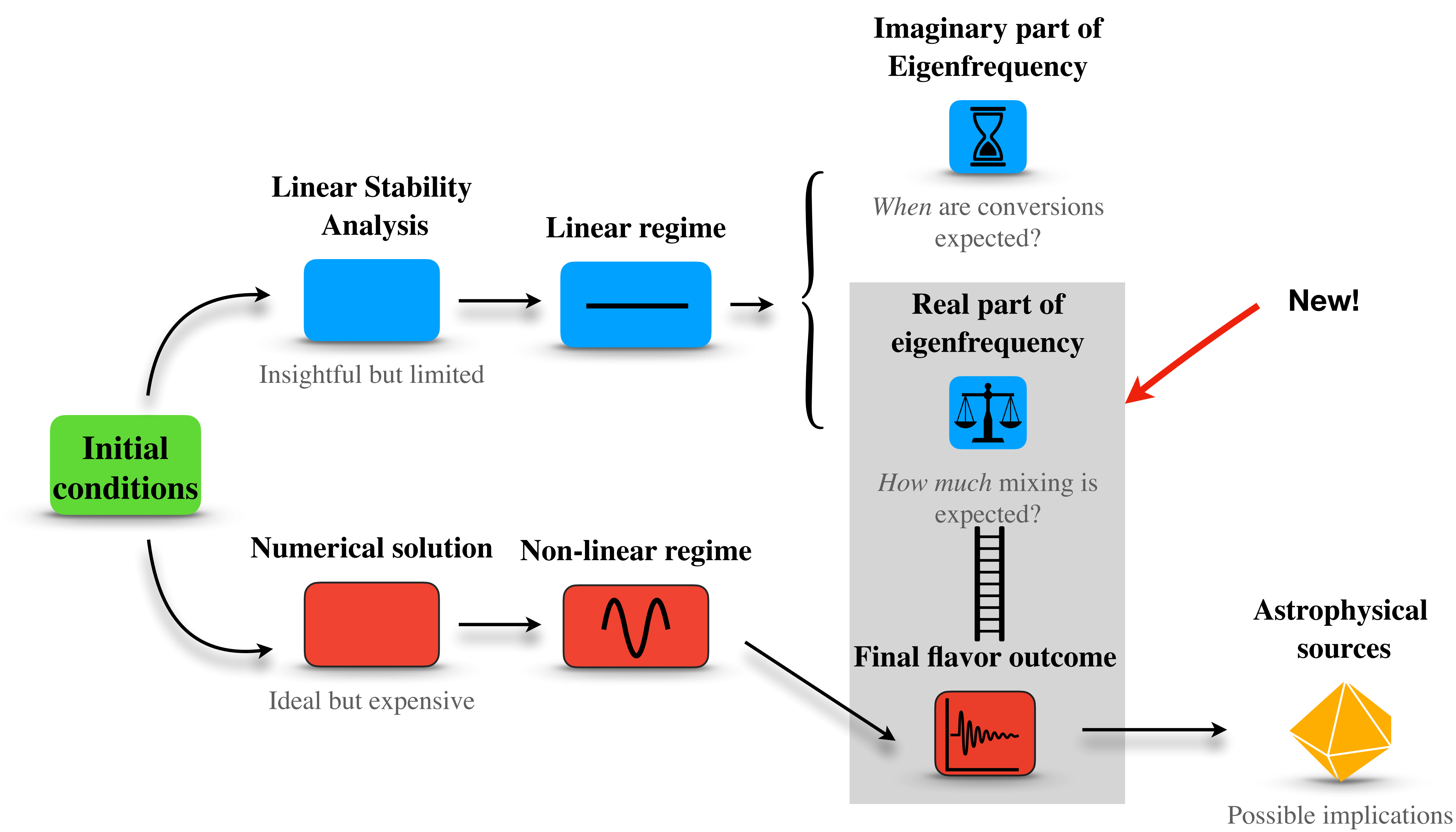
✅ Find a method to systematically and analytically predict flavor outcome based on initial ELN spectra

Can we gauge the amount of conversions without evolving the equations of motion?

How do we investigate neutrino self-interaction?



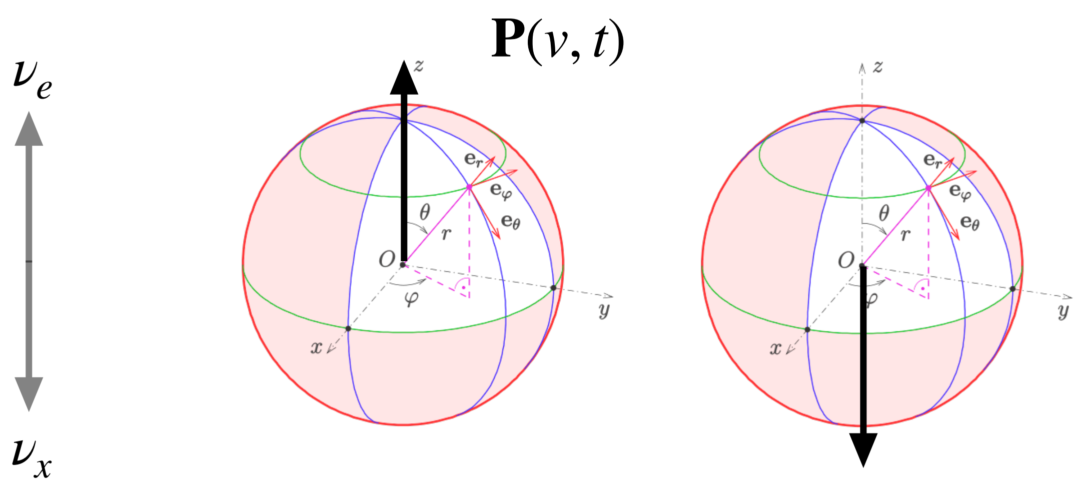
What is new?



Imaginary part of Eigenfrequency
When are conversions expected?
Linear Stability Analysis
Linear regime
Insightful but limited
Real part of eigenfrequency
How much mixing is expected?
New!
Initial conditions
Numerical solution
Non-linear regime
Final flavor outcome
Astrophysical sources
Ideal but expensive
Possible implications

Flavor polarization vectors

- Geometric representation



$P_a = \mathrm{Tr}(\rho \sigma_a)$

Lepton number

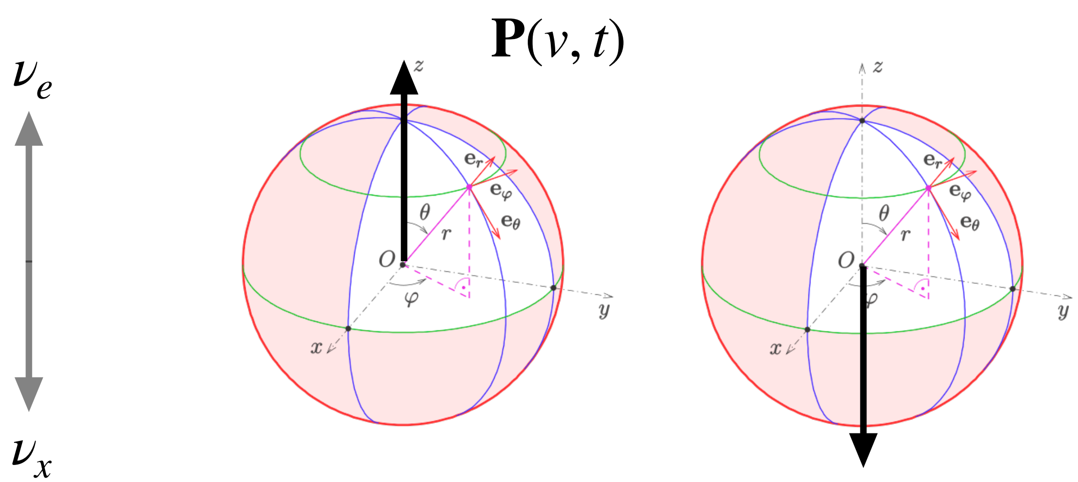
$$\mathbf{D}(v, t) = \mathbf{P}(v, t) - \bar{\mathbf{P}}(v, t)$$

Particle number

$$\mathbf{S}(v, t) = \mathbf{P}(v, t) + \bar{\mathbf{P}}(v, t)$$

Flavor polarization vectors

- Geometric representation



$$P_a = \mathrm{Tr}(\rho\sigma_a)$$

Lepton number

$$\mathbf{D}(v,t) = \mathbf{P}(v,t) - \bar{\mathbf{P}}(v,t)$$

Particle number

$$\mathbf{S}(v,t) = \mathbf{P}(v,t) + \bar{\mathbf{P}}(v,t)$$

- Then, the EOMs look simpler

$$\dot{\mathbf{D}}(v) = \mu v \mathbf{D}(v) \times \mathbf{D}_1$$

$$\dot{\mathbf{S}}(v) = \mu v \mathbf{S}(v) \times \mathbf{D}_1$$

- And convenient because a special role is played by

<u>Lepton number density</u>

$$\mathbf{D_0} = \int_{-1}^{+1} dv\, \mathbf{D}(v,t)$$

conserved

<u>Lepton number flux</u>

$$\mathbf{D_1} = \int_{-1}^{+1} dv\, v\, \mathbf{D}(v,t)$$

dynamic

Gyroscopic pendulum in flavor space

Linearly independent functions:

Mechanical analogy

"Gravity" = Lepton-number density vector

$$\boldsymbol{G} \;=\; \boldsymbol{D}_0 \;=\; \int dv\, \boldsymbol{D}_v(t)$$

Pendulum = Lepton-number flux vector

$$\boldsymbol{R}(t) \;=\; \boldsymbol{D}_1(t) = \int dv\, v\boldsymbol{D}_v(t)$$

Total angular momentum

$$\boldsymbol{J}(t) = \int dv\, w_v \boldsymbol{D}_v(t)$$

Gyroscopic pendulum in flavor space

Linearly independent functions:

Mechanical analogy

"Gravity" = Lepton-number density vector

$$\boldsymbol{G} = \boldsymbol{D}_0 = \int dv\, \boldsymbol{D}_v(t)$$

Pendulum = Lepton-number flux vector

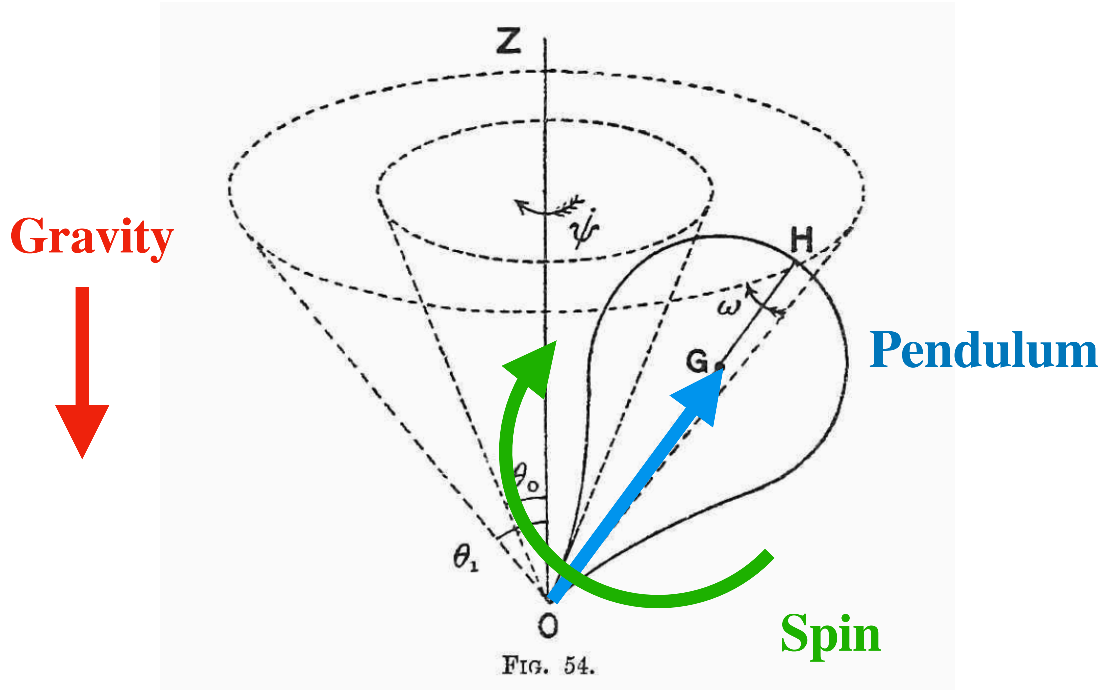
$$\boldsymbol{R}(t) = \boldsymbol{D}_1(t) = \int dv\, v\boldsymbol{D}_v(t)$$

Total angular momentum

$$\boldsymbol{J}(t) = \int dv\, w_v \boldsymbol{D}_v(t)$$

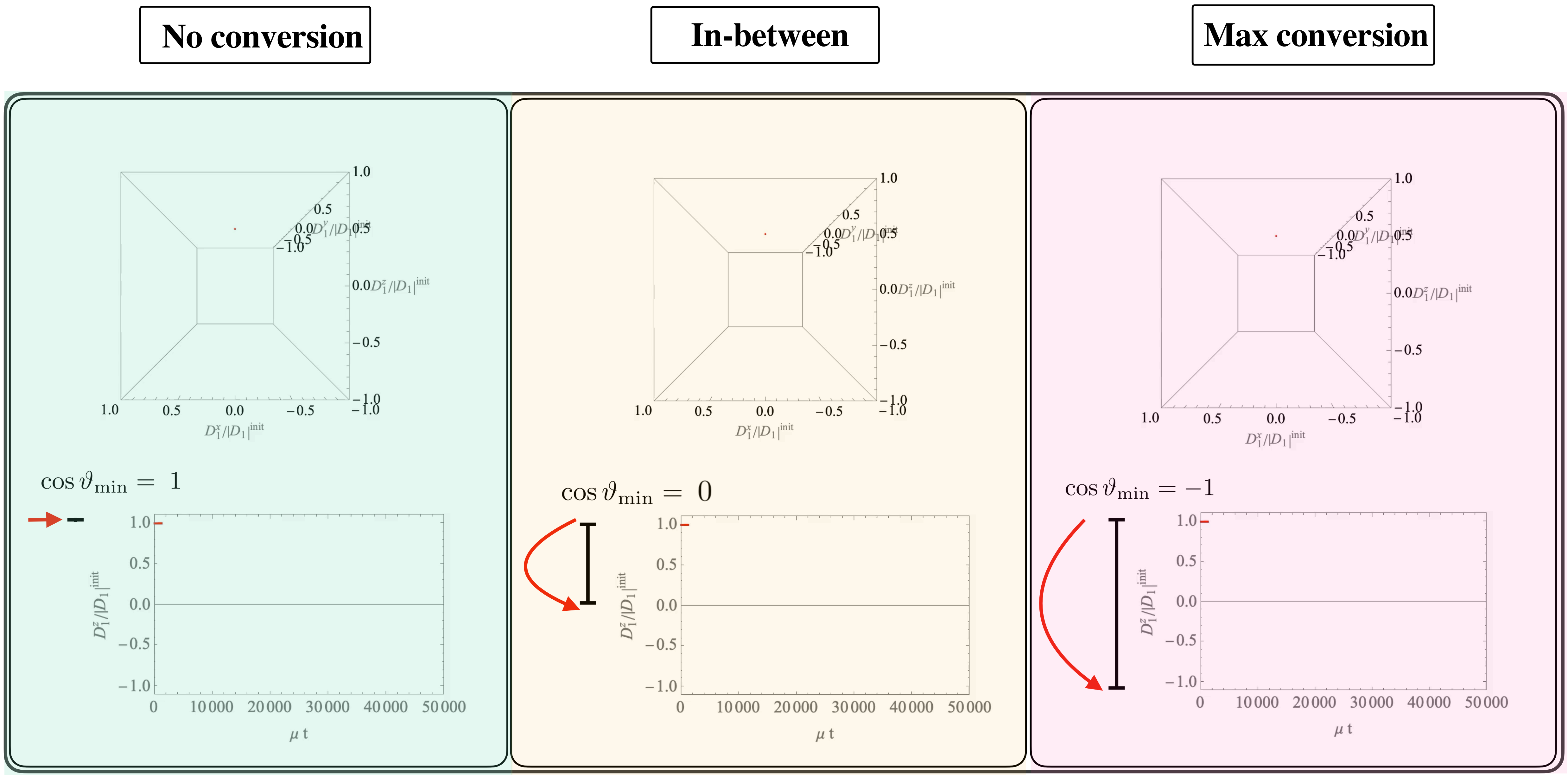
EOMs of a gyroscopic pendulum

$$\dot{\boldsymbol{G}} = 0\,, \quad \dot{\boldsymbol{R}} = \mu \boldsymbol{J} \times \boldsymbol{R} \quad \text{and} \quad \dot{\boldsymbol{J}} = \gamma \boldsymbol{G} \times \boldsymbol{R}\,.$$



Harold Crabtree (1909)

A mechanical analogy



Our criterion in practice

The linear stability analysis gives us:

1. The growth rate
2. Real part eigenfrequency (new)

$$\omega = \omega_{\mathrm{P}} \pm i\Gamma$$

Prediction for amount of flavor mixing (lowest point)

$$\cos\vartheta_{\min} = -1 + 2\,\frac{\omega_{\mathrm{P}}^2}{\omega_{\mathrm{P}}^2 + \Gamma^2}$$

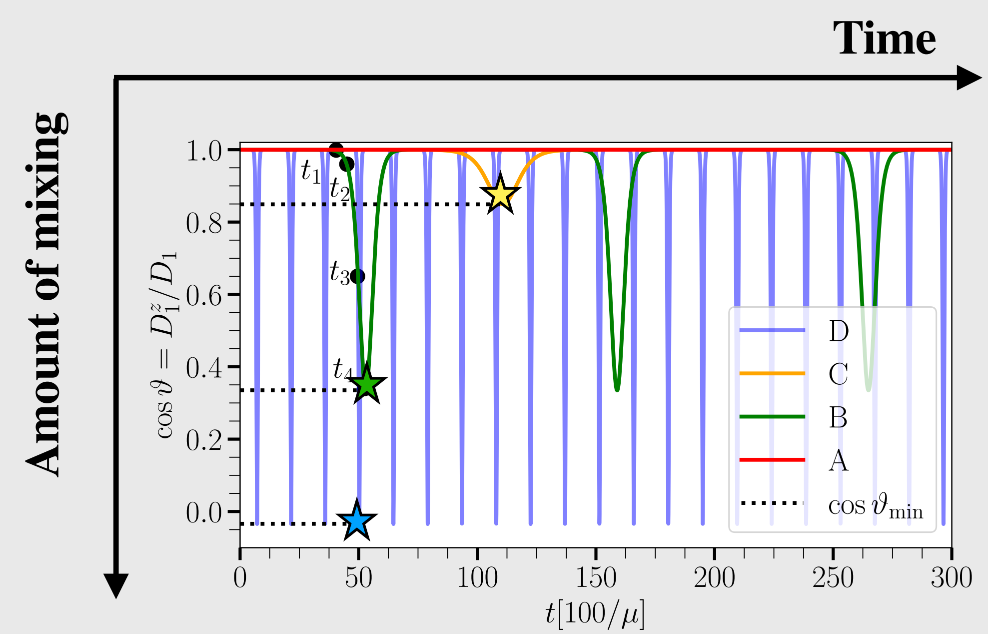
Lowest points

$\cos\vartheta_{\min}$
—
+0.335
+0.849
−0.034

- Stability if: $\Gamma = 0$
- Max conversion if: $\omega_{\mathrm{P}} = 0$

Excellent agreement: analytical vs numerical

dotted = analytical
solid = numerical



Take-home messages!

- Amount of flavor mixing can be estimated analytically

- One can identify:

  1. **Growth rate of instability** ~ imaginary part of eigenfrequency

  2. **Amount of mixing** ~ real part of eigenfrequency

- ELN spectrum → *when* and *how much* flavor conversion occurs

Ian Padilla-Gay, Irene Tamborra, Georg G. Raffelt, *Neutrino Flavor Pendulum Reloaded: The Case of Fast Pairwise Conversion*, Phys. Rev. Lett. 128 121102, arXiv:2109.14627

Ian Padilla-Gay, Shashank Shalgar, Irene Tamborra, *Multi-Dimensional Solution of Fast Neutrino Conversions in Binary Neutron Star Merger Remnants*, JCAP01(2021)017, arXiv:2009.01843

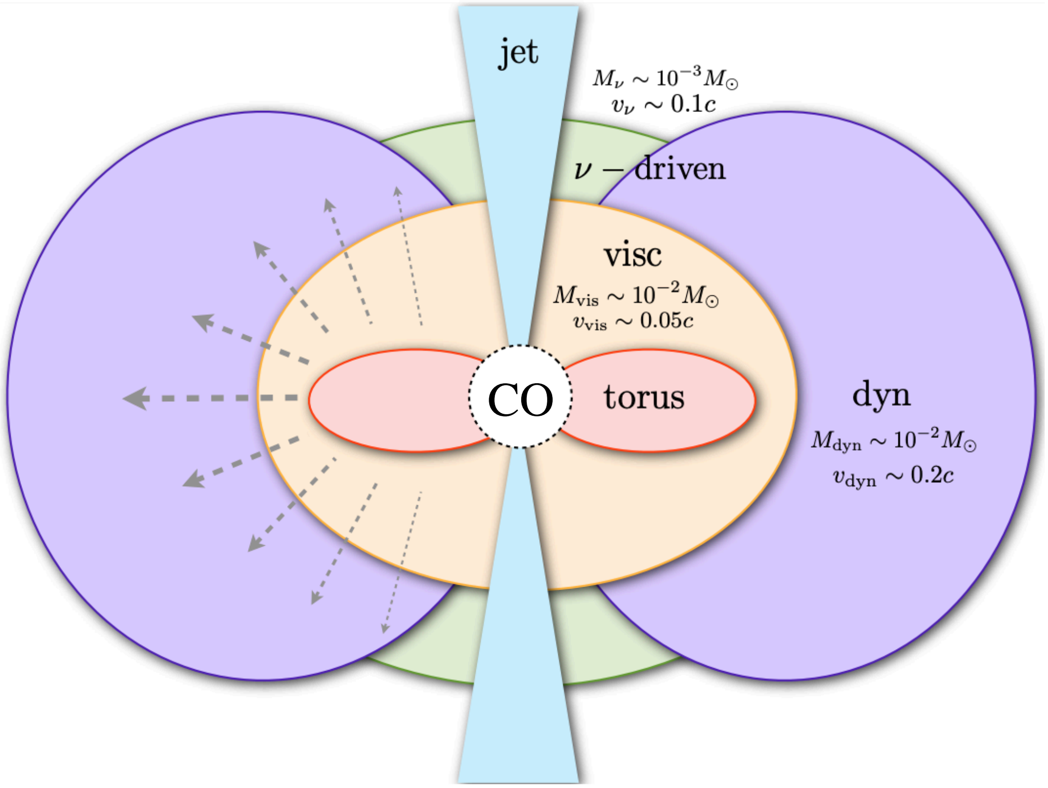
✓ Design a model of the flavor evolution of neutrinos in merger remnants

Where, when and how much flavor conversion take place?

Neutrino conversion in compact binary mergers

Mergers can host:

- MSW resonant conversion (km)
- Matter-neutrino resonances (km)
- Fast flavor conversion in the proximity of decoupling regions (m)

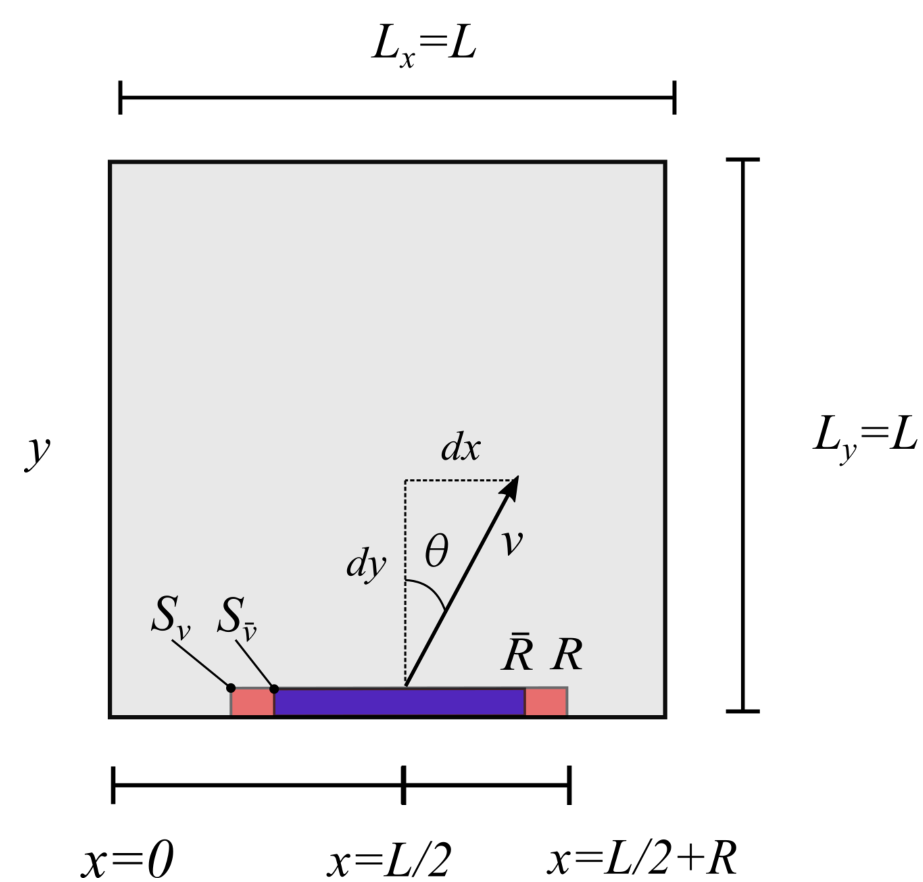


MR Wu et al 2017

Neutron star merger remnant

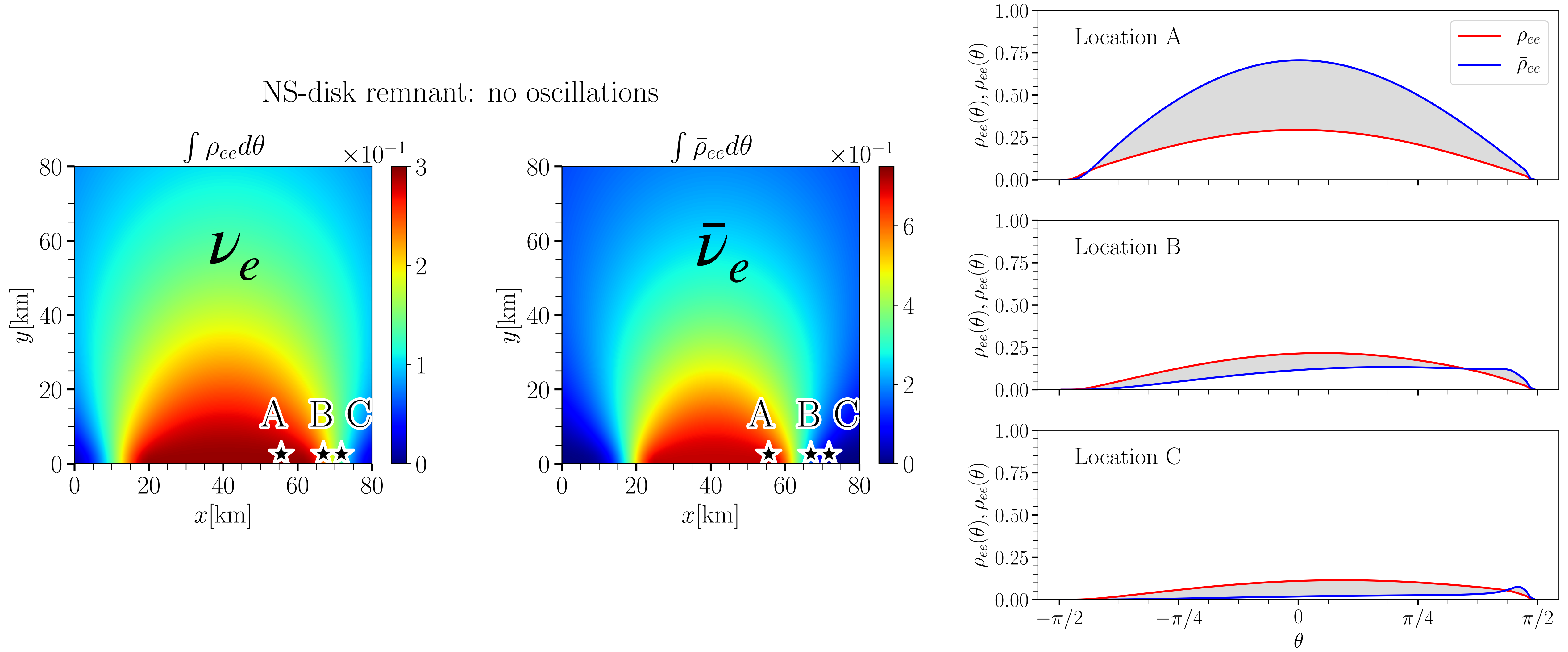
- Numerical solution of the neutrino flavor evolution above the remnant disk
- Dense neutrino gas with ν emission properties characteristic of a NS-disk merger remnant

Merger remnant disk setup



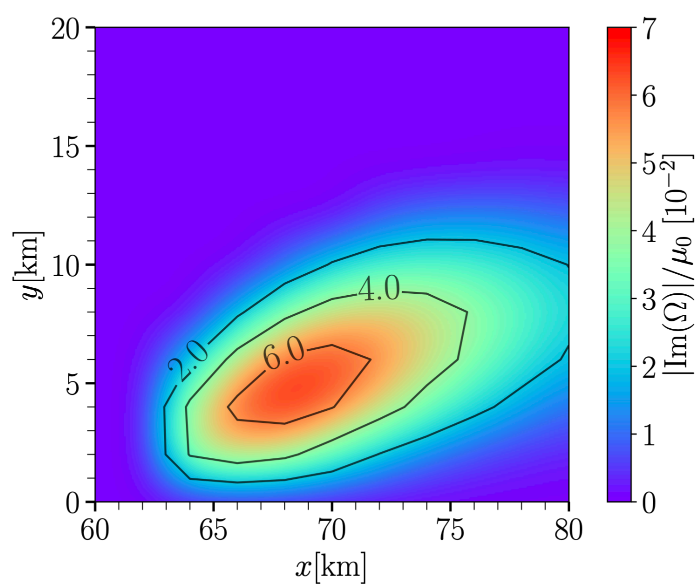
Neutron star merger remnant

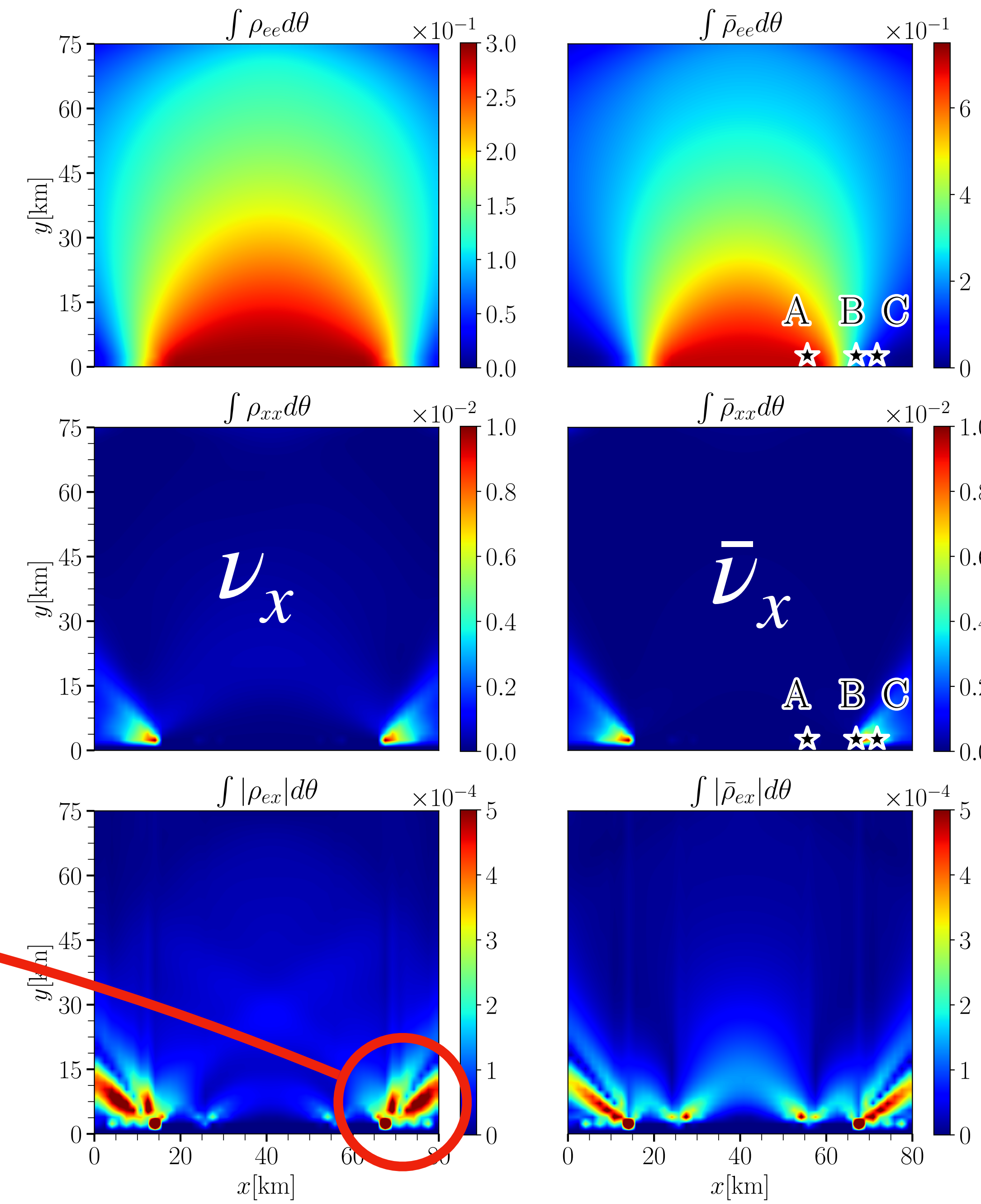
- Initial 2D setup as a function of x, y, θ
- Geometry + protonization of the merger remnant ($\bar{\nu}_e > \nu_e$ are emitted)



Flavor evolution

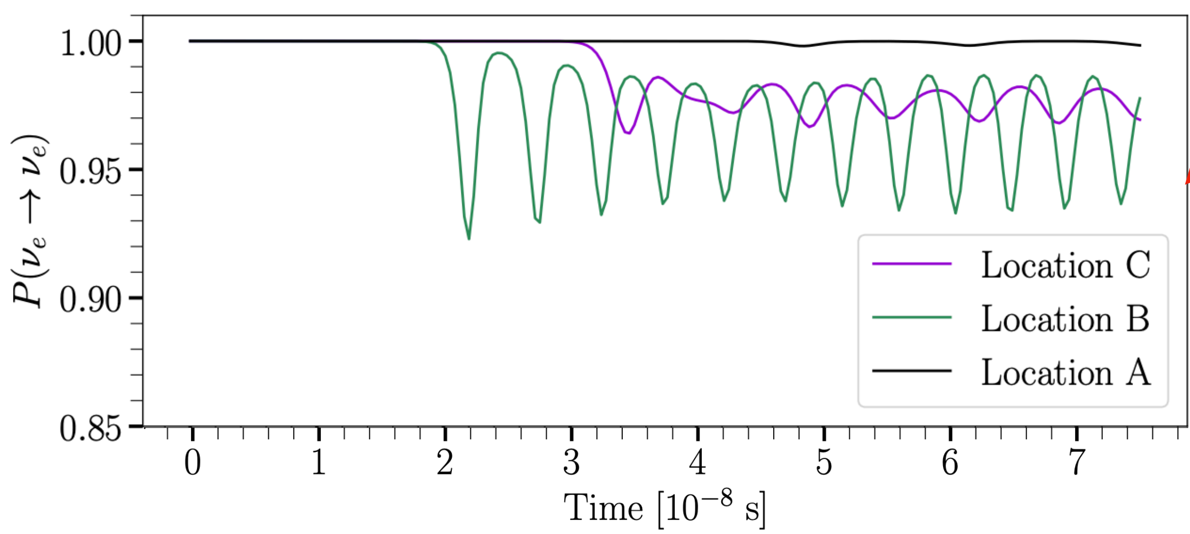
- Growth rates in agreement with previous studies
- Flavor evolution mainly occurring along the equator

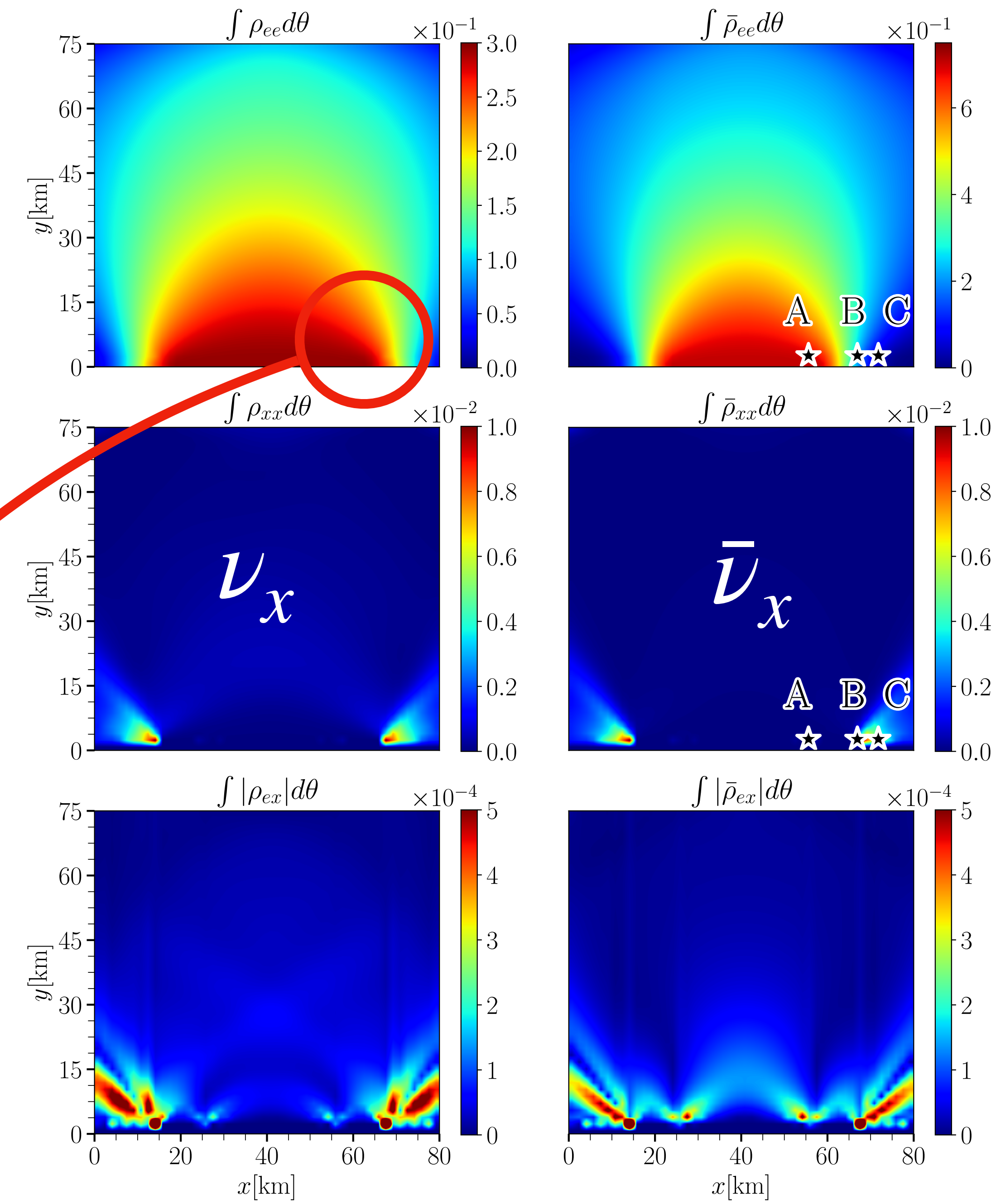




Flavor evolution

- Minimal neutrino flavor conversion
- Roughly 1% of non-electron flavors





Take-home messages!

- Fast flavor conversion occurs in compact binary merger remnants
- Within this model: minimal flavor conversion

Conclusions

- Neutrino flavor evolution in dense astrophysical environments is very complex
- Non-trivial role of neutrino-neutrino interaction in the equations of motion
- Fast flavor conversion likely to have a key role in the inner workings of core-collapse supernovae and neutron star merger remnants
- Implications for realistic astrophysical sources remain to be understood

Thank you!

🤓