η bound-virtual states from reactions with d, ^{3}He, ^{4}He

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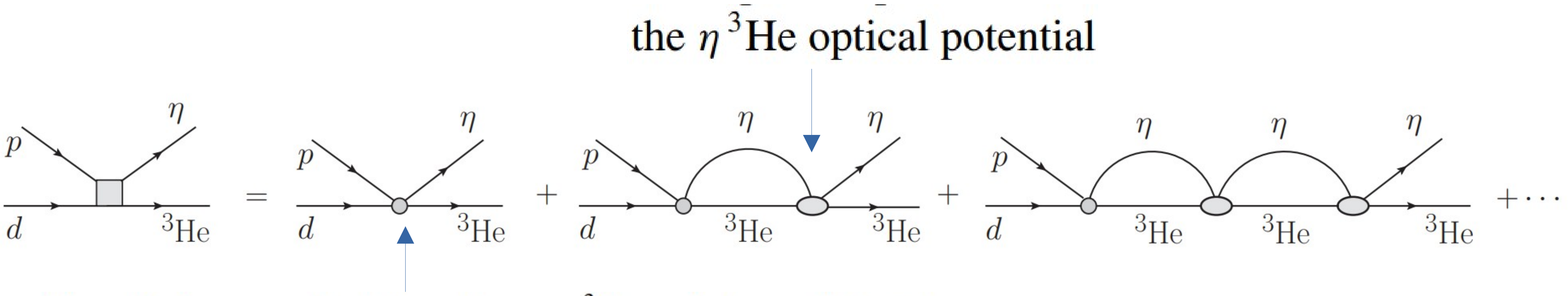
The p d $\rightarrow$ η ^{3}He reaction

The d d $\rightarrow$ η ^{4}He reaction

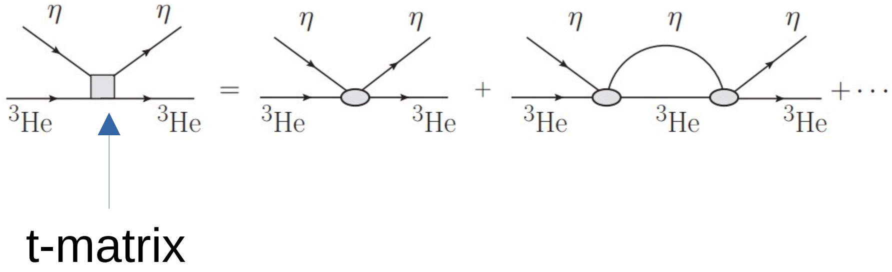
The γ d $\rightarrow$ π^0 η d reaction

The process $pd \to \eta\,^3\mathrm{He}$ considering explicitly the $\eta\,^3\mathrm{He}$ rescattering



contains all diagrams that do not have $\eta\,^3\mathrm{He}$ as intermediate state.

$$V(\vec{r}) = 3t_{\eta N}\tilde{\rho}(\vec{r})$$



$$V(\vec{p}_\eta, \vec{p'}_\eta) = 3t_{\eta N}\int d^3\vec{r}\,\tilde{\rho}(\vec{r})e^{i(\vec{p}_\eta - \vec{p'}_\eta)\cdot\vec{r}}$$
$$= 3t_{\eta N}F(\vec{p}_\eta - \vec{p'}_\eta),$$

$$F(\vec{q}) = \int d^3\vec{r}\,\tilde{\rho}(\vec{r})e^{i\vec{q}\cdot\vec{r}} \qquad F(\vec{q}) = e^{-\beta^2|\vec{q}|^2}$$

S-wave projection

$$V(\vec{p}_\eta,\vec{p}_\eta^{\,\prime}) = 3t_{\eta N}\frac{1}{2}\int_{-1}^{1} d\cos\theta\, e^{-\beta^2(|\vec{p}_\eta|^2+|\vec{p'}_\eta|^2-2|\vec{p}_\eta||\vec{p'}_\eta|\cos\theta)}$$

$$= 3t_{\eta N} e^{-\beta^2|\vec{p}_\eta|^2} e^{-\beta^2|\vec{p'}_\eta|^2}$$

$$\times\left[1+\frac{1}{6}(2\beta^2|\vec{p}_\eta||\vec{p'}_\eta|)^2+\cdots\right]. \quad (($$

Separable potential

$\tilde{V}$

negligible

$$T(\vec{p}_\eta,\vec{p'}_\eta) = \tilde{T} e^{-\beta^2|\vec{p}_\eta|^2} e^{-\beta^2|\vec{p'}_\eta|^2}$$

$$\tilde{T} = \tilde{V} + \tilde{V} G \tilde{T}$$

$$G = \frac{M_{^3\mathrm{He}}}{16\pi^3}\int \frac{d^3\vec{q}}{\omega_\eta(\vec{q})E_{^3\mathrm{He}}(\vec{q})}\frac{e^{-2\beta^2|\vec{q}|^2}}{\sqrt{s}-\omega_\eta(\vec{q})-E_{^3\mathrm{He}}(\vec{q})+i\epsilon}$$

$pd \to \eta\,{}^3\mathrm{He}$ transition

$$V_P = A\vec{\epsilon}\cdot\vec{p} + i\,B(\vec{\epsilon}\times\vec{\sigma})\cdot\vec{p}$$

p, momentum of the proton

After η-^{3}He final state interaction

$$t_{pd\to\eta\,^3\mathrm{He}} = V_P e^{-\beta^2|\vec{p}_\eta|^2} + V_P G\tilde{T} e^{-\beta^2|\vec{p}_\eta|^2}$$
$$= V_P e^{-\beta^2|\vec{p}_\eta|^2}(1+G\tilde{T}) = \frac{V_P e^{-\beta^2|\vec{p}_\eta|^2}}{1-\tilde{V}G}$$

$$V_{1P} = C\vec{\epsilon}\cdot\vec{p}_\eta + i\,D(\vec{\epsilon}\times\vec{\sigma})\cdot\vec{p}_\eta$$

Asymmetry $$\alpha = \frac{d}{d\cos\theta_\eta}\ln\left(\frac{d\sigma}{d\Omega}\right)\bigg|_{\cos\theta_\eta=0}$$

$$A = B \text{ and } C = D$$

C. Wilkin et al., Phys. Lett. B 654, 92 (2007)

$$a'_{\eta N} = \frac{1}{4\pi} \frac{m_N}{\sqrt{s_{\eta N}}} \frac{\tilde{V}}{3} \bigg|_{\sqrt{s_{\eta N}} = m_N + m_\eta}$$

$$a_{\eta\,{}^3\mathrm{He}} = \frac{1}{4\pi} \frac{M_{{}^3\mathrm{He}}}{\sqrt{s}} T \bigg|_{\sqrt{s} = M_{{}^3\mathrm{He}} + m_\eta}$$

perform six-parameter [$A = B = r_A$, $C = D = r_C e^{i\theta}(1 + \gamma Q)$, and $\tilde{V} = \mathrm{Re}(V) + i\,\mathrm{Im}(V)$] χ^2 fits

r_A (MeV^{-2})	$(9.43 \pm 0.17) \times 10^{-7}$
r_C (MeV^{-2})	$(6.85 \pm 0.31) \times 10^{-6}$
θ (degree)	347 ± 2
γ (MeV^{-1})	$(-5.25 \pm 0.15) \times 10^{-2}$
Re(V) (MeV^{-1})	$(-14.57 \pm 0.42) \times 10^{-2}$
Im(V) (MeV^{-1})	$(-5.36 \pm 0.14) \times 10^{-2}$

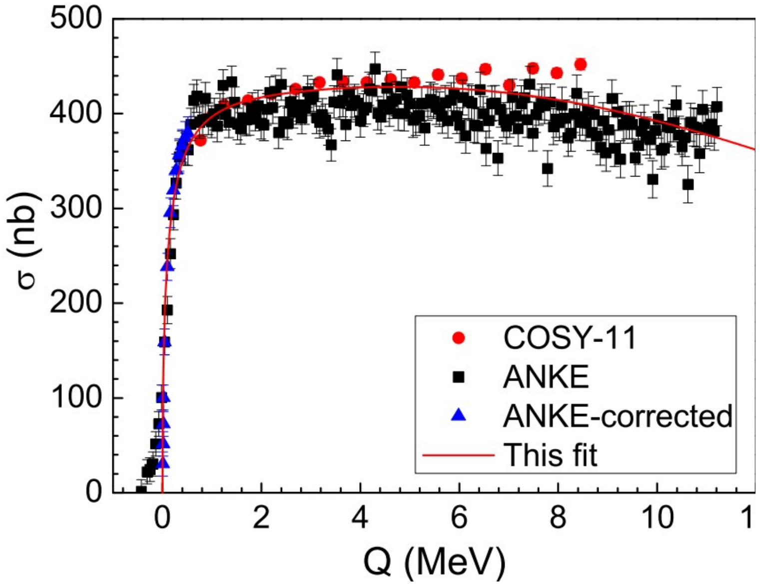
$$a'_{\eta N} = [-(0.48 \pm 0.05) - i(0.18 \pm 0.02)]\ \mathrm{fm}$$

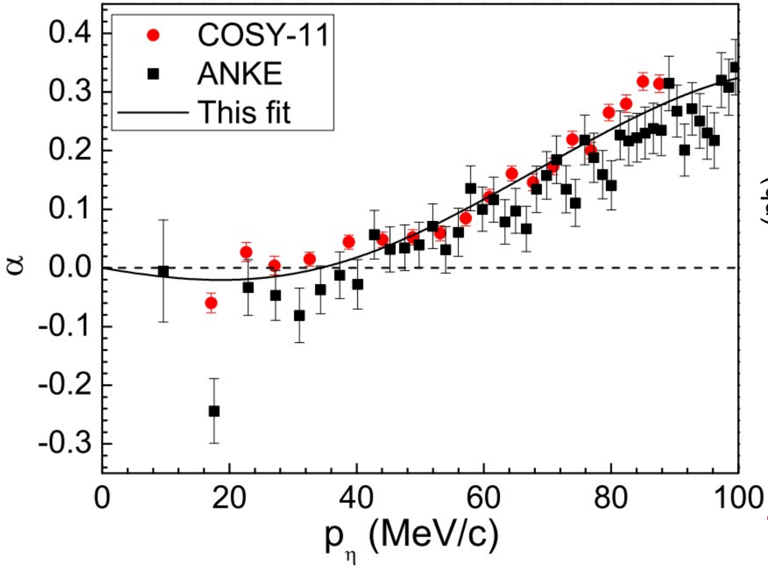
By definition should be similar to $a_{\eta N}$

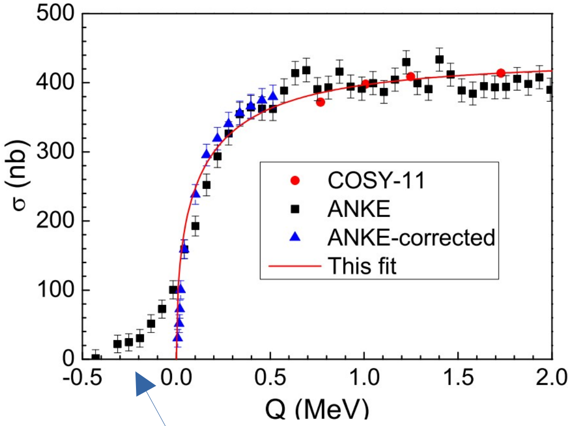
$$a_{\eta\,^3\mathrm{He}} = [(2.23 \pm 1.29) - i(4.89 \pm 0.57)]\ \mathrm{fm}$$

A. Sibirtsev, J. Haidenbauer, C. Hanhart, and J. A. Niskanen,Eur. Phys. J. A 22, 495 (2004).

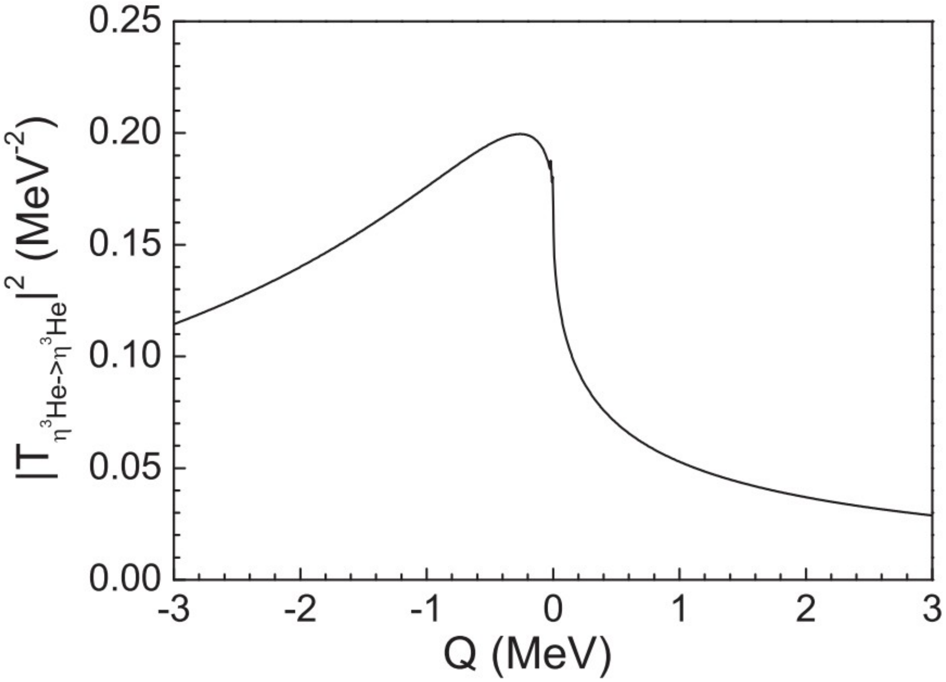
$$a_{\eta\,^3\mathrm{He}} = [(-4.3 \pm 0.3) - i(0.5 \pm 0.5)]\ \mathrm{fm}$$

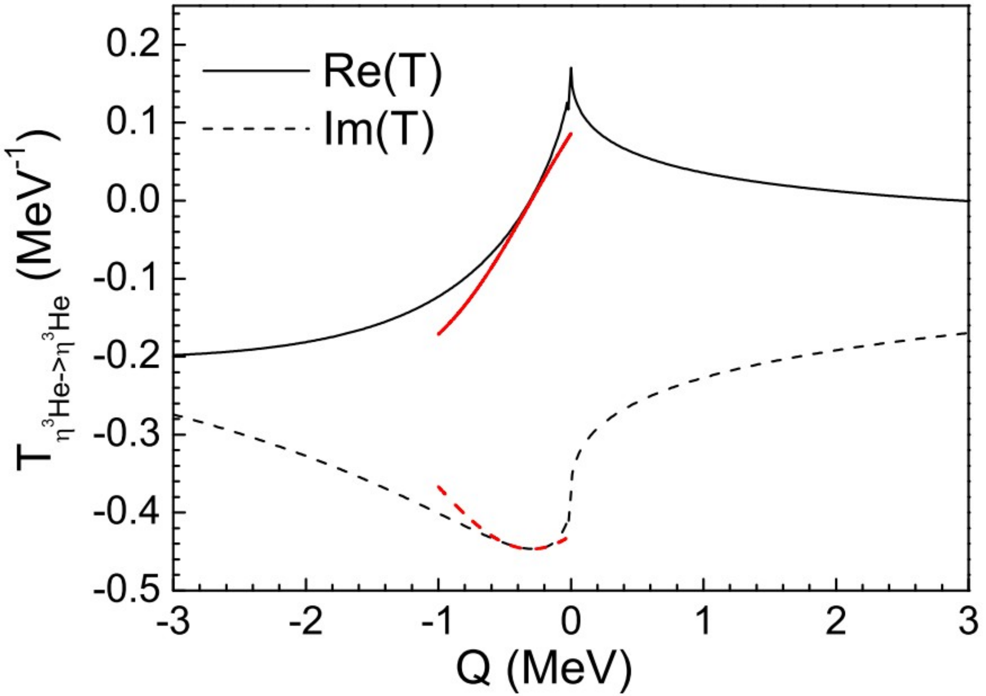






Threshold data corrected by ANKE

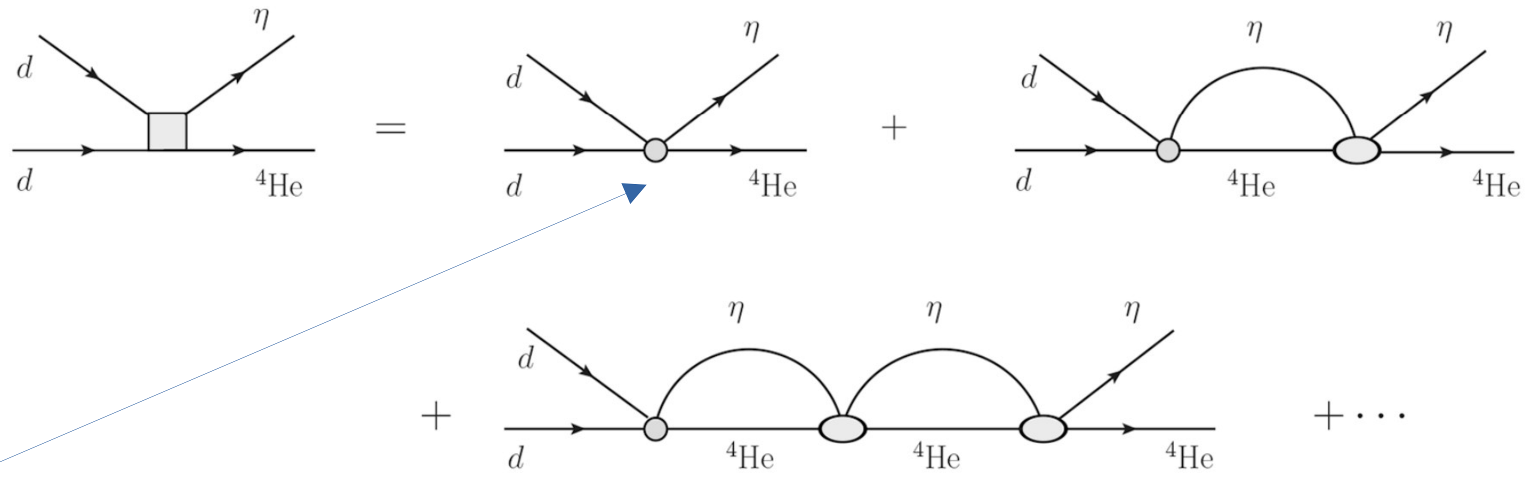




Concerning the η ^{3}He T-matrix, there is a structure suggestive of a bound state with less than 1 MeV binding, but we do not find a pole in the bound region, but above threshold. Technically there is no bound state. The reaction studied evidences this structure!

η-^{4}He interaction from the dd → η^4He reaction near threshold

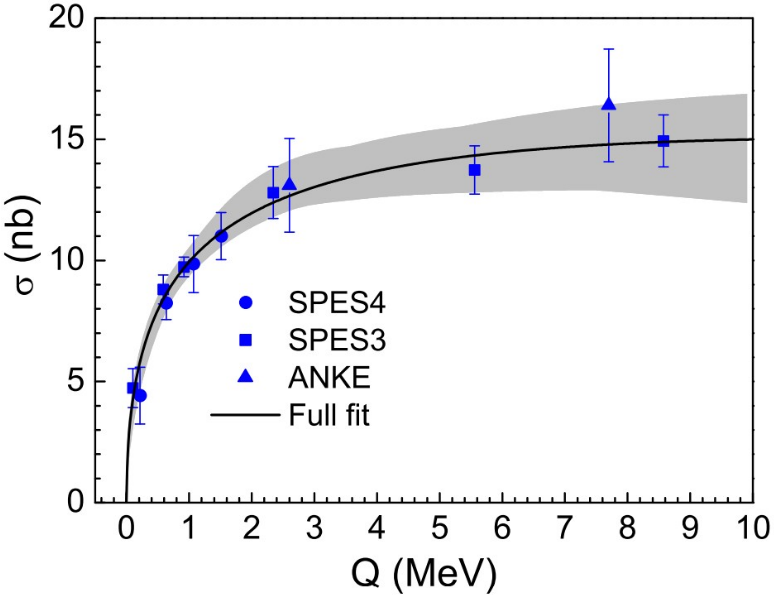
Same formalism as before



$$V_P = A(\vec{\epsilon}_1 \times \vec{\epsilon}_2) \cdot \vec{p}_d$$

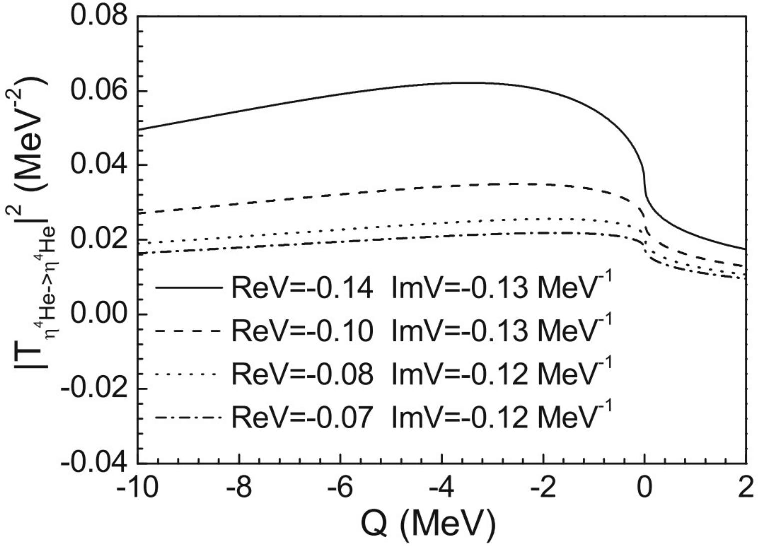
|A| = rA

Parameter	Fitted value
r_A [MeV$^{-5/2}$]	$(7.6 \pm 2.3) \times 10^{-9}$
Re(V) [MeV^{-1}]	$(-12.3 \pm 18.4) \times 10^{-2}$
Im(V) [MeV^{-1}]	$(-13.7 \pm 5.4) \times 10^{-2}$

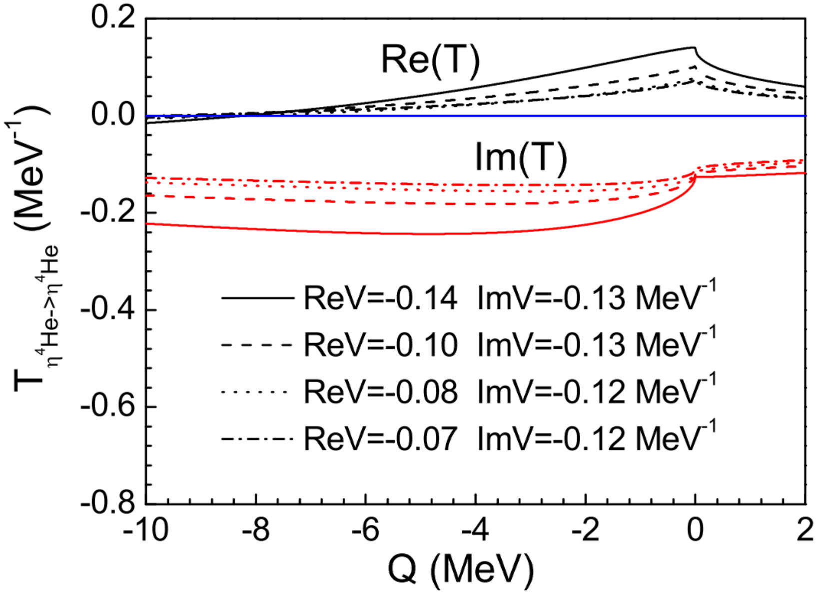


$$a'_{\eta N} = [-(0.39 \pm 0.19) - i(0.23 \pm 0.12)]\,\mathrm{fm}$$

$$a_{\eta^4\mathrm{He}} = [(2.11 \pm 1.07) - i(1.21 \pm 0.67)]\,\mathrm{fm}$$



Once again, structure below threshold but pole above it.

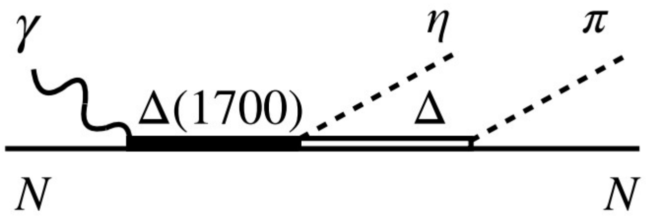


Theoretical study of the $\gamma d \to \pi^0 \eta d$ reaction

A. Martínez Torres[1,3a], K. P. Khemchandani[2,3b], and E. Oset[3c]

2205.00948

Basic element in the $\gamma p \to \pi^0 \eta p$ reaction

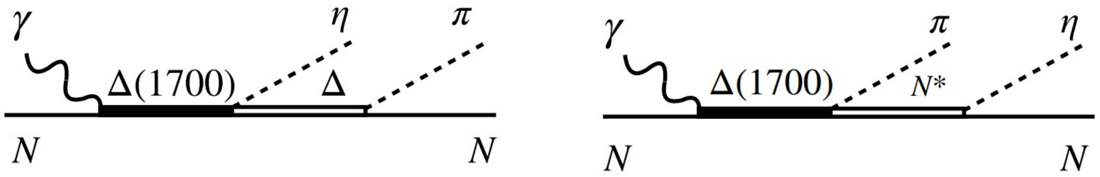


The Δ(1700) is dynamically generated
From the

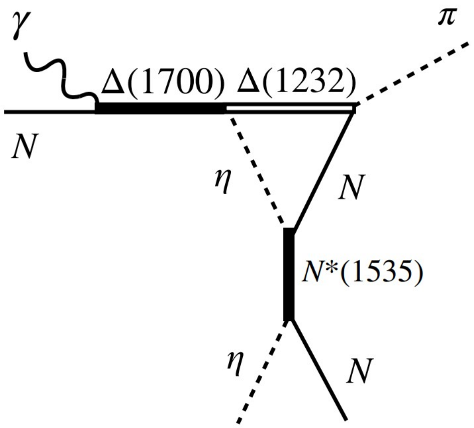
$\Delta\pi$, $\Sigma^* K$ and the $\Delta\eta$ states in coupled channels.

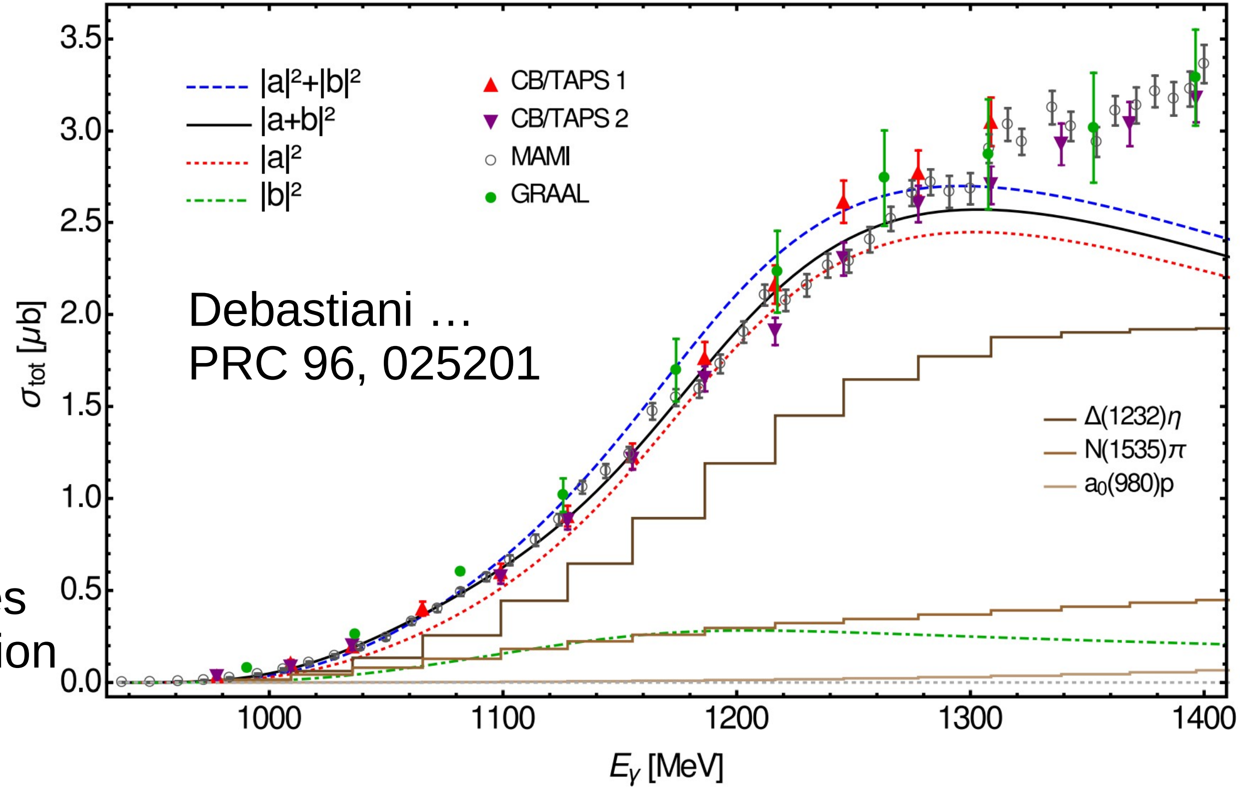
S. Sarkar et al. Nuclear Physics A 750 (2005) 294–323

A. Fix, V. L. Kashevarov, A. Lee, and M. Ostrick, Phys. Rev. C **82**, 035207 (2010)



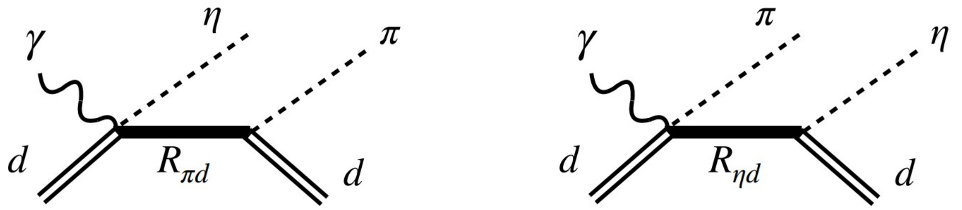
We generate the N*(1535) through a triangle singularity, as





Coherent sum of Δη and N* π does not practically increase cross section due to Schmid theorem.

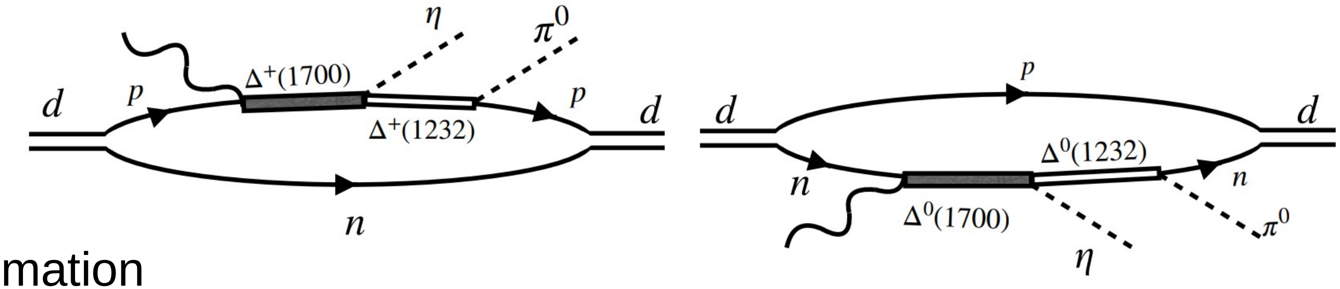
Empirical model for the deuteron reaction suggested in Ishikawa.. Phys.Rev.C 105 (2022) 045201



Our model

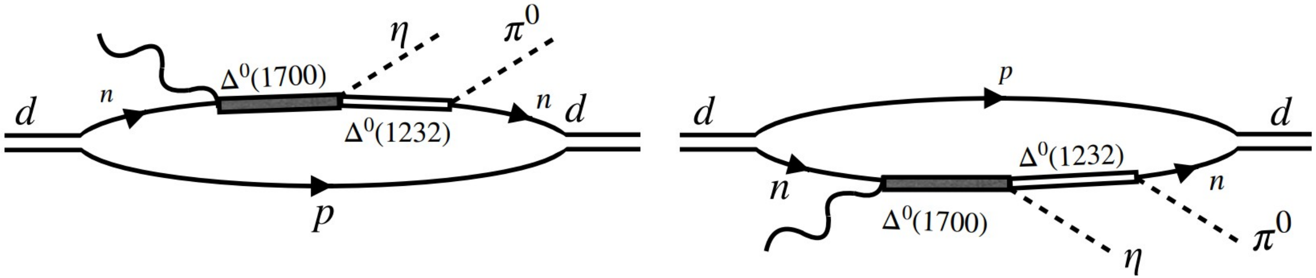
$$|d\rangle = \frac{1}{\sqrt{2}}[|pn\rangle - |np\rangle]$$

$$-it_{\gamma p \Delta^*} = g_{\gamma p \Delta^*} \vec{S}^\dagger \cdot \vec{\epsilon},$$



Impulse approximation

(a) **(b)**



Simplified deuteron
wave function. Improved later.

$$
\begin{aligned}
-it_{4a+4b+4c+4d} = & -2i\sqrt{\frac{2}{3}}\int\frac{d^3q}{(2\pi)^3}\left(\frac{f^*}{m_\pi}\vec{S}\cdot\vec{p}_\pi\right)\left(g_{\gamma p\Delta^*}\vec{S}^\dagger\cdot\vec{\epsilon}\right)\left(g_{\eta\Delta\Delta^*}\right)\left[g_d\,\theta\left(q_{max}-\left|\frac{\vec{p}_d}{2}-\vec{q}\right|\right)\right] \\
& \times\left[g_d\,\theta\left(q_{max}-\left|\frac{\vec{p}_d+\vec{k}-\vec{p}_\eta-\vec{p}_\pi}{2}-\vec{q}\right|\right)\right]\frac{M_N}{E_N(\vec{q})}\frac{M_N}{E_N(\vec{p}_d-\vec{q})}\;\frac{1}{p_d^0-E_N(\vec{q})-E_N(\vec{p}_d-\vec{q})+i\epsilon} \\
& \times\frac{M_{\Delta^*}}{E_{\Delta^*}\left(\vec{p}_d-\vec{q}+\vec{k}\right)}\;\frac{1}{p_d^0-E_N(\vec{q})+k^0-E_{\Delta^*}\left(\vec{p}_d-\vec{q}+\vec{k}\right)+i\epsilon} \\
& \times\frac{M_\Delta}{E_\Delta\left(\vec{p}_d-\vec{q}+\vec{k}-\vec{p}_\eta\right)}\frac{1}{p_d^0-E_N(\vec{q})+k^0-p_\eta^0-E_\Delta\left(\vec{p}_d-\vec{q}+\vec{k}-\vec{p}_\eta\right)+i\epsilon} \\
& \times\frac{M_N}{E_N\left(\vec{p}_d-\vec{q}+\vec{k}-\vec{p}_\eta-\vec{p}_\pi\right)}\frac{1}{p_d^0-E_N(\vec{q})+k^0-p_\eta^0-p_\pi^0-E_N\left(\vec{p}_d-\vec{q}+\vec{k}-\vec{p}_\eta-\vec{p}_\pi\right)+i\epsilon}.
\end{aligned}
$$

We substitute

$$\left[g_d\,\theta\left(q_{max}-\left|\frac{\vec{p}_d}{2}-\vec{q}\right|\right)\right]\frac{M_N}{E_N\left(\vec{q}\right)}\frac{M_N}{E_N\left(\vec{p}_d-\vec{q}\right)}\frac{1}{p_d^0-E_N\left(\vec{q}\right)-E_N\left(\vec{p}_d-\vec{q}\right)+i\epsilon}$$

and

$$\left[g_d\,\theta\left(q_{max}-\left|\frac{\vec{p}_d+\vec{k}-\vec{p}_\eta-\vec{p}_\pi}{2}-\vec{q}\right|\right)\right]\frac{M_N}{E_N\left(\vec{q}\right)}\frac{M_N}{E_N\left(\vec{p}_d-\vec{q}+\vec{k}-\vec{p}_\eta-\vec{p}_\pi\right)}$$

$$\times\frac{1}{p_d^0-E_N\left(\vec{q}\right)+k^0-p_\eta^0-p_\pi^0-E_N\left(\vec{p}_d-\vec{q}+\vec{k}-\vec{p}_\eta-\vec{p}_\pi\right)+i\epsilon}$$

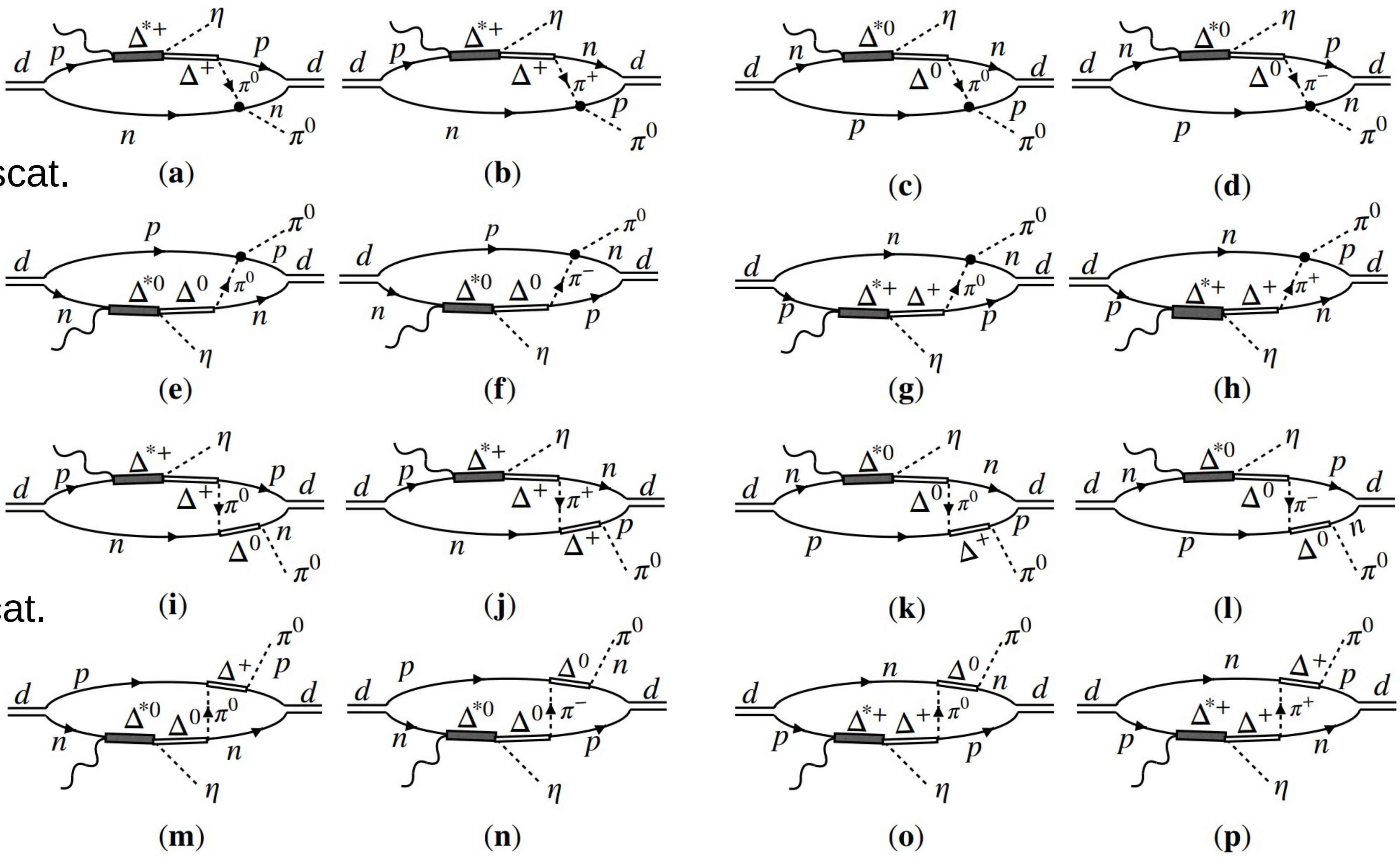
By

$$-\left(2\pi\right)^{3/2}\psi\left(\frac{\vec{p}_d}{2}-\vec{q}\right)\text{ and }-\left(2\pi\right)^{3/2}\psi\left(\frac{\vec{p}_d+\vec{k}-\vec{p}_\eta-\vec{p}_\pi}{2}-\vec{q}\right),\text{ respectively,}$$

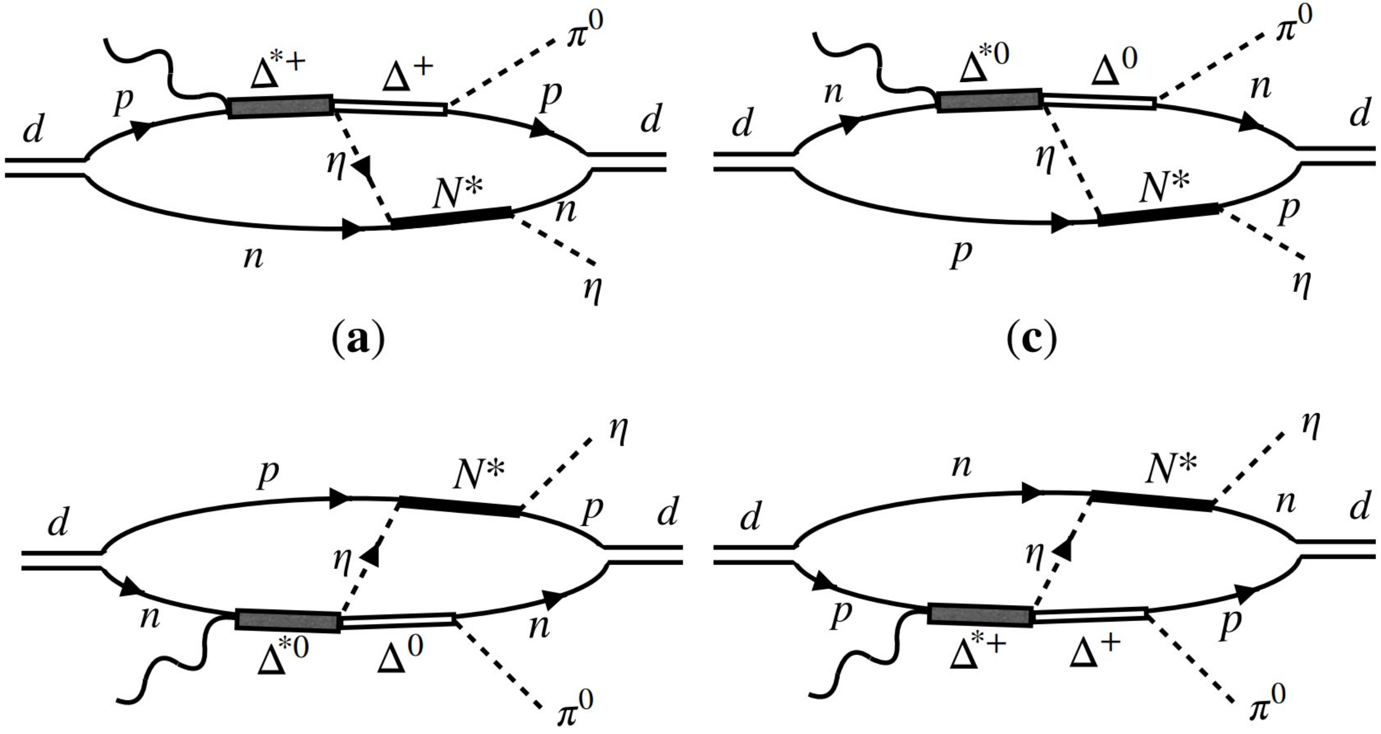
Rescattering diagrams

S-wave rescat.

P-wave rescat.

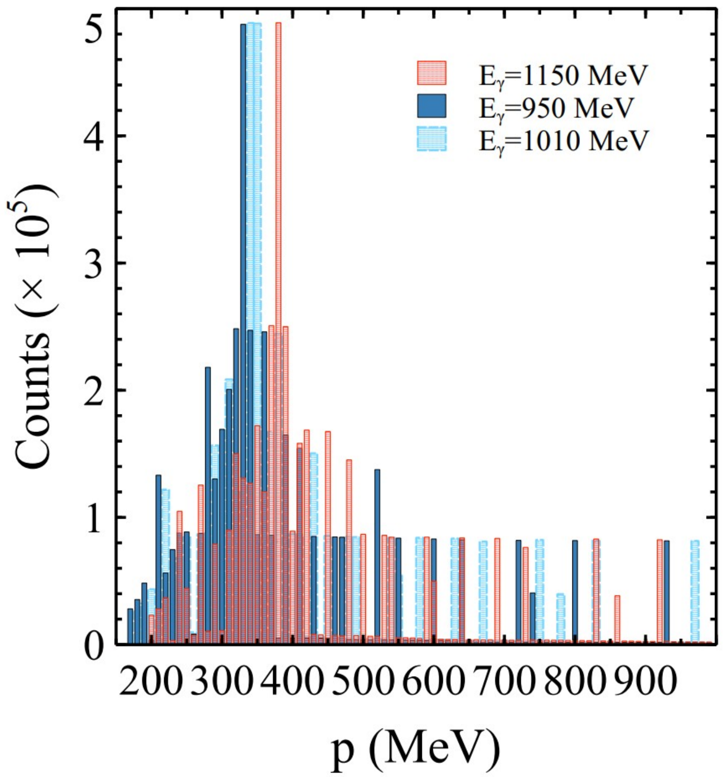


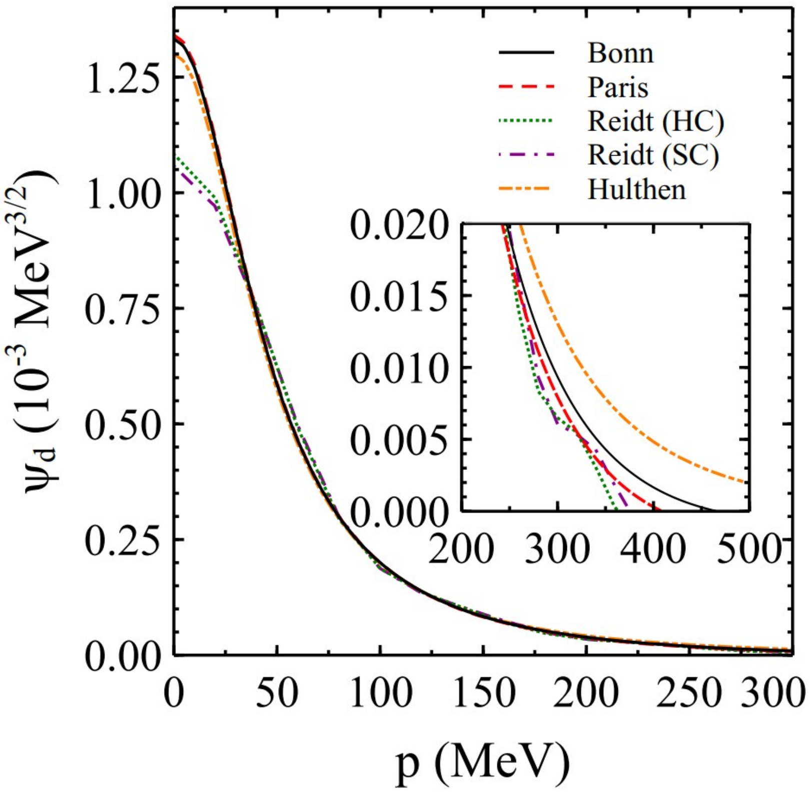
η rescattering



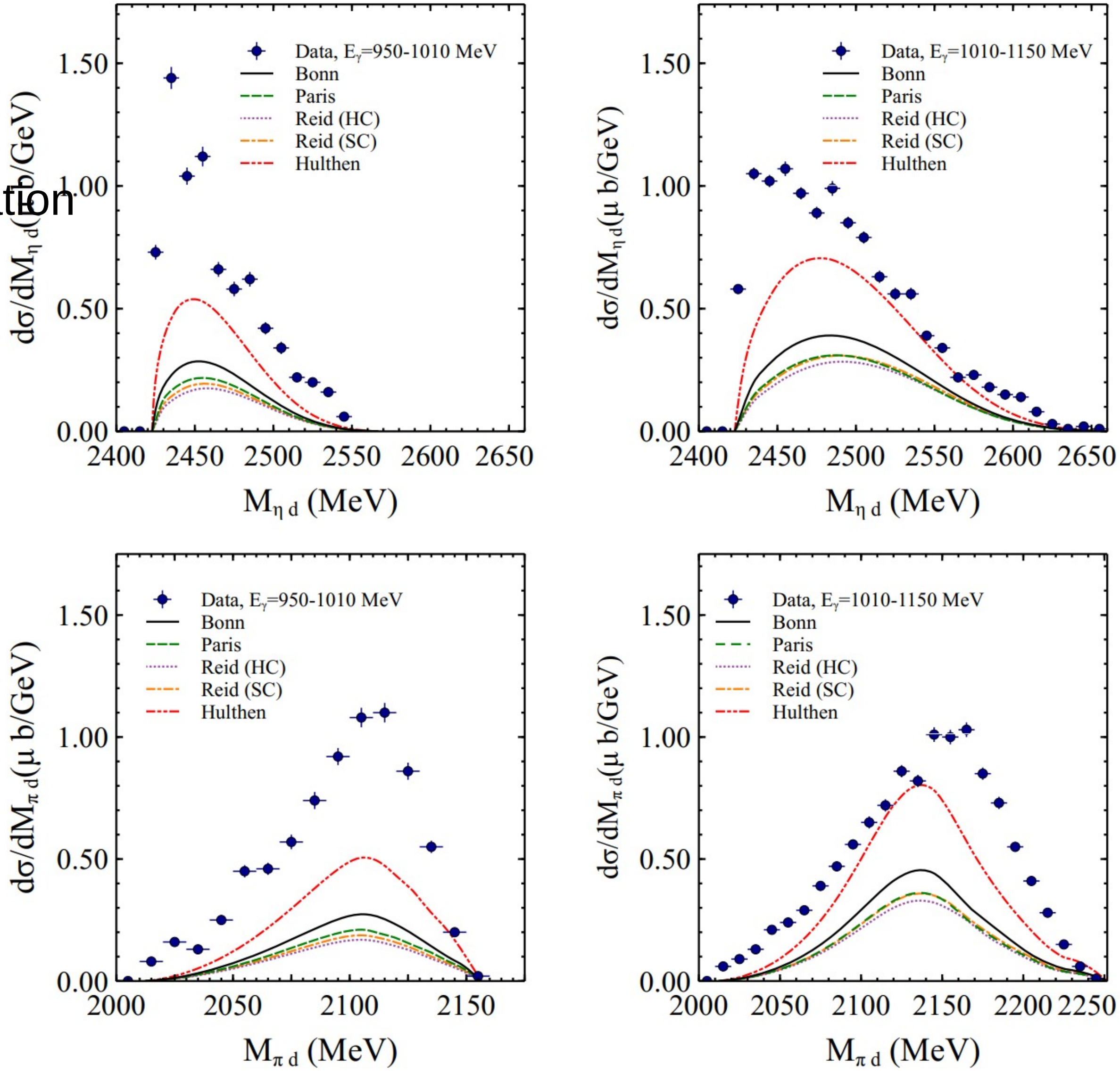
$$\frac{d\sigma}{dM_{\eta d}} = \frac{M_d^2}{8|\vec{k}|s}\frac{1}{(2\pi)^4}|\vec{p}_\pi||\vec{p}_\eta^{R\eta d}|\int dcos\theta_\pi \int d\Omega_\eta^{R\eta d}\overline{\sum_{\mu,\lambda}}\sum_{\mu'}|t^\lambda_{\mu,\mu'}|^2$$

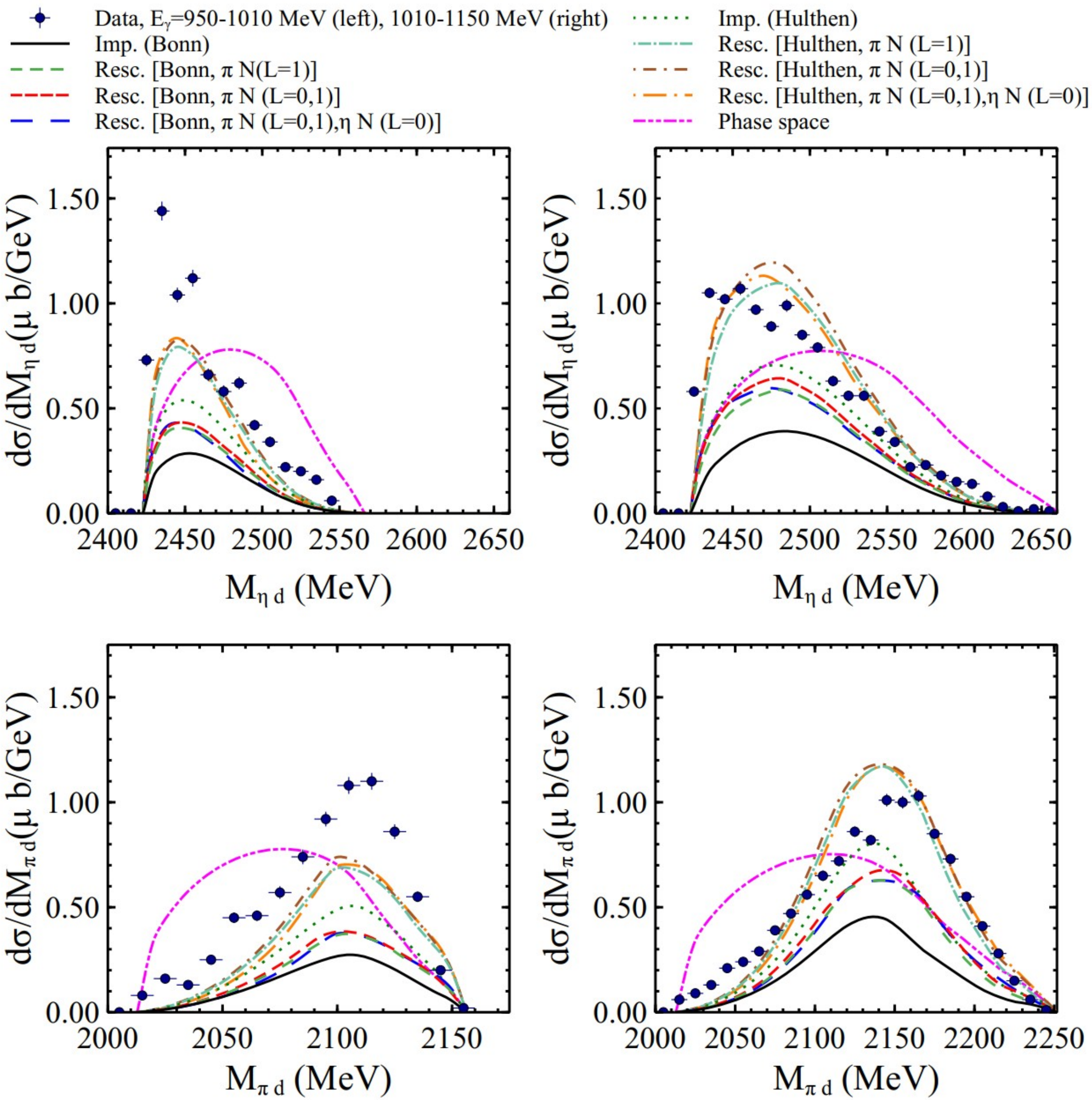
$$\frac{d\sigma}{dM_{\pi^0 d}} = \frac{M_d^2}{8|\vec{k}|s}\frac{1}{(2\pi)^4}|\vec{p}_\eta||\vec{p}_\pi^{R\pi d}|\int dcos\theta_\eta \int d\Omega_\pi^{R\pi d}\overline{\sum_{\mu,\lambda}}\sum_{\mu'}|t^\lambda_{\mu,\mu'}|^2$$





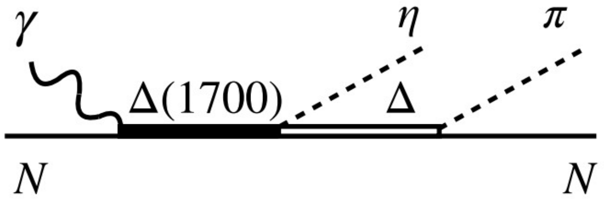
Impulse approximation





Important result

The mechanisms that we have shift the invariant mass in agreement with experiment. There is no need to introduce dibaryons to accomplish it.



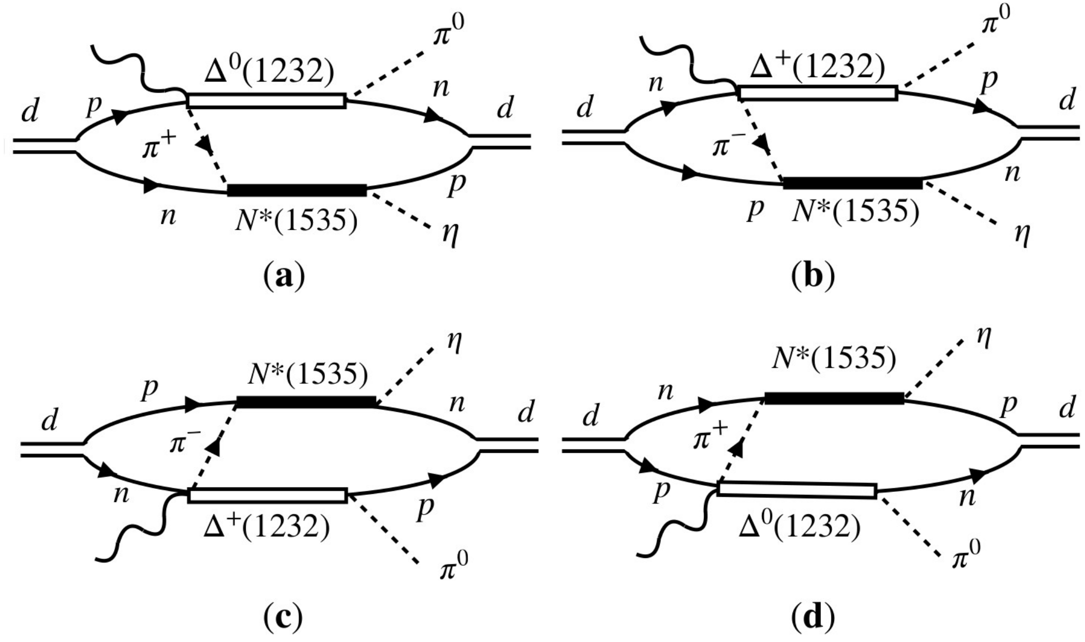
$$-it_{\Delta\to\pi N} = -\frac{f^*}{m_\pi}\vec{S}\cdot\vec{p}_\pi\ T^\lambda,$$

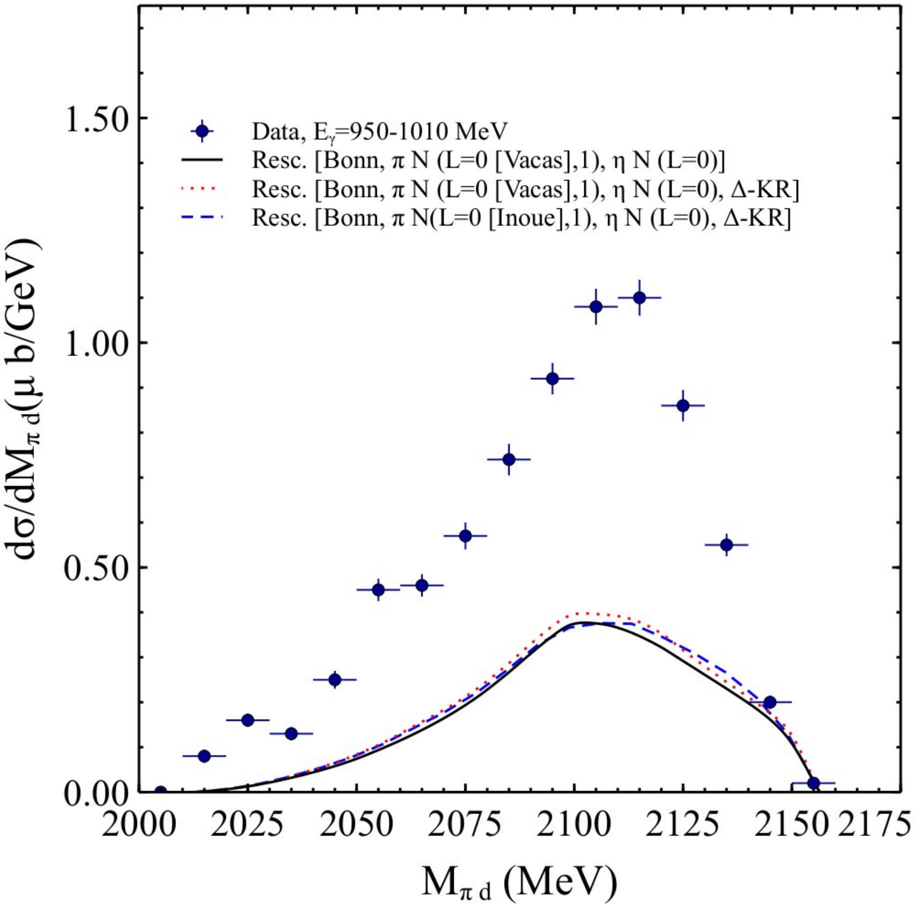
Bigger pion momenta are favored →

π d invariant mass bigger

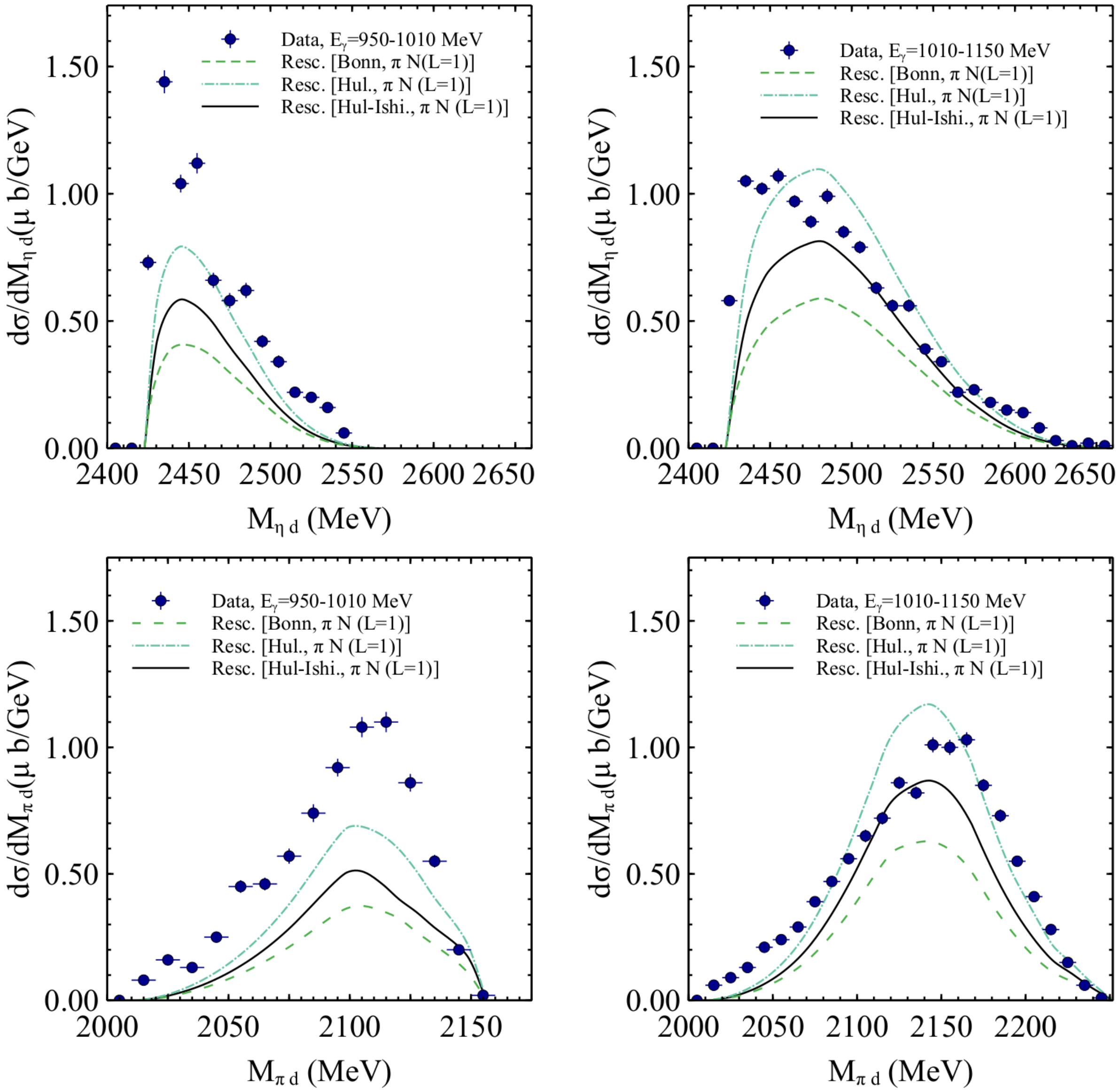
η d invariant mass smaller

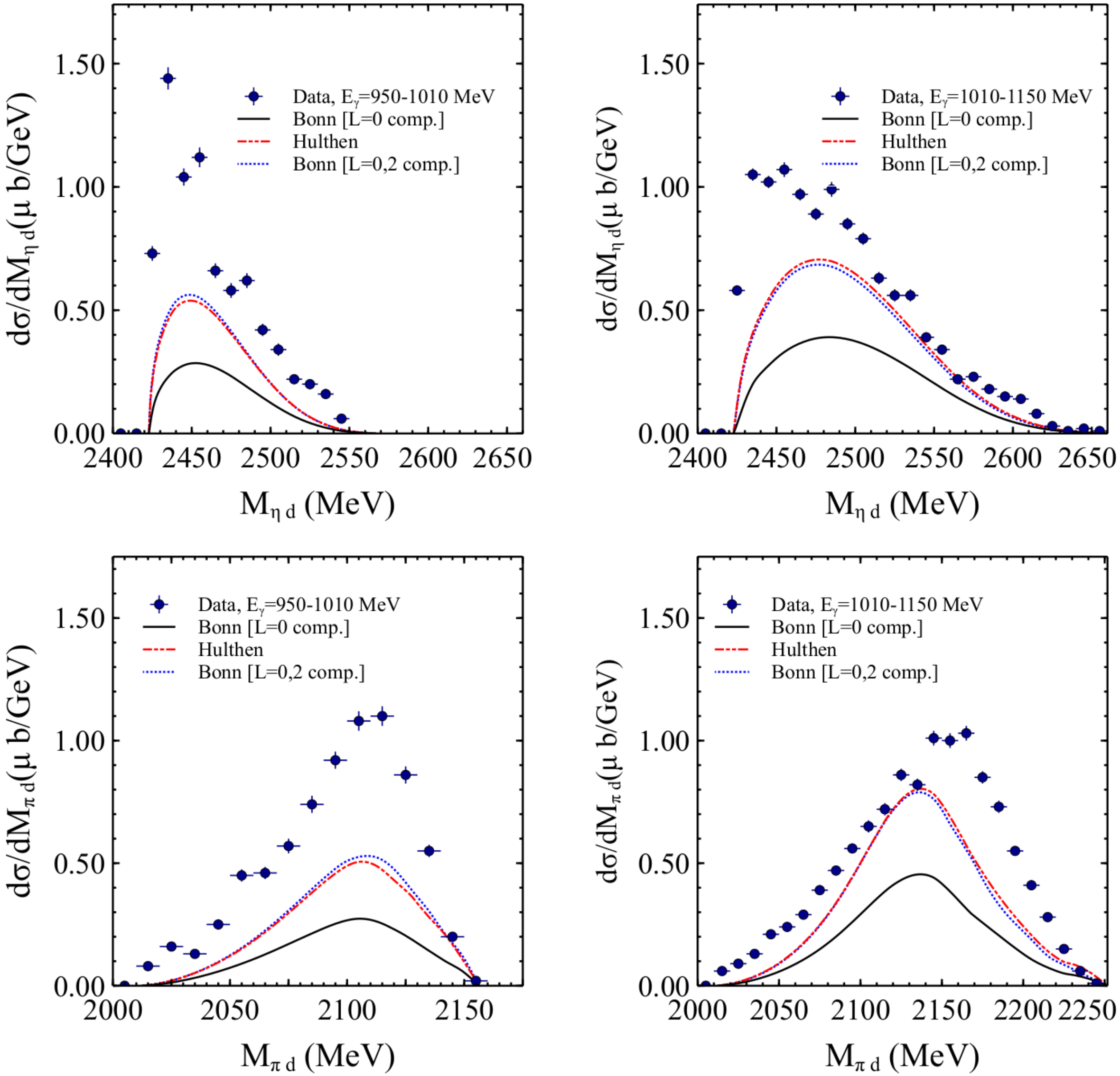
Other rescattering mechanisms: from γ N → ππ N followed by π N → η N rescattering . PRACTICALLY NEGLIGIBLE





Different deuteron wave functions

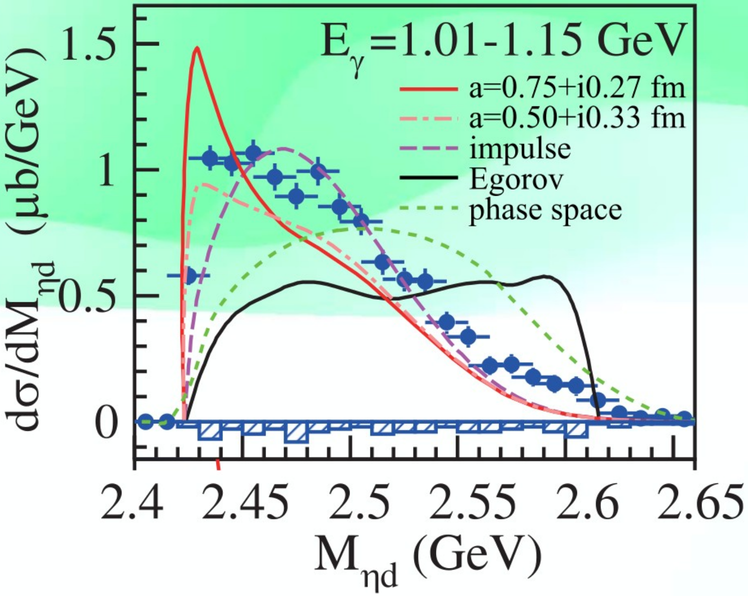


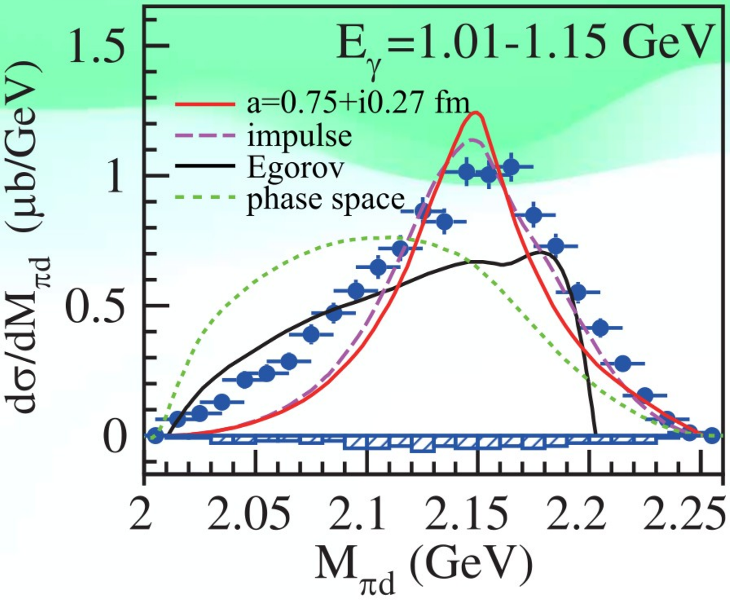


Effect of d-wave in the Deuteron wave function

Results with d-wave practically the same as with simple Hulthen wave function, three slides before

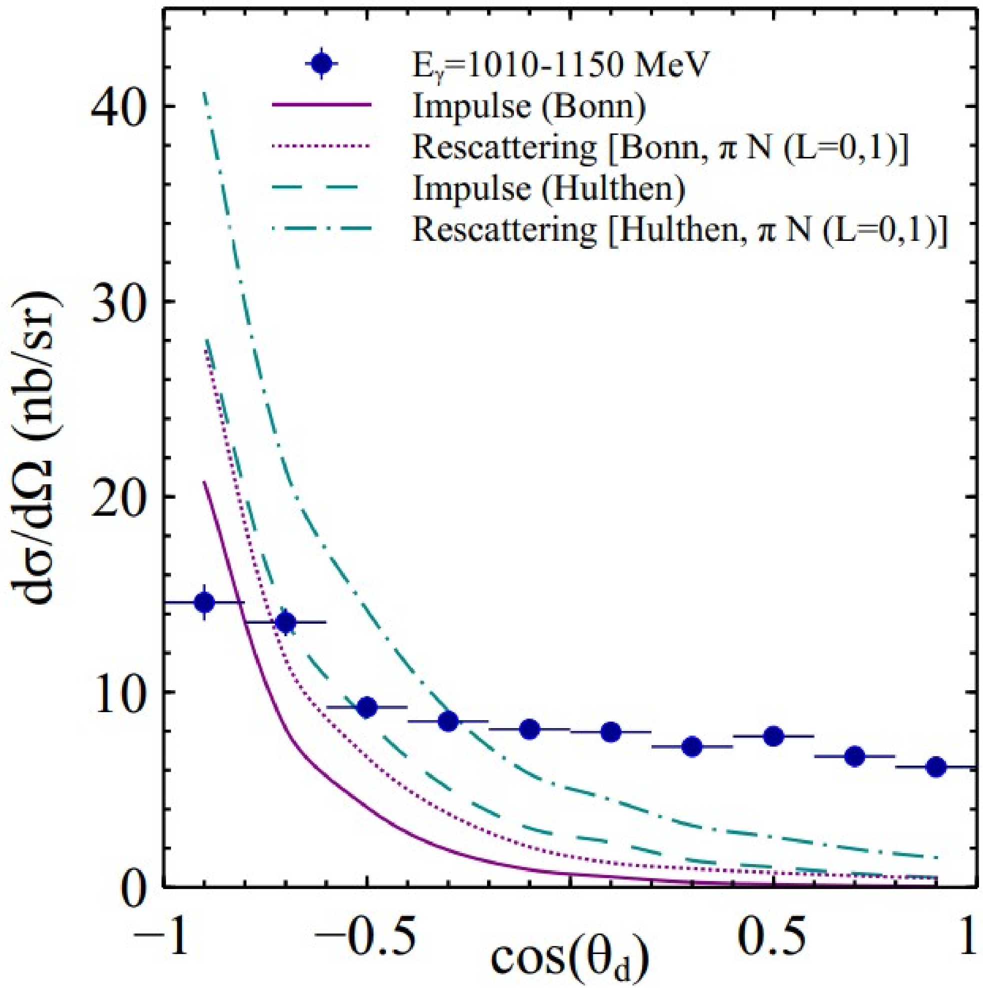
- Study of the $\gamma d \rightarrow \pi^0 \eta d$ process[1]: clear shift with respect phase space.

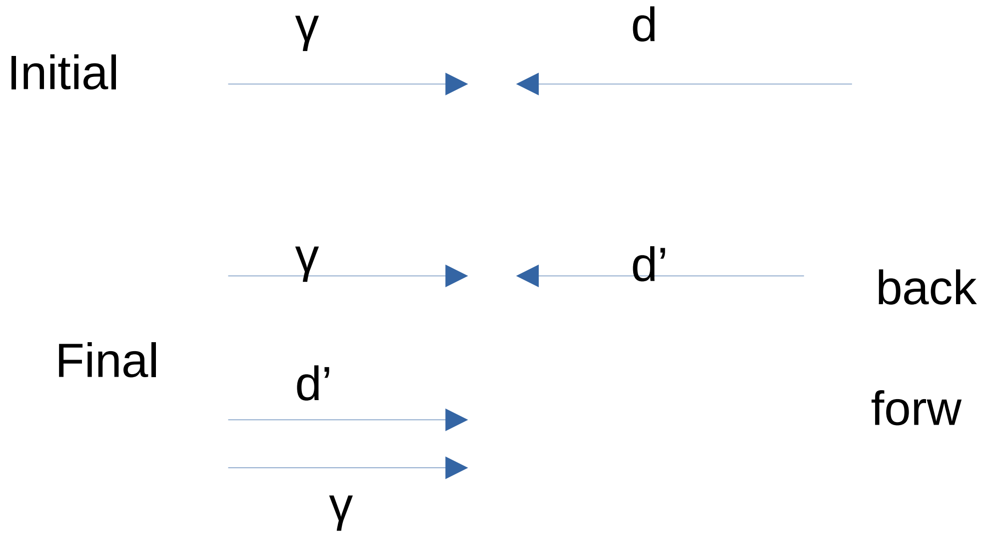




M. Egorov, Phys. Rev. C101, 065205 (2020);

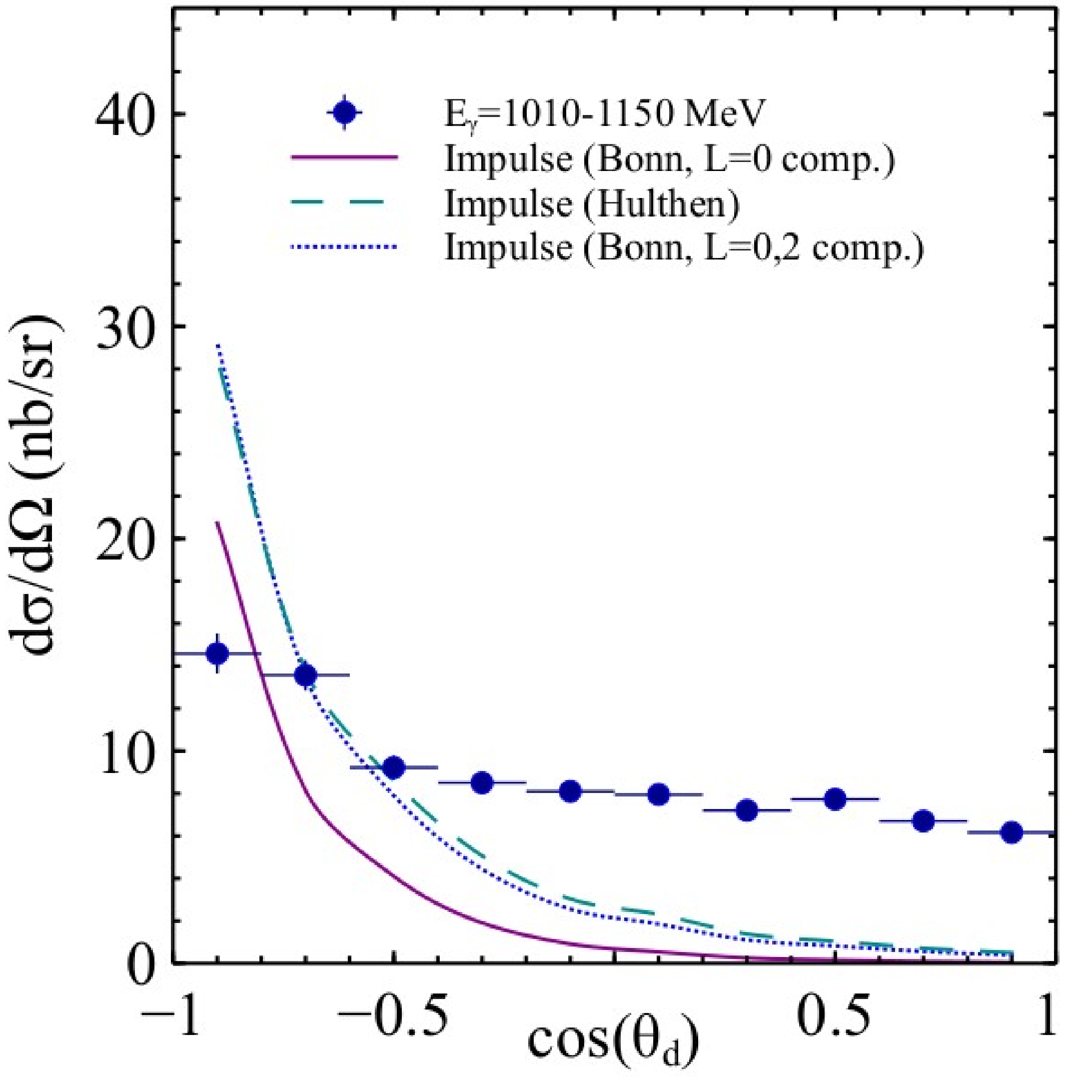
M. Egorov and Fix, Phys. Rev. C88, 054611 (2013).





Forward, the momentum transfer is very large
The deuteron wave functions kill $d\sigma/d\Omega$

Effect of d-wave in angular distribution



Conclusions

The data from p d → ^{3}He η and d d → ^{4}He η close to threshold indicate that there is a structure in the η ^{3}He and η ^{4}He close to threshold indicating the proximity of a bound state, but the poles are above threshold.

We induce from there that η d is even less bound

We studied the γ d → ηπ^0 d reaction and found a reasonable description of the mass distributions. No need of an eta d bound state to reproduce the shifts of the π d and η d mass distributions.

Angular distribution not explained. NO MODEL DOES. ?????