General Relativistic Multidimensional Flux-Limited Diffusion Scheme for Neutrino Transport and its Astrophysical Applications

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Outline

- Motivation.
- General relativistic hydrodynamics and neutrino transport.
- Hypermassive neutron star (HMNS) evolution (preliminary).
- Core collapse simulations of very massive stars (VMSs).

Motivation

- Core-collapse supernovae (CCSNe).
- Evolution of compact remnant and ejecta from the compact binary star merger.

General Relativistic Hydrodynamics-GR

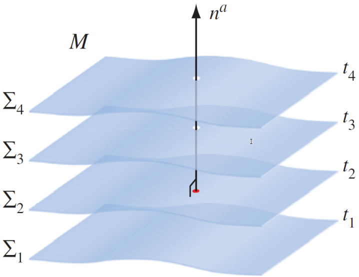


Figure 1: Space-like hyper-surface and time-like normal (Baumgarte and Shapiro 2010).

- Einstein equation is solved dynamically in 3+1 decomposition.

$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{1}$$

- Hyperbolic formalism by Baumgarte, Shapiro, Shibata and Nakamura.
- Solved in Spherical polar coordinate.
- Time integration is done by 2nd order partially implicit runge-kutta method.

General Relativistic Hydrodynamics-Hydro

- Local conservation of matter current density, energy-momentum tensor, and lepton number.

$$\begin{aligned} \nabla_a J^a &= 0 \\ \nabla_a T^{ab} &= 0 \\ \nabla_a J^a_e &= S_\nu . \end{aligned} \tag{2}$$

- Finite difference method (Montero 2013).
- Solved by High resolution shock capturing method of Harten Lax and van Leer.
- Micro-physical equation of state.

Neutrino Transport-1

- Co-moving orthonormal frame.
- Solves Flux limited diffusion (FLD) scheme for neutrino transport.
- In FLD, 0th moment of neutrino distribution function, $\mathcal{J}$, is dynamically evolved.
- Neutrino Flux $\mathcal{H}_i = -\lambda D \partial_i \mathcal{J}$.
- Flux limiter, λ, smoothly interpolates between diffusive and free-streaming region.
- Flux limiter ensures causality is not violated. Flux: $F_i \leq cJ$.
- GR correction to FLD flux:

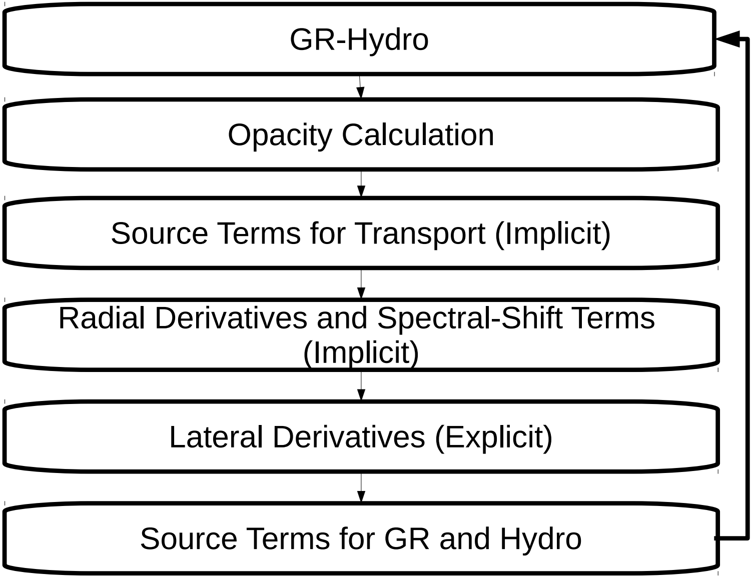
$$\mathcal{H}^{\hat{i},\alpha} \;=\; -\lambda D e^{i\hat{i}} \alpha^{-3} \partial_i (\alpha^3 \mathcal{J}) \,. \tag{3}$$

Neutrino Transport-2

$$
\begin{aligned}
&\partial_t(\sqrt{\gamma}\mathcal{J}) + \partial_j(\alpha\sqrt{\gamma}v^j\mathcal{J}) - \partial_j(\alpha^{-2}\sqrt{\gamma}\lambda D\partial^j(\alpha^3\mathcal{J})) \\
&+[R_e - \frac{\partial}{\partial e}(eR_e)] = \alpha\sqrt{\gamma}\kappa_a(\mathcal{J}^{eq} - \mathcal{J})
\end{aligned}
\tag{4}
$$

- 2nd term(lhs): advection.
- 3rd term(lhs): diffusion term.
- 4th term: velocity dependent term responsible for doppler and gravitational redshift.
- 5th term(rhs): absorption and emission source term.

Neutrino Transport-3 (Flowchart)


GR-Hydro

Opacity Calculation

Source Terms for Transport (Implicit)

Radial Derivatives and Spectral-Shift Terms (Implicit)

Lateral Derivatives (Explicit)

Source Terms for GR and Hydro

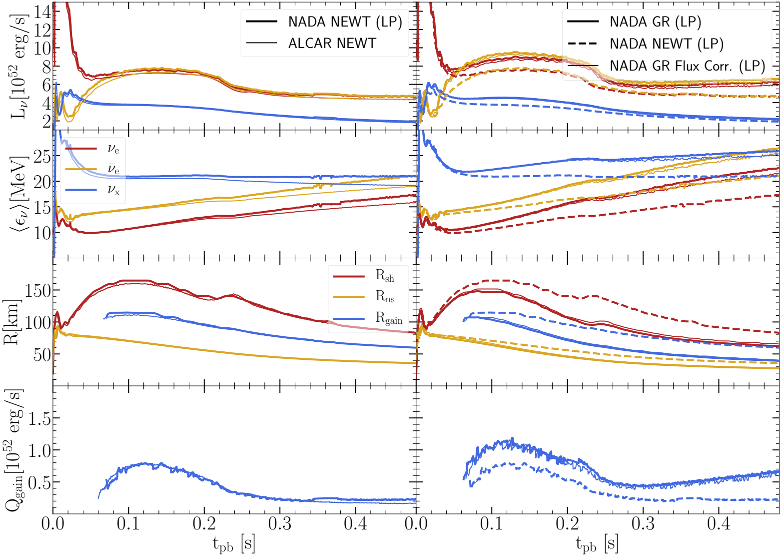
Neutrino Transport Test-CCSN (Setup)

- 1D Radiation-Hydrodynamics simulations with general relativistic and Newtonian gravity.
- $20\mathrm{M}_\odot$ ZAMS (Woosley and Heger, 2007).
- Radial grid points = 600
- Nuclear EOS SFHo (Steiner et al. 2012)

Reaction	Neutrino
$\nu + A \leftrightarrow \nu + A$	$\nu_\mathrm{e}, \bar{\nu}_\mathrm{e}, \nu_x$
$\nu + N \leftrightarrow \nu + N$	$\nu_\mathrm{e}, \bar{\nu}_\mathrm{e}, \nu_x$
$\nu_e + A \leftrightarrow e^- + A'$	ν_e
$\nu_e + n \leftrightarrow e^- + p$	ν_e
$\bar{\nu}_e + p \leftrightarrow e^+ + n$	$\bar{\nu}_e$
$\nu + \bar{\nu} \leftrightarrow e^- + e^+$	ν_x
$\nu + \bar{\nu} + N + N \leftrightarrow N + N$	ν_x

Table 1: Neutrino opacities used for the CCSN simulations. “N” denotes nucleons and “A” and “A’ ” denote nuclei.

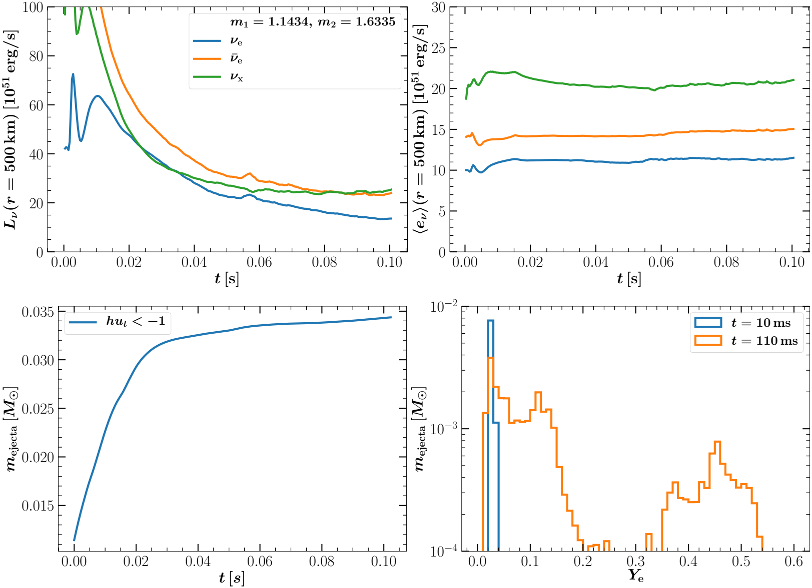
Neutrino Transport Test-CCSN (Code Comparison)



HMNS evolution (preliminary)

- Binary neutron stars merger simulation by CFC+SPH code.
- Data from the CFC+SPH code mapped to the Full GR grid based NADA-FLD code.
- HMNS evolutions are studied in two dimensions.

HMNS evolution-neutrino and ejecta properties (preliminary)



Very massive stars and pulsational pair-instability supernova

- 70–140$\mathrm{M}_{\odot}$ ZAMS.
- Mass loss due to the pulsational pair-instability supernovae (PPISNe).
- Final mass 30–50$\mathrm{M}_{\odot}$.
- PPISNe can explain the Great Eruption of the Eta Carinae.
- Core collapse of VMS can form a rapidly rotating black hole or a magnetized neutron star.
- Result of VMS core-collapse, shock revival? Final remnant? BH or NS?

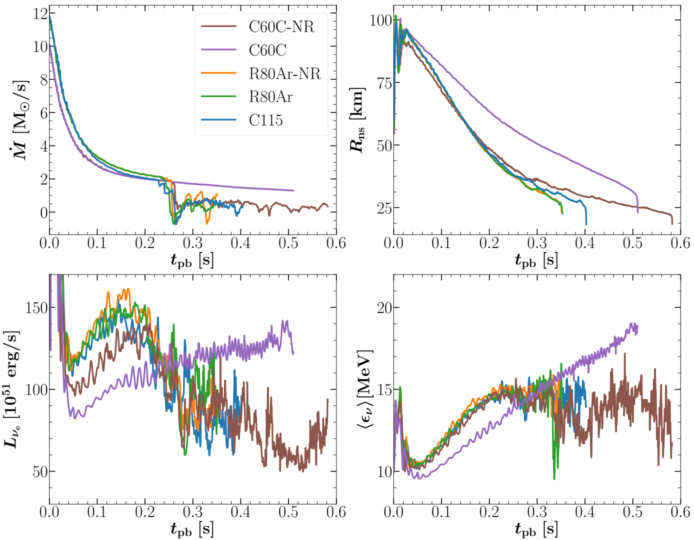
Progenitor and Setup

Model	$\mathrm{M}_{\mathrm{ZAMS}}$ ($\mathrm{M}_{\mathrm{prog}}$)	M_{Fe}	$\mathrm{J}_{\mathrm{prog}}$ ($\mathrm{a}_{\mathrm{prog}}$)	$t_{\mathrm{sh-rev}}$	t_{BH}
	$[\mathrm{M}_\odot]$	$[\mathrm{M}_\odot]$	$[10^{50}$ erg s$]$	[s]	[s]
C60C-NR	60 (41.54)	2.35		0.250	0.580
C60C	60 (41.54)	2.35	105 (0.55)		0.510
R80Ar-NR	80 (47.64)	2.74		0.246	0.350
R80Ar	80 (47.64)	2.74	14 (0.07)	0.237	0.350
C115	115 (45.50)	2.44		0.220	0.400

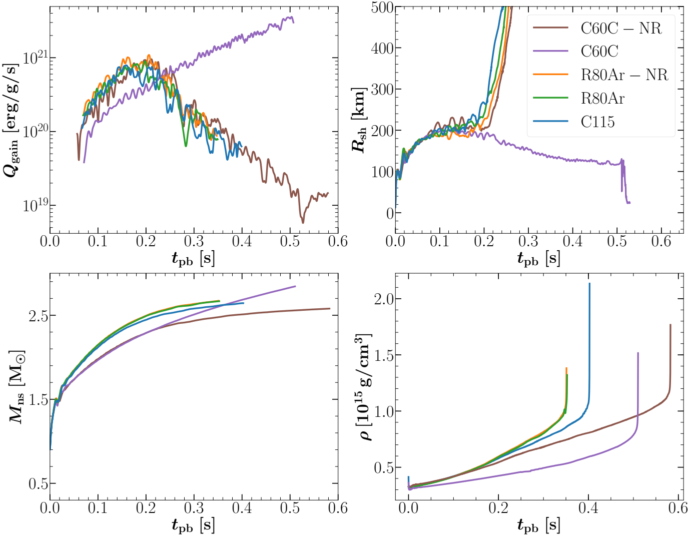
Table 2: Progenitor properties. All model have zero-age-main-sequence (ZAMS) metallicity of 10% $\mathrm{Z}_\odot$.

- 2D GR-Hydro-Transport simulation.
- Radial grid points = 500. Angular resolution = 1.4°

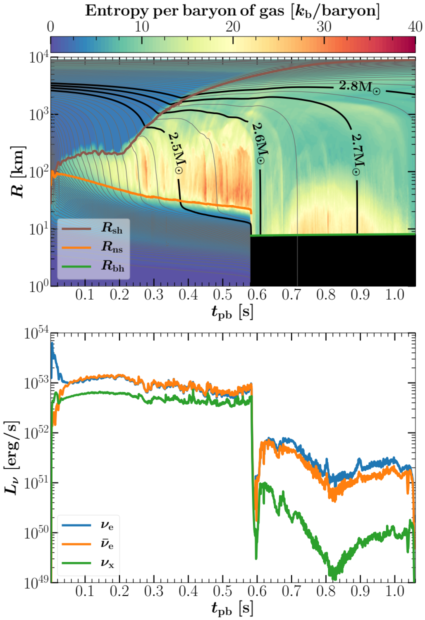
VMS results-before BH formation



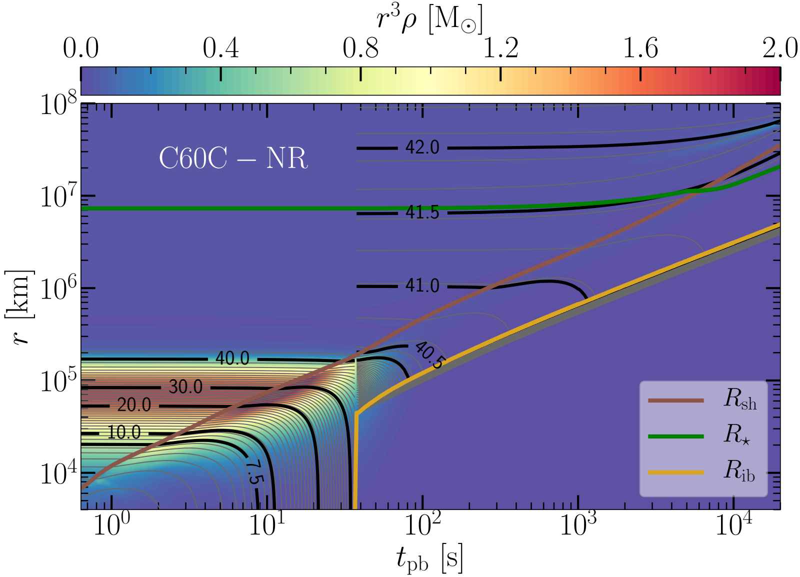
VMS results-before BH formation



VMS results-after BH formation



VMS Results-After BH formation-Mass shell plot



Conclusion

- Multidimensional general relativistic FLD.
- Most global properties agree well between FLD based NADA and M1 based ALCAR code.
- Alteration of ejecta properties by the weak interaction.
- Core collapse simulations of VMSs beyond BH formation.
- We observe continuation of shock expansion after BH formation, however, only for C60C-NR model shock reaches stellar surface.
- We predict ν properties after BH formation.