BJÖRN SCHENKE - BROOKHAVEN NATIONAL LABORATORY

AB-INITIO APPROACHES IN HEAVY-ION COLLISIONS

6/2/2022
EMMI Rapid Reaction Task Force
Heidelberg

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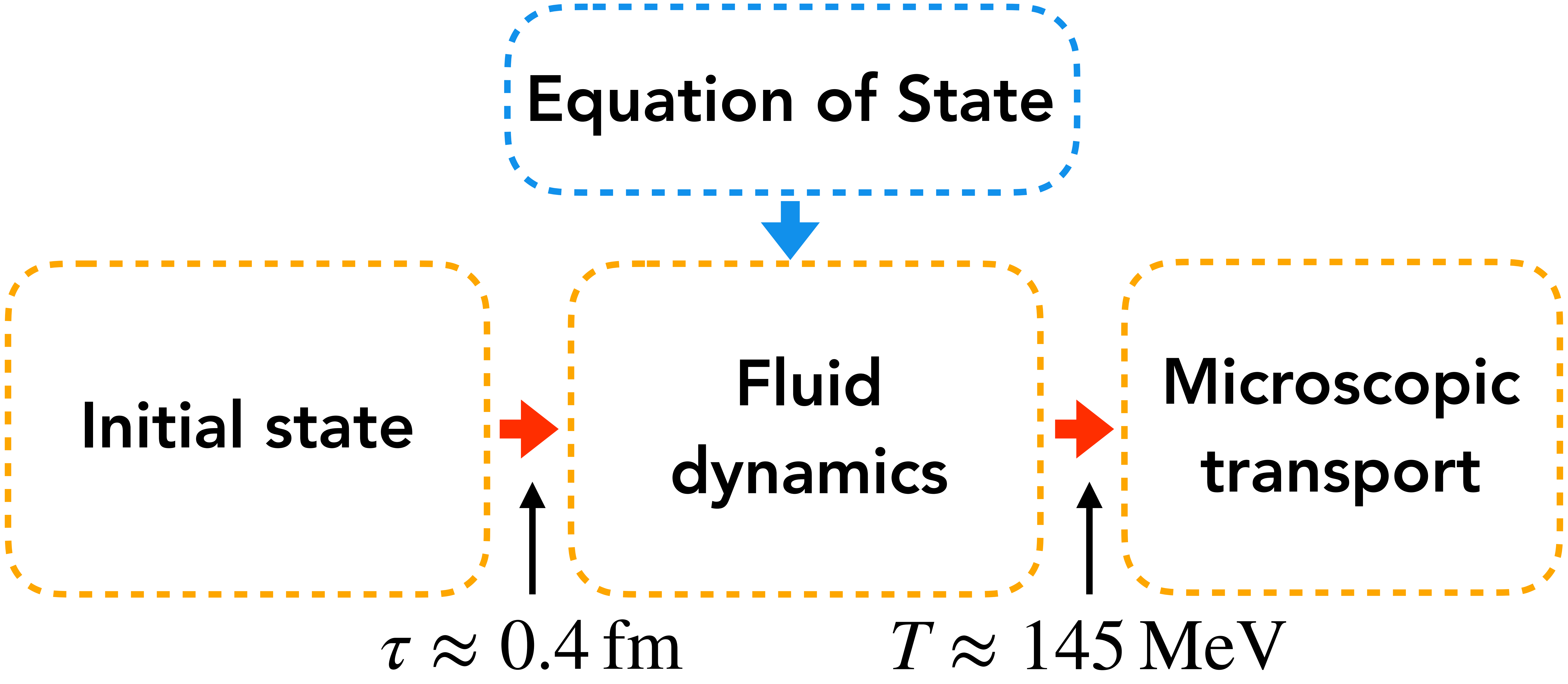
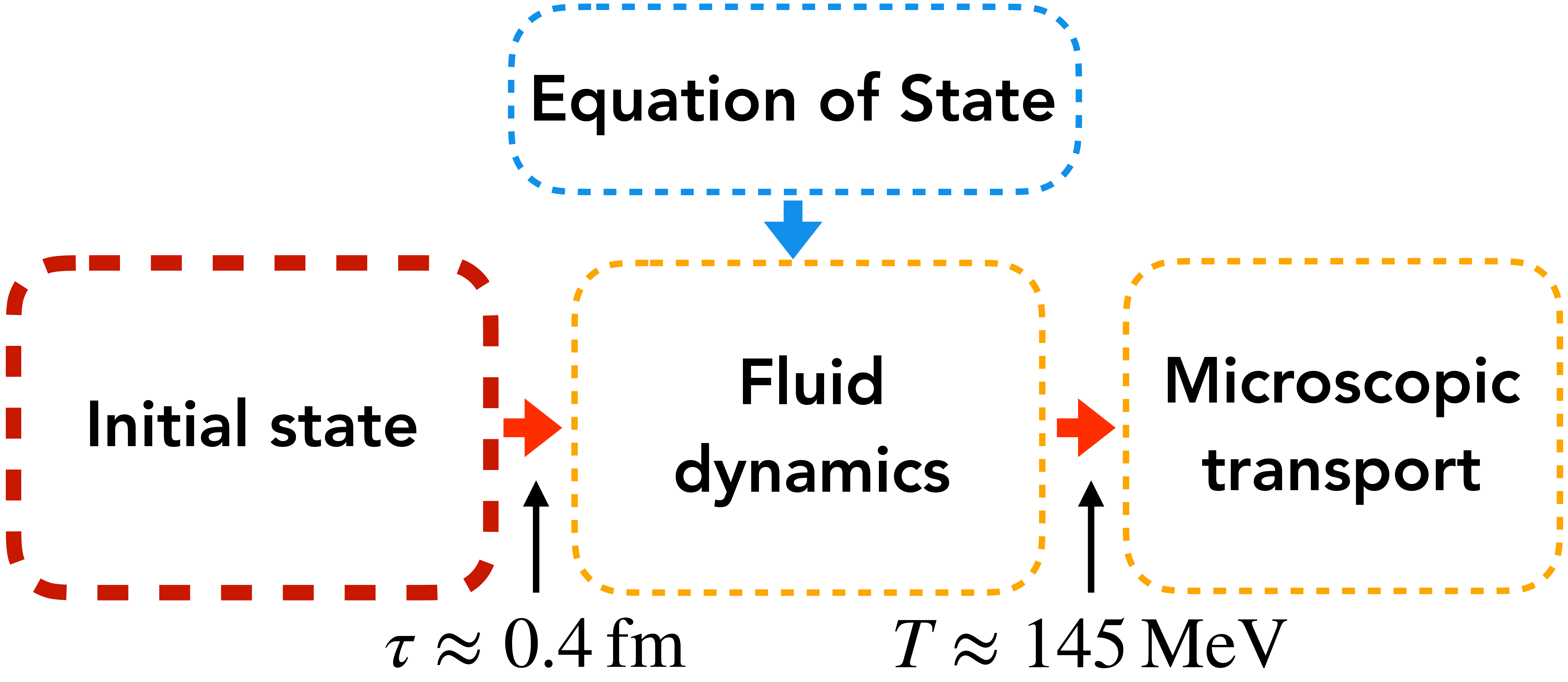
TYPICAL MODEL FRAMEWORK



see e.g.
B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905

- **Exactly match $T^{\mu\nu}$ when switching from one part to the next**

TYPICAL MODEL FRAMEWORK



see e.g.
B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905

- **Exactly match $T^{\mu\nu}$ when switching from one part to the next**

OPTIONS FOR HEAVY ION INITIAL STATE CALCULATIONS

- High energy, weak coupling:
  - Color Glass Condensate, like used in IP-Glasma model (will focus on this)
  - Mini jet (few-GeV particle) production, calculated from perturbative QCD (pQCD).
    - EKRT model: Includes a saturation criterion which dominates the distribution of initial energy density K. J. Eskola, K. Kajantie, P. V. Ruuskanen, and Kimmo Tuominen, Nucl. Phys. B, 570:379–389, 2000.
- Strong coupling:
  - Use duality between a conformal field theory (CFT) and gravity in Anti-de-Sitter (AdS) space (AdS/CFT)
- Purely geometry based models like Monte-Carlo Glauber models and Trento (not subject of this discussion)

COLOR GLASS CONDENSATE

My Little Pony® Hasbro, Inc.

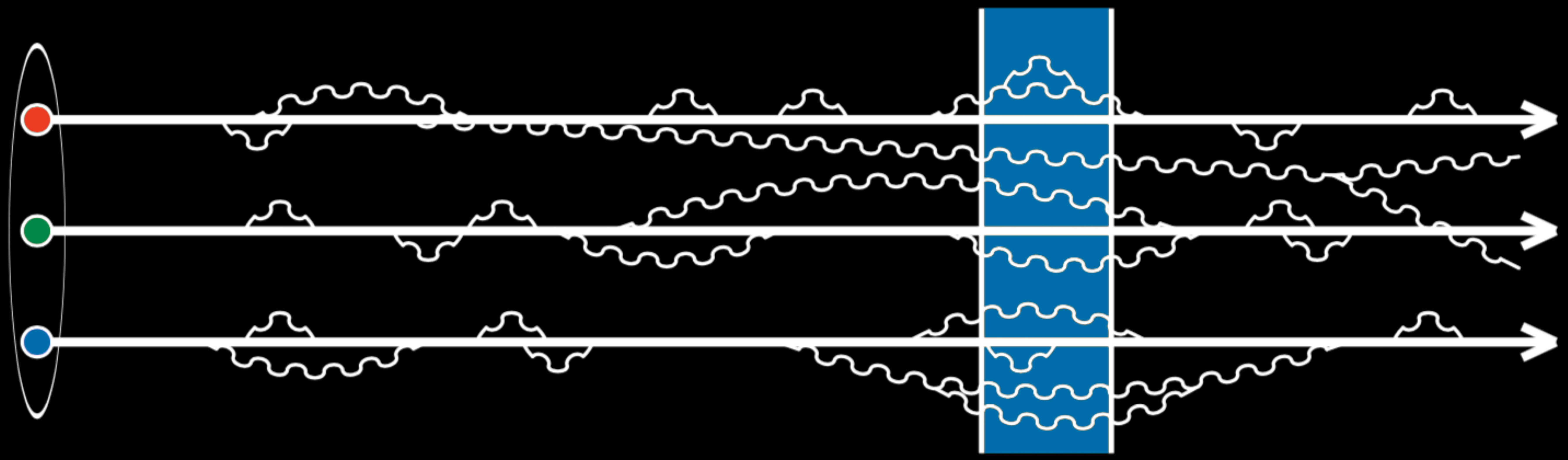


Figure from F. Gelis

Nucleon at high energy:

- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales longer than the characteristic time-scale of the probe → The constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe. Nucleon appears denser at high energy (contains more gluons)
- Pre-existing fluctuations are totally frozen over the time-scale of the probe, and act as static sources of new partons

PARTON SATURATION

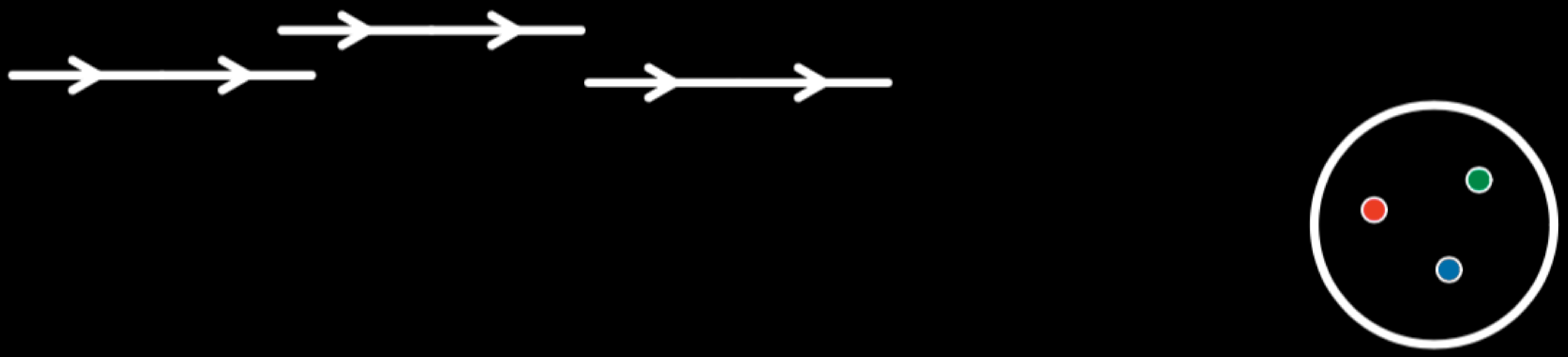


Figure from F. Gelis

At low energy, only valence quarks in the hadron wave function

PARTON SATURATION

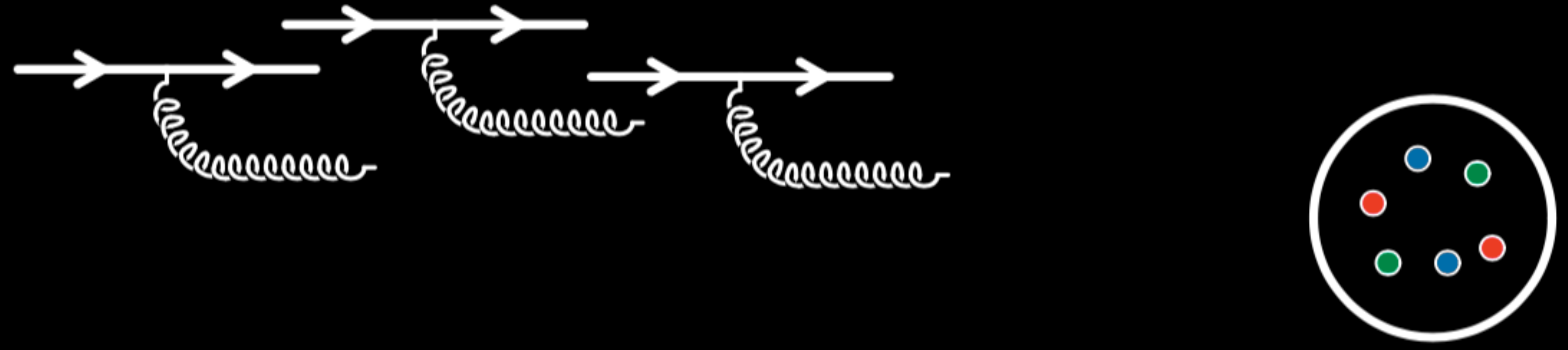


Figure from F. Gelis

- When energy increases, new partons are emitted
- The emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(1/x)$
  with *x* the longitudinal momentum fraction of the gluon
- At small *x* (i.e. high energy), these logs need to be resummed

PARTON SATURATION

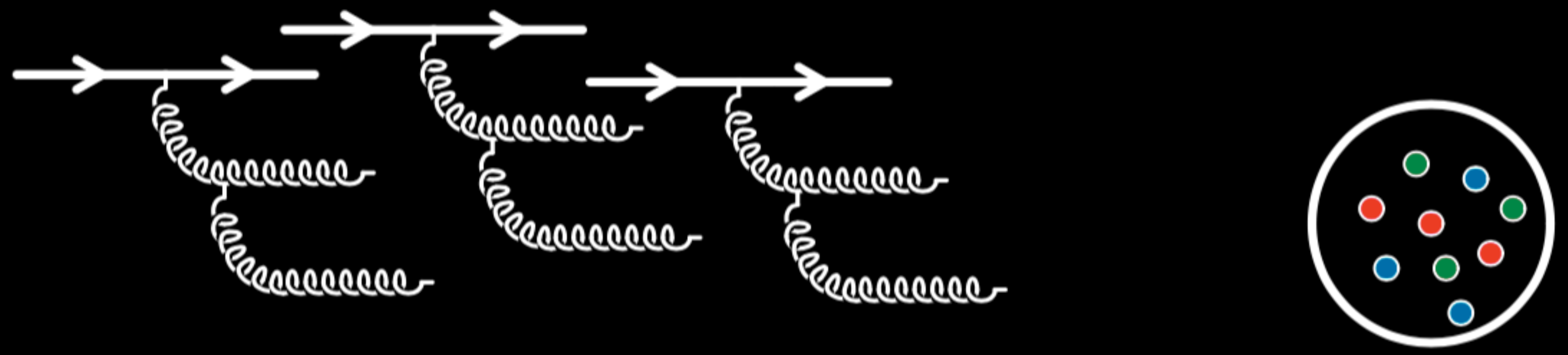


Figure from F. Gelis

- As long as the density of constituents remains small, the evolution is linear:
  The number of partons produced
  at a given step is proportional to
  the number of partons at the previous step (BFKL)

Balitsky-Fadin-Kuraev-Lipatov: L. N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 642;
V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B60 (1975) 50;
Sov. Phys. JETP 44 (1976) 443; 45 (1977) 199;
Ya. Ya. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822

PARTON SATURATION

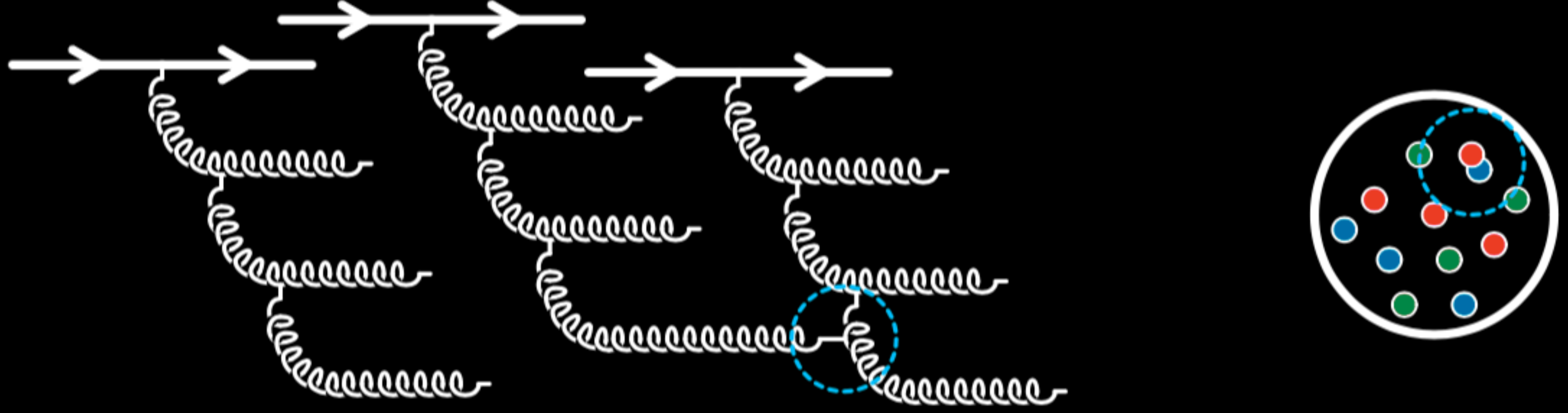


Figure from F. Gelis

- Eventually, the partons start overlapping in phase-space → parton recombination (happens when gluon density $\sim 1/\alpha_s$; for gluons with a transverse momentum below a scale Q_s)

- Then the evolution becomes non-linear:
  The number of partons created at a given step depends non-linearly on the number of partons present previously

Balitsky (1996), Kovchegov (1996,2000)
Jalilian-Marian, Kovner, Leonidov, Weigert (1997,1999)
Iancu, Leonidov, McLerran (2001)

FIRST PRINCIPLES CALCULATIONS, BUT

... approximations are necessary:

- $N_f = 0$ - only gluons
- weak coupling limit - $\alpha_s(Q^2)$ is very small at some relevant scale Q^2
- more approximations and inputs to come, but for now we have:

- Gluon fields follow Yang-Mills equations of motion

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

with $D_\mu = \partial_\mu + igA_\mu$ and $F_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

INCOMING CURRENTS AND SOLUTIONS OF YM EQUATIONS

Use light cone coordinates $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$

Projectile and target currents are then

$$J_P^{\nu} = \delta^{\nu +}\rho_P^a(x^-, \mathbf{x}_\perp)t^a \quad \text{and} \quad J_T^{\nu} = \delta^{\nu -}\rho_T^a(x^+, \mathbf{x}_\perp)t^a$$

assuming color sources (‘valence quarks’) are static in light cone time

Plug currents into Yang-Mills equations and solve in Lorenz gauge $\partial_\mu A^\mu = 0$

For projectile use ansatz $A^\mu = \delta^{\mu +}A_a^\mu(x^-, \mathbf{x}_\perp)t^a$ one finds that A^+ is independent of x^+ because $\partial_+ A^+ = 0$

Then field strength tensor only has this component $F^{i+} = \partial^i A^+$

INCOMING CURRENTS AND SOLUTIONS OF YM EQUATIONS

Then field strength tensor only has this component $F^{i+} = \partial^i A^+$

plugged into YM equations yields $\partial_i \partial^i A^+ = J^+$, the Poisson equation in the transverse plane:

$$-\Delta_\perp A^+(x^-, \mathbf{x}_\perp) = \rho^a(x^-, \mathbf{x}_\perp) t^a$$

where $\Delta_\perp = \sum_i \partial_i^2$.

Formal solutions to this are

$$A_P^+(x^-, \mathbf{x}_\perp) = -\frac{\rho_P^a(x^-, \mathbf{x}_\perp) t^a}{\Delta_\perp}, \text{ and } A_T^-(x^+, \mathbf{x}_\perp) = -\frac{\rho_T^a(x^+, \mathbf{x}_\perp) t^a}{\Delta_\perp}$$

Now comes something not first principles: include an infrared regulator m....

INCOMING CURRENTS AND SOLUTIONS OF YM EQUATIONS

The infrared regulator m gets rid of Coulomb tails:

$$A^{\pm}_{P/T}(x^{\mp}, \mathbf{x}_\perp) = -\frac{\rho^a_{P/T}(x^{\mp}, \mathbf{x}_\perp)t^a}{\Delta_\perp - m^2}$$

This concludes the first principles calculation of the incoming gluon fields.

But, note that $\rho^a_{P/T}(x^{\mp}, \mathbf{x}_\perp)$ has not been specified yet.

Doing so will not be "first principles" at all…

Aside:

BEYOND CLASSICAL FIELDS (NOT OFTEN USED IN HICS)

The gluon fields are defined at a certain momentum fraction x.

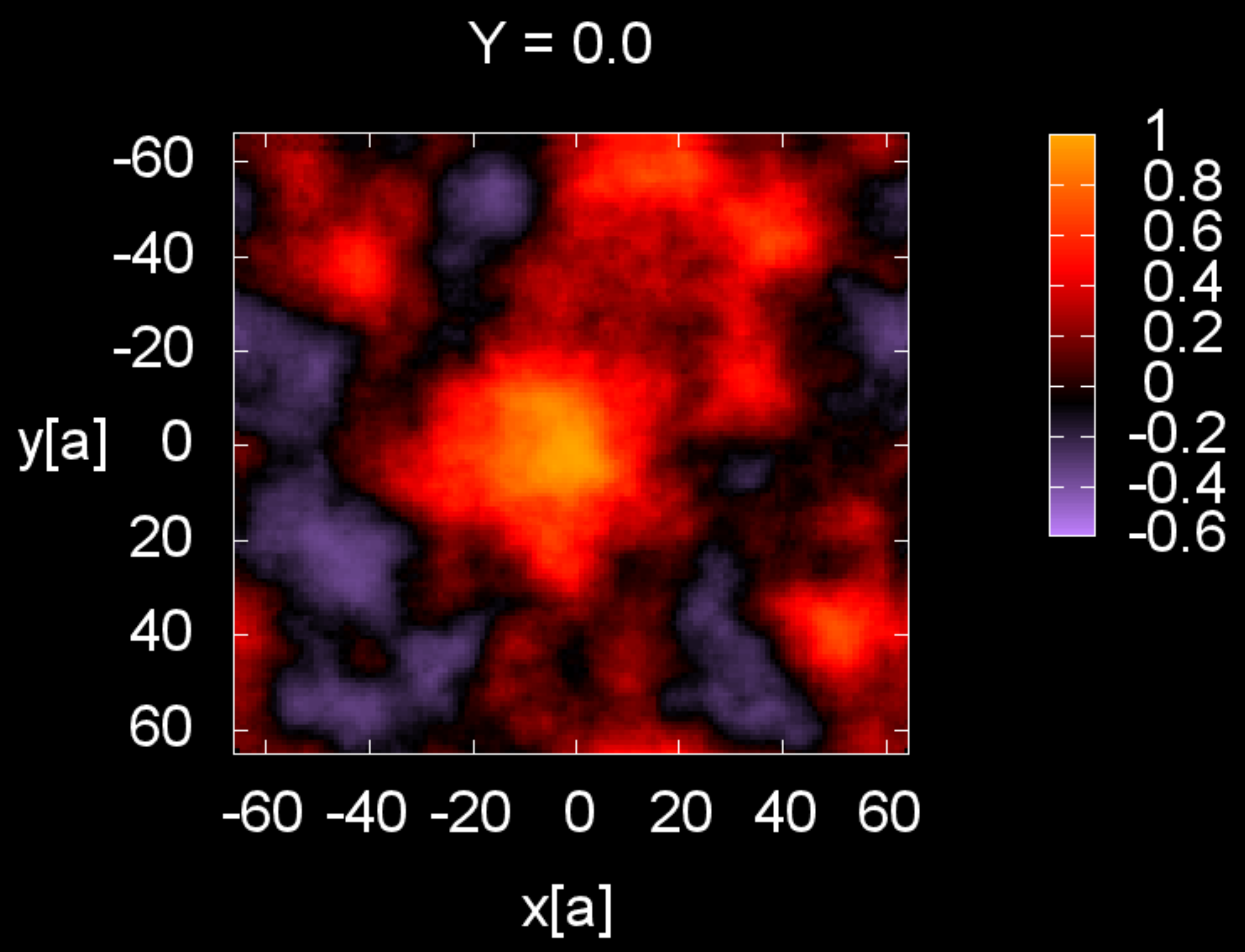
The x-dependence can be computed using the JIMWLK equations (functional renormalization group equations considering the non-linear evolution from large to small x)

Example on the right:

Wilson line

$$V^\dagger(x^-, \mathbf{x}_\perp) = \mathscr{P} \exp\left(-ig\int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_\perp)\right)$$

correlator for constant color charge density and Gaussian distributed ρ^a



THE COLLISION - LIGHT CONE GAUGE

- Computing gluon fields after collision is more conveniently done in light cone gauge $A^+ = 0$ for a right moving nucleus, $A^- = 0$ for a left moving nucleus

gauge transformation: $$A_\mu(x) \to V(x)\left(A_\mu(x) - \frac{i}{g}\partial_\mu\right)V^\dagger(x)$$

To achieve $A^+ = 0$ the Wilson line must fulfill

$$\partial\_V^\dagger(x^-, \mathbf{x}_\perp) = -igA^+(x^-, \mathbf{x}_\perp)V^\dagger(x^-, \mathbf{x}_\perp)$$

which is solved by $V^\dagger(x^-, \mathbf{x}_\perp) = \mathscr{P}\exp\left(-ig\int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_\perp)\right)$

Also $\partial_+ V^\dagger = 0$, so $A^- = 0$ as well.

THE COLLISION - LIGHT CONE GAUGE

- The transverse components are not zero but pure gauge fields:

$$A^i(x^-, \mathbf{x}_\perp) = \frac{1}{ig} V(x^-, \mathbf{x}_\perp)\partial^i V^\dagger(x^-, \mathbf{x}_\perp)$$

- Color current in light cone gauge is

$$J^+_{\mathrm{LC}}(x^-, \mathbf{x}_\perp) = \rho_{\mathrm{LC}}(x^-, \mathbf{x}_\perp) = V(x^-, \mathbf{x}_\perp)\rho(x^-, \mathbf{x}_\perp)V^\dagger(x^-, \mathbf{x}_\perp)$$

- Again, REALLY high energy approximation: $\rho(x^-, \mathbf{x}_\perp) = \delta(x^-)\rho(\mathbf{x}_\perp)$ (thin sheet)
  A^+ has the same support and Wilson line is only nontrivial if x^- integration limit is >0.

- So $A^i(x^-, \mathbf{x}_\perp) = \theta(x^-)\alpha^i(\mathbf{x}_\perp)$ with $\alpha^i(\mathbf{x}_\perp) = \frac{1}{ig} V(\mathbf{x}_\perp)\partial^i V^\dagger(\mathbf{x}_\perp)$ and $V^\dagger(\mathbf{x}_\perp) = V^\dagger(x^- \to \infty, \mathbf{x}_\perp)$

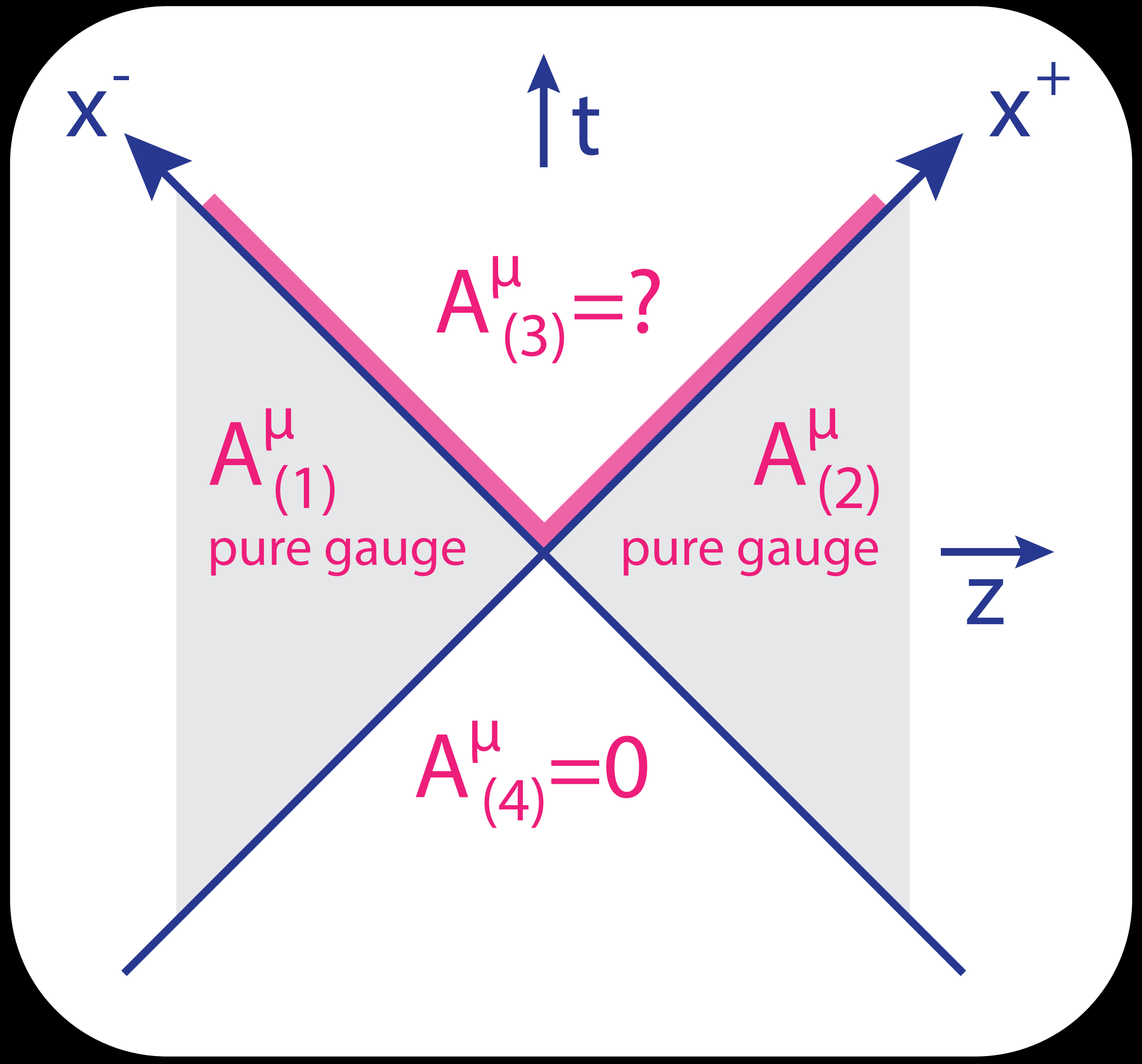
THE COLLISION - LIGHT CONE GAUGE

- Doing the same for the left moving nucleus and choosing the gauge field to be zero for the quadrant where $x^- < 0$ and $x^+ < 0$ one finds

$$A^i(\mathbf{x}_\perp) = \theta(x^-)\theta(-x^+)\alpha_P^i(\mathbf{x}_\perp) + \theta(x^+)\theta(-x^-)\alpha_T^i(\mathbf{x}_\perp)$$

$$= \theta(x^-)\theta(-x^+)\frac{1}{ig}V_P(\mathbf{x}_\perp)\partial^i V_P^\dagger(\mathbf{x}_\perp) + \theta(x^+)\theta(-x^-)\frac{1}{ig}V_T(\mathbf{x}_\perp)\partial^i V_T^\dagger(\mathbf{x}_\perp)$$

for the fields before the collision (all transverse)



- Now for the forward light cone. We choose Fock-Schwinger gauge: $x^+A^- + x^-A^+ = 0$

BOOST INVARIANCE

- At super high energy, currents are proportional to delta functions

$$J^\nu = \delta^{\nu+}\rho_P(\mathbf{x}_\perp)\delta(x^-) + \delta^{\nu-}\rho_T(\mathbf{x}_\perp)\delta(x^+)$$

In this case, the current, and the solutions for the gluon fields are invariant under longitudinal boosts (“boost invariant”), meaning invariant under

$$x^\pm \to x'^\pm = e^{\pm\beta}x^\pm$$
$$J^\pm(x) \to J'^\pm(x') = e^{\pm\beta}J^\pm(x)$$

with β the longitudinal boost parameter (no mixing, just rescaling!).

- Current does not change its form: $J^\pm_{P/T}(x^\mp, \mathbf{x}_\perp) \to e^{\pm\beta}\delta(e^{\pm\beta}x'^\mp)\rho_{P/T}(\mathbf{x}_\perp) = \delta(x'^\mp)\rho_{P/T}(\mathbf{x}_\perp)$

BOOST INVARIANCE

In the future light cone define $x^+ = \frac{\tau}{\sqrt{2}} e^{+\eta}$, and $x^- = \frac{\tau}{\sqrt{2}} e^{-\eta}$

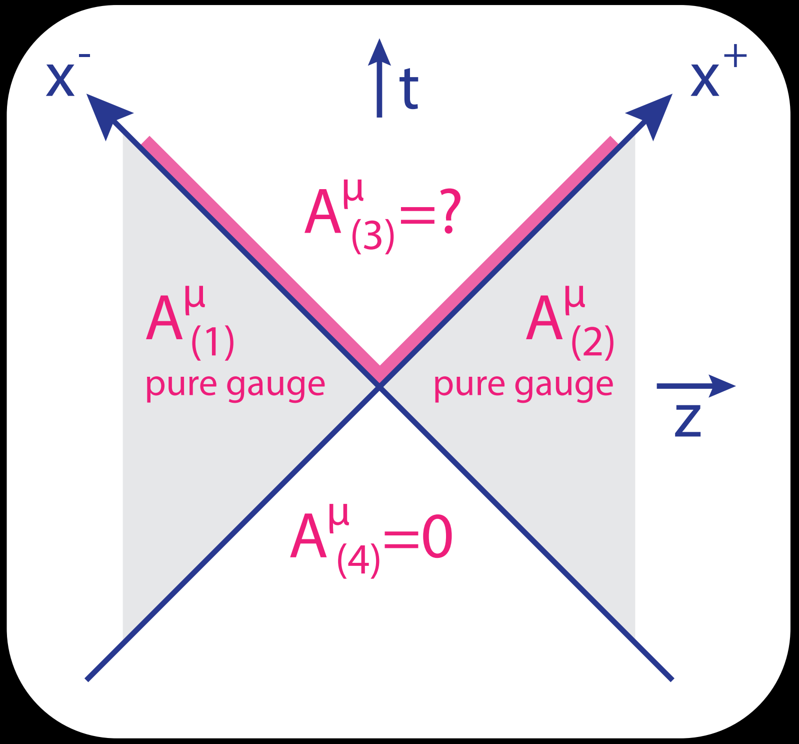
or inverted $\tau = \sqrt{2x^+x^-}$, and $\eta = \frac{1}{2} \ln\left(\frac{x^+}{x^-}\right)$

The gauge field transforms as $A'^{\mu}(x') = \frac{\partial x'^{\mu}}{\partial x^{\nu}} A^{\nu}(x)$, leading to

$$A^{\tau} = \frac{1}{\tau}(x^-A^+ + x^+A^-) = A_{\tau} \quad \text{and} \quad A^{\eta} = \frac{1}{\tau^2}(x^-A^+ - x^+A^-) = -\frac{1}{\tau^2}A_{\eta}$$

All fields are independent of η

SOLUTION IN THE FORWARD LIGHT CONE



Finally, to find the solution in the forward light cone, write general expression

$$A^i(x) = \theta(x^+)\theta(x^-)\alpha^i(\tau,\mathbf{x}_\perp) + \theta(x^-)\theta(-x^+)\alpha_P^i(\mathbf{x}_\perp) + \theta(x^+)\theta(-x^-)\alpha_T^i(\mathbf{x}_\perp)$$

$$A^\eta(x) = \theta(x^+)\theta(x^-)\alpha^\eta(\tau,\mathbf{x}_\perp)$$

$A^\tau = 0$, **because we chose Fock-Schwinger gauge** $x^+A^- + x^-A^+ = 0$

Plugging above ansatz into YM equations leads to singular terms on the boundary $\tau \to 0$ **from derivatives of** θ**-functions.**

Requiring that the singularities vanish leads to the solutions

$$\alpha^i = \alpha_P^i + \alpha_T^i \qquad \alpha^\eta = -\frac{ig}{2}\left[\alpha_{Pj},\alpha_T^j\right] \qquad \begin{aligned}\partial_\tau\alpha^i &= 0\\ \partial_\tau\alpha^\eta &= 0\end{aligned}$$

INITIAL CONDITIONS FOR HEAVY ION COLLISIONS

From the gluon fields in the forward light cone compute the energy momentum tensor

$$T^{\mu\nu} = -g^{\mu\alpha}g^{\nu\beta}g^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta} + \frac{1}{4}g^{\mu\nu}g^{\alpha\gamma}g^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}$$

$$T^{\tau\tau} = \frac{1}{2}(E^{\eta})^2 + \frac{1}{2\tau^2}[(E^x)^2 + (E^y)^2] + \frac{1}{2}F_{xy}F_{xy} + \frac{1}{2\tau^2}(F_{x\eta}^2 + F_{y\eta}^2)$$

transverse electric field

longitudinal electric field

longitudinal magnetic field

transverse magnetic field

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INITIAL ENERGY DENSITY - HOW DOES IT GO?

$$\varepsilon_0 = \frac{1}{2}[(E^\eta)^2 + (B^\eta)^2] = -\frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\Big([\alpha_P^i, \alpha_T^j][\alpha_P^k, \alpha_T^l]\Big)$$

$$= \frac{g^2}{2} f_{abe} f_{cde} (\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) \langle \alpha_P^{ai} \alpha_P^{ck} \rangle_{\rho_P} \langle \alpha_T^{bj} \alpha_T^{dl} \rangle_{\rho_T}$$

$$\propto g^2 \mu_P^2 \propto T_P \qquad \propto g^2 \mu_T^2 \propto T_T$$

$$\Rightarrow \varepsilon_0 \propto T_P T_T$$

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NOW FOR THE "NOT SO AB-INITIO" PART

Remember, we have not defined $\rho^a_{P/T}(x^\mp, \mathbf{x}_\perp)$ yet.

Usually, one assumes a really large nucleus, where the color charges are locally Gaussian correlated: $\langle \rho^a(\mathbf{b}_\perp)\rho^b(\mathbf{x}_\perp)\rangle = g^2\mu^2(x, \mathbf{b}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{b}_\perp - \mathbf{x}_\perp)$ and have zero mean.

That's the McLerran-Venugopalan model (originally with constant μ).

Here, $g^2\mu(x, \mathbf{b}_\perp)$ depends on the longitudinal momentum fraction x and the transverse position $\mathbf{b}_\perp$, both functional dependencies that need to be modeled.

In the IP-Glasma model, that modeling is done in part by using the IPSat model and in part by a Monte Carlo model for the fluctuating geometry of a nucleus or proton projectile

IPSAT

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

- The total cross section for a small dipole passing through a dilute gluon cloud is proportional to the dipole area, the strong coupling constant, and the number of gluons in the cloud: L. Frankfurt, A. Radyushkin, and M. Strikman, Phys. Rev. D55, 98 (1997)

$$\sigma_{q\bar{q}} = \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2)$$

where $xg(x, \mu^2)$ is the gluon density at some scale μ^2

- From that we get the Glauber-Mueller dipole cross section in a dense gluon system

$$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2(1 - \mathrm{Re}S(b)) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T(b)\right)\right]$$

- $T(\mathbf{b})$ and $xg(x, \mu^2)$ are determined from fits to HERA DIS data (b, x, and initial scale μ_0^2 dependence) and DGLAP evolution in μ^2

GEOMETRY

- The thickness function $T(b)$ is modeled
- For a nucleon use a Gaussian or a collection of smaller Gaussians (substructure)

$$T(b) = \frac{1}{2\pi B_G} \exp\left(\frac{-b^2}{2B_G}\right)$$

- Usually B_G is assumed to be energy independent and fit yields $\sim 4\mathrm{GeV}^{-2}$
- It is related to the average squared gluonic radius $\langle b^2 \rangle = 2B_G$
- b is smaller than the charge radius: b=0.56 fm (c.f. R_p = 0.8751(61) fm)
- For a nucleus, do as in MC Glauber and sample nucleon positions from a nuclear density distribution (e.g. a Woods-Saxon distribution)
- Sum all nucleon $T(\vec{b})$ to get the total nuclear $T(\vec{b})$

GEOMETRY

from Schenke, Shen, Tribedy, Phys.Rev.C 102 (2020) 4, 044905

$$\rho(r,\theta) = \frac{\rho_0}{1+\exp[(r-R'(\theta))/a]}\,, \tag{9}$$

with $R'(\theta) = R[1+\beta_2 Y_2^0(\theta) + \beta_4 Y_4^0(\theta)]$, and ρ_0 the nuclear density at the center of the nucleus. R is the radius parameter, a the skin depth.

Nucleus	R [fm]	a [fm]	β_2	β_4
^{238}U	6.81	0.55	0.28	0.093
^{208}Pb	6.62	0.546	0	0
^{197}Au	6.37	0.535	-0.13	-0.03
^{129}Xe	5.42	0.57	0.162	-0.003
^{96}Ru	5.085	0.46	0.158	0
^{96}Zr	5.02	0.46	0	0

Smaller nuclei, such as ^{16}O, and ^{3}He are described using a variational Monte-Carlo method (VMC) using the Argonne v18 (AV18) two-nucleon potential +UIX interactions [63]. In practice we use the ^{3}He and ^{16}O configurations available in the PHOBOS Monte-Carlo Glauber distribution [64, 65].

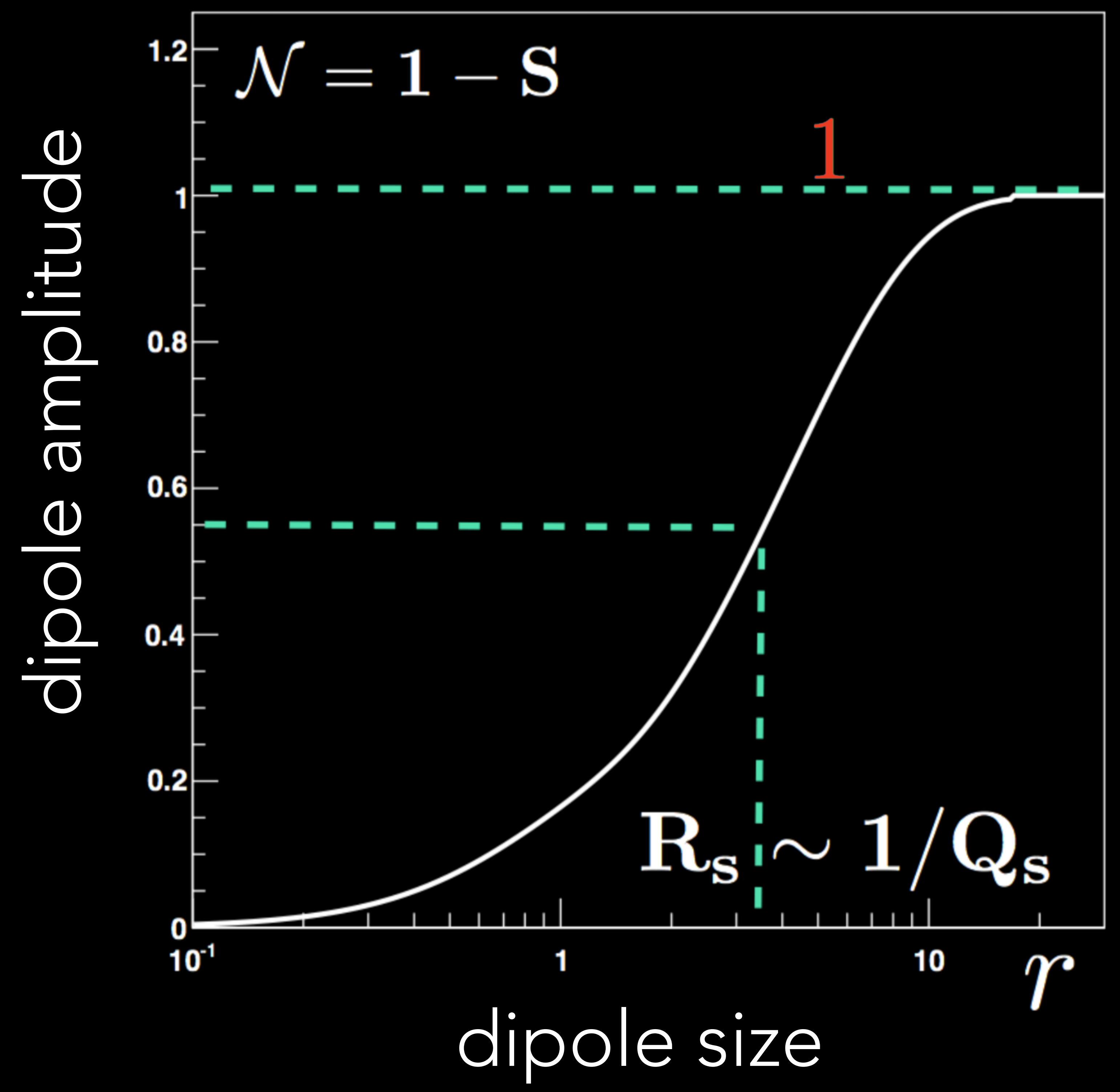
For the results we will show involving the deuteron, we employ a simple Hulthen wave function of the form [66]

$$\phi(d_{\rm pn}) = \frac{\sqrt{a_H b_H(a_H+b_H)}}{b_H - a_H}\,\frac{e^{-a_H d_{\rm pn}} - e^{-b_H d_{\rm pn}}}{\sqrt{2\pi}d_{\rm pn}}\,, \tag{10}$$

where $d_{\rm pn}$ is the separation between the proton and the neutron, and the parameters are experimentally determined to be $a_H = 0.228\,{\rm fm}^{-1}$ and $b_H = 1.18\,{\rm fm}^{-1}$.

SUCH MODELING YIELDS COLOR CHARGE DENSITY

$$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2(1 - \mathrm{Re}S(b)) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$



Dipole amplitude saturates at 1!

Q_s is defined as the inverse scale where saturation effects begin

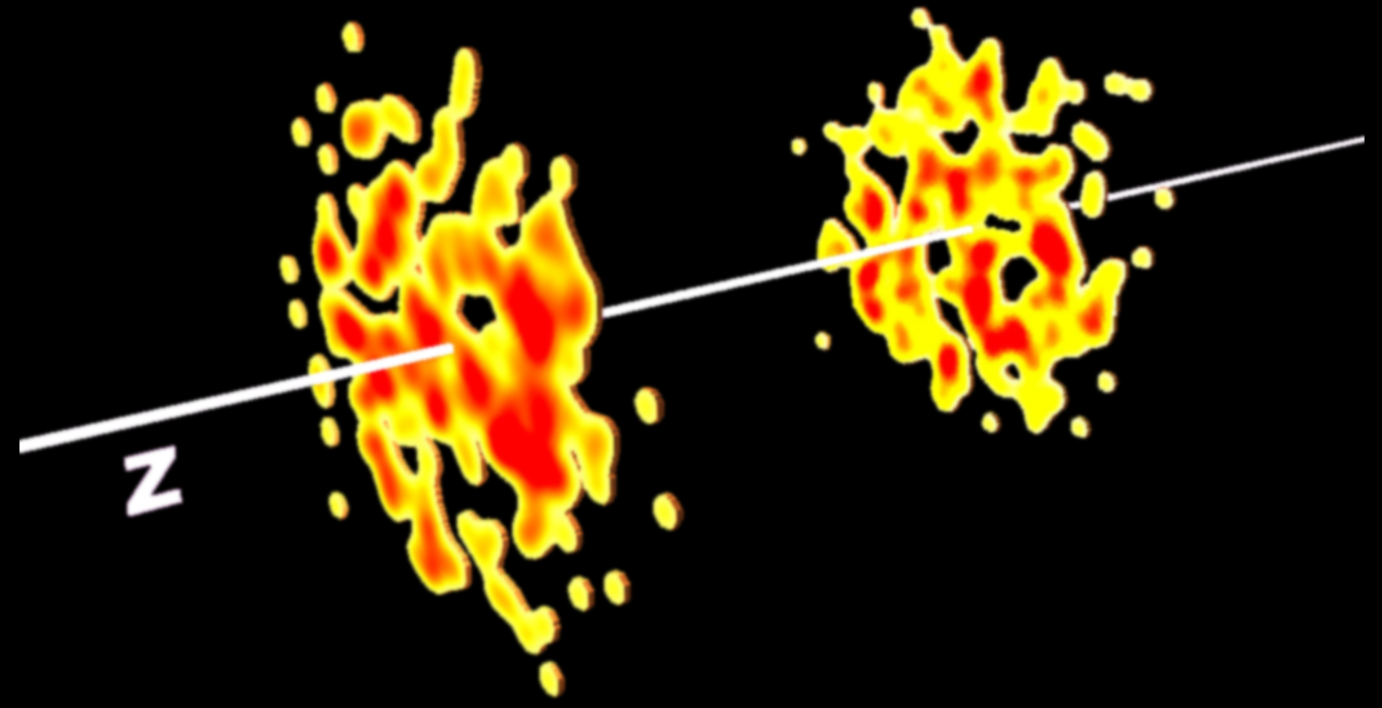
$$N(R_s, x, b) = 1 - e^{-1/2}$$

$$Q_s^2 = 2/R_s^2$$

Then assume $g^2\mu^2(x,\mathbf{b}) \propto Q_s^2(x,\mathbf{b})$ and use

$$\langle \rho^a(\mathbf{b}_\perp)\rho^b(\mathbf{x}_\perp)\rangle = g^2\mu^2(x,\mathbf{b}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{b}_\perp - \mathbf{x}_\perp)$$

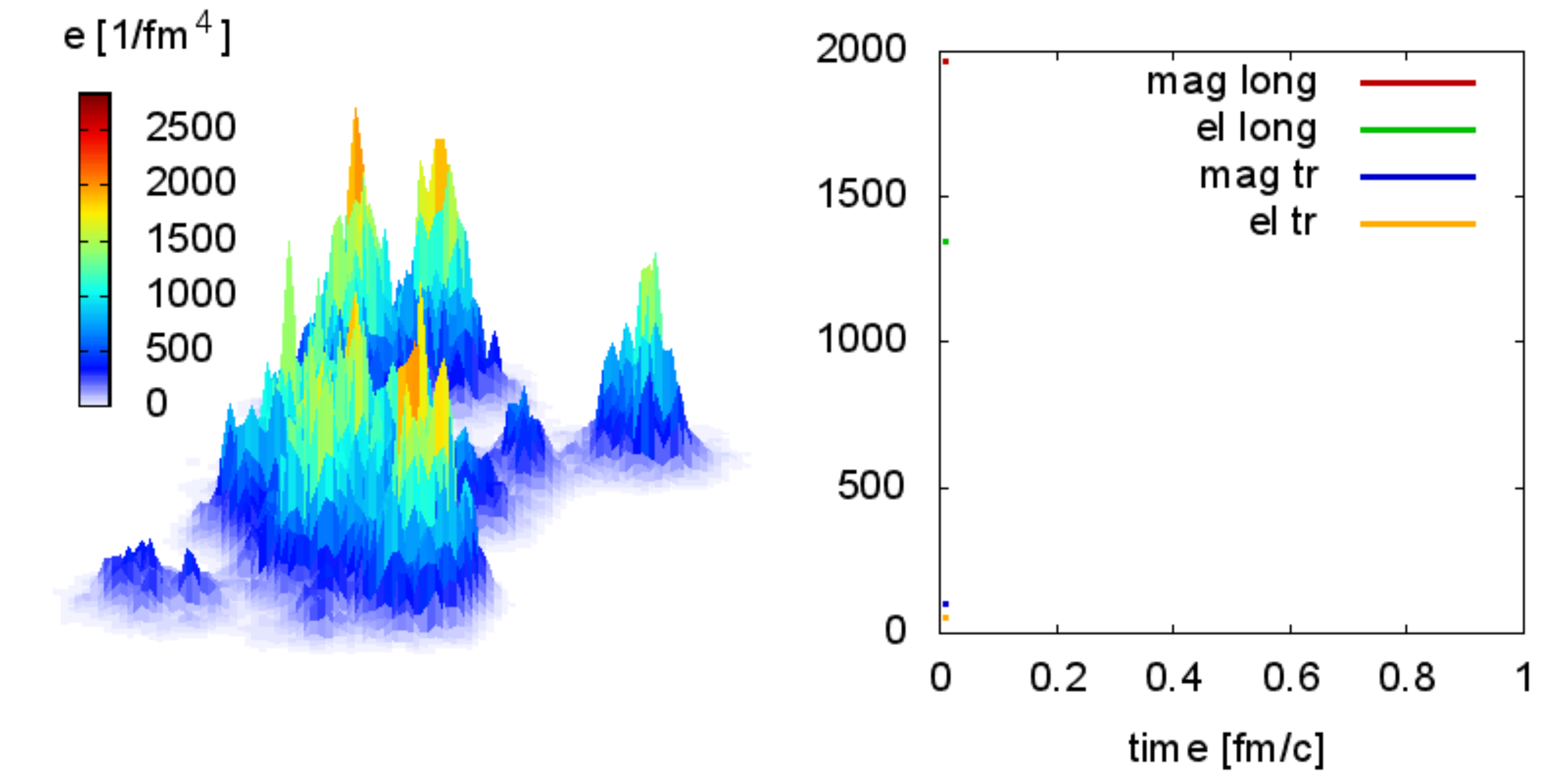
SUMMARY OF THE PROCESS



- Incoming nuclei described within color glass condensate: large x d.o.f. are color sources, small x classical gluon fields
- Incoming currents need to be constructed first:
  - Sample nucleons from nuclear density distributions
  - Add the $T(b)$ at every transverse position
  - Extract Q_s from the IPSat dipole amplitude
  - Obtain the color charge density: $g^4\mu^2 \sim (Q_s)^2$
  - Sample color charges ρ^a from local Gaussian distributions
- The color charges generate the eikonal color current that sources the small-x classical gluon fields
- Gluon fields are determined from the Yang-Mills equations
- Solve for the gluon fields in the forward light cone
- Solve source-less YM equations forward in time
- Compute the energy momentum tensor - this is your initial condition for hydrodynamics

INITIAL CONDITIONS FOR HEAVY ION COLLISIONS

$$T^{\tau\tau} = \frac{1}{2}(E^\eta)^2 + \frac{1}{2\tau^2}[(E^x)^2 + (E^y)^2] + \frac{1}{2}F_{xy}F_{xy} + \frac{1}{2\tau^2}(F_{x\eta}^2 + F_{y\eta}^2)$$



Solve the source free YM equations in time

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HOW TO IMPROVE?

- Constrain color charge density better at moderate x
  - Here input from nuclear structure calculations can be valuable
  - Sample more parameters from distributions? (e.g. β_2 etc.)
- Include non-Gaussian correlations for the color charges (important for small nuclei)
- Use small-x evolution to determine incoming Wilson lines
- Go beyond boost invariance (full 3D YM, initial conditions much harder to get)
- ...

SMALL X EVOLUTION

- Does it modify deformation? Should reduce it.

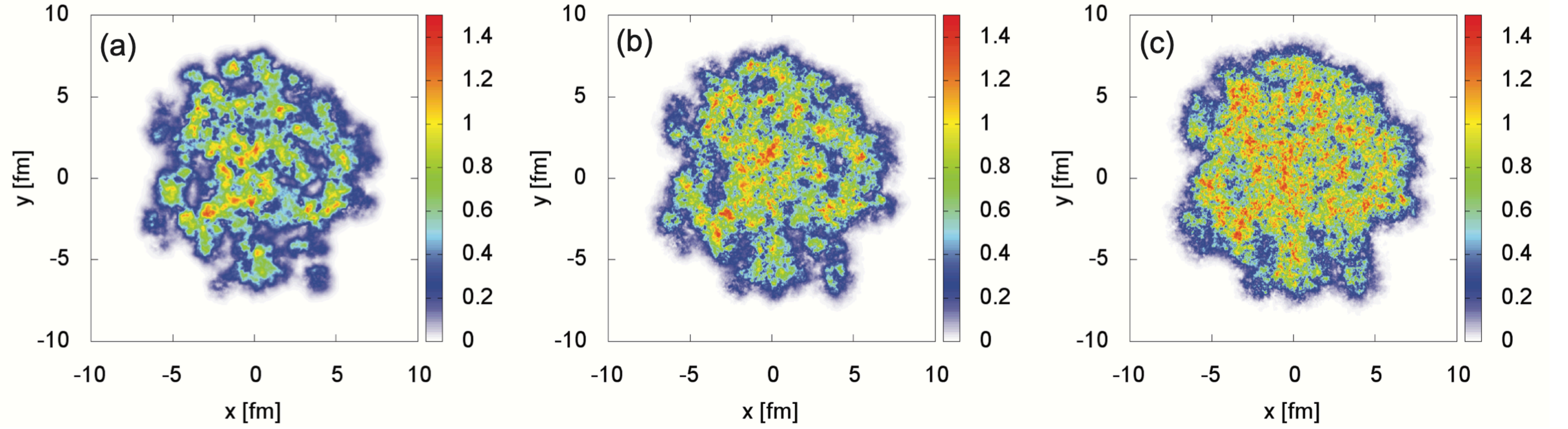


FIG. 2. JIMWLK evolution of the gluon fields in one nucleus for $m = 0.4\,\mathrm{GeV}$ and $\alpha_s = 0.3$. Shown is $1 - \mathrm{Re}[\mathrm{tr}(V_{\mathbf{x}})]/N_c$ in the transverse plane at rapidities $Y = -2.4$ ($x \approx 2\times10^{-3}$) (a), $Y = 0$ ($x \approx 2\times10^{-4}$) (b), and $Y = 2.4$ ($x \approx 1.6\times10^{-5}$) (c) to illustrate the change of the typical transverse length scale with decreasing x. The global geometry clearly remains correlated over the entire range in rapidity.

BACKUP

SATURATION CRITERION

L.V. Gribov, E.M. Levin and M.G. Ryskin, Physics Reports 100, Nos. 1 & 2 (1983) 1—150

- Number of gluons per area:

$$\rho \sim \frac{xG(x, Q^2)}{\pi R^2}$$

- Recombination cross section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

- Recombination important when $\rho\sigma_{gg\to g} \gtrsim 1, \text{i.e. } Q^2 \lesssim Q_s^2$

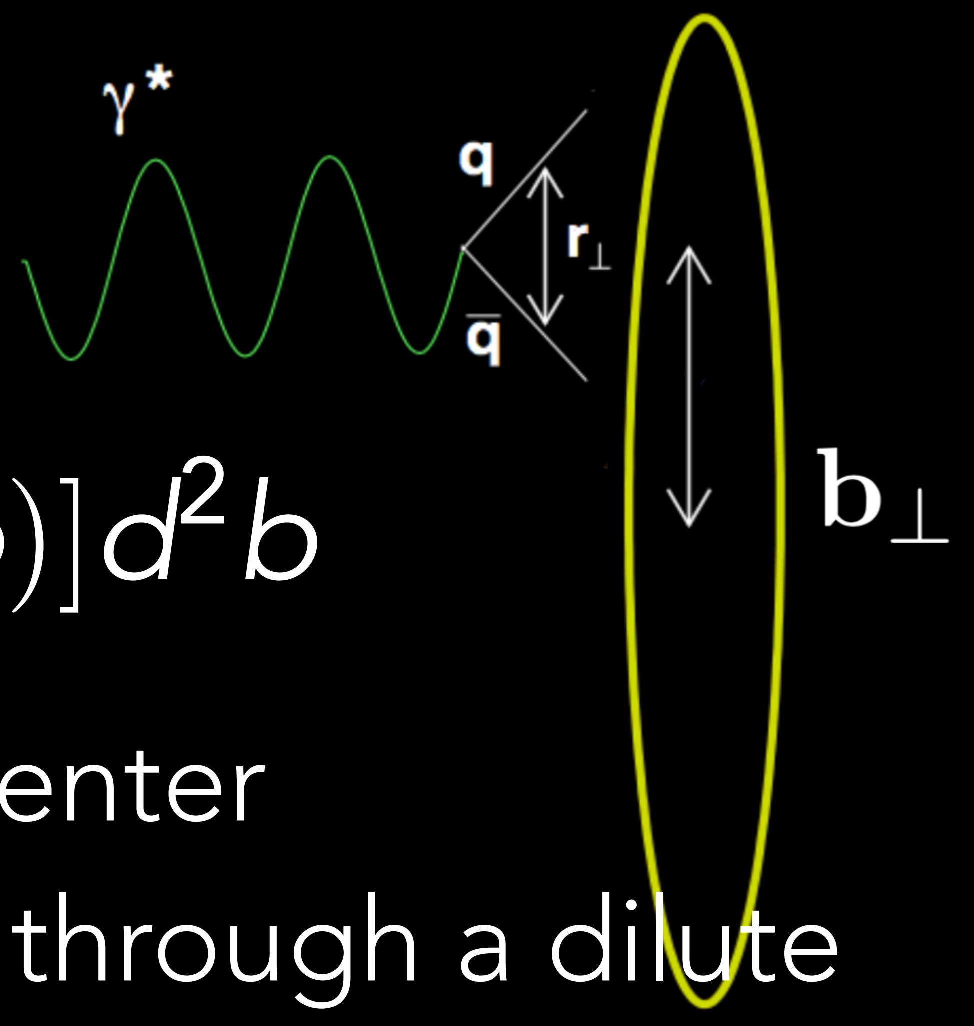
$$\text{with } Q_s^2 \sim \frac{\alpha_s xG(x, Q_s^2)}{\pi R^2} \sim A^{1/3}x^{-0.3}$$

- At saturation the phase-space density is:

$$\frac{dN_g}{d^2x_\perp d^2p_\perp} \sim \frac{\rho}{Q_s^2} \sim \frac{1}{\alpha_s}$$

IPSAT MODEL

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005



- $\sigma_{q\bar{q}}(x,r) = \mathrm{Im}\, iA_{\mathrm{el}}^{q\bar{q}}(x,r,0) = 2\int [1 - \mathrm{Re}S(b)]d^2b$
- Here *S(b)* is the S-matrix at distance b from the center
- The total cross section for a small dipole to pass through a dilute gluon cloud is proportional to the dipole area, the strong coupling constant, and the number of gluons in the cloud

  L. Frankfurt, A. Radyushkin, and M. Strikman, Phys. Rev. D55, 98 (1997)

$$\sigma_{q\bar{q}} = \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2)$$

where $xg(x,\mu^2)$ is the gluon density at some scale μ^2

- If the target is dense, the probability that the dipole does not scatter inelastically at impact parameter b is

$$P(b) = 1 - \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2)\rho(b,z)dz$$

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total prob. for no inel. interaction

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$$
\begin{aligned}
P(-L < z \leq L) &= \lim_{n\to\infty} \Pi_{i=0}^{n-1} P(z_i < z \leq z_{i+1}) \\
&= \lim_{n\to\infty} \Pi_{i=0}^{n-1} (1 - \sigma_{q\bar{q}}\rho(b, z_i < z \leq z_{i+1})dz) \\
&= \lim_{n\to\infty} \Pi_{i=0}^{n-1} \exp(-\sigma_{q\bar{q}}\rho(b, z_i < z \leq z_{i+1})dz) \\
&= \exp(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \sigma_{q\bar{q}}\rho(b, z_i < z \leq z_{i+1})dz) \\
&= \exp\left(-\int_{-L}^{L} \sigma_{q\bar{q}}\rho(b, z)dz\right) \\
&= \exp(-\sigma_{q\bar{q}} T(b)) = P_{\mathrm{tot}}(b) \qquad \text{letting } L \to \infty
\end{aligned}
$$

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- So the probability for the dipole not to interact inelastically passing through the entire target is:

$$|S(b)|^2 = P_{\rm tot}(b) = \exp\left(-\frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)$$

- Assuming the S-matrix element is predominantly real, we have

$$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2(1 - {\rm Re}S(b)) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)\right]$$

- This is the Glauber-Mueller dipole cross section

A.H. Mueller, Nucl. Phys. B335, 115 (1990)

- $T(b)$ and $xg(x,\mu^2)$ are determined from fits to HERA DIS data (b, x, and initial scale $(\mu_0)^2$ dependence) and DGLAP evolution in μ^2

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- The impact parameter dependent function $T(b)$ for a proton is assumed to be Gaussian:

$$T(b) = \frac{1}{2\pi B_G} \exp\left(\frac{-b^2}{2B_G}\right)$$

  B_G is assumed to be energy independent and fit yields ~4 GeV^{-2}

- It is related to the average squared gluonic radius $\langle b^2 \rangle = 2B_G$
  b is smaller than the charge radius: b=0.56 fm
  (c.f. R_p = 0.8751(61) fm)

- We will later discuss how additional sub-nucleonic fluctuations of this shape affect observables