Determination of the neutron skin of atomic nuclei

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Nuclear physics confronts relativistic collisions of isobars

Heidelberg, May 30th to June 3rd, 2022

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Elastic electron scattering and parity violating electron scattering; access to the isotope shift in charge radii also via laser spectroscopy.

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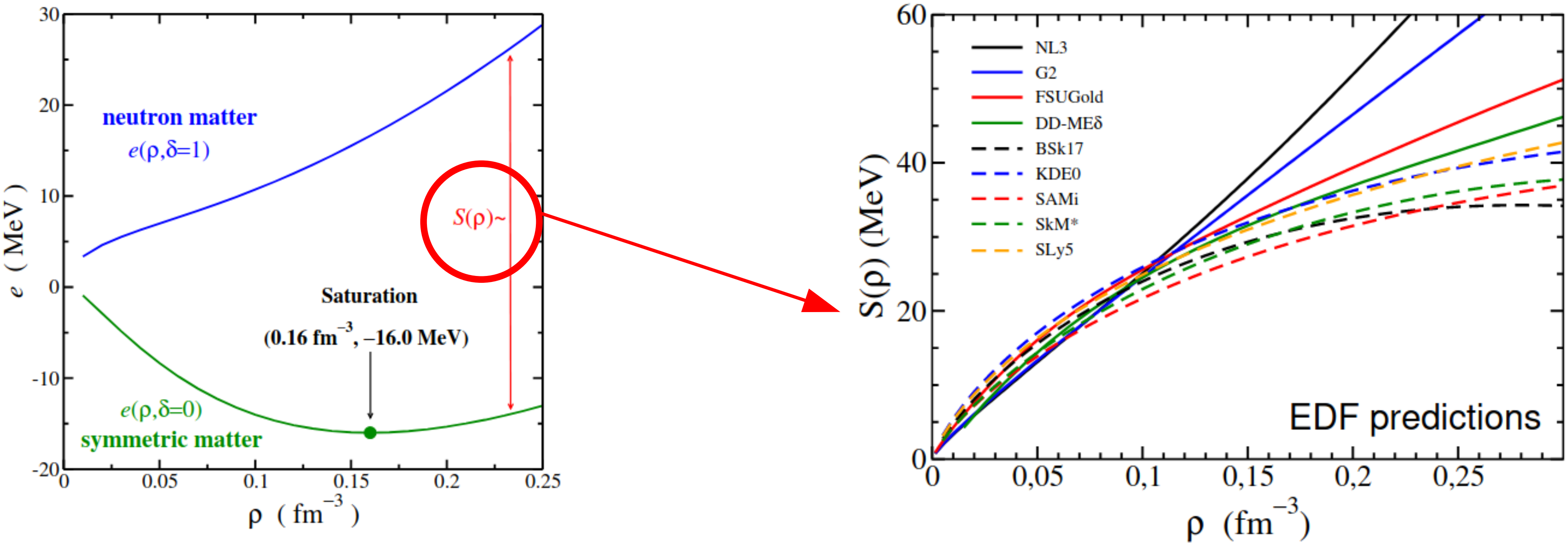
Atomic parity violation and BSM, Dipole polarizability, Giant Dipole and Quadrupole resonances and their low lying strength, Isobaric Analog State, Charge radii in mirror nuclei, Spin Dipole Resonance …

Implications for nuclear astrophysics from precision neutron skin measurements.

Nuclear Equation of State, Mass-Radius relation and deformability of a (light) neutron star, Composition of the crust of a neutron star …

Nuclear Equation of State

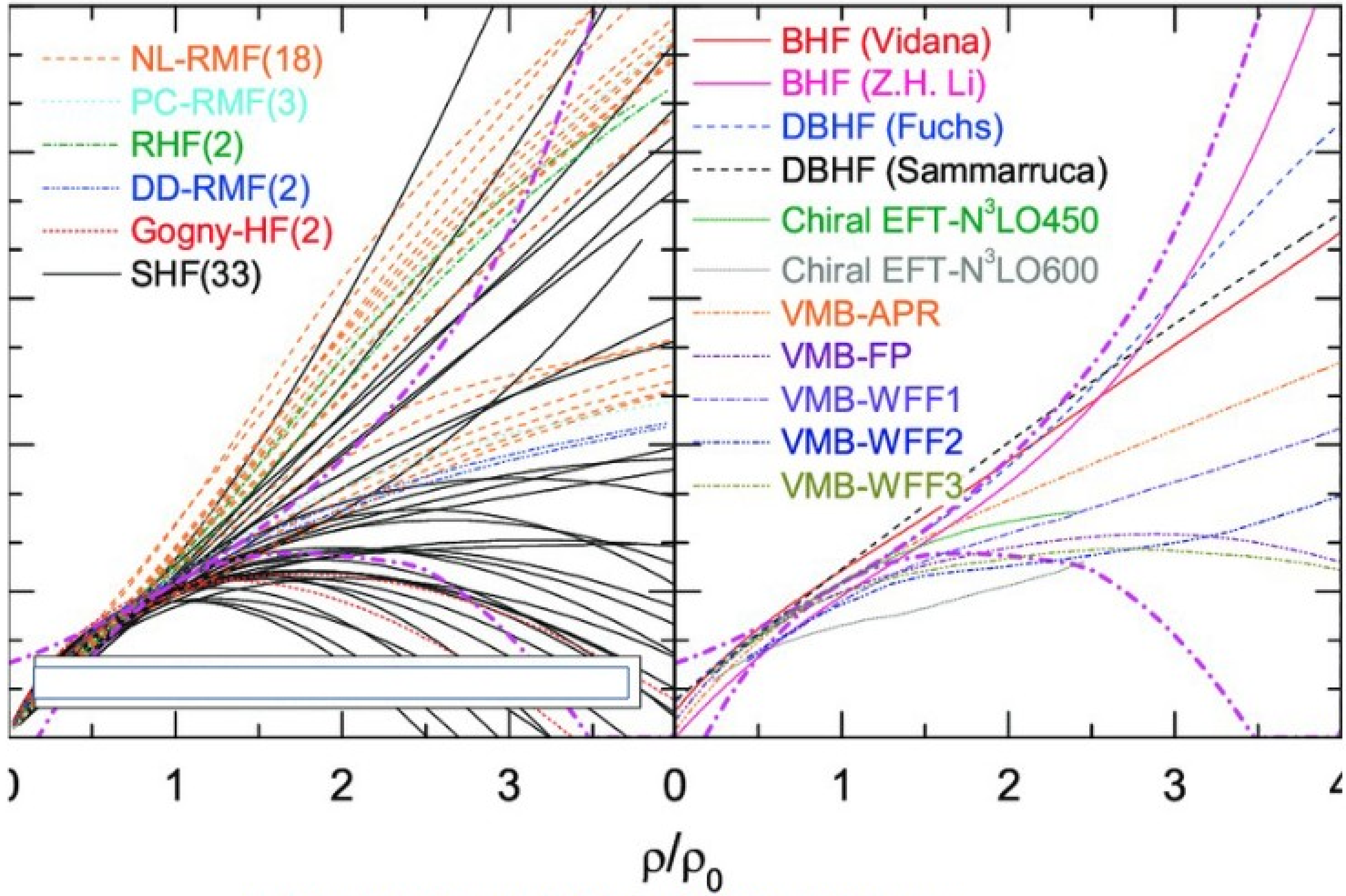
-**Isovector properties** not well determined in current **EDFs**



$$e(\rho,\beta) = e(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where} \;\; \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

$$\text{where } J \equiv S(\rho_0) \text{ and } L \equiv 3\rho_0 \frac{\partial S}{\partial \rho}\Big|_{\rho_0} = 3\rho_0 p_0^{\mathrm{neut}}$$

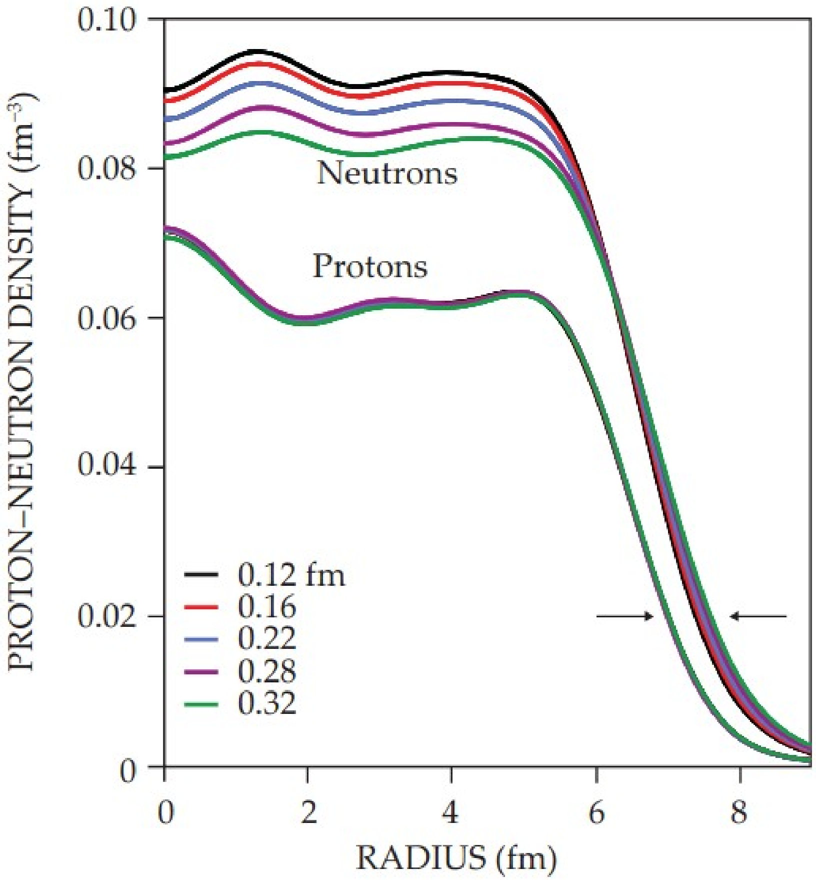
Nuclear Equation of State

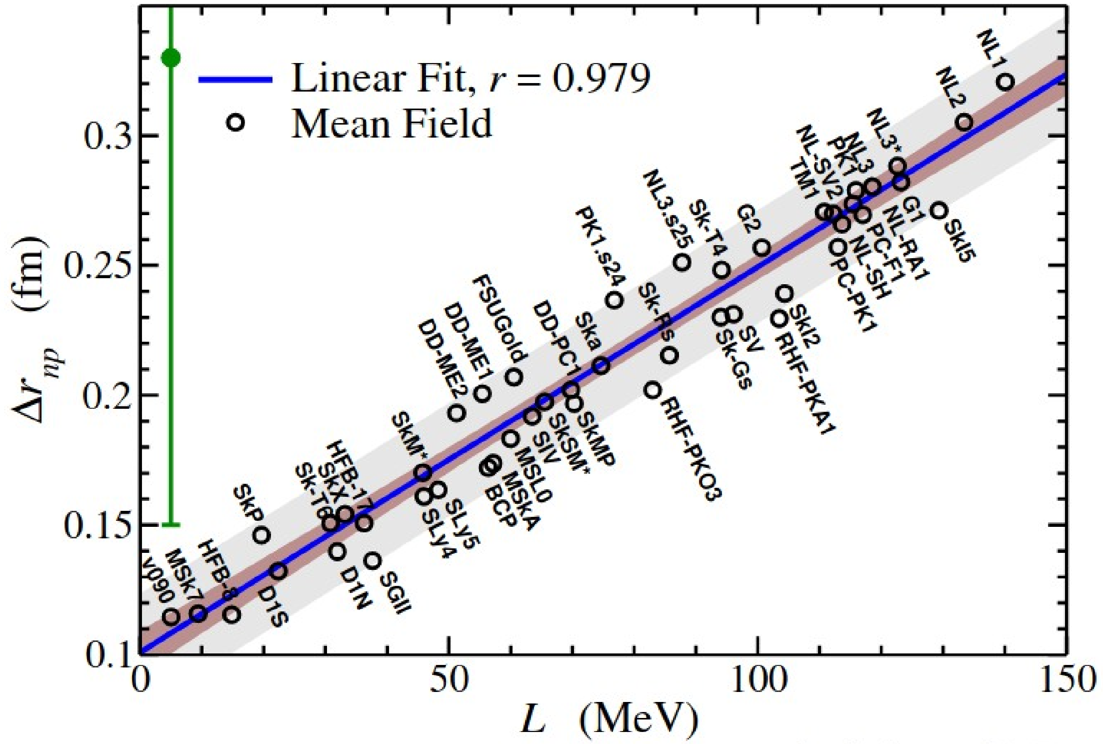


Towards understanding astrophysical effects of nuclear symmetry energy

Bao-An Li, Plamen G. Krastev, De-Hua Wen & Nai-Bo Zhang EPJ A 55, 117 (2019)

Neutron skin thickness





$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12} \frac{N-Z}{A} \frac{R}{J} L$$

Physics Today **72**, 7, 30 (2019)

X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda
Phys. Rev. Lett. **106**, 252501 (2011) - Published 21 June 2011

Elastic electron scattering: $\rho_{ch} \rightarrow \rho_p$

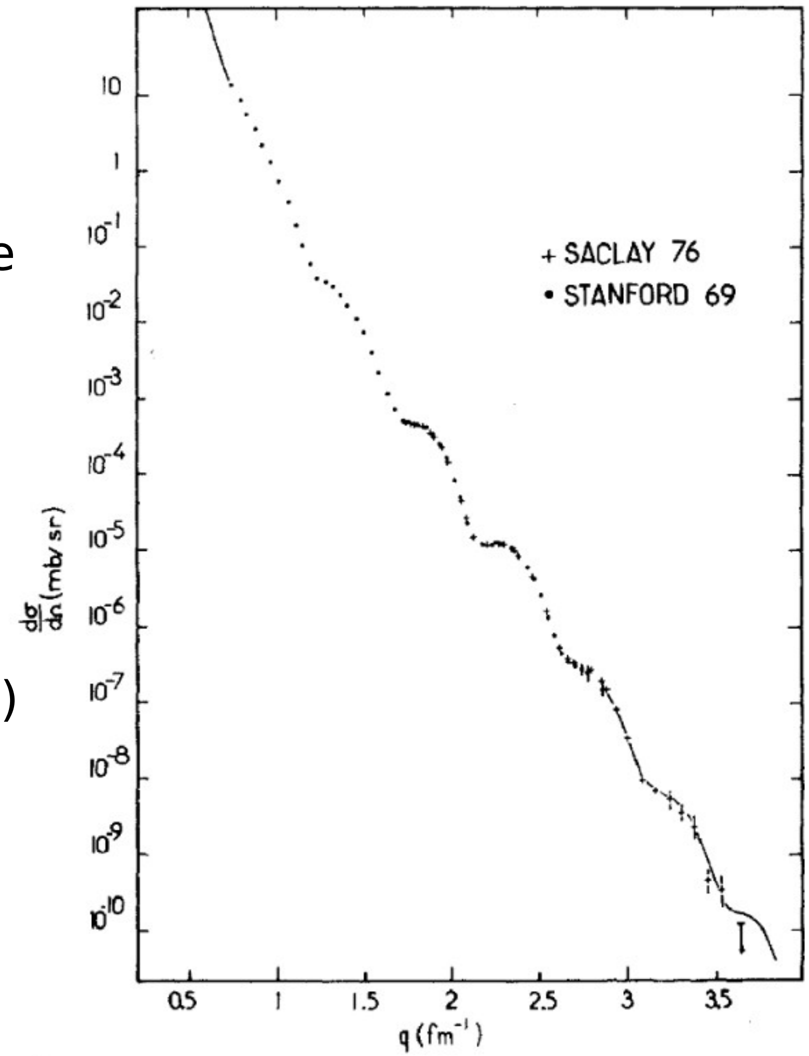
$$E_{\mathrm{beam}} \sim \frac{2\pi\hbar c}{\lambda_{\mathrm{nuclear}}} \sim 10^2\mathrm{MeV}$$

The **scattering of relativistic electrons** by the **Coulomb field** V(r) is completely **described** by the direct scattering amplitude, **f(θ)**, and the spin-flip scattering amplitude, **g(θ)**.

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2$$

f(θ) and g(θ) determined from the **solutions of the Dirac equation** for the central potential V(r)

$$V_{\mathrm{nucl.elec.}} = 4\pi Z_0 e^2 \left\{ \frac{1}{r}\int_0^r \rho_{\mathrm{ch}}(u) u^2 du + \int_r^\infty \rho_{\mathrm{ch}}(u) u du \right\}$$



Elastic electron scattering: $\rho_{ch} \rightarrow \rho_p$

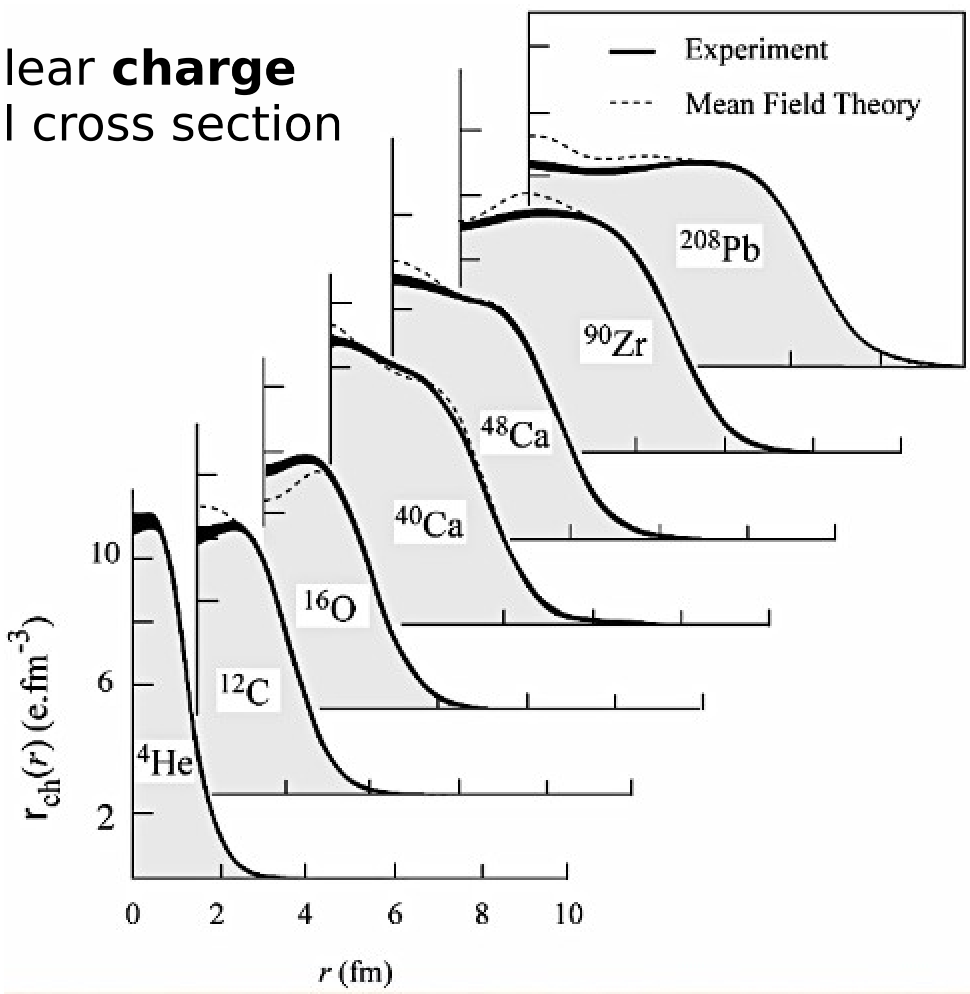
Experimentally one can access the nuclear **charge form factor** dividing by the differential cross section of a point nucleus with charge Z

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} |F(q)|^2$$

The low momentum transfer behavior of the form factor determines the **charge radius**

$$F_{ch}(q) = Z\left(1 + \frac{q^2\langle r^2_{ch}\rangle}{3!} + [O]q^4\right)$$

The **Fourier transform** of **F_{ch}(q)** gives access to **ρ_{ch}(r)**.



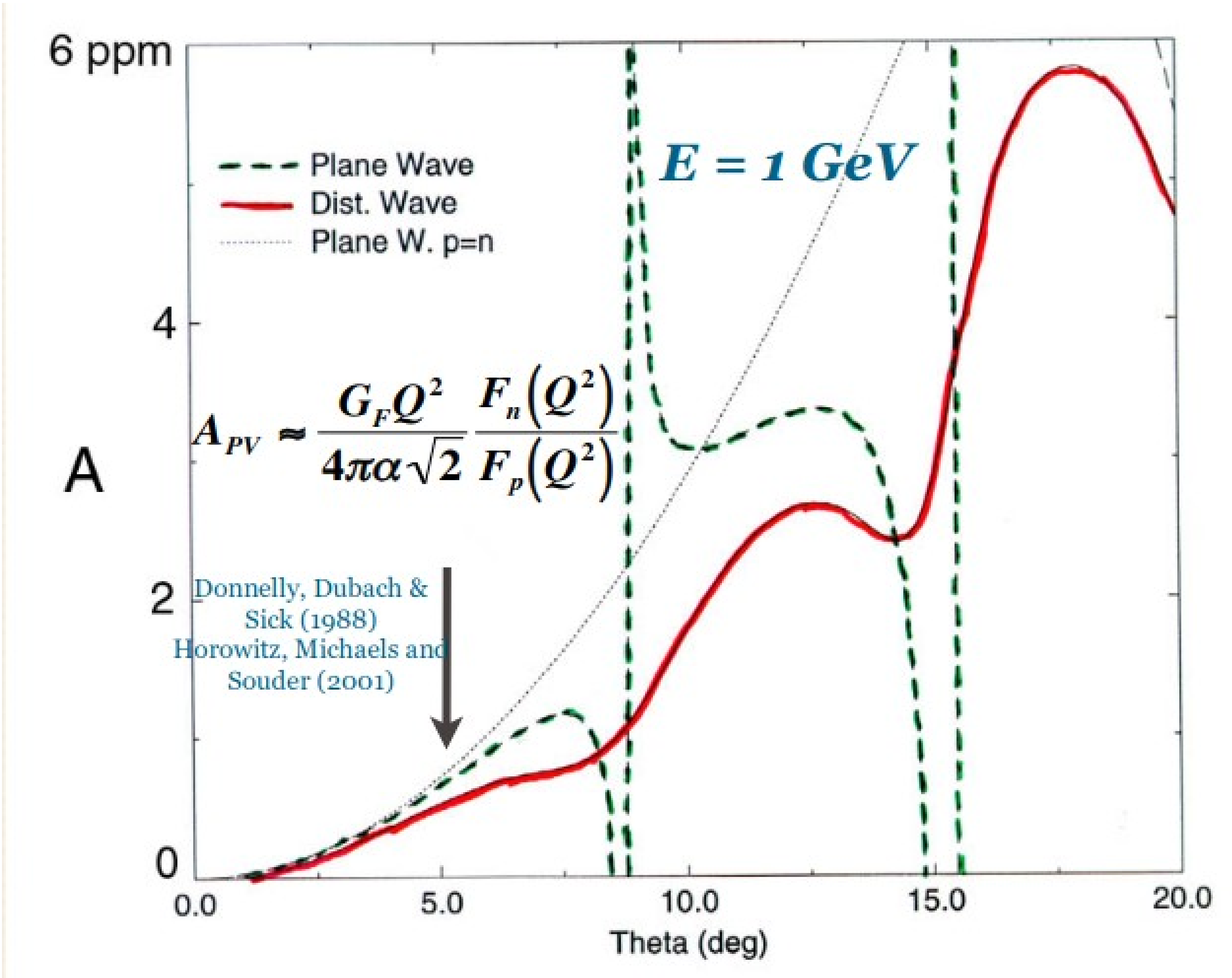
Parity Violating electron elastic scattering: $\rho_W \rightarrow \rho_n$

- **Electrons** interact by exchanging a γ or a Z_0 boson.
- While **protons** couple basically to γ, **neutrons** do it to Z_0.
- **Ultra-relativistic electrons**, depending on their helicity, interact with the nucleons $V_{\pm} = V_{\mathrm{Coulomb}} \pm V_{\mathrm{Weak}}$.
- **Ultra-relativistic electrons** moving under the effect of $V_{\pm}$ where **Coulomb distortions** are important $\Rightarrow$ solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).
- **Input for the calculation: ρ_n and ρ_p**

(as well as nucleon electromagnetic and weak form factors)

$$\mathbf{A}_{\mathbf{pv}} = \left(\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}\right) \Big/ \left(\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}\right)$$

Parity Violating electron elastic scattering: A_{pv} at the PREx kinematics



F_{ch}(q) and **A_{pv}(q)** if measured a different **q** allow to **determine ρ_{ch}(r)** and **ρ_w(r)**

Current situation (^{208}Pb)

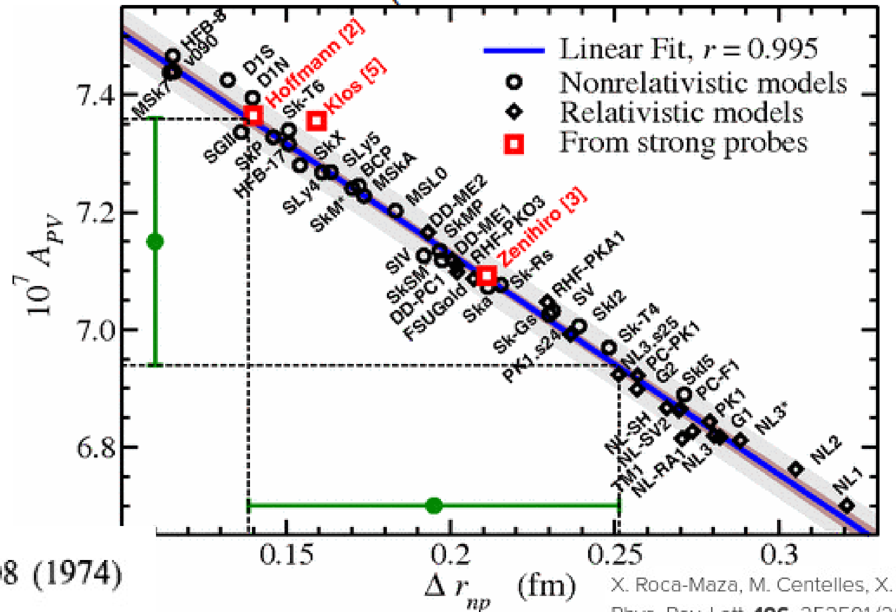
While **ρ_{ch}(r)** has been **determined** in **different nuclei, ρ_W(r)** has **not** been **determined** since **A_{pv}** has only been measured at a **single q** for ^{208}Pb and ^{48}Ca

nucleus	^{206}Pb		^{208}Pb	
rms [fm]	5.490		5.503(2)	
i	R_i	Q_i	R_i	Q_i
1	0.6	0.010615	0.1	0.003845
2	1.1	0.021108	0.7	0.009724
3	2.1	0.000060	1.6	0.033093
4	2.6	0.102206	2.1	0.000120
5	3.1	0.023476	2.7	0.083107
6	3.8	0.065884	3.5	0.080869
7	4.4	0.226032	4.2	0.139957
8	5.0	0.000005	5.1	0.260892
9	5.7	0.459690	6.0	0.336013
10	6.8	0.086351	6.6	0.033637
11	7.2	0.004589	7.6	0.018729
12	8.6	0.000011	8.7	0.000020
ref.	Fr83		Fr77a	
q-range [fm^{-1}]	0.51- 2.99		0.44- 3.70	
data-sets	Eu78,Fr83 μ		He69,Ni69, Eu76a,Fr77a μ	
RP [fm]	1.70		1.70	

ATOMIC DATA AND NUCLEAR DATA TABLES **14**, 479–508 (1974)

In PWBA for small momentum transfer:

$$A_{pv} \approx \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 \langle r_p^2 \rangle^{1/2}}{3F_p(q)}\Delta r_{np}\right)$$

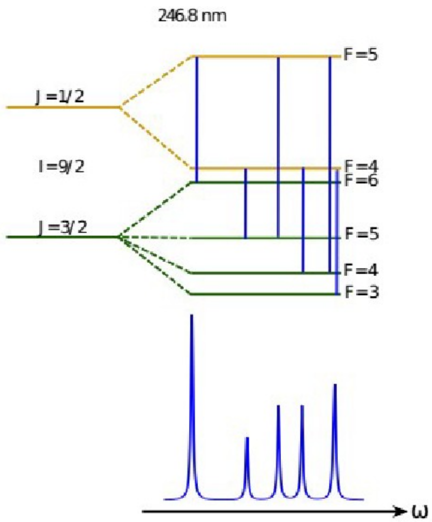


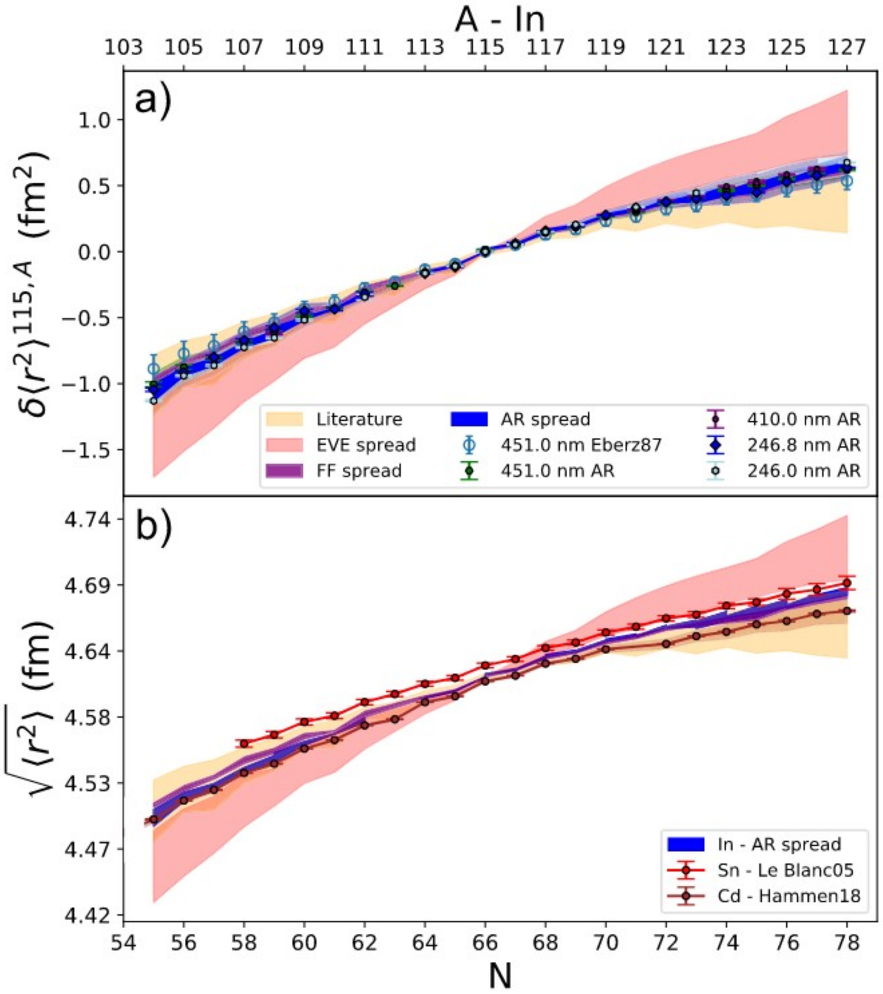
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda
Phys. Rev. Lett. **106**, 252501 (2011) - Published 21 June 2011

Laser spectroscopy: hyperfine structure

→ **Atomic energy levels** are **split** by the interaction of atomic electrons with the **nuclear magnetic dipole moment** and by **nuclear electric quadrupole moment**

→ **Isotope shifts** give changes in mean square charge radii **$\delta\langle r_{ch}^2\rangle$**





B K Sahoo *et al* 2020 *New J. Phys.* **22** 012001

Implications for nuclear structure from precision neutron skin measurements

→ Atomic pairty non conservation (Q_W and R_W)

→ Neutrino coherent [F(q→0)→1] elastic scattering (Q_W and R_W)

→ Dipole polarizabilty (J, Δr_{np})

→ Isobaric Analog State (V_{ISB}, Δr_{np})

→ Spin Dipole Resonance (Δr_{np}, ...)

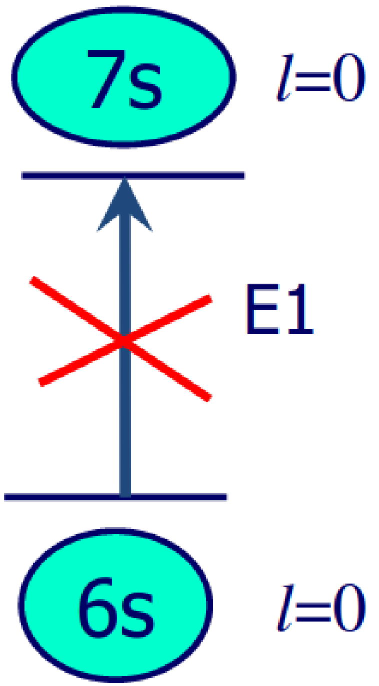
→ Charge radii in mirror nuclei (Δr_{np}) [?]

→ Giant Dipole and Quadrupole resonances (Δr_{np}, ...)

→ Among other observables !!

Atomic parity non conservation

Cs atom



Transition allowed only by **parity violation,** otherwise forbidden

Neutron density!!

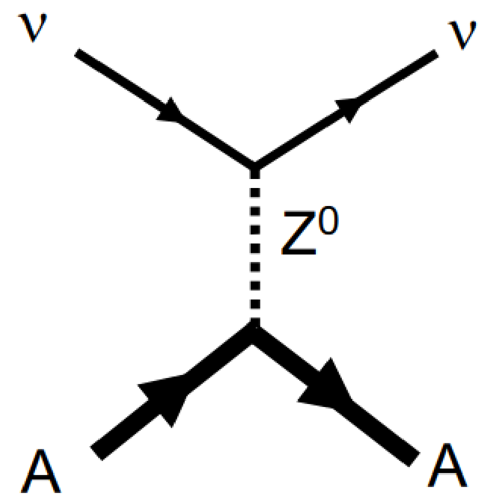
$$H_{PNC}^{(1)} = \frac{G_F}{2\sqrt{2}} Q_W \gamma_5 \rho(r)$$

$$E_{PNC} = E_{PNC}^{theory} Q_W^{\text{inferred}}$$

RMP 90, 025008 (2018)

Coherent neutrino nucleus elastic scattering

Neutrinos with E < 50 MeV exchange a Z_0 boson and the nucleus recoils as a "whole"



$$\frac{d\sigma}{dT} \simeq \frac{G_F^2 M}{2\pi} \frac{\boxed{Q_W^2}}{4} \boxed{F^2(Q)} \left(2 - \frac{MT}{E_\nu^2}\right)$$

E_ν: neutrino energy
T: nuclear recoil energy
M: nuclear mass
Q = √ (2 M T): momentum transfer

Form factor: F=1 ➔ full coherence

weak nuclear charge

$$Q_W = (1 - 4\sin^2\theta_W)Z - N$$

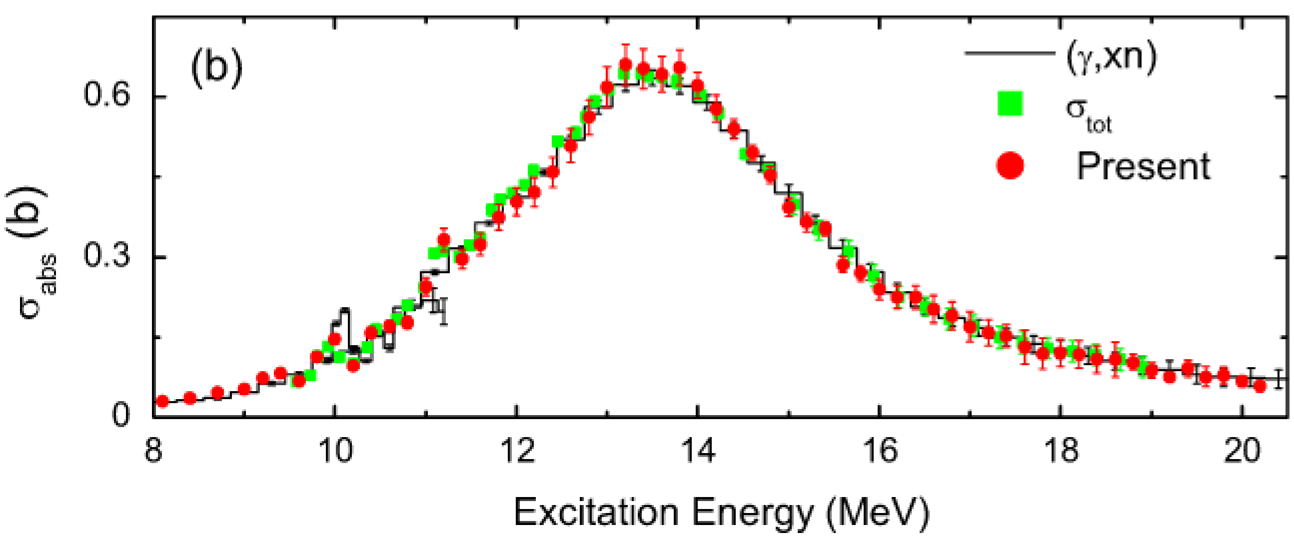
Taken form a presentation by **Kate Scholberg,** Duke University

Dipole polarizability (α_D)

The electric **polarizability** measures the tendency of the nuclear **charge distribution to be distorted**

$$\alpha_D \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{\text{ph. abs.}}(E)}{E^2} dE$$



Phys. Rev. Lett. **107**, 062502 – Published 3 August 2011

Dipole polarizability (α_D): SIMPLE MODEL

The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda \mathcal{D}$.

Adopting the Droplet Model ($m_{-1} \propto \alpha_D$):

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

Bulk - First derived by Migdal

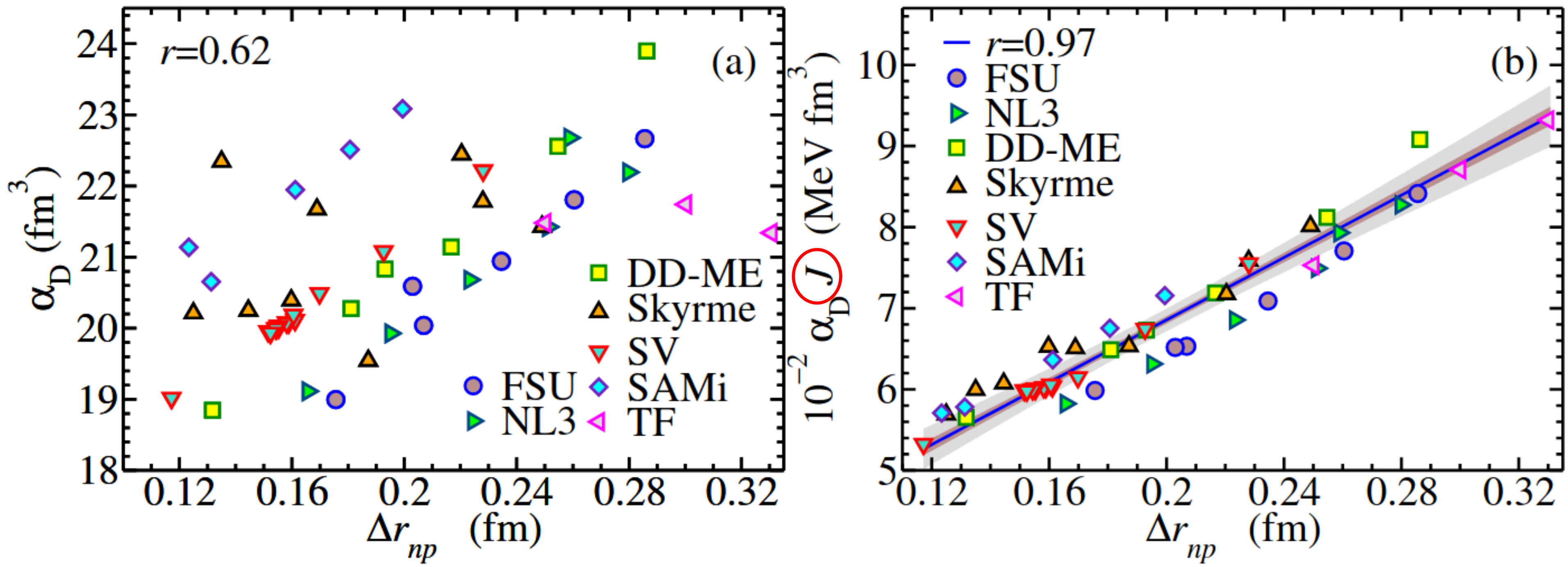
Surface correction - first derived by J. Meyer, P. Quentin, and B. Jennings, Nucl. Phys. A 385, 269 (1982)

within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Dipole polarizability (α_D): EDFs

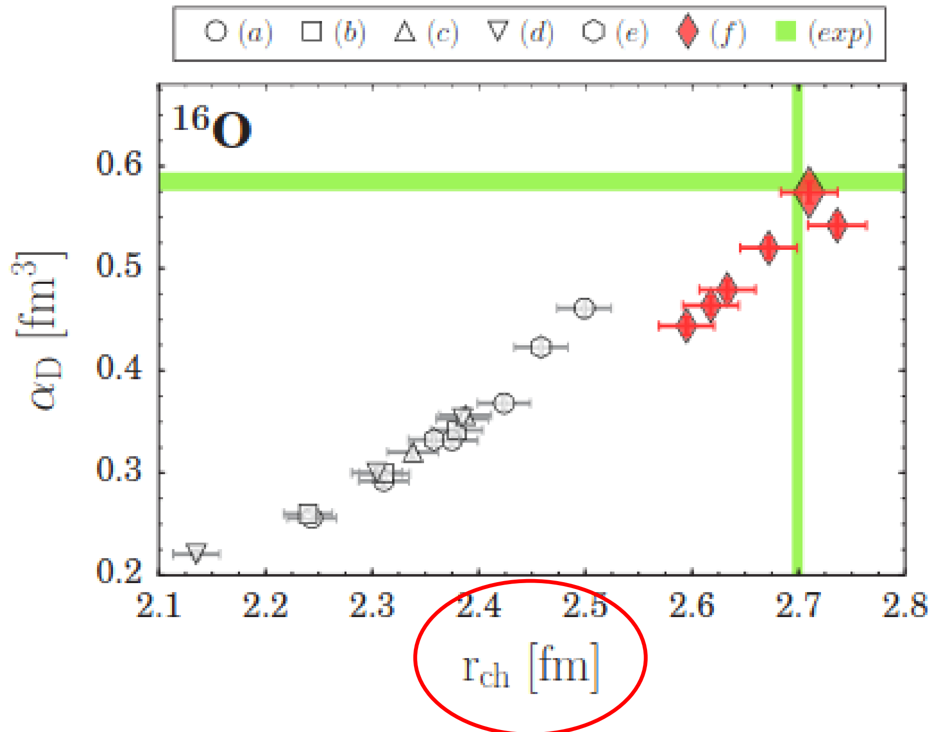
Dipole polarizability: microscopic results HF+RPA



X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

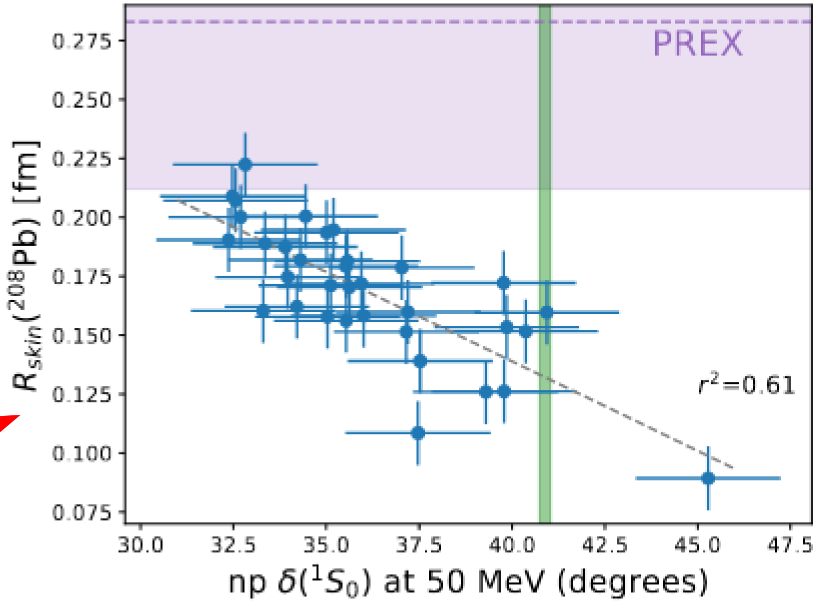
Dipole polarizability (α_D): ab initio



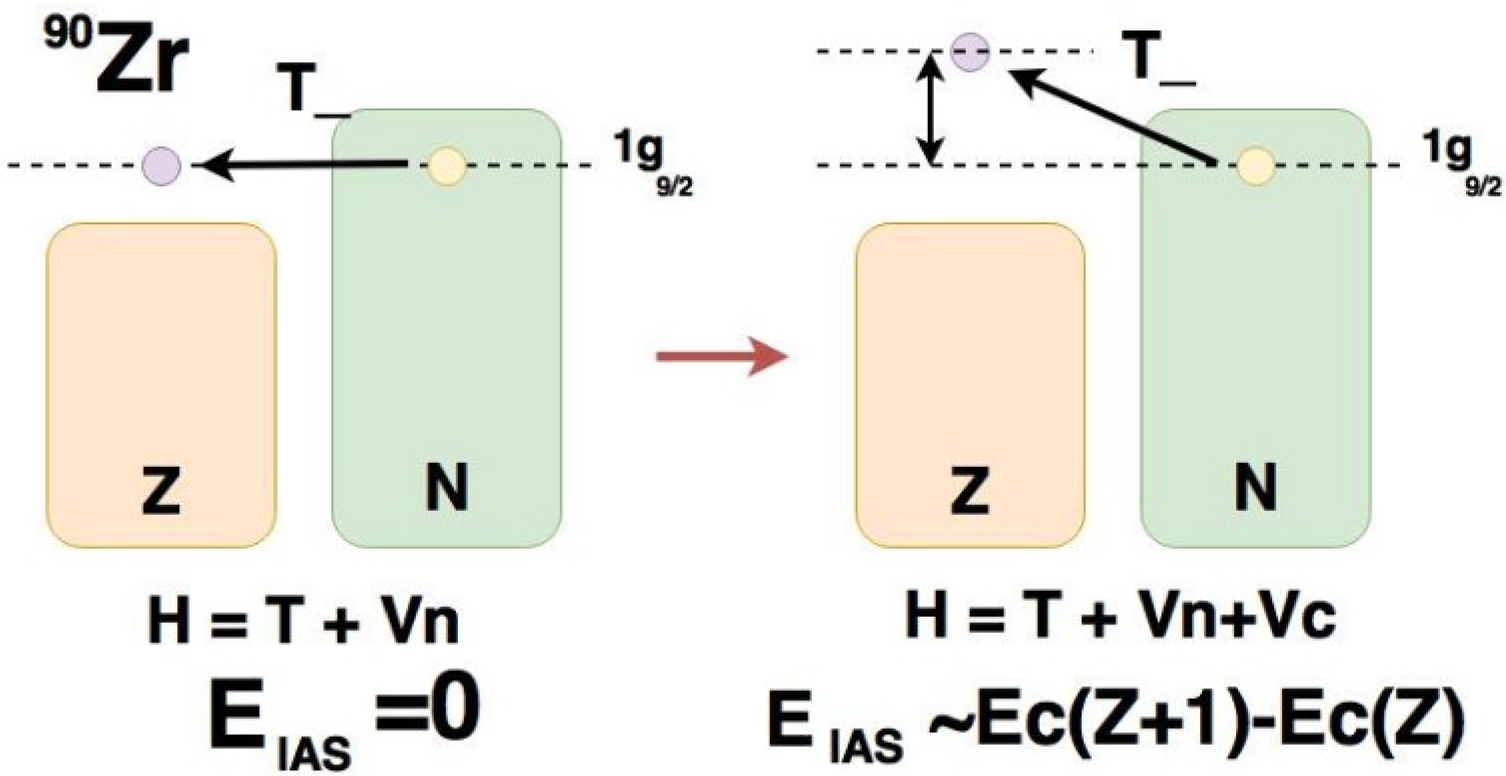
PHYSICAL REVIEW C **94**, 034317 (2016)

(a) SRG evolved EM Λ = 500
(b) SRG evolved EM Λ = 600
(c) SRG evolved CD-BONN
(d) Vlow−k evolved CD-BONN potentials
(e) Vlow−k -evolved AV18
(f) refer to calculations that include 3NF: The large one is from NNLOsat

Dependence on Rch since Δr_{np} is better determined in ab initio due to the fitting of the $^1S_0$ partial wave in np scattering



Isobaric Analog State



$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

Isobaric Analog State: SIMPLE MODEL

- Assuming indepentent particle model and good isospin for $|0\rangle$ $(\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z)$

$$E_{IAS} \approx E_{IAS}^{C,\mathrm{direct}} = \frac{1}{N-Z}\int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{\mathrm{direct}}(\vec{r})d\vec{r}$$

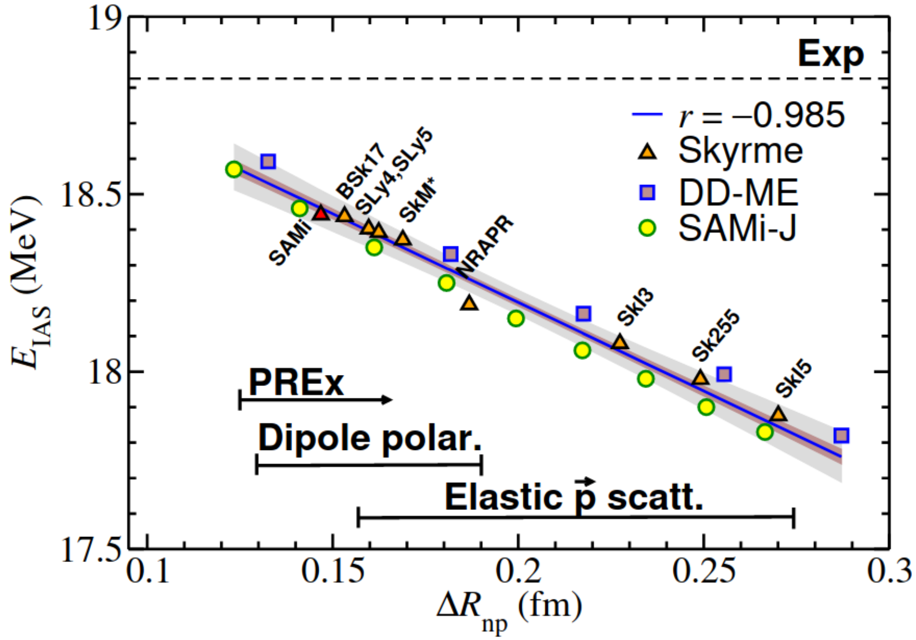
where $U_C^{\mathrm{direct}}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1)d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

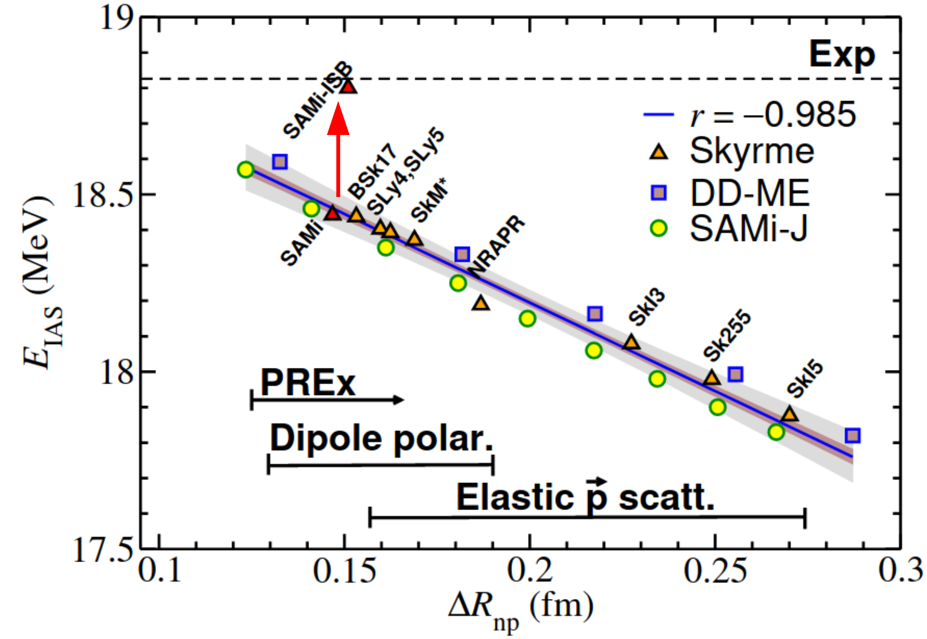
$$E_{IAS} \approx E_{IAS}^{C,\mathrm{direct}} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

Isobaric Analog State: EDFs



Phys. Rev. Lett. 120, 202501 (2018)



Phys. Rev. Lett. 120, 202501 (2018)

Nuclear models (EDFs) where the nuclear part is isospin symmetric and U_{ch} is calculated from the ρ_p

Measurement of $\Delta r_{np} \rightarrow$ determine ISB in the nuclear medium

Treatment of Coulomb + QED corrections + **nuclear Isospin symmetry Breaking**

Spin-Dipole Resonance: sum rule

Excitation operator:

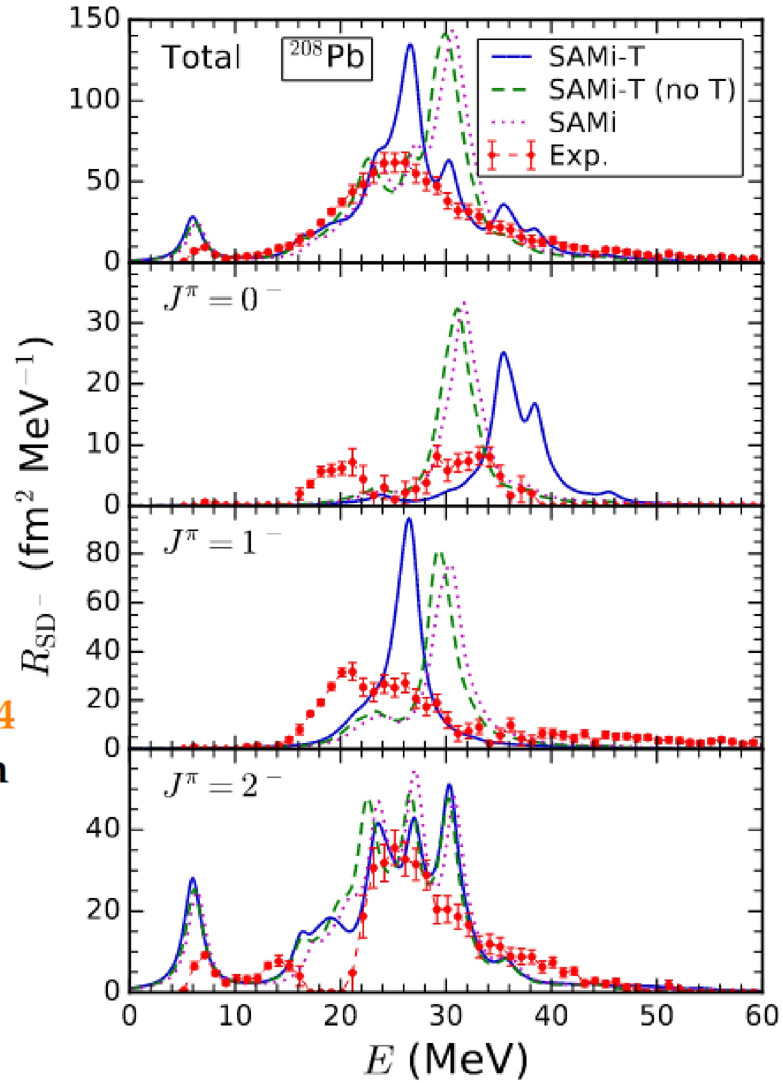
$$\hat{O}_{\rm SDR} = \sum_{i=1}^{A}\sum_{M} \tau_{\pm}(i) r_i^L [Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i)]_{JM}.$$

Non energy weighted sum rule:

$$\int [R_{SD^-}(E) - R_{SD^+}(E)]dE = \frac{9}{4\pi}(N\langle r_n^2\rangle - Z\langle r_p^2\rangle)$$

$$\approx (N-Z)\langle r_p^2\rangle \left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

- Experimental NEWSR in ^{208}**Pb** is 1004^{+24}_{-23} **fm^2**; SAMi is **1224 fm^2**; and **SAMi-T 1260± 10 fm^2** **(some strength is missing in the experimental measurement ?** $\Delta r_{np} \approx 0.05$ **fm)**.
- Experimental NEWSR in ^{90}**Zr** is 148 ± 12 **fm^2**; SAMi is **150 fm^2**; and SAMi-T 147 ± 1 **fm^2** ⇒ **neutron skin should be properly determined by SAMi and SAMi-T**

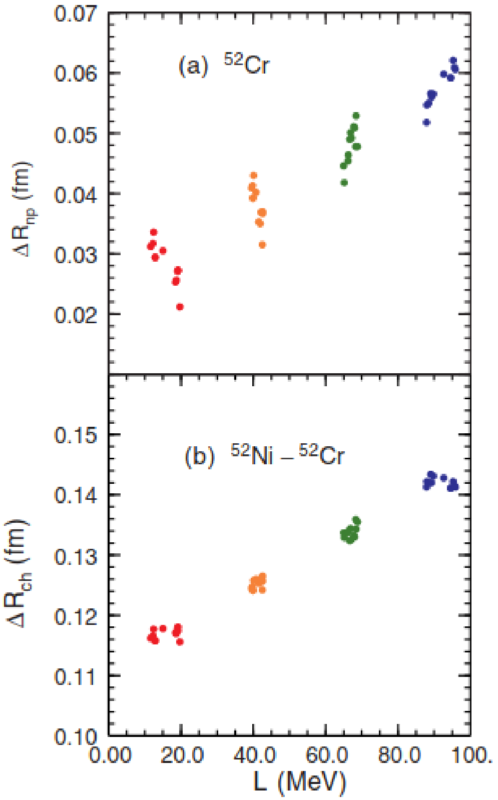


Shihang Shen (申时行), Gianluca Colò, and Xavier Roca-Maza
Phys. Rev. C **99**, 034322 – Published 20 March 2019

Charge radii in mirror nuclei

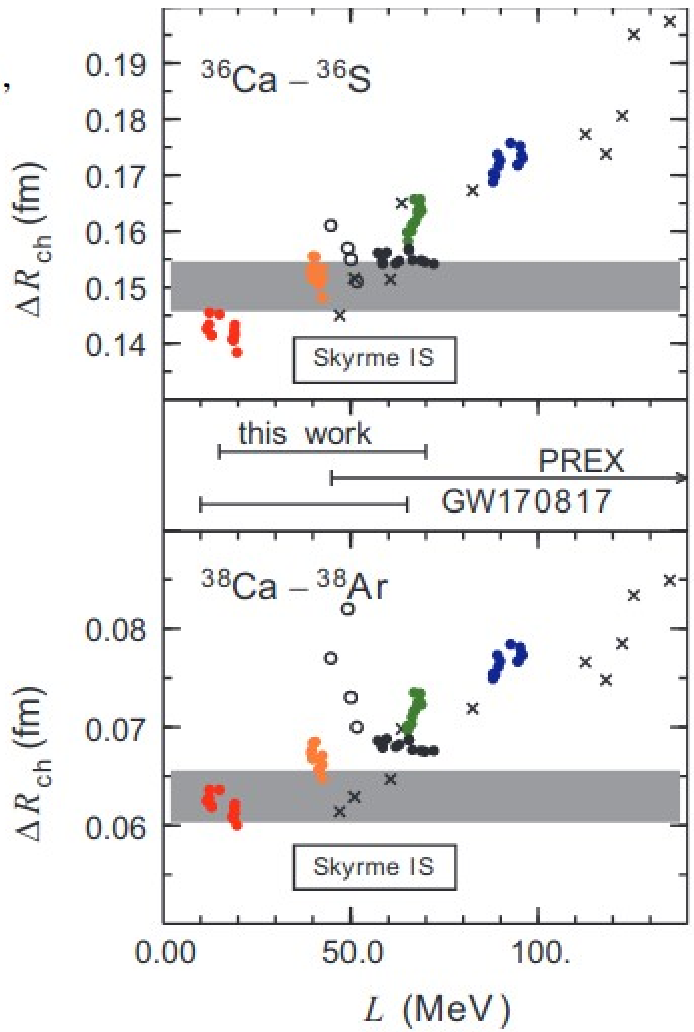
Isospin symmetry:

$$\Delta R_{\mathrm{np}} = R_{\mathrm{ch}}\left({}^{A}_{Z}X_{N}\right) - R_{\mathrm{ch}}\left({}^{A}_{N}Y_{Z}\right) = \Delta R_{\mathrm{ch}},$$

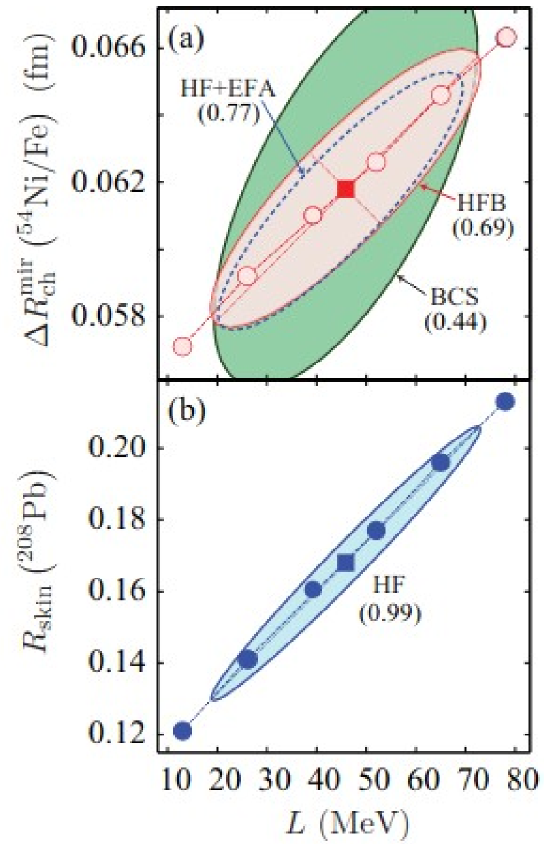


B. Alex Brown
Phys. Rev. Lett. **119**, 122502 – Published 22

PHYSICAL REVIEW RESEARCH **2**, 022035(R) (2020)



PHYSICAL REVIEW C **105**, L021301 (2022)



Large theoretical uncertainties due to pairing correlations in the presence of low-lying proton continuum

Isovector giant resonances

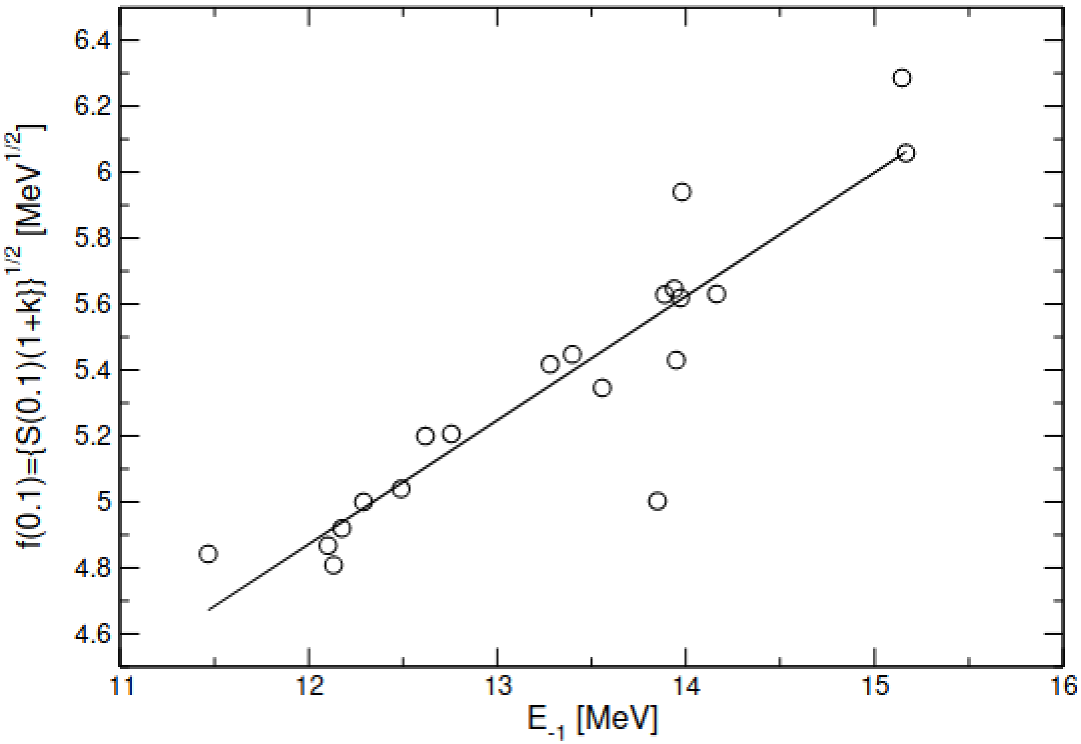
- → In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**
- → **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"
- → The **excitation energy** **(E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{\mathbf{S}(\rho)}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Giant Dipole Resonance

$(E_x \approx f(0.1) \propto \sqrt{S(0.1\mathrm{fm}^{-3})})$



Physical Review C **77**, 061304 (2008)

The larger the symmetry energy at an average density of a finite heavy nucleus, the larger the excitation energy of the Giant Dipole Resonance (GDR).

Giant Quadrupole Resonance

X. Roca-Maza, M. Brenna, B. K. Agrawal, P. F. Bortignon, G. Colò, Li-Gang Cao, N. Paar, and D. Vretenar
Phys. Rev. C **87**, 034301 – Published 1 March 2013

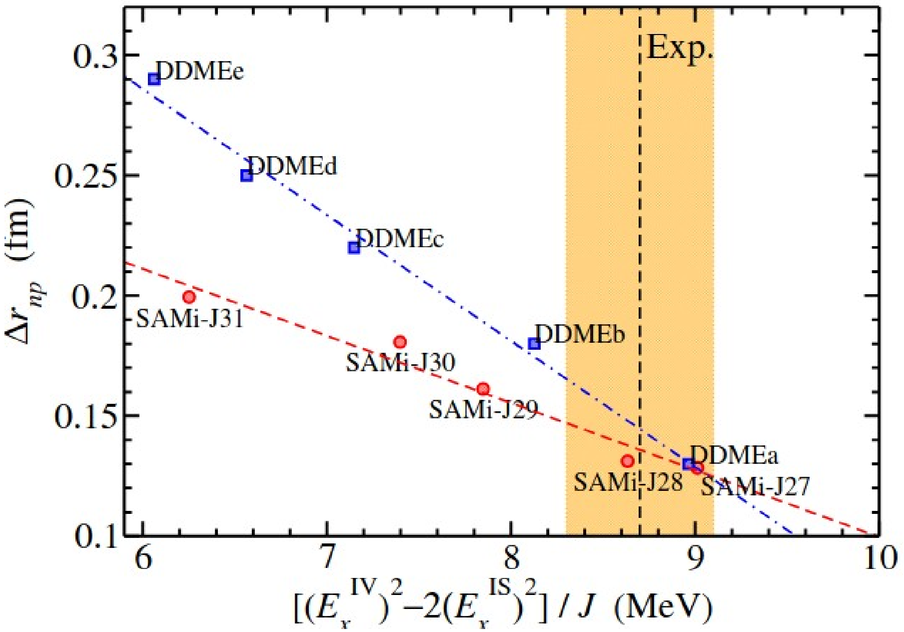
Within the Quantum Harmonic Oscillator approach

$$E_x^{IV} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{sym}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$$

and EDF calculations, one can deduce

$$V_{sym} \approx 8(S(\rho_A) - S^{kin}(\rho_0))$$

$$S^{kin}(\rho_0) \approx \varepsilon_{F_0}/3 \text{ (Non-Rel)}$$



$$S(\rho_A) \approx J - L\frac{\rho_0 - \rho_A}{3\rho_0} \approx \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

The larger the neutron skin in ^{208}Pb, the smallest the difference between the IS and IV excitation energies in GQRs.

Implications for nuclear astrophysics from precision neutron skin measurements

→ **Composition of the crust of a neutron star**

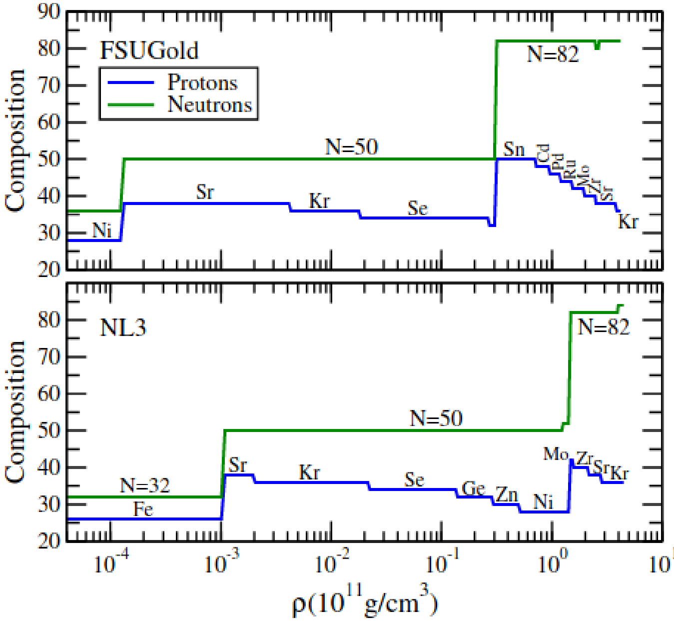
→ **Mass-Radius relation of a neutron star**

→ **Deformability of a neutron star**

→ **Among other neutron star properties and astrophysical processes …**

Outer crust of a neutron star (sub-saturation densities relevant)

- span 7 orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g/cm to the **neutron drip** $\sim 10^{11}$ g/cm)
- it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- $T \sim 10^6$ K ~ 0.1 keV $\rightarrow$ one can treat **nuclei and electrons at $T = 0$ K**
- At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent $N-$*plateau* $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{F_e}}{8 a_{sym}}$ where $(A_0, Z_0) = {}^{56}\mathrm{Fe}_{26}$
- The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$).** $[M(N,Z) + m_n < M(N+1, Z)]$

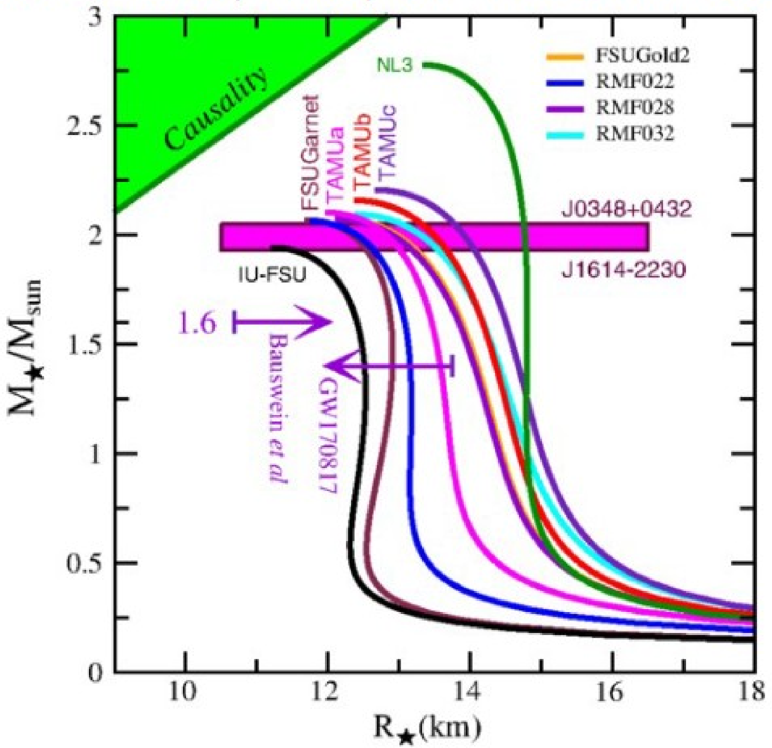


Physical Review C **78**, *025807 (2008)*

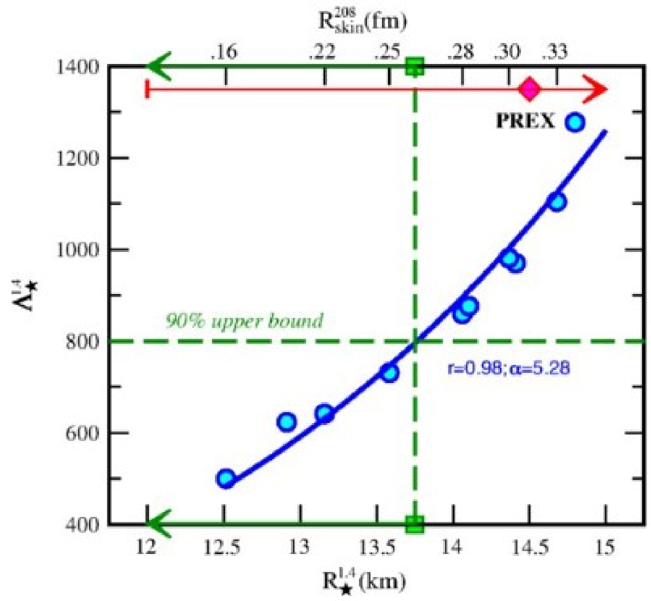
The larger the neutron skin of ^{208}Pb ($L \uparrow$), the more exotic the composition of the outer crust.

Mass-Radius relation and deformability of a Neutron Star

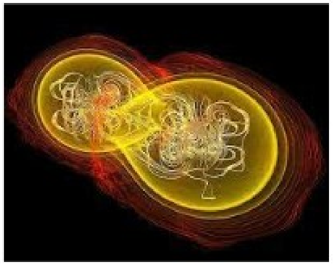
GW170817 from the binary neutron star merger → **constraint** neutron star **radius** and, thus, the **nuclear EoS**



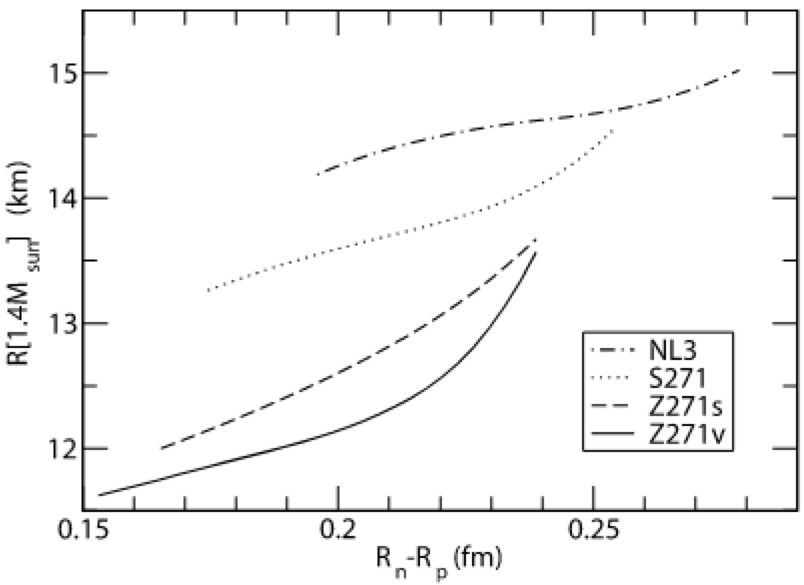
Neutron Skins and Neutron Stars in the Multimessenger Era
F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

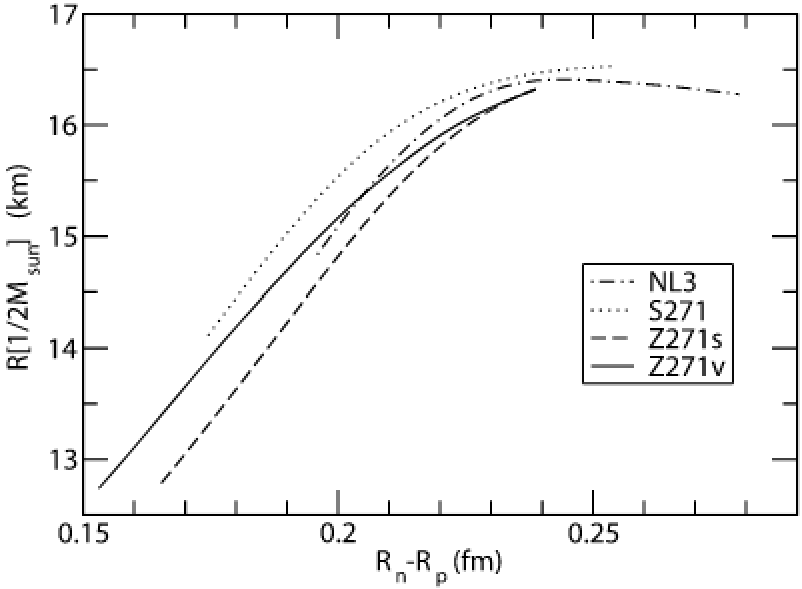


Tidal deformability (Λ) is a quadrupole deformation inferred from **GW signal →** proportional to **restoring force.** Hence, sensitive to the **nuclear EoS**



Radius of a Neutron Star





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Crust-core interface

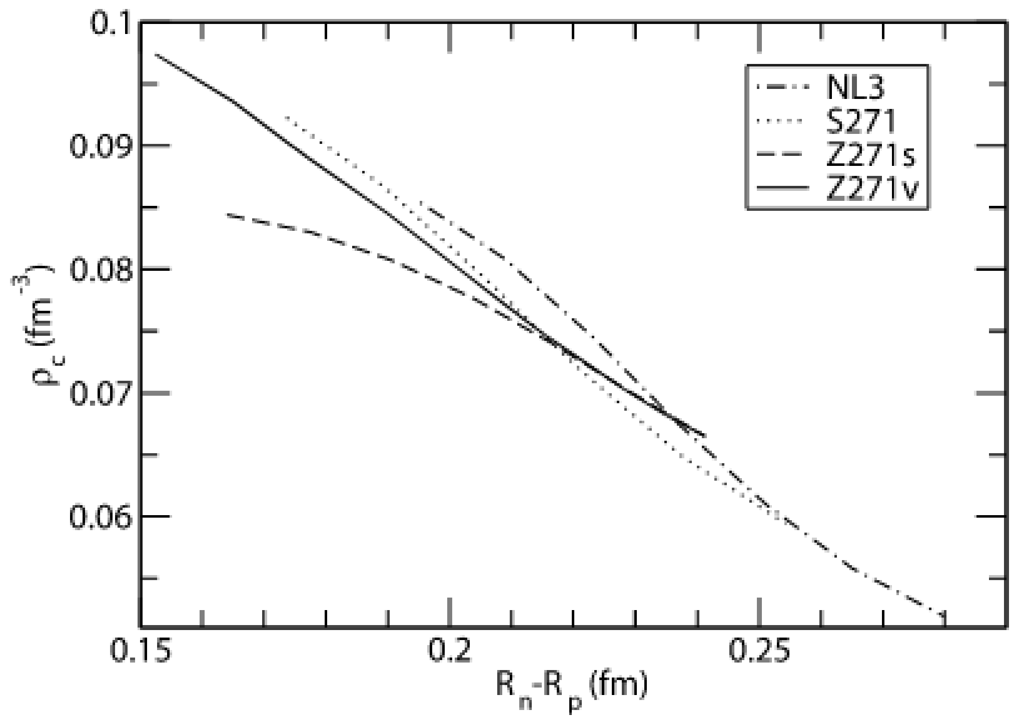

FIG. 1.—Transition density ρ_c at which uniform matter becomes unstable to density oscillations as a function of the neutron skin in ^{208}Pb. The solid curve is for the Z271 parameter set with $\Lambda_v \neq 0$ while the dashed curve uses Z271 with $\Lambda_s \neq 0$. The dotted curve is for the S271 set and the dot-dashed curve for NL3, both of these with $\Lambda_v \neq 0$.

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THANK YOU!