# On $\eta$ and $\eta'$ as QCD bound states

## and their relation to confinement, chiral symmetry breaking and $U_A(1)$ anomaly

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# Motivation: Why studying hadrons?

- Understanding hadrons from QCD
  - is a fascinating challenge by itself!
  - will solve long-standing puzzles as, *e.g.*, confinement!
  - will provide deepened insight into fundamental physics!
  - . . .
  - is needed for
    astrophysics, cosmology, high-energy particle physics, etc..
- Precision experiments to detect BSM physics rely:
  - on precise theoretical predictions within the SM,
  - and in turn on determination of SM parameters,
  - which implies, directly and indirectly, the need for
    precise calculations of the effects of hadron structure
    (*i.e.*, minimise “hadronic uncertainties”).

Estimating the reliability of a non-perturbative calculation (and thus provide an estimate of the systematic error) can only be achieved based on an understanding of the investigated physics.

# $\eta$ and $\eta'$ mesons

**Thanks to Steven and Stefan for their excellent talks!**

…will focus here on some complementary aspects …

# Motivation: Why studying $\eta$ and $\eta'$ mesons?

The $\eta$ and $\eta'$ mesons are special:

- "Relatives" to the Goldstone bosons of Dynamical Chiral Symmetry Breaking (D$\chi$SB), *i.e.*, pions and kaons
- Properties, in particular the masses, influenced by a specific property of the gluon vacuum via the axial anomaly / $U_A(1)$ anomaly.

Two competing effects with different flavour structure:
[Shore, Witten, Veneziano, . . . ]

- Mixing of $\eta$ and $\eta'$ mesons
  s.t. avoided crossing occurs
  as function of interaction strength related to $U_A(1)$ anomaly

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With isospin breaking taken into account: $\eta$ and $\eta'$ mix with $\pi^0$.

# Motivation: Why studying $\eta$ and $\eta'$ mesons?

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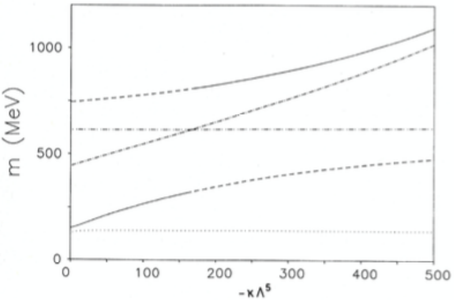


Fig. 1. The mass of the $\eta'$-meson (full line), the $\eta$-meson (dashed line) and the $\pi^0$-meson (dotted line) as functions of the coupling strength $\kappa$ of the 't Hooft determinant. Shown is also the $\eta_0$-"mass" (dash-dotted line) and the $\eta_8$-"mass" (dash-double-dotted line). [The "mass" of the state $(u\bar{u}-d\bar{d})/\sqrt{2}$ is not shown because on this scale it is not distinguishable from the $\pi^0$-mass except for very small $\kappa$.]

H. Reinhardt & RA, PLB **207** (1988) 482

# Motivation: Why studying $\eta$ and $\eta'$ mesons?

- The specific realisation of D$\chi$SB and the U$_A$(1) anomaly in QCD, *i.e.*, for the given quark current masses / Higgs-Yukawa couplings, is related to certain aspects of <u>confinement</u>.

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Colour confinement vs. "separation-of-charge" confinement [J. Greensite et al.]:

- Colour confinement is a consequence of gauge invariance, (more precisely: unbroken BRST(-like) charge) also present in Higgs phase and thus electroweak theory (*cf.* FMS mechanism)
- Dynamical "separation-of-charge" confinement refers to a strong counter-action if one tries to separate colour charges. (Hadrons locally colour-less!)

# Basic Facts about $U_A(1)$ anomaly

- $U_A(1)$ always anomalous in Poincaré-invariant vector-like gauge theories with vanishing vacuum angle ($\Theta = 0$)
  (C. Vafa & E. Witten, NPB234 (1984) 173; PRL 53 (1984) 535)
- But: Strength is determined by specific dynamics which is based on gauge group and matter content.

Topological approach:

Randomly distributed winding number spots with top. susc. $\neq 0$

$\stackrel{\textit{Index theorem}}{\Longrightarrow}$ percolating quark zero modes $\Longrightarrow$ $U_A(1)$ anomalous.

Functional approach:

For momentum-space Green functions encoded in **infrared behaviour** because related to boundary conditions in (Euclidean) space-time.

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# Basic Facts about $U_A(1)$ anomaly

“Zoo” of topological gauge-field configurations:

- Instantons with integer winding number Q, localised in spacetime
- Kran–van-Baal Calarons with integer Q, several centers
- Chromomagnetic Monopoles with integer Q, (world)lines in spacetime
- Writhing points of Center Vortices with non-integer Q (1/4), correlated along lines and surfaces
- ...

If any of these are present in sufficient density: D$\chi$SB.

Non-vanishing autocorrelation of winding number spots, called topological susceptibility $\chi$, provides contribution to $U_A(1)$ anomaly.

From $\eta$ and $\eta'$ mass: $\chi \approx (180 \text{ MeV})^4$.

# Basic Facts about $\mathrm{U}_A(1)$ anomaly

No gluonic long-range correlation for instantons

$\Longrightarrow$ Instantons are not confining!

(The attached quark zero modes percolate $\Longrightarrow \chi \neq 0$.)

Center vortices: Indications that they are confining field configurations.

Thus: Relation of $\mathrm{U}_A(1)$ anomaly to confinement unclear in topological approach.

# Basic Facts about $U_A(1)$ anomaly

How to describe $U_A(1)$ anomaly in functional approaches as, *e.g.* Dyson-Schwinger–Bethe-Salpeter equations or FRG with dynamical hadronisation?

Adding topological field configurations on top of functional equations is double counting!

*Cf.* Bloch waves, $QED_2$ (Schwinger model), etc..

# QCD correlation functions

In complete, translationally-invariant and local gauge:

Gluon propagator is positivity violating!

Coloured gluons are confined not confining!

In Landau gauge:
Quark-gluon vertex, including its D$\chi$SB-induced parts,
is decisive ingredient to make **4-quark function infrared divergent.**
(Can be shown for so-called scaling solution in 3PI approach at 3-loop level.)

**"Separation-of-charge" confinement and D$\chi$SB**

# Functional approaches: A note on current status

From a first-principle Dyson-Schwinger–Bethe-Salpeter calculation for SU(3) Yang-Mills theory in 3PI approach at 3-loop level:

Glueball masses and amplitudes (“wave functions”) determined.

[M.Q. Huber, C.S. Fischer, H. Sanchis-Alepuz, 2020 - 2022]

- Very good agreement with lattice calculations.
- Prediction of higher-spin glueballs.[1]

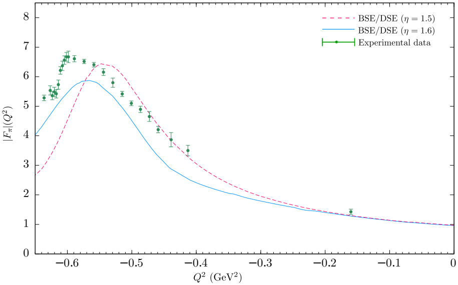
NB: Lowest-lying pseudoscalar glueball, $m \approx$2.6 GeV, likely to massive to play a significant role for the $\eta$ and $\eta'$.

[1] In a Poincaré-invariant continuum approach spin is a good quantum number whereas lattice has hypercubic symmetry.

# Functional approaches: A note on current status

Dyson-Schwinger–Bethe-Salpeter calculation with some assumptions and some modelling but genuine QCD d.o.f. up to a level where intermediate resonances and two-pion states are consistently back-coupled:

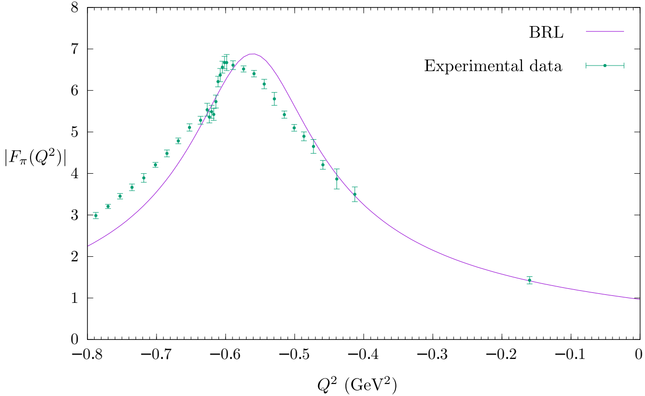
Pion time-like form factor [A. Miramontes, H. Sanchis-Alepuz, RA, 2021]




Absolute value of the pion form factor in the time-like $Q^2 < 0$ domain for $\eta = 1.5$ and $\eta = 1.6$, only leading $\pi$ BS amplitude in 2-$\pi$-exchange kernels.

# Pion time-like form factor

Absolute value of the pion form factor, $Q^2 < 0$, $\eta = 1.5$ with all BS amplitudes in 2-$\pi$-exchange kernels taken into account (requires 10 $\times$ CPU time) [A. Miramontes, unpublished]




extracted pole position:
$M_\rho = 755$ MeV, $\Gamma_\rho = 123$ MeV (before: 100 MeV, expt.: 149 MeV).

# $U_A(1)$ anomaly and $\eta'$ mass from IR divergent correlation functions

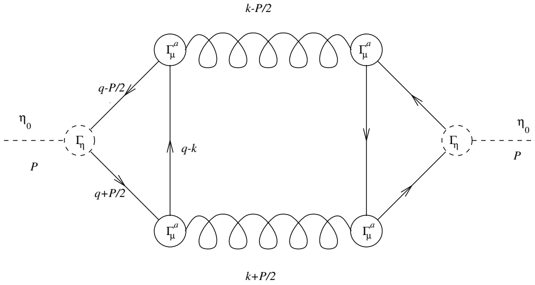
R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

$U_A(1)$ symmetry anomalous $\Rightarrow \eta$ and $\eta'$ masses $\gg \pi$ mass

**Where is this encoded in the Green functions?**

(J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.)

E.g. in:




$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

# $\eta'$ mass from IR divergent 4q correlation functions

However: Infinitely many diagrams (*n*-gluon exchange) contribute!

Nevertheless:
Calculate contribution from **diamond diagram only** employing DSE results for the gluon and quark propagators and quark-gluon vertex (provides correct pseudoscalar and vector meson masses):

$$\boxed{\chi^2 \approx (160\mathrm{MeV})^4 \text{ vs. phenomenological value } (180\mathrm{MeV})^4,}$$

together with BS amplitudes for $\eta$ and $\eta'$ analogous to pions and kaons

$$\boxed{\text{results in: } m_\eta = 479\text{MeV},\ m_{\eta'} = 906\text{MeV},\ \theta = -23^0.}$$

Conclusion:
(Fluct.) topologically non-trivial fields $\Leftrightarrow$ IR singular correlations!
Mutually consistent ways to generate Witten-Venezanio mechanism

Quark confinement $\Rightarrow$ D$\chi$SB **and** U$_A$(1)anomaly!

# Conclusions & Outlook

- ☺ Two ways to determine topological susceptibility in QCD, and thus to calculate the $\eta$ and $\eta'$ masses and their mixing:
  - Determine the autocorrelation of fluctuations around topologically non-trivial gluon field configurations.
  - Employ a functional approach with infrared divergent four quark correlation function.
- ☺ Both ways elucidate the relation to D$\chi$SB.
- ☺ Functional approach indicates relation to confinement.

Further studies:

- Anomalous $\gamma$-3$\pi$ form factor for space-like and time-like $s, t, u$ (on-going; A. Miramontes, G. Eichmann, C.S. Fischer, RA)
- Redo computation of diamond diagram with recent results for quark-gluon vertex and integrate into “unquenching the glueballs”, *i.e.*, a calculation of mesons and glueballs including their mixing.

# The up/down quark mass ratio from the RG or "Why has the down quark twice the mass of the up quark?"

- Up & down Higgs-Yukawa couplings / masses "run" equally, *i.e.*, depend equally on RG scale $k$ under strong and weak interactions but:
- $e_u = \frac{2}{3}e$ & $e_d = -\frac{1}{3}e$ implies **different flow under QED**.
- Encoded in anomalous mass dimensions

$$\frac{\partial \log m_q}{\partial \log k} = \gamma^m_{QCD} + \gamma^m_W + \gamma^m_{QED} + \text{ mixed terms}$$

$$\text{with } \gamma^m_{QED} = -\frac{3\mathbf{e_q^2}}{8\pi^2}\left(1 - \frac{m_q^2}{k^2}\log(1+\frac{k^2}{m_q^2}\right).$$

- $\frac{\partial \log(m_u/m_d)}{\partial \log k} = 0_{\text{ from QCD}} + 0_{\text{ from W}} - \frac{e^2}{8\pi^2} + \frac{e^2}{12\pi^2 k^2}\left(4m_u^2 - m_d^2\right) + \ldots$
- As $m_{u,d} \ll \Lambda_{QCD}, v_{\text{Higgs}}$ neglect mixed terms.
- $\Longrightarrow \frac{m_u}{m_d} \approx \frac{1}{2}k^{-e^2/8\pi^2} \approx \frac{1}{2} - \frac{e^2}{16\pi^2}\log k.$
- with renormalised $\hat{m}_q = Z_m m_q \Longrightarrow \hat{m}_u/\hat{m}_d = 1/2.$