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Probing Subnuclear Physics in Heavy ion Collisions

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Agenda:

- **Peripheral quasi-elastic and inelastic heavy ion reactions**
- **The excited nucleon in nuclear matter**
- **Reaction theory of peripheral heavy ion charge exchange reactions**
- **Nuclear response functions with resonances: N*RPA**
- **Applications to the recent FRS-data**
- **Perspectives on decay spectroscopy at WASA@(Super)FRS**
- **Astrophysics: resonances in neutron stars**

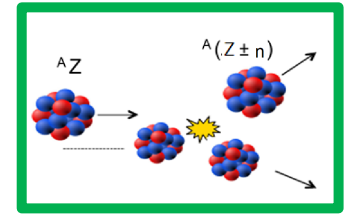
Experiment and Theory:

Rodriguez et al.,

Phys. Lett. B 807, 135565 (2020) & Physical Review C 106:014618 (2022)

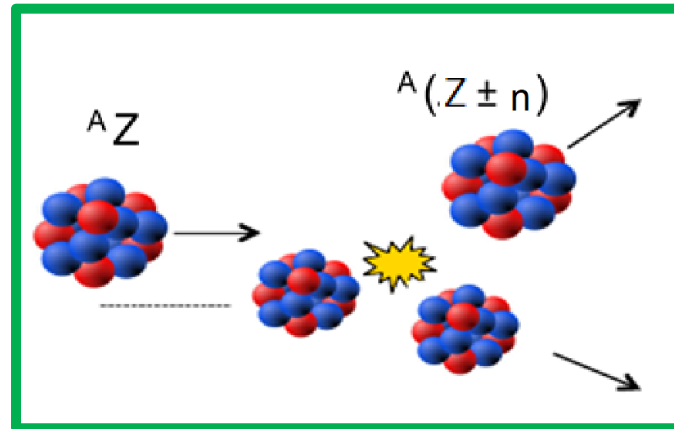
Background Reading:

H. Lenske, M. Dhar, Th. Gaitanos, Xu Cao, Prog.Part.Nucl.Phys. 98: 119 (2018)

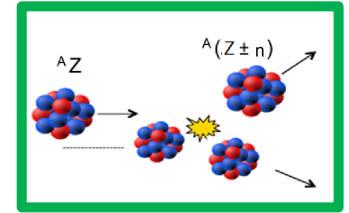


Peripheral Charge Exchange Heavy Ion Reactions

Characteristics of Peripheral Heavy Ion Charge Exchange Reactions



- **Grazing collisions and selective excitations** leaving the reacting nuclei intact
 - **Quasi-elastic and (deep-)inelastic reactions** under controlled conditions
 - **Nuclear spectroscopy** at large momentum and energy transfers
 - **Probing N^*N^{-1} modes** in $n=1$ single charge exchange (SCE) reactions
 - **Decay spectroscopy** with WASA@(Super)FRS as an exciting new perspective
 - **Probing double-beta decay** in $n=2$ double charge exchange (DCE) reactions
- (H. Lenske, F. Cappuzzello, M. Cavallaro, M. Colonna, Prog.Part.Nucl.Phys. 109 (2019) 103716)



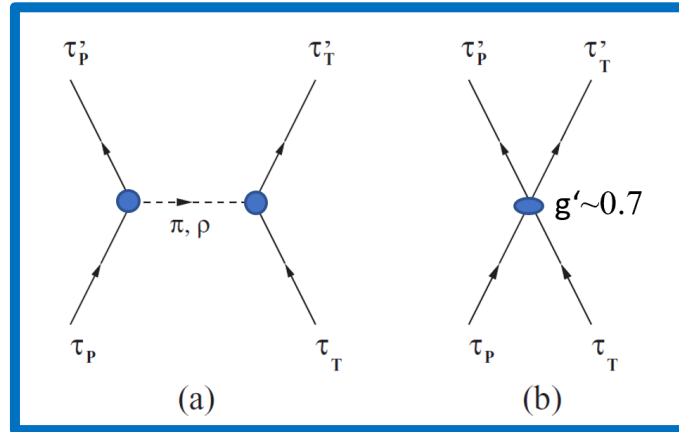
Theory of Peripheral Heavy Ion Charge Exchange Reactions: Cross Sections

Separability Hypothesis:
Quasi-Elastic and Inelastic Processes do not Interfere

$$\frac{d\sigma}{dE_b} = \frac{d\sigma}{dE_b} \Big|_{\text{qe}} + \frac{d\sigma}{dE_b} \Big|_{\text{in}}.$$

...implying the incoherent superposition of cross sections.

Quasi-elastic Cross Section – Target $N'N^{-1}$ SCE transitions



- $\pi, \rho \rightarrow \Delta S=1$ Gamow-Teller modes, $\pi_j = (-)^{j+1}$
- $\delta \sim [\pi\eta]_{I=1} \rightarrow \Delta S=0$ Fermi modes, $\pi_j = (-)^j$

„Collision Number“
→ Glauber Theory

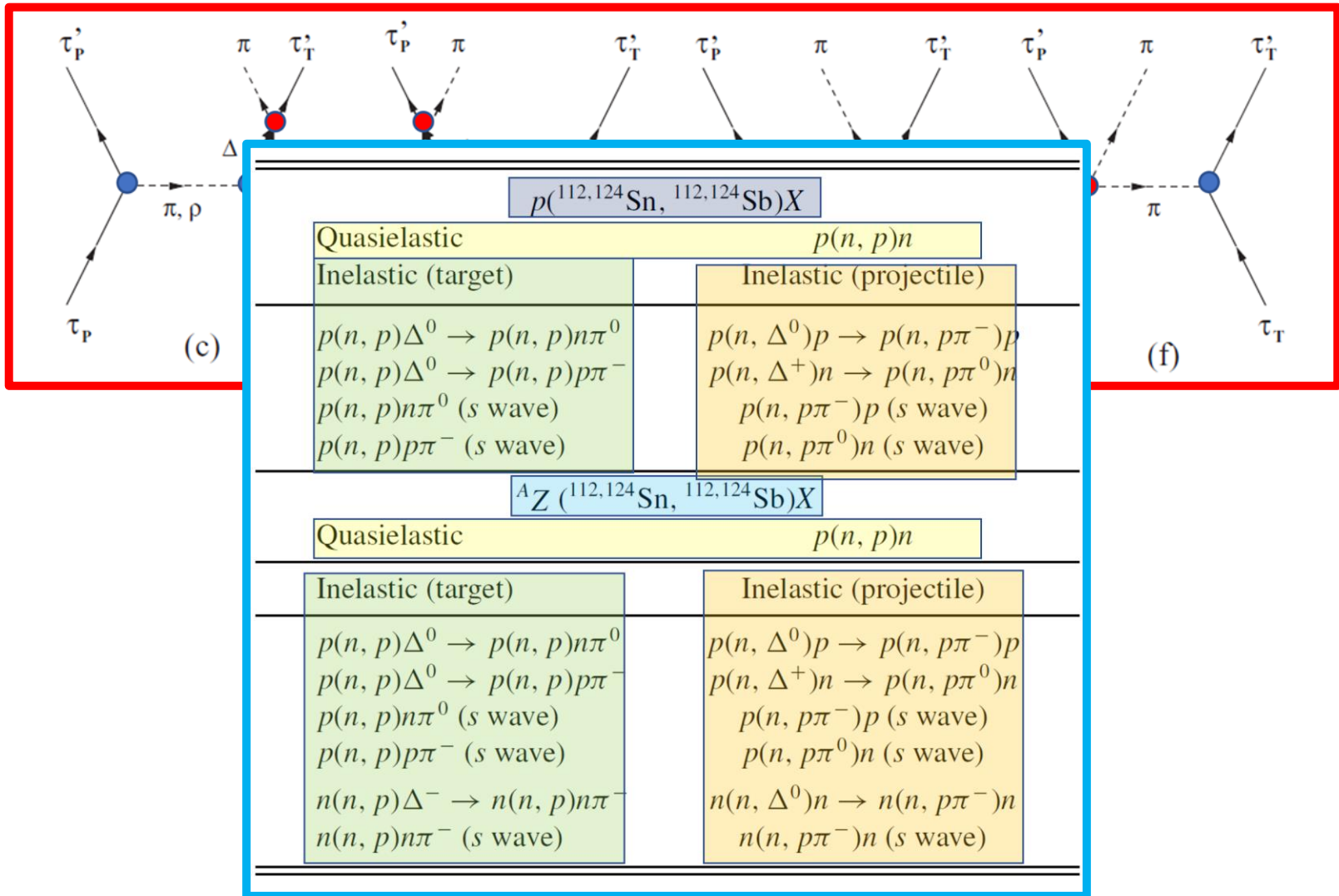
Target Response Function

NN isovector-SCE Amplitude

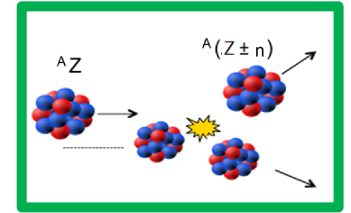
$$\frac{d\sigma}{dE_b} \Big|_{\text{qe}} = \langle N_{\tau_p \tau_T} \rangle |\mathcal{M}_{\text{qe}}|^2$$

$$|\mathcal{M}_{\text{qe}}|^2 = \frac{2|\vec{p}_b|}{(2\pi)^2} \frac{m^4}{\lambda^{1/2}(s, m^2, m^2)} \int \frac{d\vec{q}}{(2\pi)^3} \frac{d\Omega_b}{E_B} R_N^{(A)}(\sqrt{s} - E_b, \vec{q}) |M_{\text{qe}}(\vec{q})|^2$$

Inelastic Cross Section - N^*N^{-1} SCE transitions

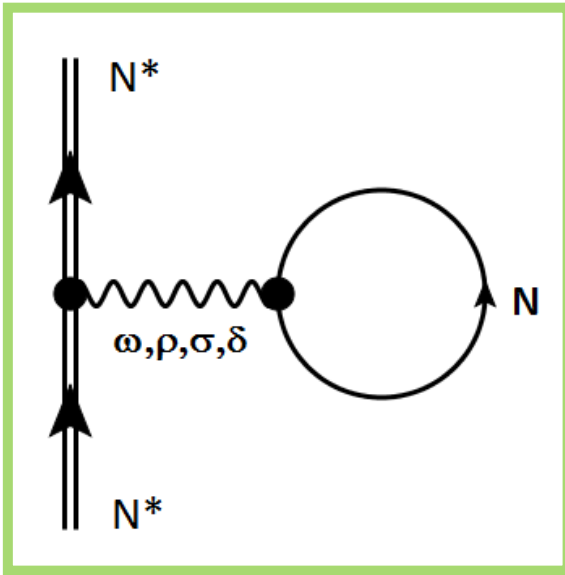


Inelastic cross sections – see PRC 106:014618 (2022)



Theory of Peripheral Heavy Ion Charge Exchange Reactions: Spectral Distributions and Response Functions

N* Self-Energies in Nuclear Matter

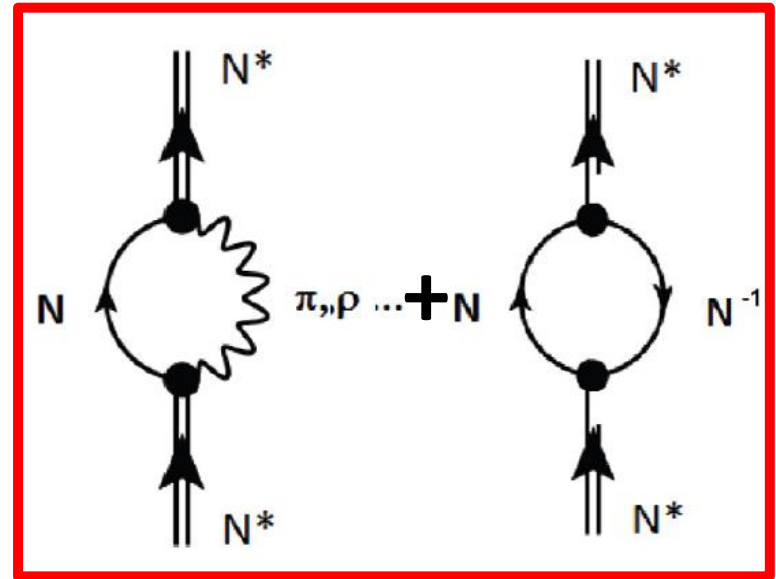


Diffractive Self-Energy
→ Hartree-Potential

$$U_{N^*}^{(H)} = U_0 + U_1 \tau_{N^*} \cdot \tau_N$$

$$U_{N^*}^{(H)} \sim U_0 + U_1 t_z^{(N^*)} \cdot \frac{N-Z}{A}$$

+



Polarization Self-Energy
→ dispersive (optical) potential

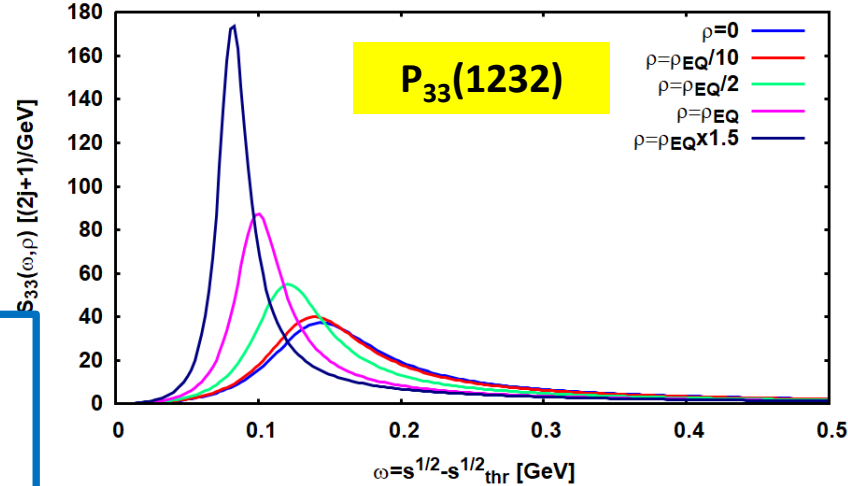
$$\Sigma_{N^*}^{(pol)} \sim \Sigma_0 + \Sigma_1 t_z^{(N^*)} \frac{N-Z}{A}$$

$$\Sigma_{0,1} = V_{0,1} - iW_{0,1} ; W_{0,1} = \frac{1}{2} \Gamma_{0,1}$$

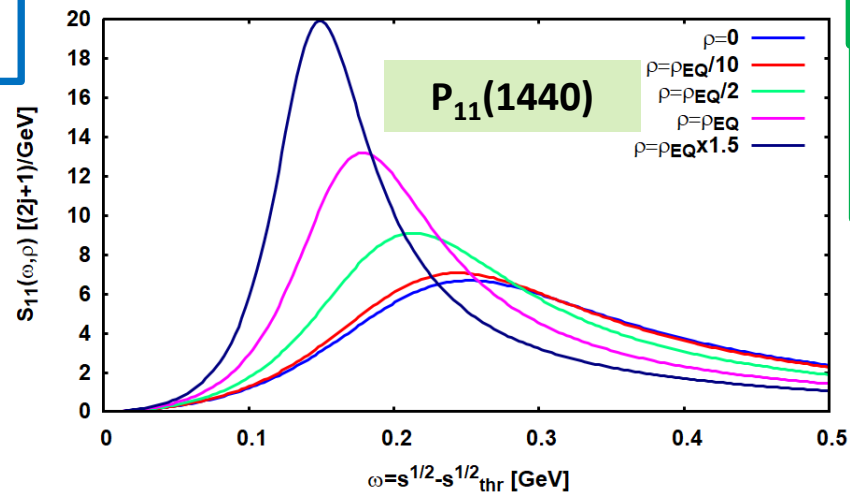
...for the dispersive part see e.g.:

E. Oset, L.L. Salcedo, NPA 468 (1987) 631; G.E. Brown, W. Weise, Phys. Rept. 22 (1975) 279

N* Spectral Distributions in Nuclear Matter



- Mean-field potentials
- Effective Masses
- Pauli-Blocking
- Pion Absorption
- Coupling to NN⁻¹ modes



...with increasing density:

- Downward mass shift
- Reduction of width

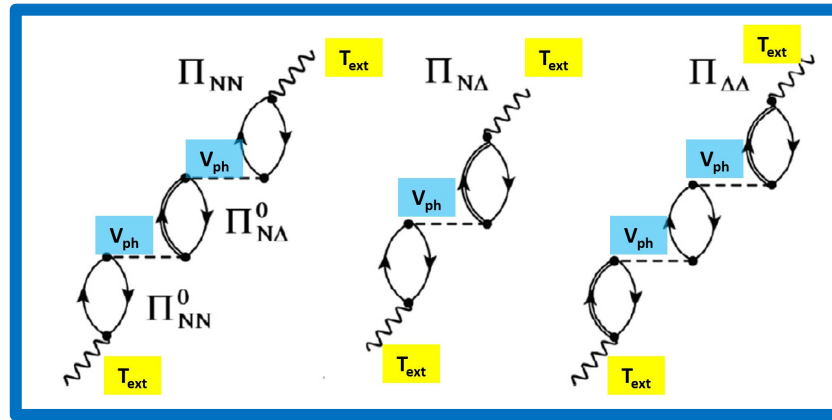
...if a pre-formed N* resonance would be captured by a nucleus!

Substitutional N^*N^{-1} Excitations in Nuclei - „N*RPA“

$$\Pi = \Pi^0 + \Pi^0 \hat{V} \Pi$$

$$\begin{pmatrix} \Pi_{NN} & \Pi_{N\Delta} \\ \Pi_{\Delta N} & \Pi_{\Delta\Delta} \end{pmatrix} = \begin{pmatrix} \Pi_{NN}^0 & 0 \\ 0 & \Pi_{\Delta\Delta}^0 \end{pmatrix} + \begin{pmatrix} \Pi_{NN}^0 & 0 \\ 0 & \Pi_{\Delta\Delta}^0 \end{pmatrix} \begin{pmatrix} V_{NN} & V_{N\Delta} \\ V_{\Delta N} & V_{\Delta\Delta} \end{pmatrix} \begin{pmatrix} \Pi_{NN} & \Pi_{N\Delta} \\ \Pi_{\Delta N} & \Pi_{\Delta\Delta} \end{pmatrix}$$

Coupled $N^*N^{-1} \leftrightarrow \Delta N^{-1}$ Dyson Equation including N and N^* self-energies



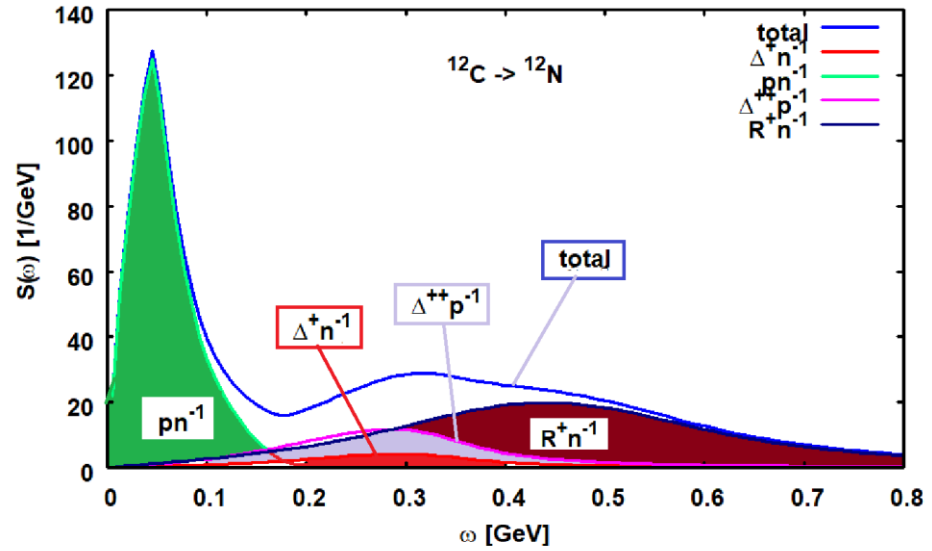
V_{ph} by pion, rho, delta/ a_0 - meson exchange and „short range“ g'

Polarization Tensor and Nuclear Response (H=N, Δ ...)

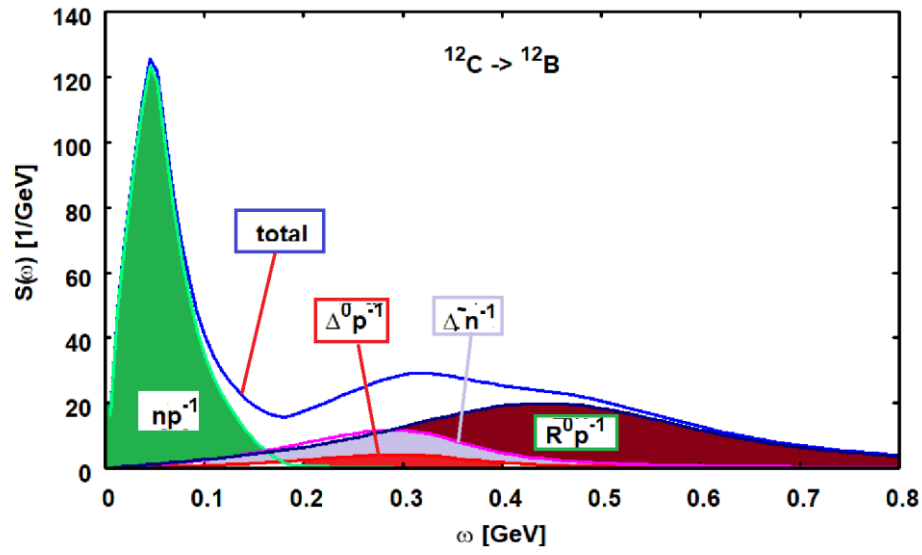
$$R_H^{(X)\mu\nu}(\omega, \vec{q}) = -\frac{1}{\pi} \text{Im} \left(\Pi_{HH}^{(X)\mu\nu}(\omega, \vec{q}) \right) = -\frac{1}{\pi} \text{Im} \left(\langle X | T_{ext}^{\dagger\mu}(\vec{q}) G_{HH}(\omega) T_{ext}^{\nu}(\vec{q}) | X \rangle \right)$$

N*N⁻¹ Spectral Distributions for ¹²C – P₃₃(1232) and P₁₁(1440) Longitudinal Response@q=300 MeV/c

τ_+ operator : pn^{-1} – type
 external probe: $\vec{\sigma} \bullet \vec{p}$

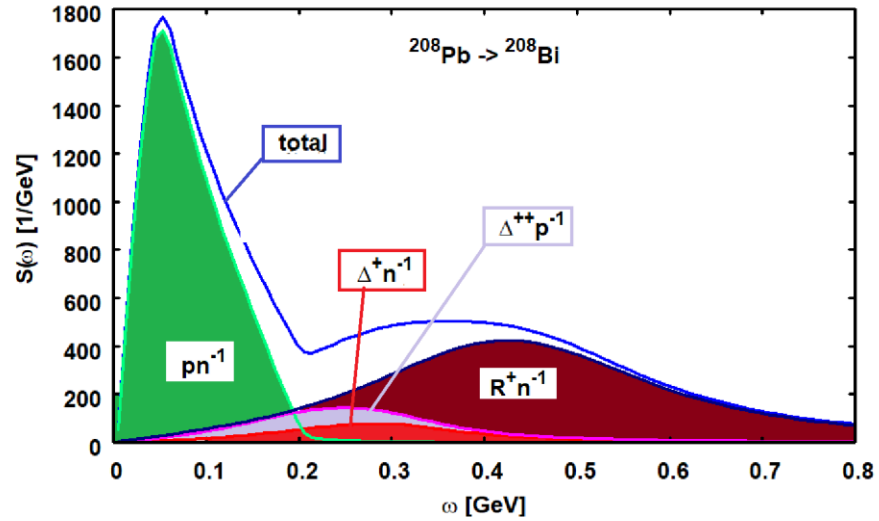


τ_- modes : np^{-1} – type
 external probe: $\vec{\sigma} \bullet \vec{p}$

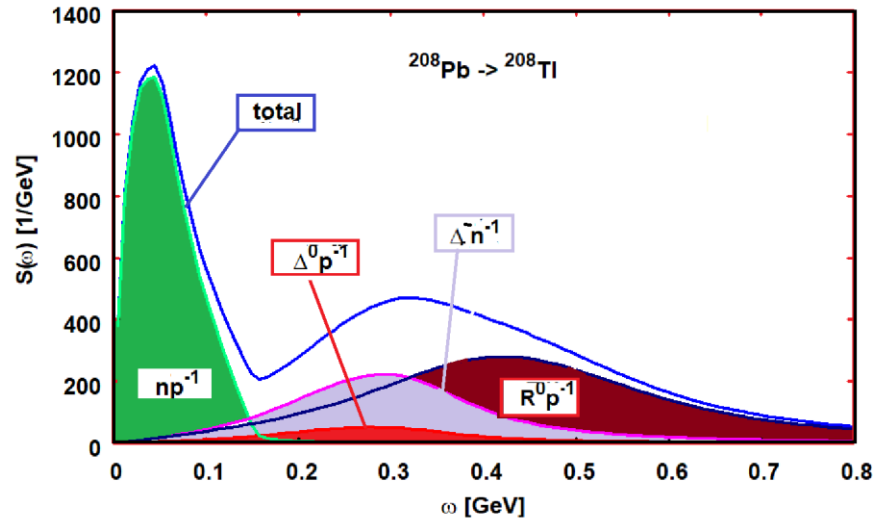


N*N⁻¹ Spectral Distributions for ²⁰⁸Pb – P₃₃(1232) and P₁₁(1440) Longitudinal Response@q=300 MeV/c

τ_+ operator : pn^{-1} – type
external probe: $\vec{\sigma} \bullet \vec{p}$

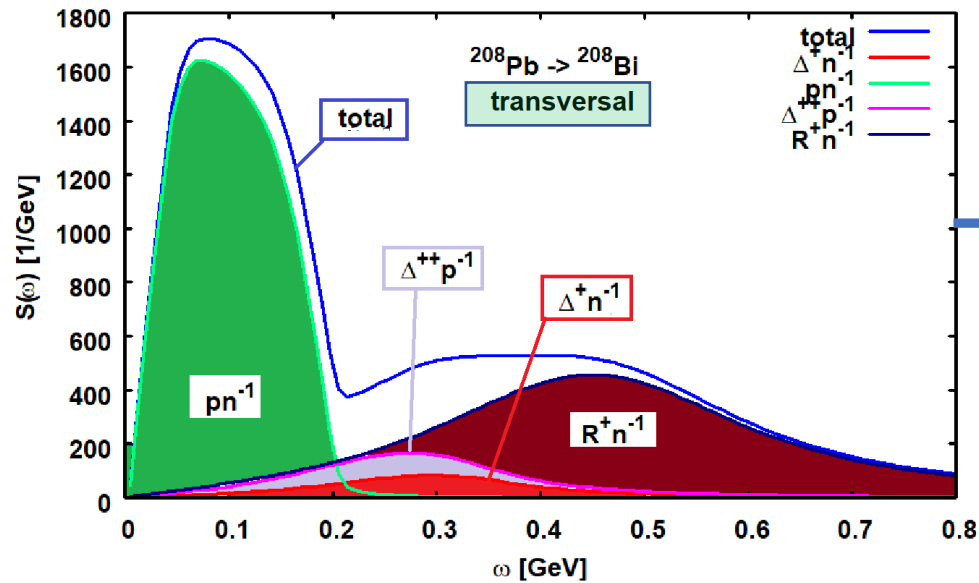
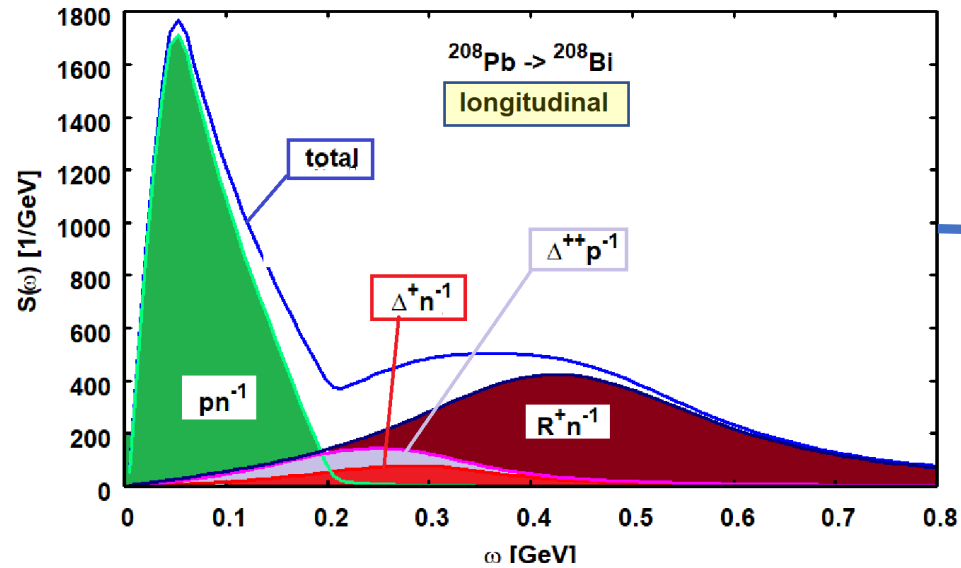


τ_- modes : np^{-1} – type
external probe: $\vec{\sigma} \bullet \vec{p}$

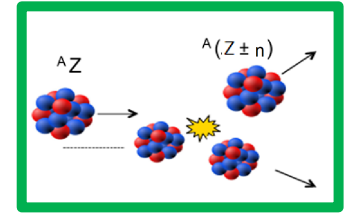


N^*N^{-1} Spectral Distributions for $^{208}\text{Pb} \rightarrow ^{208}\text{Bi}$

Longitudinal and Transversal Response @ $q=300 \text{ MeV}/c$



Comparison to the FRS-Data



Theoretical Results for ^{112}Sn on Pb, Cu, ^{12}C , and Proton Targets

Rodriguez et al., PHYSICAL REVIEW C 106, 014618 (2022)

$^{112}\text{Sn}(0^+) \rightarrow ^{112}\text{Sb}(3^+)$

$S(p) = 2.948 \text{ MeV}$

States $E_x < S(p)$:

$1^+ \leq J^\pi \leq 12^-$

$[p_p n_p^{-1}]$

$Z_p \rightarrow Z_p + 1$

\leftrightarrow

$Z_T \rightarrow Z_T - 1$

$[n_T p_T^{-1}]$

$^{112}\text{Sn}(0^+) \rightarrow ^{112}\text{In}(1^+)$

$S(p) = 6.027 \text{ MeV}$

States $E_x < S(p)$:

$0^+ \leq J^\pi \leq 18^\pm$

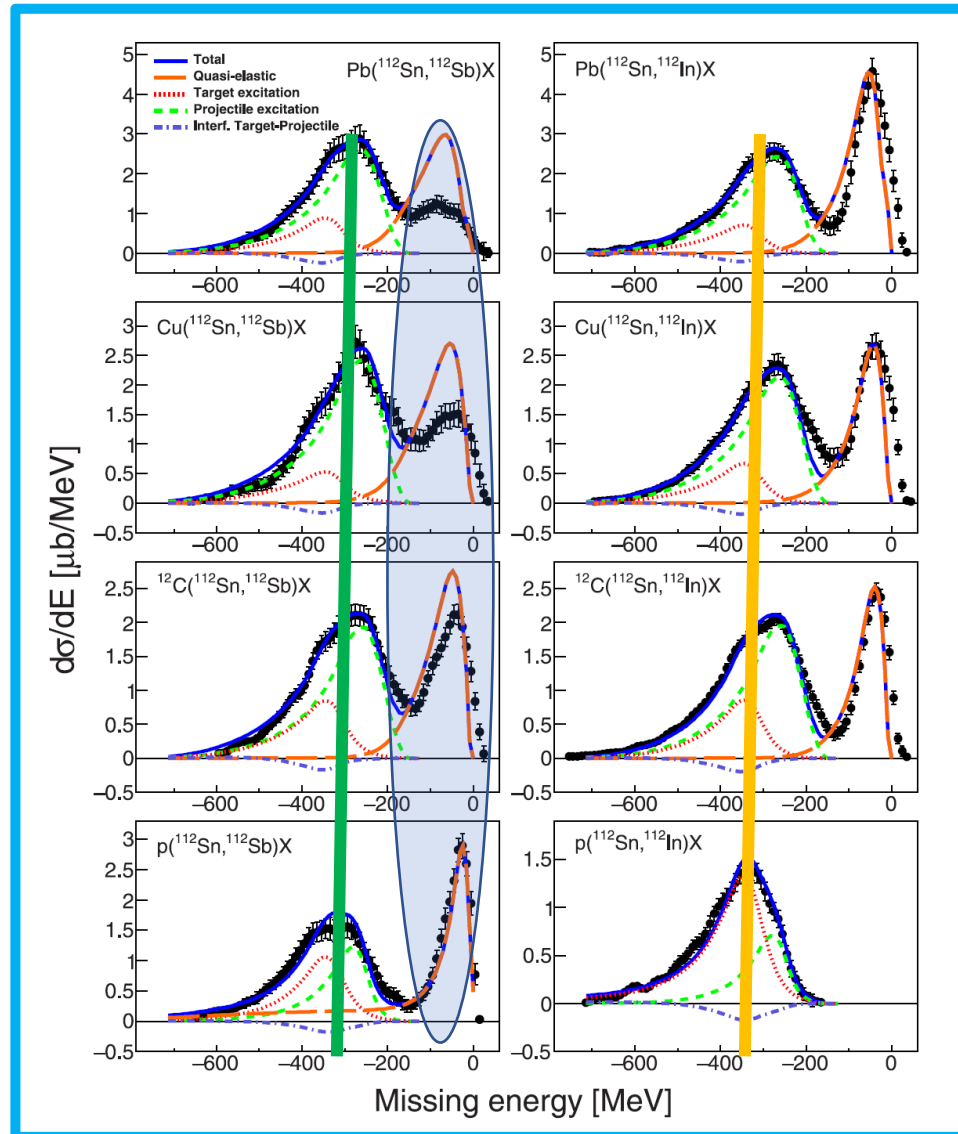
$[n_p p_p^{-1}]$

$Z_p \rightarrow Z_p - 1$

\leftrightarrow

$Z_T \rightarrow Z_T + 1$

$[p_T n_T^{-1}]$

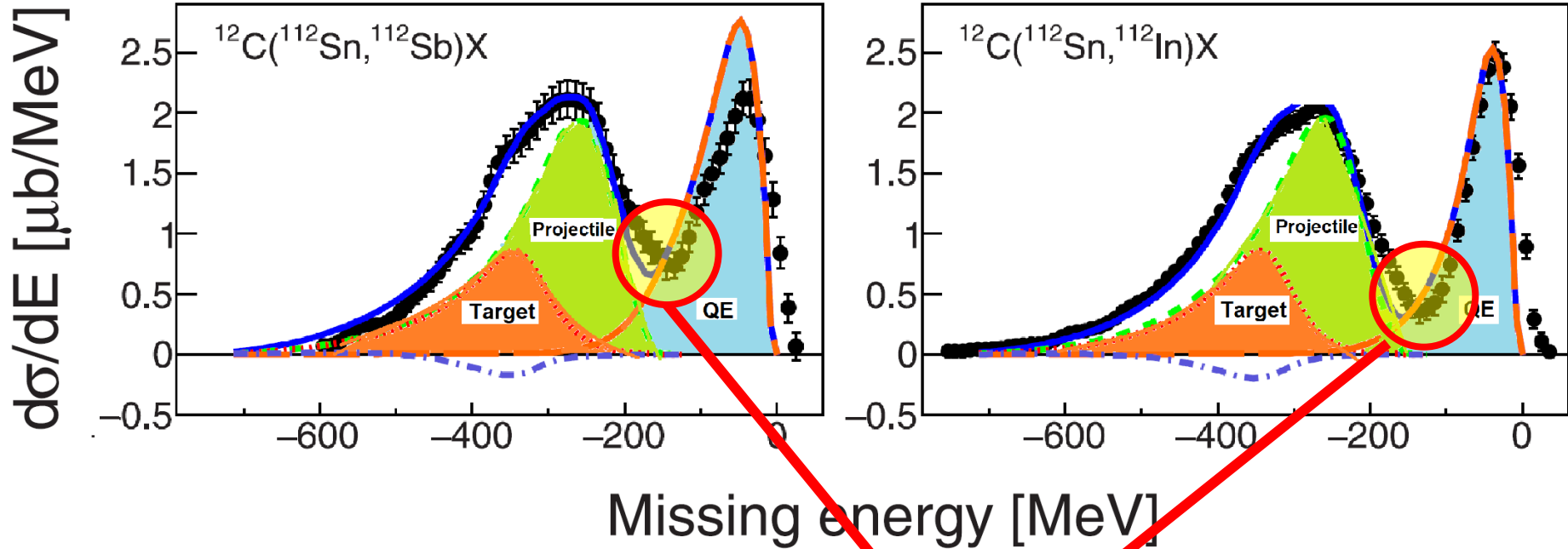


$^{112}\text{Sn}@1\text{GeV}$ on ^{12}C

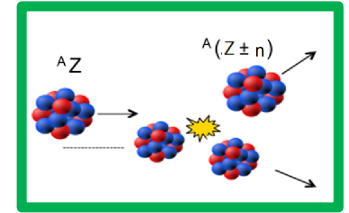
Quasi-elastic, Projectile, and Target Excitations

$X = ^{12}\text{C}(np^{-1}) \sim \text{„}^{12}\text{B}\text{“}$

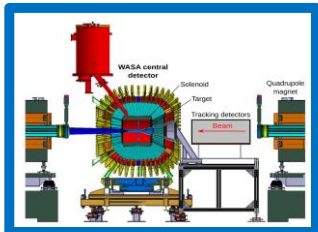
$X = ^{12}\text{C}(pn^{-1}) \sim \text{„}^{12}\text{N}\text{“}$



Interference Effect?

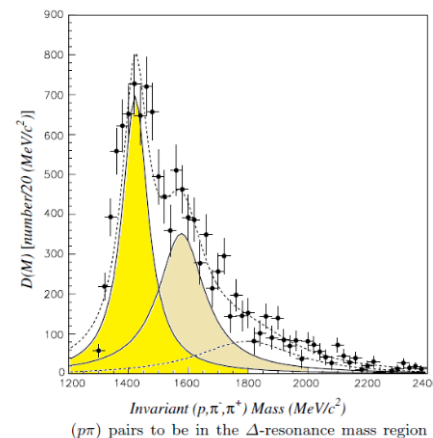
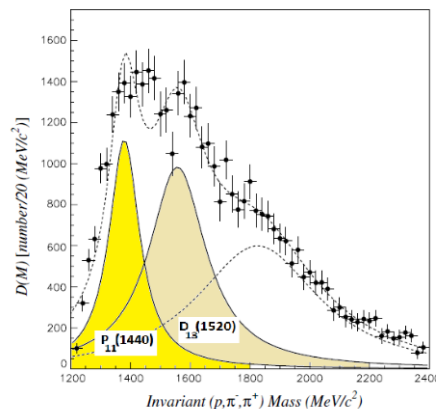
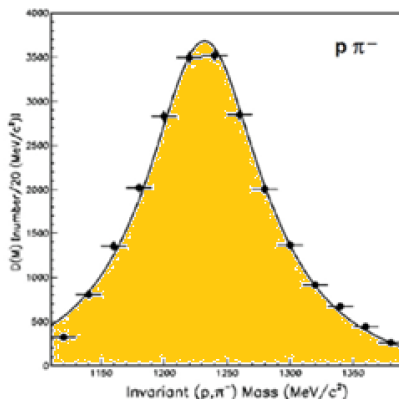
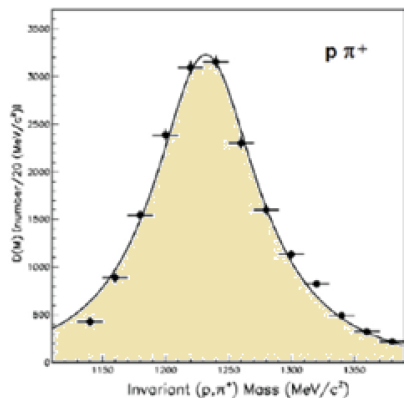
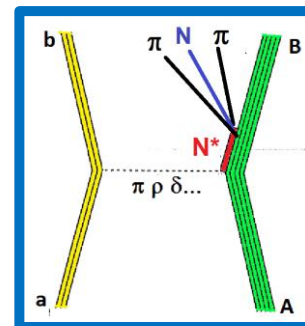
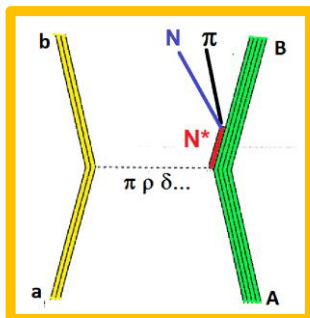


Perspectives of Resonance Studies at WASA@**(Super)FRS**



Invariant $N\pi/N\pi\pi$ Mass Spectroscopy at the SynchroPhasotron@DUBNA

$^{12}\text{C} + ^{12}\text{C}$ at $T_{\text{lab}} \sim 4 \text{ AGeV}$



Reconstruction of the Delta-resonance from $\Delta^{++} \rightarrow p\pi^+$ (right) and $\Delta^0 \rightarrow p\pi^-$ (left) mass spectroscopy

D. Krpic et al., Phys. Rev. C 65 (2002) 034909

$N^* \rightarrow p\pi^+\pi^-$ two-pion spectroscopy. Expected N^* states are indicated (left).

D. Krpic et al., Eur. Phys. J. A 20 (2004) 351

see: H. Lenske et al. / PPNP 98 (2018) 119

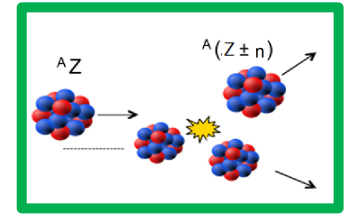
Summary and Outlook

- Resonance excitations in peripheral heavy ion collisions
- Resonance reaction dynamics and (semi-)quantal reaction theory
- Resonance spectral functions in nuclear matter
- Resonances in nuclei and N^*N^{-1} response functions
- Resonances for astrophysics: N^* in neutron stars
- Next steps:
 - Resonance tagging on WASA@FRS – experiment and theory
 - Resonances beyond $\Delta(1232)$ and $P_{11}(1440)$ – coupled channel schemes

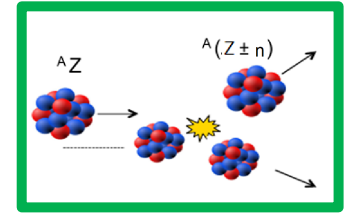
**Credits to
Jose Luis Rodriguez and the experiment team
and
Isaac Vidana for modelling the reaction**

Supported by DFG, AvH, and HFHF

Backups

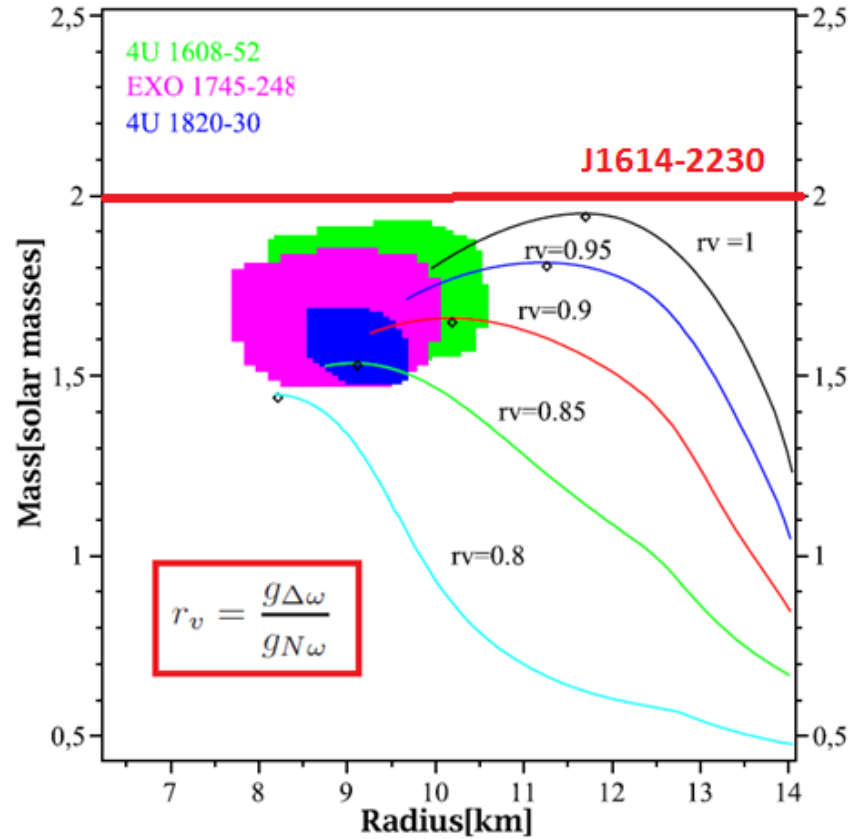
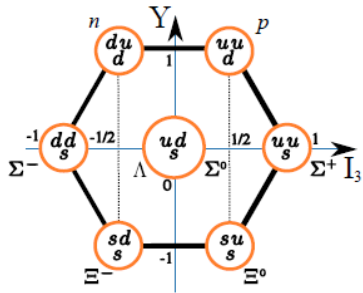


N* Resonances in Neutron Stars

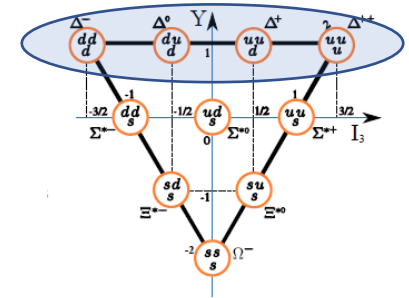


Δ's in Neutron Stars

Baryon Octet:



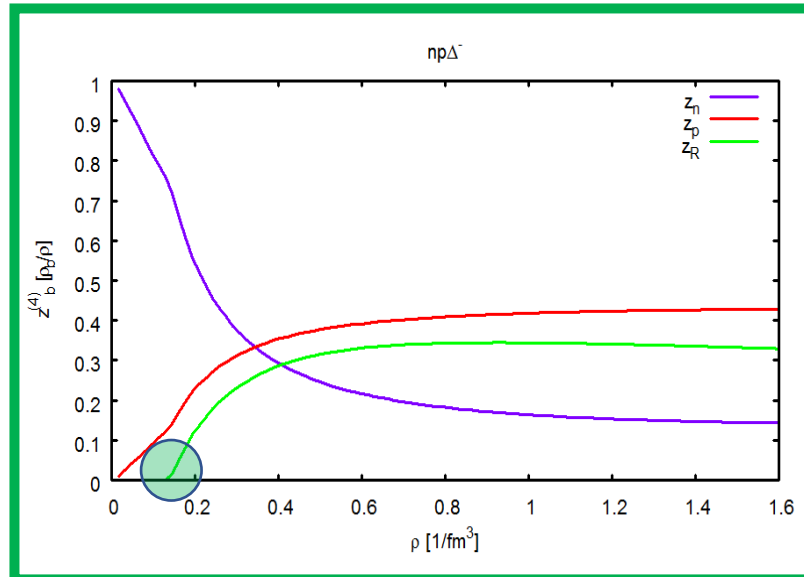
Baryon Decuplet:



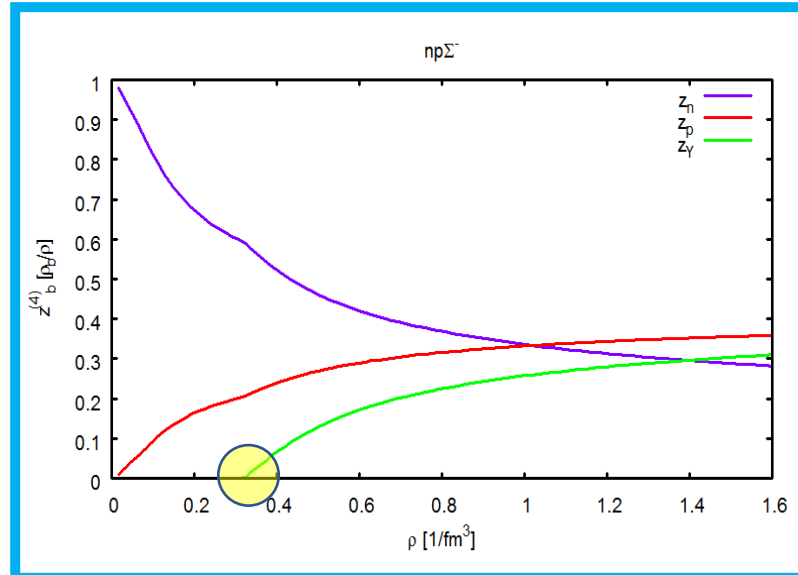
Mass-Radius-relationship of Neutron stars for various couplings of the Δ resonances, starting from $r_v = 1$ (upper line) to 0.8 (lowest line). Also included are the $1-\sigma$ errorbars for measured neutron stars . The black diamond on each curve represents the maximum stable configuration

Δ^- and Σ^- in Neutron Star Matter

$n\rho\mu\Delta^-$ matter
in beta-equilibrium:
onset at $\rho \sim \rho_{\text{SAT}}$



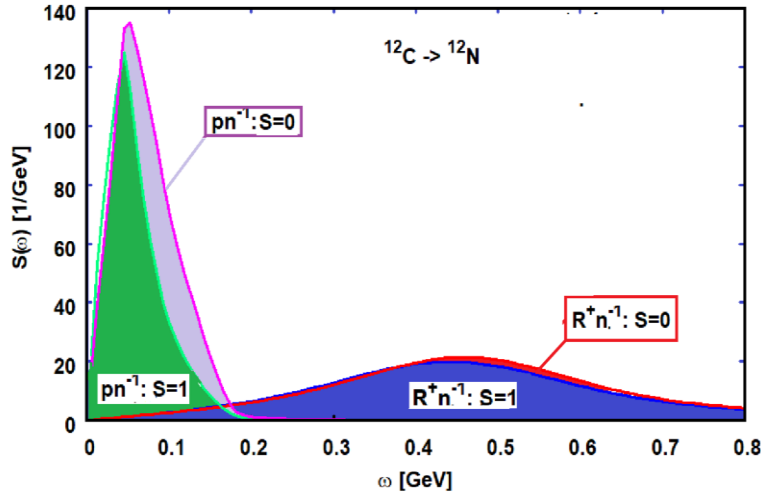
$n\rho\mu\Sigma^-$ matter
in beta-equilibrium:
onset at $\rho \sim 2\rho_{\text{SAT}}$



Spin-Scalar S=0 vs. Spin-Vector S=1 Response Functions

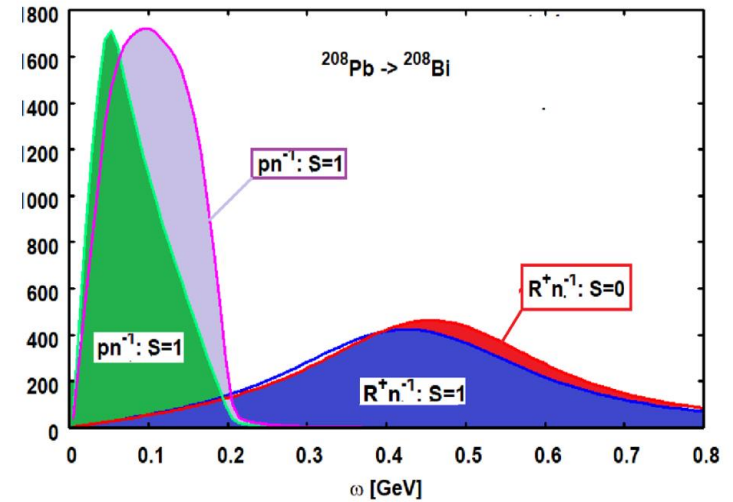
Longitudinal Response@q=300 MeV/c

^{12}C

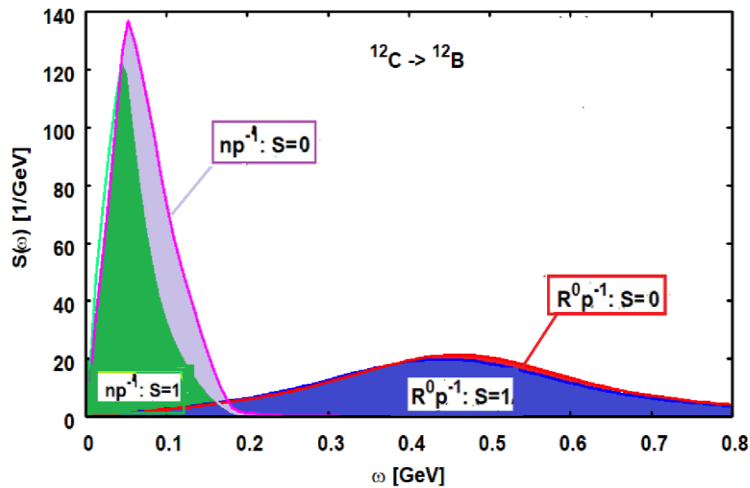


τ_+ operator: pn^{-1} - type
external probes: $\vec{\sigma} \cdot \vec{p}$ and l

^{208}Pb

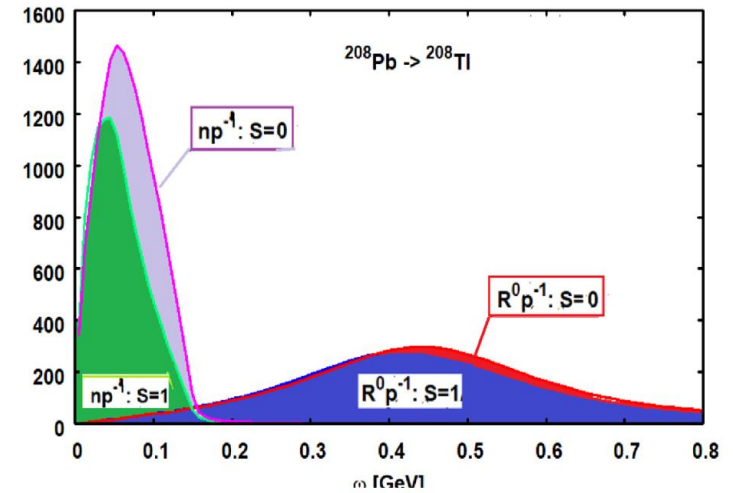


^{12}C



τ_- modes: np^{-1} - type
external probes: $\vec{\sigma} \cdot \vec{p}$ and l

^{208}Pb



Structure of the Inelastic Cross Sections

$$\left. \frac{d\sigma}{dE_b} \right|_{\text{in}} = \begin{cases} \langle N_{nn} \rangle |\mathcal{M}_{\text{in}}^{(nn \rightarrow pn\pi^-)}|^2 + \langle N_{np} \rangle |\mathcal{M}_{\text{in}}^{(np \rightarrow pp\pi^-)}|^2 + \langle N_{np} \rangle |\mathcal{M}_{\text{in}}^{(np \rightarrow pn\pi^0)}|^2 & (n, p)\text{-type} \\ \langle N_{pp} \rangle |\mathcal{M}_{\text{in}}^{(pp \rightarrow np\pi^+)}|^2 + \langle N_{pn} \rangle |\mathcal{M}_{\text{in}}^{(pn \rightarrow nn\pi^+)}|^2 + \langle N_{pn} \rangle |\mathcal{M}_{\text{in}}^{(pn \rightarrow np\pi^0)}|^2 & (p, n)\text{-type} \end{cases}$$

$$\left| \mathcal{M}_{\text{in}}^{(\tau_p \tau_T \rightarrow \tau'_p \tau'_T \pi)} \right|^2 = \frac{1}{S} \frac{|\vec{p}_b|}{(2\pi)^5} \frac{m^4}{\lambda^{1/2}(s, m^2, m^2)}$$

$$\times \int \frac{d\vec{q}}{(2\pi)^3} \frac{d\vec{p}_\pi d\Omega_b}{E_B E_\pi} R_{\Delta}^{(A)}(\sqrt{s} - E_b - E_\pi, \vec{q} - \vec{p}_\pi) \left| \mathcal{M}_{\text{in}}^{(\tau_p \tau_T \rightarrow \tau'_p \tau'_T \pi)}(\vec{q} - \vec{p}_\pi) \right|^2$$

„Collision Numbers“ by Glauber-Model

$$\left. \begin{aligned} \langle N_{np} \rangle &= (A_P - Z_P) \frac{\sigma_T}{\sigma_{PT}}, \\ \langle N_{pp} \rangle &= Z_P \frac{\sigma_T}{\sigma_{PT}}, \\ \langle N_{nT} \rangle &= (A_T - Z_T) \frac{\sigma_P}{\sigma_{PT}}, \\ \langle N_{pT} \rangle &= Z_T \frac{\sigma_P}{\sigma_{PT}}, \end{aligned} \right\} \begin{aligned} \sigma_{P(T)} &= \int d\vec{b} (1 - [1 - T_{P(T)}(b) \sigma_{NN}]^{A_P(A_T)}), \\ T_{P(T)}(b) &= \int dz \rho_{P(T)}(b, z), \\ \sigma_{PT} &= \int d\vec{b} (1 - [1 - T_{PT}(b) \sigma_{NN}]^{A_P A_T}), \\ T_{PT}(b) &= \int d\vec{s} T_P(|\vec{s} - \vec{b}|) T_T(s), \end{aligned}$$

„Collision Numbers“ by Glauber-Theory

$$\langle N_{\tau_P \tau_T} \rangle = \langle N_{\tau_P} \rangle \times \langle N_{\tau_T} \rangle, \quad \tau_P = n, p, \quad \tau_T = n, p,$$

$$\langle N_{np} \rangle = (A_P - Z_P) \frac{\sigma_T}{\sigma_{PT}},$$

$$\langle N_{pp} \rangle = Z_P \frac{\sigma_T}{\sigma_{PT}},$$

$$\langle N_{nT} \rangle = (A_T - Z_T) \frac{\sigma_P}{\sigma_{PT}},$$

$$\langle N_{pT} \rangle = Z_T \frac{\sigma_P}{\sigma_{PT}},$$

$$\sigma_{P(T)} = \int d\vec{b} (1 - [1 - T_{P(T)}(b)\sigma_{NN}]^{A_{P(T)}}),$$

$$T_{P(T)}(b) = \int dz \rho_{P(T)}(b, z),$$

$$\sigma_{PT} = \int d\vec{b} (1 - [1 - T_{PT}(b)\sigma_{NN}]^{A_P A_T}),$$

$$T_{PT}(b) = \int d\vec{s} T_P(|\vec{s} - \vec{b}|) T_T(s),$$

→ Nuclear form factors and elementary NN-cross sections

„Collision Numbers“ by Glauber-Theory

$$\langle N_{\tau_P \tau_T} \rangle = \langle N_{\tau_P} \rangle \times \langle N_{\tau_T} \rangle, \quad \tau_P = n, p, \quad \tau_T = n, p,$$

$$\langle N_{np} \rangle = (A_P - Z_P) \frac{\sigma_T}{\sigma_{PT}},$$

$$\langle N_{pp} \rangle = Z_P \frac{\sigma_T}{\sigma_{PT}},$$

$$\langle N_{nT} \rangle = (A_T - Z_T) \frac{\sigma_P}{\sigma_{PT}},$$

$$\langle N_{pT} \rangle = Z_T \frac{\sigma_P}{\sigma_{PT}},$$

$$\sigma_{P(T)} = \int d\vec{b} (1 - [1 - T_{P(T)}(b) \sigma_{NN}]^{A_{P(T)}}),$$

$$T_{P(T)}(b) = \int dz \rho_{P(T)}(b, z),$$

$$\sigma_{PT} = \int d\vec{b} (1 - [1 - T_{PT}(b) \sigma_{NN}]^{A_P A_T}),$$

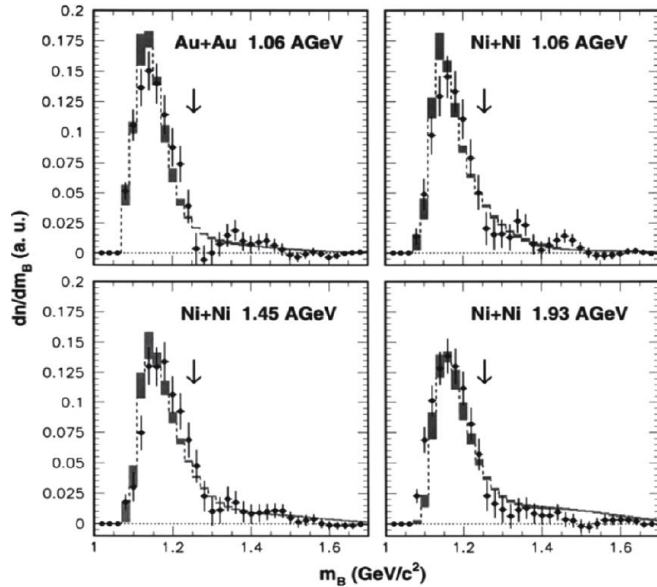
$$T_{PT}(b) = \int d\vec{s} T_P(|\vec{s} - \vec{b}|) T_T(s),$$

→ Nuclear form factors and elementary NN-cross sections

Discussion

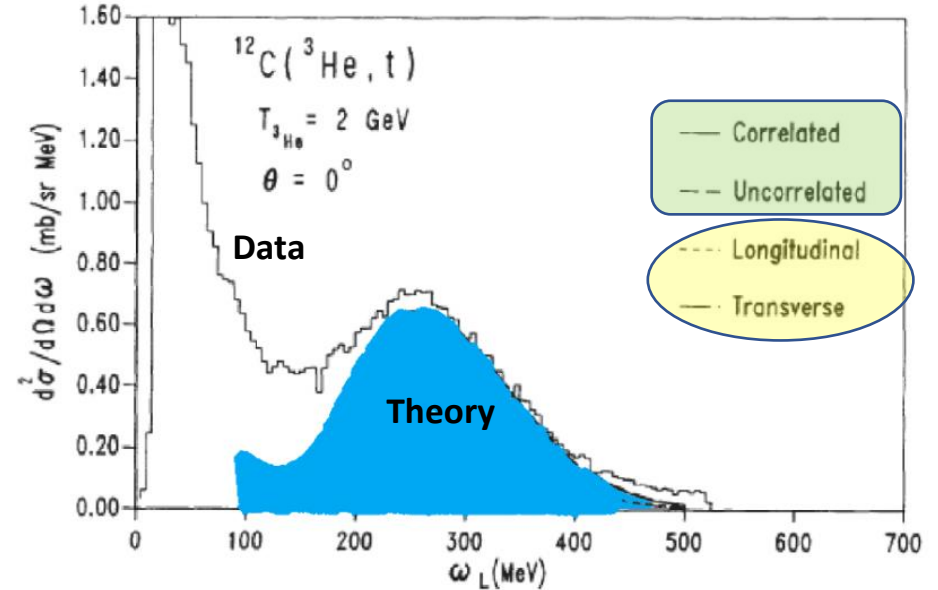
Prelude: Early Resonance Studies with Heavy Ions

Central Heavy Ion Collisions FOPI@GSI



Filled symbols: transverse momentum spectra of π^\pm .
Filled circles: measured $p\pi^\pm$ pairs.
Arrows: Centroid of the free $\Delta(1232)$ mass distribution
 M. Eskef, et al., Eur. Phys. J. A 3 (1998) 335

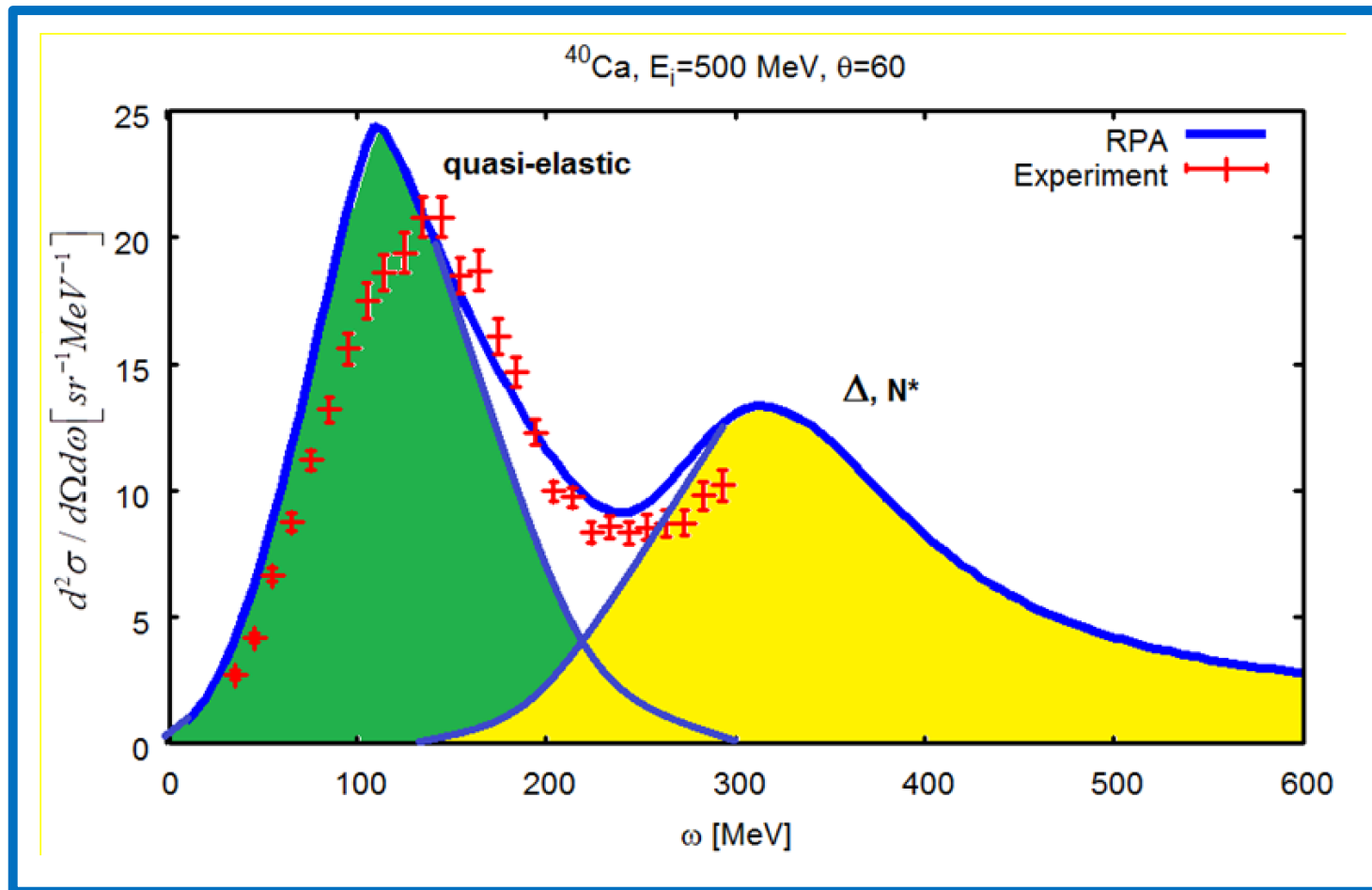
Peripheral Charge Exchange Collisions DIOGENE@SATURNE



Theory: T. Udagawa et al., Phys. Rev. C 49 (1994) 3162
Data: D. Contardo, et al., Phys. Lett. B 168 (1986) 331

See also: **EOS@BEVALAC:** E.L. Hjort, et al., PRL. 79 (1997) 4345 ; **E814@AGS,** J. Barrette *et al.*, PLB **351**, 93 (1995) ; **Synchro-Phasotron@Dubna:** D. Krpic et al., PRC 65 (2002) 034909...

Photo-Excitation of Resonances in Inclusive (e,e') Scattering



Theory (GiEDF): NN^{-1} & N^*N^{-1} Longitudinal Response Functions by „N*-RPA“
Data (Bates): Williamson et al., Phys.Rev.C 56 (1997) 3152

Cross Section for SCE Reactions at Relativistic Energies

$$d^2\sigma_{aA\rightarrow bB} \sim \sum_{\mu\nu} D_{aA,bB}(\sqrt{s}, \vec{q}) R_{ab,\mu\nu}^\dagger(\omega - \omega_A, \vec{q} - \vec{q}_A) R_{AB}^{\mu\nu}(\omega_A, \vec{q}_A) d\omega d\Omega$$

Distortion Coefficient \leftrightarrow Ion-Ion Kinematics and elastic ISI/FSI:

$$D_{aA,bB}(\sqrt{s}, q) = \langle \chi_{bB}^{(-)} | e^{iq\cdot r} | \chi_{aA}^{(+)} \rangle$$

...at AGeV properly treated by Eikonal or Glauber Theory

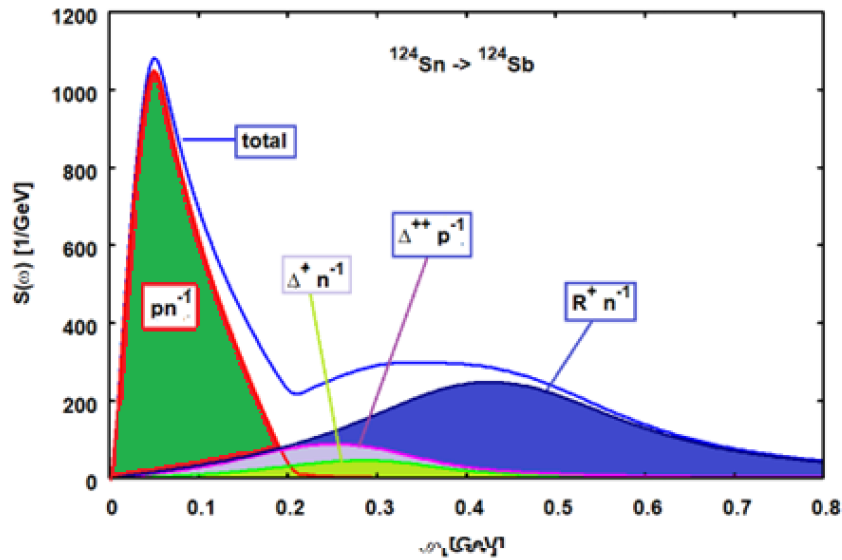
Hadronic Tensor \leftrightarrow Nuclear Response and Interactions:

$$R_X^{\mu\nu}(\omega, \vec{q}) = -\frac{1}{\pi} \text{Im} \left(\sum_{\alpha=L,T} F_\alpha(\sqrt{s}, \vec{q}) \Pi_{\alpha X}^{\mu\nu}(\omega, \vec{q}) \right)$$

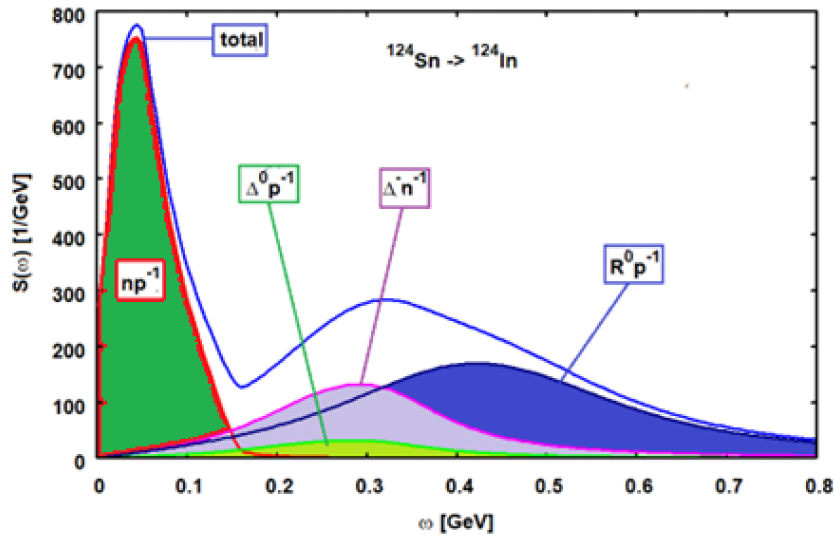
(X=ab;AB)

Composition of N^*N^{-1} Spectral Distributions – $P_{33}(1232)$ and $P_{11}(1440)$ Longitudinal@ $q=300$ MeV/c

τ_+ operator : pn^{-1} – type
external probe: $\square \vec{\sigma} \bullet \vec{p}$



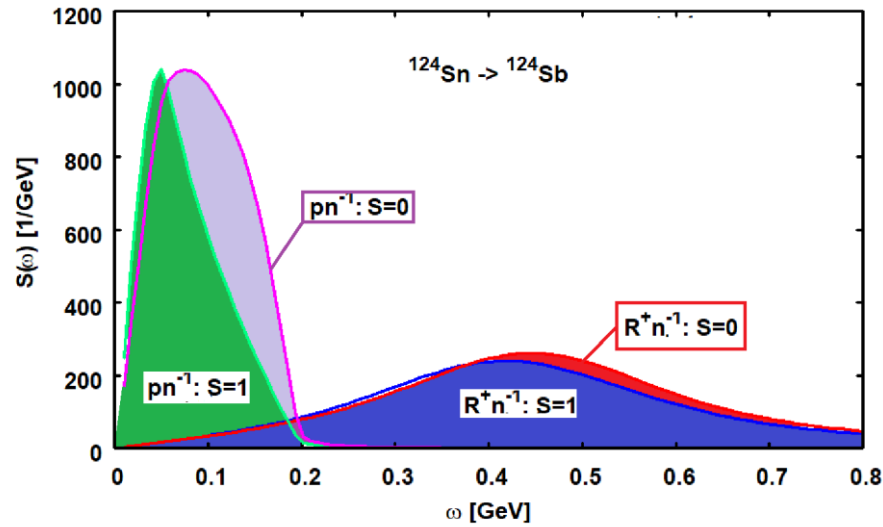
τ_- modes : np^{-1} – type
external probe: $\square \vec{\sigma} \bullet \vec{p}$



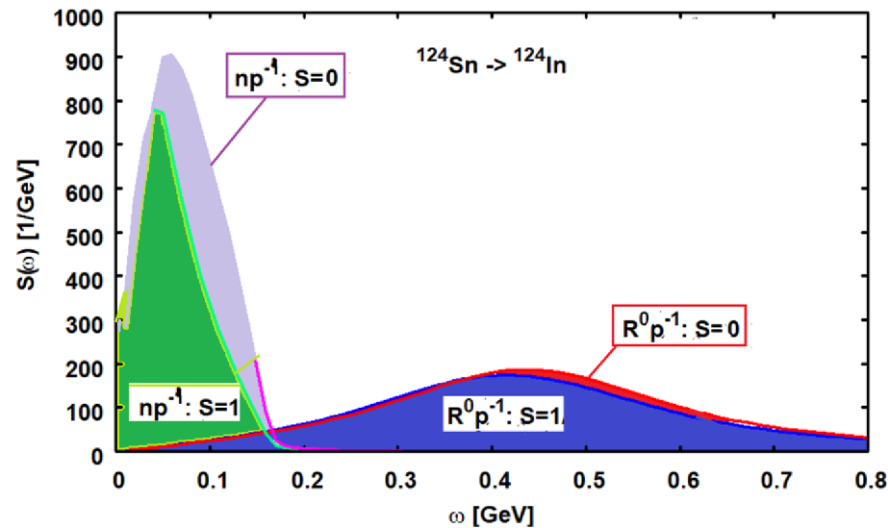
Spin-Scalar S=0 vs. Spin-Vector S=1 Modes

Longitudinal@q=300 MeV/c

τ_+ operator: pn^{-1} - type
 external probes: $\square \vec{\sigma} \bullet \vec{p}$ and 1



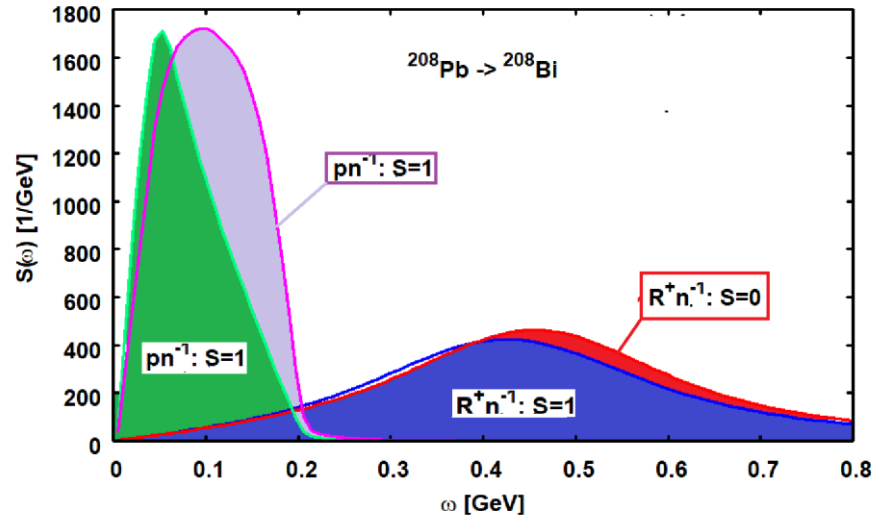
τ_- modes: np^{-1} - type
 external probes: $\square \vec{\sigma} \bullet \vec{p}$ and 1



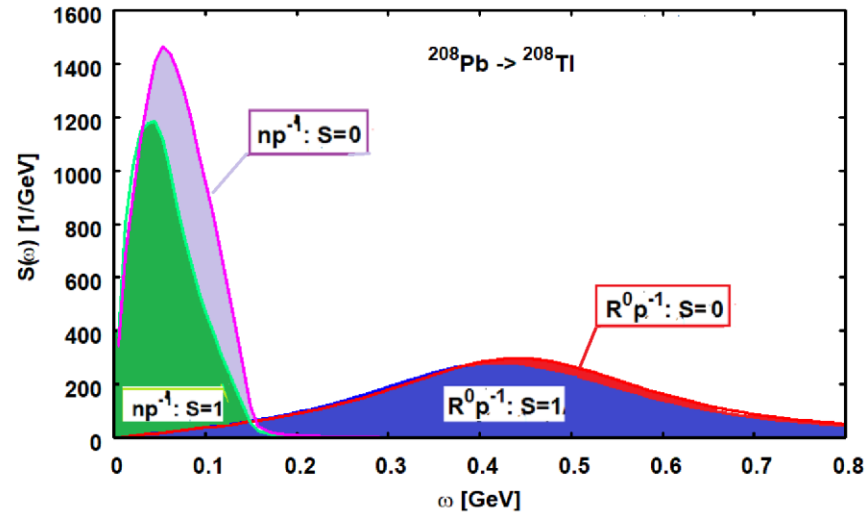
Spin-Scalar S=0 vs. Spin-Vector S=1 Modes

Longitudinal@q=300 MeV/c

τ_+ operator : pn^{-1} – type
 external probes: $\square \vec{\sigma} \cdot \vec{p}$ and 1



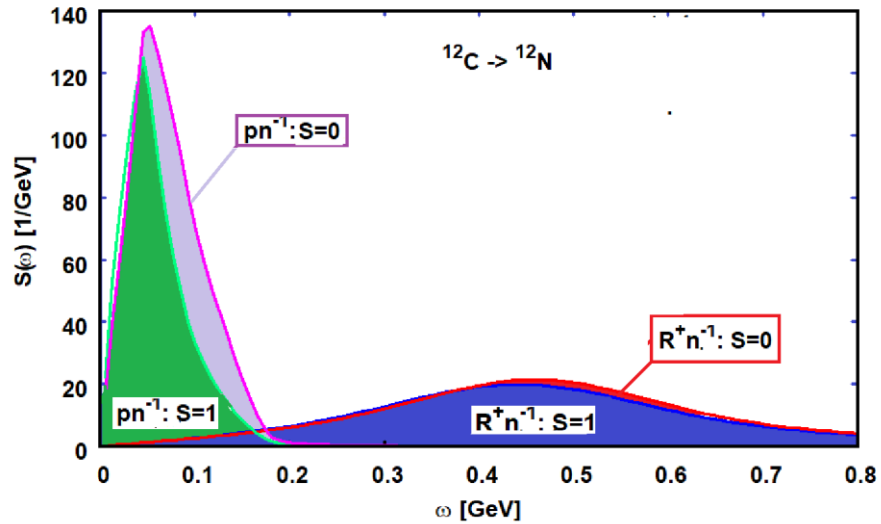
τ_- modes : np^{-1} – type
 external probes: $\square \vec{\sigma} \cdot \vec{p}$ and 1



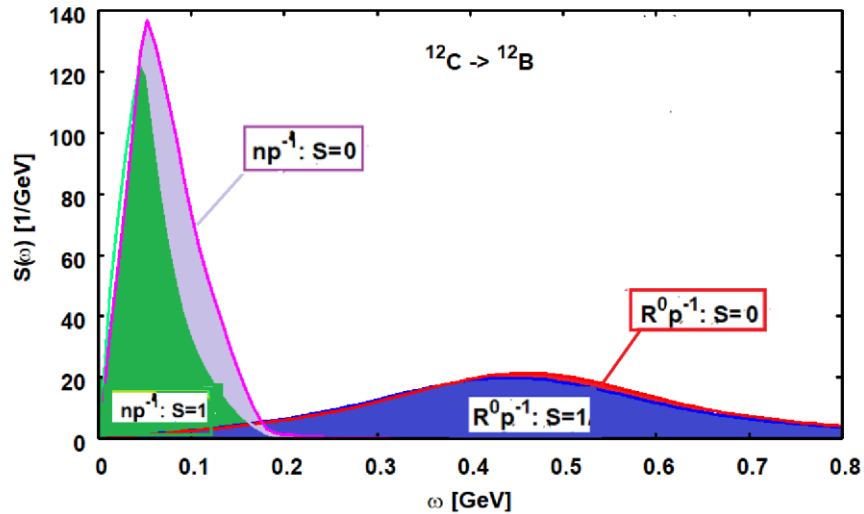
Spin-Scalar S=0 vs. Spin-Vector S=1 Modes

Longitudinal@q=300 MeV/c

τ_+ operator : pn^{-1} - type
 external probes: $\square \vec{\sigma} \bullet \vec{p}$ and 1

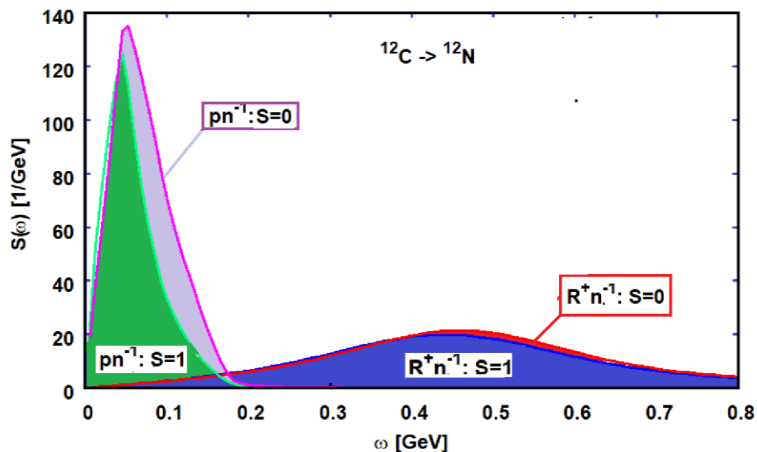


τ_- modes : np^{-1} - type
 external probes: $\square \vec{\sigma} \bullet \vec{p}$ and 1

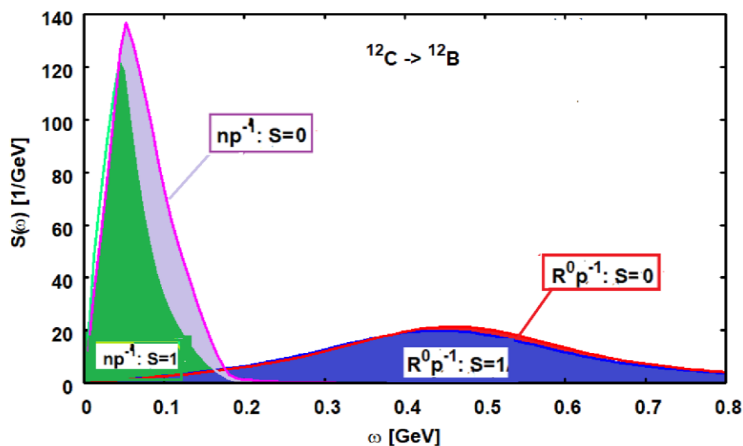
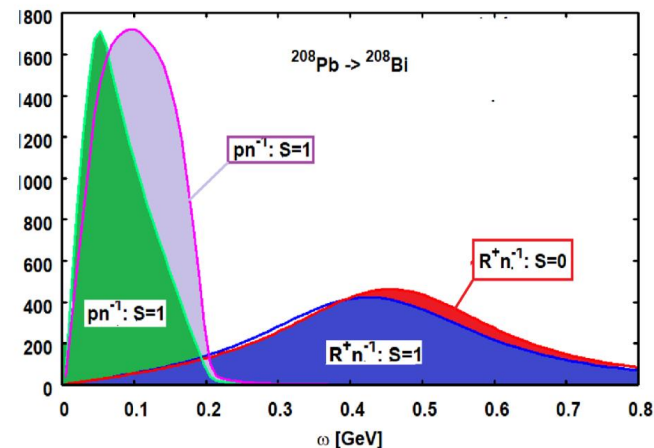


Spin-Scalar S=0 vs. Spin-Vector S=1 Modes

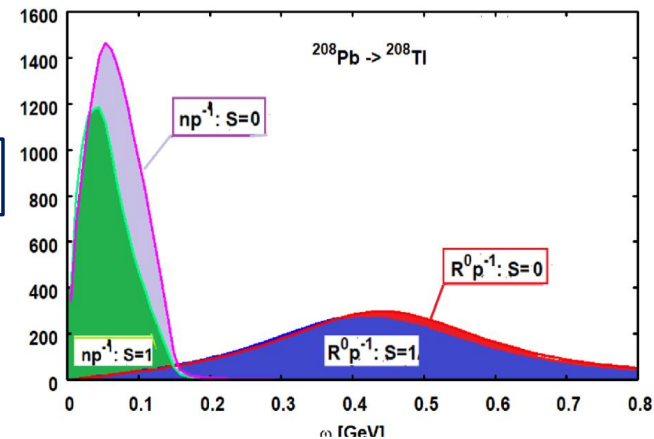
Longitudinal@q=300 MeV/c



τ_+ operator: pn^{-1} -type
external probes: $\square \vec{\sigma} \cdot \vec{p}$ and 1

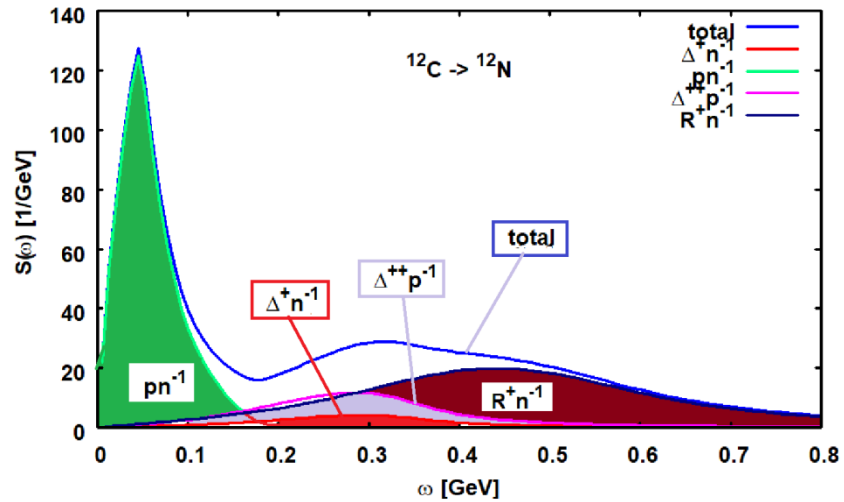


τ_- modes: np^{-1} -type
external probes: $\square \vec{\sigma} \cdot \vec{p}$ and 1

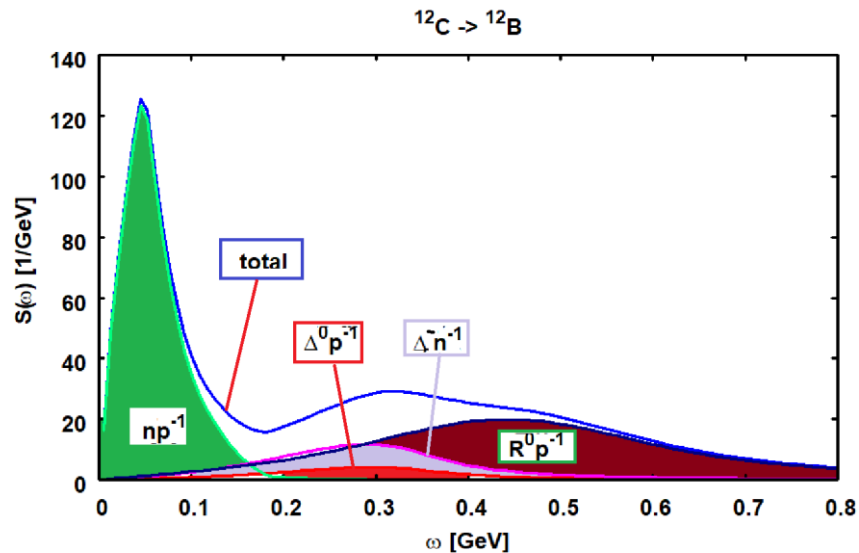


N*N⁻¹ Spectral Distributions – P₃₃(1232) and P₁₁(1440) Longitudinal@q=300 MeV/c

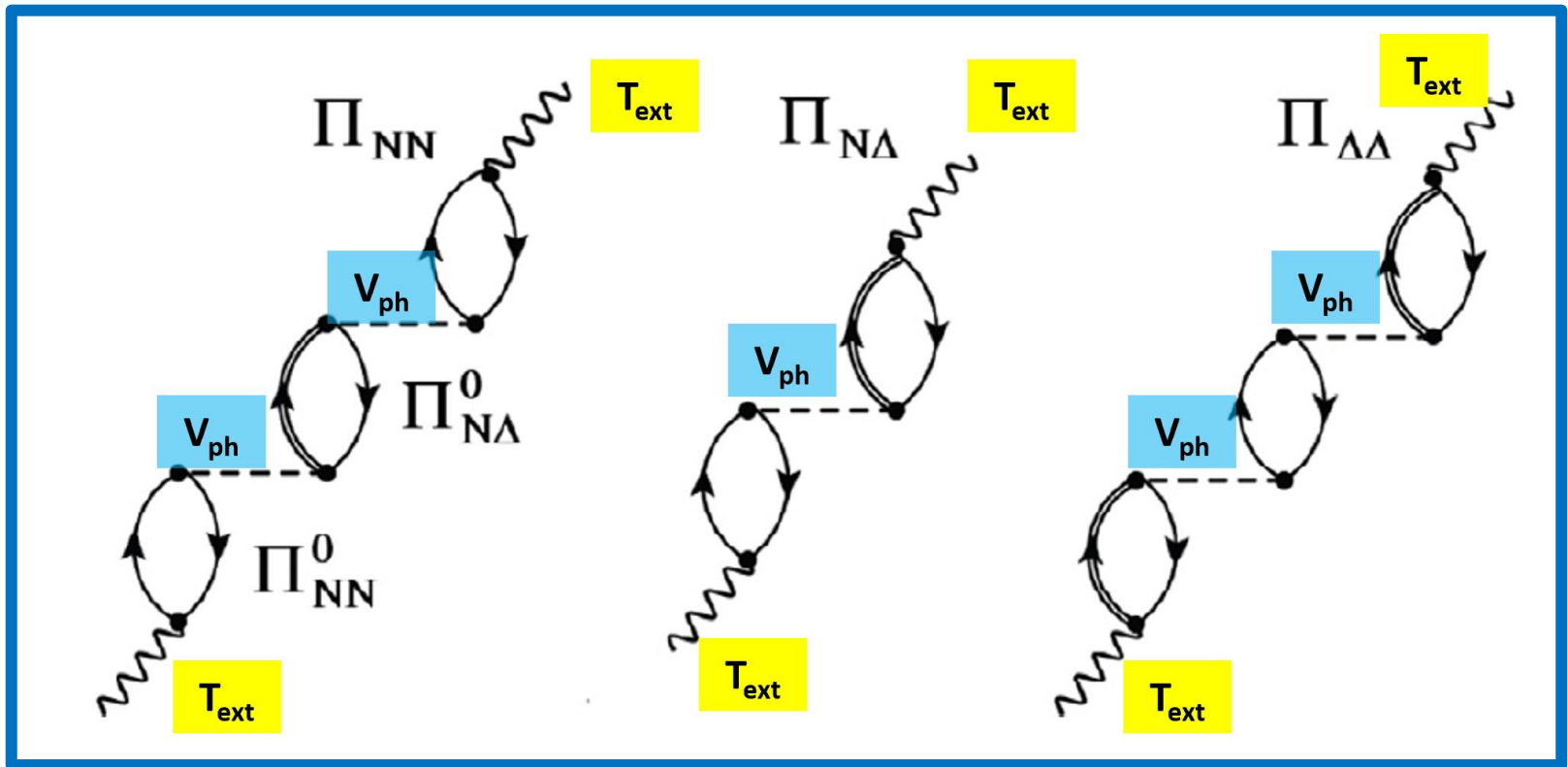
τ_- modes : np^{-1} – type
external probes: $\square \vec{\sigma} \bullet \vec{p}$ and 1



τ_+ operator : pn^{-1} – type
external probes: $\square \vec{\sigma} \bullet \vec{p}$ and 1



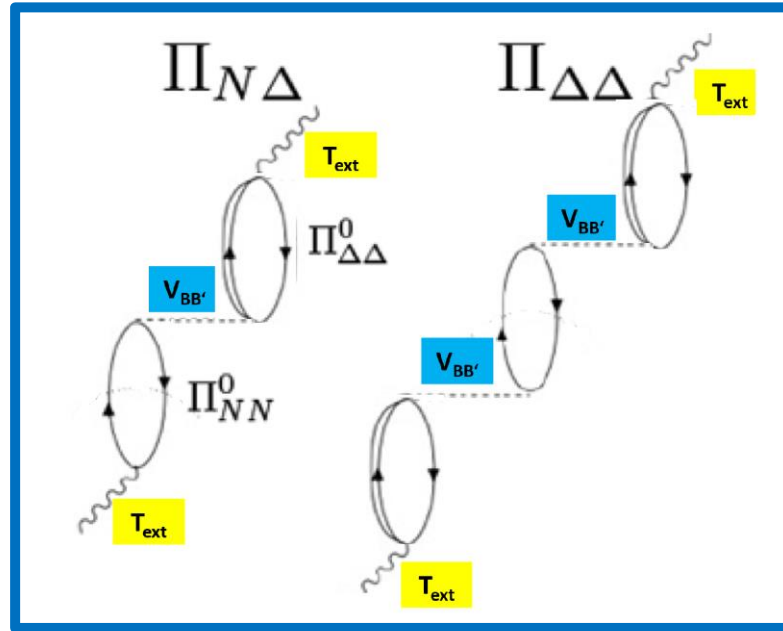
Polarization Tensor and Nuclear Response



$V_{ph} \sim \pi, \rho, \delta$ exchange and „short-range“ g'

$$R_X^{\mu\nu}(\omega, \vec{q}) = -\frac{1}{\pi} \text{Im} \left(\Pi_X^{\mu\nu}(\omega, \vec{q}) \right) = -\frac{1}{\pi} \text{Im} \left(\langle X | T_{ext}^{\dagger\mu}(\vec{q}) G_X(\omega) T_{ext}^{\nu}(\vec{q}) | X \rangle \right)$$

Polarization Tensor and Nuclear Response



$$V_{N\Delta} = (V_\pi \sigma_N \cdot q S_{N\Delta} \cdot q + V_\rho \sigma_N \times q \cdot S_{N\Delta} \times q + g'_{N\Delta} \sigma_N \cdot S_{N\Delta}) \tau_N \cdot T_{N\Delta}$$

$$V_{NR} = (V_\pi \sigma_N \cdot q \sigma_R \cdot q + V_\rho \sigma_N \times q \cdot \sigma_R \times q + g'_{NR} \sigma_N \cdot \sigma_{NR} + U_\rho + U_\delta) \tau_N \cdot \tau_R$$

$$R_X^{\mu\nu}(\omega, \vec{q}) = -\frac{1}{\pi} \text{Im}(\Pi_X^{\mu\nu}(\omega, \vec{q})) = -\frac{1}{\pi} \text{Im}(\langle X | T_{ext}^{\dagger\mu}(\vec{q}) G_X(\omega) T_{ext}^\nu(\vec{q}) | X \rangle)$$

Example: Elementary Modes in pn^{-1} -type ($^A\text{Sn}, ^A\text{Sb}$) Reactions

$p(^{112,124}\text{Sn}, ^{112,124}\text{Sb})X$	
Quasielastic Inelastic (target)	$p(n, p)n$ Inelastic (projectile)
$p(n, p)\Delta^0 \rightarrow p(n, p)n\pi^0$	$p(n, \Delta^0)p \rightarrow p(n, p\pi^-)p$
$p(n, p)\Delta^0 \rightarrow p(n, p)p\pi^-$	$p(n, \Delta^+)n \rightarrow p(n, p\pi^0)n$
$p(n, p)n\pi^0$ (<i>s</i> wave)	$p(n, p\pi^-)p$ (<i>s</i> wave)
$p(n, p)p\pi^-$ (<i>s</i> wave)	$p(n, p\pi^0)n$ (<i>s</i> wave)
$^AZ (^{112,124}\text{Sn}, ^{112,124}\text{Sb})X$	
Quasielastic Inelastic (target)	$p(n, p)n$ Inelastic (projectile)
$p(n, p)\Delta^0 \rightarrow p(n, p)n\pi^0$	$p(n, \Delta^0)p \rightarrow p(n, p\pi^-)p$
$p(n, p)\Delta^0 \rightarrow p(n, p)p\pi^-$	$p(n, \Delta^+)n \rightarrow p(n, p\pi^0)n$
$p(n, p)n\pi^0$ (<i>s</i> wave)	$p(n, p\pi^-)p$ (<i>s</i> wave)
$p(n, p)p\pi^-$ (<i>s</i> wave)	$p(n, p\pi^0)n$ (<i>s</i> wave)
$n(n, p)\Delta^- \rightarrow n(n, p)n\pi^-$	$n(n, \Delta^0)n \rightarrow n(n, p\pi^-)n$
$n(n, p)n\pi^-$ (<i>s</i> wave)	$n(n, p\pi^-)n$ (<i>s</i> wave)