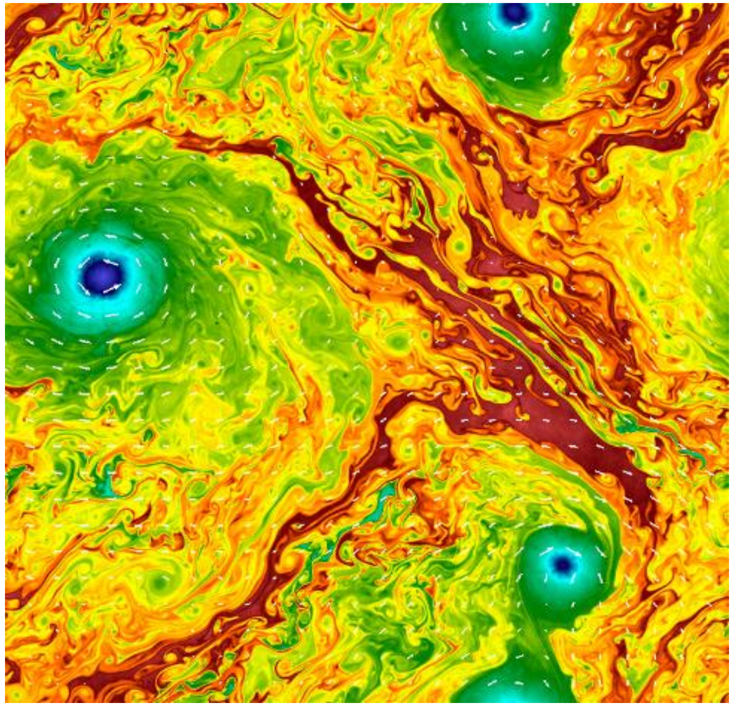
TURBULENT MAGNETIC FIELD AMPLIFICATION IN BINARY NEUTRON STAR MERGERS

Ricard Aguilera-Miret
Carlos Palenzuela, Daniele Viganò
Federico Carrasco, Borja Miñano,
Riccardo Ciolfi, Wolfgang Kastaun,
Jay Vijay Kalinani



Remnants of neutron-star mergers – Connecting hydrodynamics models to nuclear, neutrino, and kilonova physics, *October 18th*

Computational resources:
«LESBNS» project (20th PRACE Regular Call)
MareNostrum BSC

Long LES BNS project (21th PRACE Regular Call)
MareNostrum BSC

Universitat de les Illes Balears
IAC3 Institute of Applied Computing & Community Code.
PRACE

Modeling MHD turbulence

Turbulent magnetic field amplification in binary neutron star mergers C Palenzuela, R Aguilera-Miret, F Carrasco, R Ciolfi, JV Kalinani, ... Physical Review D 106 (2), 023013	3	2022
Universality of the turbulent magnetic field in hypermassive neutron stars produced by binary mergers R Aguilera-Miret, D Viganò, C Palenzuela The Astrophysical Journal Letters 926 (2), L31	6	2022
Turbulent magnetic-field amplification in the first 10 milliseconds after a binary neutron star merger: Comparing high-resolution and large-eddy simulations R Aguilera-Miret, D Viganò, F Carrasco, B Miñano, C Palenzuela Physical Review D 102 (10), 103006	20	2020
General relativistic MHD large eddy simulations with gradient subgrid-scale model D Viganò, R Aguilera-Miret, F Carrasco, B Miñano, C Palenzuela Physical Review D 101 (12), 123019	15	2020
Gradient subgrid-scale model for relativistic MHD large-eddy simulations F Carrasco, D Viganò, C Palenzuela Physical Review D 101 (6), 063003	15	2020
Extension of the subgrid-scale gradient model for compressible magnetohydrodynamics turbulent instabilities D Viganò, R Aguilera-Miret, C Palenzuela Physics of Fluids 31 (10), 105102	16	2019

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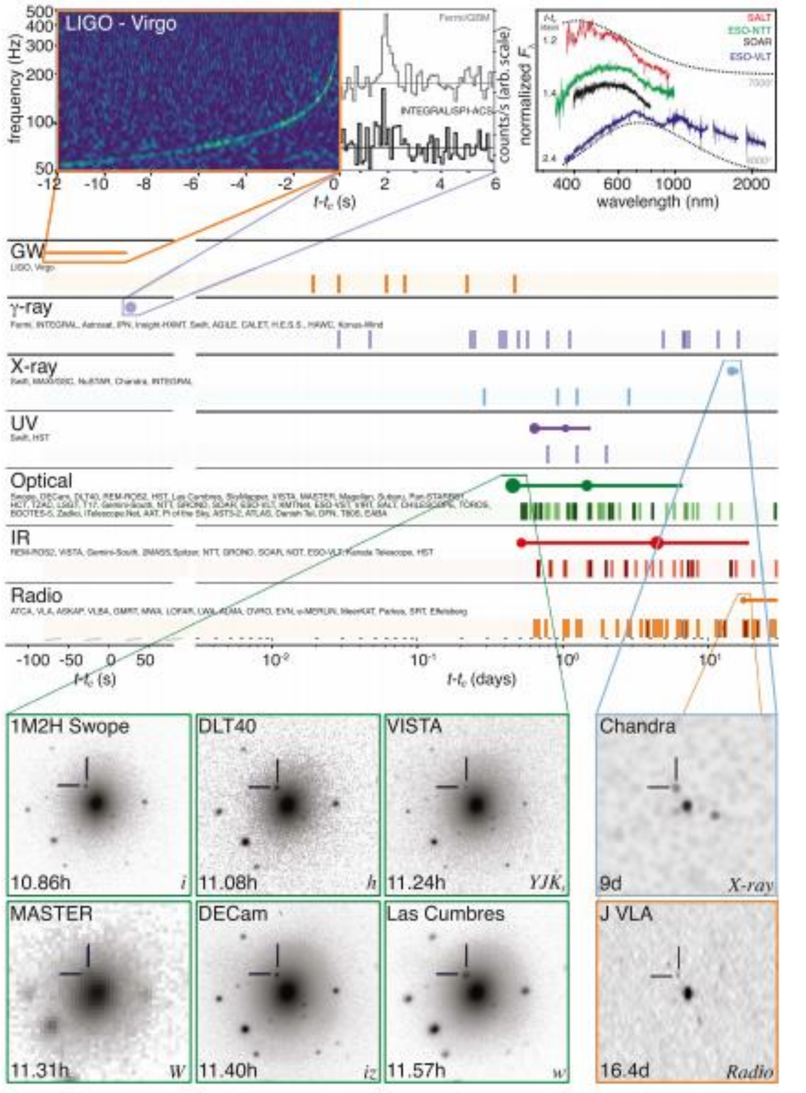
- **Introduction**
  - GW170817
  - Magnetic field amplification mechanisms
  - Importance of the initial magnetic field topology and strength?
- **Large Eddy Simulations (LES)**
  - Filtering
  - Gradient model
  - Compressible non-relativistic MHD evolution equations
  - GRMHD evolution equations
- **BNS mergers**
  - Effects of LES in BNS mergers
  - Importance of the magnetic field topology
- **Conclusions**

Binary Neutron Star Mergers



GW170817:
the beginning of the multi-messenger era

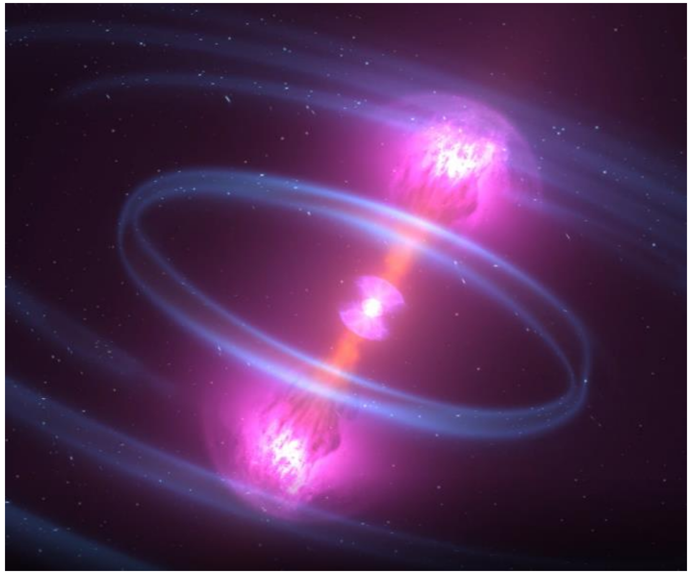
Things that we can learn:

- Test General Relativity (or alternative teories to GR)
- Internal properties of NSs (eq. state)
- **Magnetic field amplification mechanisms**
- Production of heavy elements
- EM counterpart (short GRB, kilonova)
- Formation of massive NS and/or light BH

Magnetic field amplification in mergers

- **PROCESSES DURING AND AFTER THE MERGER:**

  - Kelvin-Helmholtz instability (small scale)
  - Winding up (large scale)
  - Magneto-rotational instability (large scale)



Jet appear during the merger → Seems to need a **strong large-scale magnetic field**

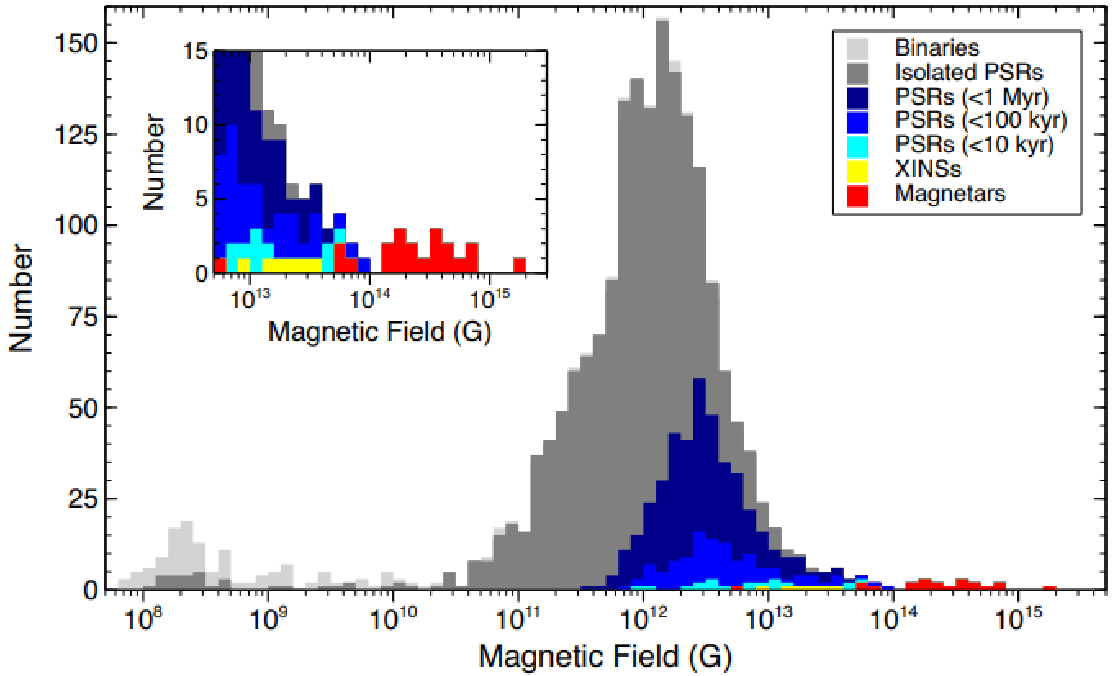
sGRB

Simulate these mechanisms via BNS merger simulations

Magnetic strenght of neutron stars

What are the typical magnetic fields expected for merging neutron stars?

Most works for simplicity (and for convenience) start with unrealistic magnetar-like values of purely dipolar fields (**10^{15} G**), either in the pre-merger or directly in the post-merger stage



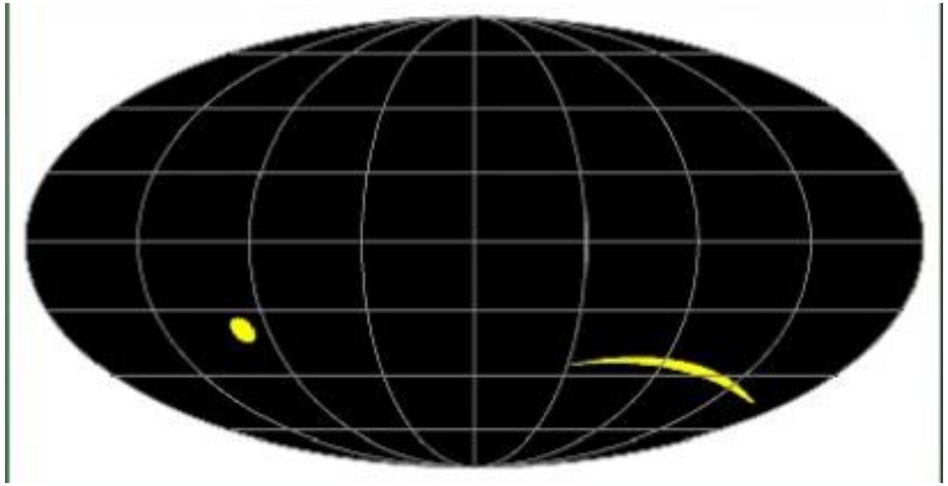
$B_{BNS} \sim 10^{8} – 10^{12}$ G

[Olausen & Kaspi 2014]

Magnetic field topology of neutron stars

What is the typical magnetic topology expected for neutron stars?

Most works for simplicity (and for convenience) start with unrealistic magnetar-like values of **purely dipolar** fields (10^{15} G), either in the pre-merger or directly in the post-merger stage



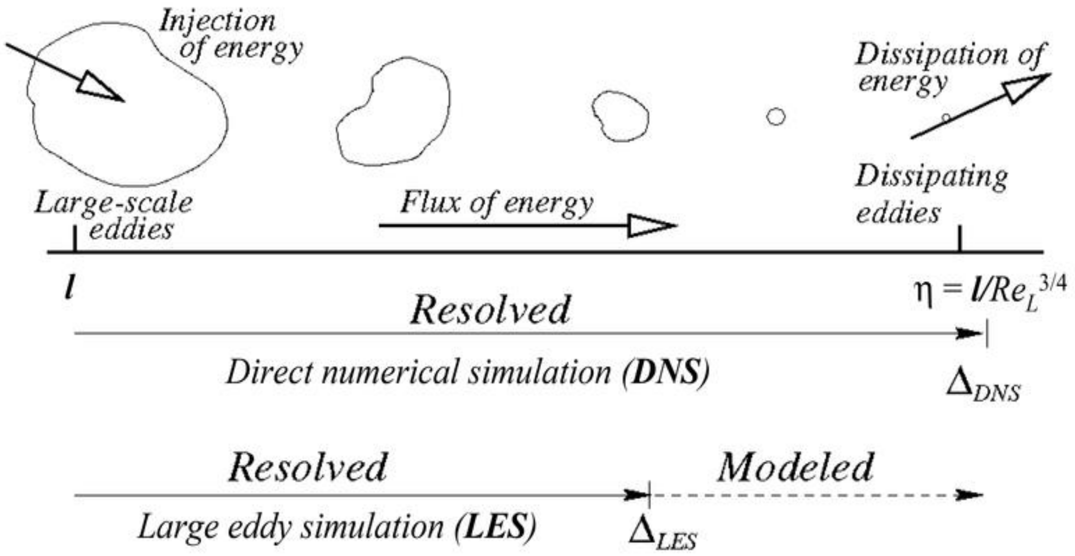
NICER results
[Riley++ 2019, Bogdanov++ 2019, Miller++ 2019]

Strong indications of a multipolar structure in NS
→
Assuming a strong dipolar magnetic field topology is unsupported by the NICER results.

Does the initial magnetic field strength and topology matter at all?

Simulating large eddies, modeling small eddies

Resolve all the scales → Costs lots of computational resources



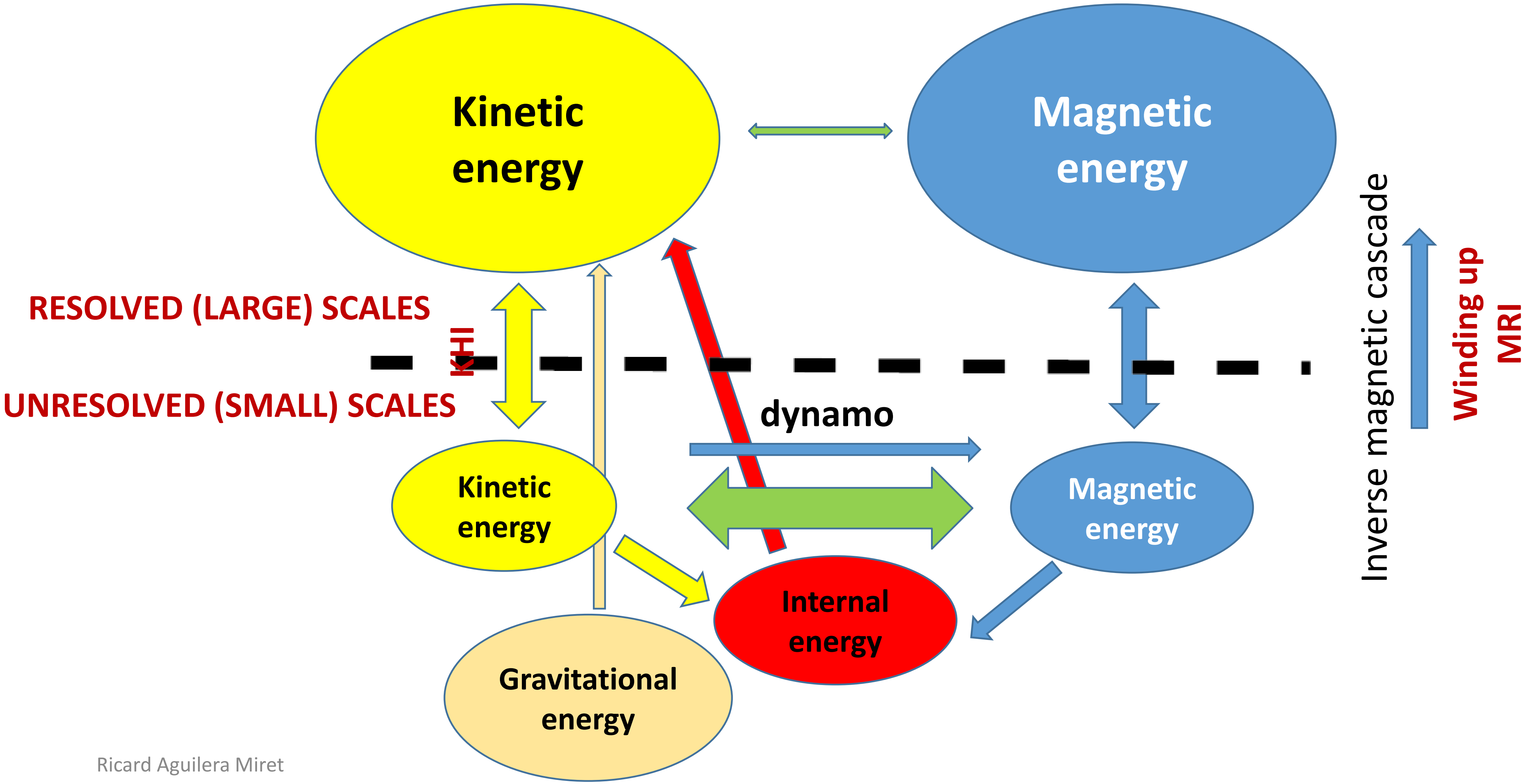
[Foroozani 2015]

The finite resolution of a simulation corresponds to an effective spatial filter for the fields:

$$f(\vec{x},t) = \overline{f}(\vec{x},t) + f'(\vec{x},t)$$

$$\bar{f}(\mathbf{x},t) = \int_{-\infty}^{\infty} G(\mathbf{x}-\mathbf{x}')f(\mathbf{x}',t)d^3x'.$$

Magnetic field amplification: MHD (Large Eddy Simulations)



The simplest example

Take the simplest non-linear evolution equation, Burgers:

$$\partial_t u + \frac{1}{2}\partial_x u^2 = 0$$

Apply the filter and **express your discretized equations only in terms of the resolved evolved fields:**

$$\partial_t \overline{u} + \frac{1}{2}\partial_x \overline{u}^2 = \frac{1}{2}\partial_x \overline{\tau} \qquad \overline{\tau} \equiv \overline{u}^2 - \overline{u^2}$$

The new sub-filter-scale tensor is not known, by definition. It needs to be modelled (or ignored)

Models can be explicit (Sub-Grid Scale/residual-based models), or implicitly given by the numerical dissipation (implicit LES, non-controllable and intrinsic to the scheme used)

Magnetic field amplification: MHD

DISCRETIZED EQUATIONS

$$\partial_t(\overline{\rho}\tilde{v}^i) + \partial_k\left[\overline{\rho}\tilde{v}^k\tilde{v}^i - \overline{B}^k\overline{B}^i + \delta^{ki}\left\{p(\overline{\rho},\tilde{e}) + \frac{1}{2}\overline{B}^2\right\}\right] =$$

$$= \partial_k\left[\overline{\tau}^{ki}_{\rm kin} - \overline{\tau}^{ki}_{\rm mag} + \delta^{ki}\delta_{lm}\left(\overline{\tau}^{lm}_{\rm p} + \frac{1}{2}\overline{\tau}^{lm}_{\rm mag}\right)\right] \quad (13)$$

$$\partial_t\overline{B}^i + \partial_k\left[\tilde{v}^k\overline{B}^i - \tilde{v}^i\overline{B}^k\right] = \partial_k\overline{\tau}^{ki}_{\rm ind} \quad (14)$$

$$\partial_t\overline{U} + \partial_k\left[\widetilde{\Theta}\,\tilde{v}^k - (\tilde{v}_j\overline{B}^j)\overline{B}^k\right] = \partial_k[\overline{\tau}^k_{\rm adv} - \overline{\tau}^k_{\rm hel}]$$

UNKNOWN SFS TERMS

$$\overline{\tau}^{ki}_{\rm kin} = \overline{\rho}\,\tilde{v}^k\,\tilde{v}^i - \overline{\rho v^k v^i}$$

$$\overline{\tau}^{ki}_{\rm mag} = \overline{B}^k\,\overline{B}^i - \overline{B^k B^i}$$

$$\overline{\tau}^{ki}_{\rm ind} = (\tilde{v}^k\,\overline{B}^i - \tilde{v}^i\,\overline{B}^k) - (\overline{v^k B^i} - \overline{v^i B^k})$$

$$\overline{\tau}_{\rm pres} = \widetilde{p} - \overline{p} + \frac{1}{2}\overline{\tau}^{jm}_{\rm mag}\delta_{jm}$$

$$\overline{\tau}^k_{\rm adv} = \widetilde{\Theta}\,\tilde{v}^k - \overline{\Theta v^k}$$

$$\overline{\tau}^k_{\rm hel} = (\tilde{v}_j\,\overline{B}^j)\,\overline{B}^k - \overline{(v_j B^j) B^k}$$

Discretization (i.e., filtering) makes you lose some information contained in the Sub-Filter Scales, in the non-linear terms of the fluxes.

How to model them as a function of the known filtered values?

Subgrid-Scale modeling

$$\bar{f}(\mathbf{x},t) = \int_{-\infty}^{\infty} G(\mathbf{x}-\mathbf{x}')f(\mathbf{x}',t)d^3x'.$$

- Ideally, with infinite resolution, the kernel function is a Krönecker delta: $\delta(\mathbf{x}-\mathbf{x}')$
- **BUT:**
  - Our world is not ideal ☹
  - Simulations have a grid cell size (Δ)
- Simplest kernel function: Step function: $G_i(|\mathbf{x}-\mathbf{x}'|) = \begin{cases} 1/\Delta_f & \text{if } |\mathbf{x}-\mathbf{x}'| \leq \Delta_f/2 \\ 0 & \text{otherwise} \end{cases}$
- **BUT:**
  - It is not suitable for analytical calculations involving derivatives…

Subgrid-Scale modeling: gradient model

- The finite resolution of a simulation can be thought as a filter of conserved equations:

$$\partial_t \bar{U}^a + \partial_k F^{ka}(\bar{U}) = \partial_k \bar{\tau}^{ka}, \qquad \bar{\tau}_F^{ka} := F^{ka}(\bar{U}) - \overline{F^{ka}(U)}.$$

- The gradient model assumes a Gaussian kernel:

$$\bar{f}(\mathbf{x}, t) = \int_{-\infty}^{\infty} G(\mathbf{x} - \mathbf{x}') f(\mathbf{x}', t) d^3x'. \qquad G_i(|x_i - x_i'|) = \left(\frac{1}{4\pi\xi}\right)^{1/2} \exp\left(\frac{-|x_i - x_i'|^2}{4\xi}\right),$$

Moment (μ)	Step function kernel	Gaussian kernel
μ_0	1	1
μ_1	0	0
μ_2	$\Delta^2/12$	$2\,\xi$
μ_3	0	0

$$\xi = \Delta^2/24$$

We obtain a Gaussian function that ressambles to a step function up to the **third moment!**

Subgrid-Scale modeling: gradient model

- Its inverse Fourier transform, expanded in series of ξ , is:

$$\frac{1}{\hat{G}(\mathbf{k})} = \sum_{n=0}^{\infty} \frac{1}{n!} (\xi \mathbf{k}^2)^n.$$

- First-order Taylor series expansion of the Gaussian filter in the Fourier space.

$$f \equiv G^{-1} * \bar{f} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (\xi \nabla^2)^n \bar{f}.$$

- For a generic filtered product, we can **approximate its unknown filtered value with the filtered fields and their gradients (that we know)**:

$$\overline{fg} \simeq \overline{f}\,\overline{g} + 2\,\xi\,\nabla\overline{f} \cdot \nabla\overline{g}$$

The new terms vanish by construction in the continuous limit (as most SGS models do).

Compressible non-relativistic MHD with the gradient model

[Viganò+ 2019]

DISCRETIZED EQUATIONS

$$\partial_t(\overline{\rho}\tilde{v}^i) + \partial_k \left[\overline{\rho}\tilde{v}^k\tilde{v}^i - \overline{B}^k\overline{B}^i + \delta^{ki}\left\{p(\overline{\rho},\tilde{e}) + \frac{1}{2}\overline{B}^2\right\}\right] =$$

$$= \partial_k\left[\tau^{ki}_{\rm kin} - \tau^{ki}_{\rm mag} + \delta^{ki}\delta_{lm}\left(\tau^{lm}_{\rm p} + \frac{1}{2}\tau^{lm}_{\rm mag}\right)\right] \qquad (13)$$

$$\partial_t\overline{B}^i + \partial_k\left[\tilde{v}^k\overline{B}^i - \tilde{v}^i\overline{B}^k\right] = \partial_k\tau^{ki}_{\rm ind} \qquad (14)$$

$$\partial_t\overline{U} + \partial_k\left[\widetilde{\Theta}\,\tilde{v}^k - (\tilde{v}_j\overline{B}^j)\overline{B}^k\right] = \partial_k[\tau^k_{\rm adv} - \tau^k_{\rm hel}]$$

UNKNOWN SFS TERMS

$$\tau^{ki}_{\rm kin} = \overline{\rho}\,\tilde{v}^k\,\tilde{v}^i - \overline{\rho v^k v^i}$$

$$\tau^{ki}_{\rm mag} = \overline{B}^k\,\overline{B}^i - \overline{B^k B^i}$$

$$\tau^{ki}_{\rm ind} = (\tilde{v}^k\,\overline{B}^i - \tilde{v}^i\,\overline{B}^k) - (\overline{v^k B^i} - \overline{v^i B^k})$$

$$\tau_{\rm pres} = \tilde{p} - \overline{p} + \frac{1}{2}\tau^{jm}_{\rm mag}\delta_{jm}$$

$$\tau^k_{\rm adv} = \widetilde{\Theta}\,\tilde{v}^k - \overline{\Theta v^k}$$

$$\tau^k_{\rm hel} = (\tilde{v}_j\,\overline{B}^j)\,\overline{B}^k - \overline{(v_j B^j)B^k}$$

GRADIENT SGS MODEL TERMS

$$\tau^{ki}_{\rm kin} = -2\xi\,\overline{\rho}\,\partial_j\tilde{v}^k\,\partial^j\tilde{v}^i$$

$$\tau^{ki}_{\rm mag} = -2\xi\,\partial_j\overline{B}^k\,\partial^j\overline{B}^i$$

$$\tau^{ki}_{\rm ind} = -2\xi\left[\partial_j\tilde{v}^k\left(\partial^j\overline{B}^i - \frac{\overline{B}^i}{\overline{\rho}}\partial^j\overline{\rho}\right) - \partial_j\tilde{v}^i\left(\partial^j\overline{B}^k - \frac{\overline{B}^k}{\overline{\rho}}\partial^j\overline{\rho}\right)\right].$$

$$\tau^k_{\rm adv} = \tau_{\rm pres}\tilde{v}^k - 2\xi\left(\partial_j\widetilde{\Theta}\partial^j\tilde{v}^k - \frac{\Theta}{\overline{\rho}}\partial_j\overline{\rho}\partial^j\tilde{v}^k\right)$$

$$\tau^k_{\rm hel} = -2\xi\left[\partial_j(\tilde{v}_m\overline{B}^m)\partial^j\overline{B}^k + \overline{B}^k\partial^j\tilde{v}^m\left(\partial_j\overline{B}_m - \frac{\overline{B}_m}{\overline{\rho}}\partial_j\overline{\rho}\right)\right]$$

Gradient model for general relativistic MHD

[Viganò+ 2020]

$$\partial_t(\sqrt{\gamma}\bar{D}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{D} + \alpha\sqrt{\gamma}(\tilde{N}^k - \bar{\tau}^k_N)] = 0,$$
$$\partial_t(\sqrt{\gamma}\bar{S}_i) + \partial_k[-\beta^k\sqrt{\gamma}\bar{S}_i + \alpha\sqrt{\gamma}(\tilde{T}^k_i - \gamma_{ij}\bar{\tau}^{jk}_T)] = \sqrt{\gamma}\overline{R^S}_i,$$
$$\partial_t(\sqrt{\gamma}\bar{U}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{U} + \alpha\sqrt{\gamma}(\tilde{S}^k - \bar{\tau}^k_S)] = \sqrt{\gamma}\overline{R^U},$$
$$\partial_t(\sqrt{\gamma}\bar{B}^i) + \partial_k[\sqrt{\gamma}(-\beta^k\bar{B}^i + \beta^i\bar{B}^k) + \alpha\sqrt{\gamma}(\gamma^{ki}\bar{\phi} + \tilde{M}^{ki} - \bar{\tau}^{ki}_M)] = \sqrt{\gamma}\overline{R_B}^i, \quad (20)$$

$$\partial_t(\sqrt{\gamma}\bar{\phi}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{\phi} + \alpha c_h^2\sqrt{\gamma}\bar{B}^k] = \sqrt{\gamma}\overline{R^\phi}, \quad (21)$$

$$\tau^k_N = -\mathcal{C}\xi H^k_N, \qquad \tau^{ki}_T = -\mathcal{C}\xi H^{ki}_T,$$
$$\tau^k_S = 0, \qquad \tau^{ki}_M = -\mathcal{C}\xi H^{ki}_M.$$

$$H^k_N = 2\nabla\bar{D}\cdot\nabla\tilde{v}^k + \bar{D}H^k_v, \quad (48)$$

$$H^{ki}_T = 2[\nabla\tilde{\mathcal{E}}\cdot\nabla(\tilde{v}^i\tilde{v}^k) + \tilde{\mathcal{E}}(\tilde{v}^{(i}H^{k)}_v + \nabla\tilde{v}^i\cdot\nabla\tilde{v}^k)] + \tilde{v}^i\tilde{v}^k H_{\mathcal{E}} - 2[\nabla\bar{B}^i\cdot\nabla\bar{B}^k + \nabla\tilde{E}^i\cdot\nabla\tilde{E}^k + \tilde{E}^{(i}H^{k)}_E] + \delta^{ki}[H_p + \nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j + \tilde{E}_j H^j_E], \quad (49)$$

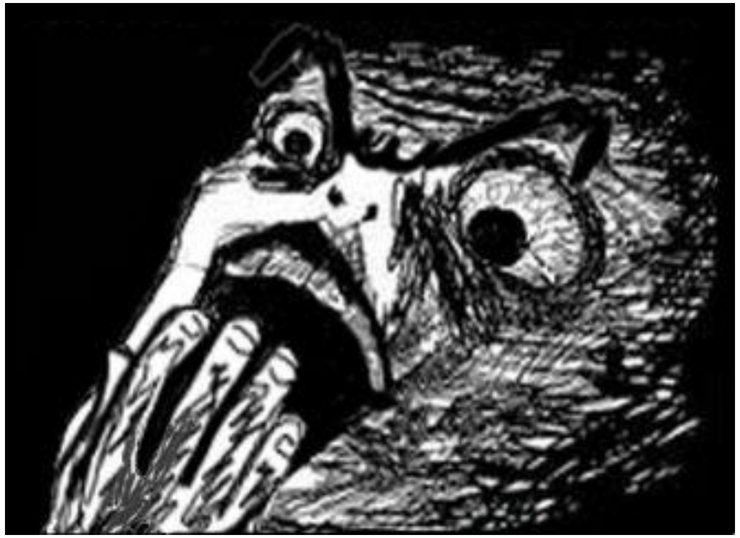
$$H^{ki}_M = 4\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + 2\bar{B}^{[i}H^{k]}_v, \quad (50)$$

$$H_{\mathcal{E}} = H_p - \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \tilde{E}_k H^k_E,$$
$$H_\Theta = \tilde{\Psi}_\Theta + \frac{\tilde{\Theta}}{\tilde{\Theta}-\tilde{E}^2}H_p,$$
$$H^k_v := \tilde{\Psi}^k_v - \left(\tilde{v}^k + \frac{\tilde{v}\cdot\bar{B}}{\tilde{\mathcal{E}}}\bar{B}^k\right)\frac{H_\Theta}{\tilde{\Theta}}.$$

$$\tilde{\Psi}^k_v = \frac{2}{\tilde{\Theta}}\left\{\nabla(\tilde{v}\cdot\bar{B})\cdot\nabla\bar{B}^k - \nabla\tilde{\Theta}\cdot\nabla\tilde{v}^k + \frac{\bar{B}^k}{\tilde{\mathcal{E}}}[\tilde{\Theta}\nabla\bar{B}_j\cdot\nabla\tilde{v}^j + \bar{B}_j\nabla\bar{B}^j\cdot\nabla(\tilde{v}\cdot\bar{B}) - \bar{B}_j\nabla\tilde{v}^j\cdot\nabla\tilde{\Theta}]\right\},$$
$$\tilde{\Psi}^{ki}_M = \frac{4}{\tilde{\Theta}}[\tilde{\Theta}\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + \bar{B}^{[i}\nabla\bar{B}^{k]}\cdot\nabla(\tilde{v}\cdot\bar{B}) - \bar{B}^{[i}\nabla\tilde{v}^{k]}\cdot\nabla\tilde{\Theta}],$$
$$\tilde{\Psi}_\Theta = \frac{\tilde{\Theta}}{\tilde{\Theta}-\tilde{E}^2}\{\nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \bar{B}_{[i}\tilde{v}_{k]}\tilde{\Psi}^{ki}_M\},$$
$$\tilde{\Psi}_A = \tilde{W}^2\left(\tilde{p}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\rho}^2\frac{d\tilde{p}}{d\tilde{\rho}}\right),$$

[Carrasco+, 2020]

$$\frac{H_p}{\tilde{\Theta}-\tilde{E}^2} = \frac{\tilde{\mathcal{E}}\tilde{W}^2}{(\tilde{\rho}\,\tilde{\mathcal{E}}-\tilde{\Psi}_A)(\tilde{\Theta}-\tilde{E}^2)\tilde{W}^2 + \tilde{\Psi}_A\tilde{\Theta}}\left\{\tilde{\rho}\left(\nabla\frac{d\tilde{p}}{d\tilde{\rho}}\cdot\nabla\tilde{\rho} + \nabla\frac{d\tilde{p}}{d\tilde{\epsilon}}\cdot\nabla\tilde{\epsilon}\right) - 2\frac{d\tilde{p}}{d\tilde{\epsilon}}\nabla\tilde{\rho}\cdot\nabla\tilde{\epsilon}\right.$$
$$-\left(\tilde{\mathcal{E}}\frac{d\tilde{p}}{d\tilde{\epsilon}} - \tilde{\Psi}_A\right)\left[\frac{\tilde{W}^2}{4}\nabla\tilde{W}^{-2}\cdot\nabla\tilde{W}^{-2} + \nabla\tilde{W}^{-2}\cdot\nabla(\ln\tilde{\rho})\right] - \frac{2}{\tilde{W}^2}\frac{d\tilde{p}}{d\tilde{\epsilon}}[\nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{W}^2\cdot\nabla\tilde{h}]$$
$$\left.-\left(\tilde{\mathcal{E}}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\Psi}_A\right)[\tilde{v}_k\tilde{\Psi}^k_v + \nabla\tilde{v}_j\cdot\nabla\tilde{v}^j + \tilde{W}^2\nabla\tilde{W}^{-2}\cdot\nabla\tilde{W}^{-2}] + \frac{1}{\tilde{\mathcal{E}}}\left[\left(\tilde{\mathcal{E}}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\Psi}_A\right)(\tilde{\Theta}-\tilde{E}^2) - \frac{\tilde{\Psi}_A\tilde{\Theta}}{\tilde{W}^2}\right]\frac{\tilde{\Psi}_\Theta}{\tilde{\Theta}}\right\},$$



Gradient model for general relativistic MHD

[Viganò+ 2020]

Assumptions & Caveats

- **The space-time metric is not "turbulent"**, i.e., the gradient terms arising from metric components in the fluid equations are neglected (verified by a-priori tests under typical conditions)
- Similarly, **the SGS terms arising in the Einstein equations are not included**, i.e., the steepness (derivatives) of MHD fields are dominating the non-linearity of the turbulence.
- The **SGS modelling mimics the dynamics down to finite "depths"** inside the cell: if physical dynamics qualitatively differ at much smaller scales, there is nothing one can do.

Methods & Models considered

[Palenzuela+ 2022]

MHDuet code generated with Simflowny sowftware

Einstein equation 4th order accurate finite differences
Kreiss-Oliger 6th order dissipation
Fluid MP5 reconstruction scheme + Lax-Friedrichs flux splitting formula
LES 4th order differential operators for SGS terms
4th order Runge-Kutta

Numerical methods

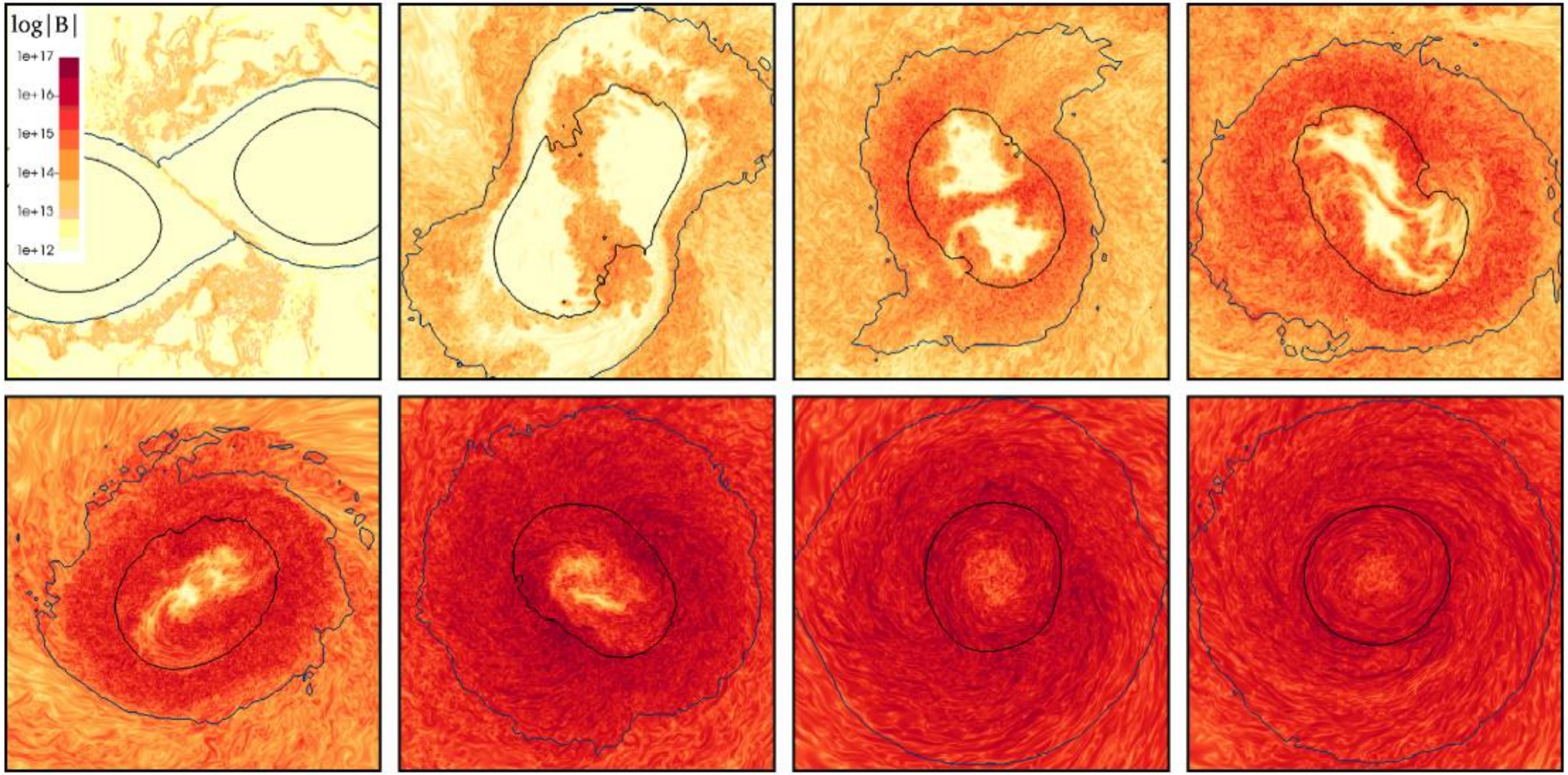
CCZ4 formulation of Einstein equations.
Initial data by Lorene code, equal masses (1.3 Msun), quasi-circular orbits separated by 37 km
Magnetic fields initially **10^{11} G**, confined to each star
Hybrid EoS: piecewise APR4 + ideal

Physical setup

Case	$C_{\mathcal{M}}$	Refinement levels	ΔL_{min} [km]	Δ_{min} [m]
LR	0	7 FMR	[-28,28]	120
MR	0	7 FMR + 1 AMR	[-13,13]	60
HR	0	7 FMR + 2 AMR	[-11,11]	30
LR LES	8	7 FMR	[-28,28]	120
MR LES	8	7 FMR + 1 AMR	[-13,13]	60
HR LES	8	7 FMR + 2 AMR	[-11,11]	30
MR BO	8	7 FMR + 1 AMR	[-13,13]	60

Binary NS evolution

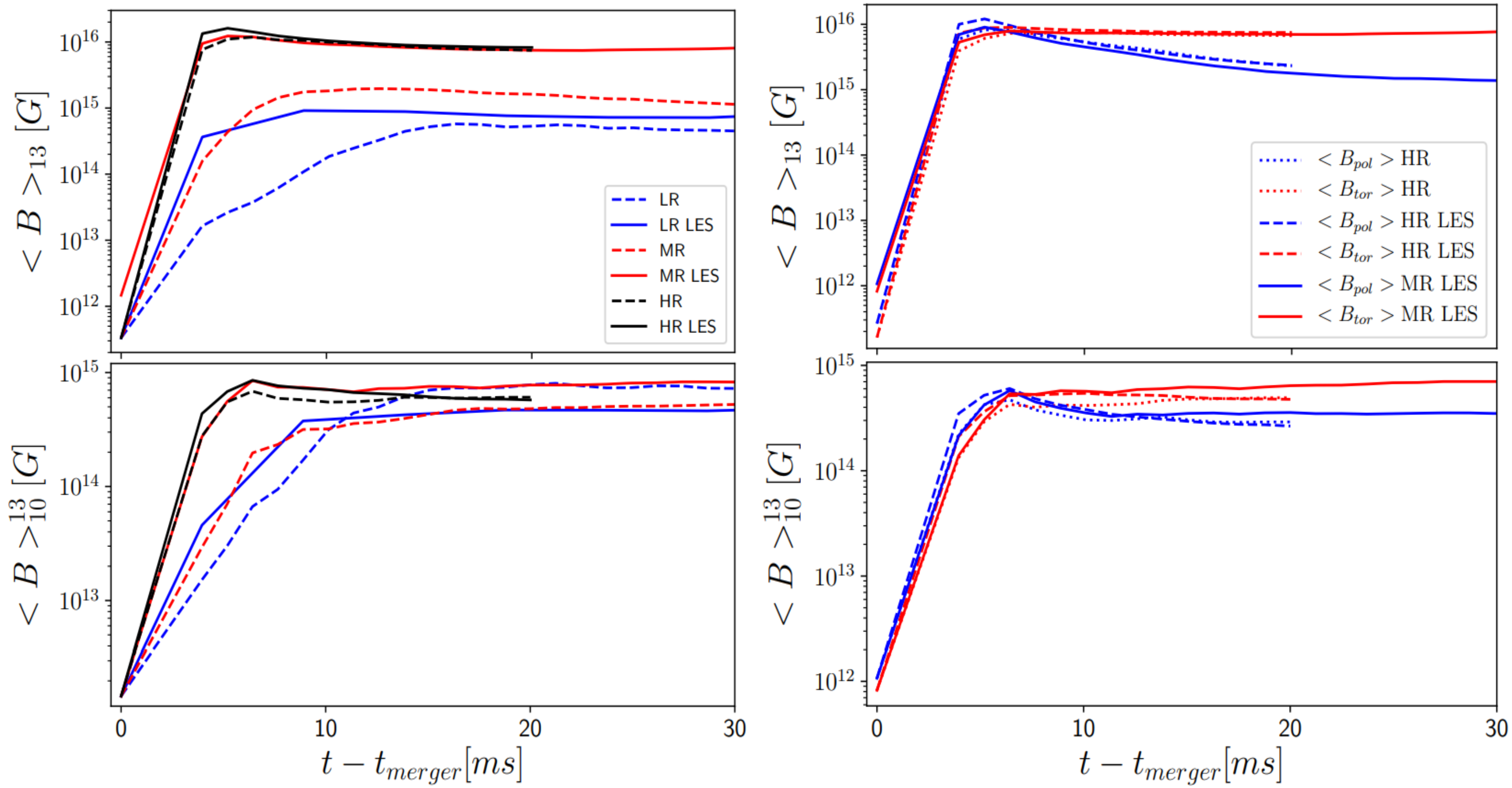
[Palenzuela+ 2022]



Magnetic field strength at t = (0.5, 1.5, 2, 2.5, 3.5, 5, 10, 15) ms
Constant density surfaces in 10^{13} and 5×10^{14} g / cm^3

Binary NS evolution

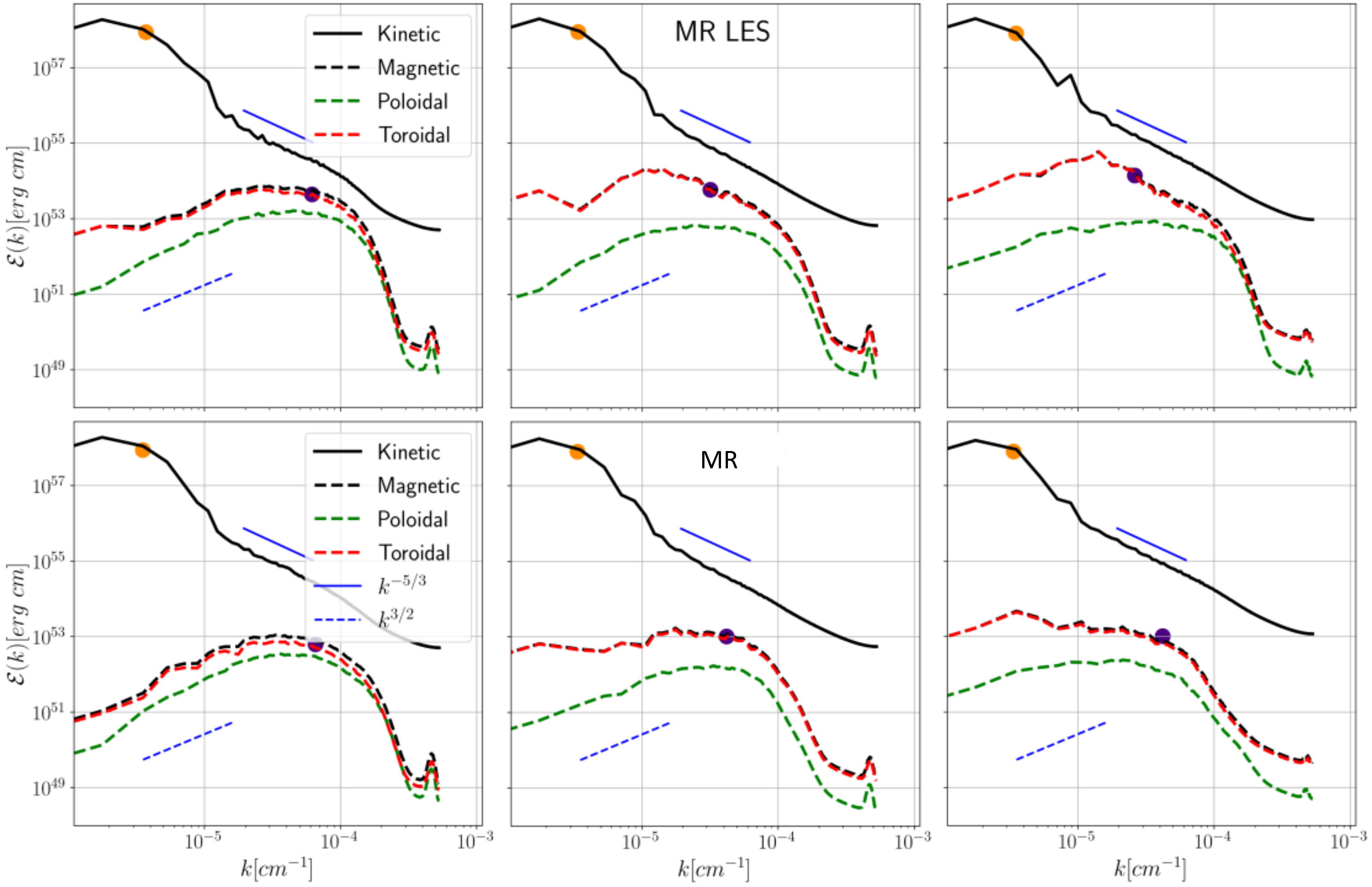
[Palenzuela+ 2022]



Saturation of magnetic field in t < 5 ms and convergence of averaged magnetic field strength and components!!

Binary NS evolution

[Palenzuela+ 2022]



t=10 ms **t=30ms** **t=50ms**

Magnetic field characteristic structure:

t = 10 ms → 700 m
t = 50 ms → 2 km

Spectra of the kinetic and magnetic energy (poloidal and toroidal)

Binary NS evolution (diferent topologies)

[Aguilera-Miret+ 2022]

Does the initial magnetic field strength and topology matter at all?

Dependence on the initial magnetic field topology

Consider different initial topologies of the magnetic field inside the star before the merger

1) Dipolar magnetic field <B> ~ 10^{11} G **(Dip)**

$$A_\phi \propto r^2(P - P_{cut})$$

2) Dipolar magnetic field <B> ~ 10^{14} G **(Bhigh)**

3) Dipolar with magnetic moment perpendicular to the z-axis <B> ~ 10^{11} G **(Missaligned)**

4) Multipolar magnetic field <B> ~ 10^{11} G **(Multipolar)**

$$A_\phi \propto \sin^4\theta\,(1+\cos\theta)\,r^2(P - P_{cut}).$$

Binary NS evolution (diferent topologies)

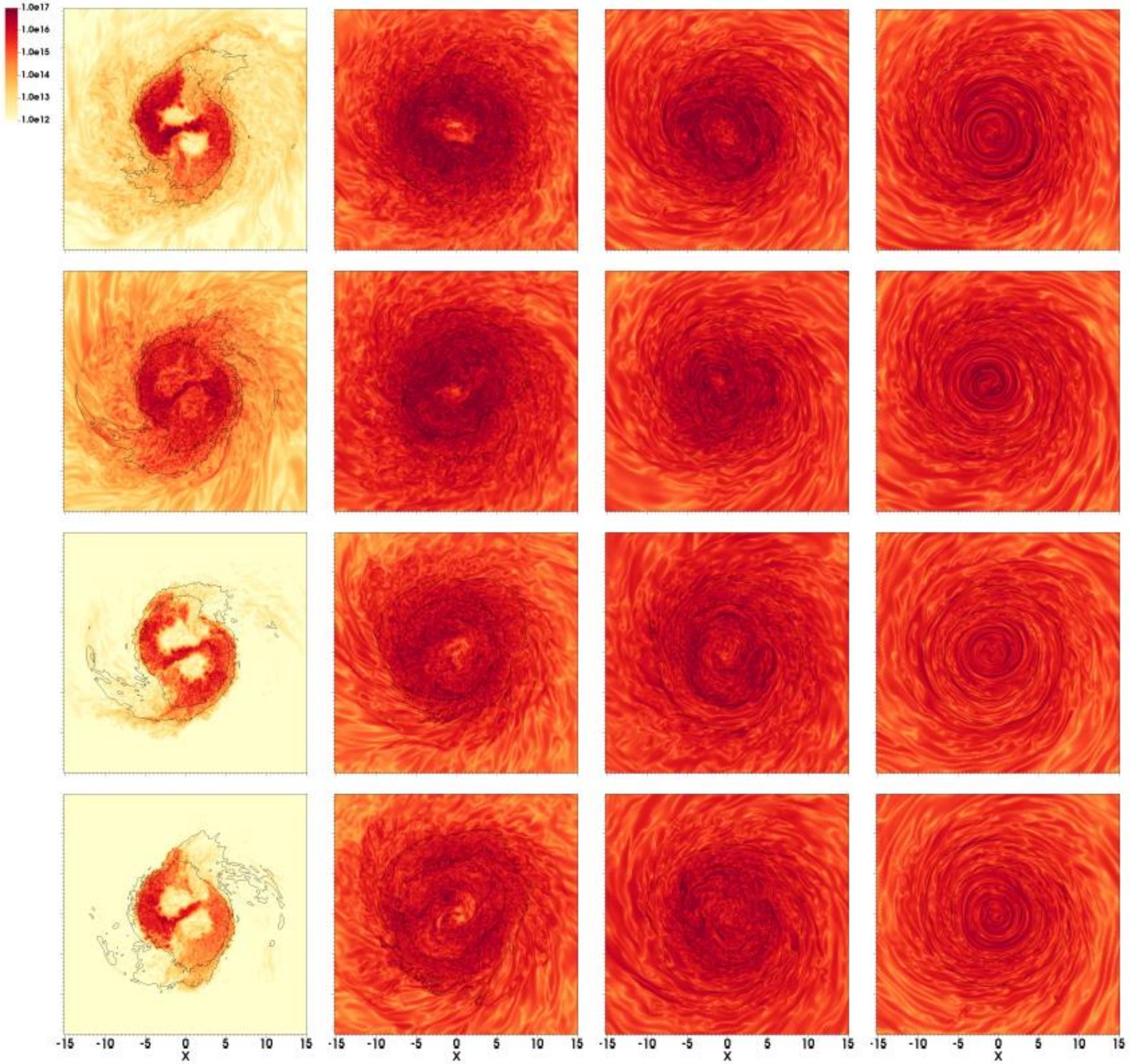
[Aguilera-Miret+ 2022]

Dip

Bhigh

Missaligned

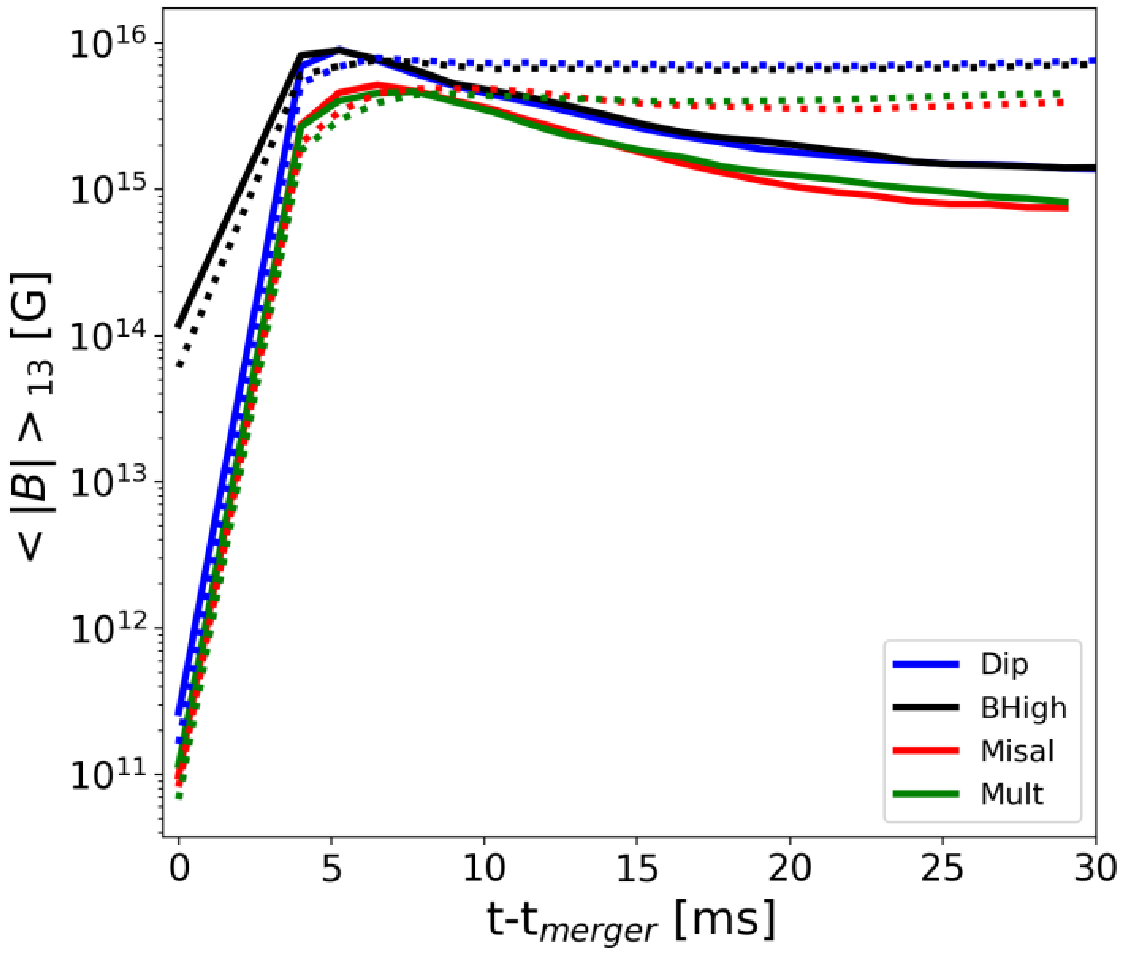
Multipolar



t = 2 ms | t = 5 ms | t = 10 ms | t = 20 ms

Binary NS evolution (diferent topologies)

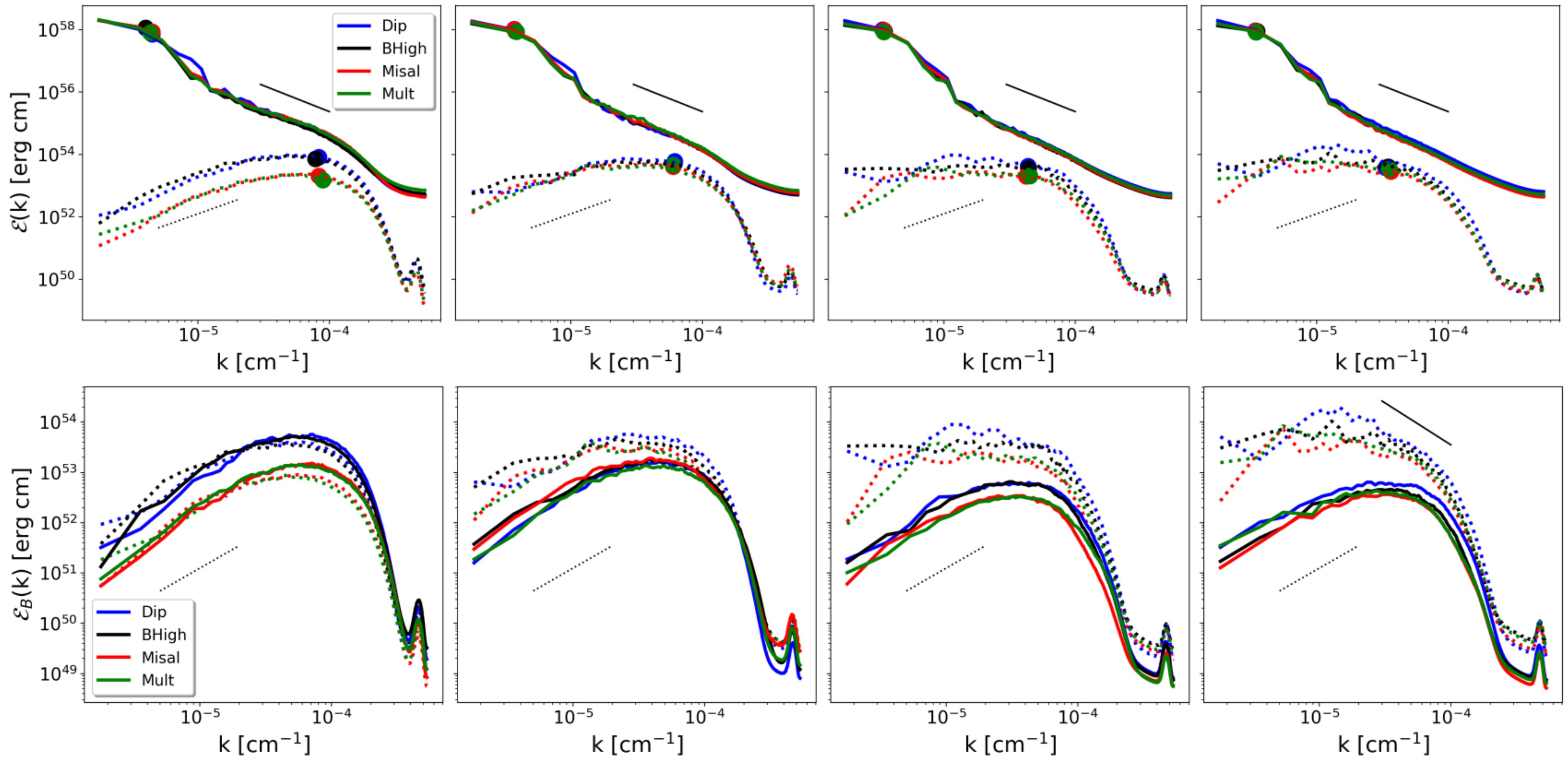
[Aguilera-Miret+ 2022]



Comparable averaged magnetic fields!!

Binary NS evolution (diferent topologies)

[Aguilera-Miret+ 2022]



Comparable magnetic field spectra (both toroidal and poloidal part)!!

1. Turbulent MHD in BNS cannot be fully captured with DNS in the near future in long simulations → affects the dynamics of the remnant and jet/kilonova → LES

2. Average magnetic fields are amplified $\langle B \rangle \sim 10^{11}$ G → 10^{16} G in $t < 5$ ms after merger (bulk)

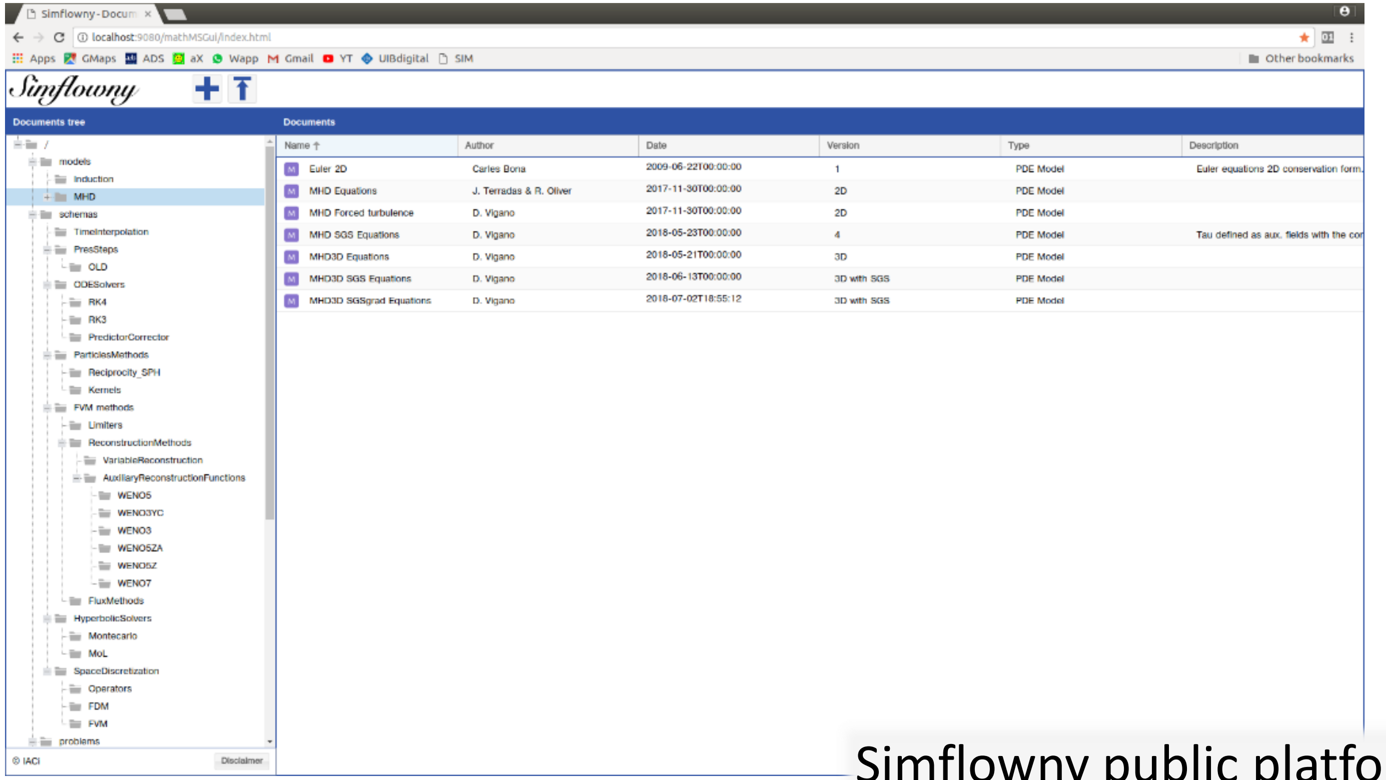
3. The winding up effect change the magnetic spectra after the KHI from Kazantsev (3/2) to Kolmogorov (-5/3) power law

4. The LES + SGS models allows to include part of the unresolved dynamics, effectively increasing the accuracy of the solution and saving of computational time → **quick (convergent) saturation of the magnetic field during the merger**.

5. **The initial magnetic field strength and topology DOES NOT MATTER at all**… as long as you can resolve the KHI that causes a turbulent amplification of the magnetic field. The turbulent magnetic field is isotropic and **erases any dependence on the initial magnetic field topology and strength.**

6. The formulation is general and **can be applied to BNS post-merger or any scenario where the small scales are important.**

Simflowny platform (developed by IAC3)



[Arbona+ 2018,
Palenzuela+ 2018,
Viganò+ 2019]

https://bitbucket.org/iac3/simflowny/

Simflowny public platform:

- High-order accuracy methods
- Any evolution equation (time dependent PDE)
- Adaptive Mesh Refinement + Parallelization by SAMRAI

Sub-grid-scale modeling: dissipative

Turbulent viscosity term in the momentum equation (prop. to strain rate) and turbulent resistive term (prop. to current) in the induction equation.

$$\tau^{ki}_{\rm kin} = \Delta^2 \overline{\rho} |\tilde{S}| \tilde{S}^{ki}$$

$$\tau^{ki}_{\rm ind} = \Delta^2 \frac{|\overline{J}|}{\sqrt{\overline{\rho}}} \overline{J}^{ki}$$

Applied also in GRHD mergers [Radice 2017-2020, Shibata & Kouichi 2017]:

$$\tau_{ij} = -2\nu_T (e+p) W^2 \left[\frac{1}{2} \left(D_i \overline{v_j} + D_j \overline{v_i} \right) - \frac{1}{3} D_k \overline{v^k} \gamma_{ij} \right],$$

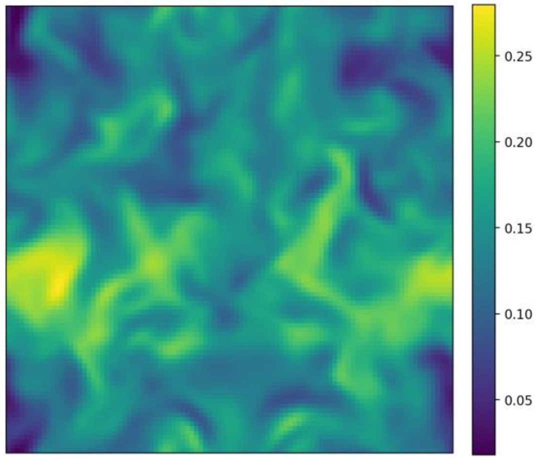
$$\nu_T = \ell_{\rm mix}\, c_s\,,$$

$$\ell_{\rm mix} = \alpha\, c_s\, \Omega^{-1},$$

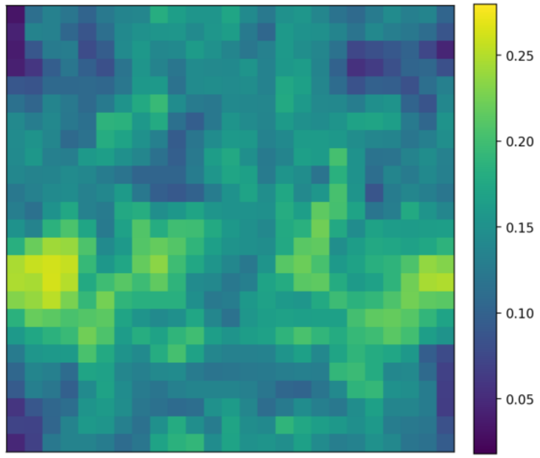
It **only allows transfer from large to small scales**. It can simulate the effective viscous magnetic force but not inverse (non-local) cascades.

Large Eddy simulations

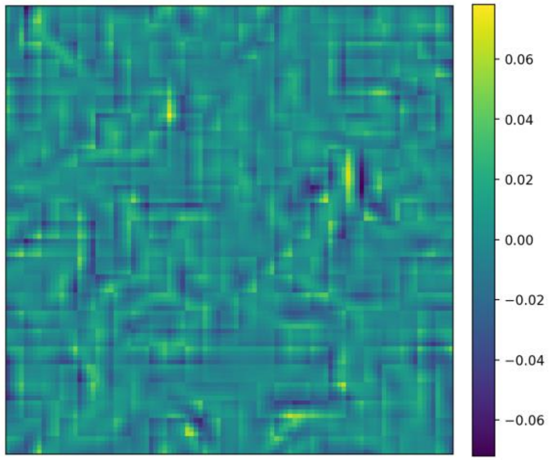
Simulating large eddies, modeling small eddies



Original resolution



Filtered resolution



Sub-Filter Scale loss

$$f(\vec{x},t) = \overline{f}(\vec{x},t) + f'(\vec{x},t)$$

$$\bar{f}(\mathbf{x},t) = \int_{-\infty}^{\infty} G(\mathbf{x}-\mathbf{x}')f(\mathbf{x}',t)d^3x'.$$

How is it applied with non-linear evolution equations?

Sub-grid-scale modeling: gradient model

Consider evolution equations with fluxes and conserved variables as a function of primitive variables P

$$\partial_t \bar{C}^a + \partial_k F^{ka}(\tilde{P}) = \partial_k \bar{\tau}^{ka}, \qquad C^a = f^a(P), \qquad P^a := (f^{-1})^a(C) \equiv g^a(C).$$

$$\tilde{P}^a := g^a(\bar{C})$$

The SFS residuals are

$$\bar{\tau}_F^{ka} := F^{ka}(\tilde{P}) - \overline{F^{ka}(P)},$$

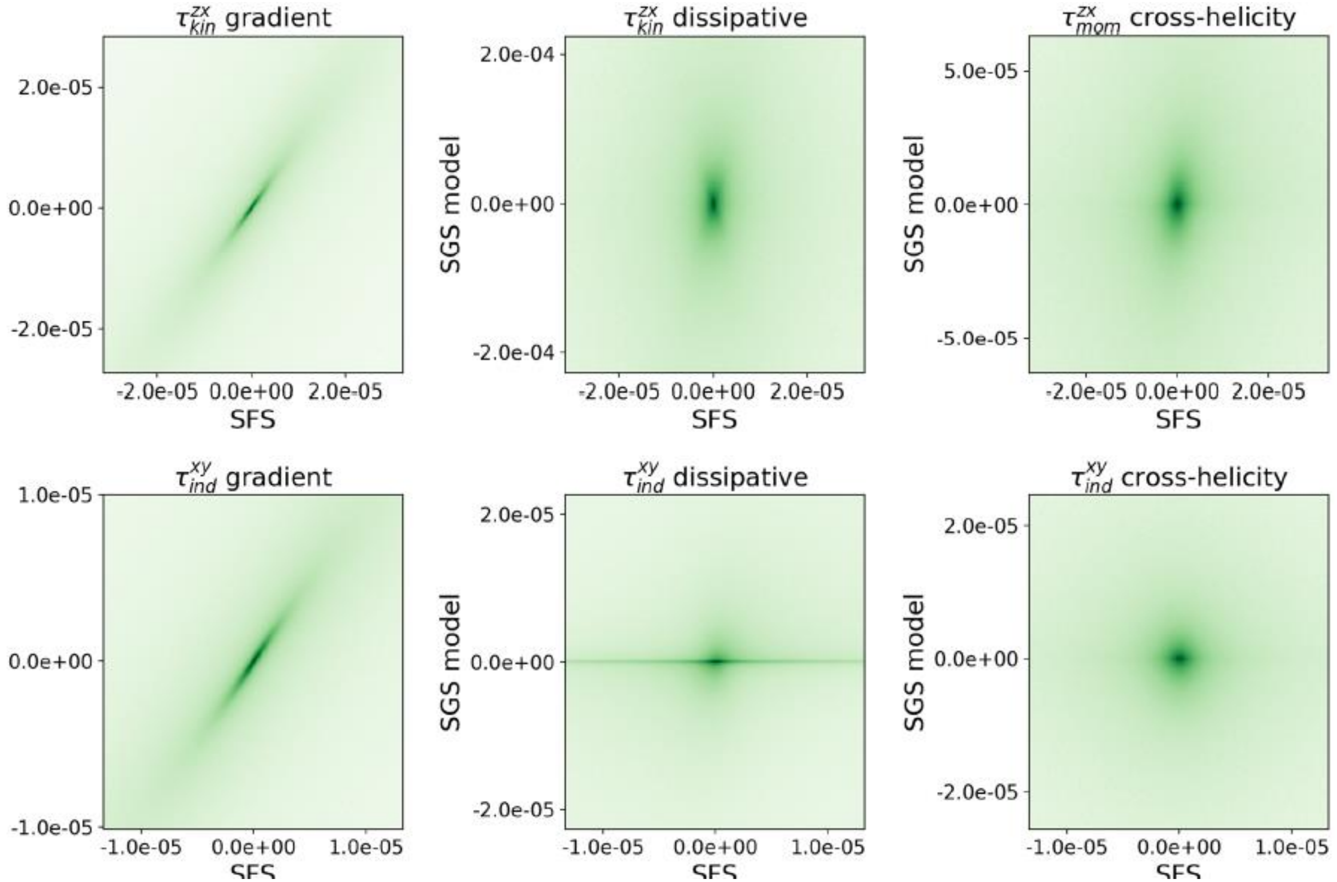
and can be modelled by alternative formulations:

$$\tau_F^{ka} = \xi \left(\frac{dF^{ka}}{d\tilde{P}^b} \frac{d\tilde{P}^b}{d\bar{C}^e} \nabla^2 \bar{C}^e - \nabla^2 F^{ka}(\tilde{P}) \right),$$

$$\tau_F^{ka} = -\xi \nabla \frac{dF^{ka}}{d\bar{C}^b} \cdot \nabla \bar{C}^b.$$

Depending on the non-linear form in fluxes, expressions can be cumbersome

Compressible MHD: a-priori test for different SGS models



Gradient model outperforms the other models tested for all:

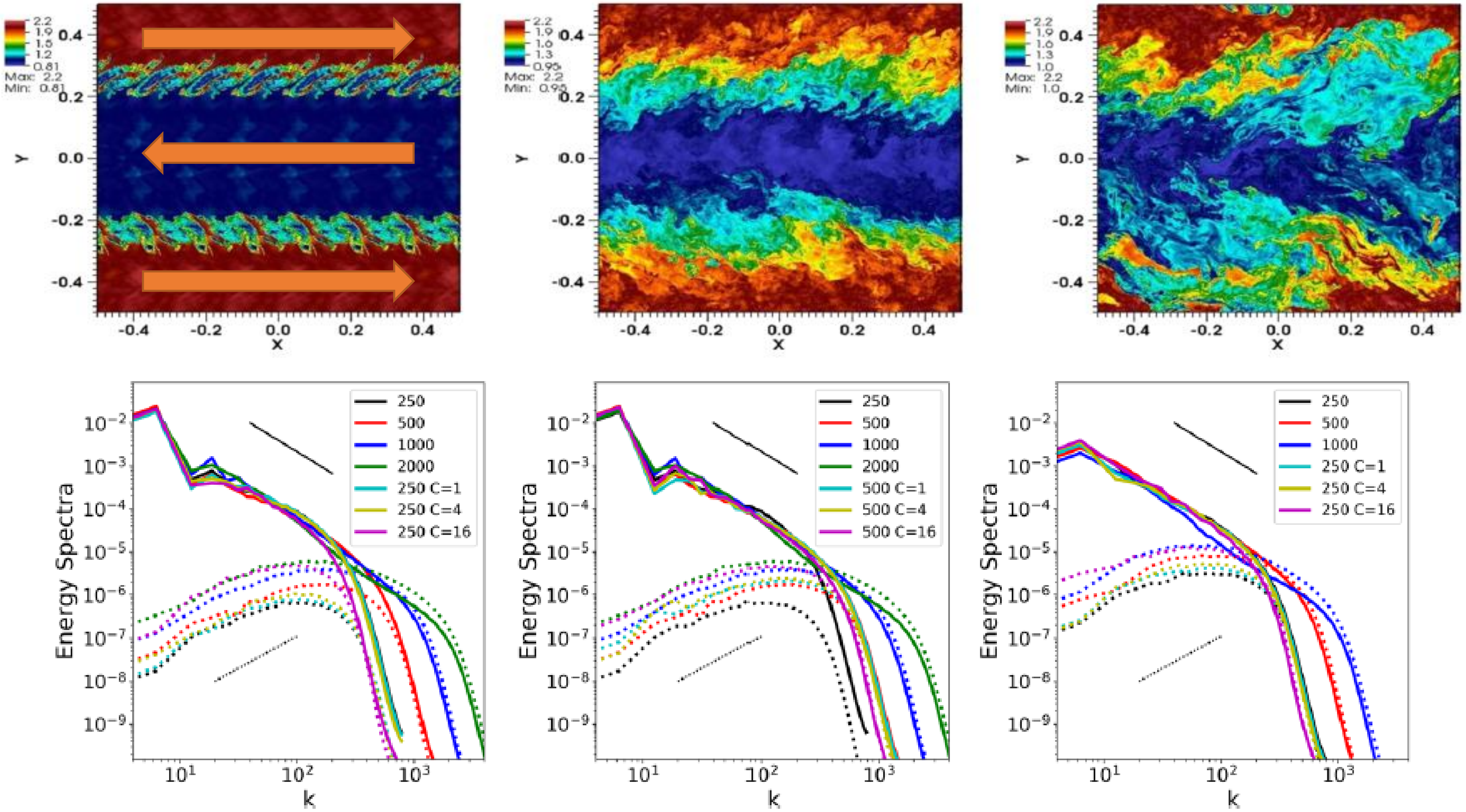
- Tensor components
- Resolutions
- Filter sizes
- Initial conditions
- Times

[Viganò+ 2019]

See also Machine Learning SGS models applied to the same problem in 2D can perform better [Rosofsky & Huerta 2020] (but how costly is to train them?)

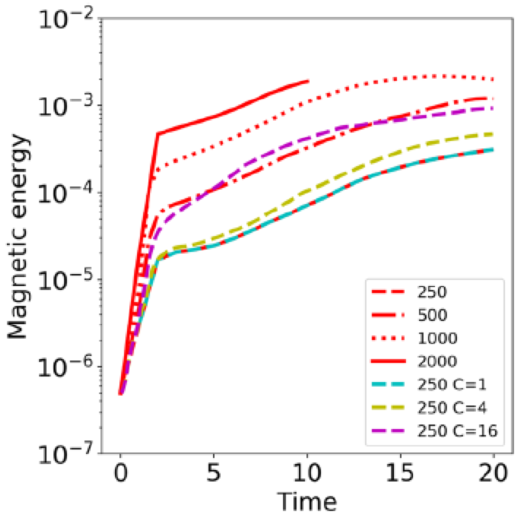
Assessment with KHI box simulations

[Viganò+ 2019]



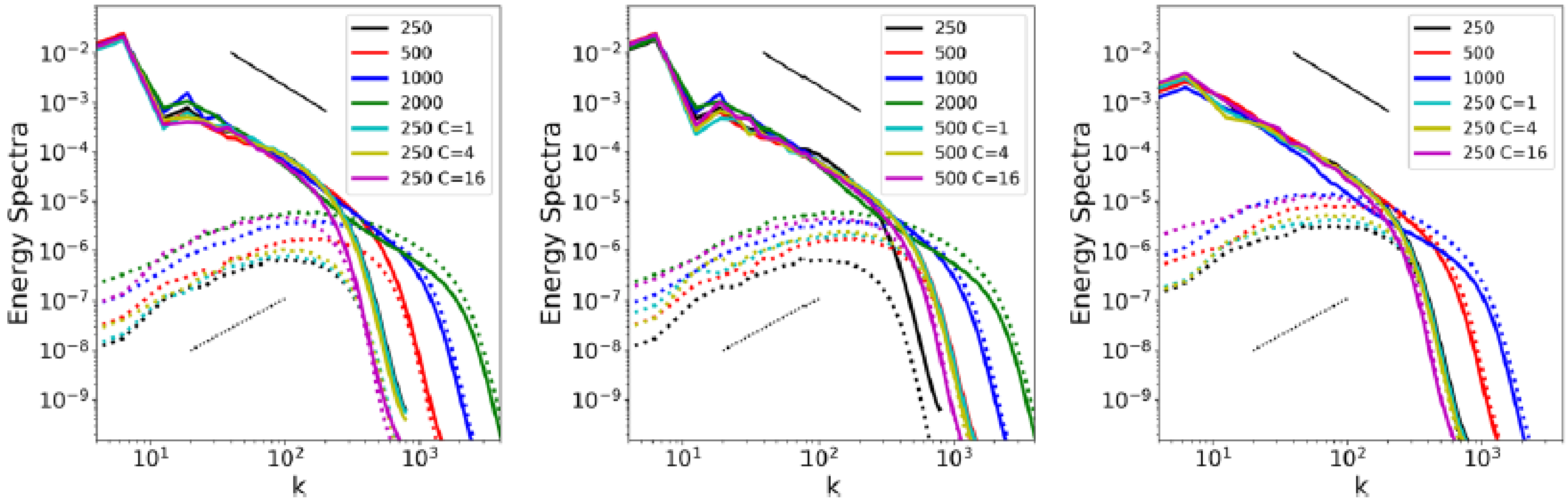
Effective resolution x8 !!

Compressible MHD: a-posteriori test *[Viganò+ 2019]*



Compare high resolution runs with low resolution + SGS.

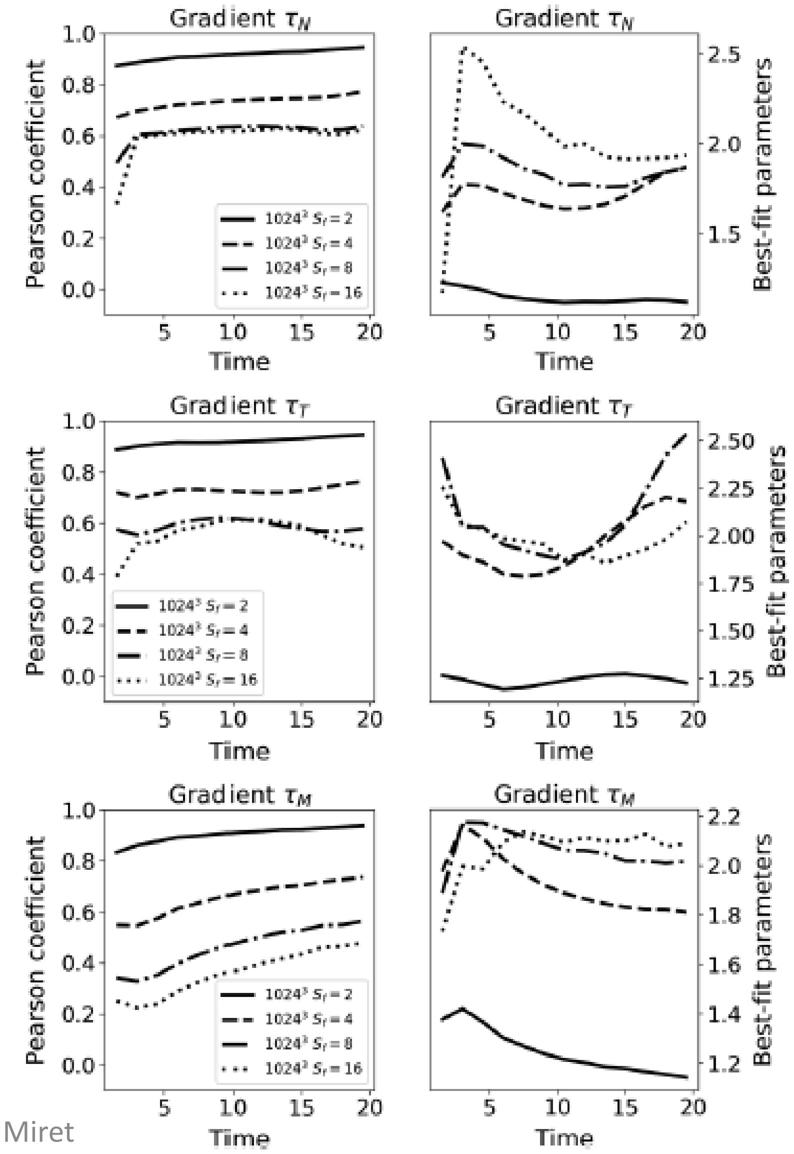
Need for large values of the free parameter C. Probably due to the numerical dissipation of the scheme.



Effective resolution x8 !!

Gradient model for different filtering

[Carrasco+ 2020]



Higher filter: more information loss: more difficult to fit.

Applying the SGS gradient model partially includes the physics which would appear with an effective resolution higher by a factor of a few.

Gradient model for general relativistic MHD *[Viganò+ 2020]*

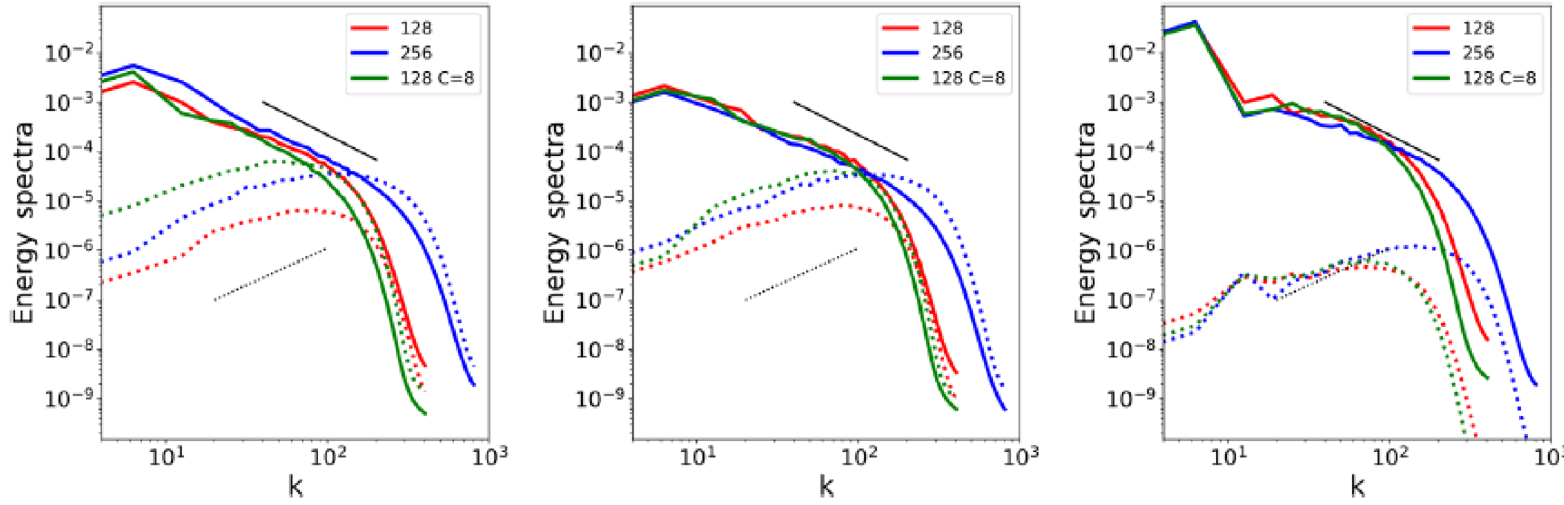
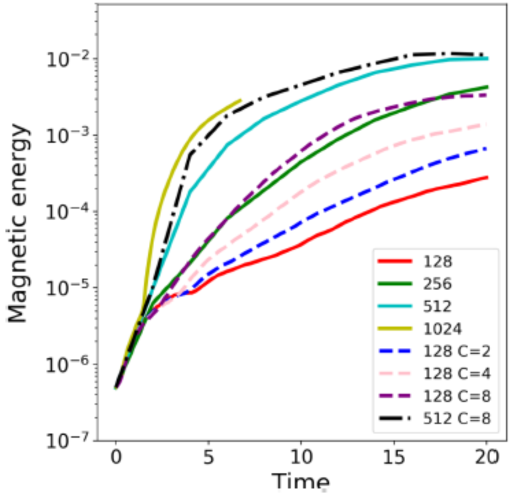


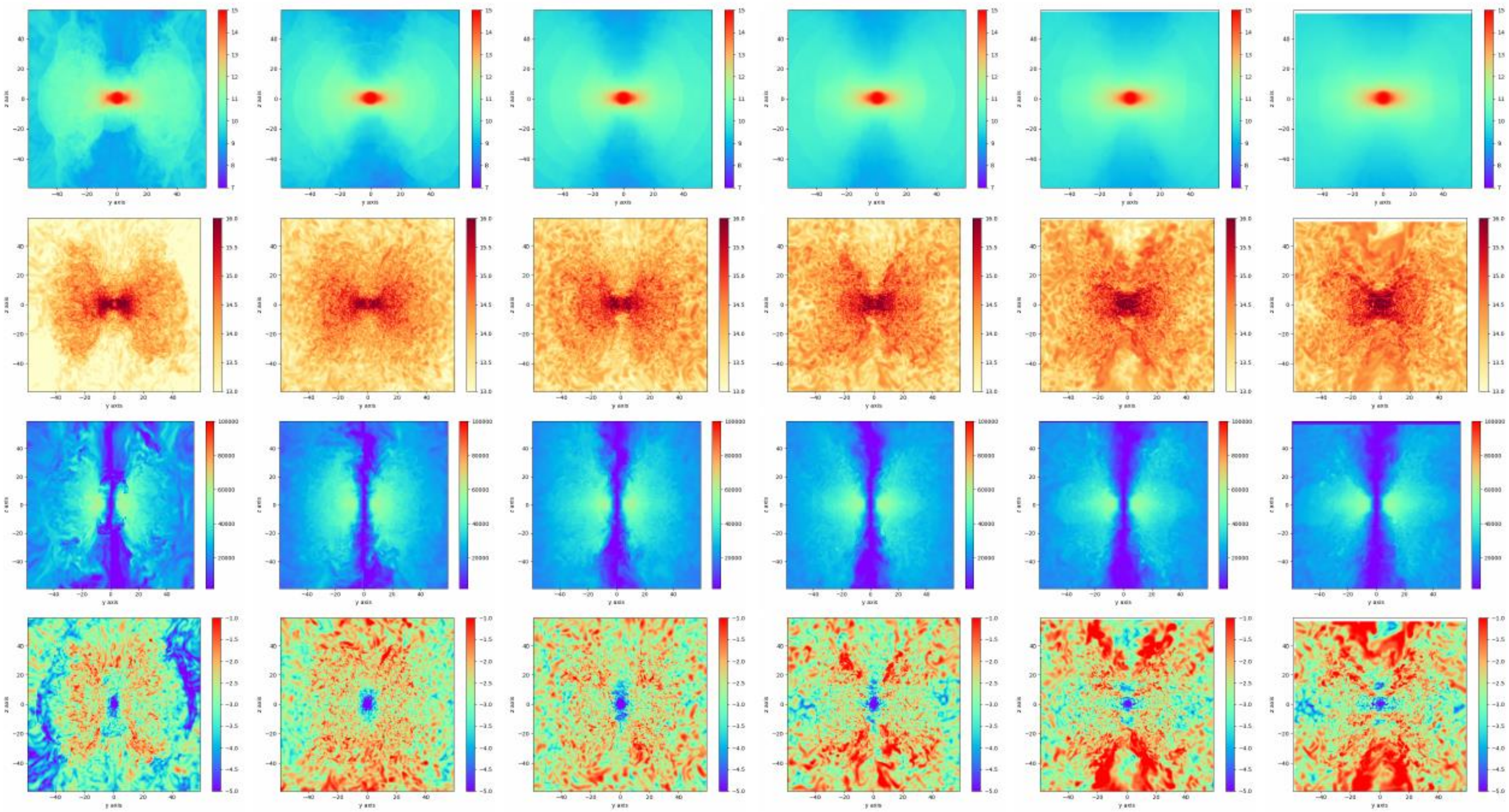
FIG. 6. Box simulations in 3D: *A posteriori* tests in curved background. Kinetic (solid lines) and magnetic (dashed lines) energy spectra at $t = 20$ for $\chi = 1$ (left panel), $\chi = [0.20 - 0.90]$ (middle panel), and $\chi = [0.20 - 0.24]$ (right panel) of a box simulation, compared with higher resolution and $C = 8$.

We again find excellent results for a variety of fixed space-time backgrounds.



Binary NS evolution

[Palenzuela+ 2021]

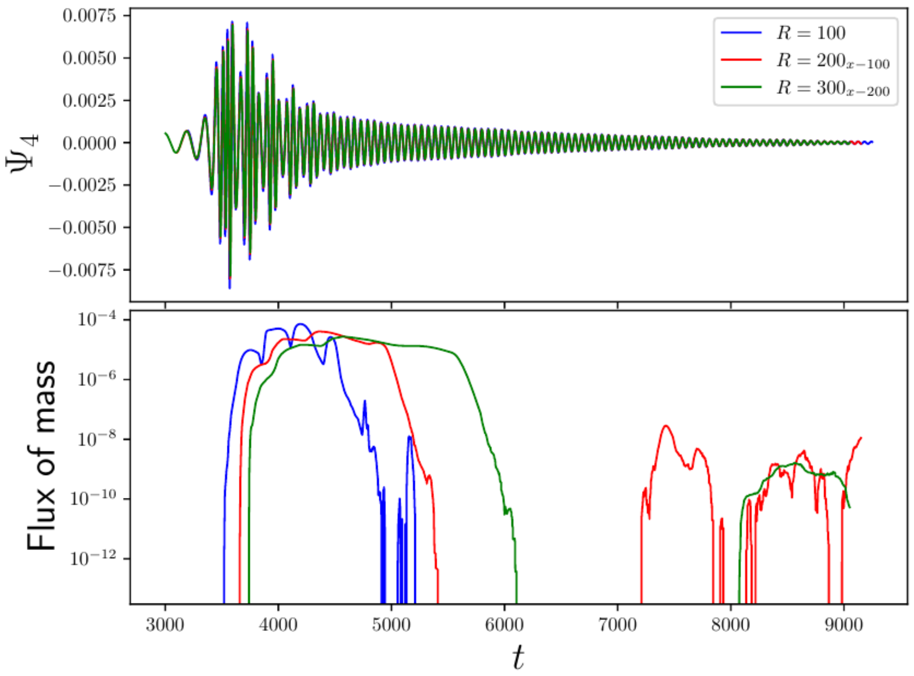


t=5ms t=10ms t= 20ms t=30ms t=40ms t=50ms

Density, magnetic field, angular velocity and $\beta^{-1} = P_{mag}/P$ (meridional plane)

Binary NS evolution

[Palenzuela+ 2021]



Gravitational waves (Ψ_4)
and dynamical mass ejecta