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Universality in Hypernuclei

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HFHF

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DFG Deutsche Forschungsgemeinschaft

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Outline

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- Universality
- Threshold bound states, the unitary limit, and Efimov physics
- Applications to the Hypertriton
- Summary and Outlook

References:

HWH, Nucl. Phys. A **705** (2002) 173

Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002, ibid. **102** (2020) 039901(E)

Universality in Physics

Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

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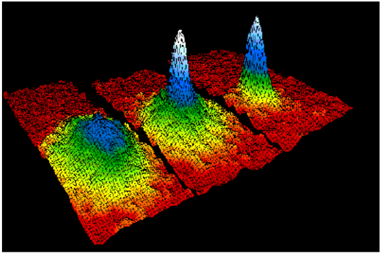
- Ultracold atoms: interaction at sufficiently low energies described by scattering length *a*
- Properties of dilute homogeneous BEC: $\rho a^3 \ll 1$



(Source: http://jilawww.colorado.edu/bec/)

$$\frac{E}{N} = \frac{2\pi\hbar^2}{m} a\rho \left(1 + \mathcal{O}((\rho a^3)^{1/2})\right)$$

$$\rho = \rho_0 \left(1 + \mathcal{O}((\rho a^3)^{1/2})\right)$$

Tail wagging the dog

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- Universality: low-energy physics controlled by tail of wave function

- Tail wagging the dog

  (idiomatic) A minor or secondary part of something controlling the whole.
  (cf. http://en.wiktionary.org/wiki/tail_wagging_the_dog)

Physics Near the Unitary Limit

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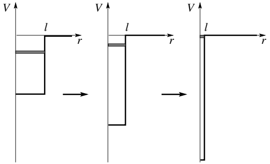
- Consider short-ranged, resonant *S*-wave interactions
- Unitary limit: $a \to \infty$, $\ell \sim r_e \to 0$

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \sim i/k$$



  - Scattering amplitude scale invariant, saturates unitarity bound

Physics Near the Unitary Limit

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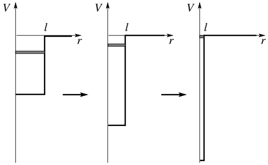

  - Scattering amplitude scale invariant, saturates unitarity bound
- Use as starting point for description of few-body physics
  - Natural expansion parameter: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots \quad \Rightarrow \quad \ell/|a|, k\ell, \ldots$
  - Universal dimer with energy $E_d = -1/(ma^2) \quad (a > 0)$
  - Reproduce tail of the wave function: $\psi(r) \propto \dfrac{e^{-r/a}}{r}$
  - Corrections in higher orders

Broken Scale Invariance

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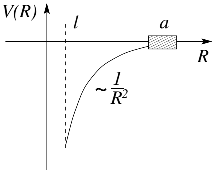
- Three-boson system near the unitary limit (Efimov, 1970)
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2 \kappa^2}{m}}_{E} f(R)$$



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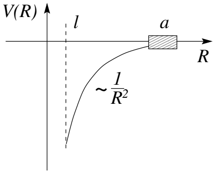


- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

$\Longrightarrow$ "3-body force"

$\Longrightarrow$ scale invariance is anomalous

$\Longrightarrow$ observables depend on boundary condition and a

- Universality concept must be extended for such systems

Three-Body Force

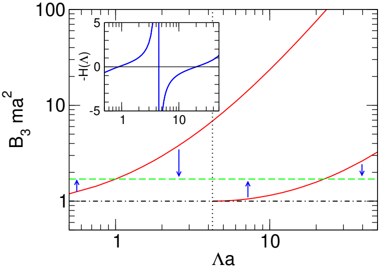
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda)$ $(\Lambda \sim 1/R)$

- $H(\Lambda)$ periodic: limit cycle

  $$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

  (cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \quad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Three-body parameter: $\Lambda_*, \ldots$
- Limit cycle $\Longleftrightarrow$ Discrete scale invariance $\Longleftrightarrow$ Efimov physics

Limit Cycle: Efimov Physics

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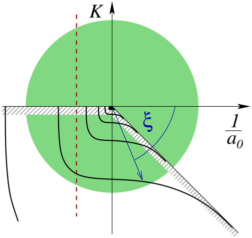
- Universal spectrum of three-body states (Efimov, 1970)



- Window of universality
- Discrete scale invariance for fixed angle ξ
- Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a \to 0} e^{2\pi/s_0} = 515.035...$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances
- Nuclei $\Longrightarrow$ universal correlations and scaling relations
  - Applications: ^{3}H, ^{3}He, ^{4}He, halo nuclei

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- Extension of nuclear chart to third dimension: strangeness

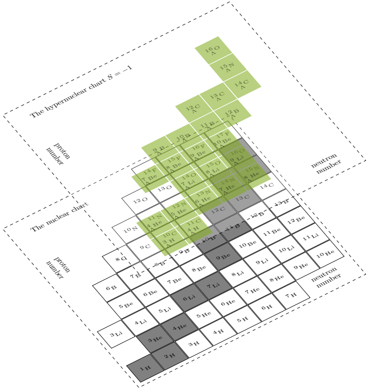

Figure: F. Hildenbrand

Λd-System and the Hypertriton

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- Hypertriton
  - $np\Lambda$ bound state with $J^P = \frac{1}{2}^+, I = 0$
  - Λd separation energy: $B^\Lambda = (0.13 \pm 0.05)$ MeV
  - total binding energy: $B_3^\Lambda = 2.35$ MeV
- EFT for large scattering lengths

  $\Longrightarrow$ shallow hypertriton follows naturally
- Leading order EFT $\Rightarrow$ S-wave interactions
  - ${}^3S_1(NN) + \Lambda \longrightarrow a_d \sim 1/\gamma_d$
  - ${}^3S_1(\Lambda N) + N \longrightarrow a_3 \sim 1/\gamma_3$
  - ${}^1S_0(\Lambda N) + N \longrightarrow a_1 \sim 1/\gamma_1$
- Scattering lengths large compared to interaction range

  ($NN \rightarrow \pi$-exchange, $\Lambda N \rightarrow 2\pi$-exchange)

Low-Energy ΛN-System

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- ΛN system unbound
- (Old) effective range analyses inconclusive (few data at relatively high energies)

$0 > a_1 > -15$ fm $\qquad\qquad 0 < r_1 < 15$ fm

-0.6 fm $> a_3 > -3.2$ fm $\qquad\qquad 2.5$ fm $< r_3 < 15$ fm

- Extractions using hyperon-nucleon potentials

$a_1 \approx -2.9$ fm, $\qquad a_3 \approx (-1.5 \ldots -1.7)$ fm, $\qquad \gg R \sim 1/(2m_\pi)$

(NLO chiral EFT: Haidenbauer et al., Nucl. Phys. A **915** (2013) 24)

- Characteristic three-body momentum

$$\gamma_3^\Lambda \sim 2\sqrt{|MB_3^\Lambda - \gamma_d^2|/3} \approx 14\text{ MeV} \ll \sqrt{m_\Lambda(m_\Sigma - m_\Lambda)} \approx 300\text{ MeV}$$

$\Rightarrow$ $\Lambda\Sigma$ conversion is short range $\quad\Longrightarrow\quad$ captured in ΛNN three-body force

Integral Equations

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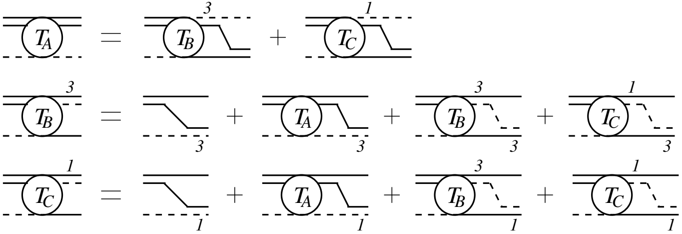
- Integral equations for hypertriton ($I = 0$)



HWH, Nucl. Phys. A **705** (2002) 173; Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Strong cutoff dependence

  $\Longrightarrow$ renormalize with Λnp three-body force

- Side remark: similar behavior for $I = 1$ $\Rightarrow$ Λnn not excluded “a priori” in pionless EFT

Renormalization

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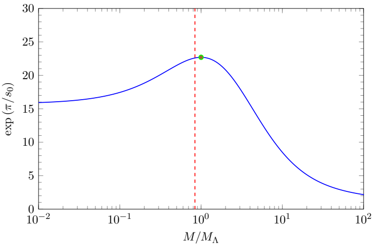
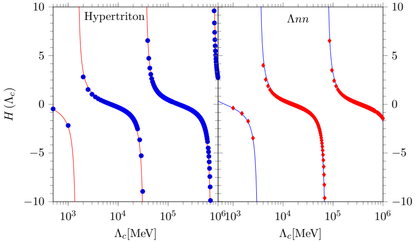
- Scaling factor and three-body force





- $M/M_\Lambda \approx 0.84 \quad \Rightarrow \quad$ limit cycle with $s_0 = 1.0076$
- Scaling factor: $\exp(\pi/s_0) \approx 22.60$
- Three-body parameter: $B_3^\Lambda = 2.22 + 0.13$ MeV $\quad \Rightarrow \quad \Lambda_*^{I=0} = 6.372$ MeV
- No room for excited states....

Λd Scattering

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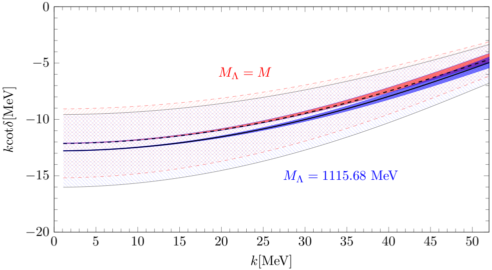

Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002, ibid. **102** (2020) 039901(E)

- Exact value of γ_i not determined by B_3^Λ
- Phase shifts independent of γ_i $\Longleftarrow$ shallowness of hypertriton
- Low-energy parameters:

$$a_{\Lambda d} = 15.4 \text{ fm} \quad \text{and} \quad r_{\Lambda d} = 1.3 \text{ fm}$$

Λnp **Phillips Line**

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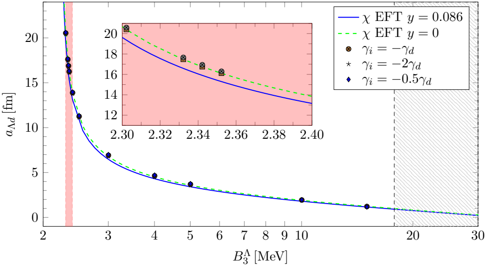
- Correlation between hypertriton triton binding energy and $S = 1/2\ \Lambda d$ scattering length (cf. Phillips ’68)



Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002, ibid. **102** (2020) 039901(E)

- Sensitivity to specific values of γ_i only for deeper binding
- Hypertriton wave function can also be extracted $\Rightarrow$ matter radii

Hypertriton Wave Function

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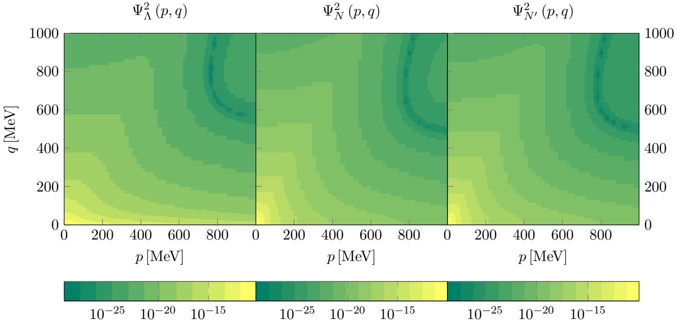
- Hypertriton wave function for different spectator particles



Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Next step: calculate matter radii

Hypertriton Radii

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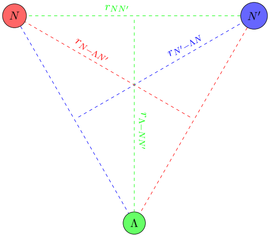


- Hypertriton wave function can also be extracted
- Matter radii for the hypertriton ($B_3^\Lambda = 2.35$ MeV)

$\sqrt{\langle r^2_{\Lambda-NN'}\rangle}$[fm]	$\sqrt{\langle r^2_{N'-\Lambda N}\rangle}$[fm]	$\sqrt{\langle r^2_{N-N'\Lambda}\rangle}$[fm]	$\sqrt{\langle r^2_{NN'}\rangle}$[fm]
10.79	3.96	4.02	2.96
$+3.04/-1.53$ $+0.03/-0.02$	$+0.40/-0.25$ $+0.03/-0.03$	$+0.41/-0.25$ $+0.03/-0.03$	$+0.06/-0.05$ $+0.03/-0.04$

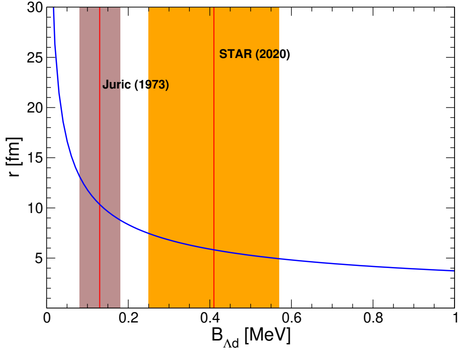
Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Two-body Λd EFT: $\sqrt{\langle r^2_{\Lambda-NN'}\rangle} = 10.3$ fm $\implies$ works very well!

Λd **Universality**

- Low-energy aspects of hypertriton can be described in EFT with Λd degrees of freedom

$\Longrightarrow$ simple correlation between size and B_Λ



Summary

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- Universality in unitary limit
- Discrete Scale Invariance $\Leftrightarrow$ Efimov physics
  - Effective field theory for hypertriton
  - ....
- Three-body calculation of hypertriton
  - Hypertriton can be considered Efimov state
  - Little sensitivity to exact values of ΛN scattering lengths
  - $\Lambda\Sigma$ conversion $\Longrightarrow$ ΛNN three-body force
  - Matter radius well described in EFT with Λd dof
- Low-energy aspects of Hypertriton can be described in EFT with Λd degrees of freedom
  - Hypertriton lifetime (Hildenbrand, HWH, Phys. Rev. C **102** (2020) 064002)

    $\Longrightarrow$ talk by F. Hildenbrand on Friday